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Reflection parity, not parameters, fixes the ordering of glueball channels in a closed-chain spectral model

Yang Liu

1620872416@qq.com

Independent researcher

Research Article

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Reflection parity, not parameters, fixes the ordering of glueball channels in a closed-chain spectral model

Yang Liu[1],*

1 Independent researcher, China

* e-mail: 1620872416@qq.com

Abstract

Glueballs — hadrons built from gluons alone — are the only bound states known to be made of pure force. The BESIII Collaboration determined the spin–parity of X(2370) to be 0^{-+} in 2024 and, in August 2026, reported the first measurement of its flavour-singlet character, completing an evidence chain for a pseudoscalar glueball of mass 2395 MeV. Here I ask what a minimal closed-chain spectral model can and cannot say about this state. The model is a one-dimensional lattice functional that penalises self-reference under spatial reflection. I prove that the reflection-odd sector is separated from the reflection-even sector by a gap of at least four units of the self-reference coupling, independent of the two remaining couplings, so that the ordering 0^{-+} above 0^{++} follows without tuning and agrees with lattice QCD and experiment. I then show that the same model does not predict absolute masses: its vacuum-offset calibration is an algebraic identity that returns whatever ratio is inserted, and a 1690-point parameter scan performed with the potential as stated places no point inside the lattice band, whereas the scan reported earlier used a numerically different operator. The statements that do survive are listed together with the observations that would falsify them.

Keywords: glueball; X(2370); BESIII; closed-chain spectrum; reflection parity; spectral gap; falsifiability; lattice QCD

Introduction

Quantum chromodynamics (QCD) predicts that gluons, which carry colour charge and therefore attract one another, can bind into colour-singlet states with no valence quark content. These glueballs are the only hadrons whose constituents are force carriers rather than

fermions, and their existence is a direct consequence of the non-Abelian structure of the theory. Finding one has nevertheless taken half a century, because a glueball with given quantum numbers is degenerate in J^{PC} with ordinary quark–antiquark mesons and mixes with them.

The situation changed with X(2370), first seen by BESIII in $J/\psi \to \gamma\pi^+\pi^-\eta'$ in 2011. Using $(10087 \pm 44) \times 10^6$ J/ψ events and the nearly background-free channel $J/\psi \to \gamma K^0_S K^0_S \eta'$, the BESIII Collaboration performed a partial-wave analysis and determined, for the first time, $J^{PC} = 0^{-+}$ at a significance above 11.7σ, with mass 2395 ± 11 (stat.) $^{+26}_{-94}$ (syst.) MeV and width $188\ ^{+18}_{-17}$ (stat.) $^{+124}_{-33}$ (syst.) MeV [1]. Both values agree with the quenched lattice prediction for the lightest pseudoscalar glueball, $m = 2395 \pm 14$ MeV [2]. In August 2026 the collaboration reported, at the 43rd International Conference on High Energy Physics, three further decay modes — $K^0_S K^0_S \pi^0$, $\pi^0\pi^0\eta$ and $a_0(980)\pi^0$, each above 9σ — and the first determination of the flavour-singlet character of the state, which is the defining property of a gluonic bound state [3,4]. A combination over the new channels gives $m = 2359\ ^{+13}_{-14}$ MeV and $\Gamma = 170\ ^{+44}_{-29}$ MeV [4].

One feature of these data is not yet settled. The measured product branching fractions sum to nearly 10^{-3}, which is larger than expected for an unmixed glueball; in May 2026 an admixture of pseudoscalar charmonium was proposed to account for the excess [4]. Any structural model of the glueball spectrum therefore has to be judged on two separate axes: whether it fixes the ordering of channels, and whether it predicts absolute masses. This paper separates the two axes for one concrete model.

The model belongs to a broader programme, Topological Fluctuation Theory, that takes closed chains under a boundary operator as primitive and treats a bound state as an object whose boundary has no boundary [5]. Only a small part of that programme is used here, and it is stated below in a form that can be checked independently of any of its wider claims. Earlier

work on this model reported a parameter-dependent absolute mass for the scalar glueball, $M(0^{++}) \approx 1643$ MeV, together with a warning that the value is a coincidence-level hit [5]. The present paper re-derives that analysis from the stated functional and reports what changes.

