Fibonacci LC Transmission Lines: Transfer-Matrix Spectra, Scale-Invariance Controls, Gamma-Ray-Burst Dispersion Bounds and a Benchtop Test Protocol

An Electromagnetic Extension of the Fractal Quantization Model

A standalone study building on the earlier FQM preprint

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Abstract

Building on the Fractal Quantization Model preprint (FQM v2, DOI 10.21203/rs.3.rs-10424451/v2), we study one-dimensional lossless LC transmission-line lattices whose cells follow substitution rules (Fibonacci, silver mean, Tribonacci), with a periodic control. Exact $ABCD$ transfer matrices, an FDTD scheme made equivalent to the cell chain, and a standard-library Python suite give: the Fibonacci trace-map recurrence with an analytically proved closed-form Fricke–Vogt invariant $I = (\omega^2 L\Delta C)^2$; progressive spectral fragmentation of approximants (bands $8 \to 84$, allowed measure $51.6\% \to 37.7\%$ versus 2 bands and 70.5% periodic); homogenization $c_{\text{eff}}/c_0 = (1 + \delta\Phi^{-3})^{-1/2}$; and a scale-invariance lemma that holds for any cell-length ratio, hence not specific to Φ. The quadratic dispersion $\Delta v_g/c = -(ka)^2/8$ maps to $E_{\text{QG},2} = \sqrt{12}\,\hbar c/a$ and, with the LHAASO limit, to $a \leq 9.3 \times 10^{-28}$ m. The closed-form invariant $I = (\omega^2 L\Delta C)^2$ is established analytically; the supplied earlier experiment archive reports agreement between the circuit model, `ngspice-42` and `scikit-rf`; the present release independently re-runs four Stage 6 finalists in `ngspice-47` with discrepancies below 5.0×10^{-7} dB; and two proposed extensions are turned into falsification tests. A supplied catalogue analysis of 1003 adjacent exoplanet period ratios reports no pre-specified golden-ratio excess (the exact catalogue snapshot is not archived in this release) ($z = -0.14$) while a $3{:}2$ resonance control is detected ($z = +7.9$) alongside the known pile-up just wide of $2{:}1$; a time-of-flight pipeline shows an intrinsic source lag mimics a quadratic delay in 38% of synthetic trials unless a calibrated rule is used; the relaxation law $\gamma_n \to \Phi^{-4}$ is untestable without an independent clock; and mean-field and DMRG (`TeNPy`) calculations find no systematic pairing advantage of Fibonacci over random geometry. Nothing here supports a physical role for Φ; the contribution is a reproducible, falsifiable test bed. The study also adds a reproducible multiobjective search and a source audit over 21-cell, composition-matched LC sequences. A seeded evolutionary search evaluated 1508 unique sequences and identified a robust candidate whose mean reflective-stopband fraction was 68.6% under independent uniform $\pm 5\%$ component perturbations ($n = 24$ per candidate and tolerance), compared with 56.6% for Fibonacci and 62.4% for the previous 250-sample winner. Four finalists were independently re-simulated in ngspice 47 over 3401 frequencies; the maximum $|S_{21}|$ discrepancy against the independent ABCD implementation was below 5.0×10^{-7} dB. These are model-specific computational results, not experimental measurements or evidence for a universal Fibonacci or FQM law.

Keywords: Fibonacci transmission line; LC lattice; $ABCD$ transfer matrix; trace map; Fricke–Vogt invariant; discrete scale invariance; FDTD; quadratic dispersion; gamma-ray bursts; falsifiability.

1 Introduction and scope

The unified FQM preprint [1] consolidated a research program around the Perron–Frobenius eigenvalue $\Phi = (1+\sqrt{5})/2$ of the Fibonacci substitution matrix and discrete scale invariance [12], with strict epistemic separation between algebraic identities, reproductions of established results, effective assumptions and open hypotheses. It concluded that FQM supports an auditable computational test bed and not a universal law. This article applies the same discipline to a concrete wave-physics question: how do discrete lossless LC transmission lines behave when their cells follow Fibonacci-type substitution rules, and what can be tested on a bench and in the sky?

Fibonacci multilayers and quasiperiodic transmission media are an established subject [6, 7, 8, 9], and the trace-map formalism is classical [2, 3, 4]. The contributions here are modest and specific: a self-contained LC formulation with machine-checkable identities and an exact closed-form Fricke–Vogt invariant; explicit negative controls; a cross-validated time-domain scheme; a quantitative conversion of the lattice dispersion into the convention used by gamma-ray-burst (GRB) analyses; and a bench protocol with loss and tolerance included.

Relationship to the earlier FQM preprint. This manuscript is a separate research contribution, not a numbered revision of the earlier FQM preprint cited above. The earlier work supplies the broader research context; the present article develops and tests a specific electromagnetic LC-lattice model. Its contributions are the analytical closed-form trace-map invariant (Section 2), independent circuit-model cross-checks (Section 8), explicit negative controls, and quantitative falsification tests of proposed extensions (Section 9). The results do not establish a universal FQM law or a physical role for Φ. The designation "FQM v2" in the bibliography refers exclusively to the earlier Research Square preprint and is retained as a citation to prior work; it is not the version number of this manuscript.

Scope. We do not claim that three-dimensional spacetime is a static LC grid, which would violate boost invariance. The vacuum analogy ($c_0 = 1/\sqrt{\mu_0\varepsilon_0}$, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$) is used only to define a falsifiable effective model, and Section 6 bounds it.

2 Transfer-matrix formalism

A symmetric T-section (series $Z_s/2$, shunt Y, series $Z_s/2$; standard $ABCD$ conventions [11]) with $Z_s = j\omega L + R_s$ and $Y = j\omega C$ has transfer matrix

$$M = \begin{pmatrix} 1 + \frac{1}{2}Z_sY & Z_s\left(1 + \frac{1}{4}Z_sY\right) \\ Y & 1 + \frac{1}{2}Z_sY \end{pmatrix}. \tag{1}$$

For $R_s = 0$ and $x = \omega^2 LC$: $A = D = 1 - x/2$, $B = j\omega L(1 - x/4)$, $C = j\omega C$, $\det M = 1$, and $\mathrm{Tr}\, M = 2 - x$. For a cascade $M = \prod_i M_i$ between real reference impedances Z_{ref},

$$S_{21} = \frac{2}{A + B/Z_{\mathrm{ref}} + CZ_{\mathrm{ref}} + D}, \qquad S_{11} = \frac{A + B/Z_{\mathrm{ref}} - CZ_{\mathrm{ref}} - D}{A + B/Z_{\mathrm{ref}} + CZ_{\mathrm{ref}} + D}. \tag{2}$$

In the lossless case $|S_{21}|^2 + |S_{11}|^2 = 1$ identically; we use $\max|T + R - 1|$ only as a numerical-consistency audit, not as physical validation (Table 11, row 1).

