Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

A Formal Model of Wealth–Power Feedback with Eight Classes: Thresholds of Elite Capture, Class Mobility and Policy Counter-Levers, with an Illustrative Calibration for Urban Punjab

Dr. Muhmmad Muqeem Islam Muqeem PhD

muqeemz@gmail.com

National school of public policy https://orcid.org/0009-0004-2005-4951

Research Article

Keywords: wealth–power feedback; elite capture; class structure; rent seeking; contest success functions; social mobility; political economy of inequality; Pakistan; Punjab

Posted Date: November 23rd, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11812202/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: The authors declare no competing interests.

RESEARCH ARTICLE

A Formal Model of Wealth–Power Feedback with Eight Classes: Thresholds of Elite Capture, Class Mobility and Policy Counter-Levers, with an Illustrative Calibration for Urban Punjab

Dr Muhammad Muqeem ul Islam Muqeem[1*]

[1]**PhD in Public Policy and Governance (NDU); MPhil in Economics, Certification in Economics for Public Policy, Blavatnik School of Government, University of Oxford; Certification in Economic Policy Design through Structural Adjustment Programs, IMF Institute, Washington, DC.** Former Assistant Professor of Economics, School of Economics, IIU Islamabad; Former Research Economist / Assistant Director, Monetary & Fiscal Research Department, State Bank of Pakistan (Central Bank); Former Director General, National Vocational and Technical Training Commission (NAVTTC); Former Director General, KIM; Head, Senior Management Wing, National Institute of Public Administration (NIPA), National School of Public Policy, Peshawar, Pakistan.

*** Corresponding author:** Dr Muhammad Muqeem ul Islam Muqeem | Email: muqeemz@gmail.com | Tel: +92 343 5090648

Cite this article

Muqeem, M. M. U. I. (2026). A formal model of wealth–power feedback with eight classes: Thresholds of elite capture, class mobility and policy counter-levers, with an illustrative calibration for urban Punjab. **[Preprint]** *Scholar Square Springer Nature*

JEL classification: D31, D72, D74, H23, O15, P16

Abstract

This paper develops a dynamic political-economy model in which wealth and political power reinforce one another across an ordered structure of eight economic classes. Each class converts population, wealth and organisational capacity into political power; power is converted into policy rents through a contest whose decisiveness is a property of institutions; and rents, together with power-dependent privileged returns, feed back into the distribution of wealth. Class mobility responds to the concentration of power, closing a second loop between who governs and who can rise. The model yields a single summary statistic, the Wealth–Power Feedback Index Λ, which combines the wealth elasticity of power (β), the decisiveness of rent contests (θ), the share of wealth reallocated through politics (ρ), the strength of market income (μ), the privileged-return premium (η) and the rate at which captured rents are clawed back and redistributed (φ). We prove that a pluralist steady state exists for all parameter values, that it is stable if and only if $\Lambda < 1$ in the symmetric benchmark, and that when the rent elasticity $k = \beta\theta$ exceeds one in the purely political economy, the plural steady state is unstable and a single class absorbs the rent stream. We further prove that concentration of power shifts the stationary class distribution downward in the sense of first-order stochastic dominance, hollowing out the upper and middle classes. An illustrative calibration to a stylised eight-class urban Punjab economy shows that the observed distribution can sit on a knife-edge ($\Lambda_{cal} \approx 1.00$), that heterogeneity in organisational capacity lowers the capture threshold relative to the symmetric benchmark, and that a rent levy that does not reduce k can rotate rather than dissolve elite dominance. Institutional reforms that lower the decisiveness of discretionary allocation and the money-to-influence elasticity are the most robust counter-levers. The framework provides a formal backbone for an eight-class approach to socio-economic public policy and a programme for empirical estimation.

Keywords: *wealth–power feedback; elite capture; class structure; rent seeking; contest success functions; social mobility; political economy of inequality; Pakistan; Punjab*

1. Introduction

Economic inequality and political inequality are rarely independent. Wealth buys access, organisation and voice; access, organisation and voice shape the rules under which wealth is subsequently earned, taxed and protected. When this loop is weak, market forces and democratic competition keep the distribution of wealth roughly stable. When it is strong, small initial advantages compound into durable oligarchy. The central question of this paper is therefore quantitative rather than merely descriptive: *how strong must the feedback between wealth and power be before it overwhelms the forces that keep a society plural, and which policy levers move a society back across that line?*

The question has particular force in Pakistan. The United Nations Development Programme's National Human Development Report on inequality estimated that economic privileges accruing to the country's most powerful groups — through favourable pricing, lower taxation and preferential access to land, capital and infrastructure — amounted to about PKR 2,660 billion, or roughly US$17.4 billion, in 2017–18, equivalent to some six to seven per cent of GDP (UNDP, 2021). The same report found that the richest one per cent received around nine per cent of national income while the poorest one per cent received about 0.15 per cent. These figures describe a stock of privilege; they do not by themselves explain why such privilege persists or under what conditions it would expand or contract. That requires a dynamic model.

Most formal models of the political economy of inequality work with two or three groups — rich and poor, or elite, middle class and poor (Meltzer & Richard, 1981; Bénabou, 2000; Acemoglu & Robinson, 2006). This is analytically convenient but blunt. In a city such as Lahore, Faisalabad or Rawalpindi, the daily-wage labourer, the small shopkeeper, the salaried government officer, the doctor or engineer, the property-holding trader, the industrialist and the politically connected dynastic family do not form three coherent blocs. They hold different assets, rely on different channels of influence, and face different mobility prospects. An eight-class structure is fine enough to capture these distinctions while remaining coarse enough to be tractable and to map directly onto policy instruments.

This paper makes four contributions.

1. **A formal eight-class wealth–power model.** We specify a discrete-time dynamical system in which each class's political power is a Cobb–Douglas aggregate of its population (the numbers channel), its wealth (the money channel) and its organisational capacity (the positional channel). Power is converted into rents through a Tullock-type contest whose decisiveness parameter θ is interpreted as a property of institutions: how discretionary, winner-take-all and unreviewable public allocation is.

2. **A threshold theory of elite capture.** We show that the strength of the loop is governed by the product $k = \beta\theta$ — the elasticity of a class's rent share with respect to its wealth share — and, more generally, by a Wealth–Power Feedback Index Λ. A pluralist steady state is stable when $\Lambda < 1$ and unstable when $\Lambda > 1$. In the purely political benchmark, $k > 1$ implies winner-take-all dynamics in which one class absorbs the rent stream, and $k \to 1^{-}$ produces extreme amplification of small organisational differences.

3. **A mobility channel.** We embed a birth–death model of class mobility in which upward transitions become rarer and downward transitions more frequent as political power concentrates. We prove that higher concentration shifts the stationary population

distribution downward in the sense of first-order stochastic dominance, formalising the intuition that oligarchy hollows out the middle.

4. **An illustrative calibration and policy map.** Calibrating the model so that a stylised urban Punjab distribution is a steady state, we simulate five policy regimes. The exercise yields three lessons: the observed distribution may sit close to the capture threshold; heterogeneity in organisational capacity makes capture easier than symmetric analysis suggests; and redistribution alone can rotate the identity of the dominant class without removing the underlying instability. Lowering θ and β through institutional reform is the most robust remedy.

The remainder of the paper is organised as follows. Section 2 situates the model in the literature. Section 3 sets out the eight-class structure. Section 4 presents the model. Section 5 derives the analytical results. Section 6 reports the calibration and simulations. Section 7 translates the parameters into policy instruments. Section 8 sets out an empirical strategy for estimating the key parameters. Section 9 discusses limitations and extensions, and Section 10 concludes. Proofs and computational details are collected in the appendices.

2. Related Literature

Redistribution and the median voter. The canonical model of Meltzer and Richard (1981) predicts that greater inequality raises demand for redistribution because the median voter becomes poorer relative to the mean. Empirically this prediction often fails: highly unequal societies frequently redistribute less. Bénabou (2000) resolves part of the puzzle by allowing political influence to rise with wealth, generating multiple steady states in which unequal societies sustain low redistribution. Our power function nests this logic: when the money channel β is large relative to the numbers channel α, the decisive political weight sits above the median.

Elites, institutions and persistence. Acemoglu and Robinson (2006, 2008) distinguish de jure power, allocated by institutions, from de facto power, acquired through wealth and organisation, and show how elites can preserve their position even when formal institutions change. Acemoglu, Johnson and Robinson (2005) place this distinction at the centre of long-run development. Our organisational capacity term O_i and the contest decisiveness θ are, respectively, reduced-form representations of de facto and de jure power, and the model makes precise how they interact with wealth.

Rent seeking and contests. Krueger (1974) and Tullock (1980) model the competitive dissipation of resources in pursuit of politically created rents. Skaperdas (1996) provides an axiomatic foundation for the ratio-form contest success function used here, in which the decisiveness parameter governs how sharply small advantages in effort translate into large advantages in prizes. Esteban and Ray (2006) show that when lobbying capacity correlates with wealth, government allocation can be systematically distorted towards the rich. We take these micro-foundations as given and focus on their dynamic, distributional consequences when rents feed back into lobbying capacity.

