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Farzana Anjum Esha

farzana.esha09@gmail.com

Independent Researcher https://orcid.org/0009-0006-4316-3960

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Bistability and Boundary Bifurcations in a Mean-Field Replicator Model of Strategic Complementarity in Agent Banking

Farzana Anjum Esha
Independent Researcher
Dhaka, Bangladesh
farzana.esha09@gmail.com

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Abstract

Roughly two-thirds of Bangladesh's agent banking outlets report no outstanding loans, even though the network has expanded rapidly. This paper models the share of outlets with positive outstanding loans, $x(t)$, as a replicator-type mean-field ODE driven by the profit differential between lending and non-lending outlets, with strategic complementarity entering through a logistic loan-volume function of x. The profit differential is strictly increasing in x, so the interior equilibrium is unique and unstable: it is a coordination threshold (separatrix) between a low-lending trap and a full-participation state. The analysis derives the separatrix in closed form and two critical risk costs, $\underline{\kappa}$ and $\overline{\kappa}$, at which the separatrix reaches the boundaries of the state space; local normal forms show that these are transcritical, not saddle-node, bifurcations. The model is calibrated to Bangladesh Bank agent-banking aggregates for December 2024 (outlets, deposits, outstanding loans) and the share of outlets with no outstanding loans reported by the International Growth Centre, with assumed curvature, inflection and float-cost parameters and a maintained assumption that the observed share lies in the low-lending basin. Whether cost reduction alone can remove the trap depends on the unobserved float cost relative to a critical value of about 3% of deposits. If the float cost is high (5%), the lower threshold is negative, no non-negative risk cost removes the trap, and eliminating the risk cost entirely lowers the separatrix only from 0.33 to about 0.14. If it is low (2%), the trap can be removed, but only by a reduction of roughly 58–63% in the effective cost. In both cases, information-side or liquidity-side changes can substitute for cost reduction. The quantitative results are conditional model-based counterfactuals, not estimates of the support actually required, and the paper does not establish empirically that the network is in the trap.

Keywords: Nonlinear dynamical systems, Mathematical modelling, Replicator dynamics, Strategic complementarity, Bistability, Transcritical bifurcation, Agent banking

1 Introduction

1.1 Expansion without credit deepening

Bangladesh's agent banking network presents a puzzle of financial access without equivalent depth in credit intermediation. The network has expanded to more than 20,000 outlets and mobilised hundreds of billions of takas in deposits [1]. Yet, approximately two-thirds of these outlets had no outstanding loans as of December 2024 [2]. This contrast suggests that the expansion of financial access and deposit mobilisation has not translated into similarly widespread participation in lending. Although branchless banking channels are commonly associated with broader financial inclusion and improved access to financial services [11, 12], the uneven distribution of lending activity raises a further question: how can low participation persist within an expanding financial network?

Existing studies of agent banking in Bangladesh have primarily examined financial inclusion, service adoption, customer acceptance, and the opportunities and constraints associated with agent-based delivery. Nisha et al. [6], for example, investigate the relationship between agent banking and financial inclusion, while Barua et al. [7] examine its opportunities and adoption-related challenges. Related survey-based research also considers the factors influencing agents' acceptance of these services [19]. More recently, the International Growth Centre documents the uneven distribution of lending activity across outlets and the prevalence of outlets with zero outstanding loans [2]. While this study establishes the importance of agent banking and documents the limited reach of credit activity, it leaves open the dynamic question of how lending participation evolves across outlets and under what conditions a low-participation state can persist.

1.2 Strategic complementarity and participation dynamics

A possible explanation for persistent low participation is strategic complementarity: the return to an activity may increase with the number of other agents undertaking it. When the profitability of lending at an outlet depends on the lending activity of other outlets, participation decisions become interdependent. Such interactions can generate coordination problems and multiple equilibria, in which different initial conditions lead to distinct long-run outcomes [9, 23, 10]. This possibility connects the agent-banking problem to a broader class of nonlinear population models in which aggregate behaviour feeds back into individual incentives.

In an agent banking network, several channels could motivate this type of interaction. Greater lending activity across outlets may support the circulation of borrower information, improve screening through accumulated local experience, and strengthen familiarity and trust between agents and their communities. Research on credit-market information sharing shows that information exchange can affect lenders' ability to distinguish between borrowers [17]. Evidence from Kenya also shows that the expansion of mobile-money agent networks can improve financial access and reduce transaction costs, enabling households to share risk more effectively through remittances [13]. Related work on relationship lending highlights the value of locally acquired soft information in supporting credit decisions [18]. Taken together, these studies motivate possible channels through which lending activity may reinforce itself,

although they do not establish that such effects operate across Bangladesh's agent banking outlets.

The present paper represents this possibility through a network-wide lending share, denoted by x, and assumes that the average loan volume per lending outlet, $L(x)$, increases with that share. Formally, the model imposes $L'(x) > 0$ and specifies loan volume using a logistic function. This monotonicity condition is a modelling hypothesis capturing strategic complementarity, rather than an empirically established relationship. It implies that a higher lending share raises the loan volume associated with lending outlets and strengthens the relative profitability of lending. Consequently, aggregate participation feeds back into the incentives governing further participation.

1.3 Equilibrium Structure and Bifurcation Analysis

The mathematical implications of strategic complementarity depend on the structure of the resulting dynamical system. In nonlinear population models, aggregate interactions can produce threshold behaviour, unstable interior equilibria, and multiple attracting states. Replicator dynamics provide a standard framework for describing the evolution of population shares according to relative payoffs [21], while mean-field approaches represent aggregate interactions through a population-wide state variable [22, 24]. These frameworks offer a natural basis for examining how interdependent participation decisions can generate persistent differences in long-run outcomes.

A related literature establishes that credit-market imperfections can also produce rationing and equilibrium multiplicity. In particular, information asymmetries and imperfect screening may prevent some borrowers from obtaining credit even when they are willing to borrow at prevailing interest rates [14, 16, 15]. The mechanism considered here is distinct. Rather than generating multiplicity primarily through information asymmetries between lenders and borrowers, it examines how complementarity in the intermediation network itself can make the profitability of lending depend on aggregate participation.

This distinction leads to the specific mathematical problem addressed in the paper: how does a monotone network-dependent profit differential shape the equilibrium structure of lending participation, and how do changes in lending costs alter the location and stability of these equilibria? In particular, it is important to establish whether the system admits an interior coordination threshold, whether that threshold can collide with the boundary equilibria, and what types of bifurcations are possible under the assumed monotonicity condition. Although replicator dynamics and local bifurcation analysis are established mathematical tools [21, 20], their application to a network-dependent loan-volume mechanism provides a way to characterize these questions in the context of agent-banking credit participation.

The paper addresses this problem through a mean-field replicator model in which the state variable is the share of outlets with positive outstanding loans. The model focuses on the evolution of lending participation rather than on individual borrower-level loan approval or repayment decisions. Its central analytical feature is that the profit differential is strictly increasing in the participation share. This restriction permits a complete characterization of the equilibrium structure and rules out interior saddle-node bifurcations under the maintained assumptions.

1.4 Analytical contributions and scope

The analysis makes three contributions. First, it derives the complete equilibrium structure of a mean-field replicator model with logistic loan volume. The unique interior equilibrium is obtained in closed form and acts as an unstable coordination threshold separating the two stable boundary states in the bistable parameter regime. The analysis also derives two parameter thresholds, $\underline{\kappa}$ and $\overline{\kappa}$, at which the interior equilibrium collides with the boundary equilibria. Local bifurcation analysis establishes that these boundary collisions are transcritical and explains why an interior saddle-node bifurcation cannot arise under the imposed monotonicity condition. It further identifies the type of change in the loan-volume relationship that would be required to permit interior equilibrium multiplicity.

Second, the model is conditionally calibrated using publicly available Bangladesh banking aggregates. The calibration identifies the loan volume implied at the observed lending share and makes explicit which parameters cannot be independently identified from the available aggregate data, including the risk cost κ, the curvature b, and the float cost λ. The implied cost benchmark $\hat{\kappa}$ is interpreted conditionally on the maintained assumption that the observed lending share lies in the low-lending basin. Thus, the calibration illustrates the model's implications without treating the assumed network-complementarity mechanism or the existence of an actual low-lending trap as empirically established.

