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Influence of Spatial Variability on the Bearing Capacity of Deep Soil Mixing Columns

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Influence of Spatial Variability on the Bearing Capacity of Deep Soil Mixing Columns

Abstract

Deep soil mixing (DSM) is widely used to improve the engineering properties of soft and weak soils. However, the performance of DSM columns is influenced by spatial variability, in-situ soil exhibits variability in moisture content, density, and particle size distribution that may influence the strength of DSM . Variability such as differences in binder distribution and degree of mixing leads to varriations in DSM's strength, stiffness, and geometry, affecting the performance of DSM columns. This study examined effects of spatial variability in the strength parameters and column diameter on the bearing capacity of laboratory-scale DSM columns. Using a finite volume model and Monte Carlo simulation, the study analyzed the effects of spatial variability on the mechanical response of DSM columns.

The results highlighted a reduction in bearing capacity of DSM columns due to spatial variability. Spatial variability in the strength parameters reduced the peak failure stress of the DSM column from approximately 570 kPa to 507 kPa, corresponding to a decrease of about 11% relative to the case with uniform strength properties. Variations in column diameter reduced the average peak stress to approximately 520 kPa. These findings demonstrate that neglecting spatial variability may lead to an overestimation of DSM column performance.

Keywords: Ground improvement, Deep soil mixing, Spatial variability, Bearing Capacity

1 Introduction

Ground improvement techniques are widely used in geotechnical engineering applications to improve the engineering properties of soft and compressible soils. Common methods include stone columns and the Deep Soil Mixing technique [1]. Several studies have examined the performance of stone columns, including their ability to increase bearing capacity, reduce settlement, and improve soil consolidation [2,3,4]. Other studies have explored how factors such as column spacing, stiffness, reinforcement, and variations in soil properties influence their effectiveness in improving ground performance and reducing settlement [5,6,7]. DSM has gained traction due to its relatively rapid construction process and ability to be employed over a wide range of soil conditions. Unlike conventional excavation and replacement methods, DSM improves the in-situ soil by mechanically mixing it with cementitious binders, thereby reducing the amount of material requiring removal and disposal. This not only lowers construction costs but also minimizes environmental impacts associated with waste handling and transportation [8,9].

The DSM technique involves blending the in-situ soil with cement, lime, slag, or other binding agents to create columns of improved soil. Depending on site conditions and project requirements, either wet or dry mixing techniques may be used. In wet mixing, a slurry binder is injected into the soil, whereas dry mixing relies on the introduction of powdered binders during

the mixing process [8,10]. Once cured, the treated columns provide increased strength and stiffness to the surrounding ground and can be installed in various configurations, including walls, blocks, and grid systems, to support foundations, embankments, retaining structures, and other geotechnical applications [10]. DSM columns, however, have more heterogeneity as compared to jet grouting [11].

For transportation infrastructure, the performance of improved ground is particularly important where embankments, approach fills, and foundation systems must remain serviceable over long periods [12]. Soft-ground deformation can affect both the stability of supported structures and their long-term settlement response. DSM is therefore relevant not only as a means of increasing soil strength but also as a method for controlling deformation beneath infrastructure constructed over weak deposits. Previous studies have examined the behavior of cement-mixed ground supporting embankments and the response of column-supported systems under applied loads [13,14,15]. In these applications, evaluating the average strength of treated material alone may not be sufficient when individual column sections respond differently under loading. Yano and Sadeghiamirshahidi [16], for example, showed that spatial variability in soil treatment can influence the strength and stress–strain response of stabilized soils, emphasizing the importance of considering non-uniform treatment when evaluating ground improvement performance.

DSM column performance depends largely on achieving a uniform distribution of binder throughout the treated soil. In the real world, however, achieving uniform mixing is difficult [17]. Variations in natural soil conditions (e.g., differences in moisture content and/or particle size distribution) can influence the distribution of binder within the treated column [18]. In addition to in situ soil conditions, operational factors such as binder injection rates, mixing efficiency, and equipment operation can lead to non-uniform distribution of the binder within the column [13,19]. As a result, the mechanical properties of the treated soil may vary significantly within the same column, producing zones with different levels of strength and stiffness.

Several researchers have reported that spatial variability is an inherent characteristic of DSM columns due to the complexity of the mixing process and the natural variability of soil properties [20,21,22]. Xiao et al. [23] investigated the influence of spatial variability in soil stiffness on tunnelling-induced ground settlement using random finite difference analysis. Their findings demonstrated that neglecting spatial variability in Young's modulus can underestimate the risk of excessive ground settlement, highlighting the importance of accounting for material heterogeneity in geotechnical numerical analyses.Variations in binder distribution can result in localized differences in cohesion, friction angle, and stiffness, which subsequently affect the load-bearing capacity of DSM columns [15,24,25]. This heterogeneous nature of DSM may create weaker regions within the treated soil, leading to uneven stress transfer and reduced bearing capacity under applied loads.

Importantly, spatial variability is not limited to the variation in measured soil properties but also encompasses the spatial distribution of stronger and weaker materials within the soil column,

including the extent to which these materials are clustered or dispersed. Two columns with similar average strength may therefore develop different stress distributions and deformation patterns if their weaker zones occur at different locations. Probabilistic studies of foundation soils have demonstrated the importance of accounting for uncertain strength properties in bearing-capacity assessments [24]. For DSM columns, this highlights the need for a numerical modeling approach that accounts for the spatial variability of cohesion, friction angle, and elastic modulus within the treated column, rather than assuming uniform average values for these properties.

