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Interpolation Error of SPRTs at the Melting Point of Gallium and Its Application

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Abstract: The present study explores interpolation errors at the melting point of gallium for four sub-ranges above the triple point of water covering from 0.01 °C to 660.323 °C in the temperature range of standard platinum resistance thermometers. A substitution of actual measurement for standard thermometers quality assessment is proposed on the grounds of error analysis. Interpolated value at the melting point of gallium by the ITS-90 interpolation from 0.01 °C to 419.527 °C is selected as the metric for assessing the quality of standard thermometers This approach would substantially reduce the number of measurements required at the melting point, thereby lowering measurement costs in the relevant subranges.

1. Introduction

For temperature ranges above the triple point of water (TPW, 0.01 °C), realisation of the International Temperature Scale of 1990 (ITS-90) [1] with standard platinum resistance thermometers (SPRTs), the measurement of resistance ratio at the melting point of gallium (MPGa, 29.7646 °C) W_{Ga} ought to be carried out since it is a quality control for thermometers. A SPRT is considered qualified when its W_{Ga} is equal to or greater than 1.11807; conversely, a W_{Ga} below 1.11807 results in rejection of such thermometer. However, Subranges of TPW to the freezing point of indium (FPIn, 156.5985 °C), TPW to the freezing point of tin (FPSn, 231.928 °C), TPW to the freezing point of zinc (FPZn, 419.527 °C) and TPW to the freezing point of aluminium (FPAl, 660.323 °C) are specified without MPGa. Therefore, interpolation errors between the interpolated value and actual measured value of MPGa arise. For any overlapping subrange between TPW to MPGa and one of the four aforementioned subranges, discrepancy between the interpolated and actual measured values of MPGa, i.e. interpolation error at MPGa, is the maximum of relevant overlapping subrange inconsistencies (SRIs). SRIs are of a very small magnitude, as are the interpolation

errors. Hence, it would be reasonable to assess the quality of a SPRT using interpolated value in stead of actual measurement value at MPGa.

This study assembles 38 SPRTs and conducts an analysis of interpolation errors at MPGa for the four relevant subranges. A discrimination method is proposed for quality evaluation and only a minuscule fraction of thermometers with interpolation values proximate to the quality threshold of 1.11807 need to be measured at MPGa.

2. Interpolation errors at MPGa of subranges covering from TPW to FPAl and their analysis

In the sub-range from TPW to FPIn, interpolation equation defined by the ITS-90 is

$$W(t) - W_r(t) = a_{In}[W(t) - 1] \quad (1)$$

where $W(t)$ is the resistance ratio defined as following

$$W(t) = R(t)/R_{H_2O} \quad (2)$$

$R(t)$ represents the resistance value of a thermometer at temperature t, and R_{H_2O} represents the resistance value of the thermometer at TPW. In equation (1), $W_r(t)$ refers to the reference function while a_{In} represents the coefficient of linear interpolation determined by resistance ratio W_{In} at FPIn.

In subrange from TPW to FPSn, interpolation equation defined by the ITS-90 is

$$W(t) - W_r(t) = a_{Sn}[W(t) - 1] + b_{Sn}[W(t) - 1]^2 \quad (3)$$

where a_{Sn} and b_{Sn} represent the interpolation coefficients determined by W_{In} and W_{Sn}.

In subrange from TPW to FPZn, interpolation equation defined by the ITS-90 is

$$W(t) - W_r(t) = a_{Zn}[W(t) - 1] + b_{Zn}[W(t) - 1]^2 \quad (4)$$

where a_{Zn} and b_{Zn} represent the interpolation coefficients determined by W_{Sn} and W_{Zn}.

In subrange from TPW to FPAl, interpolation equation defined by the ITS-90 is

$$W(t) - W_r(t) = a_{Al}[W(t) - 1] + b_{Al}[W(t) - 1]^2 + c_{Al}[W(t) - 1]^3 \quad (5)$$

where a_{Al}, b_{Al} and c_{Al} represent the interpolation coefficients determined by W_{Sn}, W_{Zn} and W_{Al}.