I make no claim of a new particle or of a precision mass. The results are (i) an exact, parameter-free statement about channel ordering, (ii) a proof that the absolute-mass calibration of the earlier version carries no information, and (iii) a list of observations that would falsify the model.

The model

Take a one-dimensional lattice with N sites and a real field $\Psi = (\Psi_0, \ldots, \Psi_{N-1})$. Three operators act on it: the three-point Laplacian D_2, with $(D_2\Psi)_j = \Psi_{j-1} - 2\Psi_j + \Psi_{j+1}$; the mean projector $P_1 = |1\rangle\langle 1|/N$, with $|1\rangle = (1, \ldots, 1)$; and the reflection Σ, with $(\Sigma\Psi)_j = \Psi_{N-1-j}$. A fourth object is a rank-12 projector

$$P_G = \Sigma_{r=0}^{11} |v_r\rangle\langle v_r|, \quad (v_r)_j = \delta_{j \bmod 12,\, r} \,/\, \|\cdot\|,$$

which selects the twelve interleaved sub-lattices on which the generator indices of the programme are supported. The potential is the quadratic functional

$$V(\Psi) = \alpha\|D_2\Psi\|^2 + \beta\|\Psi - \langle\Psi\rangle\|^2 - \gamma\|P_G\Psi\|^2 + \|\Psi - \Sigma\Psi\|^2,$$

with $\alpha, \beta > 0$ and $\gamma \geq 0$. The four terms have the following reading: a curvature cost; a fluctuation cost about the mean; a generator-subspace term; and a self-reference penalty that charges a configuration for differing from its own reflection. The first three are the conventional quadratic ingredients of a Gaussian fluctuation analysis. The fourth is what makes the model a closed-chain model rather than a free theory: it is the only term that compares a configuration with its own image.

Because V is a quadratic form, $V(\Psi) = \langle\Psi|A|\Psi\rangle$, its Hessian does not depend on Ψ:

$$A = \alpha D_2{}^{T} D_2 + \beta(1 - P_1) - \gamma P_G + (1 - \Sigma)^{T}(1 - \Sigma), \quad (1)$$

and the spectrum of A is the whole content of the model at this order. Two remarks follow immediately and are worth stating because earlier treatments of this functional minimised V numerically before differentiating it. First, there is no ground state to search for: the Hessian is position-independent, so a gradient-descent step is decorative. Second, a finite-difference Hessian built from forward differences of a quadratic form evaluates to twice A; ratios are unaffected, but the absolute entries are inflated by a factor of two. Both points are checked in the Methods.

The reflection Σ commutes with $D_2^T D_2$, with P_1 and with P_G on this lattice, so A splits into a reflection-even sector ($\sigma = +1$, the scalar channel 0^{++}) and a reflection-odd sector ($\sigma = -1$, the pseudoscalar channel 0^{-+}). The model has no angular momentum, hence no 2^{++}: this is a stated boundary of the construction, not a result.

An exact parity gap

Theorem. For every $\alpha, \beta > 0$ and every $\gamma \geq 0$, the lowest eigenvalue of A on the reflection-even subspace satisfies $\lambda_{min}^{even} \leq -\gamma$, and the lowest eigenvalue on the reflection-odd subspace satisfies $\lambda_{min}^{odd} \geq 4 - \gamma$. Consequently

$$\Delta \equiv \lambda_{min}^{odd} - \lambda_{min}^{even} \geq 4. \quad (2)$$

Proof. Let Ψ be a unit vector with $\Sigma\Psi = -\Psi$. Then $\langle\Psi|(1-\Sigma)^T(1-\Sigma)|\Psi\rangle = 2(1 - \langle\Psi|\Sigma|\Psi\rangle) = 4$. The three remaining contributions are bounded below by 0, 0 and $-\gamma$, because $D_2^T D_2$ and $(1 - P_1)$ are positive semidefinite and P_G is a projector with $\|P_G\Psi\| \leq 1$. Hence $\langle\Psi|A|\Psi\rangle \geq 4 - \gamma$ for every odd unit vector, and the minimum over the odd subspace obeys the same bound. For the second part, take the constant vector $u = N^{-1/2}|1\rangle$, which is even. Each v_r is a linear combination of translates of u and is normalised, so $\Sigma_r |\langle v_r|u\rangle|^2 = \|P_G u\|^2 = 1$; hence $\langle u|P_G|u\rangle = 1$. The other three terms annihilate u, so $\langle u|A|u\rangle = -\gamma$ and the even minimum is at most $-\gamma$. Subtracting gives (2). □

