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Bubble Dynamics for Warm Dense Matter

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Abstract

It has been experimentally demonstrated that a high-temperature, high-pressure plasma—with a temperature of approximately 12,000 K and a pressure of about 3.2 Mbar—can be generated by utilizing a 1.4 J driver to induce the violent collapse of a laser-generated vapor bubble. In this study, the state changes of the vapor inside a collapsing bubble were calculated by applying bubble dynamics equations derived from the mass, momentum, and energy equations for the vapor within the bubble and the liquid in the vicinity of the bubble wall. The calculation results indicated that a high-pressure, high-temperature plasma—referred to as "warm dense matter"—could be generated, which was similar to what has been obtained through experimental methods. During the bubble collapse phase, it was assumed that the forces acting on the bubble remained constant.

Keywords: warm dense matter, bubble collapse, bubble dynamics

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1. Introduction

It is possible to concentrate energy into the gas contained within a microbubble. A single bubble with an equilibrium radius of 4–5 μm can be trapped and induced to oscillate by applying an ultrasonic frequency of 26.5 kHz and an amplitude of 1.35 bar.; during the contraction phase of a bubble, the temperature and pressure of the gas inside the bubble rise rapidly. At the point of minimum radius, light is emitted through collisions between electrons and ions originating from the gas bubble; this phenomenon is known as 'sonoluminescence' [1, 2]. The estimated temperature and pressure at the point of contraction are 10,000–20,000 K and 10,000–15,000 bar, respectively [3]. The light emitted from the bubbles is characterized by flashes with a continuous spectrum, with duration of several hundred picoseconds [4]. To increase the intensity of sonoluminescence, Thomas et al. [5] experimented with an array of eight ultrasonic transducers, rather than attaching just two transducers to opposite ends of a water-filled spherical glass vessel.

Warm dense matter [WDM] refers to a state of condensed matter characterized by a broad temperature range—from 10,000 K to 1,000,000 K—pressures ranging from ambient pressure to the megabar (Mbar) scale, and densities spanning from 0.01 g/cm^3 to 100 g/cm^3. In nature, WDM occurs in astrophysical objects such as solar core, white dwarf [6]. The warm dense matter, which is a subject of research in astrophysics and in inertial confinement fusion [7] was created by isochoric heating of Au foil [8] by pulse laser or Cu slab [9] by an intense laser-driven proton beam.

To enhance the intensity generated during the collapse of vapor bubble with a radius of 1.8 mm, Ramsey and Pitz [10] utilized an array of eight ultrasonic transducers, similar to the setup employed by Thomas et al. [5]. Using this tabletop device, it was possible to generate a

plasma state at the point of collapse with a radius of 28 μm, a temperature exceeding 12,000 K, and a pressure of 3.2 Mbar. A plasma with such high temperature and pressure can be classified as 'warm dense matter.'

In this study, bubble dynamics equations—derived from the mass, momentum, and energy equations for the gas or vapor inside a bubble and the surrounding liquid—were applied to a vapor bubble with a radius of 1.8 mm, located at the center of a stainless steel vessel where a pressure of 22 bar is generated by a driving energy of 1.4 J. During the bubble collapse phase, it was assumed that the forces acting on the bubble remained constant. According to the calculation results, when the initial pressure of a bubble is 0.45 bar and its initial temperature is 298 K, the pressure, temperature, and density of the collapsed bubble are approximately 3.04 Mbar, 14,310 K, and 46.1 g/cm³, respectively—values that fall within the range of warm dense matter. During the final collapse stage, the velocity of the bubble wall exceeds 7 km/s, which generates a shock wave directed outward from the bubble wall [11].

2. Bubble Dynamics

2.1 Energy equation

Figure 1 shows a vapor bubble with a radius of 1.8 mm, generated by laser irradiation near the optical breakdown threshold at the center of a stainless-steel vessel filled with degassed, deionized water at room temperature. The pressure inside the bubble is assumed to be 0.45 bar. The pressure inside a bubble can be determined by the applied laser energy—a variable that dictates the size of the bubble [12]. The arrows represent eight piezoelectric

transducers (PZT), which generate a uniform pressure increase of 22 bar at the center of the vessel over a duration of 45 μs. The duration for which the applied pressure is sustained is longer than the time it takes for the bubbles to collapse of 32.4 μs. Under these initial conditions, the evolution of the bubbles will be calculated using the bubble dynamics described below.

