FQM Paper 2 — Advanced Simulation Addendum (Stage 3)

Independent numerical cross-validation and expanded robustness ensemble

Scope: normalized 1D LC transmission-line model derived from the manuscript's symmetric T-cell transfer matrix. This addendum is a computational cross-check, not experimental validation.

1. Independent nodal-analysis cross-validation

The same finite ladder was solved in two mathematically independent ways: (i) cascading the manuscript's ABCD/T-cell matrices and (ii) solving the complex nodal-admittance system. The comparison covers N = 1, 2, 5, 13 and 21 cells, over 301 frequencies from ω = 0.03 to 1.8, for ideal, inductor-loss, and loss-plus-capacitor-ESR/parasitic-capacitance cases.

N	Model	Max |ΔS21|	Median relative error
1	ideal	4.52e-16	1.24e-16
1	inductor_loss	4.65e-16	1.44e-16
1	loss_esr_cp	4.97e-16	1.46e-16
2	ideal	1.55e-15	2.30e-16
2	inductor_loss	3.26e-15	2.43e-16
2	loss_esr_cp	1.54e-15	2.53e-16
5	ideal	6.55e-15	3.99e-16
5	inductor_loss	5.12e-15	4.70e-16
5	loss_esr_cp	9.84e-15	4.82e-16
13	ideal	2.11e-14	9.59e-16
13	inductor_loss	1.91e-14	1.16e-15
13	loss_esr_cp	2.21e-14	1.35e-15
21	ideal	3.47e-14	1.55e-15
21	inductor_loss	2.91e-14	1.86e-15
21	loss_esr_cp	4.15e-14	2.04e-15

Across all cases, the maximum absolute complex S21 discrepancy is approximately 4.15×10■¹■; median relative errors are approximately 10■¹■ to 10■¹■. This is strong internal numerical agreement between two formulations of the same circuit model. It does not validate the physical parameter values or the circuit against a real device.

2. Expanded tolerance ensemble

A new Monte Carlo run used 500 realizations per topology (1,000 total), N = 21, ±5% independent cell-level L and C variations, QL = 60, QC = 200, Cp/C = 2%, and 301 sampled frequencies over ω = 0.03–1.8. The reported metric is the fraction of that sampled frequency interval with |S21| < 0.316 (–10 dB). These are normalized illustrative parameters.

Topology	n	Mean fraction	SD	P05–P95	Mean min. S21 (dB)
Fibonacci	500	0.1926	0.0148	0.1694–0.2193	-21.26
Periodic_AB	500	0.2073	0.0156	0.1860–0.2392	-22.00

The periodic AB control exceeds the Fibonacci mean by 0.0147 (about 1.47 percentage points); bootstrap 95% interval for this difference: [0.0129, 0.0166]. In this particular loss-and-tolerance configuration, Fibonacci is therefore not superior on the chosen

−10 dB coverage metric. The distributions overlap, and this conclusion is specific to the modeled frequency window, loading, topology, and normalized component assumptions.

3. Interpretation and implications for the manuscript

• The ABCD implementation is independently reproduced by nodal analysis to near floating-point precision. This supports implementation consistency, not physical validation.
• The expanded Monte Carlo result is a counterexample to any broad claim that Fibonacci ordering always improves attenuation robustness. The paper should report topology-dependent trade-offs and avoid a universal superiority statement.
• The metric |S21| < −10 dB measures attenuation coverage, not stopband count, bandwidth quality, insertion loss, group delay, or device utility. A multi-objective comparison is required.
• This expanded ensemble uses a separately specified normalized setup; it should not be merged numerically with earlier Monte Carlo results unless all model assumptions, seeds, grid definitions and source/load conventions are reconciled.

4. Remaining validation boundary

No ngspice cross-check of this new parasitic model, full-wave 2D/3D electromagnetic simulation, fabricated benchtop measurement, or real GRB photon-data analysis was performed in this stage. The available runtime did not provide ngspice or an installed full-wave solver. The next defensible step is to export a SPICE netlist and compare AC transfer functions, then calibrate parasitic values from component datasheets or measured impedance data before any hardware claims.

Reproducibility files: independent_nodal_crosscheck_metrics.csv; monte_carlo_500_per_topology.csv; monte_carlo_500_summary.csv; monte_carlo_500_distribution.png. Random seed: 20261120 for the ensemble; 20261121 for bootstrap resampling.