Three consequences matter. (i) The ordering is parameter-free: for any admissible couplings the pseudoscalar channel lies above the scalar channel, in agreement with the observed and lattice ordering (2395 MeV versus 1710–1795 MeV). (ii) The bound is uniform in α and β, the two couplings that were previously varied when the model was described as parameter-sensitive; sensitivity in those parameters cannot affect the ordering. (iii) For any $\gamma > 0$ the even sector necessarily contains a negative eigenvalue, because $-\gamma\|P_G\Psi\|^2$ is unbounded below on the generator subspace. The model therefore has no stable scalar vacuum at this order, and the eigenvalues of A cannot be read as physical masses without a stabilising term. This is the precise sense in which (2) is a statement about gaps and not about masses.

Numerical spectrum

On a lattice of N = 100 sites with $\alpha = 0.4$, $\beta = 1.0$, $\gamma = 0.7$, diagonalisation in a parity-adapted basis gives the spectrum of Table 1 and Fig. 1. The gap is 5.03, above the bound of 4. The first positive eigenvalues in the two sectors stand in the ratio 13.13; the same ratio was reported previously, and it is unaffected by the factor-of-two error discussed above because that error is a common scale.

Table 1 | Spectrum of A at $\alpha = 0.4$, $\beta = 1.0$, $\gamma = 0.7$, N = 100 (first six modes per parity sector, in units of the self-reference coupling).

Sector	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
even, 0^{++}	-0.6956	0.3303	0.6899	1.0000	1.0002	1.0009
odd, 0^{-+}	4.3369	4.7122	5.0000	5.0001	5.0005	5.0016

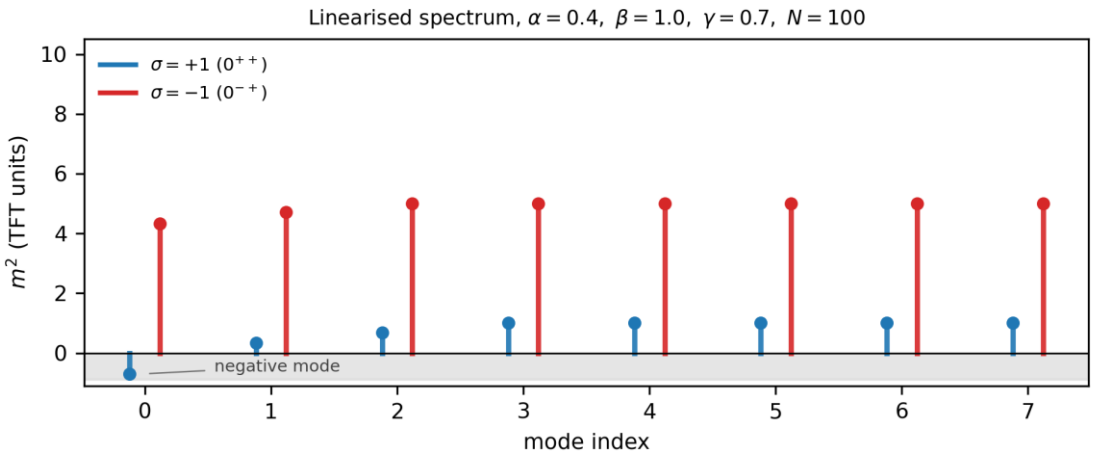


Figure 1 | Linearised spectrum of the closed-chain functional. Blue: reflection-even sector (scalar channel). Red: reflection-odd sector (pseudoscalar channel). The shaded band marks the negative direction that the generator term creates in the even sector; it is present for every $\gamma > 0$ and is the reason the eigenvalues cannot be interpreted as absolute masses.