Lobbying and policy capture. Grossman and Helpman (1994) model organised interest groups bidding for policy with contribution schedules; Bardhan and Mookherjee (2000) analyse capture at local and national levels; Olson (1965, 1982) explains why small, concentrated groups organise more effectively than large, diffuse ones. These ideas motivate the separate role of organisational capacity, which allows a numerically tiny class to wield disproportionate power.

Wealth concentration and its dynamics. Piketty (2014) emphasises the gap between the return to capital and the growth rate as a driver of concentration; Gabaix, Lasry, Lions and Moll (2016) show that heterogeneous returns, and especially return differentials that scale with wealth, are needed to match the speed of observed changes in top inequality. Our privileged-return parameter η captures a political version of this mechanism: returns that rise with access to power. Stiglitz (2012) and Winters (2011) argue that concentrated wealth becomes a political resource that defends itself; Gilens and Page (2014) and Bartels (2008) document unequal policy responsiveness in the United States.

Class analysis and elite theory. Pareto's account of the circulation of elites (Pareto, 1916/1935) and Mills's (1956) analysis of an interlocking power elite describe the phenomena our model formalises: the persistence of a dominant stratum, and the replacement of one dominant group by another without the dissolution of dominance itself. Section 6 shows that the latter emerges endogenously in the model when redistribution is not accompanied by institutional reform.

Pakistan. Alavi (1972) described an "overdeveloped" post-colonial state mediating between landed, industrial and military-bureaucratic interests. Husain (1999) characterised Pakistan's political economy as that of an elitist state in which growth benefits accrue to a narrow group. Jalal (1995) traced the interplay of authoritarianism and democracy in South Asia, and Zaidi (2015) surveyed the structural features of the economy. Cheema, Khwaja and Qadir (2006) examined the politics of local government reform, and Mohmand (2019) showed how rural voters and local elites interact under inequality. The UNDP (2021) report quantified the privileges accruing to the corporate sector, landowners, the political class and state institutions. These works provide rich qualitative and descriptive foundations but, to our knowledge, no formal dynamic model that links wealth, power and mobility across a disaggregated class structure. This paper aims to fill that gap.

3. The Eight-Class Structure

The economy is partitioned into eight ordered classes indexed $i = 1, \dots, 8$. The ordering is by per-capita wealth in the baseline, but the classes are defined by their characteristic asset base and their dominant channel of political influence, not merely by income cut-offs. This matters because two classes with similar incomes — for example, a senior salaried professional and a mid-sized trader — can differ sharply in their access to discretionary state allocation.

Table 1 gives a stylised operationalisation for urban Punjab. The analytical results in Sections 4 and 5 hold for any ordered partition into eight classes, so the labels and shares can be replaced by alternative definitions without changing the theory. The population and wealth shares are illustrative calibration targets chosen to be broadly consistent with the qualitative pattern of urban household survey data; they are not estimates.

Table 1. Stylised eight-class structure and illustrative calibration targets for urban Punjab

Class	Label	Indicative profile (urban Punjab)	Principal asset base	Dominant power channel	Pop. share n_i (%)	Wealth share x_i (%)
C1	**Extreme-poor**	Destitute, irregular informal work, no assets	None or minimal	Numbers only (weak)	12.0	1.0
C2	**Working poor**	Daily-wage, construction, domestic and transport labour	Labour	Numbers; patronage voting	22.0	4.0
C3	**Vulnerable lower-middle**	Small shopkeepers, low-grade salaried staff, informal micro-enterprise	Labour, small stock	Numbers; *biradari* networks	22.0	8.0
C4	**Core middle**	Salaried professionals, mid-grade civil servants, teachers, skilled technicians	Human capital, housing	Numbers; professional associations	20.0	14.0
C5	**Upper-middle professional**	Doctors, engineers, senior managers, established lawyers	Human capital, property	Expertise; bar and professional bodies	13.0	17.0
C6	**Rich: Propertied-commercial**	Wholesale traders, mid-sized business, urban real-estate holders	Property, trade capital	Trade associations; local office	7.0	20.0
C7	**Very Rich: Corporate-industrial**	Large manufacturers, financial and corporate groups	Industrial and financial capital	Lobbying; chambers; regulatory access	3.5	18.0
C8	**Extreme Rich: Power elite**	Dynastic political–business–landed families; apex of state networks	Diversified wealth plus office	Office, networks, coercive and positional access	0.5	18.0

Two features of Table 1 drive the dynamics. First, per-capita wealth rises steeply across classes: C8 holds 36 times the average per-capita wealth, while C1 holds less than one-tenth. Second, the dominant power channel changes along the ladder, from numbers at the bottom to money and position at the top. The model represents this through three distinct inputs into political power.

4. The Model

4.1 Environment

Time is discrete, $t = 0,1,2,\ldots$, with a period interpreted as one year. Let $n_{i,t} \geq 0$ denote the population share of class i and $x_{i,t} \geq 0$ its share of aggregate wealth, with $\sum_i n_{i,t} = \sum_i x_{i,t} = 1$. The state of the economy is the pair $(x_t, n_t) \in \Delta \times \Delta$, where Δ is the seven-dimensional unit simplex. Working with shares rather than levels isolates distribution from aggregate growth.

4.2 Political power

Each class converts three resources into political power: population (votes, street presence, the capacity to be mobilised), wealth (campaign finance, lobbying, media, the ability to reward and punish) and organisational or positional capacity $O_i > 0$ (embeddedness in parties, bureaucracy, security institutions, kinship networks and business associations). The *political power share* of class i is

$$\pi_i(x,n) = \frac{n_i^{\alpha}\, x_i^{\beta}\, O_i}{\sum_{j=1}^{8} n_j^{\alpha}\, x_j^{\beta}\, O_j}, \qquad \alpha \geq 0,\ \beta > 0. \tag{1}$$

The elasticity α measures the weight of numbers — it is high where elections are competitive, turnout is broad and votes cannot be bought. The elasticity β measures the conversion of money into influence — it is high where campaign finance is unregulated, asset declarations are unenforced and media ownership is concentrated. The term O_i captures organisational and positional advantages not reducible to numbers or money; following Olson (1965), small groups typically have higher O_i. A useful summary of concentration is the Herfindahl index $H(\pi) = \sum_i \pi_i^2 \in [1/8,1]$.

4.3 Rents and the decisiveness of institutions

A fraction $\rho \in (0,1)$ of aggregate wealth is reallocated each period through politically mediated channels: tax exemptions and statutory concessions, subsidised credit, administered prices, land allotments, public procurement, regulatory protection, and discretionary licensing. The share of this rent stream captured by class i is determined by a contest:

$$s_i(x,n) = \frac{\pi_i^{\theta}}{\sum_{j=1}^{8} \pi_j^{\theta}}, \qquad \theta > 0. \tag{2}$$

Equation (2) is the ratio-form contest success function of Tullock (1980), axiomatised by Skaperdas (1996), with political power as the effective lobbying input. The *decisiveness* parameter θ is a property of institutions. When allocation is rule-based, transparent and subject to review, a class with twice the power of another receives roughly twice the rents ($\theta \approx 1$) or less ($\theta < 1$). When allocation is discretionary, winner-take-all and unreviewable, small advantages in access translate into disproportionately large prizes ($\theta > 1$).

Substituting (1) into (2), and holding population fixed, the rent share of class i is proportional to $a_i\, x_i^{k}$, where

$$k \equiv \beta\theta, \qquad a_i \equiv (n_i^{\alpha} O_i)^{\theta}. \tag{3}$$

The composite parameter k is the **wealth–power elasticity of rents**: the percentage change in a class's rent share induced by a one per cent increase in its wealth share, other things equal. It is the single most important parameter in the model.

A fraction $0 \leq \varphi < 1$ of the rent stream is clawed back — through taxation of rents, withdrawal of exemptions, or auctioning of previously allotted assets — and redistributed equally per capita, so that class i receives φn_i of the levied amount.

4.4 Privileged returns and market income

Power also raises the private return on wealth already held, through preferential access to credit, inside information, protection from enforcement and the ability to shift losses onto the public. Class i's gross return is

$$g_i = g_0 + \eta\, \pi_i, \qquad \eta \geq 0. \tag{4}$$

Finally, a fraction $0 \leq \mu < 1 - \rho$ of wealth is renewed each period through market income — wages, professional earnings and competitive profits — distributed according to market earning

power: $m_i = n_i \ell_i / \sum_j n_j \ell_j$, where ℓ_i is the per-capita earning capacity of class i. The parameter μ measures the weight of competitive markets relative to political allocation.