The temperature at the center of the bubble is $T_o(t)$, and heat transfer to the surrounding liquid occurs through the bubble wall, which is at a temperature T_{bl}. The pressure indie the bubble is assumed to be uniform. The energy equation for the vapor inside a bubble can be expressed in terms of internal energy and enthalpy as follows without viscous dissipation [13].

$$\rho_v C_{v,b} \frac{DT_b}{Dt} = -P_b \nabla \cdot \mathbf{u} - \nabla \cdot \mathbf{q} \tag{1}$$

$$\rho_v C_{p,b} \frac{DT}{Dt} = \frac{dP_b}{dt} - \nabla \cdot \mathbf{q} \tag{2}$$

Here ρ_v is the density of vapor, and $C_{v,v}$ and $C_{p,v}$ are constant-volume specific heat and constant-pressure specific heat for the vapor, respectively. T_b and P_b denote temperature and pressure, respectively, and $\mathbf{u}$ and $\mathbf{q}$ are the radial velocity and heat flux inside the bubble, respectively. From these two energy equations one may obtain a heat transport equation for the vapor inside the bubble by eliminating the DT/Dt term [13], which is given by

$$(\gamma - 1)\nabla \cdot \mathbf{q} = \frac{dP_b}{dt} - \gamma P_b \nabla \cdot \mathbf{u} \tag{3}$$

where γ is the specific heat ratio. Assuming a radial velocity distribution within the bubble, such as [13]:

$$u = \frac{\dot{R}_b}{R_b} r \tag{4}$$

With the velocity profile, Eq. (3) can be written as follows:

$$(\gamma - 1)\nabla \cdot \mathbf{q} = -\frac{dP_b}{dt} - \frac{3\gamma P_b}{R_b}\frac{dR_b}{dt} \tag{5}$$

where R_b is the instantaneous bubble radius. The heat transport equation based on the ideal gas assumption and the radial velocity profile provide the temperature distribution within a microbubble subjected to ultrasound [14]. Furthermore, the equations of motion for Newtonian stars—such as the Sun, Jupiter, and Saturn, which are composed of typical "warm dense matter"—can also be addressed using this formulation [15]. From Eq. (5) one can obtain the temperature profile inside the bubble using Fourier law.

$$T_b = \frac{r^2}{6(\gamma - 1)k_v}\left[\frac{dP_b}{dt} + \frac{3\gamma P_b}{R_b}\frac{dR_b}{dt}\right] + T_{bo} \tag{6}$$

where k_v is the heat conductivity of vapor and T_{bo} is the temperature at the bubble center.

The temperature profile in the thermal boundary layer is assumed to be quadratic, which is described in detail as follows [16]:

$$\frac{T - T_\infty}{T_{bl} - T_\infty} = \left(1 - \frac{r - R_b}{\delta}\right)^2 \tag{7}$$

where δ is the thermal boundary layer thickness and T_{bl} is the temperature at the bubble wall. The quadratic temperature profile satisfies the appropriate boundary conditions. With this temperature profile the heat flux at the bubble wall is given by

$$\nabla \cdot \mathbf{q} = \frac{6k_l(T_{bl} - T_\infty)}{\delta R_b} \tag{8}$$

where k_l is the thermal conductivity of the liquid. With the relation given in Eq. (8), the time dependent pressure inside the bubble can be obtained from Eq. (5).

$$\frac{dP_b}{dt} = -\frac{3\gamma P}{R_b}\frac{dR}{dt} - \frac{6(\gamma - 1)k_l(T_{bl} - T_\infty)}{\delta R_b} \tag{9}$$

Since Boyle-Charles's law holds at the center of the bubble [13], the temperature at the center of the bubble as a function of time is as follows:

$$\frac{dT_{bo}}{dt} = -\frac{3(\gamma - 1)T_{bo}}{R_b}\frac{dR_b}{dt} - \frac{6(\gamma - 1)k_l T_{bo}(T_{bl} - T_\infty)}{\delta R_b P_b} \tag{10}$$

The temperature at the bubble wall can be obtained from Eq. (6) and (8):

$$T_{bl} = \left[T_\infty + \left(\frac{k_v \delta}{k_l R_b}\right)T_{bo}\right] / \left(1 + \frac{k_v \delta}{k_l R_b}\right) \tag{11}$$