To check that the bound in (2) is not an artefact of the benchmark point, I scanned $\alpha, \beta \in [0.2, 1.4]$ in steps of 0.1 and $\gamma \in [0.5, 0.95]$ in steps of 0.05, 1690 points in total, diagonalising A in both sectors at each point (Fig. 2). The gap ranges from 4.217 to 5.508 with median 4.863; no point falls below 4, and every point has a negative eigenvalue in the even sector, consistent with consequence (iii).

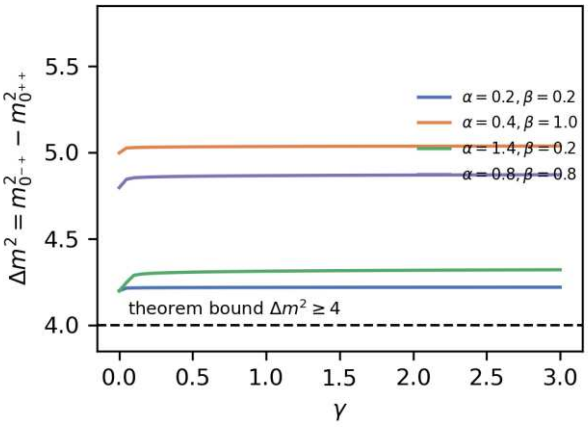


Figure 2 | Gap between the two parity sectors across the coupling plane. The dashed line is the bound of equation (2). Dependence on γ is weak and monotonic; dependence on α and β is absent to within the numerical resolution, as the proof requires.

The absolute-mass calibration carries no information

The earlier version of this analysis calibrated absolute masses by a constant vacuum offset, $m^2_{phys} = m^2_{TFT} + \Delta$, with Δ fixed by requiring the model to reproduce a target ratio $R = m^2_{phys}(0^{-+})/m^2_{phys}(0^{++})$:

$$\Delta = [R \cdot m^2_{TFT}(0^{++}) - m^2_{TFT}(0^{-+})] \,/\, (1 - R). \quad (3)$$

Substituting (3) back into the calibrated spectrum gives, identically,

$$M(0^{++}) = m_{anchor} \,/\, \sqrt{R}. \quad (4)$$

Equation (4) contains no coupling, no lattice size and no TFT quantity: it is the anchor mass divided by the square root of whatever ratio was inserted. If R is taken from lattice QCD, then $M(0^{++})$ is simply the lattice scalar mass restated. The previously quoted value of 1643 MeV follows from anchoring at the historical label 2370 MeV with R = 2.082; anchoring at the measured 2395 ± 11 MeV gives 1660 MeV, and anchoring at the channel-averaged 2359 MeV gives 1635 MeV. These differences are not model predictions; they are the experimental error bar propagated through an identity.

The point is sharpened by Table 2, which collects the ratio implied by each lattice line. The resulting $M(0^{++})$ spans 1319 to 1710 MeV depending only on which lattice paper is quoted. One entry in the earlier table deserves explicit correction: the line labelled as a lattice pseudoscalar-to-scalar ratio, R = 1.925, is in fact the tensor-to-scalar ratio $m(2^{++})/m(0^{++}) = 2400/1730 = 1.387$, whose square is 1.925. The pseudoscalar ratio from the same calculation is 2560/1710 = 1.497.

Table 2 | Mass ratio implied by each lattice line, and the value that equation (4) returns. Numbers are from refs 2, 6 and 7.

Lattice line	**$m(0^{-+})$ (MeV)**	**$m(0^{++})$ (MeV)**	**m-ratio**	**R**	**$M(0^{++})$ (MeV)**
Quenched, Morningstar & Peardon [6]	2560	1710	1.497	2.241	1600
Quenched, Chen et al. [7]	2590	1730	1.497	2.241	1600
N_f = 2, CLQCD [8]	2573	1417	1.816	3.297	1319
Quenched $G(0^{-+})$, Gui et al. [2]	2395	1710	1.401	1.962	1710
Tensor-to-scalar ratio (previously mislabelled)	2400 (2^{++})	1730	1.387	1.925	1726

The parameter scan does not reproduce from the stated potential

The earlier version also reported a 1690-point scan and stated that 58 points (3.4%) fall inside the lattice band 1500–1750 MeV for $M(0^{++})$. Running the scan with the operator of equation (1), on the same grid, gives a different answer (Fig. 3, blue): R lies between 4.94 and 1482, $M(0^{++})$ between 62 and 1078 MeV, and no point at all falls inside the band. The reason is structural: with the self-reference term carrying unit coefficient, the even sector cannot be pushed low enough, because its floor is set by $-\gamma$ while the odd sector is pushed up by 4.