4.5 Wealth dynamics

Combining these elements, the law of motion for wealth shares is

$$x_{t+1} = F(x_t, n_t) \equiv (1 - \rho - \mu)\, \tilde{x}_t + \mu\, m_t + \rho[(1 - \varphi)\, s(x_t, n_t) + \varphi\, n_t], \tag{5}$$

where $\tilde{x}_{i,t} = x_{i,t}(1 + g_{i,t}) / \sum_j x_{j,t}\,(1 + g_{j,t})$ is the distribution of retained wealth after privileged returns. The map F is a convex combination of four distributions on Δ: retained wealth, market income, captured rents and redistributed levies. It therefore maps Δ into itself.

4.6 Class mobility

Households move between adjacent classes. Let $u_i(H)$ be the annual probability of moving up from class i and $d_i(H)$ the probability of moving down, with

$$u_i(H) = u_i^0\,(1 - \lambda H), \qquad d_i(H) = d_i^0\,(1 + \lambda H), \qquad \lambda \geq 0,\ \lambda H < 1. \tag{6}$$

Concentrated power closes avenues of advancement — merit recruitment, fair access to credit, contestable markets — and exposes households to arbitrary loss. Population shares evolve according to $n_{t+1} = n_t\, M(H_t)$, where $M(H)$ is the tridiagonal (birth–death) transition matrix with off-diagonal entries $u_i(H)$ and $d_i(H)$.

4.7 The feedback loop

Figure 1 summarises the system. Wealth raises power through β; power raises rents through θ; rents return to wealth at rate $\rho(1 - \varphi)$; power raises returns through η; and power concentration shapes mobility, which in turn determines the population weights that enter power through α. Market income (μ) and the levy (φ) are the two forces that pull the distribution back towards one determined by earning capacity and population.

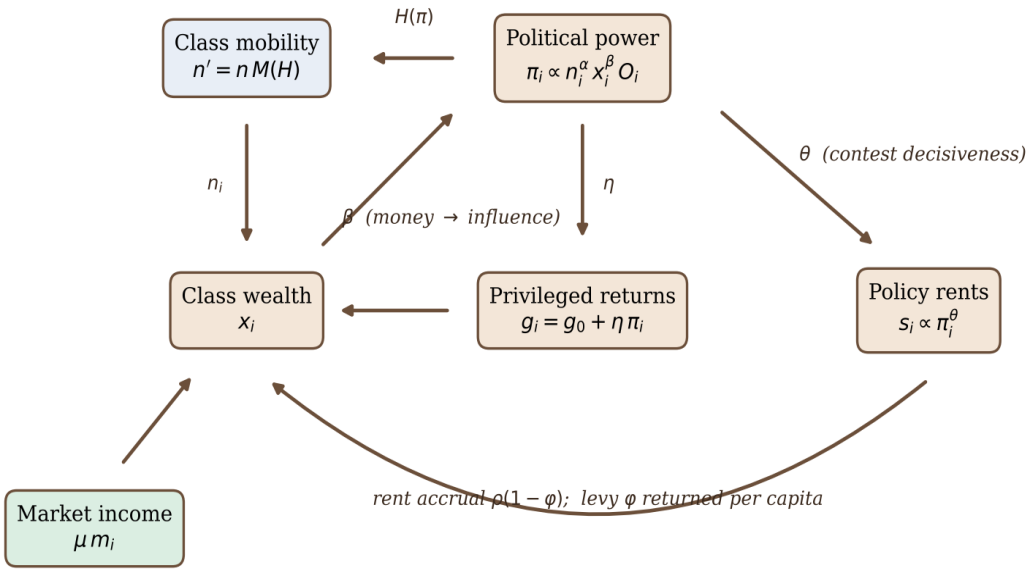


Figure 1. The wealth–power feedback loop. Green boxes denote stabilising forces; the parameters on the arrows govern the strength of each link.

Table 2 lists the parameters and their baseline values in the illustrative calibration.

Table 2. Parameters and baseline values

Symbol	Meaning	Baseline
α	Elasticity of power with respect to population (numbers channel)	0.35
β	Elasticity of power with respect to wealth (money channel)	0.60
θ	Decisiveness of rent contests (institutional discretion)	1.80
$k = \beta\theta$	Wealth–power elasticity of rents	1.08
ρ	Share of wealth reallocated through politics each year	0.10
μ	Share of wealth renewed through market income each year	0.03
g_0	Baseline return on retained wealth	0.03
η	Privileged-return premium per unit of power share	0.10
φ	Rent levy redistributed per capita	0.00
λ	Sensitivity of mobility to power concentration	1.50
O_i	Organisational–positional capacity (calibrated)	Table 3
ℓ_i	Per-capita market earning capacity	Table 3

5. Analytical Results

This section studies the wealth subsystem (5) holding population fixed, then the mobility subsystem (6) holding power fixed. Separating the two is natural because class composition changes more slowly than wealth shares; Section 6 simulates the full coupled system.

5.1 Existence

Proposition 1 (Existence of a steady state). *For any parameter values and any fixed $n \in \Delta$, the map $F(\cdot, n)$ has at least one fixed point $x^* \in \Delta$. If $\mu > 0$ and $\ell_i > 0$ for all i, or if $\varphi > 0$ and $n_i > 0$ for all i, every fixed point lies in the interior of Δ.*

Proof. Each component of F is a continuous function of x on Δ: the power shares in (1) are continuous because the denominator is strictly positive whenever at least one class holds wealth, and the rent shares in (2) are continuous compositions. Since F is a convex combination of elements of Δ, it maps the compact convex set Δ into itself. Brouwer's fixed-point theorem (see Border, 1985) yields existence. For interiority, any fixed point satisfies $x_i^* \geq \mu m_i + \rho\varphi n_i$, which is strictly positive under either condition. ▫

Existence is not the interesting question. A steady state that is unstable will not be observed for long; the substantive issue is stability.

5.2 The purely political benchmark

To isolate the core mechanism, suppose that wealth is renewed only through politics: $\mu = 0$, $\eta = 0$ and $\varphi = 0$. Then (5) reduces to

$$x_{t+1} = (1-\rho)\, x_t + \rho\, s(x_t), \qquad s_i(x) = \frac{a_i\, x_i^k}{\sum_j a_j\; x_j^k}. \tag{7}$$

Proposition 2 (Threshold at $k = 1$). *In the purely political benchmark (7):*

(i) If $k < 1$, there is a unique interior steady state

$$x_i^* = \frac{a_i^{1/(1-k)}}{\sum_j a_j^{1/(1-k)}}, \tag{8}$$

---

and it is locally asymptotically stable. The eigenvalues of the linearised map on the tangent space of Δ all equal $1 - \rho + \rho k < 1$.

(ii) If $k > 1$, the interior steady state given by (8) with exponent $1/(1-k) < 0$ exists but is unstable, with all tangent eigenvalues equal to $1 - \rho + \rho k > 1$; every vertex e_j of Δ (complete capture by class j) is locally asymptotically stable.

(iii) If $k = 1$, the class with the largest a_i asymptotically absorbs all wealth from any interior initial condition.

Proof. (i) A fixed point of (7) satisfies $x = s(x)$, i.e. $x_i \propto a_i x_i^k$, so $x_i^{1-k} \propto a_i$, which gives (8). Differentiating s,

$$\frac{\partial s_i}{\partial x_j} = k\, s_i \left(\frac{\delta_{ij}}{x_j} - \frac{s_j}{x_j} \right).$$

At a point where $s = x$ this becomes $k\left(\delta_{ij} - x_i^*\right)$, so $J_s = k\,(I - x^* \mathbf{1}^\top)$. For any tangent vector v with $\mathbf{1}^\top v = 0$, $J_s v = kv$. The Jacobian of (7) on the tangent space is therefore $(1-\rho)I + \rho k I$, with every eigenvalue equal to $1 - \rho + \rho k$, which is less than one if and only if $k < 1$. Uniqueness follows because (8) is the only solution of $x_i^{1-k} \propto a_i$ in the interior.

(ii) The same computation gives eigenvalues $1-\rho+\rho k>1$ at the interior fixed point when $k>1$, so it is unstable. Near a vertex e_j, consider the ratios $r_i = x_i/x_j$ for $i \neq j$. Then

$$r_i' = \frac{(1-\rho)x_i + \rho s_i}{(1-\rho)x_j + \rho s_j} \approx (1-\rho)\, r_i + \rho\, \frac{a_i}{a_j}\, r_i^k,$$

since x_j and s_j are close to one. Because $k>1$, the second term is $o(r_i)$, so $r_i' \leq (1-\rho+\epsilon)\, r_i$ for r_i small enough, and $r_i \to 0$ geometrically.