All these four subranges are specified without MPGa, interpolation errors are

engendered at this fixed point consequently. Let the interpolated values of the above four ITS-90 interpolations at MPGa be denoted as W_{Ga}^{In}、W_{Ga}^{Sn}、W_{Ga}^{Zn} and W_{Ga}^{Al} where superscripts correspond to relevant subranges respectively. W_{Ga} refers to actual measured value at MPGa. Then interpolation errors of these four subranges would be expressed as follows

i. interpolation error derived by equation (1) at MPGa

$$\Delta W_{Ga}^{In} = W_{Ga}^{In} - W_{Ga} \quad (6)$$

ii. interpolation error derived by equation (3) at MPGa

$$\Delta W_{Ga}^{Sn} = W_{Ga}^{Sn} - W_{Ga} \quad (7)$$

iii. interpolation error derived by equation (4) at MPGa

$$\Delta W_{Ga}^{Zn} = W_{Ga}^{Zn} - W_{Ga} \quad (8)$$

and

iv. interpolation error at derived equation (5) at MPGa

$$\Delta W_{Ga}^{Al} = W_{Ga}^{Al} - W_{Ga} \quad (9)$$

Data of 38 SPRTs are assembled for the comparison. 7 SPRTs are calibrated at Beijing Institute of Metrology (BIM) and 6 SPRTs are calibrated at National Institute of Measurement and Testing Technology, Chengdu (NIMTT). 25 SPRTs are calibrated worldwide. Information of sample SPRTs is listed in Table 1.

Table 1 Detailed information of 38 SPRTs

No.	SPRT	Calibration Institute	Data Source
1	Hart 1094	NIST	Ref. [2]
2	Hart 1098	NIST	Ref. [2]
3	Rosemount 4386	NIST	Ref. [2]
4	Hart 1032	NIST	Ref. [2]
5	Rosemount 4385	NIST	Ref. [2]
6	Hart 1030	NIST	Ref. [2]
7	Isotech 040	NIST	Ref. [2]
8	Rosemount 4727	NIST	Ref. [3]
9	Hart 1207	PTB	Ref. [4]
10	L&N 1773051	VSL	Ref. [5]
11	Isotech 054	CEM	Ref. [6]
12	Hart 1283	METAS	Ref. [7]
13	Tinsley 274686	NMI-VSL	Ref. [7]
14	YSI 4807	PTB	Ref. [7]
15	Tinsley 269586	NPL	Ref. [7]
16	Isotech 036	BNM-INM/CNAM	Ref. [7]
17	2	VNIIM	Ref. [8]

18	3-95	VNIIM	Ref. [8]

Table 1 cont'd Detailed information of 38 SPRTs

No.	SPRT	Calibration Institute	Data Source
19	s/n 948	VNIM	Ref. [9]
20	Isotech 136	KRISS	Ref. [10]
21	Isotech 160	KRISS	Ref. [10]
22	53	VNIIM	Ref. [11]
23	98-02	VNIIM	Ref. [11]
24	Rosemount 4726	PTB	Ref. [12]
25	Chino RS 93 Z-4	PTB	Ref. [12]
26	10252	BMI	Beijing
27	184267	BMI	Beijing
28	186165	BMI	Beijing
29	238697	BMI	Beijing
30	10222	BIM	Beijing
31	226298	BMI	Beijing
32	10280	BMI	Beijing
33	5084	NIMTT	Chengdu
34	5284	NIMTT	Chengdu
35	5124	NIMTT	Chengdu
36	5112	NIMTT	Chengdu
37	1620	NIMTT	Chengdu
38	1629	NIMTT	Chengdu

Comparison results show that all the interpolated values at MPGa are above 1.118074 while all the calibrated values of W_{Ga} is above the quality control of W_{Ga}, i.e., 1.11807 of this data collection. For a coherent and precise discussion, all calculation results would be displayed in two decimal places.

Two interpolation errors exceed 2.00×10^{-6} in resistance ratio which is approximately equivalent to 0.50 mK and seventeen errors exceed 1.00×10^{-6} which is approximately equivalent to 0.25 mK for subrange from the TPW to FPIn. The sample average of interpolation is 6.70×10^{-7} and standard deviation is 9.53×10^{-7}. Errors are scattered from -1.78×10^{-6} to 2.63×10^{-6}. All the interpolation errors for this subrange are illustrated in Figure 1.