Third, the analysis maps changes in lending costs into changes in the coordination threshold. It identifies conditions under which reducing the risk cost alone can eliminate the low-lending basin and conditions under which a positive lower-bound threshold prevents this outcome. It further quantifies the cost reductions required to move the separatrix below specified participation targets and examines how these requirements vary with the unobserved parameters of the loan-volume function and the float cost. This provides a conditional policy mapping between cost-side interventions and the participation thresholds implied by the model.

The scope of the analysis is deliberately limited. The high-lending outcome is represented by the full-participation boundary $x = 1$, which serves as a stylized upper-bound state rather than a forecast of the eventual structure of Bangladesh's agent banking network. Similarly, the aggregate data are used to discipline the calibration, not to test whether loan volume actually increases with the network-wide lending share. The principal contribution is therefore analytical: to characterize how an assumed strategic complementarity in an agent banking network can generate a low-lending coordination trap and to establish the parameter conditions under which cost reductions can weaken or eliminate it.

2 A Mean-Field Model of Agent Intermediation

2.1 State and Profits

Let $x(t) \in [0, 1]$ be the share of outlets with positive outstanding loan balances ("lending outlets") at time t. Non-lending outlets earn commissions only; lending outlets additionally earn interest and bear costs of lending. All flows are measured per year, so that r_L, κ and λ are annual rates, L and $\bar{D}$ are balances (stocks), and π_N, π_L are annual profits.

Definition 1 (Profits)**.** A non-lending outlet earns $\pi_N = \tau T$, with commission rate τ on annual transaction volume T. A lending outlet earns

$$\pi_L(x) = \tau T + r_L L(x) - \kappa L(x) - \lambda \bar{D}, \tag{1}$$

where $L(x)$ is the average loan balance per lending outlet, r_L is the annual lending rate, κ is the annual effective cost per unit of loan balance, and $\lambda \bar{D}$ is the annual opportunity cost of liquidity held against deposits $\bar{D}$.

The parameter κ is an *effective* cost: expected default loss, provisioning, monitoring, and any other per-unit cost of credit intermediation. It is assumed that $\lambda, \bar{D} > 0$ and $0 \leq \kappa < r_L$ (if $\kappa \geq r_L$ lending is unprofitable at every volume). The liquidity cost is modeled as a fixed amount per outlet because the reserve held to meet withdrawals depends on the deposit base, not on the loan book; any component that scales with loans is absorbed into κ.

2.2 Institutional Structure of Lending Decisions

In many agent-banking arrangements, the bank supplies loanable funds, sets lending rules and bears credit risk, while the agent outlet refers borrowers and facilitates loan servicing. Accordingly, the model does not treat outlets as independent lenders. Instead, lending status, defined by positive outstanding loans, represents a joint bank–outlet decision: the bank supports credit intermediation through the outlet, and the agent undertakes the associated activities.

The returns π_L and π_N therefore represent the joint returns from lending and non-lending activities, respectively. The cost κ captures the pair's default losses and servicing costs, while $\lambda \bar{D}$ represents the opportunity cost of supporting liquidity. The replicator equation provides an aggregate reduced-form representation of changes in lending participation. Although the qualitative results depend on the sign of F, identifying the separate incentives of banks and agents, and independently calibrating their costs, would require bank-level or contract-level data.

2.3 Network Externality

Assumption 1 (Logistic loan volume)**.**

$$L(x) = \bar{L}\, s(x), \qquad s(x) = \frac{1}{1 + e^{-b(x - x_0)}}, \qquad b > 0,\ x_0 \in (0, 1), \tag{2}$$

where $\bar{L}$ is the saturation loan volume, b the steepness of the network effect, and x_0 the inflection point (the critical mass at which the externality is strongest).

Assumption 2 (Deposits)**.** Average deposits per outlet are constant: $D(x) = \bar{D}$.

Status of the assumption $L'(x) > 0$**.** The assumption states that the average loan balance of a lending outlet rises with the network-wide lending share. This is a *modelling hypothesis* representing strategic complementarity; it is not demonstrated by the Bangladesh data and is not tested in this paper. Several distinct channels could generate it, and they

are not interchangeable: (i) *information sharing*, since more lending outlets means more repayment histories that improve borrower screening; (ii) *trust and demand*, since visible lending raises borrowers' willingness to use agents for credit; (iii) *institutional willingness*, since banks extend larger limits to channels with proven performance; and (iv) *learning* among outlets and bank staff. The model does not discriminate between these channels, and which of them operates, if any, is an empirical question; $L(x)$ is a reduced-form representation of their joint effect.

Rationale for the logistic specification. The logistic function provides a convenient bounded and strictly increasing representation of loan volume as a function of network participation, consistent with the use of logistic-type functions to represent nonlinear relationships and saturation in dynamic models [27]. Its S-shaped profile allows the marginal response of loan volume to vary with the lending share: loan volume initially responds slowly, increases most rapidly around the inflection point x_0, and gradually approaches the saturation level $\bar{L}$. The parameters $\bar{L}$, b, and x_0 respectively control the upper bound, steepness, and location of this transition. More importantly, the specification permits a closed-form expression for the interior equilibrium and its associated coordination threshold in Equation (6). The logistic form is therefore adopted primarily for analytical tractability and interpretability; the qualitative monotonicity results do not depend on this particular functional form.

Note that $L(0) = \bar{L}s(0) > 0$. This is a counterfactual limiting value representing the loan balance that a lending outlet would carry when the network-wide lending share approaches zero. It determines the lower boundary threshold $\underline{\kappa}$ and is not an observed loan volume.

2.4 Dynamics

Lending status adopts or lapses according to relative profitability. The aggregate adjustment is represented by the replicator-type equation [21]

$$\dot{x} = h(x;\kappa) \coloneqq \gamma\, x(1-x)\, F(x;\kappa), \qquad F(x;\kappa) \coloneqq \pi_L - \pi_N = (r_L - \kappa)L(x) - \lambda\bar{D}, \quad (3)$$

with adjustment speed $\gamma > 0$. The factor $x(1-x)$ keeps $[0,1]$ invariant.

Interpretation of the dynamics. Equation (3) is a reduced-form adoption mechanism, not a literal description in which each outlet independently switches strategy. It captures that the share of outlets in credit intermediation grows when the pair's payoff differential is positive and shrinks when it is negative, with the rate proportional to the number of outlets that can switch in each direction, $x(1-x)$. The decision unit is the bank–outlet pair (Section 2.2).

The state $x = 0$ is always an equilibrium, so persistence of the low-lending state depends on its *stability*, which is governed by the sign of $F(0;\kappa)$.

Definition 2 (Equilibrium, stability)**.** $x^* \in [0,1]$ is an equilibrium if $h(x^*;\kappa) = 0$. It is locally asymptotically stable if $\partial_x h(x^*;\kappa) < 0$ and unstable if $\partial_x h(x^*;\kappa) > 0$. For $x^* \in \{0,1\}$, stability is understood relative to the state space $[0,1]$, that is, with respect to one-sided perturbations.

For later use, the following quantities are defined:

$$c(\kappa) \coloneqq \frac{\lambda \bar{D}}{r_L - \kappa}, \qquad g(x) \coloneqq r_L - \frac{\lambda \bar{D}}{L(x)}. \tag{4}$$

3 Analysis

3.1 The Profit Differential is Monotone

Lemma 1. *For $\kappa < r_L$, $F(\cdot;\kappa)$ is strictly increasing on $\mathbb{R}$. For every real κ and x, $F(x;\kappa) > 0 \iff \kappa < g(x)$, and g is strictly increasing with $g(x) < r_L$.*

Hence strategic complementarity here means that lending becomes more attractive as more outlets lend, and every zero of F is a *crossing from below*.

3.2 Regimes and the Separatrix

The boundary thresholds are defined as

$$\underline{\kappa} \coloneqq g(0) = r_L - \frac{\lambda \bar{D}}{L(0)}, \qquad \overline{\kappa} \coloneqq g(1) = r_L - \frac{\lambda \bar{D}}{L(1)}, \qquad \underline{\kappa} < \overline{\kappa} < r_L. \tag{5}$$

By Lemma 1, $F(0;\kappa) < 0 \iff \kappa > \underline{\kappa}$ and $F(1;\kappa) > 0 \iff \kappa < \overline{\kappa}$. Note that $\underline{\kappa}$ can be negative; the admissible costs are $\kappa \geq 0$, so the effective bistable window is $[\max(\underline{\kappa}, 0), \overline{\kappa})$.

