Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

Growth Promotion Policies when Taxes Cannot be Raised

Katsunori Minami
Kyoto University of Advanced Science
Ryo Horii
horii@iser.osaka-u.ac.jp

The University of Osaka

Research Article

Keywords:

Posted Date: November 23rd, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11665147/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

Growth Promotion Policies when Taxes Cannot be Raised

Katsunori Minami* Ryo Horii†

This Version: November 8, 2026

Abstract

This paper examines the feasibility and effectiveness of growth promotion policies, such as R&D subsidies, when political constraints prevent the government from raising revenue. With sufficiently high life-cycle saving motives and high R&D productivity, our variety-expansion model shows that the growth rate exceeds the interest rate, and the government can finance its expenses while perpetually rolling over the debt. The economy is nevertheless dynamically efficient, because the social rate of return of R&D exceeds the growth rate. Given this situation, debt-financed R&D subsidies always accelerate technological progress in the short run. However, long-term growth can be improved only when R&D productivity exceeds another threshold. Our estimates suggest that this condition holds for most advanced countries, including the U.S., but not for low-growth countries such as Japan, Italy, and Greece. We then build a second-generation endogenous growth model that nests the baseline case, and show that low population growth is one reason why those countries do not meet the condition. We also extend the theory to include publicly funded basic research, unsustainable policies, asset income taxation, and a country premium.

Keywords: Endogenous growth, R&D, Public debt, Perpetual debt rollover, R minus G, Research subsidy, Basic research

JEL Classification Codes: O38, O41, E62

*Faculty of Economics and Business Administration, Kyoto University of Advanced Science. Email: minami.growth@gmail.com

†Correspondence: Institute of Social and Economic Research (ISER), The University of Osaka, 6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan. E-mail: horii@iser.osaka-u.ac.jp

1 Introduction

> *Once one starts to think about [growth], it is hard to think about anything else.*
> *— Robert Lucas Jr. (1988)*

As Lucas (1988) mentioned, economic growth is one of the most important objectives in economics. How can we achieve faster economic growth? Modern theories of endogenous growth provide a surprisingly simple answer. In the long run, economic growth is determined by technological change. Technological change is realized via R&D. Therefore, by promoting R&D, e.g., through government subsidies for R&D, economic growth can be accelerated. Nevertheless, many economies face slower economic growth than they desire. Even when they know that R&D promotion policies enhance growth, there are often insufficient funds to implement those policies. Governments typically have a limited ability to raise taxes, which constrains their ability to promote R&D and, therefore, economic growth.

Recent studies on the political economy suggest that it is difficult for the government to raise tax rates for political reasons. For example, Bassetto and Miller (2025) considered an upper bound on tax revenue due to political constraints on austerity. Jiang, Sargent, Wang and Yang (2022) assumed a similar constraint based on the political considerations of Keynes (1923).[1] However, existing studies of long-term growth typically disregard such constraints by implicitly assuming that the government can levy lump-sum taxes to implement policies (e.g. Romer, 1990; Grossman and Helpman, 1991a, Ch. 3). Some studies include distortions caused by factor-income taxation but still assume that the government can set any tax rate.[2]

In contrast, this paper considers an extreme situation in which political constraints do not allow the government to increase its tax revenue.[3] If the government wants to support R&D, the expenses must be financed entirely by public debt. The government also cannot raise taxes in the future to repay the debt. Therefore, the debt must be rolled over perpetually. In other words, we consider the environment in which growth-promotion policies must be financed by a Ponzi scheme. While common sense suggests that such policies would not be sustainable, O'Connell and Zeldes (1988) showed that the

[1] Through an overlapping generations political economy model, Song, Storesletten and Zilibotti (2012) demonstrated that intergenerational conflict can prevent the government from raising taxes if the political power of the old is stronger than that of the young. See also Alesina and Passalacqua (2016) and Yared (2019) for the literature review.

[2] For example, Ferraro and Peretto (2020) examined the case where the government can use a distortionary labor tax in the context of fiscal consolidation.

[3] We relax this assumption in Section 7.3, where the government is allowed to levy an asset income tax.

government can run a "rational" Ponzi scheme if the rate of economic growth is higher than the interest rate on governmental debt, i.e., $g > r$.

As we show in Section 2, the inequality $g > r$ holds widely across time and countries. Given this finding, this paper asks whether the growth rate can be increased when the government finances R&D subsidies entirely by issuing debt. The result is not obvious because such policies affect both g and r. The direct effect of R&D subsidies is to induce private firms to engage in more R&D, which will speed up technological progress. However, the government's debt increases the equilibrium interest rate in financial markets and may crowd out private R&D investments. More precisely, a higher interest rate implies that the present value of future profits realized by R&D is lower, thereby discouraging R&D.

In Section 3, we present a baseline model that combines a standard variety-expansion R&D model with a continuous-time overlapping-generations model in which individuals' labor productivity deteriorates as they age. This effect creates a life-cycle saving motive. We then show in Section 4 that $g > r$ holds in equilibrium if individuals have enough life-cycle saving motives and if the R&D productivity is high enough. However, this result does not always mean that the growth rate can be enhanced by debt-financed R&D promotion policies. For these policies to increase the long-term growth rate, the R&D productivity must be even greater. In other words, the $g - r$ gap needs to be not only positive but also greater than a positive threshold value. Our estimates of the R&D productivity show that most advanced countries, including the United States, satisfy this condition. However, the estimates from Japan, Italy, and Greece fall short of the threshold. In those countries, our theory suggests that R&D subsidies accelerate technological progress only temporarily, although output can continue to be higher than without the subsidies for several hundred years.

In Section 5, we examine whether the R&D promoted by these policies is beneficial to the economy. This question matters because the government in our model runs a rational Ponzi scheme of the kind studied by O'Connell and Zeldes (1988). What makes such a scheme feasible is not the length of individuals' planning horizons but the continuous arrival of new cohorts, which provides the economy with an infinite number of agents. In the rational bubble literature, Diamond (1965) and Tirole (1985) showed that an unproductive asset such as government debt can improve welfare when $g > r$, because that condition implies that the economy is dynamically inefficient.[4] In contrast, even when $g > r$ holds, we show that the economy in our model is dynamically efficient, but still, the government

[4]In later studies, Olivier (2000) and Hirano and Toda (2025) introduced bubbles for productive assets.

can provide research subsidies financed by a Ponzi scheme that can improve the welfare of future generations. Specifically, we show that the social rate of return of R&D, which includes the benefits from intertemporal knowledge spillovers, is always at least as high as the growth rate.

In Section 6, we examine how these results are affected when we explicitly introduce population growth or decline. We build a second-generation endogenous growth model with a changing population size, which nests the baseline model as a special case. When the population grows faster, the debt per person is diluted more quickly. Rolling over the debt then becomes easier, and the crowding-out effect of a given amount of debt weakens, so debt-financed growth-promotion policies are more likely to be effective. The opposite holds in countries where the population is shrinking. To assess the implications of this model for individual countries, we calibrate it using each country's population history since 1820. We find that Japan, Italy, and Greece cannot even roll over their debt permanently, given their current low population growth.[5]

Section 7 presents four extensions of the baseline model. According to the baseline model, Japan, Italy, and Greece cannot enhance growth through debt-financed subsidies because their research productivities are too low. In Section 7.1, we extend the model to allow funds from debt rollovers to be used on expanding basic research, aiming to improve the country's research productivity. We calibrate the extended model using the data on basic research expenditure in each country, as well as the estimate of the effectiveness of basic research on private R&D from Jaffe (1989). Then, among these three problematic countries, we find that Japan and Greece can improve long-term growth by directing funds to basic research, whereas Italy might not. We also derive the conditions that determine whether the growth-maximizing policy should direct the funds to research subsidies, basic research, or the mix of these.

Section 7.2 relaxes the setting by allowing the government to implement unsustainable policies. When the government sets a subsidy rate above the sustainable level, it will eventually face a situation in which the debt-GDP ratio is too high to justify. We assume that, at that time, there is some kind of intervention, subsidies are terminated, and the government is forced to enter a period of fiscal austerity. We show that such an unsustainable policy can nevertheless deliver higher consumption than a sustainable policy of modest scale for a long period, possibly several hundred years. The analysis shows that voters may support such a policy depending on their age and wealth holdings.

[5]We also find that Portugal can roll over its debt, but research subsidies do not raise its long-term growth rate.

Motivated by the fact that statutory rates on capital income have historically been revised more often than those on labor income, Section 7.3 relaxes the assumption that the government cannot raise any taxes, allowing it to adjust an asset income tax. It turns out that, even in this case, debt-financed research subsidies are necessary to maximize long-term growth. In Section 7.4, we confirm that the main qualitative results carry over to a small open economy in which government debt raises the country premium. The condition $g > r$ still requires that private R&D productivity is sufficiently high, and raising long-term growth by a marginal debt-financed subsidy requires an even higher productivity.

Related Literature

This paper is at the crossroads of two important strands of literature: the public finance literature and the endogenous growth one. In the public finance literature, a body of studies examined the sustainability of government debt in an environment where $g > r$ holds in equilibrium (e.g. Blanchard, 2019; Mehrotra and Sergeyev, 2021; Reis, 2021; Ball and Mankiw, 2023; Barro, 2023; Kocherlakota, 2023). While these studies are not particularly focused on how the funds from issuing debts are used, our model shows that whether debts are helpful depends on the use of the funds, even when $g > r$ holds. In our study, debts themselves are growth-reducing because they increase the equilibrium interest rates and always reduce the market value of innovation. Regarding the point that the growth rate is endogenous, our study is close to Saint-Paul (1992) and King and Ferguson (1993), who developed AK-type endogenous growth models in which $g > r$ holds in equilibrium. However, in those AK-type models, the long-term growth is determined solely by capital accumulation, while we consider an R&D-based variety-expansion growth model where the incentives for innovation matter. By explicitly modeling the R&D process, we are able to examine the effect of debt-financed R&D promotion policies on technological innovations, which unarguably serve as an important source of economic growth.

In the endogenous growth literature, starting with Romer (1990), a long line of works has explored the effects of R&D subsidies on economic growth. Among recent studies, Wan and Zhang (2021), Iwaisako (2023), Chu, Furukawa and Ji (2016), Minniti, Parello and Segerstrom (2013), and Grossmann, Steger and Trimborn (2013) examined the effects under various specifications of economic growth.[6] While these studies assume balanced budgets,

[6] Wan and Zhang (2021) examined the growth effect of R&D subsidies financed by a distortionary labor tax in a variety-expansion model and applied the result to the U.S. economy. Using a variety-expansion model in the context of heterogeneous industries, Iwaisako (2023) derived the optimal R&D subsidies. Chu et al. (2016) and Minniti et al. (2013) considered the effect of R&D subsidies in the Schumpeterian growth model. The effectiveness of R&D subsidies in semi-endogenous growth models is examined in Grossmann et al. (2013) and Cozzi (2023). See also the references in Chu and Wang (2022).

we focus on the fact that the introduction of government subsidies is not necessarily paired with increases in taxes.[7]

This paper is also related to Angeletos, Lian and Wolf (2024) in that both show that future tax increases are not necessary after debt-financed policies are implemented. While the abovementioned authors consider a short-term stimulus policy in a new Keynesian setting, we consider a long-term growth promotion policy in an R&D-based endogenous growth model.

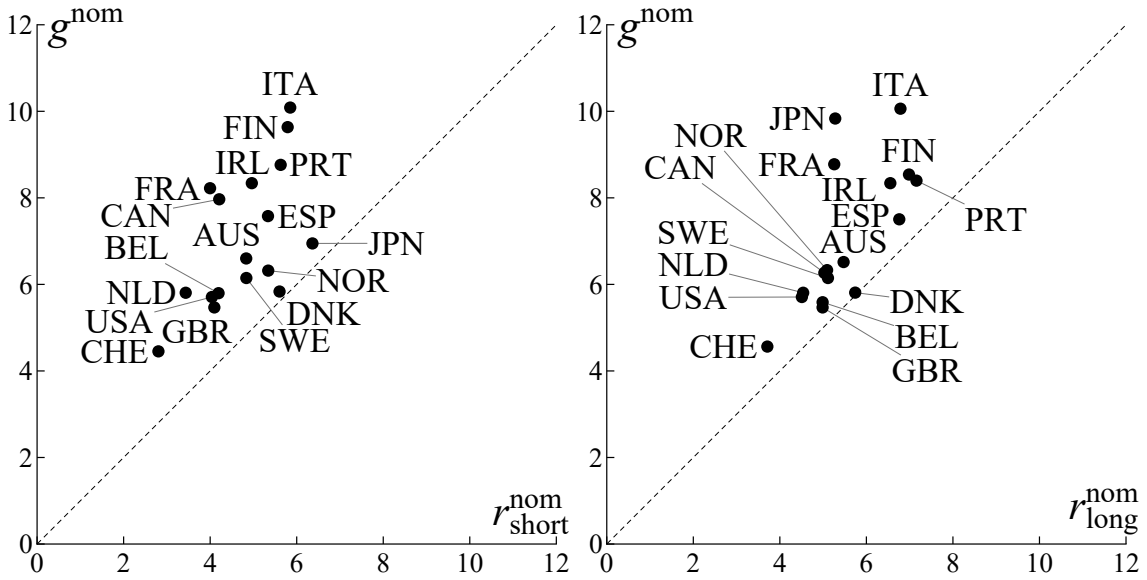
2 The $g > r$ Condition: Evidence

This section briefly presents an empirical basis for the $g > r$ condition and discusses the feasibility of financing long-term policies by exploiting the surplus generated by perpetually rolled-over debt under this condition. Figure 1 shows the cross-country scatter plot of the nominal interest rate and the nominal economic growth rate, which are averaged for the 1870-2019 period (panels a,b) and 1960-2019 period (panels c,d).[8] We use the Jordà-Schularick-Taylor Macrohistory Database (Jordà, Schularick and Taylor, 2017; Jordà, Knoll, Kuvshinov, Schularick and Taylor, 2019), which documents the macroeconomic and financial data for 18 advanced countries from 1870. Panels (a) and (c) show the relationship between the short-term interest rate and the nominal GDP growth rate.[9] For all 18 countries in the database, the nominal growth rate exceeds the short-term nominal interest rate, both for the full 1870-2019 sample and for the 1960-2019 subsample. Panels

[7] Peretto (2011) and Ferraro and Peretto (2020) also considered debt-financed fiscal policies in R&D-based growth models. Since their models have an infinitely lived representative household, the interest rate exceeds the growth rate on the balanced growth path. Hence, the debt must eventually be repaid or stabilized by raising taxes or cutting spending in the future. Our analysis complements these studies by considering the case in which taxes cannot be raised even in the future, so that the debt must be rolled over perpetually, which is possible only when $g > r$.

