FQM Paper 2 — Advanced Simulation Report (Stage 4)

High-precision spectral refinement and independent benchtop-ladder nodal analysis

Reference: FQM Paper 2 Preprint v2.1 (LC Fibonacci), especially Sections 3, 7 and 8. This report separates source-paper predictions from new calculations and from external-tool runs that remain pending.

1. High-precision refinement of spectral edges

The N=144 Fibonacci approximant was reconstructed from the manuscript's substitution convention S0=B, S1=A, S(n+1)=S(n)S(n−1), with L=1, C_A=1.4, C_B=0.6 (δ=0.40). The periodic control uses AB repeated. The 217 Fibonacci and 3 periodic edge candidates from the previous adaptive scan were re-refined with 70-digit arithmetic using mpmath, solving Tr(M)^2−4=0.

Quantity	Fibonacci N=144	Periodic AB
Refined edge roots	217	3
Allowed intervals	108	2
Allowed spectral fraction	37.49287735%	70.50344192%
Max $\lvert Tr(M)^2-4\rvert$ residual	3.63e-65	4.07e-66
Max shift from prior edge list	4.98e-12	4.39e-12

The residuals show that the refined roots satisfy the trace-edge equation to high precision for the reconstructed model. The edge shifts relative to the previous list are very small. This validates numerical root refinement, not the infinite-chain spectrum or any physical interpretation beyond the finite periodic approximant.

2. Physical-scale benchtop prototype: independent nodal MNA

A separate nodal-admittance implementation was built for the manuscript's 21-cell Prototype A: word ABAABABAABAABABAABABA; L=10 µH between nodes and 5.1 µH at both ends; C_A=5.6 nF, C_B=2.4 nF; 50 Ω source and load. The scan covers 0.1–1.8 MHz. The lossless comparison removes inductor ESR; the Q=50 case uses the manuscript's frequency-independent R=2π(1 MHz)L/Q convention. The extension adds Q_L=60, Q_C=200 and C_p/C=2%.

Topology	New nodal-MNA edges (MHz)	Manuscript prediction (MHz)
Fibonacci	0.85863–0.90279; 1.12680–1.54402	0.860–0.905; 1.127–1.545
Periodic AB	0.95517–1.45545	0.958–1.458

The independently calculated stopband edges are within a few kHz of the manuscript's tabulated values. The second Fibonacci gap is reproduced at approximately 1.127–1.544 MHz. The manuscript's comparison window in the table is stated as f<1.3 MHz, so the additional higher-frequency finite-ladder features are not included in that comparison.

3. Independent MNA versus ABCD agreement

Across 601 frequencies from 0.1 to 1.8 MHz, both topologies and three circuit cases (lossless, Q=50, and the parasitic extension), the maximum absolute complex S21 discrepancy between nodal MNA and a separately cascaded ABCD implementation was 2.59e-14. This is a strong internal cross-check of two independently assembled numerical formulations.

4. External tools: installation attempt and current status

ngspice and scikit-rf were not installed in this execution environment. apt-get update timed out; direct repository access and pip installation failed because DNS resolution was unavailable. Debian 13 (trixie) publishes ngspice 44.2+ds-1 for amd64, but the package could not be fetched in this network-restricted runtime. Six ngspice decks and a runner script were nevertheless generated for the exact Prototype A word, periodic control, lossless/Q=50 cases and the parasitic extension.

5. Conclusions and remaining boundary

• High-precision root refinement completed successfully for all 220 previously identified edge candidates.
• The physical-scale nodal MNA reproduces the manuscript's key 21-cell stopband locations within a few kHz and agrees with an independent ABCD implementation at approximately 10■[1]■ absolute complex S21 error.
• This is not an ngspice run: the decks are prepared but await execution in an environment with ngspice installed.
• No full-wave 2D/3D electromagnetic solver or physical bench measurement was run. The next external validation is to execute the supplied decks, compare their complex S21 against the CSV reference, and then calibrate ESR/parasitics using measured component impedances.

Reproducibility artifacts: high_precision_fibonacci_roots_N144.csv; high_precision_periodic_roots_N144.csv; physical_bench_mna_sweep.csv; physical_mna_vs_abcd_crosscheck.csv; physical_bench_mna_spectra.png; six ngspice_*.cir decks; run_ngspice_all.sh; INSTALL_AND_RUN_SPICE.md.