2.2 Bubble wall motion

The behavior of a bubble wall under high-pressure compression can be described by the Rayleigh equation, which can be derived from the momentum equation for the liquid in the vicinity of the bubble wall. If the velocity of the bubble wall does not exceed the speed of sound in the liquid (e.g., water), the Keller-Miksis equation [15], which accounts for fluid compressibility, can be used. However, in this case, the Keller-Miksis equation cannot be applied because the bubble wall velocity exceeds 7000 m/s near the point where the bubble radius reaches its minimum. Meanwhile, the Rayleigh equation, derived in the incompressible limit, may be used in cases where the speed of sound is infinite. The Rayleigh equation for the bubble wall subjected to high pressure compression radially can be written as

$$R_b \frac{dU_b}{dt} = \rho_l \left[P_b - P_\infty \left(R_o / R_b \right)^2 \right] - 3U_b^2 / 2 \tag{12}$$

where ρ_l is the density of the liquid, R_o is the initial bubble radius and P_∞ is the imposed pressure on the bubble. The third term in Eq. (12)—that is, the pressure acting on the bubble—was derived under the assumption that the force exerted on the bubble remains constant throughout the bubble collapse process.

The bubble wall velocity is the time derivative of the bubble radius.

$$\frac{dR_b}{dt} = U_b \tag{13}$$

2.3 Mass and energy equations in the thermal boundary layer

To determine the bubble wall temperature and the thermal boundary layer thickness—which are critical for the thermal interaction between the vapor inside a bubble and the surrounding fluid—one must solve the energy equation for the thermal boundary layer. The energy equation for the liquid under the influence of the bubble wall motion is expressed by

$$\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} = \frac{\alpha_l}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \tag{14}$$

where $\alpha_l = k_l / (\rho_\infty C_{p,\infty})$ is the thermal diffusivity. The radial velocity of the liquid due to bubble wall motion can be obtained from the mass conservation for an incompressible liquid, $\nabla \cdot \mathbf{u_r} = 0$. That is

$$u_r = \left(\frac{R_b}{r} \right)^2 U_b \tag{15}$$

Integrating Eq. (14) from $r = R_b$ to $r = R_b + \delta$, the following time dependent thermal

boundary layer thickness can be obtained [13].

$$\left[1+\frac{\delta}{R_b}+\frac{3}{10}\left(\frac{\delta}{R_b}\right)^2\right]\frac{d\delta}{dt}=\frac{6\alpha_l}{\delta}-\frac{2\delta}{R_b}\left(1+\frac{1}{4}\frac{\delta}{R_b}\right)\frac{dR_b}{dt}$$
$$-\delta\left[1+\frac{\delta}{2R_b}+\frac{3}{10}\left(\frac{\delta}{R_b}\right)^2\right]\frac{1}{T_{bl}-T_\infty}\frac{dT_{bl}}{dt} \tag{16}$$

2.4 Radiation mechanism

A possible mechanism of light emission from a laser-induced collapsing vapor bubble is the thermal bremsstrahlung and recombination radiation [18, 19]. The hemispherical spectral radiance from the light source in any medium may be described as:

$$j_\lambda^\Delta=4\pi R_b^2 e_{b\lambda}\left[1+\frac{\exp\left(-2\kappa_\lambda R_b\right)}{\kappa_\lambda R_b}+\frac{\exp\left(-2\kappa_\lambda R_b\right)-1}{2\kappa_\lambda^2 R_b^2}\right] \tag{17}$$

For a case of small absorption, the hemispherical spectral radiance may be written as follows in the limit $\kappa_\lambda R_b \le 1$ where $4\pi R_b\kappa_\lambda/3$ is the absorption coefficient of photon [20].

$$j_\lambda^\Delta=\left(\frac{4\pi}{3}\kappa_\lambda R_b\right)\cdot 4\pi R_b^2 e_{b\lambda} \tag{18}$$

where $e_{b\lambda}$ is the hemispherical emissive power from a black body source, which is given by

$$e_{b\lambda}=\frac{2\pi hc^2}{\lambda^5\left[\exp\left(\frac{hc}{\lambda k_B T_b}\right)-1\right]} \tag{19}$$

Here h is the Planck's constant, c is the velocity of light k_B is the Boltzmann constant, and λ is the wavelength of light. The total emissive power can be obtained by integrating over the entire wavelength range. That is

$$j^{\Delta} \approx \left(\frac{4\pi}{3}\kappa_{\lambda} R_b\right) \cdot 4\pi R_b^2 \cdot \sigma_{SB} T_b^4 \tag{20}$$

where σ_{SB} is the Stefan-Boltzmann constant.