In addition to material variability, geometric variability is also commonly observed in DSM construction. Variations in auger penetration rate, withdrawal speed, equipment alignment, and binder injection pressure can produce fluctuations in column diameter along the column length [10,19]. These geometric irregularities influence the surface area within the column and affect how loads are transferred through the DSM. Sections with smaller diameters are generally more susceptible to stress concentrations, while larger sections tend to provide greater resistance to applied loads [22,25,26]. Consequently, variability in column geometry may have a direct influence on the overall performance and reliability of DSM systems.

Geometric irregularity introduces a related but distinct source of uncertainty. A column may contain adequately treated material yet still exhibit changes in cross-sectional dimensions that influence the transfer of axial load. Research on variable-diameter deep mixing columns has demonstrated the engineering relevance of column configuration in soft-ground improvement [27]. However, it is useful to distinguish a deliberately designed variable-diameter column from unplanned variations that can arise during construction. The latter may introduce localized changes in stiffness and load resistance that are not represented by analyses based on an idealized, constant-diameter column. Assessing material and geometric variability separately can help identify the relative influence of these two mechanisms within a controlled numerical study.

The bearing capacity and settlement behavior of DSM columns are influenced by the mechanical properties of the treated soil and its interaction with the surrounding untreated ground. Jiao and Zhou [28] investigated the bearing capacity and settlement of deep cement-mixed ground using a bounding surface plasticity model and analytical methods. Their findings indicated that the area replacement ratio, strength of the cement-treated soil, and stiffness of the surrounding soil influenced load-bearing performance. Their study also identified limitations in conventional analytical methods, particularly their ability to account for nonlinear soil behavior and the contribution of untreated soil to bearing resistance. Spatial variability in soil strength presents an additional challenge when predicting the bearing capacity of foundations. Kawa and Puła [29] examined the bearing capacity of square and strip footings resting on spatially variable cohesive–frictional soils using three-dimensional numerical analysis and probabilistic methods. Their study represented cohesion and friction angle as spatially varying properties and evaluated the influence of spatial correlation on bearing capacity. The results showed that the scale of fluctuation influenced both the mean bearing capacity and its coefficient of variation, with these

values tending to stabilize as the correlation length increased. Their study, however, did not include the effects of spatial variability in the properties of the soil.

Although DSM technology has been widely adopted and extensively studied, comparatively limited attention has been given to the effects of spatial variability on bearing capacity. Many numerical and analytical studies assume uniform material properties and consistent column geometry throughout the treated ground. While these assumptions simplify the analysis, they may not adequately represent field conditions where some degree of variability is unavoidable. As a result, the bearing capacity predicted by idealized models may differ from the actual performance of DSM columns constructed in practice.

In this study, the influence of spatial variability in both material properties and column geometry on the bearing capacity of DSM columns subjected to axial loading were investigated. A 3D Finite Volume Model (FVM) was developed and validated using laboratory test results reported by Ni et al. [14]. The validated model was subsequently employed in a series of Random Finite Volume Model (RFVM) analyses and Monte Carlo simulations to evaluate the effects of variability in strength properties (cohesion, friction angle, and elastic modulus) and column diameter (thickness). The findings provide insight into how construction-induced variability influences the load-bearing performance of DSM columns and highlight the importance of incorporating these uncertainties into design and quality control practices.

2 Materials and Methods

Ni et al. [14] conducted a laboratory investigation to evaluate the bearing capacity of composite ground reinforced with DSM columns under axial loading. In their study, a DSM column installed in soft clay was subjected to incremental vertical loading until failure, while both stress and settlement responses were continuously monitored. The soft clay used in their experiment had a liquid limit of 74%, a plastic limit of 33%, and an undrained shear strength ranging from 7 to 11 kPa. The binder properties of the DSM column consisted of ground granulated blast-furnace slag (GGBS) and reactive magnesia (MgO), following the mix proportions proposed by Yi et al. [19]. The binder slurry was mechanically mixed with the clay to form a DSM column measuring 60 cm in height and 5 cm in diameter, representing a scaled laboratory model of a field-installed DSM column.

After curing the DSM, a sand cushion and loading plate were placed above the treated ground to simulate rigid footing conditions. Axial loading was then applied in increments of 10.6 kPa, with each load level maintained for approximately two hours. Failure of the DSM was defined when either the settlement of the current period increased five times greater than that observed during the previous loading stage or a settlement value exceeding twice that recorded in the preceding loading stage. Applied stresses were measured using pressure sensors installed on the loading plate, DSM column, and surrounding clay, while displacement transducers were used to monitor settlement throughout the test. After testing, the surrounding clay soil was excavated, and the

DSM column was recovered for further experimentation and analysis. The experimental configuration used by Ni et al. [14] is illustrated in Figure 1.