Relation to tight-binding chains and exact Fricke–Vogt invariant. Single-cell traces correspond ($\mathrm{Tr}\, T_i = E - \varepsilon_i \leftrightarrow 2 - \omega^2 L_i C_i$ at $E = 2$), but the cascades are not conjugate in general, so results for Schrödinger operators do not transfer automatically. What does transfer is purely algebraic: for any unimodular cells M_A, M_B and words $S_{n+1} = S_nS_{n-1}$ ($S_0 = B$, $S_1 = A$), the traces $x_n = \mathrm{Tr}\, M(S_n)$ satisfy $x_{n+1} = x_nx_{n-1} - x_{n-2}$ and the Fricke–Vogt quantity

$$I = x_{n+1}^2 + x_n^2 + x_{n-1}^2 - x_{n+1}x_nx_{n-1} - 4 \tag{3}$$

is constant in n [2].

Proposition (Closed-form LC Fricke–Vogt invariant). *For lossless symmetric T-section LC cells with common inductance $L_A = L_B = L$ and capacitances C_A, C_B, the invariant (3) equals $I = (\omega^2 L \Delta C)^2$, where $\Delta C = C_A - C_B$.*

Proof. Let $x_A = \omega^2 L C_A$ and $x_B = \omega^2 L C_B$, so that $x_0 = \mathrm{Tr}(M_B) = 2 - x_B$ and $x_1 = \mathrm{Tr}(M_A) = 2 - x_A$. Direct multiplication of M_A and M_B from (1) with $R_s = 0$ gives the diagonal entries

$$(M_A M_B)_{11} = \left(1 - \frac{x_A}{2}\right)\left(1 - \frac{x_B}{2}\right) - x_B\left(1 - \frac{x_A}{4}\right) = 1 - \frac{x_A}{2} - \frac{3x_B}{2} + \frac{x_A x_B}{2},$$
$$(M_A M_B)_{22} = -x_A\left(1 - \frac{x_B}{4}\right) + \left(1 - \frac{x_A}{2}\right)\left(1 - \frac{x_B}{2}\right) = 1 - \frac{3x_A}{2} - \frac{x_B}{2} + \frac{x_A x_B}{2}.$$

Summing the diagonal entries yields $x_2 = \mathrm{Tr}(M_A M_B) = 2 - 2x_A - 2x_B + x_A x_B = (2 - x_A)(2 - x_B) - 2 = x_1 x_0 - 2$. Substituting $x_2 = x_1 x_0 - 2$ into (3) at $n = 1$ gives

$$I = (x_1 x_0 - 2)^2 + x_1^2 + x_0^2 - (x_1 x_0 - 2) x_1 x_0 - 4 = (x_1 - x_0)^2 = (x_B - x_A)^2 = (\omega^2 L \Delta C)^2. \quad \blacksquare \quad (4)$$

Numerically, this exact identity is reproduced to 2.8×10^{-14} relative error at five test frequencies; the trace-map recurrence holds to 9.7×10^{-16} and the spread of I across generations is 1.2×10^{-15}. That the infinite-chain spectrum of this LC family is a Cantor set of zero Lebesgue measure is not proved here; Section 3 reports finite-generation numerical evidence only.

3 Regime A: fixed spacing, capacitance contrast

Cells share $L_i = L_0$; capacitances follow the substitution with $C_A = C_0(1 + \delta)$, $C_B = C_0(1 - \delta)$ ($C_C = C_0$ for Tribonacci). Dimensionless units are $L_0 = C_0 = 1$, $\omega_0 = 1$, $Z_{\mathrm{ref}} = 1$, $\delta = 0.40$.

A withdrawn metric. Counting contiguous regions with $T > 0.5$ on a finite chain of $N = 55$ cells gives Fibonacci 10, silver 7, Tribonacci 12, and periodic 27 bands, and the periodic count changes from 10 to 29 as the threshold moves from 0.1 to 0.9 (Fabry–Pérot ripples). The metric measures finite-size ripple, not fragmentation, and contradicts the narrative it was meant to support; it is withdrawn.

Band structure of approximants. A robust alternative is the allowed set $|\mathrm{Tr}\, M_n(\omega)| \le 2$ of the periodic approximant of length F_n, evaluated over $\omega \in (0, 2.2]$:

Table 1: Allowed Bloch bands ($|\mathrm{Tr}\, M_n(\omega)| \le 2$) and allowed spectral measure over $\omega \in (0, 2.2]$ for Fibonacci periodic approximants of length F_n versus the periodic AB cell ($\delta = 0.40$).

Length F_n	8	13	21	34	55	89	144	periodic AB
Allowed bands	8	13	21	31	42	60	84	2
Allowed measure (%)	51.6	48.0	45.2	42.9	40.9	39.1	37.7	70.5

The band number grows with generation while the allowed measure decreases slowly; both are consistent with progressive Cantor-type fragmentation and with the established behaviour of related operators [4, 5], though a finite-n table cannot establish a zero-measure limit.

Low-frequency homogenization. With asymptotic site densities $p_A = \Phi^{-1}$ and $p_B = \Phi^{-2}$, $C_{\mathrm{eff}} = C_0[1 + \delta(\Phi^{-1} - \Phi^{-2})] = C_0(1 + \delta\Phi^{-3})$, where $\Phi^{-3} = \sqrt{5} - 2$. Thus $c_{\mathrm{eff}}/c_0 = (1 + \delta\Phi^{-3})^{-1/2} = 0.95589$; the $N = 55$ chain gives 0.95584. The chain is transparent but not perfectly matched to $Z_{\mathrm{ref}} = 1$: $T(\omega = 0.01) = 0.99940$, i.e., $1 - T = 6.0 \times 10^{-4}$, the same order as the 5.1×10^{-4} predicted from the mismatch between $Z_{\mathrm{eff}} = (1 + \delta\Phi^{-3})^{-1/2}$ and $Z_{\mathrm{ref}} = 1$.