3. Calculation Results and Discussion

3.1 Calculation procedures

As shown in Fig. 1, consider a spherical vapor bubble—generated in water by laser irradiation and having a radius of 1.8 mm and a pressure and temperature of 0.45 bar and 298 K, respectively—that is then compressed by an external pressure of 22 bar. As discussed in the previous section, five time-dependent ordinary differential equations have been derived, concerning the pressure inside the bubble, Eq. (9), the temperature at the bubble center, Eq. (10), the velocity of the bubble wall, Eq. (12), the bubble radius, Eq. (13), and the thickness of the thermal boundary layer, Eq. (16). By establishing appropriate initial conditions—such as zero initial bubble-wall velocity and an assumed thickness for the thermal boundary layer of 0.03 R_o—the five aforementioned equations can be solved simultaneously using the fourth order Runge-Kutta numerical method to determine the values for the next-time step. The integration time interval is in the ns range. The thermal properties of water—such as density (1 g/cm^3), thermal conductivity (0.606 W/mK), and thermal diffusivity (0.001435 cm^2/s) of water were set to their values at 298 K and 1 atm. The specific heat ratio for water vapor was set to 1.327, and the thermal conductivity of the water vapor (0.0177 W/mK) inside the bubble was assumed to be constant.

3.2 Calculation results

Figure 2a shows the bubble radius as a function of time near the collapse point along with the measured data. Two photomultiplier tubes, each equipped with a bandpass filter, were used to measure luminous intensity and temperature. A 1 mW laser was employed to measure the temporal radius of the bubble [10]. Measured bubble radius at the collapse point is 28 μm, whereas the calculated value is approximately 34.8 μm. Before collapse, the measured bubble radius is smaller than the calculated value. After the collapse, the bubble radius values are significantly lower than the calculated values. This is because, past the collapse point, the bubble disintegrates into a cloud of microbubbles [10]. Figure 2b shows the bubble wall velocity as function of time near the collapse point. The calculated bubble wall velocity is approximately 7900 m/s near the collapse point, while the measured value is 7 km/s. As shown in Table 1, the minimum bubble radius, the bubble-wall velocity near the point of collapse, and the temperature, pressure, and density all depend on the initial vapor pressure inside the bubble. The lower the initial vapor pressure inside a bubble, the smaller the minimum bubble radius becomes, and the greater the temperature, pressure, and density at the moment of collapse. In all cases, the time elapsed from the start until the collapse occurs shows a nearly identical value of 32.4 μs. However, if the ambient pressure term in the Rayleigh equation, Eq. (11), is held constant during the collapse, the calculated results differ significantly from the experimental results as shown in the last row in Table 1.

Figure 3 shows (a) temperature, (b) pressure, and (c) density in the vicinity of the collapse point as a function of time. The maximum temperature at the time of collapse is approximately 14,300 K. The measured temperature values around the collapse point are also indicated by circles in Figure 3a. The maximum pressure calculated at the point of collapse is approximately 3.04 Mbar. In contrast, Ramsey and Pitz estimated the maximum pressure

based on the pressure following the passage of a shock wave, $p_s = \rho u_s^2$, arriving at a value of 3.2 Mbar with the velocity of shock (18 km/s) relative to the upstream fluid. The maximum density calculated at the moment of collapse is approximately 46.1 g/cm³, which is identical to the value obtained from a mass-volume relation directly, $\rho = m/(4\pi/3/R^3)$, where m represents the mass of the vapor inside the bubble.

The velocity of an outward-propagating shock wave can be determined as c+u, where c is the local shock wave velocity and u is the particle velocity. The local speed of sound can be obtained using Tait's equation of state for water, as follows [21].

$$\left(\frac{p+B}{p_\infty+B}\right)=\left(\frac{\rho}{\rho_\infty}\right)^n \tag{21}$$

For water B is approximately 3000 atm, and n is 7. The local sound velocity, c_s, at the instantaneous liquid pressure p can be obtained using Eq. (21).

$$c_s = c_\infty\left(\frac{p+B}{p_\infty+B}\right)^{(n-1)/2n} \tag{22}$$

where c_∞ is the sound velocity in an undisturbed medium, which is 1481 m/s for water. The shock velocity at the local pressure of 3.0 Mbar and particle velocity of 7.0 km/s is approximately 28446.7 m/s, which is much higher than the shock velocity estimated by Ramsey and Pitz [10]. However, shock wave velocities cannot be used to estimate the pressure inside a bubble.