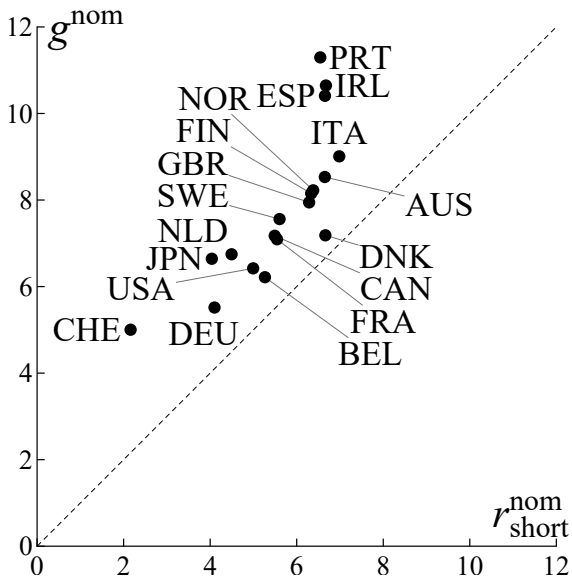
[8] For each country, the growth rate and the interest rate are averaged over the years in which both are available. Because of the hyperinflation in the early 1920s, the average growth rate of Germany lies above the vertical range of panels (a) and (b), and Germany is not shown in these panels. When the short-term interest rate is used, the average nominal growth rate of Germany is 20.6%, and the average interest rate is 4.0%. When the long-term interest rate is used, they are 13.4% and 5.1%, respectively. The two averages of the growth rate differ because the sets of years in which both are available are different.

[9] The short-term interest rate series in the database is a composite whose underlying instrument varies across countries and periods. It is the central-bank discount rate in earlier periods (for example, the Bank of Spain rate until 1924 and the National Bank of Belgium rate until 1964), the money market or interbank rate in the post-war decades for France, Germany, Spain, and Japan, the three-month Treasury bill rate in Canada and Ireland, and a commercial short-term lending rate in the United Kingdom and the United States. The database also reports the yield on short-term government bills for 16 of the 18 countries, all except Canada and Ireland. When we use this yield instead of the short-term interest rate, the average nominal growth rate exceeds it in all 16 countries, both for the full 1870-2019 sample and for the 1960-2019 subsample.

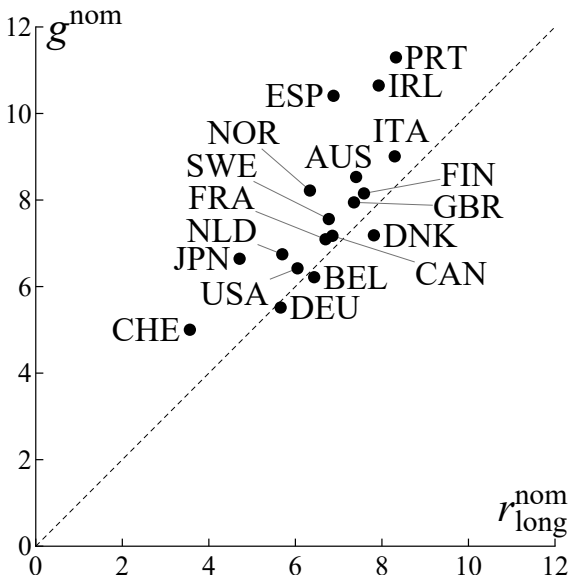


(a) short-term r, 1870-2019 average

(b) long-term r, 1870-2019 average



(c) short-term r, 1960-2019 average

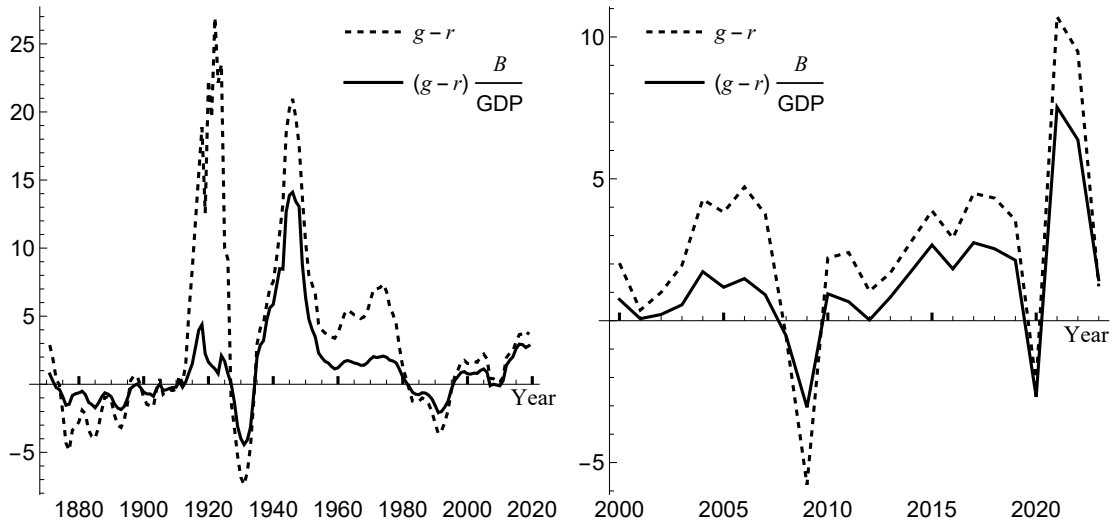


(d) long-term r, 1960-2019 average

Figure 1: The scatter plot of the nominal interest and growth rates (%)

(b) and (d) show the results when the long-term nominal interest rate is used.[10] Even though the long-term interest rate is typically higher than the short-term rate, the nominal GDP growth rate is still higher than the long-term interest rate across all 18 countries when averaged over the entire period. When we focus only on the post-1960 period, the

[10] The post-World War II series is mostly from the IMF International Financial Statistics "Government Bonds, percent per annum" yield, extended by the OECD "Long-term interest rates". For the pre-World War II period, when domestic long-term government debt is thinly recorded, the series for several countries (Australia, Canada, France, Japan, the Netherlands, Portugal, and Sweden) is taken to be the UK consol yield plus a country-specific spread following Clemens and Williamson (2004). The maturity is ten years, with two exceptions. The United Kingdom uses the twenty-year par yield on British Government Securities from 2009 onwards, and Belgium uses the yield on a perpetual-maturity debt until 1912.



(a) 18 advanced countries' average (b) 37 OECD countries' average

Figure 2: Time series of $g - r$ gap and the ratio of the surplus from debts to the GDP (%)

difference between g and r tends to be narrower, but $g > r$ still holds for 15 countries, with Germany, Belgium, and Denmark failing to meet the condition by narrow margins of 0.14, 0.22, and 0.63 percentage points, respectively. In practice, governments finance the debt through bonds of various maturities, and they can choose maturities to meet their objectives. The fact that $g > r$ holds for the short-term interest rate suggests that, on the margin, a government can finance additional debt at $r < g$ by issuing short-term bonds if reducing interest payments is its priority.

Our finding is consistent with observations in existing studies. Mehrotra and Sergeyev (2021) reported that in the 1946–2006 period, the median value of $g - r$ was 1.0% for the United States and 0.8% for the average of 17 advanced countries. Blanchard (2019) noted that $g > r$ has been the historical norm rather than the exception in the U.S. since 1950. Mauro and Zhou (2021) analyzed data on average effective borrowing costs for 55 countries over a span of up to 200 years and reported that $g > r$ prevails for both advanced and emerging economies.

The broken line in Figure 2 illustrates the time series of the cross-country average $g - r$ gap across 18 advanced countries in the Macrohistory data (panel a), and across 37 OECD countries (panel b).[11] Although $g - r$ was negative before World War I, during the Great Depression (and briefly during the Great Recession), and in the 1980s, it was positive the

[11] In both panels, we use short-term interest rates for r and GDP growth at the aggregate level for g. In panel (a), GDP and the short-term interest rate are taken from the Jordà-Schularick-Taylor Macrohistory Database (Jordà et al., 2017, 2019), covering the 18 advanced economies listed in Figure 1. The cross-country average for each year uses observations available in that year, and the plotted line is a five-year centered moving average of the cross-country average. In panel (b), GDP and the short-term interest rate are obtained from the OECD Data Explorer.

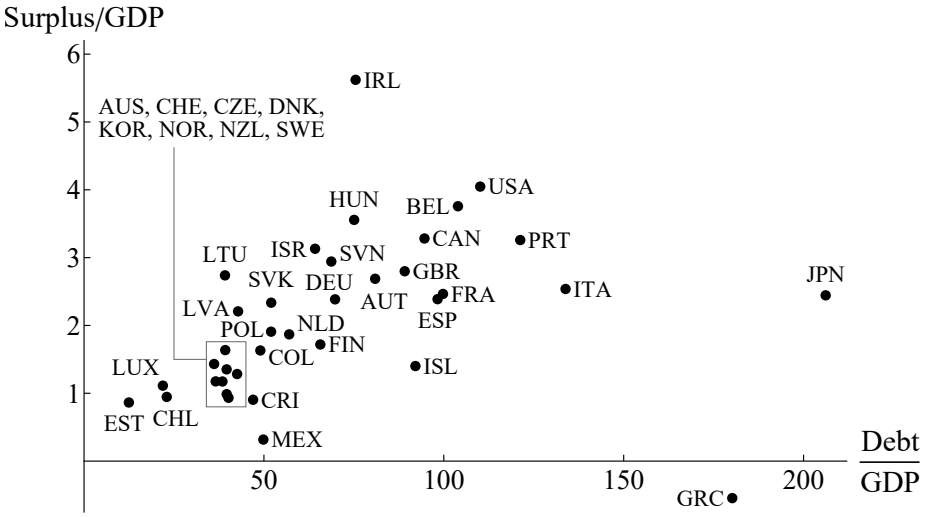


Figure 3: Debt/GDP ratio and surplus from government debt (%, 2010-2023 average)

majority of the time. When $g - r$ is positive, the debt-GDP ratio, B/GDP, shrinks by $(g - r)(B/\text{GDP})$ if the primary balance (the government revenue minus the government expenditure excluding interest payments on its debts) is zero. In other words, if $g - r$ is positive, the government can spend $(g - r)B$ over its revenue without increasing the debt-GDP ratio. The solid line in Figure 2 shows the ratio of this surplus to GDP (i.e., $(g - r)(B/\text{GDP})$).[12] The figure shows that the surplus is significant. On average, the OECD countries during 2000-2023 enjoyed 1.34% of GDP as the surplus from debts.[13]

Since the 2000s, the surplus/GDP ratio has been on the rise partly because of the increasing debt-GDP ratios among countries. The relationship between surplus and debt can be seen more precisely in Figure 3, which presents a scatter plot of B/GDP and $(g - r)(B/\text{GDP})$ from OECD countries.[14] For countries where the debt/GDP ratio is not too high (i.e., below 150%), there is a positive relationship between this ratio and the surplus/GDP ratio. This finding suggests that a moderate level of government debt can be helpful in financing long-term policies, given the government's constraint on raising taxes. Notably, however, the surplus is not necessarily large (in fact, it is negative for Greece) when the debt-GDP ratio is too high. When government debt is too high, the economy cannot maintain a positive $g - r$ gap, and the surplus vanishes. This pattern is consistent

[12]The data on the debt-GDP ratio in panel (a) are from the Jordà-Schularick-Taylor Macrohistory Database (Jordà et al., 2017, 2019). Those in panel (b) are from the IMF World Economic Outlook (General Government Gross Debt, percent of GDP).

[13]Excluding the Great Recession period (2008-2009) and the COVID-19 trough (2020), the average was 1.83% of GDP.

[14]The data sources are the same as in Figure 2 (panel b). Turkey is omitted because the OECD Data Explorer does not provide its short-term interest rate series, plausibly because the country has experienced periods of extreme inflation.

with our theory. While the condition $g > r$ enables perpetual debt financing to a certain extent, too much exploitation of the condition eventually eliminates the equilibrium with $g > r$.

3 Baseline Model

3.1 Individuals

We consider a continuous-time overlapping generations model based on the perpetual youth framework of Blanchard (1985), extended to incorporate exogenous population growth following Buiter (1988). Let N_t be the population at time t. New individuals are born at the constant rate $\beta > 0$, so that βN_t individuals are born per unit of time, and each individual faces a constant Poisson death rate $\mu > 0$. The size of the cohort born at time s and still alive at time t ($\geq s$) is $N_{s,t} = \beta N_s e^{-\mu(t-s)}$. Integrating this over all surviving cohorts gives $N_t = \int_{-\infty}^{t} \beta N_s e^{-\mu(t-s)} ds$, and differentiating it with respect to t yields $\dot{N}_t = (\beta - \mu) N_t$. The population thus grows at the constant rate $\beta - \mu$. Since β is constant, $N_s = N_t e^{-(\beta-\mu)(t-s)}$, and the cohort size can be written as

$$N_{s,t} = \beta N_t e^{-\beta(t-s)}. \tag{1}$$

Each individual supplies inelastically $e^{-\delta(t-s)}$ units of effective labor, where $t - s$ is their age.[15] The effective labor supply decreases with age at the rate of $\delta > 0$ because of the deterioration in productivity and the declining ability to work longer hours as individuals age.[16] Because of this aging effect, individuals have an incentive to make life-cycle savings. In this sense, the parameter δ represents the strength of individuals' saving motives. Then, from (1), the aggregate labor supply is

$$L_t = \int_{-\infty}^{t} N_{s,t} e^{-\delta(t-s)} ds = \frac{\beta}{\beta + \delta} N_t. \tag{2}$$

The expected utility of a generation s individual, accounting for their mortality, is given by

$$U_s = \int_{s}^{\infty} (\ln c_{s,t}) e^{-(\rho+\mu)(t-s)} dt, \tag{3}$$

[15] More precisely, $t - s$ is the period after each individual starts their economic activity. If the starting age is 20, the actual age of an individual is $20 + (t - s)$.