Theorem 1 (Regimes)**.** *Under Assumption 1 and $\kappa \geq 0$:*

(i) ***Trap.*** *If $\kappa > \overline{\kappa}$, then $F < 0$ on $[0,1]$, the state $x = 0$ is asymptotically stable and attracts every trajectory with $x(0) < 1$, and $x = 1$ is unstable.*

(ii) ***Bistability.*** *If $\underline{\kappa} < \kappa < \overline{\kappa}$, then F has exactly one zero $x_s(\kappa) \in (0,1)$, given by*

$$x_s(\kappa) = x_0 + \frac{1}{b} \ln \frac{c(\kappa)}{\bar{L} - c(\kappa)} = g^{-1}(\kappa). \tag{6}$$

It is unstable. Both $x = 0$ and $x = 1$ are asymptotically stable, with basins $[0, x_s)$ and $(x_s, 1]$. The map $\kappa \mapsto x_s(\kappa)$ is continuous and strictly increasing, with $x_s \to 0$ as $\kappa \downarrow \underline{\kappa}$ and $x_s \to 1$ as $\kappa \uparrow \overline{\kappa}$.

(iii) ***Escape.*** *If $\kappa \leq \underline{\kappa}$ (possible with $\kappa \geq 0$ only if $\underline{\kappa} \geq 0$), then $F > 0$ on $(0,1]$ and $x = 1$ attracts every trajectory with $x(0) > 0$. If $\kappa < \underline{\kappa}$, then in addition $F(0;\kappa) > 0$ and $x = 0$ is unstable with nonzero linearization; at $\kappa = \underline{\kappa}$, $F(0;\kappa) = 0$, the linearization at $x = 0$ vanishes, and $x = 0$ is repelling relative to $[0,1]$.*

At $\kappa = \overline{\kappa}$ the boundary equilibrium $x = 1$ is repelling relative to $[0,1]$ and has zero linearization; on the extended real line it is semi-stable.

Remark 1 (The full-participation state)**.** The state $x = 1$ is the *full-participation boundary state*: every outlet has positive outstanding loans. It is a consequence of the homogeneity assumption and should not be read as a realistic description of a mature network. In practice, geography, customer composition, bank policy or local demand may keep some outlets out of credit intermediation even when lending is profitable on average. Heterogeneity across outlets could produce a stable interior participation share below one; this extension is not pursued here. References in the paper to the "high-lending state" mean this boundary state, and the threshold $\overline{\kappa}$ is the cost at which full participation ceases to be stable in the homogeneous model.

Proposition 1 (Bifurcation structure)**.** *The model has no interior saddle-node bifurcation. At $\kappa = \underline{\kappa}$ and $\kappa = \overline{\kappa}$ the unstable interior equilibrium x_s meets a boundary equilibrium ($x = 0$ and $x = 1$ respectively) and exchanges stability with it in a transcritical bifurcation. If $\underline{\kappa} < 0$, the bifurcation at $\underline{\kappa}$ occurs at a non-admissible cost, and only the bifurcation at $\overline{\kappa}$ lies in the admissible range.*

Remark 2 (Local normal forms)**.** The boundary equilibria exist for all κ because of the factor $x(1 - x)$, so the transcritical claim concerns the continuation of the interior branch $x_s(\kappa)$ beyond $[0, 1]$. Near $x = 0$, expanding h gives

$$\dot{x} = \gamma F(0;\kappa)\, x + \gamma\big[F_x(0;\kappa) - F(0;\kappa)\big]x^2 + O(x^3). \tag{7}$$

With $\mu(\kappa) \coloneqq \gamma F(0;\kappa)$, which vanishes at $\underline{\kappa}$ with $\mu'(\underline{\kappa}) = -\gamma L(0) < 0$, and with quadratic coefficient $a = \gamma F_x(0;\underline{\kappa}) > 0$ at $\kappa = \underline{\kappa}$, this is the transcritical normal form $\dot{x} = \mu x + ax^2$ [20]. Its two branches are $x = 0$ and $x = -\mu/a$, the latter being the local continuation of $x_s(\kappa)$. Near $x = 1$, with $y = 1 - x$,

$$\dot{y} = -\gamma F(1;\kappa)\, y + \gamma\big[F_x(1;\kappa) + F(1;\kappa)\big]y^2 + O(y^3), \tag{8}$$

with linear coefficient vanishing at $\overline{\kappa}$ with slope $\gamma L(1) > 0$ and quadratic coefficient $\gamma F_x(1;\overline{\kappa}) > 0$, which is again transcritical. Only the branch lying in $[0, 1]$ is economically admissible.

Remark 3 (The midpoint $\kappa^\dagger$)**.** Let $\kappa^\dagger \coloneqq r_L - 2\lambda\bar{D}/\bar{L}$. Then $F(x_0;\kappa) = 0$ iff $\kappa = \kappa^\dagger$, and $x_s(\kappa) < x_0 \iff \kappa < \kappa^\dagger$. This marks where the separatrix crosses the inflection point; it is a convenient reference value but not a bifurcation: the qualitative dynamics are identical on either side.

Remark 4 (What would produce a saddle-node)**.** A saddle-node needs $F = \partial_x F = 0$, which is impossible when $L' > 0$ and $r_L > \kappa$. Genuine folds require non-monotone F, for example if the risk cost or float cost fell with x ($\kappa = \kappa(x)$, $\lambda = \lambda(x)$) fast enough to offset the volume effect. This extension is left to future work.

3.3 Comparative Statics and Required Reductions

Proposition 2 (Comparative statics)**.** *For fixed other primitives and $\kappa \in (\underline{\kappa}, \overline{\kappa})$, the separatrix x_s is strictly increasing in κ, λ, $\bar{D}$, x_0; strictly decreasing in r_L and $\bar{L}$; and*

$$\frac{\partial x_s}{\partial b} = -\frac{x_s - x_0}{b}. \tag{9}$$

Raising b moves the separatrix *toward* x_0, not toward zero. A sharper network effect helps only if the separatrix currently lies above x_0.

Corollary 1 (Required cost reduction and the separatrix floor)**.** *Let the current cost satisfy $\underline{\kappa} < \kappa < r_L$. For a target $x_T \in (0,1)$, the separatrix satisfies $x_s(\kappa') \le x_T$ iff $F(x_T;\kappa') \ge 0$ iff $\kappa' \le g(x_T)$. The target is attainable with a non-negative cost only if $g(x_T) \ge 0$, that is, $x_T \ge g^{-1}(0)$, and the required proportional reduction is then*

$$\rho(x_T;\kappa) = 1 - \frac{g(x_T)}{\kappa}. \tag{10}$$

The low-lending basin is eliminated, in the sense that every trajectory with $x(0) > 0$ converges to $x = 1$, iff $\kappa' \le \underline{\kappa}$; a strictly positive invasion payoff $F(0;\kappa') > 0$ requires $\kappa' < \underline{\kappa}$. The reduction at which the basin vanishes is $\rho_{\text{lo}}(\kappa) = 1 - \underline{\kappa}/\kappa$, which is increasing in κ, and any strictly larger reduction gives a strictly positive invasion payoff. If $\underline{\kappa} < 0$, no non-negative cost eliminates the basin and cost reduction alone cannot remove the trap: the lowest separatrix reachable is the floor $x_s(0) = g^{-1}(0) > 0$. If $\underline{\kappa} = 0$, only $\kappa' = 0$ (a 100% reduction) does.

4 Calibration

4.1 Data and Timing

Table 1 lists the aggregates. The baseline uses December 2024, the date to which the zero-loan share reported by the International Growth Centre refers, so that all quantities refer to one period; September 2025 is used as a check. Outlets, deposits and loans are from the Bangladesh Bank agent-banking summary. Loans are reported as *outstanding* balances, which is the stock concept the model uses for $L(x)$.

Item	Dec 2024 (baseline)	Sep 2025 (check)
Outlets N	21,248	20,488
Total deposit balance D_{tot} (mn BDT)	417,858.2	471,963.9
Total loan outstanding L_{tot} (mn BDT)	101,118.0	111,283.7
Loan accounts	225,158	231,171
Lending rate r_L	12.16% (all-bank avg., Sep 2025)	
Share of outlets with loans x_o	$\approx 1/3$ (IGC, Dec 2024)	

Table 1: Aggregate calibration data.