The calculated temporal emissive power (W), assuming a light emission mechanism of blackbody radiation with a finite optical thickness of 0.06 [20] along with the measured light intensity (arbitrary) is shown in Fig. 4a. As shown in Fig. 4a, the calculated full width at

half maximum (FWHM) of the light pulse generated by a collapsing bubble is approximately 4.9 ns, which is larger than the observed value of 3.5 ns. Figure 4b shows the calculated hemispherical spectral radiance (mW/nm) emitted from a collapsing bubble at wavelengths of 300 nm, 500 nm, and 700 nm, along with the observed values (arbitrary). The hemispherical spectral radiance calculated at wavelengths of 300 nm and 500 nm shows a pattern similar to the observational results; however, at a wavelength of 700 nm, the calculated spectral radiance appears flatter than the experimental results.

4. Conclusion

It has been shown that warm dense matter can be generated by using a high energy driver to induce the spherical collapse of a vapor bubble at the center of a water filled stainless steel vessel. When a vapor bubble with a radius of 1.8 mm, located at the center of a spherical flask, was compressed to a pressure of 22 bar using a driver with an energy of 1.4 J, it reached a state of "warm dense matter," characterized by temperatures of 12,000–14,000 K and a pressure of 3.2 Mbar. In this study, by simultaneously solving the bubble dynamics equations—derived from the mass, momentum, and energy equations for the vapor inside a bubble and the liquid near the bubble wall—we identified the formation of 'warm dense matter' at the collapse, characterized by a temperature of 14,000 K, a pressure of 3.0 Mbar, and a density of 46.1 g/cm^3. The pressure exerted on a collapsing bubble was determined under the assumption that a constant force acts upon the bubble. The lower the initial vapor pressure inside the bubble, the higher the temperature, pressure, and density at the moment of collapse. By utilizing the bubble dynamics presented in this research, we have successfully addressed the gravitational collapse of Newtonian stars composed of typical WDM.

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Table 1. Time to collapse, minimum bubble radius, bubble wall velocity near the moment of collapse, and the maximum temperature, pressure, and density at the instant of bubble collapse. The result in the last row corresponds to the case where a constant pressure (22 bar) is applied to the bubble.

Initial Pressure (bar)	Time to collapse point (μs)	Minimum bubble radius (μm)	Minimum bubble wall velocity (m/s)	Maximum temperature (K)	Maximum pressure (Mbar)	Maximum density (g/cm^3)
0.4	32.33	30.8	-9489	16100	4.36	58.7
0.45	32.38	34.8	-7915	14310	3.04	46.1
0.5	32.43	38.7	-6729	12880	2.20	37.1
0.6	32.54	46.6	-3736	10740	1.26	25.5
0.45	35.82	100.3	-972	5060	0.045	1.90

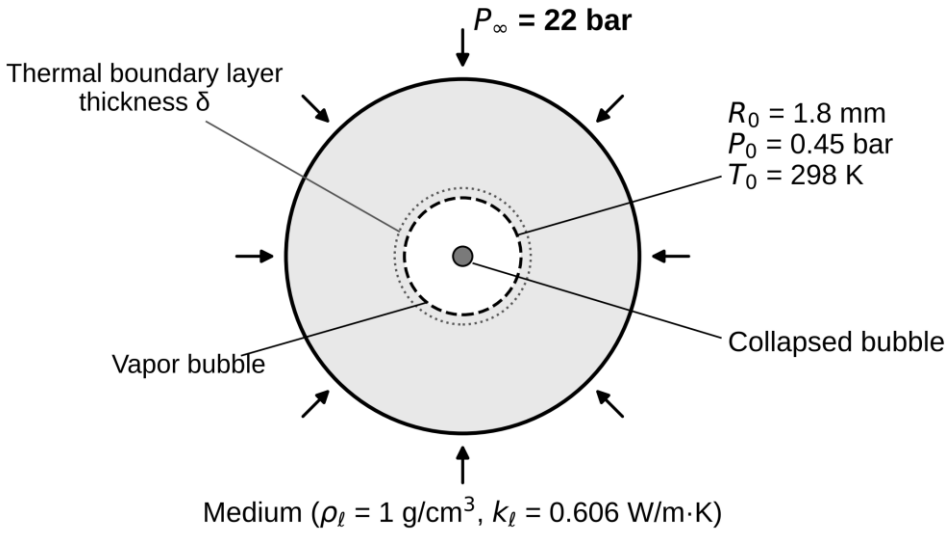

Fig 1. Schematic of the laser-generated vapor bubble at the center of a water-filled stainless steel vessel under an external pressure of 22 bar.