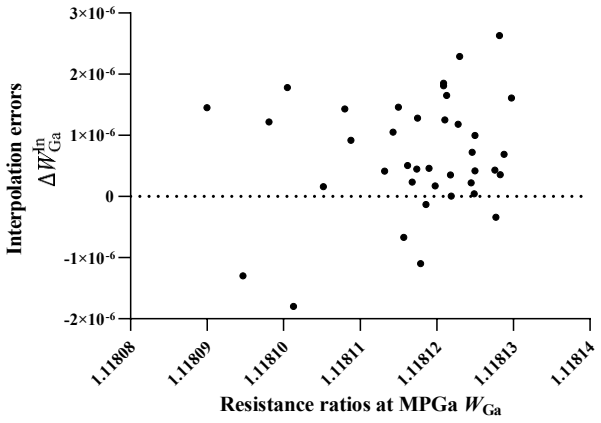


Figure 1 Interpolation errors at MPGa for the subrange from the TPW to FPIn

Six inconsistencies exceed 2.00×10^{-6} and twelve inconsistencies exceed 1.00×10^{-6} for subrange from the TPW to FPSn. The sample average of interpolation is -3.10×10^{-7} and standard deviation is 1.52×10^{-6} for this subrange and interpolation errors are scattered from -3.54×10^{-6} to 4.87×10^{-6}. All the interpolation errors for this subrange are illustrated in Figure 2.

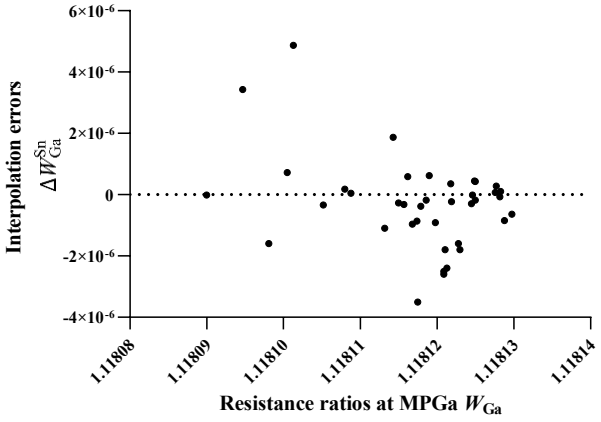


Figure 2 Interpolation errors at MPGa for the subrange from the TPW to FPSn

Two inconsistencies exceed 2.00×10^{-6} in resistance ratio and seven inconsistencies exceed 1.00×10^{-6} for subrange from the TPW to FPZn. The sample average of interpolation is -1.50×10^{-7} and standard deviation is 9.66×10^{-7} for this subrange. Interpolation errors are scattered from -2.33×10^{-6} to 2.50×10^{-6}. All the interpolation errors for this subrange are illustrated in Figure 3.

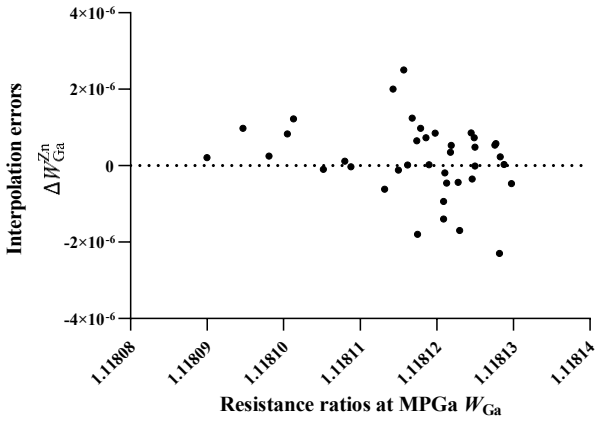


Figure 3 Interpolation errors at MPGa for the subrange from the TPW to FPZn

Three inconsistencies exceed 2.00×10^{-6} in resistance ratio and four inconsistencies exceed 1.00×10^{-6} for subrange from the TPW to FPAl. The sample average of interpolation is 9.74×10^{-7} and standard deviation is 1.37×10^{-6} for this subrange. Interpolation errors are scattered from -3.15×10^{-7} to 3.02×10^{-6}. All the interpolation errors for this subrange are illustrated in Figure 4.