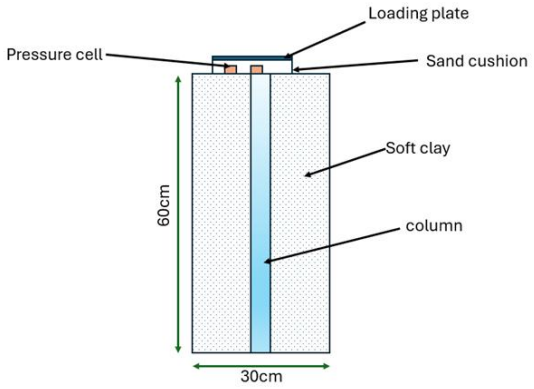


Figure 1: Schematic of lab experiment conducted by Pengpeng Ni et al. (2019) to investigate the effect of spatial variability on DSM columns

Unconfined compressive strength (UCS) tests were performed on samples collected from different locations along the column. The measured UCS values ranged from 459 to 549 kPa, indicating noticeable variability within the treated soil. This variability may be attributed to differences in binder distribution, mixing efficiency, and local curing conditions. The results highlighted the heterogeneous nature of DSM columns and emphasized the challenges associated with achieving a uniform distribution of strength throughout the treated ground.

In this research, a 3D FVM was developed using the commercial software FLAC3D to numerically simulate the geometry and loading conditions of the laboratory experiment. Experimental results reported by Ni et al. [14] were used to validate the numerical model before conducting further analyses. The geometry of the numerical model was chosen based on the experimental setup (See Figure 1) and is shown in Figure 2. The model consisted of a DSM column, sand cushion, surrounding soft clay, and a rigid loading plate. A total of 11,956 finite volumes were used in the model (160 for the DSM column, 255 for the sand cushion, and 11,541 for the surrounding clay). Fixed boundary conditions were applied along the sides and base of the model to simulate the laboratory test boundary conditions. A constant vertical displacement was imposed on the top surface to simulate the monotonic axial loading conditions used during the laboratory tests.

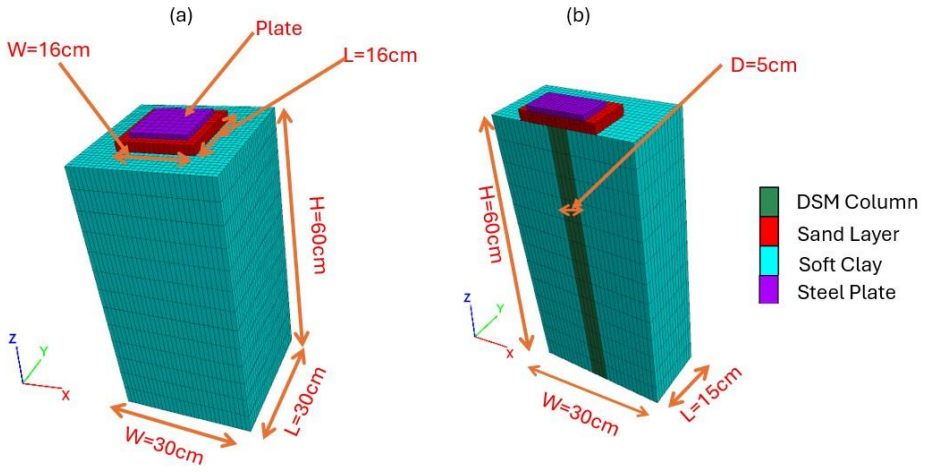


Figure 2: The 3D geometry of the validation finite volume model having uniform cohesion along the DSM column: a) shows the geometry of the 3D model, b) shows a section view cut at the midpoint of the model (revealing the DSM column)

The surrounding clay and sand cushion were modeled using the Mohr-Coulomb constitutive model. The loading plate was modeled to behave elastically. The material properties used in the numerical model simulation are summarized in Table 1.

Type	Unit Weight Υ (kg/m^3)	Elastic modulus, E (mPa)	Poisson's ratio, ν	Friction angle, ϑ	Cohesion (kPa)
Soft clay	1200	1.5	0.49	0	10
Sand	2000	20	0.3	30	0
Rigid footing	7850	210,000	0.15	N/A	N/A

Table 1: Material properties used for validation of the finite volume model

A strain-softening constitutive model was adopted for the DSM column, as cement-treated soils typically exhibit a reduction in strength after reaching peak strength. This approach allowed the model to capture both peak strength and post-peak softening observed in cement-stabilized soils. The variation in strength parameters with accumulated plastic strain is presented in Table 2. The cohesion and friction angle at the initial yielding point (plastic strain = 0) were estimated from the average of the UCS tests. The strain-softening behavior was modeled by assigning plastic strain-dependent cohesions and friction angles after the yield. The post-yield cohesion and friction angle parameters were determined through an iterative trial-and-error process to achieve agreement between the simulated and experimentally observed stress–strain responses of the DSM columns.

Mohr-coulomb		
Plastic strain	Cohesion (kPa)	Friction angle ϕ

0	120	35
0.2	125	38
0.5	125	37
1.5	125	33

Table 2: Strength softening properties used for model validation based on uniformly distributed DSM columns.

Following model validation, the numerical framework was used to investigate two forms of spatial variability commonly encountered in DSM construction. The first considered variability in material properties: cohesion (C), friction angle (ϕ), and elastic modulus (E), often associated with the non-uniform binder distribution within the column. The second examined variability in column geometry resulting from fluctuations in column diameter along the column length.