[16] Empirically, age-earnings profiles are inverted-U shaped rather than monotonically declining (Mincer, 1974; Murphy and Welch, 1990). We adopt the exponential specification because it keeps the aggregation across generations tractable. In this sense, δ is best viewed as a reduced-form proxy for the strength of the life-cycle saving motive rather than as a literal description of the age profile of productivity.

where $c_{s,t}$ is the amount of consumption by a generation-s individual at time t and where $\rho > 0$ is the discount rate. Observe that individuals further discount the future by their survival probability, $e^{-\mu(t-s)}$. We assume that the discount rate is not too high. In particular, we assume that the discounting is smaller than the sum of the aging effect and population growth ($\rho < \delta + \beta - \mu$) so that individuals have enough incentives to save for their later age (see Rachel and Summers, 2019).

Let $k_{s,t}$ be the real asset holding of a generation-s individual and r_t be the real interest rate on bonds. Following Blanchard (1985), we assume that there is a perfect market for annuities. Thus, the rate of return on annuities is $r_t + \mu$ for all generations, independent of when they are born. From $r_t + \mu > r_t$, individuals hold all their assets in the form of annuities. We also normalize the price of the final goods to one. Then, the budget constraint is given by

$$\dot{k}_{s,t} = (r_t + \mu)\, k_{s,t} + e^{-\delta(t-s)} w_t - c_{s,t}, \tag{4}$$

where w_t is the real wage per unit of effective labor. The newborn generation has zero financial assets, which means that $k_{t,t} = 0$. To summarize, each individual maximizes the expected utility (3) subject to the budget constraint (4), initial condition $k_{t,t} = 0$, and the usual non-Ponzi game condition. In this standard setting, the Euler equation for individuals is $\dot{c}_{s,t} = (r_t - \rho)c_{s,t}$.

Let us define aggregate consumption, C_t, and aggregate asset holding, K_t, as

$$C_t = \int_{-\infty}^{t} c_{s,t} N_{s,t} ds, \quad K_t = \int_{-\infty}^{t} k_{s,t} N_{s,t} ds. \tag{5}$$

In the following, we derive the dynamics for aggregate consumption in terms of C_t and K_t. Using the Leibniz integral rule, we can differentiate C_t in (5) as follows:

$$\dot{C}_t = \int_{-\infty}^{t} \dot{c}_{s,t} N_{s,t} ds + \int_{-\infty}^{t} c_{s,t} \dot{N}_{s,t} ds + N_{t,t} c_{t,t}. \tag{6}$$

On the right-hand side (RHS), the first term represents the sum of the changes in consumption by existing individuals. Using the Euler equation of individuals, $\dot{c}_{s,t} = (r_t - \rho)c_{s,t}$, this term can be written as $(r_t - \rho)C_t$. In the second term, the change in the cohort size $\dot{N}_{s,t}$ can be written $-\mu N_{s,t}$ from (1). Therefore, this term becomes $-\mu C_t$. This value represents the decline in aggregate consumption due to the death of a fraction μ of individuals per unit time. The last term is the sum of the consumption by newly born individuals. In

Appendix A.1, we show that the consumption of a newborn is

$$c_{t,t} = \frac{\beta+\delta}{\beta}\big(\bar{c}_t - (\rho+\mu)\overline{k}_t\big), \tag{7}$$

where $\bar{c}_t = C_t/N_t$ is average consumption and $\overline{k}_t = K_t/N_t$ is the average asset holding. Equation (7) can be interpreted as follows. Recall that newborns do not have financial assets ($k_{t,t} = 0$). Given the log utility, the consumption propensity out of assets is $\rho+\mu$. Therefore, the lack of financial assets induces a newborn to consume $(\rho+\mu)\overline{k}_t$ less than the average consumption, $\bar{c}_t$. However, newborns have a greater ability to work than average individuals do. A newborn's working ability (labor supply) is 1, whereas that of an average individual is $L_t/N_t = \beta/(\beta+\delta)$. Therefore, the RHS of (7) is multiplied by $(\beta+\delta)/\beta$. By substituting (7) into (6) and using $N_{t,t} = \beta N_t$, we obtain the Euler equation for aggregate consumption.

$$\dot{C}_t = (r_t - \rho + \beta - \mu + \delta)C_t - (\rho+\mu)(\delta+\beta)K_t. \tag{8}$$

The aggregate Euler equation differs from that for individuals in three respects. First, it depends on the population growth rate, $\beta-\mu$, because the size of the economy grows with population, and so does the aggregate consumption. The remaining two effects concern generational turnover. When older individuals are replaced by younger ones, does it increase or decrease the aggregate consumption? The answer depends on whether younger individuals consume more (or less) than older individuals. There are two opposing effects. Since the labor productivity of individuals declines at rate δ with age, older individuals, who are replaced by younger ones, tend to have lower income. However, those older individuals hold a portion of the aggregate asset K_t, while the new generation is born with zero assets. In sum, generational turnover replaces existing agents with new ones who have higher incomes but less financial wealth. The income effect positively affects $\dot{C}_t$ by δC_t, while the wealth effect reduces $\dot{C}_t$ by $(\rho+\mu)(\delta+\beta)K_t$.

In the baseline model developed in Sections 3–5, we focus on the case of a stationary population by setting $\beta = \mu$. The population N_t and the aggregate labor supply L_t are then constant over time, and we denote them simply by N and L. The case of a growing or shrinking population ($\beta \neq \mu$) is examined in Section 6, where we study a second-generation R&D-based growth model.

3.2 Supply Side

The supply side of this economy is purposefully close to that of the standard variety-expansion model of Grossman and Helpman (1991a). It consists of three sectors, namely,

the final goods sector, the intermediate goods sector, and the R&D sector. In the final goods sector, a representative firm competitively produces final goods X_t from a continuum of varieties of intermediate goods $x_t(i)$, where i is the index of intermediate goods. Each variety is produced in a different industry. The production function is given by

$$X_t = \left[\int_0^{n_t} x_t(i)^\alpha di\right]^{1/\alpha}, \tag{9}$$

where n_t is the number of intermediate goods available at time t and where $\alpha \in (1/2, 1)$ is a production parameter.[17] Let $p_t(i)$ be the price of intermediate good i. The representative final goods firm maximizes its profit, which is given by

$$X_t - \int_0^{n_t} p_t(i)x_t(i)di. \tag{10}$$

The first-order condition for profit maximization implies that the demand function of intermediate goods is

$$x_t(i) = p_t(i)^{-\frac{1}{1-\alpha}} X_t. \tag{11}$$

In the intermediate goods sector, there are n_t intermediate goods firms, each of which produces its own variety of goods, $x_t(i)$. The production of one unit of $x_t(i)$ requires one unit of labor; therefore, its profit is given by $\pi_t(i) = (p_t(i) - w_t)x_t(i)$. Given that $x_t(i)$ is determined by the demand function (11), profit-maximizing pricing implies

$$p_t(i) = \frac{w_t}{\alpha}, \quad x_t(i) = \left(\frac{\alpha}{w_t}\right)^{\frac{1}{1-\alpha}} X_t, \tag{12}$$

$$\pi_t(i) = (1-\alpha)\left(\frac{\alpha}{w_t}\right)^{\frac{\alpha}{1-\alpha}} X_t. \tag{13}$$

The above result shows that all intermediate goods firms produce the same amount of output. Therefore, the output of each intermediate goods firm can be written as $x_t(i) = L_t^P/n_t$ for all i, where L_t^P is the total amount of labor for the production of goods. By substituting this output into the final goods production function (9), we obtain

$$X_t = n_t^{\frac{1-\alpha}{\alpha}} L_t^P. \tag{14}$$

The R&D sector has a representative R&D firm, which competitively creates new goods

[17] As we show in (12), α represents the inverse of the markup rate. $\alpha \in (1/2, 1)$ means the rate is between 1 and 2.

using labor for R&D, L_t^R. The number of goods increases as follows:[18]

$$\frac{\dot{n}_t}{n_t} = A\frac{L_t^R}{L}, \tag{15}$$

where $A > 0$ is a parameter that measures the productivity of R&D. When 1% of the labor force is used for R&D, the number of intermediate goods increases A% per year.[19] While we treat A as an exogenous parameter in the baseline model, we endogenize it in Section 7.1.

Equation (15) implies that the creation of a new intermediate good requires L/An_t units of labor. We assume that the government subsidizes a fraction $\theta \in [0, 1)$ of the R&D cost. Then, the private cost of developing a new intermediate good is $(1-\theta)w_tL/An_t$. Let v_t be the value of an intermediate goods firm. Then, the free entry condition for R&D is

$$v_t \leq (1-\theta)\frac{w_tL}{An_t} \text{ with equality if } \dot{n}_t > 0. \tag{16}$$

3.3 Government

As explained above, the government subsidizes a fraction θ of the cost of R&D. Since the presubsidy aggregate cost of R&D is $w_tL_t^R$, the amount of government expenditure is $\theta w_tL_t^R$. We assume that the government cannot collect taxes and that all expenditures are financed by government debt. Then, the amount of government debt, B_t, evolves according to

$$\dot{B}_t = r_tB_t + \theta w_tL_t^R. \tag{17}$$

If the government holds net assets ($B_t < 0$), it may provide the subsidy without raising taxes even when $g < r$, because the returns from these assets can pay for the subsidy.[20] In practice, however, only a limited number of countries, such as oil-producing countries, hold such net assets. Therefore, in the main text, we focus on the case where the government at $t = 0$ does not hold net assets, $B_0 \geq 0$. Given this initial condition, Equation (17) implies that B_t does not become negative.

---

[18]While this baseline model sticks to the standard formulation, it is possible to generalize the R&D equation (15) by adding a curvature parameter $\zeta \in (0, 1]$ and assuming $\dot{n}_t/n_t = A(L_t^R/L)^\zeta$. The case of $\zeta < 1$ applies, for example, when too much R&D results in duplication of ideas (see Horii and Iwaisako, 2007).

[19] We can also write equation (15) as $\dot{n}_t = an_tL_t^R$, where $a = A/L$. Although the latter convention is more standard, we use A rather than a because its value is easier to interpret from the data. Also, we can interpret this formulation as a special limiting case of the second-generation endogenous growth model, where population growth is introduced. See Section 6.5 for the relationship.

[20]The analysis in Appendices A.5–A.8 includes such cases.

Since there is no tax revenue, the repayment and interest of each bond must be refinanced entirely by issuing new bonds. Therefore, the government is running a Ponzi scheme.[21] The lenders expect that their bonds will be repaid and invest in them only when the economy is on an equilibrium path that continues into the infinite future. If there is no such equilibrium path, the repayment plan will fail in finite time. When lenders expect this, they will not buy bonds even before failure, which means that under this Ponzi scheme, the repayment will also fail at that earlier time. By backward induction, the government can sell bonds only when lenders expect repayment to continue forever, which is equivalent to the economy having an equilibrium path that continues into the infinite future. As examined by O'Connell and Zeldes (1988), this scheme requires an infinite flow of new generations.[22] With the overlapping-generations structure, explained in Section 3.1, we investigate the possibility that the government uses the funds from this scheme to promote economic growth.

4 Equilibrium

4.1 The $g - r$ Gap

A necessary condition to run a rational Ponzi scheme is that the growth rate is higher than the interest rate. Given the economy described in the previous section, here, we examine the gap between the growth rate and the interest rate in equilibrium. First, we derive the real interest rate.[23] The consumers hold all their assets in the form of annuities, and the annuity company invests the assets in government bonds and the shares of intermediate goods firms. Therefore, the equilibrium in the asset market is

$$K_t = B_t + n_t v_t. \tag{18}$$

Since risks are fully diversified, the expected return on holding the shares of an intermediate goods firm should be equal to the interest rate on bonds. This no-arbitrage condition can

[21]Note that this situation is in contrast to that of individuals, who maximize their lifetime utility subject to the no-Ponzi-game condition. The difference in the ability to borrow between the government and individuals reflects reality. Without collateral, individuals usually cannot borrow large amounts of money for various reasons (e.g., the risk of running away). We can rewrite the model with a borrowing constraint for individuals, which yields the same result as the present setting in the steady state.

[22]In the analysis of O'Connell and Zeldes (1988), a rational Ponzi game is feasible only when the economy is dynamically inefficient. In our model, the government can run the game even though the economy is dynamically efficient, as we show in Section 5.

[23]Note that, in the model, g and r are real variables. However, when we calculate $g - r$ from the data, as shown in Figures 2 and 3, we use the nominal growth rate and the nominal interest rate because the result is the same.

be written as

$$r_t = \frac{\pi_t + \dot{v}_t}{v_t}, \tag{19}$$

where π_t is given by (13).[24] In the main text, we focus on the case where the amount of R&D is positive, leaving the discussion of the case of $\dot{n}_t = 0$ for the Appendix A.5. Then, v_t is given by (16) with equality. Both π_t and v_t depend on the wage level, which we now derive. Since the final goods sector is competitive, the maximized profit of the final goods firm in Equation (10) should be zero. We can eliminate $x_t(i)$ and $p_t(i)$ from the amount of profit (10) using (11) and (12). Then, the zero profit condition determines the market wage as

$$w_t = \alpha n_t^{\frac{1-\alpha}{\alpha}}. \tag{20}$$

Eliminating π_t and v_t from (19) yields the real interest rate in equilibrium.[25]

$$r_t = \frac{A}{\alpha}\left(\frac{1-\alpha}{1-\theta}\frac{L_t^P}{L} - (2\alpha - 1)\frac{L_t^R}{L}\right). \tag{21}$$

Next, we consider the growth rate. The GDP of this economy is defined as the sum of consumption expenditures C_t, private investment expenditures for R&D, $(1-\theta)w_t L_t^R$, and government expenditures, $\theta w_t L_t^R$. Note that the final output is used only for consumption; therefore, the equilibrium in the goods market means that $C_t = X_t$. Then, using Equations (14) and (20), the GDP can be written as

$$\text{GDP}_t = n_t^{\frac{1-\alpha}{\alpha}}(L_t^P + \alpha L_t^R). \tag{22}$$

Note that the labor market equilibrium requires the sum of the two types of labor inputs to coincide with the labor supply L:

$$L_t^P + L_t^R = L. \tag{23}$$

From $\alpha < 1$, the GDP is greater when more labor is used for production, given the value of n_t. The reason is that there is a positive markup in the intermediate goods sector, whereas there is no markup in the R&D sector.

Let us also define full-production GDP, Y_t, as the level of GDP that would obtain if all

[24] Equation (19) implicitly assumes that the value of the intermediate firm is differentiable with respect to time. This assumption holds as long as the underlying parameters are constant. In Section 7.2, we extend the analysis to the case where the subsidy rate has to change when the government debt reaches a critical value, at which point v_t jumps and therefore Equation (19) cannot be defined.