4 Scale-invariance lemma and its controls

Lemma. *If cell n has length a_n, $L_n = \mu_0 a_n$, and $C_n = \varepsilon_0 a_n$, then $Z_n = \sqrt{L_n/C_n} = \sqrt{\mu_0/\varepsilon_0}$, $v_n = a_n/\sqrt{L_n C_n} = 1/\sqrt{\mu_0\varepsilon_0}$, and $\omega_{c,n} = 2/\sqrt{L_n C_n} = 2c_0/a_n$. The cancellation holds for any positive sequence a_n; Φ plays no role.*

Numerically, geometric tapers $a_n = r^{-n}$ with $r \in \{1.1, 1.3, \Phi, 2, 3\}$ yield relative drifts of Z_n and v_n of at most 2×10^{-16} (floating-point level). This corrects an earlier presentation of $a_n = a_0\Phi^{-n}$ as a Fibonacci-specific result.

Finite frequency. Exact matching holds only as $\omega \to 0$. Adjacent cells have Bloch impedances $Z_0\sqrt{1-(\omega/\omega_{c,n})^2}$ that differ, so transmission falls below unity as ω rises. For 12-cell tapers the first frequency with $T < 0.99$ is $1.067, 1.036, 0.938, 0.925, 0.927$ (units of ω_0) for $r = 1.1, 1.3, \Phi, 2, 3$: smaller ratios are more transparent and Φ is not distinguished. For a two-length Fibonacci chain ($a_A = r a_B$, $N = 55$, $\omega \le 1.5$) the allowed measure falls monotonically: 100%, 77.9%, 60.5%, 49.0% for $r = 1, 1.3, \Phi, 2$; again no feature singles out Φ. Any special role of Φ in such chains would have to appear in structure-factor positions, which is outside the present test set.

5 Time-domain cross-validation (FDTD)

We integrate the telegrapher equations with a leapfrog scheme in which node i carries shunt C_i and the branch between nodes $i, i+1$ carries $L = \frac{1}{2}(L_i + L_{i+1})$, so that the interior is exactly equivalent to the symmetric T-cell chain (a scheme with branch L_i differs from the $ABCD$ model whenever L varies). Ends are matched resistors, with $\Delta t \le 0.15$ (0.04 for the micro cascade). A modulated Gaussian ($\sigma = 10$) is injected into a dark line; $E_{\text{trans}}/E_{\text{ref}}$ is the probe energy relative to a uniform reference run. The frequency-domain prediction is the $ABCD$ transmission weighted by the actual source spectrum, with the leapfrog numerical dispersion substitution $\omega \to (2/\Delta t)\sin(\omega\Delta t/2)$ and the uniform line's Bloch impedance as reference.

Table 2: Cross-validation between time-domain leapfrog FDTD energy transmission and source-spectrum-weighted $ABCD$ predictions.

Case	Pulse ω	FDTD ratio	$ABCD$ prediction
Fibonacci passband	0.25	0.99386	0.99398
Fibonacci widest interior gap (1.421–1.942)	1.682	4.1×10^{-7}	1.0×10^{-5}
5-stage Φ^{-n} cascade	0.25	0.99995	0.99999
5-stage Φ^{-n} cascade	1.00	0.9907	0.9865

Transmitting cases agree to ≤ 0.42%; in the interior gap both values are $\ll 1$ and at the numerical floor, so their ratio is not meaningful. The micro cascade at $\omega = 0.25$ is trivially transparent (five electrically small cells); the $\omega = 1.00$ row is the informative one. The earlier gap value 0.012 came from a pulse centred on the edge of a search window and is superseded.

6 Quadratic dispersion and a GRB bound

A symmetric lattice of spacing a obeys $\omega(k) = (2c_0/a)|\sin(ka/2)|$ and $v_g = c_0\cos(ka/2)$ [10], so the linear term vanishes by cell symmetry and $\Delta v_g/c_0 \simeq -(ka)^2/8$, with $ka = Ea/(\hbar c)$. Writing $E^2 = p^2c^2[\sin(ka/2)/(ka/2)]^2 \simeq p^2c^2[1-(ka)^2/12]$ and comparing with the convention $E^2 \simeq p^2c^2[1 - s(E/E_{\text{QG},n})^n]$ used in GRB analyses [13, 14, 15], for which $v_g \simeq c[1 - s\frac{n+1}{2}(E/E_{\text{QG},n})^n]$, gives for

$n = 2$, $s = +1$ (subluminal):

$$E_{\mathrm{QG},2} = \sqrt{12}\, E_{\mathrm{mesh}}, \qquad E_{\mathrm{mesh}} = \frac{\overline{h}c}{a}. \tag{5}$$

The LHAASO Collaboration's 95% CL limit from GRB 221009A is $E_{\mathrm{QG},2} > 6 \times 10^{-8} E_{\mathrm{Pl}} \simeq 7.3 \times 10^{11}$ GeV [15]; this gives the baseline bound below. A later independent DisCan analysis combining WCDA and KM2A data reports a 95% CL subluminal limit $E_{\mathrm{QG},2} > 13.7 \times 10^{11}$ GeV [17]. Under the same lattice-to-dispersion mapping, this corresponds to the tighter conditional bound $a < 4.99 \times 10^{-28}$ m. That analysis explicitly neglects intrinsic energy-dependent source emission and other astrophysical propagation delays, so we retain the collaboration result as the primary conservative reference and report the DisCan conversion separately. Hence

$$a \leq \frac{\sqrt{12}\,\overline{h}c}{E^{\mathrm{lim}}_{\mathrm{QG},2}} = 9.3 \times 10^{-28}\,\mathrm{m}. \tag{6}$$

Table 3: Illustrative quadratic arrival delay $\Delta t = T(ka)^2/8$ (10 TeV photon over 1 Gly, $T = 3.156 \times 10^{16}$ s, Euclidean) and observational status against the published LHAASO $E_{\mathrm{QG},2}$ bound.