To simulate variability in binder distribution on the DSM column, spatial variability was introduced into the material properties of the column. Random values of cohesion, friction angle, and elastic modulus were assigned to individual finite volumes within the DSM column. The variability was intended to represent the heterogeneous conditions that commonly develop during DSM construction due to differences in binder distribution and mixing efficiency. A normal probability distribution was used to generate the random material properties throughout the column. Cohesion values were assigned using a maximum value of 120 kPa with a standard deviation of 0.3. The maximum cohesion represented regions that were fully treated with binder, while lower values represented zones with reduced cement content or less effective mixing. Similarly, elastic modulus values were generated using a maximum value of 20,000 kPa with a standard deviation of 5, while friction angle values were assigned using a maximum value of 38° and a standard deviation of 1.5°. These distributions produced realistic variations in strength and stiffness within the DSM column and allowed the numerical model to capture the effects of material heterogeneity observed in practice.

An example of the spatial distribution of the randomly assigned material properties is shown in Figure 3. The generated realizations show that zones with different strength and stiffness characteristics were distributed randomly throughout the column, reflecting the variability typically associated with field-installed DSM columns. A total of 5000 random finite volume simulations were performed, with a new set of material properties generated for each realization. The resulting stress-settlement responses and peak bearing capacities were then evaluated to quantify the influence of variability in cohesion, friction angle, and elastic modulus on the overall performance of the DSM column.

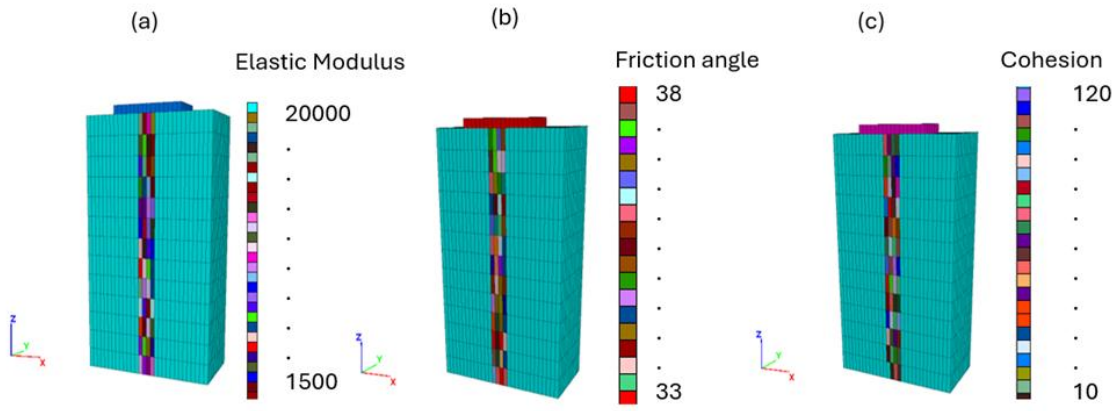


Figure 3: Random spatial variability in the DSM column model: (a) shows the distribution of elastic modulus values randomly assigned to finite volumes, ranging from 1500 kPa to 20,000 kPa; (b) illustrates the random distribution of friction angle values varying between 33° and 38°; and (c) displays the random allocation of cohesion values, ranging from 10 kPa to 120 kPa.

To evaluate the influence of geometric variability, the DSM column was divided into four segments along its length. Each segment was assigned a randomly generated height and diameter while maintaining a constant overall column height of 60 cm. The segment heights were generated such that the sum of all segments equaled the total column length. The randomly generated column diameters varied between 4 cm and 6 cm, representing realistic geometric deviations that may occur during laboratory or field installation.

Examples of the randomly generated column geometries are shown in Figure 4. A total of 5000 random finite volume simulations were performed using different combinations of segment heights and diameters. For each realization, the stress-settlement response and peak bearing capacity were recorded. The resulting dataset was subsequently used in the Monte Carlo analysis to evaluate the probabilistic effects of geometric variability on the load-bearing performance of the DSM column.

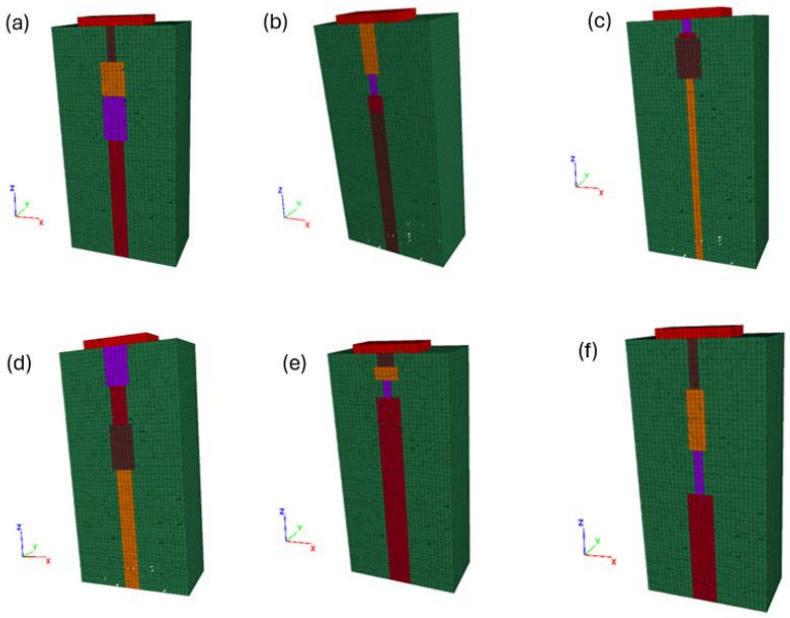


Figure 4: The 3D geometry of the finite volume model having variable column diameter and height along the DSM column: a) to f) shows possible outcomes of having different lengths and diameters of cement distribution in the column

2.1 Monte Carlo Simulations

Monte Carlo simulation is a probabilistic technique commonly used to evaluate uncertainty in systems influenced by random variables. The method involves generating a large number of realizations based on predefined probability distributions and analyzing the resulting outcomes statistically. In geotechnical engineering, Monte Carlo simulation has been widely applied to investigate the effects of variability in material properties, geometry, and loading conditions on system performance and reliability.