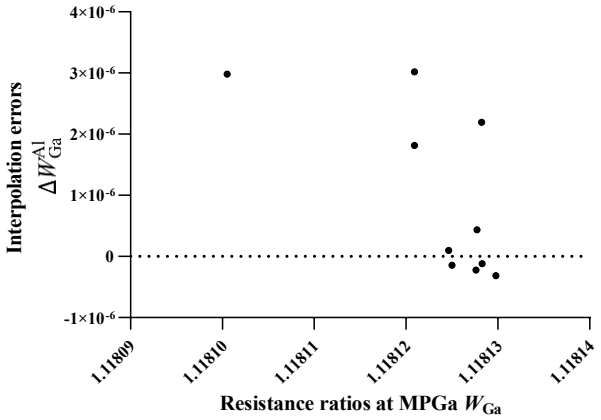


Figure 4 Interpolation errors at MPGa for the subrange from the TPW to FPAl

Outcomes of the present study are basically consistent with those of Peruzzi[13] which looked into SRIs from 83.8058 K to 660.323°C using long stem thermometers as well and inconsistencies of four relevant overlapping subranges are approximately - 0.50mK ~0.70 mK, - 0.60 mK ~0.80 mK, -0.70~0.50 mK, and - 0.50 mK~-0.05 mK at MPGa respectively. In Peruzzi's research, though interpolation from TPW to FPAl exhibits the smallest error at MPGa, data employed might be insufficient—only four SPRTs. Therefore, in the present study, considering there are only ten SPRTs in

subrange from TPW to FPAl, the best substitution for actual measured W_{Ga} would be interpolated value by the ITS-90 from TPW to FPZn, i.e. W_{Ga}^{Zn}. W_{Ga}^{Zn} possesses the smallest interpolation error mean and the second least standard deviation (only $1x10^{-8}$ larger than the smallest standard deviation for the TPW to FPIn) while maintains a robust sample size at the same time.

3. Application of the interpolation error outcomes

On the basis of this 38 SPRTs sample, W_{Ga}^{Zn} is chosen to substitute the actual measurement at MPGa. Such substitution would be applicable for quality assessment of a SPRT. For this specific subrange, maximum interpolation error at MPGa is 0.63 mK ($\Delta W_{Ga}^{Zn} = 4.87x10^{-6}$). To be prudent, estimation range is expanded to ±2.00 mK, triple of maximum error, at the quality threshold of 1.11807, then let

$$\frac{|W_{Ga} - 1.11807|}{[\frac{dW_r(t)}{dt}]_{Ga}} \leq \frac{|W_{Ga}^{Zn} - 1.11807|}{[\frac{dW_r(t)}{dt}]_{Ga}} \times 10^3 = 2 \text{ mK} \qquad (10)$$

Solving equation (10) yields $W_{Ga}^{Zn} \in$ [1.118062, 1.118078]. Therefore, while interpolated value W_{Ga}^{Zn} of a SPRT falls within the interval derived by (10), its actual measurement value of W_{Ga} would most likely falls within this same interval as well. Whether its W_{Ga} higher than the quality threshold or not is indeterminate and actual measurement should be carried out.

For feasibility verification of such process, data of 20 SPRTs were collected at National Institute of Metrology, China and only one thermometer presented a W_{Ga}^{Zn} of 1.118077 which falls within the inconclusive interval and therefore requiring actual measurement. All 20 values of W_{Ga}^{Zn} are plotted against their measurement value at FPZn, i.e. W_{Zn} in Figure 5. Shading area parallel to X-axis is zone of indecision, i.e. [1.118062, 1.118078].