a (m)	E_{mesh} (GeV)	Δt (10 TeV, 1 Gly)	$E_{\mathrm{QG},2}/E^{\mathrm{lim}}$	Status
1.616×10^{-35}	1.22×10^{19}	2.65×10^{-15} s	5.8×10^{7}	allowed
10^{-31}	1.97×10^{15}	1.01×10^{-7} s	9.3×10^{3}	allowed
10^{-27}	1.97×10^{11}	10.1 s	0.93	marginally excluded
10^{-26}	1.97×10^{10}	1013 s	0.093	excluded

Because Δt in the table omits the cosmological integral, it is illustrative; the status column uses the published $E_{\mathrm{QG},2}$ limit directly. The result is the existing observational bound re-expressed for one model: a one-dimensional, boost-violating lattice evaluated in the preferred frame. It is not a measurement and does not constrain Planck-scale spacings. A cubic three-dimensional lattice would add anisotropy and additional constraints not considered here.

7 Benchtop protocol ($50\,\Omega$)

All predictions use the lossy $ABCD$ engine of the suite with series ESR modelled as $R = 2\pi(1\,\mathrm{MHz})L/Q$ per inductor (frequency-independent, a simplification), ideal capacitors, and a $50\,\Omega$ source and load.

Prototype A (21 cells). $L = 10\,\mu$H between nodes and $5.1\,\mu$H at the two ends (the half-cell inductors); shunt $C_A = 5.6$ nF, $C_B = 2.4$ nF (E24) in the word `ABAABABAABAABABAABABA`; control: `ABAB`… with identical parts. Reference values are $\sqrt{L/C_0} = 50\,\Omega$ at $C_0 = 4.0$ nF, $f_0 = 795.8$ kHz; cell cutoffs are 1.345 (A), 1.592 (C_0), and 2.055 MHz (B).

The distinction between Fibonacci and periodic ladders at $N = 21$ is therefore real but small (one extra narrow stopband and a delay offset); lossless -20 dB stopband counts below 1.3 MHz are 2, 2, and 6 for $N = 21, 34, 55$, so $N = 55$ is the more discriminating (and lossier) choice. Because nominal predictions use ideal parts, the pass criterion should use the as-built prediction: measure every L and C with an LCR meter, rerun the suite with those values, and compare S_{21} and delay at the instrument's stated uncertainty. Disagreement beyond that uncertainty falsifies the circuit model or the build, not any FQM hypothesis.

Prototype B (5-stage Φ^{-n} cascade). Ideal $L_n = 10\,\mu\mathrm{H}\,\Phi^{-n}$ and $C_n = 4.0\,\mathrm{nF}\,\Phi^{-n}$ (cutoffs 1.59, 2.58, 4.17, 6.74, 10.91 MHz). An E24 realisation uses $L = (10, 6.2, 3.9, 2.4, 1.5)\,\mu$H and $C = (3.9, 2.4, 1.5, 0.91, 0.56)$ nF, with stage impedances 50.6–51.8 Ω.

Table 4: Pre-specified benchtop predictions for Prototype A (21-cell $50\,\Omega$ Fibonacci vs. periodic ladder).

Observable	Prediction
Low-frequency delay ratio vs. uniform 4.0 nF ladder (100 kHz phase)	1.0469 (analytic $\sqrt{\overline{C}/C_0} = 1.0465$; $c_{\text{eff}}/c_{\text{ref}} = 0.955$)
Stopbands, $\lvert S_{21}\rvert < -20$ dB lossless, $f < 1.3$ MHz	Fibonacci: 0.860–0.905 and 1.127–1.545 MHz; periodic: one band 0.958–1.458 MHz
Monte Carlo ($Q = 50$, $C, L \pm 5\%$), -10 dB stopbands < 1.3 MHz	Fibonacci median 2 (range 1–2); periodic 1
Same with $C \pm 5\%$, $L \pm 10\%$	Fibonacci median 2 (range 1–3)
Passband insertion loss at 100 kHz, $Q = 50$ ($Q = 100$)	-2.4 dB (-1.2 dB), dominated by ESR

Table 5: Predicted return loss $\lvert S_{11}\rvert$ and insertion loss $\lvert S_{21}\rvert$ of Prototype B (5-stage Φ^{-n} cascade) versus a uniform 5-cell ladder.

f (MHz)	0.1	0.5	0.7	1.0	1.3
$\lvert S_{11}\rvert$ ideal (dB)	-69.8	-29.0	-21.4	-14.0	-8.4
$\lvert S_{11}\rvert$ E24 (dB)	-46.4	-31.2	-21.6	-13.9	-8.9
$\lvert S_{11}\rvert$ uniform $5\times$ stage-0 (dB)	-58.7	-51.0	—	-18.2	—
$\lvert S_{21}\rvert$ ideal (dB)	0.00	-0.005	-0.032	-0.18	-0.68

The cascade is transparent at low frequency ($\lvert S_{21}\rvert > -0.2$ dB to 1 MHz) but is not better matched than a uniform ladder of the largest cell, and return loss reaches -20 dB only below about 0.7 MHz. Prototype B tests the lemma of Section 4 (a property of the circuit equations), not any Φ-specific physics.

8 Independent verification with external software

The original v2.1 experiment archive reports that the $ABCD$ engine was re-implemented in two external tools: `ngspice-42` (SPICE AC analysis; each T-cell built from two $L/2$ inductors, optional series ESR resistors, shunt C, $50\,\Omega$ source and load; $S_{21} = 2V_{\text{out}}/V_s$) and `scikit-rf 2.2.0` (cascaded lumped-element two-ports) [18, 21].

Table 6: Reported maximum magnitude and phase residuals in the supplied original experiment archive between the standalone $ABCD$ suite, `ngspice-42`, and `scikit-rf 2.2.0` across 701 frequencies (21 cells, $50\,\Omega$, evaluated where $\lvert S_{21}\rvert > -60$ dB).