[25] The derivation is given in Appendix A.2.

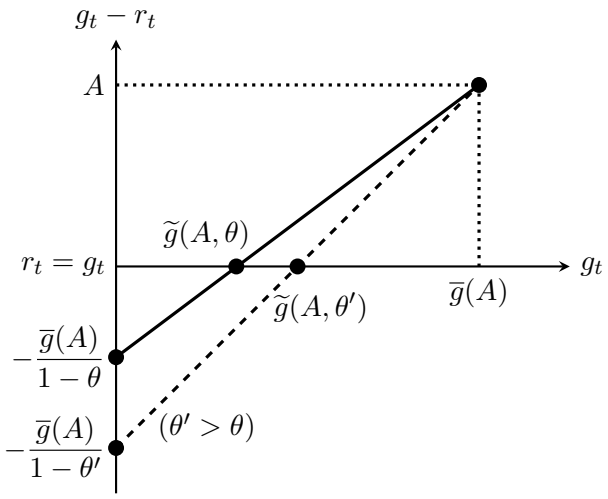


Figure 4: Relationship between the $g_t - r_t$ gap and the growth rate.

labor were used for production, given the number of intermediate goods developed by that time, n_t. With $L_t^P = L$ and $L_t^R = 0$, Equation (22) reduces to

$$Y_t = n_t^{\frac{1-\alpha}{\alpha}} L. \tag{24}$$

Since Y_t is proportional to $n_t^{\frac{1-\alpha}{\alpha}}$, it serves as an index of the technology available at time t, and it gives the upper bound for GDP_t under that technology. Note that this bound is attained only when all labor is allocated to production, which means that there is no economic growth. We use Y_t for normalization in the dynamic analysis below because it depends only on the predetermined variable n_t. From (15), the growth rate of full-production GDP in (24) is given by

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = \overline{g}(A)\frac{L_t^R}{L}, \text{ where} \tag{25}$$

$$\overline{g}(A) \equiv \frac{1-\alpha}{\alpha} A \tag{26}$$

is the maximum growth rate of full-production GDP when all labor is used for R&D. Note that on any balanced growth path (BGP), g_t is constant. Then, Equation (25) means that L_t^R is constant; the same holds for $L_t^P = L - L_t^R$. In this case, from (22) and (24), the growth rate of GDP_t coincides with $\dot{Y}_t/Y_t$. Therefore, g_t also represents the growth rate of GDP in the long run. Below, we simply call g_t the growth rate unless otherwise noted.

Now, we are ready to derive the gap between the growth rate and the interest rate. By eliminating L_t^P and L_t^R from (21) via (25) and (23), we can represent the $g - r$ gap in

terms of g_t:

$$g_t - r_t = s(\theta)(g_t - \widetilde{g}(A,\theta)), \tag{27}$$
$$\text{where } s(\theta) = \frac{1-\alpha\theta}{(1-\alpha)(1-\theta)} > 1, \quad \widetilde{g}(A,\theta) = \frac{1-\alpha}{1-\alpha\theta}\overline{g}(A) \in (0, \overline{g}(A)].$$

As shown in Figure 4, given the parameters, the $g_t - r_t$ gap is positively and linearly related to g_t, with a slope of $s(\theta) > 1$. In particular, condition $g_t > r_t$ holds when g_t is greater than the threshold at $\widetilde{g}(A,\theta)$. Therefore, when the growth rate is endogenized, we find that the possibility of running the government's Ponzi scheme depends on whether the growth rate is higher than the $\widetilde{g}(A,\theta)$ threshold.

Note that the threshold depends on the country's research productivity A and the subsidy rate θ. When the rate of R&D subsidies, θ, is increased, the thick line in Figure 4 rotates counterclockwise, which lowers $g_t - r_t$ for a given growth rate (as shown by the dashed line). Accordingly, the threshold $\widetilde{g}(A,\theta)$ in (27) increases with θ. While research subsidies may raise the growth rate, the threshold rises as well, so g_t must be even higher to keep $g_t > r_t$. The remaining question is how g_t is determined. To answer this, we need to consider the equilibrium dynamics, including the demand side of the economy.

4.2 Equilibrium Dynamics

We now derive the dynamics of the growth rate using the result from the previous subsection and the demand side of the economy, as explained in Section 3.1. In this model, final goods are used only for consumption. Therefore, the equilibrium of the goods market implies $X_t = C_t$. Combining the aggregate Euler equation (8) with the market-clearing conditions and (27), we obtain[26]

$$\dot{g}_t = (\overline{g}(A) - g_t)\big(s(\theta)(g_t - \widetilde{g}(A,\theta)) - \delta + \rho\big) + \overline{g}(A)(\rho+\mu)(\delta+\mu)\left(D_t + \frac{\alpha(1-\theta)}{A}\right), \tag{28}$$

where $D_t \equiv B_t/Y_t$ denotes the ratio of government debt to full-production GDP. We call it the debt-GDP ratio for brevity. Equation (28) shows that the dynamics of g_t depend on the debt-GDP ratio because the asset holdings of individuals affect the aggregate Euler equation as discussed at the end of Section 3.1.

Next, we turn to the dynamics of D_t. From (17), (20), (25) and (27), the time derivative of D_t is given by

$$\dot{D}_t = -s(\theta)(g_t - \widetilde{g}(A,\theta))D_t + \frac{\theta\alpha}{\overline{g}(A)}g_t. \tag{29}$$

[26]Appendix A.2 provides the formal derivation of (28) and (29) and the interpretation of each term.

The first term $-s(\theta)(g_t-\widetilde{g}(A,\theta))D_t$ equals $(r_t-g_t)D_t$, the rate at which the debt-GDP ratio would change if the primary balance were zero. The second term is the ratio of government spending on subsidies to full-production GDP. This term accelerates the increase in D_t.

Equations (28) and (29) constitute a dynamic system in the two variables g_t and D_t. Since Y_t depends only on the predetermined variable n_t, D_t is a state variable, and its initial value $D_0 = B_0/Y_0$ serves as the initial condition of the equilibrium dynamics.[27] Below, we construct a phase diagram for this system. Setting $\dot{g}_t = 0$ in (28) and $\dot{D}_t = 0$ in (29) gives the two loci

$$\dot{g}_t = 0: \quad D_t = \frac{\overline{g}(A) - g_t}{\overline{g}(A)(\rho+\mu)(\delta+\mu)}\Big(\delta - \rho - s(\theta)\big(g_t - \widetilde{g}(A,\theta)\big)\Big) - \frac{\alpha(1-\theta)}{A}, \tag{30}$$

$$\dot{D}_t = 0: \quad D_t = \frac{\alpha\theta}{\overline{g}(A)s(\theta)}\frac{g_t}{g_t - \widetilde{g}(A,\theta)}. \tag{31}$$

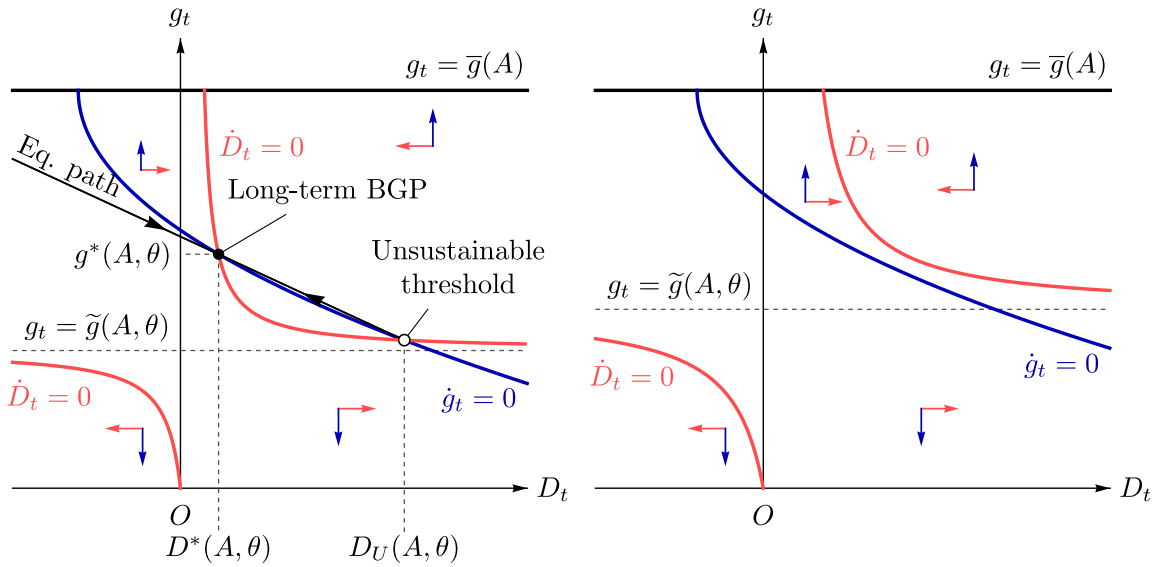
A pair of g_t and D_t represents a steady state when they satisfy (30) and (31) at the same time. Appendix A.4 explains how steady states of this economy can be calculated. Note that the equilibrium dynamics represented by (28) and (29) assume that the free entry condition for R&D, (16), holds with equality, which is true whenever $g_t > 0$. When no firm enters the market, which means $g_t = 0$, the dynamics of the economy are represented by another set of equations and steady states are defined accordingly. We examine such a case in Appendix A.5, while focusing on the case of positive R&D in the main text.

Another presumption behind the dynamics is that the government can sell bonds to households. As discussed in Section 3.3, this is possible only when households think the economy is on an equilibrium path that continues forever. This is true if and only if the dynamic system of Equations (28) and (29) (and the corresponding equations for the case of $g_t = 0$) with the initial debt-GDP ratio D_0 has a feasible solution that continues for all $t \geq 0$. Ignoring the unrealistic situation where the government holds net assets that are far larger than its GDP,[28] the following proposition shows that the existence of such an equilibrium path depends on whether the economy has steady states and whether the initial debt-GDP ratio is below a threshold.

Proposition 1 (Existence, Nonexistence, and Uniqueness of Equilibrium) *Suppose*

[27]The denominator of $D_t \equiv B_t/Y_t$ is full-production GDP rather than actual GDP. While calling D_t the debt-GDP ratio for brevity is a slight abuse of wording, this analytical choice makes the phase-diagram analysis both intuitive and tractable. Since GDP_t depends on the jump variables L_t^P and L_t^R, it is not possible to use B_t/GDP_t as an initial condition. Although it becomes more mathematically demanding, it is possible to describe the dynamics by g_t and B_t/GDP_t instead of g_t and D_t, as we explain in Appendix A.3.

[28]The lower bound in Proposition 1 is violated only when the government holds net assets exceeding $\min\{\alpha/(\delta-\rho), \alpha/(\rho+\mu)\}$ times full-production GDP, which is more than 30 times GDP under the parameter values in Table 1. Even the largest sovereign wealth funds amount to less than ten times GDP.



(a) When the two loci intersect, the economy has two steady states. From any $D_0 < D_U(A,\theta)$, the economy converges to the long-term BGP.

(b) When the $\dot{g}_t = 0$ locus and the $\dot{D}_t = 0$ locus do not intersect, there is no equilibrium path.

Figure 5: Possible Configurations of Phase Diagram in D_t and g_t.

that $D_0 > -\min\{\alpha/(\delta-\rho),\, \alpha/(\rho+\mu)\}$.

(a) Except for knife-edge cases (i.e., cases that disappear under an infinitesimal perturbation to the parameters), the economy has either two or zero steady states.

(b) Suppose that the economy has two steady states, and let $D^*(A,\theta) < D_U(A,\theta)$ *be their debt-GDP ratios.*

(i) If $D_0 < D_U(A,\theta)$*, there exists a unique equilibrium path, and it converges to the steady state with* $D^*(A,\theta)$*.*

(ii) If $D_0 > D_U(A,\theta)$*, there is no equilibrium path.*

(c) If the economy has no steady state, there is no equilibrium path.

Proof: See Appendices A.6 and A.7.

Throughout the paper, we disregard knife-edge cases that would occur only by chance. In addition to the cases mentioned in part (a) of the proposition, we do not consider the case of $D_0 = D_U(A,\theta)$ because D_0 is predetermined and unlikely to be exactly on the threshold.[29] We call the steady state with $D^*(A,\theta)$ the long-term BGP, and $D_U(A,\theta)$ the unsustainable threshold, because there is no equilibrium path when D_0 exceeds it. Figure 5 illustrates the two cases of Proposition 1 in the phase diagram. The $\dot{g}_t = 0$ locus, given by (30), is a parabola open to the right, and the $\dot{D}_t = 0$ locus, given by (31), is a hyperbola. Depending on the parameters, A and θ in particular, they do or do not intersect. Panel (a)

[29] If $D_0 = D_U(A,\theta)$ happens, the dynamic system implies that $D_t = D_U(A,\theta)$ holds for all t. However, this is an unstable situation in that if there is even an infinitesimal perturbation to a parameter or D_t, either the economy loses equilibrium, or it converges to the long-term BGP.

shows the case when the two loci intersect twice in the region $D_t \geq 0$, and the economy has two steady states. There is a unique equilibrium line (saddle path) that connects the two steady states. Given $D_0 < D_U(A, \theta)$, the initial value of g_0 is determined so that (D_0, g_0) is on the equilibrium line, and then the economy converges toward the long-term BGP. On the long-term BGP, the growth rate $g^*(A, \theta)$ is higher than the interest rate $r^*(A, \theta)$, because g_t is higher than the critical value $\widetilde{g}(A, \theta)$, as defined in (27). Therefore, the government can obtain a surplus from debt, with which it covers the expense of the R&D subsidy. Note that, even when the economy has the long-term BGP, it cannot converge to it if D_0 is larger than the threshold $D_U(A, \theta)$. In this case, there is no equilibrium path, which means that the government cannot sell bonds to lenders; i.e., it goes bankrupt. Therefore, even though a modest amount of debt could give a surplus to the government, it loses everything once the debt goes above the threshold. Panel (b) exhibits the case where the two loci do not intersect. In this case, there is no equilibrium path regardless of the initial value of D_t. As we show below, it happens when the government tries to implement subsidies that are so aggressive that they cannot be maintained even with $D_0 = 0$. The government cannot sell the bonds, and it will be forced to cancel such a policy.