(iii) With $k=1$, $s_i = a_i x_i/\bar{a}$ where $\bar{a} = \sum_j a_j\, x_j$. For any two classes,

$$\frac{x_i'}{x_j'} = \frac{x_i}{x_j} \cdot \frac{(1-\rho) + \rho\, a_i/\bar{a}}{(1-\rho) + \rho\, a_j/\bar{a}}.$$

If $a_j > a_i$, the second factor is strictly below one and bounded away from one while x_j is bounded away from zero, so $x_i/x_j \to 0$. ▫

Proposition 2 has a sharp interpretation. When $k<1$, the economy settles on a plural distribution in which each class's wealth reflects its numbers and organisation, but with a crucial amplification: the elasticity of relative wealth x_i^*/x_j^* with respect to relative political endowment a_i/a_j is $1/(1-k)$. At $k=0.5$ this elasticity is 2; at $k=0.9$ it is 10; as $k \to 1^-$ it diverges. **Near the threshold, small differences in organisational capacity are magnified into very large differences in wealth.** Above the threshold, the plural distribution cannot be sustained, and which class ends up dominant depends on history — on initial wealth and organisation — rather than on any fundamental. The model therefore generates path dependence of exactly the kind emphasised by historical institutionalists.

5.3 The Wealth–Power Feedback Index

We now restore market income, privileged returns and the rent levy. A closed-form stability condition is available in the symmetric benchmark, in which all classes have the same population, organisational capacity and earning capacity, so that the uniform distribution $\bar{x} = (1/8, \ldots, 1/8)$ is a steady state.

Definition (Wealth–Power Feedback Index). Let $N=8$, $\bar{g} = g_0 + \eta/N$ and $\varepsilon \equiv \eta\beta/[N(1+\bar{g})]$. The Wealth–Power Feedback Index is

$$\Lambda \equiv \frac{\rho\,(1-\varphi)\,k + (1-\rho-\mu)\,\varepsilon}{\rho+\mu}. \tag{9}$$

Proposition 3 (Stability of the plural steady state). *In the symmetric benchmark with population fixed, the uniform steady state is locally asymptotically stable if $\Lambda<1$ and unstable if $\Lambda>1$. Convergence or divergence is monotone (non-oscillatory).*

Proof. Perturb $x = \bar{x} + v$ with $\mathbf{1}^{\top} v = 0$. In the symmetric benchmark $\pi_i \propto x_i^{\beta}$, and the argument of Proposition 2 gives $d\pi = \beta v$ to first order. Hence $dg = \eta\beta v$. For retained wealth,

$$d\tilde{x}_i = v_i + \bar{x}_i \frac{dg_i}{1+\bar{g}} - \bar{x}_i \sum_j \bar{x}_j \frac{dg_j}{1+\bar{g}} = (1+\varepsilon)v_i,$$

because $\bar{x}_i = 1/N$ and $\sum_j d\, g_j = 0$. Rents satisfy $s_i \propto x_i^k$ in the symmetric benchmark, so $ds = kv$. Market income and the per-capita levy do not depend on x. The linearised map on the tangent space is therefore multiplication by

$$\lambda_F = (1-\rho-\mu)(1+\varepsilon) + \rho(1-\varphi)k.$$

Rearranging, $\lambda_F < 1$ if and only if $\rho(1-\varphi)k + (1-\rho-\mu)\varepsilon < \rho + \mu$, that is, $\Lambda < 1$. Since $\lambda_F \geq 1-\rho-\mu > 0$, the dynamics are monotone. ▫

The index has a transparent reading. The numerator is the strength of the destabilising forces: rents that respond more than proportionally to power, net of the levy, plus privileged returns that compound with power. The denominator is the strength of renewal: the share of wealth reallocated each period on terms that do not simply reproduce the existing distribution. **Oligarchy is the regime in which wealth-driven rent capture outpaces renewal.**

Corollary 1 (Critical values). *Holding other parameters fixed, the plural steady state loses stability when the rent elasticity exceeds*

$$k^* = \frac{(\rho+\mu) - (1-\rho-\mu)\,\varepsilon}{\rho\,(1-\varphi)}, \tag{10}$$

or, equivalently, when the levy falls below

$$\varphi^* = 1 - \frac{(\rho+\mu) - (1-\rho-\mu)\,\varepsilon}{\rho\,k}. \tag{11}$$

At the baseline values in Table 2, $\varepsilon \approx 0.0072$ and $k^* \approx 1.24$ with no levy, rising to about 1.55 with $\varphi = 0.2$ and 2.06 with $\varphi = 0.4$. We verified the analytical threshold numerically on a grid of 123 parameter combinations; in every case the simulated symmetric economy converged to the uniform distribution when $\Lambda < 1$ and diverged from it when $\Lambda > 1$.

Corollary 2 (Comparative statics). *Λ is increasing in β, θ and η, and decreasing in φ and μ. It is increasing in ρ whenever $(1-\varphi)k\mu > \varepsilon$.*

The proof is immediate from (9). The last statement is substantively important: enlarging the share of the economy allocated through politics — through more exemptions, more administered prices or more discretionary programmes — raises the feedback index, even if each programme is individually well intentioned, unless the levy or market renewal rises in step.

Note that α does not enter Λ in the symmetric benchmark, because all classes have equal numbers. The numbers channel matters through heterogeneity, as we show next.

5.4 Heterogeneity

With heterogeneous classes, the steady state is not uniform, and closed-form stability conditions are unavailable. Two analytical observations nonetheless carry over. First, by Proposition 2, heterogeneity in $a_i = (n_i^\alpha O_i)^\theta$ determines the composition of the plural steady state, with an amplification factor $1/(1-k)$. Second, the stability of a heterogeneous steady state is governed by the spectral radius of the Jacobian of F at that steady state, which we denote Λ_{cal} and compute numerically in Section 6. Raising α increases the weight of numerically large classes in a_i, which pulls the steady state towards the population distribution and away from the most organised classes.

5.5 Mobility and the hollowing of the middle

Proposition 4 (Concentration shifts the class distribution down). *Let* $u_i^0 > 0$ *for* $i < 8$ *and* $d_i^0 > 0$ *for* $i > 1$. *For each* H *with* $\lambda H < 1$, *the birth–death chain* $M(H)$ *has a unique stationary distribution* $n^*(H)$ *satisfying*

$$\frac{n_{i+1}^*(H)}{n_i^*(H)} = \frac{u_i^0}{d_{i+1}^0} \cdot q(H), \qquad q(H) = \frac{1-\lambda H}{1+\lambda H}. \tag{12}$$

If $H' > H$, *then* $n^*(H)$ *first-order stochastically dominates* $n^*(H')$: *for every* $j \geq 2$, *the population share in classes* $j, \ldots, 8$ *is strictly lower under* H'.

Proof. An irreducible birth–death chain is reversible, so its stationary distribution satisfies detailed balance, $n_i^* u_i = n_{i+1}^* d_{i+1}$, which gives (12). Iterating, $n_i^*(H) \propto c_i\, q(H)^{\,i-1}$ with $c_i = \prod_{l<i} u_l^0 / d_{l+1}^0$. Because q is strictly decreasing in H, the ratio $n_i^*(H')/n_i^*(H) \propto [q(H')/q(H)]^{i-1}$ is strictly decreasing in i: the family has the monotone likelihood ratio property. The monotone likelihood ratio property implies first-order stochastic dominance, which gives the result. ▫

Proposition 4 closes the second loop. When power concentrates, upward mobility slows and downward mobility accelerates, so the stationary population shifts towards the lower classes. Because the lower classes hold little wealth and little organisation, this weakens the countervailing power of the middle classes — except through the numbers channel. A higher population share raises n_i^{α} for the lower classes, a self-limiting force that is effective only if α is large. This gives a formal rationale for why concentrated elites invest in reducing α: through patronage-based vote capture, disenfranchisement, the postponement of local elections, or the fragmentation of mass organisations.

6. Calibration and Simulation

6.1 Calibration strategy

The calibration is illustrative: its purpose is to show how the model behaves in an economy with the qualitative features of urban Punjab, not to provide estimates. We impose the population and wealth shares of Table 1 and the parameters of Table 2, and set per-capita earning capacities $\ell = (0.3, 0.5, 0.8, 1.2, 2.2, 4.0, 9.0, 15.0)$, which imply market-income shares that are higher than wealth shares for the bottom five classes and lower for C6 and C8.

We then **back out** the organisational capacities O_i such that the Table 1 distribution is an exact steady state of (5) at the baseline parameters. This is the model analogue of "revealed organisational power": given observed wealth, numbers and the rules of the game, O_i is whatever organisational advantage is needed to rationalise the observed distribution as an equilibrium. The upward mobility rates u_i^0 are similarly backed out, given assumed downward rates d_i^0, so that the Table 1 population distribution is the stationary distribution of the mobility chain at the baseline concentration H_0. The algorithm is described in Appendix B.