In this study, Monte Carlo simulation was used in conjunction with the RFVM to evaluate the effects of spatial variability on the bearing capacity of DSM columns. Two independent datasets were generated: one consisting of 5000 peak bearing capacity values obtained from DSM columns with randomly distributed cohesion, friction angle, and elastic modulus, and a second dataset consisting of 5000 peak bearing capacity values obtained from columns with variable diameters. The datasets were subsequently analyzed to quantify the statistical characteristics of the system. Parameters, including the mean, standard deviation, variance, and coefficient of variation (COV), were calculated to assess the magnitude of variability introduced by the random input parameters. Probability density functions were then fitted to the simulation results using both normal and log-normal distributions. This analysis provided insight into the distribution of bearing capacities and allowed the reliability and expected performance of DSM columns under spatially variable conditions to be evaluated.

3 Results and Discussions

Figure 5 compares the stress-settlement response obtained from the FVM with the experimental results reported by Ni et al. [14]. The close agreement between the numerical and experimental curves demonstrates that the developed model accurately reproduces the load-bearing behavior of the DSM column under axial loading. In particular, the strain-softening constitutive model successfully captured both the peak stress and the post-peak response, providing confidence in the model's ability to simulate the behavior of cement-treated soils.

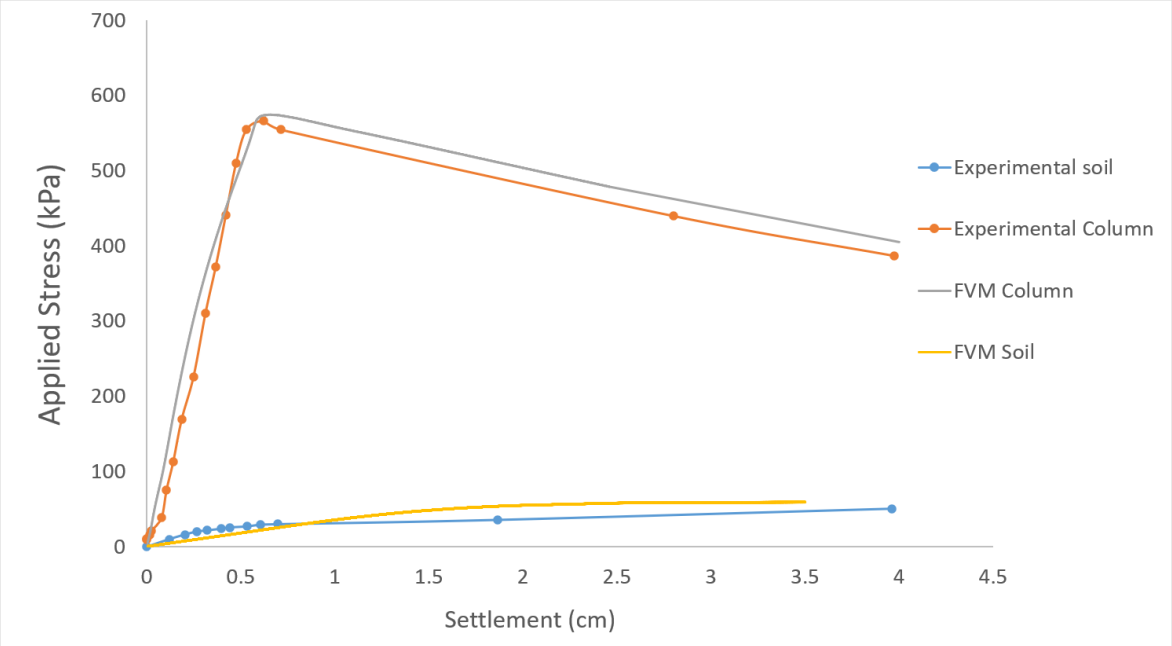


Figure 5: Experimental and FVM simulated deviatoric stress-settlement of the DSM column and soft clay soil (uniform distribution in the amount of cement in the column)

Following validation of the model, the calibrated model was used to investigate the influence of spatial variability on the bearing capacity of DSM columns. Figure 6 presents a representative stress-settlement response for a DSM column with randomly distributed strength parameters. Compared to the DSM column with uniformly distributed strength properties, the introduction of variability in strength parameters resulted in a reduction in peak bearing capacity. This reduction can be attributed to the development of weak localized zones with lower stiffness and strength, which limit the ability of the column to mobilize its full load-carrying capacity. As a result, failure occurs at lower stress levels than those observed in a column with uniform material properties, as can be observed in the graph.

A similar trend was observed when variability in column diameter was introduced (Figure 7). Variations in diameter produced a non-uniform stiffness distribution along the column length, resulting in uneven load transfer and reduced peak stress-bearing capacity. Sections within the column with smaller diameters may deform earlier under the same applied stress magnitude as compared to larger radius sections. Consequently, the overall response of the column was

governed by its weaker sections, leading to a reduction in bearing capacity compared with the uniform-diameter column.