4.3 Debt Sustainability and the Effect of Research Subsidies

Now, we are ready to answer one of the paper's main questions: When can the government roll over its debt indefinitely and use the proceeds to subsidize research? The phase diagrams presented in the previous subsection suggest that the answer depends both on the research productivity of the economy, A, and the rate at which the government subsidizes R&D, θ.

In this subsection, we first provide a division of the parameter space that answers the questions graphically, and then present formal propositions that state how such a division can be obtained. Figure 6 shows the result, which we numerically calculated using the same set of parameters as we use in Section 4.4 (parameter values are given in Table 1). The shaded area, which is below the $\overline{\theta}(A)$ curve, gives the parameter region in which the economy has a long-term BGP with $g > r$, as in Panel (a) of Figure 5. More specifically, suppose the research productivity is higher than a threshold

$$\overline{A} \equiv \frac{(\rho + \mu)(\delta + \mu)}{\delta - \rho}. \tag{32}$$

Then, the government can set a positive subsidy rate $\theta \in (0, \overline{\theta}(A))$, providing firms with research subsidies while raising no taxes, and the economy still has a long-term BGP

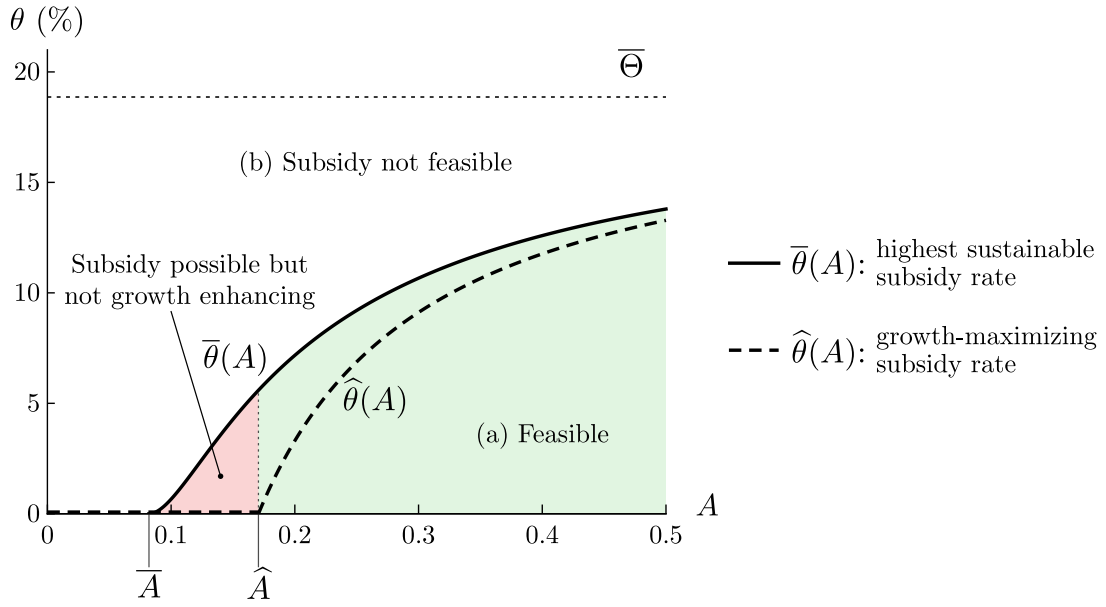


Figure 6: Sustainable and growth-maximizing subsidy rates. The economy has a long-term BGP with $g > r$ only if the pair of the research productivity (A) and the subsidy rate (θ) falls within the shaded area. Given A, the rate of subsidy that maximizes the long-term growth is given by the dashed curve.

with $g > r$. This is exactly the equilibrium in which the government rolls over its debt indefinitely and uses the proceeds to subsidize research. However, a subsidy rate above the $\overline{\theta}(A)$ curve cannot be implemented. If the government tried to set such a rate, the economy would have no long-term BGP, as in Panel (b) of Figure 5, and there would be no equilibrium path (Proposition 1). As explained in Section 3.3, lenders would then not lend to the government, and the government would be unable to obtain the funds for the subsidy. There is no $\overline{\theta}(A)$ curve when $A < \overline{A}$, which means that the economy has no long-term BGP with $g > r$ regardless of the subsidy rate. In that case, the government cannot finance the research subsidy by debt rollover.

The figure also shows the answer to the other main question: If such a policy is possible, will it increase the growth rate? The dashed curve, $\widehat{\theta}(A)$, shows the subsidy rate that maximizes the long-term growth rate, given A. Note that $\widehat{\theta}(A)$ is positive only when the research productivity exceeds a second threshold

$$\widehat{A} \equiv 2\overline{A} + \frac{\delta - \rho}{2}, \tag{33}$$

which is higher than $\overline{A}$. Only when $A > \widehat{A}$ does setting a positive research subsidy rate lead to a higher growth rate. In particular, when $A \in (\overline{A}, \widehat{A})$, providing research subsidies financed by debt rollover is feasible, but it never increases the long-term growth. In fact, setting $\theta = 0$ is the best long-term growth policy.

The following two propositions state these properties formally and give the thresholds and the two curves explicitly.

Proposition 2 (Sustainability of Debt-Financed Subsidy) *The economy has a long-term BGP with $g^*(A,\theta) > r^*(A,\theta)$ if and only if $A > \overline{A}$ and $\theta < \overline{\theta}(A)$ hold. The upper bound is*

$$\overline{\theta}(A) = \left(1 + \frac{27(1-\alpha)(\delta-\rho)A\,\overline{A}}{\Omega(A)}\right)^{-1}, \tag{34}$$

where $\Omega(A) \equiv 2\big(A^2+(\delta-\rho)(\delta-\rho-A+3\overline{A})\big)^{3/2} - 2(A+\delta-\rho)^3 + 9(\delta-\rho)(A+\delta-\rho)(A-\overline{A})$. *The upper bound is strictly increasing in A and converges to a finite $\overline{\Theta}$ as $A \to \infty$.*
Proof: See Appendix A.8.

Proposition 3 (Growth-Maximizing Subsidy) *Debt-financed research subsidies can raise the long-run growth rate if and only if $A > \widehat{A}$. When $A > \widehat{A}$, the long-run growth rate is maximized at the subsidy rate*

$$\widehat{\theta}(A) = \left(1 + 4\frac{1-\alpha}{\delta-\rho}\frac{\overline{A}}{1-\widehat{A}/A}\right)^{-1} \in (0, \overline{\theta}(A)), \tag{35}$$

which is strictly increasing in A, and converges to $\overline{\Theta}$ as $A \to \infty$. The maximized long-run growth rate is

$$\widehat{g}(A) = \overline{g}(A)\left(1 - \frac{4\overline{A}\alpha}{\delta-\rho+4\overline{A}}\right) \in \big(g^*(A,0), \overline{g}(A)\big). \tag{36}$$

Proof: See Appendix A.9.

These two propositions establish that the intuition we obtained from Figure 6 holds generally. They show that the possibility of subsidizing research by debt rollover and its effectiveness are determined by the pair of research productivity A and subsidy rate θ. While we discuss the way to estimate A in the next subsection, we can provide a useful corollary of Proposition 3 that does not require knowledge of A.

Corollary 1 (The $g-r$ Gap and the Effect of R&D Subsidy) *Suppose that the economy has a long-term BGP with $g^*(A,\theta) > r^*(A,\theta)$. Then, the gap $g^*(A,\theta) - r^*(A,\theta)$ is strictly decreasing in θ. The long-term growth can be increased by raising θ if and only if $g^*(A,\theta) - r^*(A,\theta) > (\delta-\rho)/2$.*
Proof: See Appendix A.9.

In this form, we can gauge the possibility of raising the long-term growth rate through a debt-financed research subsidy by looking at the economy's $g-r$ gap. If the gap is

Table 1: Parameters for numerical simulations

Parameter	Description	Value	Source
ρ	Discount rate	0.01	Standard
μ	Mortality rate	1/(80.3-20)	OECD (2023b)
δ	Labor deterioration rate	1/(64-20)	OECD (2023a)
α	Inverse of the markup rate	1/1.2	Ball and Mankiw (2023)
A	R&D productivity ($A > \widehat{A}$)	0.399[a]	Estimated from U.S. data
	R&D productivity ($A < \widehat{A}$)	0.120[a]	Estimated from Japanese data

[a] In the numerical simulations, we use the higher-precision values of A obtained from the estimation.

sufficiently large, the corollary suggests there is room to enhance growth by increasing θ. However, as θ gets higher, the $g - r$ gap will shrink because of the crowding-out effect of the government debt. The corollary states that it is best to stop raising the subsidy rate when the long-term $g - r$ gap reaches $(\delta - \rho)/2$, provided that the long-term growth rate is the objective. In the following subsection, we examine how these propositions and the corollary apply to data from various countries.

4.4 Numerical Simulations

In the following, we examine the effect of research subsidies via numerical simulations. Table 1 summarizes the parameters used in the simulations below. We set a standard value for the discount rate $\rho = 0.01$. The mortality rate is $\mu = 1/(80.3 - 20)$, where 80.3 is the average life expectancy in the OECD in 2021 (OECD, 2023b), and 20 is the age from which we assume that the agents start economic activities. The deterioration rate of labor productivity δ is set to $1/(64 - 20)$, where 64 is the average normal retirement age in the OECD as of 2022 (OECD, 2023a).[30] We set $\alpha = 1/1.2$ so that the markup rate of intermediate goods is $1/\alpha = 1.2$, following Ball and Mankiw (2023).[31] Given these parameters, we find that the threshold values of A are $\overline{A} = 0.0821$ and $\widehat{A} = 0.171$, and that the limit of the sustainable subsidy rate is $\overline{\Theta} = 18.9\%$.

To account for the difference in research productivity across countries, we separately estimate the value of A for each of the 24 countries that were included in the OECD by 1973. We focus on those developed countries because they can be seen as already on the BGP in the year 2000. Given that all investment in our model takes the form of R&D and

[30] Using similar data from the OECD, Rachel and Summers (2019) developed a probabilistic retirement model, whereas we consider the continuous depreciation of the working ability to simplify the analysis. The evolution of the average productivity is the same in these two specifications.

[31] In our model, α is a parameter in the production function, determining the elasticity of substitution across intermediate goods. We assume that the production function is the same across countries, and use a common value for α.

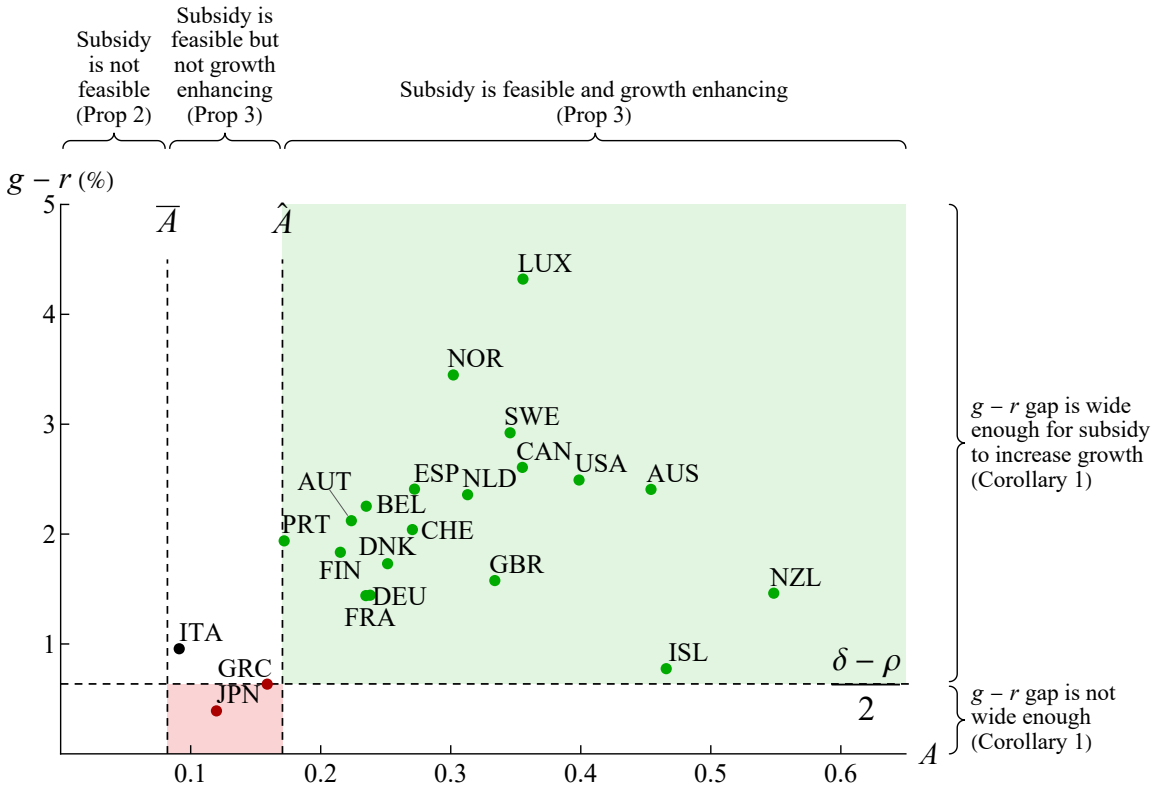


Figure 7: Estimates of research productivity A and the actual $g - r$ gaps.

that R&D uses only labor, the value of A in country i can be estimated as[32]

$$A_i = \alpha \left(1 + \frac{\alpha}{1-\alpha} \frac{1}{(wL^R/\text{GDP})_i} \right) g_i, \tag{37}$$

where we use the average real GDP growth rate and the average investment-GDP ratio of country i for the 2000–2023 period to specify g_i and $(wL^R/\text{GDP})_i$.[33] Figure 7 plots the estimates of A along with the average value of the $g - r$ gap for the same 2000–2023 period.[34] We find that all the estimates of A_i exceed the threshold value of $\overline{A} = 0.0821$. Then, Proposition 2 predicts that those economies have a long-term BGP with $g > r$. Indeed, Figure 7 confirms that the average values of $g - r$ in the 2000–2023 period are

[32]The derivation of this estimation equation is given in Appendix A.10.