Case	Max. magnitude residual, SPICE–$ABCD$ (dB)	Max. magnitude residual, scikit-rf–$ABCD$ (dB)	Max. phase residual (degrees)
Fibonacci, lossless	3.1×10^{-12}	3.0×10^{-12}	1.3×10^{-11}
Fibonacci, $Q = 50$	5.9×10^{-10}	1.2×10^{-12}	3.1×10^{-9}
Periodic, lossless / $Q = 50$	2.8×10^{-12} / 4.4×10^{-10}	8.7×10^{-12} / 7.0×10^{-13}	$\leq 3 \times 10^{-9}$

The supplied original results report that SPICE reproduces the stopbands of Section 7 (Fibonacci 0.860–0.905 and 1.128–1.545 MHz; periodic 0.958–1.458 MHz) and the Prototype B return loss

agrees with `scikit-rf` and $ABCD$ ($-69.8, -29.0, -21.4, -14.0, -8.4$ dB at $0.1, 0.5, 0.7, 1.0, 1.3$ MHz). A SPICE Monte Carlo of the benchtop protocol (40 draws, $Q = 50$, $C \pm 1\%$, $L \pm 5\%$) gives a median of 2 stopbands below 1.3 MHz at the -10 dB level for the Fibonacci ladder (range 1–2) and 1 for the periodic control (1–1), as predicted. Finally, the supplied archive reports that `astropy`'s ΛCDM integral for the time-of-flight factor (Section 6 and Test 10) reproduces $K = 1.1513 \times 10^{17}$ s to 10^{-10} relative [22].

The reported checks validate numerical implementations of the lumped circuit equations; they cannot validate any property of the physical vacuum. The ngspice-42/scikit-rf/astropy runs in the supplied archive were not independently re-executed in the present environment because the exact software environment and, for the catalogue study, source data snapshot were not supplied. The separate Stage 6 ngspice-47 cross-check is independently re-executed and documented in Section 11. Numerical agreement is not physical validation; the informative outcome of the future bench measurement remains an independently measured comparison.

9 Falsification tests of two proposed extensions

Each extension is stated and converted into a quantitative test with a decision rule encoded in the supplied scripts. The original archive supplies results and JSON outputs; the present release independently re-executes only Tests 11 and 12, with exact JSON matches. The status of the other tests is detailed in Section 12. Items marked *synthetic* validate pipelines and rules and are not analyses of real observations.

9.1 E2a: time-of-flight pipeline for a quadratic photon delay (synthetic)

Photons ($N = 3000$, $dN/dE \propto E^{-2.3}$, 0.2–13 TeV) are drawn from a two-pulse light curve with $t_{\rm obs} = t_{\rm src}(E) + pE^2$, $p = K/E^2_{\rm QG,2}$. The estimator minimises the entropy of the de-dispersed arrival-time histogram; 95% intervals come from a photon bootstrap ($B = 10$, coarse). A linear alternative ($n = 1$) is fitted too. The decision rule detects only if the interval of p excludes zero and $n = 2$ is preferred over $n = 1$ by a margin (5.4×10^{-4} nats) calibrated on separate seeds (null, lag, linear-LIV sets).

Table 7: Detection rates across 24 synthetic trials per condition for the quadratic time-of-flight pipeline.

Condition (24 trials each)	Interval only	Full pre-specified rule
Null (no delay, no lag)	0.00	0.00
Intrinsic linear lag 0.5 s/TeV, no LIV	0.38	0.08
Lattice $a = 10^{-27}$ m	1.00	0.88
Lattice $a = 5 \times 10^{-28}$ m	0.38	0.08
Lattice $a = 2.5 \times 10^{-28}$ m	0.25	0.00
Linear LIV ($n = 1$), $p_1 = 0.3$ s/TeV	0.12	0.00

An intrinsic source lag is therefore the main confounder (38% false detections without the calibrated rule), and with 3000 photons the toy sensitivity is about $a \sim 10^{-27}$ m, the scale already constrained in Section 6. Any real analysis should additionally require reproduction in independent bursts.

9.2 E2b: is there a golden-ratio signature in real planetary data?

Pre-specified test. Data: the Open Exoplanet Catalogue [23] (879 systems with ≥ 2 periods, 1003 adjacent period ratios $1.05 \leq P_{i+1}/P_i \leq 4$). Statistic: number of ratios within $\pm 1.5\%$ of a centre, against a leave-window-out Gaussian-KDE background (bandwidth 0.10 in ln ratio) with Poisson tails; then a look-elsewhere scan over 371 centres (1.15–3.0) with a null from smoothed bootstrap.

Table 8: Pre-specified excess test on 1003 adjacent exoplanet period ratios from the Open Exoplanet Catalogue.

Centre	Observed	Expected	z	Poisson p
$\Phi = 1.618$	32	32.8	-0.14	0.58
3:2 (control)	64	24.6	$+7.9$	$\ll 10^{-6}$
8:5 / 5:3	—	—	$+0.11$ / $+0.35$	—

Φ ranks 227 of 371 centres; the scan maximum is $z = 9.3$ at 1.52 (the $3{:}2$ resonance), versus a 95% null maximum of about 4.2. Two featureless centres give $z \approx 0$. While the exact $2{:}1$ centre shows no excess ($z = -0.6$), the look-elsewhere scan exhibits a statistically significant peak ($z \approx 6.0$, above the 4.2 null threshold) just wide of 2.0 near 2.03, consistent with the well-known tidal-dissipation pile-up slightly wide of exact $2{:}1$ commensurability observed in *Kepler* multi-planet systems [27]. The method therefore detects genuine, physically understood orbital architecture and finds no excess at Φ. For the eight Solar-System planets' semi-major axes (standard values), the RMS fractional residual of log-spacings to integer multiples of $\ln \Phi$ is 0.314 against a null expectation of ~ 0.29 ($p = 0.55$ a priori; the best base, 1.39, has look-elsewhere $p = 0.57$).

Why pre-registration matters. In a synthetic log-periodic search ($N = 60$, unit noise) the 5% threshold on the amplitude rises by a factor 2.45 when the period is scanned. For a true modulation of relative amplitude 0.5, a test with $P = \ln \Phi$ fixed beforehand has power 0.71 versus 0.29 for the scanned search; if the true base is 1.7, the fixed-Φ test has power 0.07. A claim is only meaningful if the base is fixed before the data are examined.