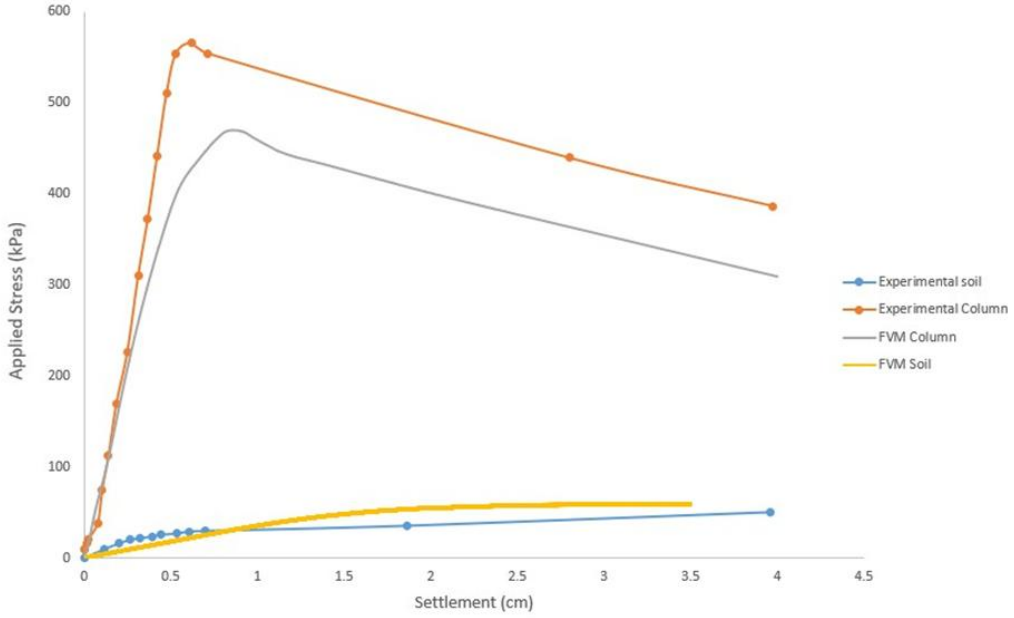


Figure 6: Experimental and FVM simulated deviatoric stress-settlement of the DSM column having spatial variability in strength parameters (cohesion, angle of internal friction, and elastic modulus) and soft clay soil.

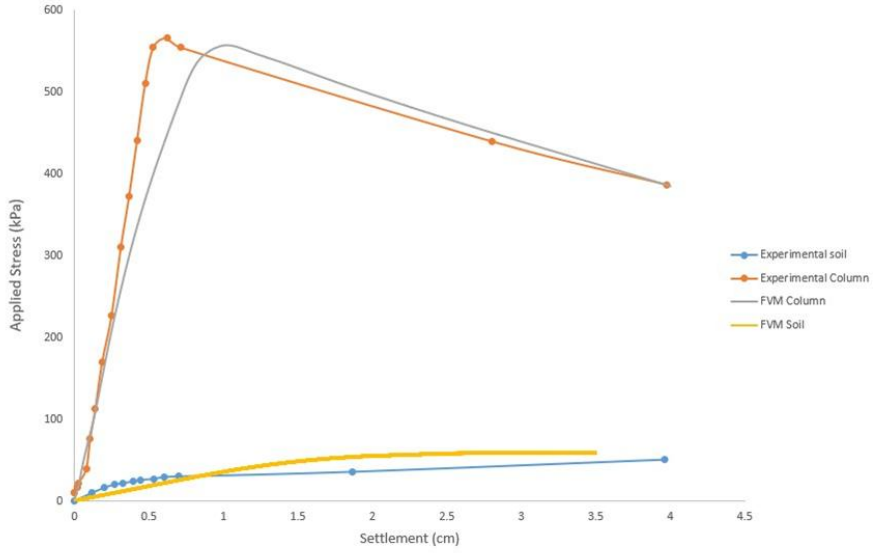


Figure 7: Experimental and FVM simulated deviatoric stress-settlement of the DSM column and soft clay soil with variability in the size of the column diameter.

Figure 8 presents the probability distributions of the randomly generated material properties and the corresponding bearing capacities obtained from the Monte Carlo simulations. The fitted probability distributions closely matched the predefined input parameters, indicating that the

numerical framework successfully represented the intended spatial variability within the DSM column. The resulting bearing capacities followed an approximately normal distribution with a mean value of 507.41 kPa, a standard deviation of 24.71 kPa, and a coefficient of variation (COV) of 4.9%.

Compared with the peak bearing capacity of approximately 570 kPa obtained for the uniform column (Figure 6), the average capacity decreased by approximately 11% when variability in material properties was considered. This reduction highlights the influence of non-uniform binder distribution on DSM column performance and suggests that analyses assuming homogeneous material strength properties may overestimate the designed field performance. Despite the reduction in mean capacity, the relatively low COV indicates that the column response remained reasonably consistent across the simulated realizations.