Table 3. Calibrated class characteristics at the baseline steady state (O_5 normalised to 1; $H_0 = 0.147$)

Class	ℓ_i	Market share m_i (%)	Calibrated O_i	Power share π_i (%)	Rent share s_i (%)	u_i^0	d_i^0
C1	0.3	2.4	0.97	2.7	0.7	0.057	—

Class	ℓ_i	Market share m_i (%)	Calibrated O_i	Power share π_i (%)	Rent share s_i (%)	u_i^0	d_i^0
C2	0.5	7.2	0.82	6.4	3.3	0.039	0.020
C3	0.8	11.6	0.83	9.9	7.3	0.036	0.025
C4	1.2	15.8	0.87	14.0	13.6	0.020	0.025
C5	2.2	18.8	1.00	15.6	16.4	0.013	0.020
C6	4.0	18.4	1.26	17.4	20.1	0.008	0.015
C7	9.0	20.8	1.56	15.9	17.1	0.002	0.010
C8	15.0	4.9	3.51	18.1	21.5	—	0.010

Table 3 shows the central feature of the calibrated economy. C8, with 0.5 per cent of the population, holds the largest single power share (18.1 per cent) and captures the largest single rent share (21.5 per cent), even though its market-income share is under 5 per cent. Its calibrated organisational capacity is 3.5 times that of the upper-middle professional class. In other words, the observed concentration of wealth at the apex can be sustained in steady state only if the apex derives a very large share of its income from political allocation rather than markets — consistent with the UNDP (2021) characterisation of elite privilege. The relatively high calibrated O_1 is a by-product of fitting the steady state and absorbs features such as targeted transfers; it should not be interpreted as genuine organisational strength.

6.2 Scenarios

We simulate five regimes over 150 years, starting from the calibrated steady state. Table 4 reports the parameter changes and the two stability measures: the symmetric-benchmark index Λ from (9), and the calibrated spectral radius Λ_{cal} of the Jacobian at the heterogeneous steady state of each regime (population held fixed).

Table 4. Scenario design and stability measures

Scenario	Description	Parameter changes from baseline	k	Λ (symmetric)	Λ_{cal}
S0	Status quo	None	1.08	0.88	1.004
S1	Deepening capture	$\theta = 2.5$	1.50	1.20	1.054
S2	Capture with rent levy	$\theta = 2.5, \varphi = 0.20$	1.50	0.97	1.025
S3	Capture with institutional reform	$\theta = 2.2, \beta = 0.45, \alpha = 0.45$	0.99	0.80	0.989
S4	Pluralist settlement	$\theta = 1.4, \varphi = 0.10$	0.84	0.63	0.967

The first striking result is in the S0 row. Although the symmetric index is comfortably below one, the calibrated spectral radius is marginally above one: **the observed distribution is a knife-edge steady state**. Heterogeneity in organisational capacity makes the economy considerably less stable than the symmetric benchmark suggests. Because Λ_{cal} is only slightly above one, the drift is slow: transferring two percentage points of wealth share to C8 from the classes below raises C8's share from 20 per cent to 22.6 per cent after 200 years and 24.8 per cent after 300 years; transferring two points away lowers it to 14.1 and 12.7 per cent respectively. The status quo persists, but it is not self-correcting — every shock leaves a permanent and growing imprint. This finding is sensitive to the value of θ, which is the principal reason why estimating θ is the first empirical priority (Section 8).

6.3 Results

Figure 2 shows the evolution of class wealth shares under each regime, and Table 5 reports summary indicators.

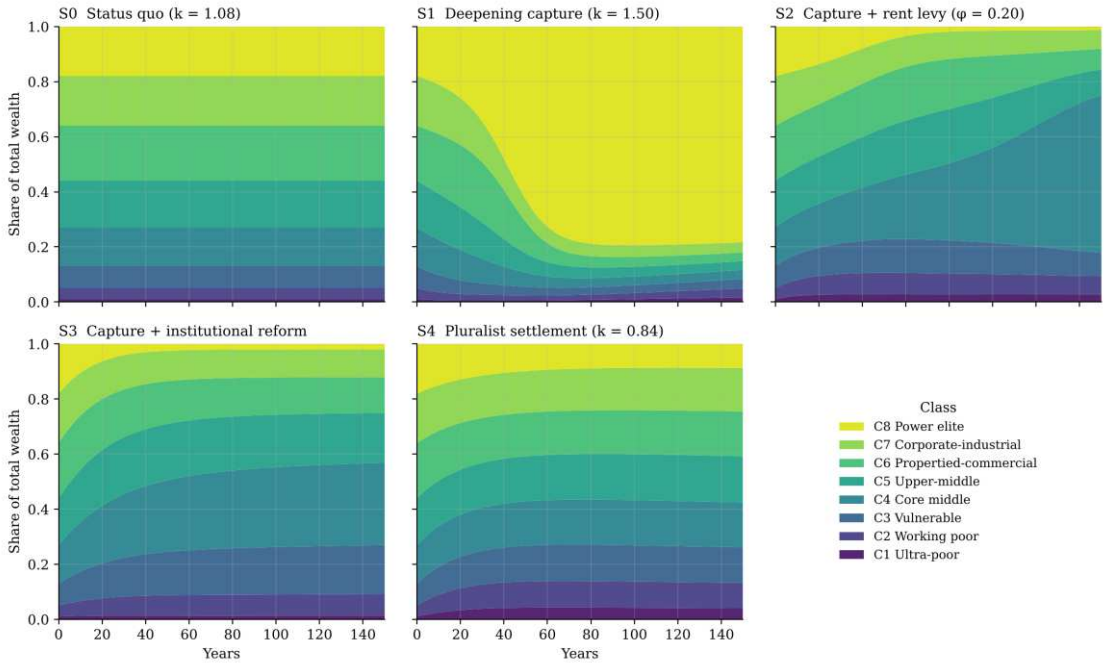


Figure 2. Wealth shares by class under five policy regimes (150 years, calibrated urban Punjab economy).

Table 5. Simulated outcomes by scenario (illustrative calibration)

Scenario	Year	C8 wealth share (%)	C7+C8 (%)	C1–C3 (%)	Middle-class population C4–C6 (%)	Between-class Gini
S0 Status quo	0–150	18.0	36.0	13.0	40.0	0.64
S1 Deepening capture	25	29.7	45.7	7.3	39.6	0.75
	50	62.7	70.9	5.5	37.9	0.85
	150	78.3	82.0	8.4	24.2	0.88
S2 Capture + levy	25	12.3	26.4	20.6	40.3	0.51
	50	5.4	17.3	22.7	40.6	0.44
	150	1.3	8.1	17.9	38.2	0.51
S3 Capture + reform	25	5.0	18.1	21.5	40.3	0.47
	50	2.5	13.5	24.5	40.4	0.41
	150	2.0	12.1	27.0	40.0	0.37
S4 Pluralist settlement	25	12.1	27.7	24.0	40.4	0.47
	50	9.8	25.0	26.7	40.9	0.42
	150	8.7	24.5	26.2	41.8	0.42

Deepening capture (S1). Raising the decisiveness of rent contests from 1.8 to 2.5 — for example, through an expansion of discretionary concessions with weaker oversight — pushes the economy past the threshold. The power elite's wealth share rises slowly for two decades and then accelerates sharply, reaching 63 per cent by year 50 and 78 per cent by year 150. The between-

class Gini rises from 0.64 to 0.88. The S-shaped path is characteristic of threshold dynamics: the early phase looks like modest drift and offers little warning of the take-off that follows.

Rent levy without institutional reform (S2). Adding a 20 per cent levy on captured rents, redistributed per capita, initially reverses concentration: C8's share falls and the Gini drops to 0.44 by year 50. But with k still at 1.5, the economy remains above its calibrated threshold. After about a century the core middle class C4, which combines large numbers with moderate organisation, becomes the new dominant class, holding 57 per cent of wealth by year 150 and 61 per cent by year 300. **Redistribution without institutional change rotates the identity of the dominant class rather than dissolving dominance** — a formal analogue of Pareto's circulation of elites. The Gini rebounds to 0.51.

Institutional reform (S3). Lowering the money-to-influence elasticity (β from 0.60 to 0.45), raising the weight of numbers (α from 0.35 to 0.45) and moderately reducing discretion (θ from 2.5 to 2.2) brings k to 0.99 and the calibrated spectral radius below one. The economy converges to a stable plural distribution with a between-class Gini of 0.37. This scenario achieves the largest and most durable reduction in concentration.

Pluralist settlement (S4). A combination of substantially lower discretion ($\theta = 1.4$) and a modest levy ($\varphi = 0.10$) produces a stable distribution in which C8 retains a per-capita advantage (8.7 per cent of wealth for 0.5 per cent of the population) and the middle classes grow. This is the most moderate reform path and illustrates that pluralism does not require the elimination of an upper class, only the breaking of the feedback loop.

Figure 3 tracks three summary indicators: the power elite's wealth share, the concentration of political power, and the population share of the middle classes C4–C6.