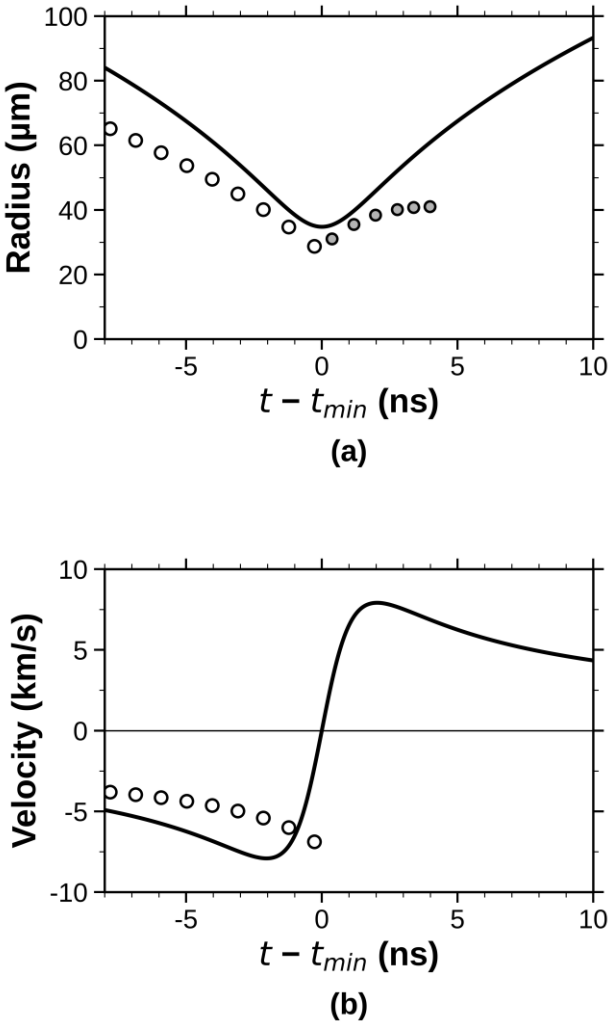

Fig. 2 Comparison of calculation results with the experimental results of Ramsey and Pitz (2013): (a) calculated (solid line) and measured (circular symbols) bubble radii, and (b) calculated (solid line) and measured (circular symbols) bubble wall velocities. Time is measured from the moment the radius reaches its minimum—that is, from the point of collapse (t_{min}).