[33] Both the real GDP growth rate and the investment-GDP ratio (the share of gross capital formation in GDP) are obtained from the Penn World Table version 11.0 (Feenstra, Inklaar and Timmer, 2015) and averaged over the 2000–2023 period. The $g - r$ gap is from the same data used in Figures 2 and 3. Alternatively, we could use the percentage of R&D expenditure in GDP (3.59% in the U.S. in 2022) to estimate $(wL^R/\text{GDP})_i$. This alternative method leads to much larger estimates of A_i, with which all 24 countries satisfy both $A_i > \overline{A}$ and $A_i > \widehat{A}$ by wide margins. We instead use all investment, which gives smaller estimates of A_i, to examine whether the conditions hold under a stricter criterion. The results for individual countries in this paper are obtained under this stricter criterion.

[34]Ireland ($A = 0.747$, $g - r = 6.11\%$) is omitted from the plot for readability. Turkey ($A = 0.858$) is not shown because its $g - r$ gap is not available due to a lack of short-term interest rate data.

positive for the countries shown.[35] Except for Japan, Italy, and Greece, all other countries exceed the threshold at $\widehat{A} = 0.171$. These countries also have $g - r$ gaps that are greater than $(\delta - \rho)/2$, the threshold in Corollary 1, which can be checked without estimating A.[36] Proposition 3 and Corollary 1 state that it is possible to increase growth via debt-financed research subsidies in those countries. We take the United States ($A_{\text{USA}} = 0.399$) as a representative of these countries and examine the effect of the policy in detail below. We take Japan ($A_{\text{JPN}} = 0.120$) as a representative of the countries with $A_i < \widehat{A}$.

Figure 8 numerically plots the long-term growth rate, interest rate, and debt-GDP ratio against the rate of research subsidies θ. Column (a) shows the baseline case of $A_{\text{USA}} = 0.399 > \widehat{A}$. The upper panel shows that the long-term growth rate, $g^*(A_{\text{USA}}, \theta)$, increases with θ until the subsidy rate reaches $\widehat{\theta}(A_{\text{USA}})$. It is possible to increase θ slightly more than that, but a rate above $\overline{\theta}(A_{\text{USA}})$ cannot be implemented, confirming the predictions of Propositions 2 and 3 and Proposition 1. The gap between $g^*(A_{\text{USA}}, \theta)$ and $r^*(A_{\text{USA}}, \theta)$ is wider than $(\delta - \rho)/2$ when $\theta = 0$, gradually shrinks as θ increases, and coincides with $(\delta - \rho)/2$ at the growth-maximizing subsidy rate $\widehat{\theta}(A_{\text{USA}})$, confirming Corollary 1.

For the case of countries with low research productivity, as represented by Japan (see Column (b)), the growth rate is always decreasing with the research subsidy. This is consistent with Proposition 3, which implies that the long-term growth-maximizing policy is $\theta = \widehat{\theta}(A_{\text{JPN}}) = 0$. In light of Corollary 1, we can understand this result by noting that $g^*(A_{\text{JPN}}, \theta) - r^*(A_{\text{JPN}}, \theta)$ is already lower than $(\delta - \rho)/2$ when $\theta = 0$. The $g - r$ gap is too low to justify the debt-financed subsidy, and the small gap comes from the fact that the research productivity is low in the first place.[37]

In the lower-left panel, $D^*(A_{\text{USA}}, \theta)$ shows the response of the long-term debt-GDP

[35] The gaps in the data tend to be larger than those in the calibrated model. One reason is that our model considers only the life-cycle saving motive under a perfect annuity market. In reality, few retirees annuitize a substantial part of their wealth (Benartzi, Previtero and Thaler, 2011), and individuals save against the risk of a long life with high medical expenses (De Nardi, French and Jones, 2010). Moreover, the elderly do not use up their wealth before they die, partly because of bequest motives (Kopczuk and Lupton, 2007; Lockwood, 2018). These saving motives, which our model abstracts from, raise the demand for assets and lower the interest rate relative to the growth rate. Our theory shows that $g > r$ can hold and debt-financed policies can promote growth even without these additional saving motives, which would make the condition $g > r$ even easier to satisfy.

[36] Note that the values for $g - r$ in Figure 7 are not the values predicted by the theory, but the actual numbers from the historical data. They are measured independently of the calibration of A. Therefore, even though Proposition 3 and Corollary 1 are theoretically equivalent if the actual economy is exactly represented by the model, they could give different predictions when applied to the calibrated values of A and the data on $g - r$. Still, our results show that the two criteria give remarkably similar predictions, with the only exception of Italy, where the $g - r$ gap is larger than $(\delta - \rho)/2$ despite $A < \widehat{A}$. This exception can be explained by the saving motives discussed in footnote 35, which make the actual gap larger than the gap in the calibrated model.

[37] In Section 7.1, we examine the possibility of enhancing the research productivity in the private sector by funding public basic research.

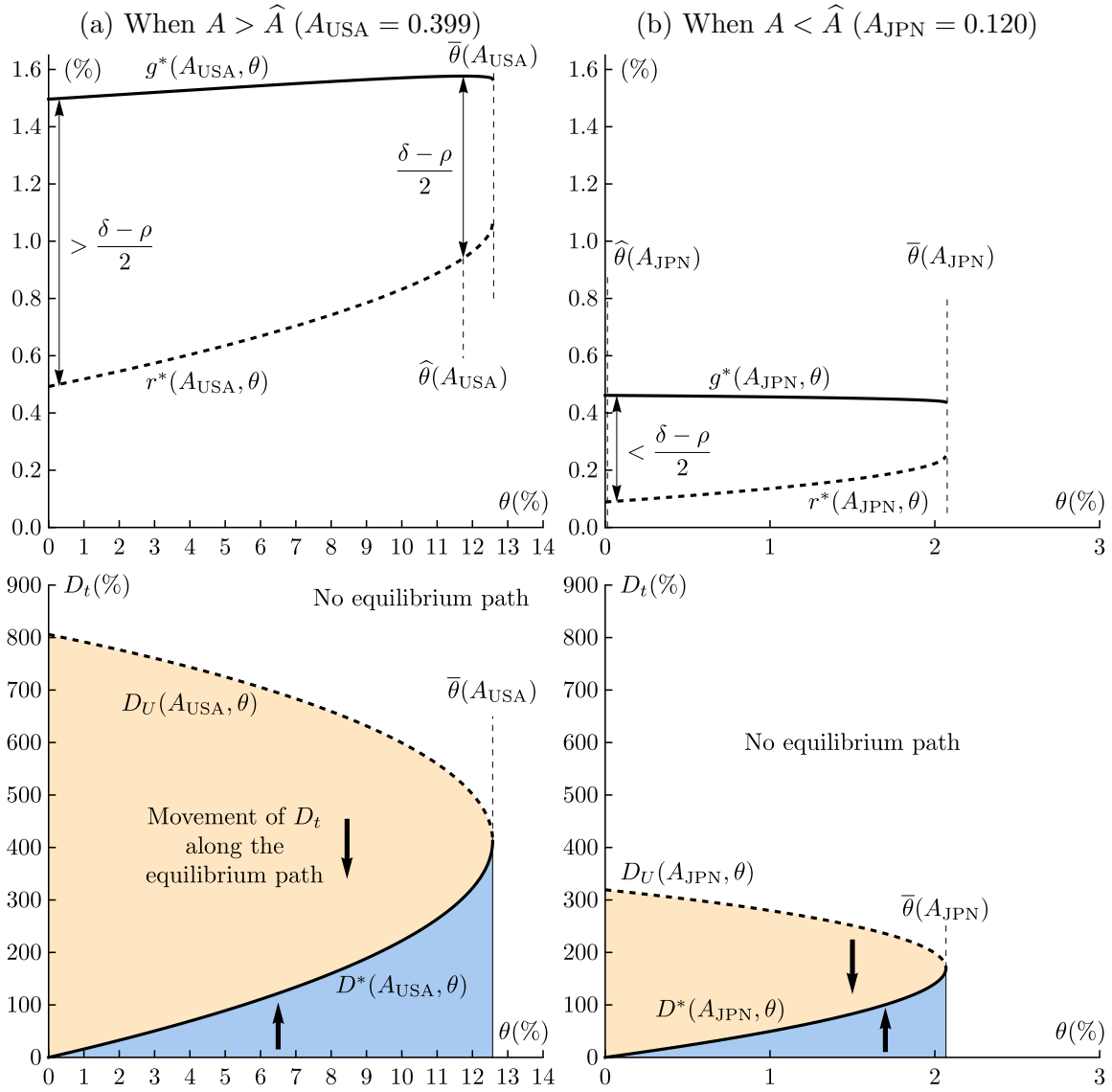


Figure 8: The long-term effect of the research subsidy rate θ on the growth rate $g^*(A, \theta)$, the interest rate $r^*(A, \theta)$, and the debt-GDP ratio $D^*(A, \theta)$ on the long-term BGP, and on the unsustainable threshold $D_U(A, \theta)$.

ratio as the subsidy rate changes.[38] The long-term debt-GDP ratio gradually increases with θ, but when the subsidy rate comes close to the upper bound, $\overline{\theta}(A_{\text{USA}})$, the slope becomes steeper, reaching infinity at $\theta = \overline{\theta}(A_{\text{USA}})$. This means that when the economy is on the verge of fiscal collapse (i.e., losing its equilibrium path), even a slight increase in government expenditure can lead to a large change in the debt-GDP ratio. In that case, the benefit of marginally increasing the subsidy to R&D will be dwarfed by the substantial increase in the interest rate, caused by the significant increase in the long-term level of

[38]Strictly speaking, the values we report in those panels are the ratio of the government debt to the full-production GDP. Conventionally, the debt-GDP ratio is defined as the ratio of the government debt to the actual GDP. Appendix A.3 shows that the conventionally defined debt-GDP ratio is higher than the values in these panels. However, the two indices yield similar values, with the difference at most 3.4%.

government debt. Therefore, setting the highest possible subsidy rate $\theta = \overline{\theta}(A_{\text{USA}})$ is not the best policy to increase growth. This is why $\widehat{\theta}(A_{\text{USA}})$ is lower than $\overline{\theta}(A_{\text{USA}})$.

The $D_U(A, \theta)$ curves in the lower panels show the unsustainable threshold. The arrows in the panels represent the direction of movement of D_t on the equilibrium path. The economy has no equilibrium path if D_t exceeds $D_U(A, \theta)$, and a subsidy rate above $\overline{\theta}(A)$ cannot be implemented (outside the shaded region in each panel). Observe that $D_U(A, \theta)$ is decreasing in θ, whereas $D^*(A, \theta)$ is increasing, and the two meet at $\theta = \overline{\theta}(A)$. A higher subsidy raises the long-term debt-GDP ratio and lowers the limit that the economy can sustain. When the two coincide, the long-term BGP disappears, and no subsidy rate above $\overline{\theta}(A)$ can be sustained. Therefore, even when a higher θ increases the growth rate, it entails two kinds of costs, namely, it increases the long-term debt-GDP ratio, $D^*(A, \theta)$, and it reduces the unsustainable threshold, $D_U(A, \theta)$.[39]

The lower-right panel shows the same for low-A countries, as exemplified by Japan. While the shape is qualitatively similar to that of the United States, it shows that the unsustainable threshold is much lower (319% for Japan and 806% for the United States when $\theta = 0$). Correspondingly, the highest sustainable subsidy rate is much lower in Japan (2.07%) than in the United States (12.57%). The United States can maintain a higher θ because its higher research productivity enables the country to sustain a larger amount of debt with a wider $g - r$ gap, which helps the government enjoy a larger surplus from the debt, $(g - r)B_t$.

4.5 Transitional Effects

Thus far, we have shown that debt-financed R&D subsidies enhance long-term growth in economies with high A (which means high $g^*(A, 0)$), but they are detrimental to long-term growth when A is low (i.e., $g^*(A, 0)$ is low). From this finding, it can be tempting to conclude that the policy will be beneficial for economies only when the growth rate before the policy is already sufficiently high. However, this conclusion may not hold for two reasons. First, the growth rate may change during the transition from the old BGP to the new long-term BGP. Second, the transition may take a very long time. If the growth rate is higher than its long-term value for a long period, the eventual difference in the GDP level can be significant. Indeed, the following proposition shows that the subsidy raises the

[39]The debt-GDP ratio could be (temporarily) affected by various kinds of shocks, although we do not explicitly consider these shocks. If the current debt-GDP ratio is close to the unsustainable threshold, $D_U(A, \theta)$, these shocks may push D_t above $D_U(A, \theta)$, which would make the government go bankrupt. Therefore, the decline in $D_U(A, \theta)$ can be viewed as a cost, as it undermines the sustainability of government debt.

growth rate of full-production GDP in the short run, regardless of A.

Proposition 4 (Short-Run Effect of the Subsidy) *From any point on an equilibrium path with positive R&D, an unanticipated permanent increase (decrease) in θ raises (reduces) g_t at that time, as long as the economy still has an equilibrium path under the new rate.*

Proof: See Appendix A.11.

Figure 9 shows the transitional response of the economy when the subsidy rate θ is permanently raised from zero to the neighborhood of the highest sustainable rate, $\overline{\theta}(A)$. At time $t = 0$, the subsidy rate is raised from 0 to 11.74% for the economy with $A_{\text{USA}} = 0.399$ (column a) and to 2% for the economy with $A_{\text{JPN}} = 0.120$ (column b).[40] The other parameters are the same as those in the previous subsection. The transitional paths of g_t and D_t are obtained in closed form, because the equilibrium path is a straight line in the (D_t, g_t) plane (see Appendix A.12). As shown in the upper panels, the growth rate of full-production GDP, g_t, increases immediately after the R&D subsidy is introduced, even when $A < \widehat{A}$, confirming Proposition 4.

The panels in the second row show the path of r_t.[41] As time passes, the growth rate gradually decreases, while the interest rate increases, shrinking the gap between g_t and r_t. It takes hundreds of years for the $g_t - r_t$ gap to converge to the long-term value. The reason for the slow convergence can be observed from the panels in the third row. The panels show that it takes a long time for the debt-GDP ratio to increase. Since g_t moves along the equilibrium path as D_t slowly changes, and r_t is determined by g_t, their moves are also sluggish.

The bottom panels show the percentage deviation of paths of full-production GDP Y_t, aggregate consumption C_t, and actual GDP, given by (22), from their respective paths without research subsidies. The formal definitions and derivations of these quantities are explained in Appendix A.12. The response of the deviation of Y_t is monotonically increasing when $A > \widehat{A}$ and is hump-shaped when $A < \widehat{A}$. Since the growth rate of Y_t is g_t, this pattern directly reflects the results shown in the upper panels. Immediately after the research subsidy is introduced, the aggregate consumption jumps down to some extent

[40] For the high-A country (calibrated to the U.S.), the subsidy rate is chosen to maximize the long-term growth rate. For the low-A country, the policy that maximizes long-term growth is $\theta = 0$, so we choose a round number that is slightly below the highest sustainable rate, $\overline{\theta}(A) = 2.068\%$.