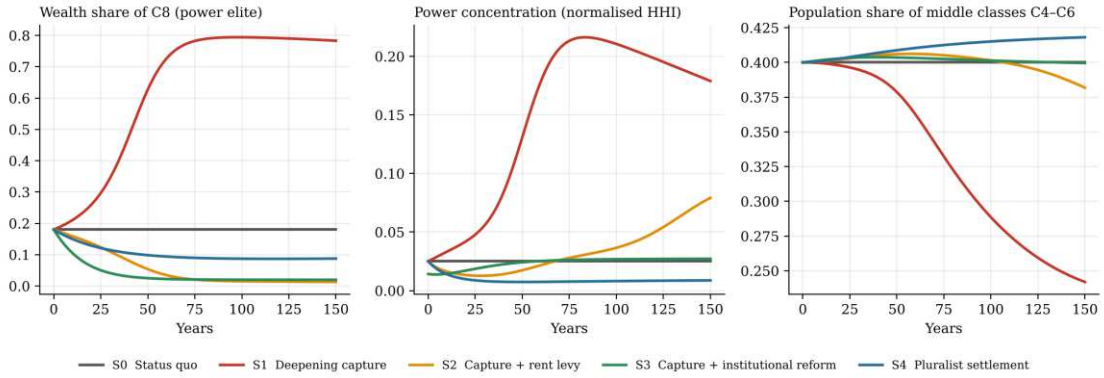


Figure 3. Summary indicators by scenario: C8 wealth share, normalised Herfindahl index of political power, and the population share of the middle classes.

The right-hand panel illustrates Proposition 4 in the coupled system. Under deepening capture, the middle-class population falls from 40 per cent to 24 per cent over 150 years as upward mobility collapses. Concentration of wealth precedes the hollowing of the middle by several decades, which means that by the time the demographic consequences are visible the wealth dynamics are already locked in.

6.4 The capture frontier

Figure 4 maps the long-run outcome across the two most policy-relevant parameters, the rent elasticity k and the levy φ, in the calibrated heterogeneous economy. Shading shows the long-run

wealth share of whichever class is dominant; green lines separate regions in which different classes dominate. The dashed line is the symmetric-benchmark threshold from Corollary 1.

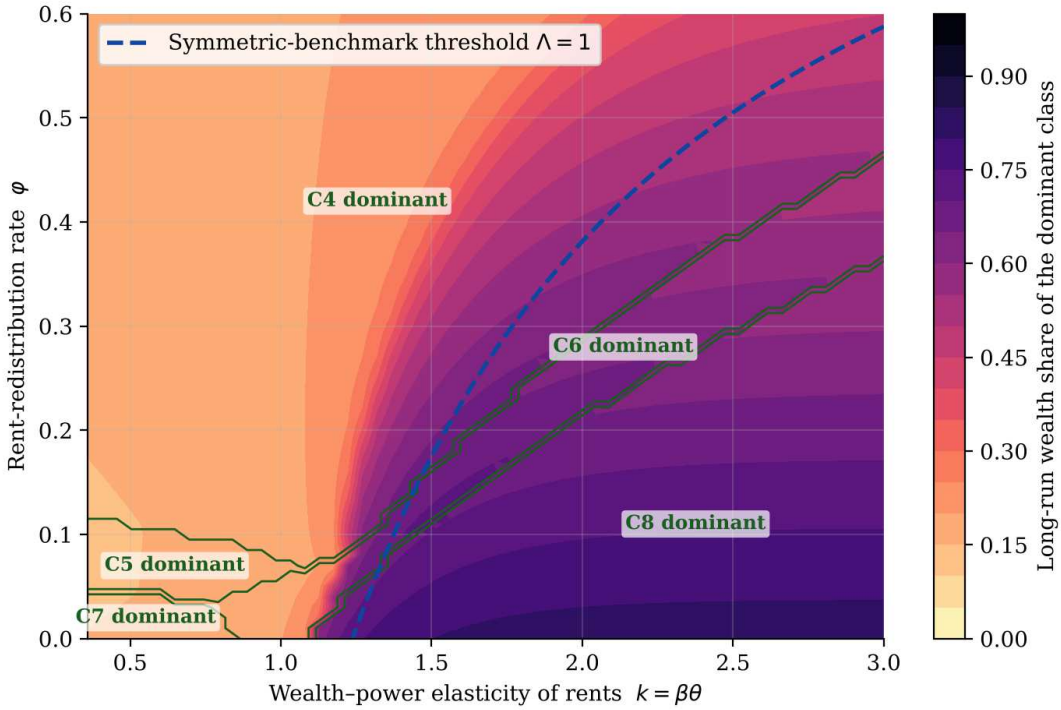


Figure 4. The capture frontier: long-run wealth share of the dominant class as a function of k and φ in the calibrated economy (mobility held fixed).

Three features stand out. First, with no levy, capture by C8 begins at $k \approx 1.08$, well below the symmetric threshold of 1.24. Heterogeneity in organisational capacity lowers the capture frontier. Second, the frontier is upward-sloping: a higher levy tolerates a higher k before capture, consistent with (10). Third, above and to the left of the C8 region lies a broad band in which C6 or C4 dominates. A levy can move the economy out of C8 dominance without moving it into pluralism, because k — not φ — governs whether *some* class dominates.

Figure 5 presents the same information as a bifurcation diagram. For each levy, the long-run wealth share of the dominant class is flat at around 15–20 per cent until a critical value of k, after which it rises steeply. The levy delays the transition and lowers the long-run plateau, but does not remove it.

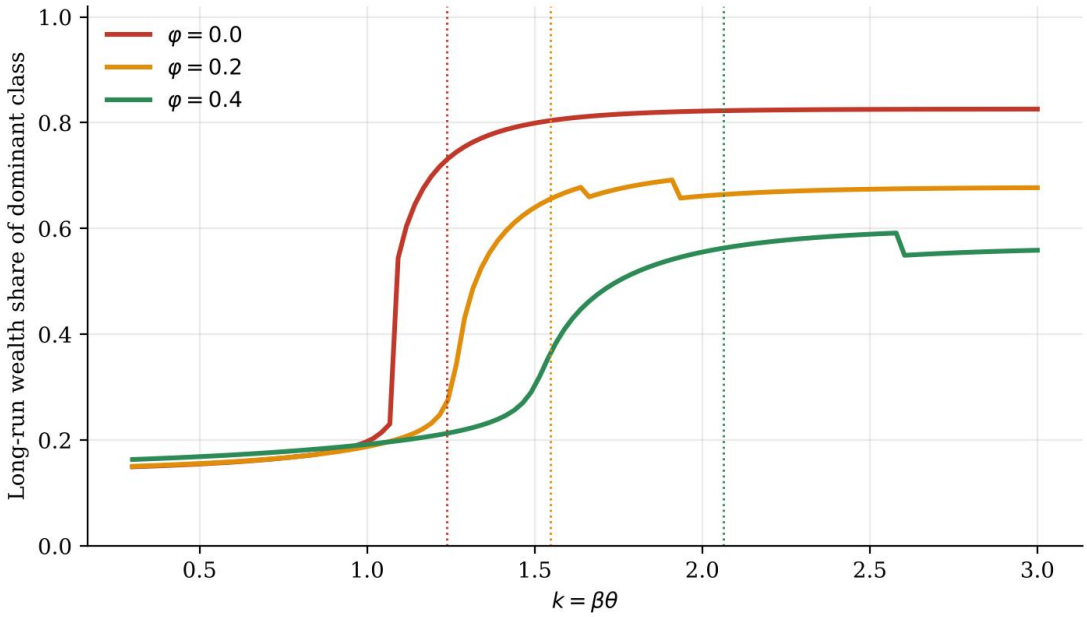


Figure 5. Bifurcation in the rent elasticity k for three values of the levy φ. Dotted lines mark symmetric-benchmark thresholds from Corollary 1.

6.5 Mobility

Figure 6 shows the stationary class distribution implied by the calibrated mobility chain for different degrees of power concentration H. At perfect equality of power ($H = 0.125$), the middle classes C4–C6 account for 43.5 per cent of the population; at the calibrated level ($H_0 = 0.147$) they account for 40 per cent; at $H = 0.25$, 23 per cent; and at $H = 0.35$, only 10 per cent, while the ultra-poor and working poor together exceed 70 per cent. The mobility channel is highly non-linear: moderate increases in power concentration produce large demographic shifts.