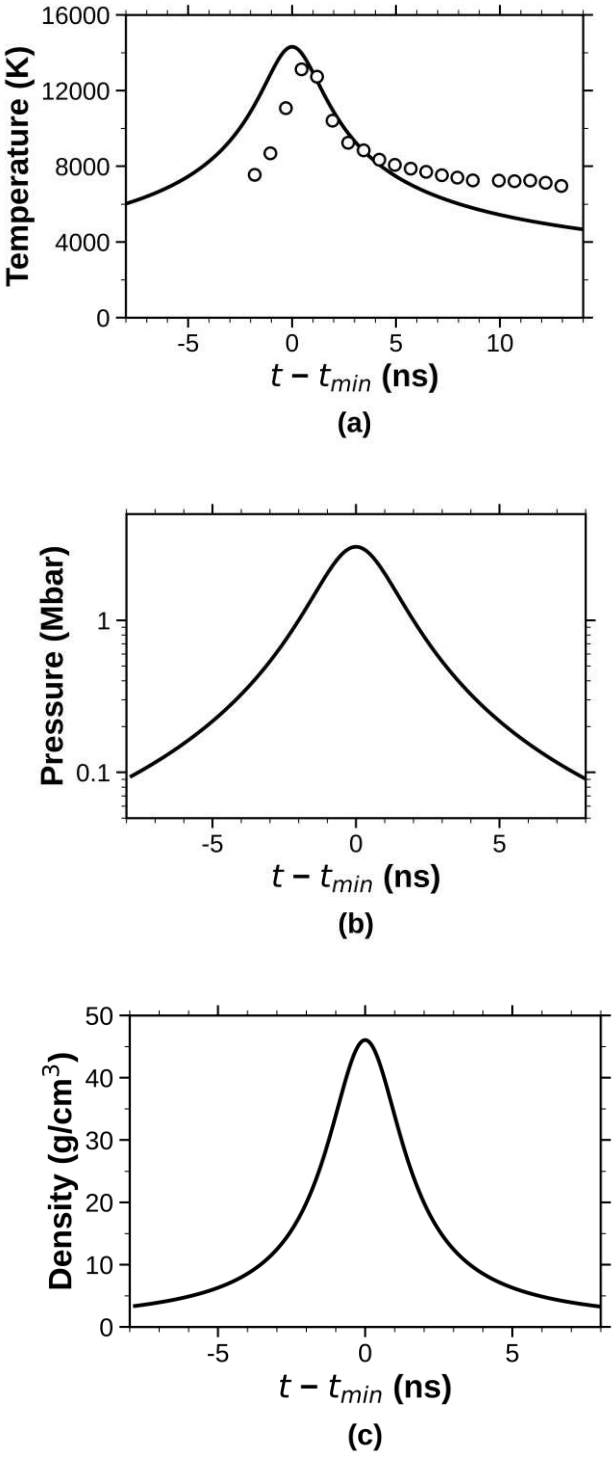

Fig. 3 (a) Calculated (solid line) and measures values (circular symbols) for the temperature at the center of the bubble, and (b) calculated values of the pressure inside the bubble, and (c) calculated values of the density inside the bubble, presented as functions of time around the moment of collapse.

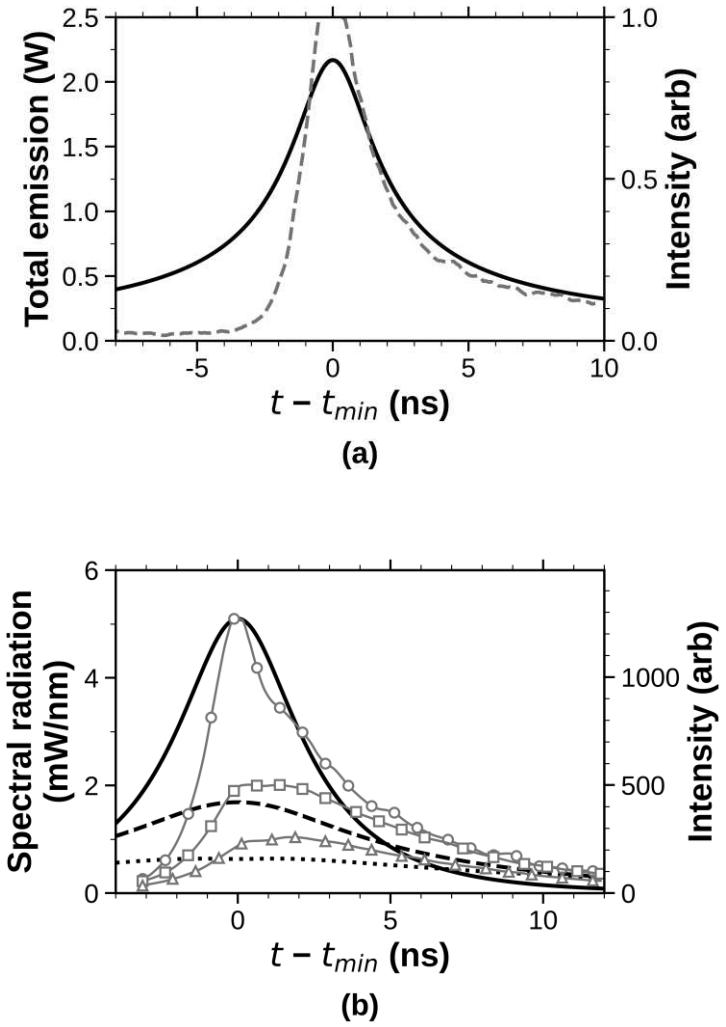

Fig.4. (a) Calculated total emission (solid line) and observed luminescence intensity (dashed line) resulting from a collapsing bubble. (b) Calculated spectral radiant emission at wavelengths of 300 nm (solid line), 500 nm (dashed line), and 700 nm (dotted line), and observed spectral radiant emission at wavelengths of 300 nm (circular symbols), 500 nm (square symbols), and 700 nm (triangle symbols).