The lending rate is the all-bank weighted average rate on advances for September 2025 [3]. The corresponding figure for the fourth quarter of 2024 was 12.05% [3]; the September 2025 value is used for both periods, which shifts $\hat{\kappa}$ by about 0.001 and does not change any conclusion. It is an average over all bank loans, not a rate specific to agent-channel loans, which may differ by product and borrower. The share x_o concerns outlets with no *outstanding* loans, which differs from outlets that have never facilitated a loan; the IGC brief reports "about two-thirds", so $x_o = 1/3$ is an approximation, and Section 6 varies it.

4.2 Identification

Aggregate-data calibration. Average deposits are $\bar{D} = D_{\text{tot}}/N = 19.67$ million BDT. The model-consistent average loan at the observed share is

$$L_{\text{obs}} = L(x_o) = \frac{L_{\text{tot}}}{N\, x_o} = 14.28 \text{ million BDT.} \tag{11}$$

Here, L_{obs} is an aggregate-implied average loan per outlet with positive outstanding loans, rather than a directly observed outlet-level average. Because L_{obs} is the loan volume at x_o, not the *maximum*, the saturation volume is $\bar{L} = L_{\text{obs}}/s(x_o) = 26.36$ million BDT, where $s(x) = 1/[1 + \exp(-b(x - x_0))]$ is the logistic component of $L(x)$. This calibration ensures that the model reproduces aggregate outstanding loans, $Nx_oL(x_o) = L_{\text{tot}}$.

Assumed model parameters. The cost κ, curvature b, inflection point x_0 and float cost λ are not directly observed. The baseline curvature and inflection point are assumed ($b = 5$, $x_0 = 0.30$) and varied in Section 6.

The float cost. The parameter λ is the annual opportunity cost, per unit of deposits, of the *additional* liquidity that an outlet engaged in credit intermediation must hold relative to an otherwise identical outlet that does not lend, for example cash and balances held to meet loan disbursements, repayment timing and withdrawals once part of the deposit base is deployed in loans. It enters $F = \pi_L - \pi_N$ as a cost of lending only and is therefore a *differential* quantity. The liquidity-cost parameter is decomposed as $\lambda = \phi_L s$, where ϕ_L denotes the lending-specific additional share of deposits held in liquid form and s is the annual return foregone per unit of such liquidity relative to lending. If liquid holdings earn a non-negative return, then $s \leq r_L$. Neither parameter is directly observed.

Regulatory reserve requirements give a reference scale, not an identification. In December 2024 banks were required to hold a cash reserve of 3.5% of deposits (reduced to 3.0% from March 2025 [4]) and a statutory liquidity ratio of 13% for conventional banks [5], a combined liquid share of about 16.5%. These requirements apply to all deposits whether or not an outlet lends, so they are not themselves the differential cost of lending, and they bind banks rather than agent outlets. They are used here only as an indication of the order of magnitude that φ_L might take.

The paper therefore treats λ as a scenario parameter and reports two cases. In Case A, $\lambda = 0.02$, which at the maximum spread $s = r_L$ corresponds to an additional liquid share of about 16% of deposits (larger if the spread is smaller). In Case B, $\lambda = 0.05$, which corresponds to about 41% at the maximum spread and is treated as a high-liquidity stress case. Neither value is an estimate. What the analysis requires is the critical value $\lambda^* = r_L L(0)/\bar{D} \approx 0.030$ at the baseline curvature and inflection point, below which $\underline{\kappa} > 0$ and cost reduction alone can remove the trap. Because $s \leq r_L$, a sufficient condition for $\lambda < \lambda^*$ is $\varphi_L < \lambda^*/r_L \approx 24\%$: if the lending-specific liquid share is below about a quarter of deposits, the trap is removable by cost reduction alone whatever the spread. The tables and figures that follow use Case B ($\lambda = 0.05$) as the computational baseline unless stated otherwise.

A maintained low-basin assumption. The calibration maintains that the observed share lies in the low-lending basin, $x_o \le x_s(\kappa)$. In the model every share in this basin declines toward 0 ($\dot{x} < 0$ for $0 < x < x_s$), so this is *not* the statement that the share is stationary: it says that, absent a shock, the network would drift toward the low-lending state, and that the observed share is at or below the critical mass. A roughly constant observed share is reconciled with the model only through slow adjustment (a small γ), offsetting inflows such as new outlets or policy programmes, or a share close to the separatrix, where the equality case $x_o = x_s$ is an unstable stationary point. This assumption is not tested by the data used here. By Theorem 1 it is equivalent to $\kappa \ge g(x_o)$. The resulting benchmark is

$$\hat{\kappa} \coloneqq g(x_o) = r_L - \frac{\lambda \bar{D}}{L_{\text{obs}}} = r_L - \frac{\lambda\, x_o\, D_{\text{tot}}}{L_{\text{tot}}} = 0.0527 \quad (\lambda = 0.05). \tag{12}$$

Under the maintained assumption, $\hat{\kappa}$ is a *conditional lower bound* on the implied effective cost, attained if the observed share sits exactly on the separatrix; it is not an independently estimated structural parameter. It does not depend on b or x_0. If λ is so large that $\hat{\kappa} < 0$ (here $\lambda > 0.088$), the assumption is infeasible for non-negative costs: the observed share would exceed the separatrix even at zero cost.

Interpretation of the bistability condition. Because g is increasing and $x_o \in (0, 1)$, $\hat{\kappa} = g(x_o) \in (\underline{\kappa}, \overline{\kappa})$ holds by construction. The inequality $\underline{\kappa} < \hat{\kappa} < \overline{\kappa}$ is therefore a mathematical consequence of the model and calibration, not independent empirical evidence that the network is bistable. The information in the calibration lies in the *location* of the thresholds, in particular the sign of $\underline{\kappa}$ and the distance from $\hat{\kappa}$ to $\overline{\kappa}$.

Parameter	Value	Source / method
$\bar{D}$	19.67 mn BDT	D_{tot}/N
L_{obs}	14.28 mn BDT	$L_{\text{tot}}/(N x_o)$; aggregate-implied
$\bar{L}$	26.36 mn BDT	$L_{\text{obs}}/s(x_o)$; model-implied
r_L	0.1216	BB weighted average lending rate, Sep 2025
x_o	1/3	IGC, Dec 2024
$x_0,\ b$	0.30, 5.0	assumed; varied in sensitivity analysis
λ	0.05 (Case B)	assumed; Case A: 0.02
γ	1	normalization
$\hat{\kappa}$	0.0527	Eq. (12); conditional lower bound
$\underline{\kappa}$	-0.0829	$g(0)$; $L(0) = 4.81$
$\overline{\kappa}$	0.0832	$g(1)$; $L(1) = 25.59$
$\kappa^\dagger$	0.0470	$r_L - 2\lambda\bar{D}/\bar{L}$
$x_s(0)$	0.137	separatrix floor at zero cost
λ^*	0.030	$r_L L(0)/\bar{D}$; $\underline{\kappa} > 0$ iff $\lambda < \lambda^*$

Table 2: Calibrated parameters and implied thresholds using December 2024 aggregate data and the September 2025 lending-rate proxy (Case B).

	Case A: $\lambda = 0.02$	Case B: $\lambda = 0.05$
$\hat{\kappa}$	0.0941	0.0527
$\underline{\kappa}$	0.0398	-0.0829
$\overline{\kappa}$	0.1062	0.0832
Trap removable by cost reduction alone	yes	no
ρ_{lo} from $\hat{\kappa}$ / from $\overline{\kappa}$	57.7% / 62.5%	n/a
Separatrix floor $x_s(0)$	0	0.137
$\rho(x_T = 0.25)$	6.9%	31.0%
$\rho(x_T = 0.20)$	12.7%	56.8%
$\rho(x_T = 0.10)$	29.7%	n/a

Table 3: Thresholds and required cost reductions under the two float-cost scenarios using December 2024 aggregate data and the September 2025 lending-rate proxy ($b = 5$, $x_0 = 0.30$, $x_o = 1/3$). The critical float cost is $\lambda^* = 0.030$ in both cases.