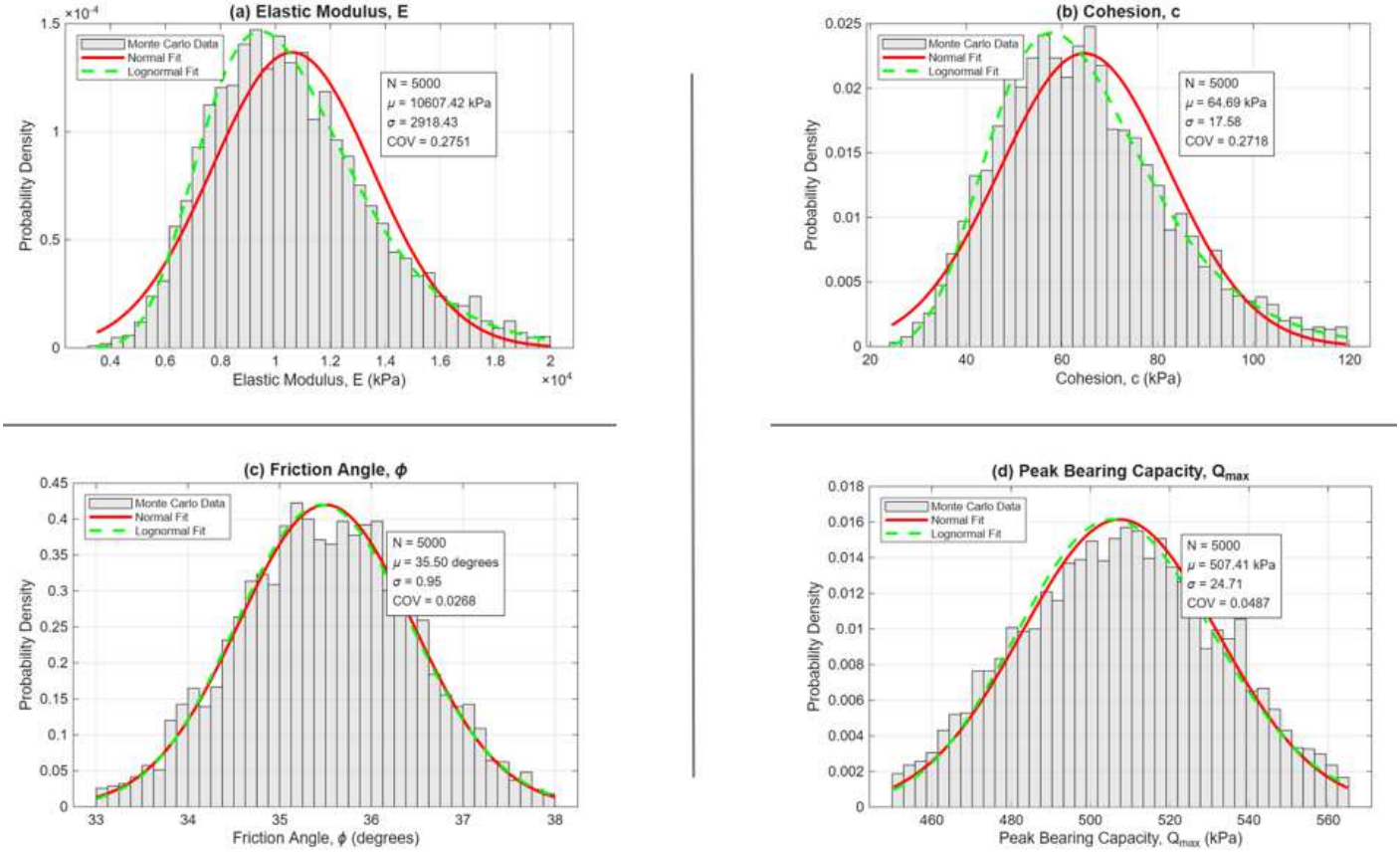


Figure 8:The results of the Monte Carlo simulation for DSM having random strength parameters (cohesion, Young's modulus, and friction angle)

The effects of simulated variability in the DSM column diameter are shown in Figure 9. The simulated bearing capacities exhibited a mean value of 519.68 kPa, a standard deviation of 19.63 kPa, and a COV of 3.8%. Although variability in column diameter also reduced the bearing capacity, its effect was less pronounced compared to that associated with variability in material properties. The average bearing capacity decreased by approximately 8.8% relative to the DSM

column with uniform diameter throughout the column height, indicating that geometric irregularities also influence the performance of the DSM columns.