9.3 E1a: is $\gamma_n \to \Phi^{-4}$ a testable physical law?

With $E_n = \kappa \epsilon_n^2/2$ and $\epsilon_n = R_n - \lambda$, the ratio $\gamma_n = \lambda^{-2}/R_n^2 \to \lambda^{-4}$ holds for every metallic-mean convergent ($m = 1, 2, 3$: $\lambda^{-4} = 0.1459, 0.0294, 0.0084$, verified numerically), so it is a generic property of continued-fraction convergence. If the map from generation to time is free, any exponential decay equals Φ^{-4} per generation for a generation length $4 \ln \Phi \, \tau = 1.925 \, \tau$ with unchanged χ^2: the hypothesis is untestable. With an independent clock it is testable; for six generations and 20% log-noise the 5%-level test rejects a true ratio 15% below (20% above) Φ^{-4} with power 0.72 (0.81), and keeps the nominal 5% false-positive rate.

9.4 E1b: does Fibonacci geometry strengthen pairing?

Mean field. A linearised real-space pairing-kernel model ($U_{\text{pair}} = +2$, $t = 1$, on-site $\pm v$, $N = 55$) with the gap equation gives $T_c(\text{Fibonacci})/T_c(\text{random shuffle of the same composition})$ between 0.74 and 1.33; Fibonacci is larger in 6 of 12 (filling, v) cases; the sign depends on filling, and the descriptive standardized difference exceeds two random-realisation standard deviations in 3 of 12 cases. This is not a conventional hypothesis-test z-score.

DMRG (`TeNPy`). With `TeNPy 1.1.1` [24, 25], validated against exact free-fermion diagonalisation at $U = 0$ (-13.12485 vs. -13.12487), we computed the spin gap $\Delta_s = E(N_\uparrow + 1, N_\downarrow - 1) - E(N_\uparrow, N_\downarrow)$ of open chains at $U = -4$ (bond dimension 80):

Table 9: DMRG spin gap Δ_s ($U = -4$, bond dimension 80) for Fibonacci, random-shuffle (mean $\pm$ sd, n realisations), and periodic $ABAB$ chains.

L ($N_\uparrow = N_\downarrow$)	v	Fibonacci	Random (mean $\pm$ sd, n)	$ABAB$	$\Delta/\sigma_{\text{random}}$
13 (4)	0.6	2.808	2.631 ± 0.323 (6)	2.372	$+0.55$
13 (4)	1.2	3.355	3.028 ± 0.389 (6)	2.842	$+0.84$
21 (6)	0.6	2.381	2.650 ± 0.150 (3)	2.177	-1.79
21 (6)	1.2	2.941	2.575 ± 0.275 (3)	2.680	$+1.33$

The clean-chain gaps are 1.876 ($L = 13$) and 1.654 ($L = 21$). Every tested binary potential raises the gap relative to the clean-chain value. The random controls are small ($n = 6$ at $L = 13$ and $n = 3$ at $L = 21$); these data do not establish a systematic Fibonacci advantage, and the descriptive difference changes sign with size. Finite-size IPR fits give Fibonacci D_2 estimates between approximately 0.58 and 0.72 for the supplied settings; without scaling uncertainties and multiple independent Fibonacci realizations, this is exploratory evidence rather than a thermodynamic proof of multifractality, and it did not establish a systematic pairing advantage; enhancement mechanisms for pairing in disordered systems are discussed elsewhere [26] and are not tested here.

Table 10: Summary outcome of each proposed FQM extension.

Proposed claim	Test	Outcome
E2: Φ signature in astrophysical data	Real exoplanet ratios, pre-specified	No excess ($z = -0.14$); 3:2 control detected ($z = +7.9$)
E2: Lattice delay detectable in GRBs	Synthetic pipeline	Feasible only near existing limit; lag confounder 38% (8% with rule)
E1: $\gamma_n \to \Phi^{-4}$ is physical	Identifiability	Untestable without independent clock; generic to metallic means
E1: Fibonacci raises pairing	Mean field and DMRG	No systematic effect; sign varies with filling and size

10 Executed results, claim registry and limitations

Established. Unimodularity of the lossless T-cell; the trace-map recurrence and analytical closed-form Fricke–Vogt invariant $I = (\omega^2 L \Delta C)^2$ for common-L cells; cancellation of a_n in Z_n, v_n (for any ratio); quadratic leading dispersion of a symmetric lattice and its conversion to $E_{\text{QG},2} = \sqrt{12}\, E_{\text{mesh}}$.

Reproduced in the present package. Tests 11 and 12 match their supplied JSON outputs exactly; the Stage 6 circuit optimization and four finalist ngspice-47 cross-checks were re-executed.

Reported in the supplied earlier archive, not independently rerun here. Low-frequency homogenization, finite-size spectral fragmentation, FDTD–$ABCD$ comparison, the ngspice-42/scikit-rf-2.2.0 comparison, and the Astropy time-of-flight integral.

Effective. The LC analogy of vacuum propagation; the one-dimensional GRB bound.

Open. Whether this LC spectrum is exactly Cantor with zero measure; physical measurement

Table 11: Executed results of `fqm_paper2_repro_suite_v2.py` (standard library only; values archived in `fqm_paper2_results_v2.json`).

Test	Metric	Result
0 Trace map	Recurrence; invariant spread; I vs. $(\omega^2 L\Delta C)^2$	9.7×10^{-16}; 1.2×10^{-15}; 2.8×10^{-14}
1 Unimodularity	max $\lvert T+R-1\rvert$; $\lvert\det M-1\rvert$ (4 topologies, $N=55$)	$\le 7.8\times10^{-14}$; $\le 6.7\times10^{-15}$
2 Band structure	Bands / measure at $F_n=8\to144$	$8\to84$; $51.6\%\to37.7\%$ (periodic: 2; 70.5%)
3 Homogenization	c_{eff}/c_0 finite / asymptotic; $1-T$	0.95584 / 0.95589; 6.0×10^{-4}
4 Scale controls	Drift of Z_n, v_n for 5 ratios; ω at $T<0.99$	$\le 2\times10^{-16}$; Φ not distinguished
5 FDTD vs. $ABCD$	Energy ratio, transmitting cases	Agree to $\le 0.42\%$
6 GRB conversion	$a_{\max}$ from LHAASO $E_{\text{QG},2}$	9.3×10^{-28} m
7 Benchtop	Delay ratio; stopbands; Monte Carlo	1.0469; 2 vs. 1; median 2 vs. 1

of Prototypes A and B; analysis of real GRB photon lists with the pre-specified rule of Section 9; larger-L many-body calculations; Lorentz-covariant discrete models.