4.3 Low-basin assumption and its implications

Table 4 reports four quarters of Bangladesh Bank aggregates. The series does not contain the share of outlets with positive outstanding loans, so it cannot test the low-basin assumption directly. A share that is falling would support the assumption, a share that is rising would contradict it, and a flat share is consistent with it only under slow adjustment or a share close to the separatrix. The aggregates show that credit depth, measured by the loan-to-deposit ratio, was essentially flat (between 23.4% and 24.6%) while deposits grew by about 13% and the number of outlets fell by about 3.6%. Loans per outlet rose about 14%, which could reflect either a higher share of lending outlets or larger loans at existing lenders, and the series cannot distinguish the two. The data are therefore uninformative about the direction of x, and a time series of the share of outlets with outstanding loans is needed.

	Dec 2024	Mar 2025	Jun 2025	Sep 2025
Outlets	21,248	21,023	20,557	20,488
Deposits (mn BDT)	417,858.2	426,329.5	452,965.8	471,963.9
Loans outstanding (mn BDT)	101,118.0	104,671.5	106,111.3	111,283.7
Loan accounts	225,158	230,037	229,393	231,171
Loans / deposits	24.2%	24.6%	23.4%	23.6%
Loans per outlet (mn BDT)	4.76	4.98	5.16	5.43
Loan accounts per outlet	10.6	10.9	11.2	11.3

Table 4: Agent banking aggregates by quarter (Bangladesh Bank) and derived ratios.

4.4 Results

In Case B, $\hat{\kappa} = 0.053$ and $\overline{\kappa} = 0.083$, while $\underline{\kappa} = -0.083$ is negative. Hence the effective bistable window over admissible costs is $[0, 0.083)$, and $x = 0$ is asymptotically stable at every admissible cost $\kappa \geq 0$ at which the system is not already in the high-lending regime.

Under the maintained assumption, the true κ lies in $[\hat{\kappa}, \overline{\kappa})$, or above $\overline{\kappa}$ (the trap regime). The observed share $x_o = 1/3$ coincides with the separatrix at $\kappa = \hat{\kappa}$ by construction. Even at $\kappa = 0$ the separatrix is $x_s(0) = 0.137$, so cost reduction alone cannot remove the low-lending basin. The same qualitative picture holds with the September 2025 data ($\hat{\kappa} = 0.051$, $\underline{\kappa} = -0.088$, $\overline{\kappa} = 0.082$). In Case A ($\lambda = 0.02$), by contrast, $\underline{\kappa} = 0.040 > 0$ and the bistable window is $(0.040, 0.106)$, so the trap can be removed by cost reduction alone (Table 3). Figure 1 shows the separatrix as a function of κ in Case B and Figure 2 shows the phase line at $\hat{\kappa}$.

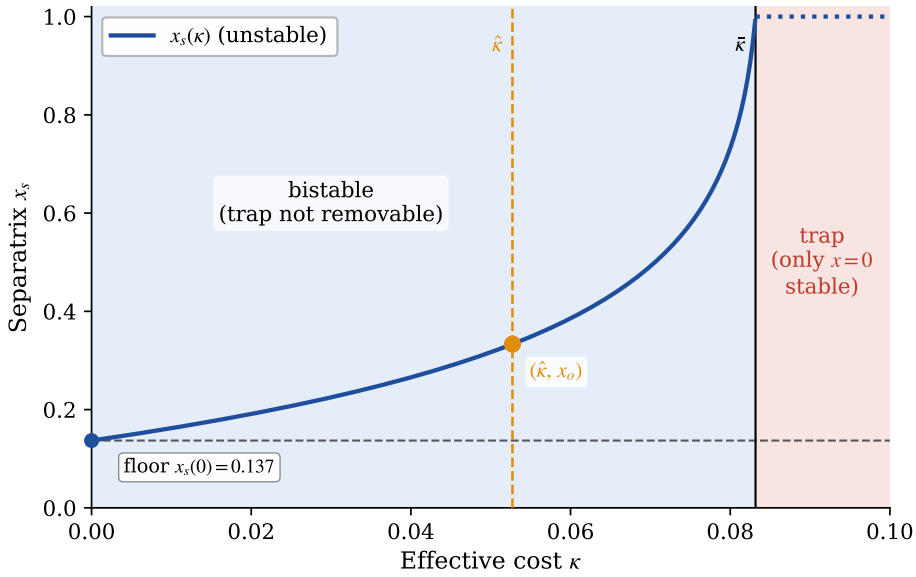


Figure 1: Separatrix x_s as a function of the effective cost κ (Case B, $\lambda = 0.05$). For $\kappa < \bar{\kappa}$ the system is bistable and the low-lending trap cannot be removed. The dashed orange line marks $\hat{\kappa}$ and the dotted branch indicates $x_s = 1$.

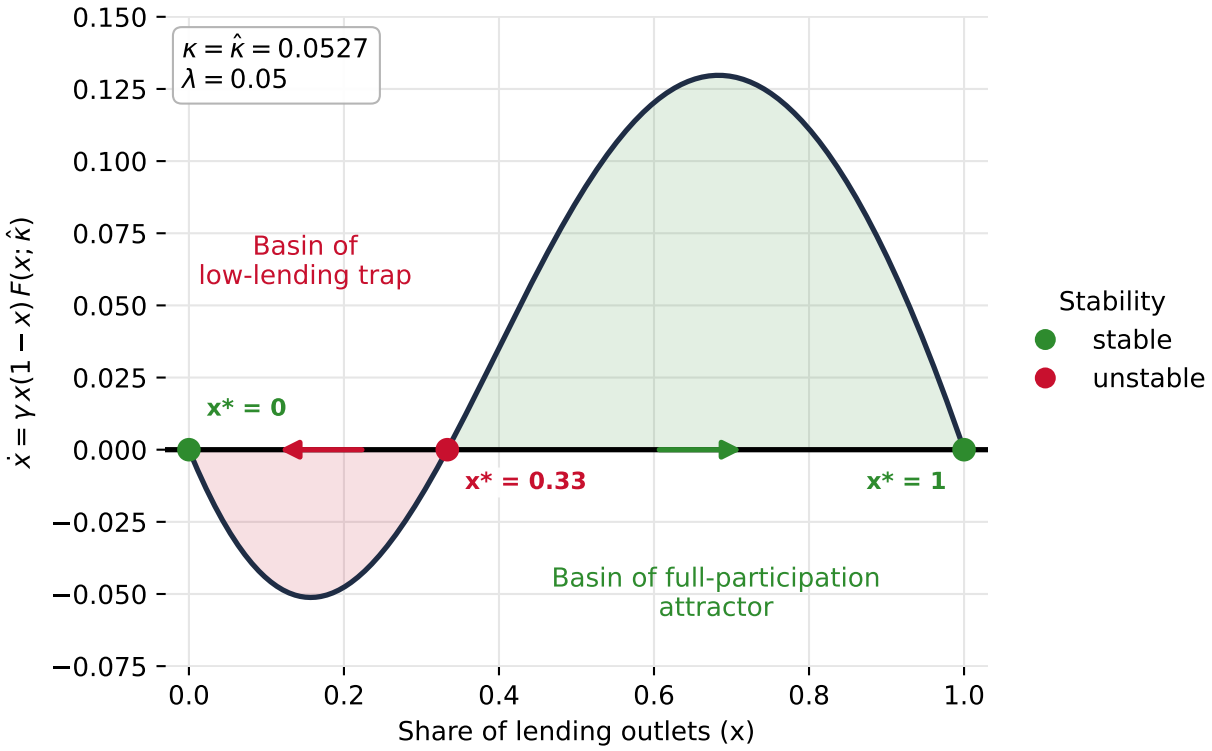


Figure 2: Phase line portrait of the bistable lending model. Two stable equilibria ($x^* = 0$ and $x^* = 1$) are separated by an unstable tipping threshold x_s.

5 Policy Counterfactuals

A risk-sharing programme that lowers the effective cost proportionally is considered, $\kappa' = (1-\rho)\kappa$. The results below are conditional model-based counterfactuals; they are not estimates of the subsidy or guarantee actually required in Bangladesh. Because κ is not identified, Table 5 reports two scenarios for the high-liquidity case ($\lambda = 0.05$, Case B): the lower bound $\kappa = \hat{\kappa}$ and the upper edge of the window $\kappa = \overline{\kappa}$.