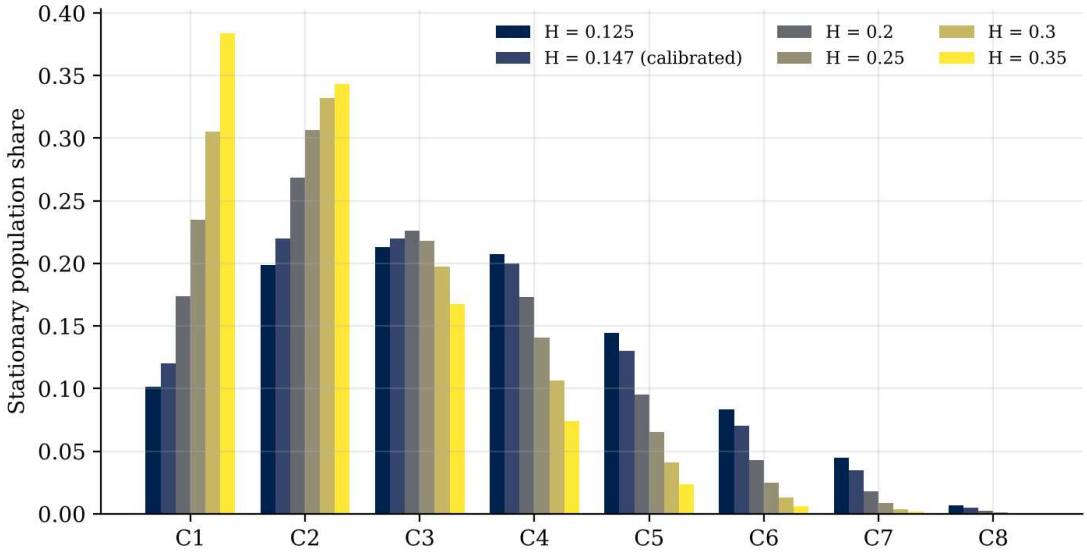


Figure 6. Stationary population distribution across classes for different levels of power concentration H (Proposition 4).

7. Policy Implications

The model translates naturally into a policy framework because each parameter corresponds to an identifiable set of institutions. Table 6 maps parameters to institutions and to illustrative instruments relevant to Pakistan and Punjab.

Table 6. From parameters to policy instruments

Parameter	Desired direction	Institutional meaning	Illustrative instruments
θ (decisiveness)	Lower	Discretion and winner-take-all allocation	Replace discretionary exemptions with legislated, sunset-bound rules; end-to-end e-procurement with open contracting data; competitive auctions of state land and licences; independent regulators; publication of tax-expenditure beneficiaries
β (money → influence)	Lower	Conversion of wealth into political leverage	Enforceable campaign-finance ceilings and audit; verified asset declarations of office-holders; lobbying registers; conflict-of-interest and cooling-off rules; diversified media ownership
α (numbers → influence)	Higher	Weight of citizens as citizens	Regular, empowered local government elections; secret-ballot integrity; freedom of association for workers and informal-sector groups; digital access to services that bypasses intermediaries
φ (levy)	Higher	Recovery of captured rents	Withdrawal of untargeted tax expenditures; progressive urban immovable-property taxation at market values; capital-gains taxation on land; redistribution through per-capita transfers and universal services
μ (market renewal)	Higher	Weight of competitive markets	Active competition enforcement; SME access to credit; simplified entry and registration; removal of protective administered prices
η (privileged returns)	Lower	Return premium from access	Rules-based credit allocation; non-discriminatory enforcement; ending preferential land conversion and utility pricing
ρ (politically allocated share)	Lower	Size of the discretionary state	Shift from discretionary to formula-based programmes; consolidate overlapping subsidy schemes
λ (mobility sensitivity)	Lower	Insulation of mobility from power	Merit-based public recruitment; universal quality schooling; portable social protection; access to justice
O_i for $i \leq 4$	Higher	Organisational capacity of mass classes	Cooperatives, producer and worker associations, residents' welfare bodies, legal aid

Four principles follow from the analysis.

Principle 1: Target the elasticity, not only the level. Fiscal redistribution addresses the level of inequality; institutional reform addresses the elasticity k that determines whether inequality is self-reinforcing. Scenario S2 shows that redistribution alone can fail in the long run. Because $k = \beta\theta$ is a product, reforms that lower both factors simultaneously are multiplicatively powerful.

Principle 2: Watch the denominator. Corollary 2 shows that expanding the share of resources allocated through politics (ρ) raises the feedback index unless market renewal and rent recovery keep pace. New subsidies, concessions and discretionary programmes should be evaluated not only on their direct merits but on their contribution to Λ.

Principle 3: Act before the take-off. Threshold dynamics produce long periods of apparent stability followed by rapid change. The early phase of S1 is nearly indistinguishable from the status quo. Indicators that track the parameters — tax-expenditure concentration, procurement competition, campaign-finance transparency — provide earlier warning than distributional outcomes.

Principle 4: Protect mobility as a stabiliser. Because concentrated power erodes mobility (Proposition 4) and eroded mobility weakens countervailing power, mobility-enhancing policies serve a political-economy function beyond their direct welfare value.

A practical implication is that a **Wealth–Power Feedback Dashboard** could be maintained for Punjab, reporting estimates of β, θ, ρ, μ and φ, and the implied Λ_{cal}, alongside conventional inequality statistics. Section 8 outlines how each component could be estimated.

8. Empirical Strategy

The model's parameters are in principle estimable from Pakistani data. This section sets out a programme of work.

Class assignment and distribution (n_i, x_i). Household Integrated Economic Survey and Pakistan Social and Living Standards Measurement microdata from the Pakistan Bureau of Statistics can be used to assign households to C1–C5 on the basis of occupation, assets and income. Because household surveys under-represent the top, C6–C8 require supplementary sources: property valuation records, the Federal Board of Revenue's published tax directories, asset declarations filed by legislators with the Election Commission of Pakistan, corporate ownership records from the Securities and Exchange Commission of Pakistan, and credit-concentration data from the State Bank of Pakistan.

Rent shares (s_i, ρ). Annual tax-expenditure statements, subsidy and administered-price records, public procurement award data, and land allotment records can be used to estimate the size of the politically allocated stream (ρ) and its incidence by class (s_i), building on the methodology of UNDP (2021).

Decisiveness (θ) and the money channel (β). Equation (2) implies the log-odds relation

$$\ln\frac{s_{i,r}}{s_{j,r}} = \theta \ln\frac{\pi_{i,r}}{\pi_{j,r}} + u_{ij,r}, \tag{13}$$

across policy domains or jurisdictions r. With power measured by observable proxies — legislative and local-council representation, holdings of key administrative posts, membership of chambers and associations — (13) identifies θ by domain. Equation (1) implies

$$\ln\pi_{i,r} = \alpha \ln n_{i,r} + \beta \ln x_{i,r} + \ln O_i + \gamma_r + e_{i,r}, \tag{14}$$

which identifies α and β with class fixed effects absorbing O_i and jurisdiction fixed effects absorbing γ_r.

Identification. Wealth and power are jointly determined, so (13)–(14) are subject to simultaneity. Three strategies are available. First, lagged and historically predetermined variation: the distribution of land and the pattern of colonial-era settlement are plausible sources of variation in initial wealth and organisation that pre-date current policy (cf. Banerjee & Iyer, 2005). Second, institutional discontinuities: changes in procurement rules, the withdrawal of

discretionary exemptions, or the timing of local government elections across districts provide variation in θ and α that can be exploited with difference-in-differences designs. Third, threshold regression (Hansen, 2000) can test directly for the change in dynamics at $\Lambda = 1$ predicted by Proposition 3, using panel data on districts or cities.

Structural estimation. The full parameter vector can be estimated by the method of simulated moments (McFadden, 1989), matching the model's predictions for wealth shares, rent shares, the mobility matrix and their changes over time. Panel data on household transitions, such as the Pakistan Panel Household Survey, would identify u_i^0, d_i^0 and λ. The output would be an estimate of Λ_{cal} with a confidence interval — a direct, quantitative answer to whether the economy is above or below the capture threshold.

9. Limitations and Extensions

Fixed class boundaries. Classes are defined ex ante and their boundaries do not shift with growth. A natural extension allows the thresholds to move with the wealth distribution, or defines classes endogenously by clustering on asset portfolios and power channels.

Within-class distribution. The model is silent on inequality within classes. For C8 in particular, within-class concentration is likely to be extreme. A nested model in which each class has its own internal wealth–power dynamics would capture this.

Coalitions. Classes act independently in the contest. In practice, coalitions — for example, between C7 and C8, or between C4 and C5 — are central to political economy. Allowing coalition formation, with power pooled within a coalition, would generate richer dynamics and a theory of which coalitions are stable.

Endogenous institutions. The decisiveness parameter θ is treated as a policy choice. In the spirit of Acemoglu and Robinson (2008), the dominant class may itself shape θ. A simple extension, $\theta_{t+1} = \theta_t + \kappa(\pi_{8,t} - \bar{\pi})$, would make institutional drift endogenous and create a third feedback loop; the threshold analysis then applies to the augmented system.

Shocks and open-economy factors. The model is deterministic and closed. Floods, commodity shocks, IMF programmes, remittances and capital flight all affect the distribution and the rent stream. Adding stochastic shocks would allow the knife-edge result of Section 6.2 to be studied as a question of the long-run distribution of outcomes.

Regional calibration. The calibration here is stylised and urban. Rural Punjab has a different class structure, with landholding and *biradari* organisation playing a larger role; calibrating the model to rural Punjab, and then to other provinces, is a natural next step that would test the generality of the eight-class framework.