[41] At the time of the introduction, r_t either jumps up or down. As shown in Equation (21), there are two forces that affect the return of R&D investment r_t immediately. First, a higher θ directly reduces the private cost, which increases the rate of return. However, when the subsidy increases the labor allocated to R&D, production shrinks, as well as profit. The panels show that the first effect dominates for $A = 0.399$, whereas the second effect is slightly larger when $A = 0.120$.

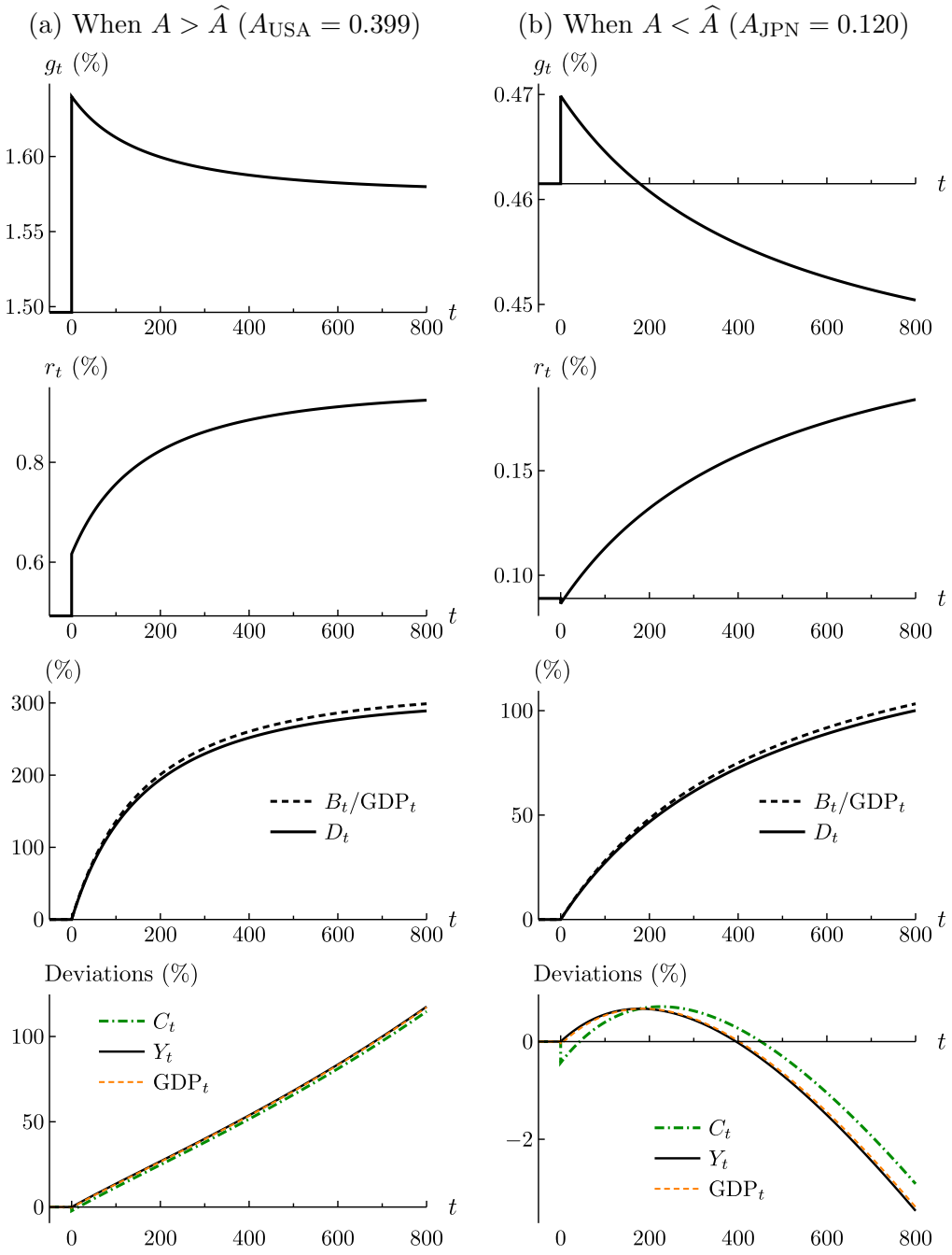


Figure 9: Transitional dynamics after research subsidies are introduced. In each panel, the horizontal axis is drawn at the level of the balanced growth path before the policy change.

because the subsidy induces more resources to be devoted to R&D than to the production of consumption goods. After about 15 years in column (a) and about 40 years in column (b), aggregate consumption overtakes the case of no subsidy, owing to increased productivity (i.e., a higher level of full-production GDP Y_t). The response of the deviation in GDP is between those of Y_t and C_t.[42]

[42]The GDP is the sum of the values of consumption and R&D. In our model, consumption goods are

Even when $A < \widehat{A}$, column (b) suggests that debt-financed R&D subsidies have sizable positive effects on the levels of GDP and consumption for several centuries. The growth rate is higher than the no-policy case for almost two centuries, and the elevated production technology keeps output higher for about four centuries. Of course, the negative effect eventually dominates the initial positive response, but it happens only centuries in the future. Given that the actual policy time horizon is far shorter, such long-term effects could be ignored in policymaking. In theory, the desirability of such a policy depends on how we discount the welfare of future generations relative to current generations.

5 Social Rate of Return of R&D and Dynamic Efficiency

In the previous sections, we have examined the conditions under which the government can roll over its debt perpetually and the conditions under which debt-financed research subsidies raise the long-term growth rate. Suppose that these conditions are met. Is such a policy desirable? Since we explicitly consider overlapping generations of consumers, there is no single welfare measure on which all generations agree. As shown in Column (b) of Figure 9, a policy can be beneficial for the current generation but detrimental to distant future generations.

We therefore consider a more modest question. Given that research subsidies increase R&D investments, we ask whether the increased investments are wasteful. In the existing literature, this question is treated as the question of dynamic efficiency. Previous studies have shown that the economy is dynamically inefficient if the rate of return on investment is lower than the rate of economic growth (e.g., Tirole, 1985; Abel, Mankiw, Summers and Zeckhauser, 1989). In this case, the aggregate consumption for all periods can be increased by reducing investment, so stimulating investment is actually welfare-reducing. For example, consider a standard neoclassical growth model in discrete time, and suppose that the economy is on a BGP where investment grows at the rate g and the social rate of return on investment is $r^s < g$. If investment is reduced by one percent in every period from some date onward, consumption at that date increases by the amount of the reduction. In each subsequent period, the investment saved in that period is larger than the output lost from the earlier reduction, and consumption increases as well.[43] Reducing investment

produced with a positive markup, whereas R&D is subject to free entry without markups. Therefore, when more resources are devoted to R&D, the GDP slightly decreases because the total value of markups decreases.

[43]The output lost in each period, including the undepreciated capital, is $1 + r^s$ times the reduction of investment in the previous period, whereas the investment saved in the same period is $1 + g$ times as large. Hence consumption increases by $(g - r^s)/(1 + g)$ percent of that period's investment.

is therefore Pareto improving when $r^s < g$. This possibility is a particular concern in the literature on rolling over debts because most papers focus on the case of $r < g$.

In this section, we show that when the research subsidy raises the long-term growth rate, the R&D investments promoted by the subsidy are not wasted, in the sense that they allow the economy to enjoy higher consumption at some future date. This claim is closely related to dynamic efficiency. Formally, we prove that on any BGP of our model, if today's R&D is reduced to increase today's consumption, for example by cutting the research subsidy, then aggregate consumption must fall at some future date. At that date, the consumption of at least one generation necessarily falls. Conversely, if the policy succeeds in increasing R&D, which requires the condition of Proposition 3 in the long run, consumption can be raised at some point in the future. This property corresponds to dynamic efficiency in the neoclassical growth model, in which a reduction in today's investment necessarily reduces consumption at some point in the future.

In our model, R&D investments have positive externalities on future R&D. In Equation (15), the output of R&D, $\dot{n}_t$, depends positively on today's stock of knowledge capital n_t, i.e., the cumulative number of ideas produced in the past. Similarly, today's R&D creates new knowledge, which will be used in the future. Therefore, the social rate of return on investment r^s, including this externality, is higher than the interest rate r. Thus, we cannot say that $r^s < g$ holds even when the interest rate on the government bond is lower than the growth rate. In fact, in the following, we show that the social rate of return of R&D investments in our model is higher than the rate of economic growth on any BGP. Following Jones and Williams (1998), we define the social rate of return as

Definition 1 *On the BGP, the social rate of return of R&D (r^s) is defined as follows:*

$$r^s \equiv \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left(\frac{dX_{t+\Delta t}}{-dX_t} - 1 \right), \tag{38}$$

where $dX_t < 0$ and $dX_{t+\Delta t} > 0$ are the changes in X_t and $X_{t+\Delta t}$, respectively, from the respective BGP values when the following operations are conducted in the model, where time is discretized with an infinitesimally small Δt.

(a) At time t, L_t^R marginally increases by a small value $dL_t^R > 0$.

(b) At time $t + \Delta t$, $L_{t+\Delta t}^R$ decreases by $dL_{t+\Delta t}^R < 0$. The magnitude of $dL_{t+\Delta t}^R$ is determined so that all variables at and after time $t + 2\Delta t$ are on the original BGP.

Note that in this definition, the final goods production X_t at time t decreases by $dX_t < 0$ because the economy allocates more resources to R&D. This result is considered an increase in the R&D investment measured in terms of final goods. At time $t+\Delta t$, the return on this

R&D investment is harvested by increasing final goods production by $dX_{t+\Delta t} > 0$ so that the reduction in R&D investment at $t + \Delta t$ exactly offsets the effect of the initial increase in R&D. The social rate of return of R&D is measured by how large the magnitude of the harvest ($dX_{t+\Delta t}$) is relative to the initial R&D cost ($-dX_t$). Since this value is the social rate of return for the duration of Δt, the rate of return for unit time is given by dividing it by Δt, as in (38). We determine the value of r^s as follows.

Proposition 5 (Social Rate of Return) *In any BGP of this model, the social rate of return of R&D, defined by Definition 1, is $r^s = \overline{g}(A)$; therefore, $r^s \geq g$ holds.*
Proof: See Appendix B.1.

Proposition 5 holds regardless of the value of θ, as long as the BGP exists. Note that $\overline{g}(A)$ is given by (26), which represents the maximum rate of economic growth when all labor is used for R&D. Therefore, the social rate of return $r^s = \overline{g}(A)$ is always higher than the rate of economic growth g. This result implies that the economy satisfies a necessary condition for dynamic efficiency ($r^s \geq g$), even though $r < g$. The following proposition establishes the essence of dynamic efficiency in a more direct way.

Proposition 6 (Dynamic Efficiency) *For any BGP in this model, it is not possible to increase the aggregate consumption for a short period while keeping the aggregate consumption outside this interval unchanged.*
Proof: See Appendix B.2.

Given the state of technology (i.e., n_t), the only way to increase aggregate consumption is by reducing R&D and allocating more resources to production. If the amount of R&D investments is reduced today, then the proof of the proposition shows that it is inevitable to reduce aggregate consumption sometime in the future. This property holds in any BGP, including the BGP with positive R&D subsidies. Therefore, a reduction in the research subsidy today (which reduces R&D investments) will lead to a reduction in consumption in the future. In other words, the R&D investments induced by the subsidy are never a waste of resources.

These results can be contrasted with the rational Ponzi games studied by O'Connell and Zeldes (1988). In their analysis, what enables the government to roll over its debt is not the length of individuals' planning horizons but the entry of an infinite number of new agents. The same structure is present in our model. The government is the only agent that lasts forever, while households are replaced by newborn cohorts in the overlapping-generations structure, and this is what makes perpetual debt rollover possible. In their

analysis, however, the rollover is feasible only when the economy is dynamically inefficient. The same condition characterizes the emergence of rational bubbles in Diamond (1965) and Tirole (1985).

Why is the rollover feasible in our model even though the economy is dynamically efficient? The reason is that the interest rate and the social rate of return of R&D are two different rates. The interest rate is kept below the growth rate by the life-cycle saving motives of individuals, and this is all that the rollover requires. Dynamic efficiency instead concerns the social rate of return, which exceeds the private return because of the intertemporal knowledge spillovers and remains above the growth rate by Proposition 5.[44] Hence the rolled-over debt itself does not improve welfare, unlike the bubbles in the studies above, and whether the debt benefits the economy depends on how the proceeds are used.

6 Effects of Population Growth

In Sections 3–5, we examined the case of the constant population by assuming that the birth rate β is the same as the mortality rate μ. This section studies how the results change when we relax this simplification. Population growth (or decline) matters for two reasons. First, a growing population expands the size of the economy and changes the rate at which a given stock of debt is diluted by GDP growth. Second, in the overlapping-generations environment, population growth changes the share of young households and the age composition of effective labor. This affects aggregate saving and the equilibrium $g - r$ gap. To examine these additional effects in a tractable manner, we present a simple second-generation endogenous growth model, which includes both horizontal expansion and vertical innovation. We then show that the baseline model is a special case of this new framework.

6.1 A Simple Second-Generation Endogenous Growth Model: Setup

The household side is identical to that in Section 3.1, except that we no longer impose $\beta = \mu$. As we have shown in that section, the rate of population growth becomes $g_L = \beta - \mu$. We assume that β and μ do not change over time, which means that g_L is also constant. As in the baseline model, the final goods are produced from the intermediate goods of different industries. The model differs from the baseline model in two respects. First,

[44] Recent studies (e.g., Blanchard, 2019; Ball and Mankiw, 2023; Barro, 2023) obtain dynamic efficiency despite $r < g$ in neoclassical settings, where the wedge between the marginal product of capital and the interest rate arises from risk premiums or markups on capital production. The wedge in our model comes instead from the knowledge spillovers of R&D.

the quality of the intermediate good of each industry is improved by R&D, which we call vertical innovation. Second, new industries are created on the basis of new ideas, which occur to individuals, and we call this process horizontal innovation.