Reduction ρ	κ' (from $\hat{\kappa}$)	x_s if $\kappa = \hat{\kappa}$	x_s if $\kappa = \overline{\kappa}$	Trap
0%	0.0527	0.333	≈ 1	present
10%	0.0475	0.302	0.575	present
20%	0.0422	0.276	0.448	present
30%	0.0369	0.252	0.372	present
40%	0.0316	0.231	0.316	present
50%	0.0264	0.212	0.273	present
100% ($\kappa' = 0$)	0	0.137	0.137	present

Table 5: Separatrix after a proportional reduction in κ (Case B). Since $\underline{\kappa} < 0$, no reduction eliminates the trap, and the separatrix cannot fall below $x_s(0) = 0.137$.

Three points follow. First, any reduction lowers the critical mass, but the trap remains because $\underline{\kappa} < 0$; what changes is the size of the low-lending basin, and there is a floor $x_s(0) = 0.137$ below which cost reduction alone cannot go. Second, the benefit depends on how far the true κ is from $\hat{\kappa}$: a 20% reduction leaves a margin $x_o - x_s$ of 0.06 if $\kappa = \hat{\kappa}$ but a separatrix of 0.45 if $\kappa = \overline{\kappa}$. Third, eliminating the trap needs a lower float burden λ or a more favorable loan-volume function (lower x_0 or b), as Section 6 shows; for $\lambda < \lambda^* \approx 0.030$ cost reduction alone can eliminate it. Table 6 gives the reduction needed to reach a target separatrix.

Target x_T	$g(x_T)$	ρ if $\kappa = \hat{\kappa}$	ρ if $\kappa = \overline{\kappa}$
0.25	0.0364	31.0%	56.2%
0.20	0.0228	56.8%	72.6%
0.15	0.0053	89.9%	93.6%
0.10	−0.0171	not attainable	not attainable

Table 6: Required proportional reduction in κ to place the separatrix at or below x_T (Case B; Corollary 1). Targets below $x_s(0) = 0.137$ cannot be reached by cost reduction.

Case A. If the float cost is at the low-liquidity scenario ($\lambda = 0.02$), $\underline{\kappa} = 0.040 > 0$ and the trap can be removed. From $\kappa = \hat{\kappa} = 0.094$ a reduction of 57.7% (62.5% from $\overline{\kappa} = 0.106$) eliminates the low-lending basin, while reductions of 6.9%, 12.7% and 29.7% bring the separatrix to 0.25, 0.20 and 0.10 respectively (Table 3).

6 Sensitivity Analysis

Since b, x_0 and λ are assumed, the saturation volume $\bar{L}$ is recalibrated to maintain $L(x_o) = L_{\text{obs}}$. Table 7 reports the one-at-a-time sensitivity of $\hat{\kappa}$, the boundary thresholds, and the required cost reductions. Here, $\rho_{\text{lo}} = 1 - \underline{\kappa}/\hat{\kappa}$ denotes the reduction needed to eliminate the trap, while $\rho_{0.10}$ denotes the reduction required to bring the separatrix to 0.10. "n/a" indicates that the corresponding target is unattainable with a non-negative cost.

Parameter	Value	$\hat{\kappa}$	$\underline{\kappa}$	$\overline{\kappa}$	ρ_{lo}	$\rho_{0.10}$
Baseline (Dec 2024, Case B)	–	0.0527	−0.0829	0.0832	n/a	n/a
Sep 2025 data	–	0.0509	−0.0882	0.0822	n/a	n/a
λ	0.01	0.1078	0.0807	0.1139	25.2%	13.0%
	0.02 (Case A)	0.0941	0.0398	0.1062	57.7%	29.7%
	0.03	0.0803	−0.0011	0.0985	n/a	52.2%
	0.04	0.0665	−0.0420	0.0909	n/a	84.0%
	0.07	0.0252	−0.1647	0.0678	n/a	n/a
b	2	0.0527	0.0212	0.0772	59.8%	37.5%
	3	0.0527	−0.0035	0.0810	n/a	62.9%
	7	0.0527	−0.2307	0.0829	n/a	n/a
	10	0.0527	−0.7244	0.0814	n/a	n/a
x_0	0.10	0.0527	−0.0175	0.0685	n/a	68.6%
	0.20	0.0527	−0.0476	0.0753	n/a	98.0%
	0.40	0.0527	−0.1196	0.0914	n/a	n/a
	0.50	0.0527	−0.1534	0.0990	n/a	n/a
x_o	0.25	0.0699	−0.0024	0.0983	n/a	46.4%
	0.30	0.0596	−0.0483	0.0897	n/a	89.3%
	0.40	0.0390	−0.1604	0.0686	n/a	n/a

Table 7: One-at-a-time sensitivity with $\bar{L}$ recalibrated. Unlisted parameters are at the December 2024 baseline (Case B).

Because parameter effects may interact, Figure 3 examines the joint sensitivity of the required cost reductions to λ, b, and x_0.

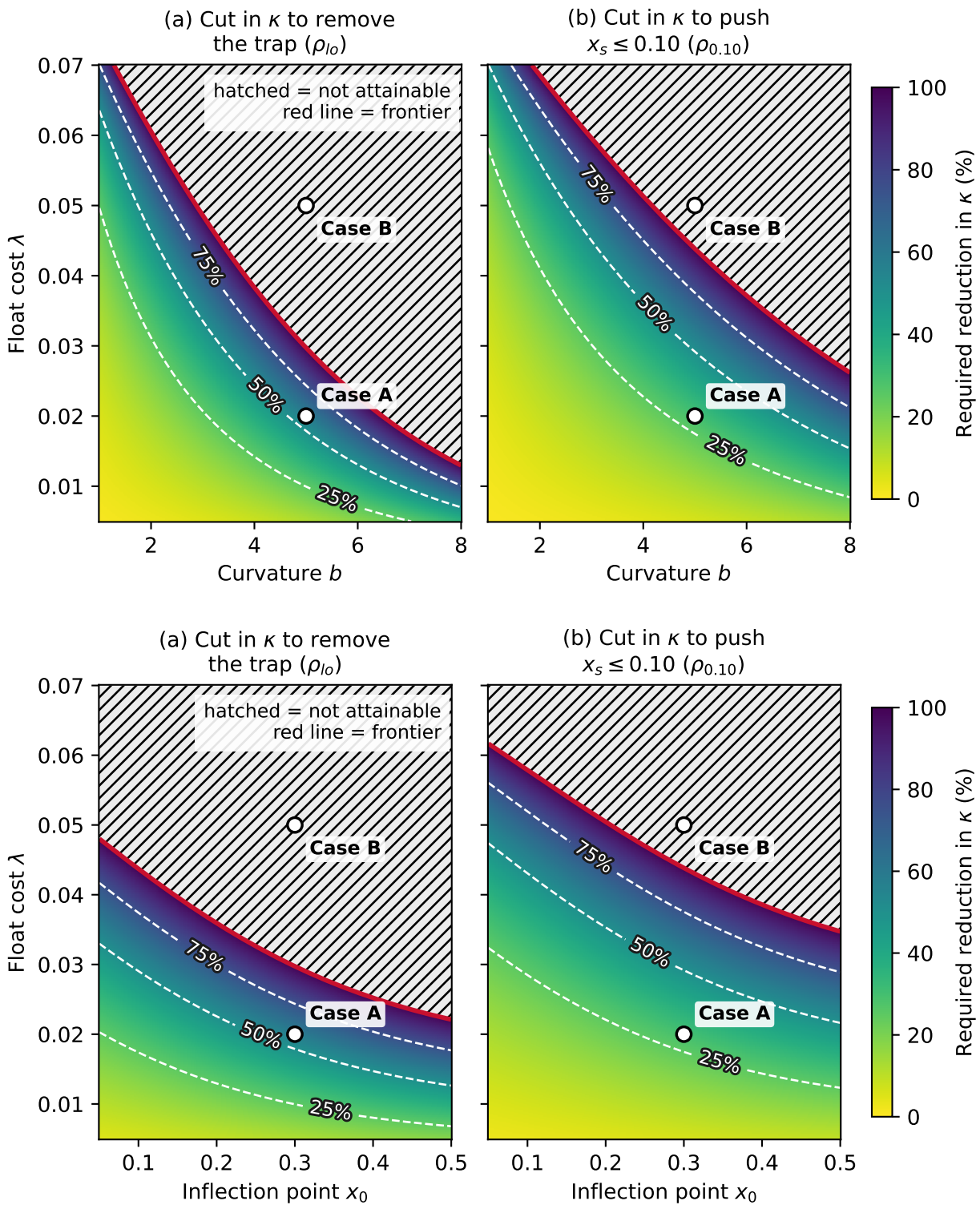


Figure 3: Required proportional reduction in κ to remove the low-lending trap (left panel of each figure) and to push the tipping threshold to $x_s \le 0.10$ (right panel). Top: heatmaps over curvature b and float cost λ. Bottom: heatmaps over inflection point x_0. Hatched regions are not attainable; the red line marks the frontier.