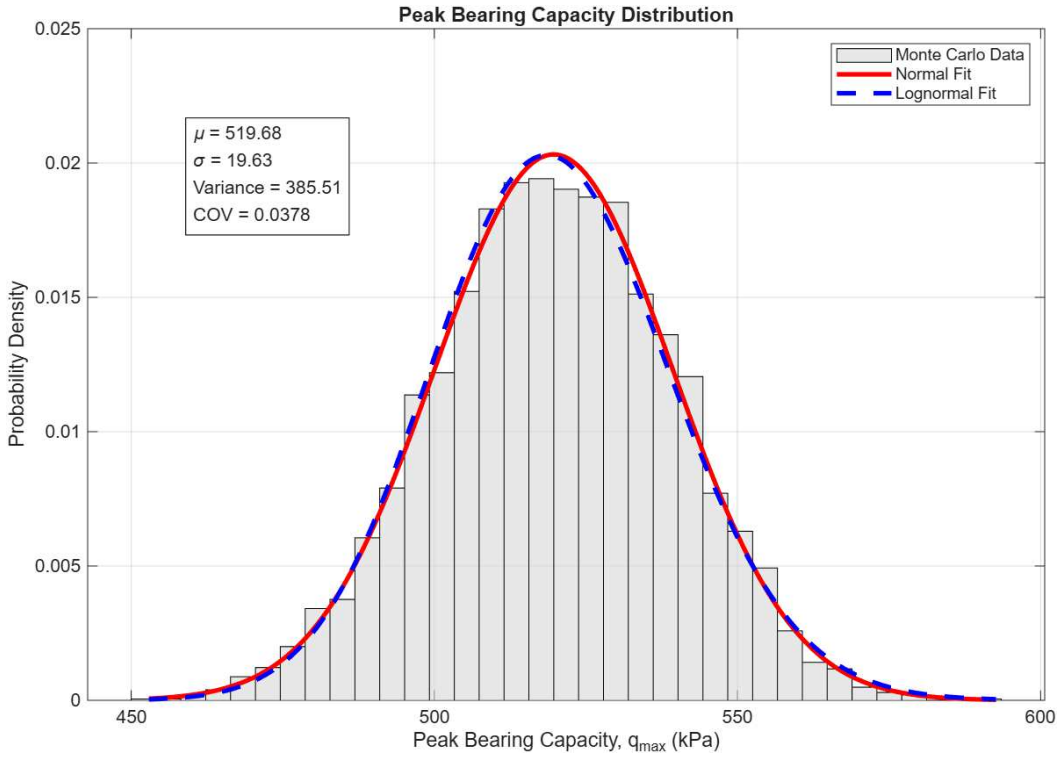


Figure 9:Monte Carlo simulation for DSM column having random column diameters but uniform Strength parameters

Overall, the results demonstrate that both material and geometric variability reduce the load-bearing capacity of DSM columns. However, variability in strength parameters had a slightly greater influence on DSM column bearing capacity performance than variability in diameter. These findings emphasize the importance of achieving uniform binder distribution during construction and suggest that quality control measures aimed at minimizing material heterogeneity may have a greater impact on performance than minor variations in column geometry.

It is important to note that the present study considered variability in material properties and variability in column geometry independently. In practice, however, these sources of uncertainty occur simultaneously and may interact in ways that further influence the mechanical response of DSM columns. A column exhibiting non-uniform strength properties may also experience variations in diameter along its length, resulting in a more complex load-transfer mechanism than that captured by either source of variability independently. The simultaneous variability in strength properties and column geometry was not investigated in this study and should be considered in future studies.

4 Conclusions

This study investigated the effects of spatial variability on the bearing capacity of DSM columns using FVM and Monte Carlo simulations. The results showed that both material and geometric variability influence the load-bearing capacity of DSM columns. Variability in strength parameters, represented by random distributions of cohesion, friction angle, and elastic modulus, resulted in a slightly greater reduction in bearing capacity as compared to variability in column diameter. This suggests that the mechanical response of DSM columns is more sensitive to non-uniform binder distribution than to moderate changes in column geometry. Localized zones with reduced strength and stiffness can limit the ability of the column to mobilize its full load-carrying capacity, resulting in lower bearing capacities than those predicted using uniform material properties and geometry.Variability in material properties had a greater effect on bearing capacity than changes in column diameter. When variations in cohesion, friction angle, and elastic modulus were introduced, the mean peak bearing capacity decreased from approximately 570 kPa to 507.41 kPa, representing a reduction of about 11%. This reduction suggests that the performance of a DSM column can be influenced by weaker sections within the treated soil. Even when most of the column has relatively high strength, areas with lower strength and stiffness may affect how the column responds to loading and ultimately limit its bearing capacity.

Variations in column diameter also affected bearing capacity, although the reduction was slightly smaller. The mean peak bearing capacity decreased to 519.68 kPa, approximately 8.8% lower than that of the uniform column. Changes in diameter alter the cross-sectional area available to resist applied loads, which may affect stress distribution along the column. Sections with smaller diameters may therefore become more susceptible to deformation under loading. Although the results showed that material variability had a greater influence, they also indicate that changes in column geometry should not be overlooked when evaluating DSM column performance.

These findings are relevant to ground improvement projects where DSM columns are used to support embankments and other transportation infrastructure constructed on soft soils. In practice, differences in soil conditions, binder distribution, and mixing efficiency make it difficult to achieve uniform properties throughout an installed column. Similarly, maintaining a constant column diameter along its entire length can be challenging. The results suggest that assuming uniform material properties and geometry may lead to an overestimation of bearing capacity. Considering these variations during numerical analysis and project design may therefore provide a better understanding of how DSM columns are likely to perform under actual construction conditions.

There are, however, limitations to the study. The numerical model was developed using laboratory-scale experimental data, and material and geometric variability were investigated separately. In the field, these variations may occur together, and their combined effects could differ from those observed in this analysis. The results are also specific to the soil properties, column dimensions, and ranges of variability considered. Further research could examine the

combined influence of material and geometric variability, the effects of spatial correlation, and how the numerical results compare with measurements from field-installed DSM columns.

Overall, the study shows that spatial variability has a measurable influence on the bearing capacity of DSM columns. Both material and geometric variations reduced bearing capacity, with material variability producing the greater reduction under the conditions examined. The findings reinforce the need to consider the non-uniform nature of DSM columns when predicting their performance. Incorporating spatial variability into numerical analyses can provide a more realistic basis for evaluating bearing capacity and may help engineers make better-informed decisions during the design and construction of ground improvement systems.

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