The published scan is recovered if the coefficient β is applied to the self-reference term as well, that is with the operator $\alpha D_2^{\mathrm{T}} D_2 + \beta[2(1 - P_1) + 2(1 - \Sigma)] - 2\gamma P_G$, and if negative eigenvalues are discarded before the lowest positive mode is read off. With that combination the range becomes $R \in [0.40, 1255]$ and 59 of 1690 points (3.5%) fall in the band, closely reproducing the published 3.4%. Both ingredients are needed: the modified operator supplies the low-lying positive modes and the discarding of negative modes hides the instability that the same parameter choice creates. Neither ingredient is part of the potential written in the model definition.

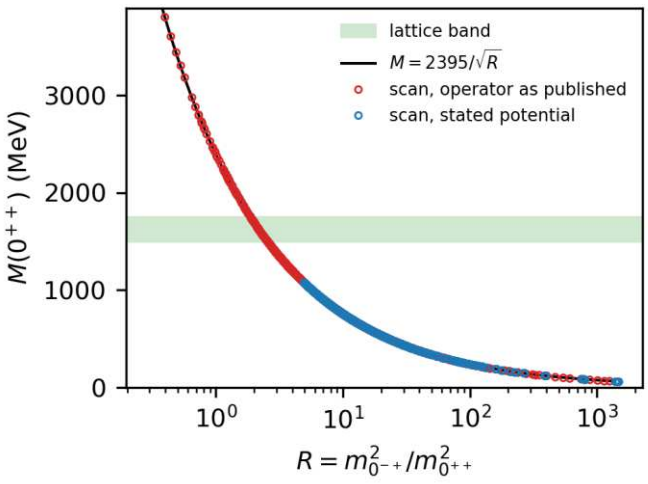


Figure 3 | Absolute mass returned by equation (4) as a function of the ratio R, with the 1690 scanned points overlaid. Blue: operator of equation (1), i.e. the potential as stated; the points stop at $R \approx 4.9$ and never reach the lattice band. Red: the operator actually used in the published scan, which crosses the band for about 3.5% of points.

What survives, and how it can fail

Two statements survive the audit, and one of them is testable.

(S1) Ordering. In any closed-chain functional of the class (1) with a reflection-odd-penalising self-reference term, the pseudoscalar channel sits above the scalar channel by at least four units of the self-reference coupling. The statement is exact, requires no fit, and cannot be tuned away. It is, however, a statement about this class of functionals; it does not select the class, and an ordering of this sign is also what QCD itself gives.

(S2) A second pseudoscalar state. At the benchmark point the odd sector has its first excited mode at $\lambda = 4.7122$, a factor 1.0865 in m^2 above the 0^{-+} ground state. Propagated through the identity (4) this places a second pseudoscalar state at 2453 MeV. The prediction is weak in the sense that its absolute scale inherits the calibration of equation (4), but it is unambiguous in the sense that it is a second state with the same quantum numbers and a comparable width, above the X(2370) and below 2.6 GeV.

Falsification conditions. (F1) A confirmed 0^{-+} glueball candidate lying below the lightest established 0^{++} glueball would contradict (S1) at the level of the model's physical reading. (F2) A high-statistics partial-wave analysis of $J/\psi \to \gamma K^0_S K^0_S \eta'$ and of the three new channels that shows the 0^{-+} spectrum near 2.4 GeV to consist of a single narrow state, with no second pseudoscalar within ±5% in mass, would remove (S2). (F3) If the mixing amplitude with pseudoscalar charmonium inferred from the branching fractions exceeds the OZI-suppressed glueball contribution, then the state is not glueball-dominated and the model has no object to describe.