A full grid of 300 parameter combinations, varying λ, b, x_0, and x_o, is summarized in Table 8. The equilibrium ordering $\underline{\kappa} < \hat{\kappa} < \overline{\kappa}$ holds throughout, as implied by the monotonicity of g; it is therefore a structural result rather than independent empirical evidence of bistability. The feasibility of eliminating the trap nevertheless varies across the grid: it holds in 54% of combinations, with required reductions ranging from 3% to 95%. The less demanding target $x_s \leq 0.10$ is attainable in 71% of combinations, with a median required reduction of 22%. Thus, reducing the size of the low-lending basin is feasible over a wider range of parameter configurations than eliminating it entirely. These results are conditional on the assumed parameter values and do not represent statistical uncertainty or empirical estimates of intervention requirements.

Statistic	Value
Parameter combinations	300
Share with $\underline{\kappa} > 0$ (trap removable by cost reduction)	54%
Median ρ_{lo} (among $\underline{\kappa} > 0$)	29%
Minimum / maximum ρ_{lo} (among $\underline{\kappa} > 0$)	3% / 95%
Share with $g(0.10) > 0$ (target 0.10 attainable)	71%
Median $\rho_{0.10}$ (among attainable)	22%
Minimum / maximum $\rho_{0.10}$ (among attainable)	1% / 99%

Table 8: Robustness summary over the 300-combination parameter grid (December 2024 data).

7 Discussion and Limitations

7.1 Interpreting the Low-Lending Trap

The model shows how low lending participation can persist even when lending is individually profitable for some outlets. The key mechanism is the dependence of loan volume on the aggregate lending share. When relatively few outlets lend, the associated loan volume and lending returns remain limited, weakening the incentive for other outlets to enter credit intermediation. Once the lending share exceeds the unstable interior equilibrium, the direction of adjustment reverses and the system moves towards the high-participation state. The interior equilibrium therefore represents a coordination threshold rather than a stable intermediate outcome. This interpretation is consistent with models of network externalities in which the benefits of adoption depend on the number of other adopters [25].

An important implication is that the persistence of low participation does not necessarily indicate that lending is unprofitable at every outlet. Rather, it may reflect a self-reinforcing configuration in which limited participation constrains the returns available to participating outlets. The model consequently distinguishes between the profitability of lending at a given network state and the ability of the network to move towards a higher-participation state. However, this mechanism remains conditional on the assumed positive relationship between loan volume and the aggregate lending share; the available Bangladesh data do not establish that relationship.

Economic interpretation of the low-lending trap. The persistence of the low-lending state reflects a coordination problem rather than the absence of lending opportunities. When the share of lending outlets is small, the model implies a relatively low network-wide loan volume. This may limit the benefits that individual outlets derive from lending, including opportunities for information sharing, borrower screening, institutional learning, and the development of local lending relationships. Consequently, the incentive to initiate or continue lending remains weak, reinforcing the initially low level of participation.

The threshold x_s captures the participation level required to overcome this coordination problem. Below it, the negative feedback associated with insufficient network activity sustains the low-lending state; above it, the positive relationship between network participation and loan volume supports movement towards the high-lending state. The parameter x_0, by contrast, determines the inflection point of the assumed loan-volume function and should not be interpreted as an equilibrium or an empirically identified critical participation level.

7.2 Policy Implications: Cost Reduction and Network Development

The results distinguish between interventions that reduce the cost of lending and those that alter the conditions supporting network-wide participation. Risk-sharing facilities and credit guarantees can be represented as reductions in the risk cost κ. In Case B, however, the lower threshold $\underline{\kappa}$ is negative. Consequently, even eliminating the risk cost entirely cannot remove the low-lending basin, and the separatrix remains at approximately 0.137. Risk-cost reduction alone therefore has a limit under the high-liquidity parameterization.

Interventions that reduce the liquidity burden operate through λ, while measures that strengthen the information and trust environment may alter the loan-volume function, represented by b and x_0. For example, improved borrower information, more effective screening, and digital credit infrastructure could potentially strengthen the conditions under which outlets participate in lending. These effects are not automatic: their representation as changes in the model parameters would require empirical validation. Nevertheless, the sensitivity analysis indicates that changes in λ and the shape of the loan-volume function can shift the coordination threshold and alter the cost reduction required to eliminate the low-lending basin.

The distinction between eliminating the trap and reducing its severity is also important. Even when cost reductions cannot eliminate the low-participation equilibrium, they may still lower the critical participation share required to move towards the high-lending state. This suggests that policy effectiveness should not be assessed solely by whether an intervention eliminates the low-lending equilibrium. Reducing the size of its basin of attraction may also represent meaningful progress. More broadly, the results highlight the importance of considering network structure and interaction effects when evaluating interventions in financial intermediation systems [26].

7.3 Model Scope and Limitations

The findings should be interpreted in light of several modelling and identification limitations.

First, the model treats lending status as the outcome of a bank–outlet decision and represents outlets as identical agents responding to the network-wide lending share. It does not explicitly model differences in bank–agent contracts, local borrower characteristics, or the distribution of lending activity across individual banks. The mean-field specification also abstracts from local interactions and spatial clustering, which may be important in a heterogeneous network [26].

Second, the strategic complementarity condition $L'(x) > 0$ is imposed rather than estimated. The analysis establishes the implications of this mechanism but does not demonstrate that it operates in Bangladesh's agent banking network. Outlet-level panel data would be required to examine whether changes in network participation are associated with changes in loan volume and lending profitability.

Third, the quantitative results depend on parameters that are not independently identified from the available aggregate data. In particular, κ is an implied residual, while b, x_0, and λ are assumed or constructed for calibration. The interpretation of $\hat{\kappa}$ also depends on the maintained low-basin assumption. Consequently, the calibrated thresholds and policy magnitudes should be viewed as conditional model implications rather than direct empirical estimates of the actual costs or participation dynamics of agent banking outlets.

Finally, the model assumes that κ and λ are independent of the lending share and does not identify the adjustment speed γ. Allowing costs to vary with network participation, introducing outlet heterogeneity, and incorporating local interactions could produce richer dynamics, including equilibrium structures not possible under the present monotonicity assumptions. These extensions would require more disaggregated data and are left for future research.

8 Conclusion

This paper investigates the emergence of low-lending traps in agent banking networks through a replicator-type mean-field model incorporating strategic complementarity in loan volume. Since the profit differential is strictly increasing in the lending share, the system has a unique interior equilibrium that is unstable. This equilibrium acts as a coordination threshold, separating the zero-lending state from the full-participation state. The boundaries of the bistable region, $\underline{\kappa}$ and $\overline{\kappa}$, correspond to transcritical bifurcations; an interior saddle-node bifurcation cannot occur under the model's monotonicity condition.

Conditional on the maintained low-basin assumption and the assumed curvature and inflection parameters, calibration to December 2024 Bangladesh Bank aggregates places the upper boundary of the bistable region at approximately 0.083 in the high-liquidity case and 0.106 in the regulatory-liquidity case. The analysis also identifies a critical float-cost threshold of approximately 3% of deposits, below which risk-cost reduction alone can eliminate the low-lending trap under the baseline parameterization. In the high-liquidity case, even eliminating the risk cost entirely leaves a separatrix of approximately 0.14. By contrast, in the regulatory-liquidity case, eliminating the trap requires a risk-cost reduction of roughly 58–63%. These results suggest that cost-side interventions may need to be complemented by measures that reduce the liquidity burden or lower the trust threshold.

Because the quantitative magnitudes depend on unobserved and assumed parameters,

the primary contribution of the model is a structural mapping between policy targets and the proportional cost reductions required to achieve them. The analysis demonstrates that a low-lending trap is a mathematically coherent outcome of strategic complementarity in this market structure; it does not establish that Bangladesh's agent banking network is currently trapped. Future research should estimate the loan-volume function and underlying cost structure using outlet-level and time-series data to test the proposed mechanism and refine the model's policy implications.