Excluded. A role for Φ in the physical vacuum; Fibonacci specificity of the scale-invariance lemma; threshold band counting; any claim that numerical coincidences with the Planck length are physical evidence; a golden-ratio excess in real exoplanet period ratios (not found); a systematic pairing advantage of Fibonacci over random geometry in the models tested (not found); $\gamma_n\to\Phi^{-4}$ as a physical law without an independent generation clock.

Limitations. One-dimensional, linear, mostly lossless models; ESR modelled as constant with frequency; no capacitor loss, parasitic capacitance or self-resonance; distance in the GRB table is Euclidean; no physical measurement is reported. The benchtop protocol is a pre-specified plan, not a result. The time-of-flight pipeline uses synthetic photons and a coarse bootstrap ($B=10$); real GRB photon lists were not accessible. The DMRG study uses $L\le21$, bond dimension 80, three to six random realisations and a one-dimensional chain (no true T_c; the spin gap is a proxy); the mean-field study is one system size. The exoplanet catalogue is heterogeneous (detection biases), and the Solar-System planetary semi-major axes were entered from standard tabulated values.

11 Robust multiobjective sequence optimization (Stage 6)

Stage 5 showed that S_{21} attenuation alone conflates reflection and dissipation. Stage 6 therefore retains the power-based metrics and asks whether a heuristic search can identify sequences with a larger robust reflective stopband than the Fibonacci ordering, for a fixed 21-shunt / 22-series ladder and a fixed composition of 13 type-A and 8 type-B capacitors.

The model retains $L_{\text{end}}=5.1\,\mu$H, $L_{\text{int}}=10\,\mu$H, $C_A=5.6\,$nF, $C_B=2.4\,$nF, $R_S=R_L=50\,\Omega$, $Q_L=60$, $Q_C=200$, $C_p/C=0.02$, and a 1-MHz Q reference. The design interval is 300 kHz–1.8

MHz; the low-frequency guard interval is below 300 kHz. We define

$$f_{\mathrm{refl}} = \langle \mathbf{1}[S_{21} < -10\,\mathrm{dB} \ \wedge \ S_{11} > -3\,\mathrm{dB}] \rangle, \quad f_{\mathrm{abs}} = \langle \mathbf{1}[S_{21} < -10\,\mathrm{dB} \ \wedge \ P_{\mathrm{diss}} > 0.5] \rangle,$$

where $P_{\mathrm{diss}} = 1 - |S_{11}|^2 - |S_{21}|^2$ for the matched two-port. The scalar evolutionary score is $f_{\mathrm{refl}} - 0.25 f_{\mathrm{abs}} - 0.10\,\max(0, -1 - \overline{S_{21,\mathrm{low}}})$, with the low-band mean in dB. A separate Pareto set retains the three objectives (maximize reflective fraction, minimize absorptive fraction, maximize low-band mean S_{21}).

The search used a population of 72, 30 generations, seed 20261124, composition-preserving crossover and swap mutations. It evaluated 1508 unique sequences on a 301-point frequency grid. The top nominal candidates and four fixed reference sequences were then assessed with 24 common-random-number tolerance realizations per candidate at each of $\pm 1\%$, $\pm 5\%$, and $\pm 10\%$. Each realization independently perturbs all 22 inductors and 21 capacitors using uniform bounds. These bounds are hypothetical sensitivity assumptions, not measured component distributions. The search is stochastic and finite; neither global optimality nor uniqueness is claimed.

Table 12: Stage 6 comparison. Robust means are averages across 24 tolerance realizations per topology and tolerance.

Sequence	Nominal f_{refl}	Nominal f_{abs}	$\pm 5\%$ f_{refl}	$\pm 5\%$ f_{abs}	$\pm 10\%$ f_{refl}
New robust candidate	69.43%	1.13%	68.58%	1.16%	65.30%
Fibonacci	56.98%	9.43%	56.57%	10.83%	56.34%
Stage 5 random winner	62.26%	2.26%	62.37%	2.25%	63.65%
Balanced 13A/8B control	55.09%	4.91%	55.31%	6.29%	56.01%
Alternating control (11A/10B)	45.66%	12.83%	49.10%	8.74%	51.92%

The best robust candidate is `AABABBABAAABBAABAABAA` (13 A cells and 8 B cells). Its $\pm 5\%$ mean reflective fraction is 68.58%, compared with 56.57% for Fibonacci and 62.37% for the Stage 5 sampled winner. The $\pm 10\%$ result remains competitive under the broader hypothetical tolerance bound. The alternating control has a different composition (11A/10B), so it is descriptive rather than a strictly composition-matched comparison. The candidate's advantage is conditional on this circuit, loss model, frequency window, objective and search budget; other objective weights or real component models may change the ranking.

The four finalist spectra were recalculated on 3401 frequencies and checked against independently generated ngspice 47 AC analyses. Each deck produced 3401 points; the maximum absolute S_{21} magnitude discrepancy was 4.93×10^{-7} to 5.00×10^{-7} dB. This is numerical cross-validation of the same lumped circuit model, not physical validation. ngspice batch execution follows its documented batch workflow [19].

Table 13: Independent ngspice 47 cross-check of Stage 6 finalists.

Deck	Points	Maximum absolute error (dB)	Mean absolute error (dB)
Finalist 1 (robust candidate)	3401	4.93×10^{-7}	3.40×10^{-8}
Finalist 2 (Fibonacci)	3401	4.97×10^{-7}	2.67×10^{-8}
Finalist 3 (Stage 5 winner)	3401	5.00×10^{-7}	3.70×10^{-8}
Finalist 4 (balanced control)	3401	4.97×10^{-7}	2.64×10^{-8}

Falsifiable prediction. If the new ordering is fabricated using components whose effective values and losses are independently characterized, its measured reflective-stopband fraction over 300 kHz–1.8 MHz should exceed that of a composition-matched Fibonacci ladder built and measured under the same port calibration and tolerance protocol. Failure to exceed it within

measurement uncertainty would reject the predicted advantage for the tested hardware. No such physical measurement is included in this release.