10. Conclusion

This paper has developed a formal model of the feedback between wealth and political power across eight economic classes. Its central message is that the stability of a plural society depends on a measurable quantity — the Wealth–Power Feedback Index Λ — which compares the strength of wealth-driven rent capture with the strength of renewal through markets and redistribution. Below the threshold, the economy settles on a stable distribution in which, nevertheless, organisational advantages are amplified by a factor that grows without bound as

the threshold approaches. Above the threshold, plural distributions are unstable and one class absorbs the rent stream, with history determining which.

The mobility channel compounds these effects. Concentration of power shifts the stationary class distribution downward in the sense of first-order stochastic dominance, hollowing out the middle classes whose numbers and organisation provide the main counterweight.

The illustrative calibration to a stylised urban Punjab economy suggests that the observed distribution may be close to the threshold, that heterogeneity in organisational capacity makes capture easier than symmetric reasoning suggests, and that redistribution unaccompanied by institutional reform can rotate the identity of the dominant class without ending dominance. The most robust remedies lower the decisiveness of discretionary allocation and the conversion of money into influence, while raising the political weight of numbers and the organisational capacity of the mass classes.

The eight-class structure is not merely descriptive. It allows the policy instruments that act on each link of the feedback loop to be targeted at the classes through which they operate, and it provides a framework within which socio-economic public policy can be evaluated by its effect on the stability of the class structure itself. Estimating the model's parameters, beginning with the decisiveness of rent contests, is the most important next step.

Appendix A. Notation

Table A1. Notation

Symbol	Definition
$i = 1, \ldots, 8$	Class index, ordered by baseline per-capita wealth
n_i, x_i	Population share and wealth share of class i
Δ	Unit simplex in $\mathbb{R}^8$
O_i	Organisational–positional capacity
π_i	Political power share, equation (1)
$H(\pi)$	Herfindahl index of power shares, $\sum_i \pi_i^2$
s_i	Share of politically allocated rents, equation (2)
a_i	Political endowment, $(n_i^{\alpha} O_i)^{\theta}$
k	Wealth–power elasticity of rents, $\beta\theta$
g_i	Gross return on retained wealth, equation (4)
m_i, ℓ_i	Market-income share and per-capita earning capacity
ε	Privileged-return term, $\eta\beta/[N(1+\bar{g})]$
Λ	Wealth–Power Feedback Index, equation (9)
Λ_{cal}	Spectral radius of the Jacobian of F at a heterogeneous steady state
u_i, d_i	Annual upward and downward mobility probabilities, equation (6)

Appendix B. Computational Details

Steady-state calibration of O_i. Given (x^0, n^0) from Table 1 and the baseline parameters, the required rent shares are obtained by inverting (5):

$$s^{req} = \frac{x^0 - \mu\, m - (1 - \rho - \mu)\, \tilde{x}(x^0)}{\rho},$$

which must be strictly positive; this holds for the calibration because $x_i^0/m_i > \mu/(\rho+\mu)$ for every class. The required power shares are $\pi^{req} \propto (s^{req})^{1/\theta}$ and the implied organisational capacities are $O_i \propto \pi_i^{req} / \left(n_i^{\alpha} (x_i^0)^{\beta}\right)$. Because $\tilde{x}$ depends on π through g, the procedure is iterated with damping until O converges (tolerance 10^{-12}), normalising $O_5 = 1$.

Mobility calibration. Downward base rates are assumed to be

$$d^0 = (0,\ 0.020,\ 0.025,\ 0.025,\ 0.020,\ 0.015,\ 0.010,\ 0.010).$$

Upward rates are set so that detailed balance holds at the baseline concentration, $u_i(H_0) = n_{i+1}^0 d_{i+1}(H_0)/n_i^0$, and then deflated by $(1 - \lambda H_0)$ to obtain u_i^0.

Simulation. Each period, power (1), rents (2), returns (4) and market shares are computed from the current state; wealth is updated by (5) and population by $n_{t+1} = n_t M(H_t)$. Phase and bifurcation diagrams hold population fixed and run 600–800 periods to approximate long-run outcomes.

Spectral radius. For each scenario, the interior steady state of (5) with population fixed is located by a Newton-type root finder on the simplex. The Jacobian is computed by central finite

differences, projected onto the tangent space using the basis $\{e_i - e_8\}_{i=1}^{7}$, and its spectral radius is reported as Λ_{cal}.

Verification of Proposition 3. With $n_i = 1/8$, $O_i = \ell_i = 1$, the economy was initialised at a small perturbation of the uniform distribution and simulated for 3,000 periods on a grid of $\theta \in [1,3]$ and $\varphi \in \{0,0.2,0.4\}$ (123 combinations). The simulated outcome agreed with the sign of $\Lambda - 1$ in every case.

Software. All computations were carried out in Python (NumPy, SciPy and Matplotlib). The code is available on request.

References

Acemoglu, D., Johnson, S., & Robinson, J. A. (2005). Institutions as a fundamental cause of long-run growth. In P. Aghion & S. N. Durlauf (Eds.), *Handbook of economic growth* (Vol. 1A, pp. 385–472). Elsevier.

Acemoglu, D., & Robinson, J. A. (2006). *Economic origins of dictatorship and democracy*. Cambridge University Press.

Acemoglu, D., & Robinson, J. A. (2008). Persistence of power, elites, and institutions. *American Economic Review, 98*(1), 267–293.

Alavi, H. (1972). The state in post-colonial societies: Pakistan and Bangladesh. *New Left Review, 74*, 59–81.

Banerjee, A., & Iyer, L. (2005). History, institutions, and economic performance: The legacy of colonial land tenure systems in India. *American Economic Review, 95*(4), 1190–1213.

Bardhan, P., & Mookherjee, D. (2000). Capture and governance at local and national levels. *American Economic Review, 90*(2), 135–139.

Bartels, L. M. (2008). *Unequal democracy: The political economy of the new gilded age*. Princeton University Press.

Bénabou, R. (2000). Unequal societies: Income distribution and the social contract. *American Economic Review, 90*(1), 96–129.

Border, K. C. (1985). *Fixed point theorems with applications to economics and game theory*. Cambridge University Press.

Cheema, A., Khwaja, A. I., & Qadir, A. (2006). Local government reforms in Pakistan: Context, content and causes. In P. Bardhan & D. Mookherjee (Eds.), *Decentralization and local governance in developing countries* (pp. 257–284). MIT Press.

Esteban, J., & Ray, D. (2006). Inequality, lobbying, and resource allocation. *American Economic Review, 96*(1), 257–279.

Gabaix, X., Lasry, J.-M., Lions, P.-L., & Moll, B. (2016). The dynamics of inequality. *Econometrica, 84*(6), 2071–2111.

Gilens, M., & Page, B. I. (2014). Testing theories of American politics: Elites, interest groups, and average citizens. *Perspectives on Politics, 12*(3), 564–581.

Grossman, G. M., & Helpman, E. (1994). Protection for sale. *American Economic Review, 84*(4), 833–850.

Hansen, B. E. (2000). Sample splitting and threshold estimation. *Econometrica, 68*(3), 575–603.

Husain, I. (1999). *Pakistan: The economy of an elitist state*. Oxford University Press.

Jalal, A. (1995). *Democracy and authoritarianism in South Asia: A comparative and historical perspective*. Cambridge University Press.

Krueger, A. O. (1974). The political economy of the rent-seeking society. *American Economic Review, 64*(3), 291–303.

McFadden, D. (1989). A method of simulated moments for estimation of discrete response models without numerical integration. *Econometrica, 57*(5), 995–1026.

Meltzer, A. H., & Richard, S. F. (1981). A rational theory of the size of government. *Journal of Political Economy, 89*(5), 914–927.

Mills, C. W. (1956). *The power elite*. Oxford University Press.

Mohmand, S. K. (2019). *Crafty oligarchs, savvy voters: Democracy under inequality in rural Pakistan*. Cambridge University Press.

Olson, M. (1965). *The logic of collective action: Public goods and the theory of groups*. Harvard University Press.

Olson, M. (1982). *The rise and decline of nations: Economic growth, stagflation, and social rigidities*. Yale University Press.

Pareto, V. (1935). *The mind and society* (A. Livingston, Ed.; A. Bongiorno & A. Livingston, Trans.). Harcourt, Brace. (Original work published 1916)

Piketty, T. (2014). *Capital in the twenty-first century* (A. Goldhammer, Trans.). Harvard University Press.

Skaperdas, S. (1996). Contest success functions. *Economic Theory, 7*(2), 283–290.

Stiglitz, J. E. (2012). *The price of inequality*. W. W. Norton.

Tullock, G. (1980). Efficient rent seeking. In J. M. Buchanan, R. D. Tollison, & G. Tullock (Eds.), *Toward a theory of the rent-seeking society* (pp. 97–112). Texas A&M University Press.

United Nations Development Programme. (2021). *Pakistan National Human Development Report 2020: The three Ps of inequality — Power, people, and policy*. UNDP Pakistan.

Winters, J. A. (2011). *Oligarchy*. Cambridge University Press.

Zaidi, S. A. (2015). *Issues in Pakistan's economy: A political economy perspective* (3rd ed.). Oxford University Press.