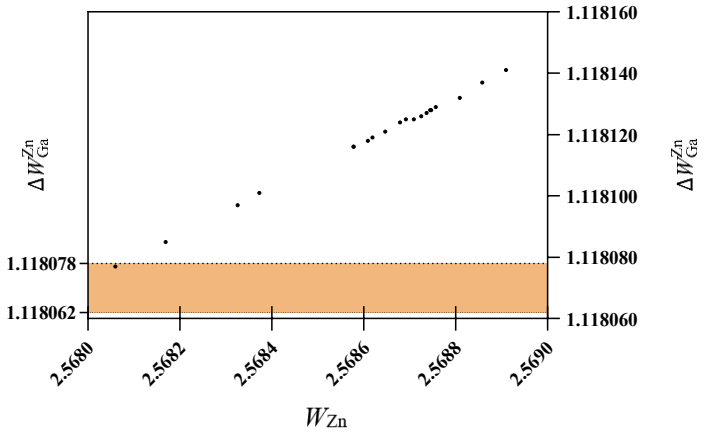


Figure 5 W_{Ga}^{Zn} for a collection of 20 SPRTs

4. Conclusion

Interpolation errors of four subranges TPW to FPIn, TPW to FPSn, TPW to FPZn and TPW to FPAl at MPGa are investigated in this study. Interpolated value of MPGa by the ITS-90 interpolation TPW to FPAl is selected to substitute actual measurement of this fixed point to determine whether a specific SRPT is qualified or not due to its outperforming comparison result and robust sample size. Suggested procedures would be that interpolated value of W_{Ga}^{Zn} is compared with the estimated interval of [1.118062, 1.118078] and: i) should $W_{Ga}^{Zn} \geq 1.118078$, it can be concluded that the measured value of relevant SPRT at MPGa is most likely higher than the quality threshold of 1.11807; ii) should $W_{Ga}^{Zn} \leq 1.118062$, actual measured W_{Ga} of relevant SPRT is most likely lower than the quality threshold of 1.11807; iii) should W_{Ga}^{Zn} falls within interval of [1.118062, 1.118078], actual measurement at MPGa then must be carried out. A sample set of 20 SPRTs is assessed with respect of this process and only one SPRT falls within the indeterminate zone. This prosess would therefore reduces measurement costs for the four ITS-90 subranges specified without MPGa from 0.01°C to 660.323°C.

Declarations

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References

[1] Preston-Thomas H 1990 The international temperature scale of 1990 (ITS-90) *Metrologia* **27** 3–10

[2] B W Mangum *et al* 2002 Summary of Comparison of Realizations of ITS-90 over the Range 83.8058K to 933.473K: CCT Key Comparison CCT-K3 *Metrologia* **39** 179-205

[3] Santiago, Josias Noe & Petkovic, S. & Teixeira, Renato & Noatsch, U. & Thiele-Krivoj, B 2003 Comparison of Fixed Point Realisations between Inmetro and PTB *Temperature: Its measurement and Control in Science and Industry (AIP Conf. Proc.)* **684** 849-854

[4] Murat Kalemci, Aliye Kartal Dogan, Sevilay Ugar, Ute Noatsch, Erich Tegeler and Bettina Thiele-Krivoj 2003 A Bilateral Comparison between UME and PTB *Temperature: Its measurement and Control in Science and Industry (AIP Conf. Proc.)* **684** 873-877

[5] E Renaot *et al* 2007 Final Report on EUROMET.T-K3: Regional key comparison of the realisations of the ITS-90 from 83.8058 K to 692.677 K *Metrologia* **44** 03001

[6] Dolores del Campo and Adrián Solano 2012 Final report on EURAMET.T-K3.3: Report on the comparison of the realizations of the ITS-90 over the range 83.8058 K to 933.473 K between Centro Español de Metrología (CEM) and Laboratorio Costarricense de Metrología (LACOMET) *Metrologia* **49** 03007

[7] U. Noatsch *et al* 2003 Intercomparison of the Realization of the ITS-90 in the Temperature Range -40°C to 420 °C *Temperature: Its Measurement and Control in Science and Industry (AIP Conf. Proc.)* **684** 855–860

[8] Sasho Nedialkov *et al* 2013 Final report on EURAMET.T-K3.1: Bilateral comparison of the realisations of the ITS-90 at the fixed points of Hg, H_2O, Ga, Sn and Zn *Metrologia* **50** 03002

[9] Duan Yuning *et al* 2005 CCT-K3: Analysis of the Results of Temperature key comparison of China *Modern Measurement and Laboratory Management* **3** 10-16.

[10] W Joung *et al* 2016 APMP.T-K3.4: key comparison of realizations of the ITS-90 over the range -38.8344 °C to 419.527 °C *Metrologia* **53** 03006

[11] Pokhodun A I *et al* 2020 Realization of the ITS-90 from 273.16 K to 692.677 K *Metrologia* **57** 03005

[12] Pokhodun A I *et al* 2019 Realizations of the ITS-90 from 273.16 K to 933.473 K (COOMET.T-K3.3) *Metrologia* **56** 03001

[13] Peruzzi A *et al* 2021 Survey of sub-range inconsistency of long-stem standard platinum resistance thermometer *Metrologia* **58** 1–13