12 Supplementary source audit and reproducibility limits

This standalone study preserves the supplied numerical record and includes the supplementary archive used for the source audit. The audit distinguishes a reported result from a result independently re-executed in the present package build. The supplied scripts and JSON files are retained unchanged in the supplementary directory; the corrections below apply to interpretation and provenance, not to the numerical values in those source files.

Test 10: synthetic GRB time-of-flight pipeline. The supplied script uses 24 trials per condition and only $B = 10$ photon-bootstrap resamples per trial. These settings are adequate for a pilot demonstration of the decision logic but are too small for precise confidence-interval coverage or stable detection-power estimates. The reported 38% interval-only response to an injected intrinsic linear lag falls to 8% under the combined decision rule in the supplied run. This is a synthetic pipeline result, not an analysis of GRB 221009A photon events. The calibration margin is estimated from finite synthetic calibration sets; the term "pre-specified" describes the code-level decision rule and should not be read as evidence of a time-stamped external preregistration. A confirmatory study should use substantially more trials, at least thousands of bootstrap resamples, and independent burst-level data.

Test 11: planetary and exoplanet analyses are separate. The script `fqm_test11_dsi_base_scan.py` analyses the logarithmic semi-major-axis spacings of the eight Solar-System planets and a synthetic log-periodic detection problem. Its supplied JSON reports RMS residual 0.314 at Φ, an a-priori null probability 0.553, and a best scanned base 1.39 with look-elsewhere probability 0.567. The distinct script `expB_exoplanet_phi.py` reports the catalogue analysis of 879 systems and 1003 adjacent period ratios. The catalogue snapshot `oec.xml.gz`, its retrieval date, revision identifier and checksum are not included; the script also contains a machine-specific absolute input path. Consequently, the exoplanet result is preserved as a supplied result, but cannot be regenerated from the uploaded archive alone. In addition, its look-elsewhere null uses only 40 bootstrap replicates and evaluates a downsampled grid (`grid[::4]`) rather than the full 371-centre observed scan. The quoted global threshold is therefore coarse and should be recalibrated with a much larger null ensemble and identical scan coverage before confirmatory inference.

Test 12: relaxation identifiability. The supplied script and JSON were re-executed in this environment; the generated JSON matched the supplied file exactly. The result is mathematical/model-based: the λ^{-4} limit generalises across the tested metallic-mean convergents, while mapping generations to physical time freely makes the proposed Φ^{-4} rate non-identifiable. No physical relaxation measurement is supplied.

Test 13: pairing toy models and DMRG. The mean-field script uses a positive pairing-kernel coefficient $U_{\mathsf{pair}} = +2$ in the linearised gap equation. This is not the same sign convention as the attractive-Hubbard Hamiltonian used by the separate DMRG script, where $U = -4$. The manuscript now states this distinction explicitly. The reported z quantity in the mean-field JSON

is $(T_c^{\mathrm{Fib}} - \overline{T_c^{\mathrm{rand}}})/\sigma_{\mathrm{rand}}$; it is a descriptive standardised difference, not a conventional inferential z-test. The DMRG random-control counts are only 6 ($L = 13$) and 3 ($L = 21$). TeNPy is not installed in the present execution environment, and the supplied DMRG driver contains a machine-specific import path. These results are therefore retained as exploratory model outputs, not independent reproductions or evidence of a thermodynamic pairing enhancement.

Independent execution status. The exact supplied Test 11 and Test 12 scripts were executed; their JSON outputs matched the supplied results exactly. Test 10 and Test 13 did not complete within the available execution window in this package-build environment. The inherited external-software experiment reports ngspice 42 and scikit-rf 2.2.0; the supplied exoplanet analysis requires Astropy and the missing catalogue snapshot; the DMRG analysis requires TeNPy 1.1.1. These dependencies and inputs are not silently replaced by the Stage 6 ngspice 47 validation. Stage 6 remains a separate, successfully executed circuit-optimization workflow.

What would upgrade the evidence. For the exoplanet analysis, archive the exact catalogue snapshot and hash, record filtering decisions, and use at least 10^4 full-grid null simulations. For Test 10, increase both the number of synthetic trials and bootstrap replicates, then analyse public photon-event data with source-intrinsic lag nuisance models. For Test 13, pin the TeNPy and BLAS environment, save convergence logs and all random-control realizations, increase the number of independent shuffles, and perform finite-size scaling with uncertainty estimates. No result in this section constitutes a physical bench measurement or a direct test of a fundamental FQM law.

13 Reproducibility

The Stage 6 source is `scripts/stage6_multiobjective_search.py` (Python 3.10+, NumPy, pandas, Matplotlib; seed `20261124`). From the package root, execute the search with `python scripts/stage6_multiobjective_search.py --population 72 --generations 30 --robust-trials 24 --seed 20261124`. Then execute `spice/run_stage6_ngspice.sh` to run the four ngspice 47 decks. The package preserves the tolerance factors, per-trial results, spectra, netlists, logs, environment manifest and SHA-256 checksums.

The inherited Stage 1–5 archive contains numerical outputs, figures, netlists, logs and prior reports, but not all original source scripts. This release therefore does not claim end-to-end re-execution of every historical analysis. The older GRB, exoplanet, DMRG and FDTD results in Sections 8–9 are retained as results reported in the supplied original archive and have not been rerun in Stage 6. Stage 6 is the fully executable reproduction path in this release. The interactive HTML simulator remains exploratory: its $ABCD$ engine matches Regime A, but its FDTD uses branch inductance L_i rather than the cell average and its metrics are illustrative.

Declarations

License: CC BY 4.0. **Competing interests:** none declared. **AI use:** Large language models were used as computational, auditing and editorial assistants (re-running and correcting scripts, verifying algebra, drafting text). Scientific claims, epistemic boundaries and decisions remain the author's responsibility. **Data and code:** supplied with this preprint as listed above; the exoplanet data come from the Open Exoplanet Catalogue [23] (snapshot of 19 November 2026).

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