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A Proofs

Lemma 2 (Logistic loan volume)**.** $L'(x) = b\,L(x)\big(1 - L(x)/\bar{L}\big) > 0$ *for all* x*;* L *is strictly increasing, with* $L(x) \in (0, \bar{L})$*.*

Proof. With $s = (1 + e^{-b(x-x_0)})^{-1}$, $s' = b\,s(1-s)$, so $L' = \bar{L}s' = b\,L(1 - L/\bar{L})$, positive since $0 < s < 1$. □

Proof of Lemma 1. $F' = (r_L - \kappa)L' > 0$ when $\kappa < r_L$. Since $L > 0$, $F > 0 \iff (r_L - \kappa)L > \lambda\bar{D} \iff \kappa < r_L - \lambda\bar{D}/L(x) = g(x)$; this also covers $\kappa \geq r_L$, where $F < 0$ and $\kappa < g(x) < r_L$ fails. Finally $g' = \lambda\bar{D}L'/L^2 > 0$ and $g < r_L$. □

Proof of Theorem 1. For $x \in (0,1)$, $\operatorname{sign} h = \operatorname{sign} F$. Also $h'(0) = \gamma F(0;\kappa)$, $h'(1) = -\gamma F(1;\kappa)$, and at an interior zero $h' = \gamma x(1-x)F_x$. By Lemma 1, $F(0;\kappa) < 0 \iff \kappa > \underline{\kappa}$ and $F(1;\kappa) > 0 \iff \kappa < \overline{\kappa}$. If $\kappa \geq r_L$, then $F < 0$ and case (i) holds, so assume $\kappa < r_L$.

(i) $\kappa > \overline{\kappa}$ implies $F(1) < 0$; since F is increasing, $F < 0$ on $[0,1]$. Then $h'(0) = \gamma F(0) < 0$, $h'(1) = -\gamma F(1) > 0$, and $\dot{x} < 0$ on $(0,1)$, so every trajectory with $x(0) < 1$ decreases to 0.

(ii) $\underline{\kappa} < \kappa < \overline{\kappa}$ gives $F(0) < 0 < F(1)$. By continuity and strict monotonicity there is exactly one zero $x_s \in (0,1)$. It solves $L(x_s) = c(\kappa)$, i.e. $1 + e^{-b(x_s - x_0)} = \bar{L}/c$; note $c < L(1) < \bar{L}$. Hence $x_s = x_0 + b^{-1}\ln\big(c/(\bar{L} - c)\big)$; equivalently $g(x_s) = \kappa$, so $x_s = g^{-1}(\kappa)$, which is continuous and strictly increasing since g is, with $g^{-1}(\underline{\kappa}) = 0$ and $g^{-1}(\overline{\kappa}) = 1$. Stability: $h'(0) < 0$, $h'(1) < 0$, $h'(x_s) = \gamma x_s(1-x_s)F_x(x_s) > 0$. Since $\dot{x} < 0$ on $(0, x_s)$ and $\dot{x} > 0$ on $(x_s, 1)$, every trajectory is monotone and bounded, and converges to 0 if $x(0) < x_s$ and to 1 if $x(0) > x_s$.

(iii) $\kappa \leq \underline{\kappa}$ gives $F(0) \geq 0$ and, since F is strictly increasing, $F > 0$ on $(0,1]$. Then $\dot{x} > 0$ on $(0,1)$, so every trajectory with $x(0) > 0$ increases to 1, and $h'(1) = -\gamma F(1) < 0$. If $\kappa < \underline{\kappa}$ then $F(0) > 0$ and $h'(0) > 0$; if $\kappa = \underline{\kappa}$ then $F(0) = 0$ and $h'(0) = 0$, with $x = 0$ repelling relative to $[0,1]$. On the extended line, $F < 0$ for $x < 0$ gives $\dot{x} = \gamma x(1-x)F > 0$ there at $\kappa = \underline{\kappa}$, so trajectories approach 0 from the left and leave it to the right: $x = 0$ is semi-stable.

At $\kappa = \overline{\kappa}$: $F(1) = 0$ and $F < 0$ on $[0,1)$, so $\dot{x} < 0$ on $(0,1)$ and $x = 1$ is repelling relative to $[0,1]$; on the extended line it is semi-stable. □

Proof of Proposition 1. A saddle-node at (x^*, κ^*) with $x^* \in (0,1)$ requires $h = h_x = 0$, i.e. $F = F_x = 0$. But $F_x = (r_L - \kappa)L' > 0$ by Lemma 2 for $\kappa < r_L$, a contradiction. For the transcritical claim, the formula (6) defines the equilibrium $x_s(\kappa)$ for all κ with $0 < c(\kappa) < \bar{L}$,

including values slightly below $\underline{\kappa}$ for which $x_s < 0$ lies outside $[0, 1]$. At $\underline{\kappa}$, $x_s = 0$ coincides with the boundary equilibrium. The local expansion in Remark 2 gives $\dot{x} = \mu x + ax^2 + O(x^3)$ with $\mu(\underline{\kappa}) = 0$, $\mu'(\underline{\kappa}) = -\gamma L(0) \neq 0$ (since $\partial_\kappa F(0; \kappa) = -L(0) < 0$) and $a = \gamma F_x(0; \underline{\kappa}) > 0$. This is the transcritical normal form: the two branches $x = 0$ and $x \approx -\mu/a$ cross at $\mu = 0$ and exchange stability, the eigenvalue at $x = 0$ changing from negative to positive as κ decreases through $\underline{\kappa}$, while the eigenvalue at the continued branch changes sign when it crosses 0. The argument at $\overline{\kappa}$ is symmetric, using $y = 1 - x$. If $\underline{\kappa} \leq 0$, the bifurcation value lies outside the admissible range $\kappa \geq 0$. □

Proof of Remark 3. $F(x_0; \kappa) = (r_L - \kappa)\bar{L}/2 - \lambda\bar{D} = 0 \iff \kappa = r_L - 2\lambda\bar{D}/\bar{L} = \kappa^\dagger$. Since $x_s = g^{-1}(\kappa)$ with g increasing and $g(x_0) = \kappa^\dagger$, $x_s < x_0 \iff \kappa < \kappa^\dagger$. □

Proof of Proposition 2. Write $x_s = x_0 + b^{-1}[\ln c - \ln(\bar{L} - c)]$. Then $\partial x_s/\partial c = \bar{L}/(b\, c(\bar{L} - c)) > 0$ and $\partial x_s/\partial \bar{L} = -1/(b(\bar{L} - c)) < 0$. Moreover $\partial c/\partial \kappa = c/(r_L - \kappa) > 0$, $\partial c/\partial \lambda = c/\lambda > 0$, $\partial c/\partial \bar{D} = c/\bar{D} > 0$, $\partial c/\partial r_L = -c/(r_L - \kappa) < 0$, and $\partial x_s/\partial x_0 = 1$. Finally $\partial x_s/\partial b = -b^{-2} \ln \frac{c}{\bar{L} - c} = -(x_s - x_0)/b$. □

Proof of Corollary 1. By Lemma 1, $F(x_T; \kappa') \geq 0 \iff \kappa' \leq g(x_T)$, and by monotonicity of F in x this is equivalent to $x_s(\kappa') \leq x_T$. A non-negative κ' with $\kappa' \leq g(x_T)$ exists only if $g(x_T) \geq 0$, equivalently $x_T \geq g^{-1}(0)$ since g is increasing. Writing $\kappa' = (1 - \rho)\kappa$ gives (10). By Lemma 1, $F(0; \kappa') \geq 0 \iff \kappa' \leq g(0) = \underline{\kappa}$ and $F(0; \kappa') > 0 \iff \kappa' < \underline{\kappa}$. By Theorem 1(iii) the basin of $x = 0$ is eliminated iff $\kappa' \leq \underline{\kappa}$, giving $\rho_{\text{lo}} = 1 - \underline{\kappa}/\kappa$, which is increasing in κ. If $\underline{\kappa} < 0$, no $\kappa' \geq 0$ satisfies $\kappa' \leq \underline{\kappa}$, and the smallest attainable separatrix is $x_s(0) = g^{-1}(0)$; if $\underline{\kappa} = 0$, only $\kappa' = 0$ does. □