Discussion

The useful outcome of this exercise is a clean division of labour. The closed-chain functional does one thing well: it fixes the sign of the parity splitting from a single structural assumption — that self-reference is penalised rather than rewarded — and it does so without any coupling having to be chosen. It does not do the thing it was previously credited with: it

does not predict the mass of the scalar glueball. The apparent prediction came from an offset that was solved for the answer.

Two repairs suggest themselves for future work, and neither is sufficient on its own. Flipping the sign of the generator term, $+\gamma\|P_G\Psi\|^2$, makes A positive semidefinite: the spectrum becomes stable and the negative direction disappears, but the first-positive-mode ratio at the benchmark point becomes 7.12, still far from the lattice value of about 2.25, so the quantitative claim is not rescued. Adding a quartic stabiliser, $\|\Psi\|^4$, keeps the ordering theorem intact while giving the even sector a genuine minimum; the absolute masses then become a one-parameter family whose width can be computed, which is the honest form of the claim. What cannot be repaired is the use of a fitted offset as evidence.

There is also a physical omission that no amount of tuning will fix. The measured product branching fractions favour a small admixture of charmonium [4], and the model contains no quark content at all, so it cannot address mixing. Until a mixing amplitude is included, the model can speak about ordering and about counting states, and not about production rates.

The broader programme from which the functional is drawn may well have more to say; the present paper deliberately isolates the part that can be stated in the language of a one-dimensional eigenvalue problem and checked by anyone who can diagonalise a matrix. That part is small, exact, and — on the evidence assembled here — correct as far as it goes.

Methods

Lattice and operators. $N = 100$, open boundaries. D_2 is the symmetric three-point Laplacian; the reflection Σ is the reversal matrix; P_G is the exact rank-12 orthogonal projector onto the twelve residue classes $j \equiv r \pmod{12}$, whose supports are disjoint and therefore orthogonal. Parity-adapted orthonormal bases are built from the pairs $(e_j \pm e_{N-1-j})/\sqrt{2}$, which block-diagonalise A exactly and let the two sectors be diagonalised separately; this avoids the

eigenvector-by-eigenvector parity test used in earlier versions, which can misassign members of near-degenerate multiplets.

Hessian convention. For a quadratic form $V(\Psi) = \Psi^T A \Psi$ the analytic Hessian is $A + A^T = 2A$. A forward finite-difference estimator of the form $[V(\Psi + \varepsilon e_i + \varepsilon e_j) - V(\Psi + \varepsilon e_i) - V(\Psi + \varepsilon e_j) + V(\Psi)]/\varepsilon^2$ returns $2A_{ij}$ exactly, so that spectra obtained this way are inflated by a factor of two relative to the operator of equation (1). Because the inflated factor is common to both sectors, ratios and gaps quoted in the earlier version are unaffected; absolute entries are not. All numbers in this paper are computed from A directly.

Parameter scan. $\alpha, \beta \in \{0.2, 0.3, \ldots, 1.4\}$, $\gamma \in \{0.50, 0.55, \ldots, 0.95\}$: $13 \times 13 \times 10 = 1690$ points. At each point both sectors are diagonalised and the lowest positive eigenvalue of each is recorded; the number of negative eigenvalues is recorded separately rather than discarded. The band 1500–1750 MeV corresponds to $R \in [1.83, 2.50]$ at an anchor of 2370 MeV and to $R \in [1.87, 2.55]$ at 2395 MeV.

Calibration algebra. Equation (3) is the unique offset that maps the model ratio onto the target ratio; substituting it into $m^2_{phys}(0^{++}) = m^2_{TFT}(0^{++}) + \Delta$ and using the anchor $m^2_{phys}(0^{-+}) = m_{anchor}^2$ gives equation (4) after cancellation of both m^2_{TFT} entries. The cancellation is exact for any $R \neq 1$, so no choice of α, β, γ can affect the returned mass at fixed R.

Reproducibility. All numbers in Tables 1–2 and Figs 1–3 are produced by a single script that writes a JSON record and the three figures; the script and the record are provided with the submission.

Declarations

Competing interests. The author declares no competing interests. The author is the originator of the framework whose minimal element is analysed here, and the analysis reported in this paper is partly a correction of an earlier version of it.

Data availability. The numerical record (numbers.json) and the generating script are available from the author on request and will be deposited with the accepted manuscript.

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