We assume that genuinely new ideas that lead to the creation of new industries occur to individuals at a small rate of $f > 0$ per unit of effective labor and per unit of time. Since the aggregate supply of effective labor is L_t, the rate of creation of new industries is fL_t. At the same time, each industry faces a risk of becoming obsolete. Such obsolescence occurs at the rate of $\varepsilon > 0$, and we assume that $g_L + \varepsilon > 0$. Hence, the number of industries evolves according to

$$\dot{n}_t = fL_t - \varepsilon n_t. \tag{39}$$

Given that the rate of population growth is constant and that the economy has a sufficiently long history, the effective labor per industry becomes[45]

$$\frac{L_t}{n_t} = \frac{g_L + \varepsilon}{f}. \tag{40}$$

In the following, we consider the economy after this ratio has converged.

Let $x_t(j)$ and $q_t(j)$ denote the quantity and the quality of the good of industry $j \in [0, n_t]$. A representative firm competitively produces the final goods according to

$$X_t = \left[\int_0^{n_t} \big(q_t(j)x_t(j)\big)^{\alpha} dj\right]^{1/\alpha}, \tag{41}$$

where $\alpha \in (1/2, 1)$ as in the baseline model. We assume that a new industry starts with the average quality of the existing industries. Let $p_t(j)$ denote the price of the good of industry j. Then the demand for this good is $x_t(j) = q_t(j)^{\alpha/(1-\alpha)} p_t(j)^{-1/(1-\alpha)} X_t$, and its price elasticity is $1/(1-\alpha)$. Equation (41) differs from the production function (9) of the baseline model only in that each good has a quality. In the baseline model, the quality of a good cannot be improved, and R&D is conducted to create new industries.

In each industry, the good is produced by a single firm, the leader, which owns the core technology of the industry.[46] The leader hires workers, and one unit of effective labor produces one unit of the good. In addition, workers sometimes obtain ideas for new businesses in the course of their work. Since workers do not have the resources to

[45] From $\dot{L}_t/L_t = g_L$ and (39), we obtain a differential equation for n_t/L_t. Solving this differential equation yields $n_t/L_t = f/(g_L+\varepsilon) + \big(n_0/L_0 - f/(g_L+\varepsilon)\big)e^{-(g_L+\varepsilon)t}$. When t is sufficiently large, it becomes $f/(g_L+\varepsilon)$.

[46] When former leaders also exist, we follow Assumption 1 of Akcigit and Kerr (2018) on pricing. In a two-stage game, each firm pays a fee that is arbitrarily close to zero in the first stage, and the firms that paid the fee announce their prices in the second stage. Since only the leader can recover the fee, only the leader enters the second stage and sets the monopoly price.

commercialize such ideas by themselves, they sign an employment contract with the firm, under which they hand over the ideas to the firm and, in return, their wage is higher by the expected value of the ideas. For simplicity, we assume that commercializing an idea requires no additional cost.[47] Let v_t^{new} denote the value of a firm when a new business is spun off and becomes the leader of a new industry. The firm whose workers find a new idea earns this amount by selling the shares in the new firm. When the leader of industry j hires $\ell_t^P(j)$ units of effective labor, it produces $x_t(j) = \ell_t^P(j)$, and its expected profit is $p_t(j)x_t(j) + fv_t^{\text{new}}\ell_t^P(j) - w_t\ell_t^P(j)$, where w_t is the wage per unit of effective labor. Maximizing this profit with respect to $\ell_t^P(j)$ gives the price $p_t(j) = (w_t - fv_t^{\text{new}})/\alpha$, and the maximized expected profit is $\pi_t(j) = (1-\alpha)p_t(j)x_t(j)$. Since individuals receive w_t per unit of effective labor, the analysis in Section 3.1, including the aggregate Euler equation (8), holds without change.

Outsiders conduct R&D to take over the position of the leader. When an outsider uses $\ell_t^R(j)$ units of effective labor for R&D in industry j, its research yields a core technology that the leader cannot follow with a Poisson arrival rate of $a\ell_t^R(j)$. In that case, the outsider becomes the new leader of the industry, and the previous leader loses its profit. The parameter a measures how likely the research is to yield such a core technology. It depends not only on the nature of technology but also on institutions and the degree of competition. In an environment where it is very difficult to take over the position of the leader, a is low. In a market where leaders are frequently replaced because of institutions or the environment, a is high. As in the baseline model, the government subsidizes a fraction $\theta \in [0, 1)$ of the wage paid for R&D and finances the subsidy by issuing debt, so that its budget constraint is (17). For each unit of effective labor that an outsider hires for R&D in industry j, it obtains the value of the leader, $v_t(j)$, with probability a per unit time, and new ideas worth v_t^{new} with probability f. The cost of hiring the worker net of the subsidy is $(1-\theta)w_t$. Since outsiders can freely enter R&D, the following free-entry condition holds in equilibrium:

$$av_t(j) + fv_t^{\text{new}} \leq (1-\theta)w_t, \quad \text{with equality if } \ell_t^R(j) > 0. \tag{42}$$

Regardless of whether it yields a core technology, the research of outsiders raises the technology level of the industry, that is, the quality of its good, and its results are also used in other industries. This is because whether the leader can follow a new technology is not

[47] The results do not change if commercializing an idea requires additional costs. As long as these costs are lower than the expected cost of replacing the leader of an existing industry, an idea is commercialized as soon as it is obtained. In that case, the increase in the wage is smaller by the cost of commercialization.

known until the technology is disclosed. Outsiders therefore disclose all their technologies, and in most cases, the leader can easily adopt them without being replaced. As a result, the quality rises according to

$$\frac{\dot{q}_t(j)}{q_t(j)} = \omega_{\text{in}} \ell_t^R(j) + \omega_{\text{out}} \frac{L_t^R}{n_t}, \tag{43}$$

where $L_t^R \equiv \int_0^{n_t} \ell_t^R(j) dj$ is the aggregate effective labor used for R&D. The first term represents spillovers within the industry, and ω_{in} measures their strength. The second term represents spillovers from other industries, which arise because the research in each industry spills out to other industries. The parameter ω_{out} measures the strength of these spillovers.[48]

Lemma 1 (Symmetric Equilibrium) *Suppose that* $\omega_{\text{in}} \in \big(0, a(1-\alpha)/\alpha\big)$.

(a) In any equilibrium, leaders do not conduct research.

(b) If, in addition, $q_0(j)$ *is the same for all industries, then in any equilibrium,* $\ell_t^R(j)$ *and* $q_t(j)$ *are the same for all industries at every* t.

Proof: See Appendix C.1.

The first result holds because what a leader gains from its own research is smaller than what an outsider gains from the same research.[49] The second result holds because the value of the leader is lower in an industry where outsiders conduct more research. In the following, we assume that $\omega_{\text{in}} \in \big(0, a(1-\alpha)/\alpha\big)$.[50] By Lemma 1, $q_t(j)$, $\ell_t^R(j)$, and $x_t(j)$ are the same for all industries, and we omit the index j in the following. In particular, $v_t(j) = v_t^{\text{new}}$ for all j, and writing v_t for this common value, the free-entry condition (42) becomes

$$v_t \leq \frac{(1-\theta) w_t}{a+f}, \quad \text{with equality if } L_t^R > 0. \tag{44}$$

[48] The spillovers increase with the aggregate R&D, L_t^R, but decrease with the number of industries, n_t. As emphasized by Peretto and Smulders (2002), when there are more industries, the technologies of different industries are farther apart, and it becomes more difficult for an industry to make use of the results of research in other industries.

[49] As in standard models such as Aghion and Howitt (1992) and Grossman and Helpman (1991a), leaders do not conduct research in our model. Some models, such as Peretto (1998), assume that leaders conduct research, but leaders are not replaced in those models. Our setting is complementary to them.

[50] If $\omega_{\text{in}} > a(1-\alpha)/\alpha$, the value of the leader is higher in an industry where outsiders conduct more research. Then, even if research is divided among several industries, a small perturbation that creates a difference in research makes the quality and the value of the leader higher in the industry with more research, and all outsiders move to that industry. Research is therefore concentrated in a single industry, and the economy effectively consists of a single industry. Since this is unrealistic, we exclude this case.

6.2 Equilibrium Dynamics

Let L_t^P denote the aggregate effective labor used for production. By Lemma 1, each leader produces $x_t = L_t^P/n_t$, and outsiders use $\ell_t^R = L_t^R/n_t$ units of effective labor for R&D in each industry. The labor market clears when $L_t^P + L_t^R = L_t$, the goods market clears when $C_t = X_t$, and the asset market clears when $K_t = B_t + n_t v_t$. Substituting $x_t = L_t^P/n_t$ into (41) gives

$$X_t = n_t^{\frac{1-\alpha}{\alpha}} q_t L_t^P. \tag{45}$$

The demand for each good then implies $p_t = n_t^{(1-\alpha)/\alpha} q_t$. Combining this price with $p_t = (w_t - f v_t)/\alpha$ gives

$$w_t - f v_t = \alpha n_t^{\frac{1-\alpha}{\alpha}} q_t, \tag{46}$$

and the profit of each leader is $\pi_t = (1-\alpha) p_t x_t = (1-\alpha) X_t/n_t$. In this section, we define full-production output, Y_t, as the output of the final goods when all effective labor is used for production:[51]

$$Y_t = n_t^{\frac{1-\alpha}{\alpha}} q_t L_t. \tag{47}$$

We define two combined parameters. The first is the relative propensity for vertical innovation, $\kappa \equiv a/f$. When κ is larger, replacing leaders through vertical R&D is easier relative to horizontal innovation, which requires new ideas that arise only in proportion to population. Hence, κ affects the direction of innovation. The second is the step size, defined as $\lambda \equiv (\omega_{\text{in}} + \omega_{\text{out}})/a$. Suppose that each industry uses one unit of effective labor for R&D. Then the replacement of leaders occurs with probability of a, which means the average duration between leader replacements is $1/a$. During this time, the quality increases by $\lambda = (\omega_{\text{in}} + \omega_{\text{out}})/a$ both due to within- and across-industry spillovers.[52]

By Lemma 1, the rate of quality growth in (43) is the same in all industries and equals $\lambda a L_t^R/n_t$. Substituting (40) into this rate, we obtain the rate of vertical innovation,

$$g_{q,t} \equiv \frac{\dot{q}_t}{q_t} = \overline{g_q}(\kappa, g_L) \frac{L_t^R}{L_t}, \qquad \overline{g_q}(\kappa, g_L) \equiv \lambda\kappa(g_L + \varepsilon), \tag{48}$$

where $\overline{g_q}(\kappa, g_L)$ is the rate of vertical innovation when all effective labor is used for R&D. By (40), the rate of horizontal innovation, $\dot{n}_t/n_t$, equals the population growth rate g_L.

[51] In this section, we call Y_t the full-production output rather than the full-production GDP because GDP now includes the value of new ideas, as discussed in Appendix C.9.

[52] While we use an example of $\ell_t^R = 1$, the step size does not depend on ℓ_t^R. Precisely, the quality increases by $e^\lambda - 1$, which is approximately equal to λ when λ is small. In the quality ladder model of Grossman and Helpman (1991a), the step size is the ratio of the qualities of successive leaders, which corresponds to e^λ in our model.

From (47), the growth rate of full-production output is then

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = \frac{g_L}{\alpha} + g_{q,t}. \tag{49}$$

Horizontal innovation contributes $(1-\alpha)g_L/\alpha$ to growth, and vertical innovation contributes $g_{q,t}$. Their sum is the growth rate per capita, and adding the population growth rate g_L gives g_t.

As in the baseline model, the expected return on holding the shares of a leader must equal the interest rate. A leader loses its position when an outsider obtains a new core technology, which occurs at the rate $a\ell_t^R$, or when its industry becomes obsolete, which occurs at the rate ε. The no-arbitrage condition is therefore

$$r_t = \frac{\pi_t + \dot{v}_t}{v_t} - a\ell_t^R - \varepsilon. \tag{50}$$

When vertical R&D is conducted, the free-entry condition (44) holds with equality. Using it together with (46) and (48) to eliminate π_t and v_t from (50), we can represent the $g-r$ gap in terms of $g_{q,t}$:[53]

$$\begin{aligned} &g_t - r_t = \frac{1-\alpha}{\alpha\lambda}\, s(\theta^e)\big(g_{q,t} - \widetilde{g}_q(\kappa, g_L, \theta)\big), \\ &\text{where } \widetilde{g}_q(\kappa, g_L, \theta) = \frac{1-\alpha}{1-\alpha\theta^e}\,\overline{g_q}(\kappa, g_L) - \frac{\alpha\lambda(1-\theta^e)}{1-\alpha\theta^e}\,(g_L+\varepsilon). \end{aligned} \tag{51}$$

The function $s(\cdot)$ is the same as in (27), but it is evaluated at

$$\theta^e \equiv \frac{\theta w_t}{w_t - f v_t} = \frac{(a+f)\theta}{a+\theta f}, \tag{52}$$

which we call the effective subsidy rate on vertical R&D. Of the wage w_t that an outsider pays for a unit of effective labor in R&D, the outsider recovers fv_t as the expected value of new ideas. The cost of vertical R&D is therefore $(w_t - fv_t)L_t^R$, whereas the government subsidizes a fraction θ of the whole wage $w_t L_t^R$. The effective subsidy rate is the ratio of the subsidy to this cost. The second equality in (52) follows from the free-entry condition (44) with equality. As in the baseline model, the $g-r$ gap increases linearly with $g_{q,t}$, and $g_t > r_t$ holds if and only if $g_{q,t} > \widetilde{g}_q(\kappa, g_L, \theta)$.

When vertical R&D is conducted, the government budget constraint (17), the aggregate Euler equation (8), and the market-clearing conditions give the dynamics of this system in

[53]The derivation is given in Appendix C.2.

$D_t = B_t/Y_t$ and $g_{q,t}$,[54]

$$\dot{g}_{q,t} = \big(\overline{g_q}(\kappa, g_L) - g_{q,t}\big)\Big(\frac{1-\alpha}{\alpha\lambda}\, s(\theta^e)\big(g_{q,t} - \widetilde{g_q}(\kappa, g_L, \theta)\big) - (\delta - \rho + g_L)\Big) + \overline{g_q}(\kappa, g_L)(\rho + \mu)(\delta + \beta)\Big(D_t + \frac{\alpha(1-\theta^e)}{\kappa(g_L + \varepsilon)}\Big). \tag{53}$$

$$\dot{D}_t = -\frac{1-\alpha}{\alpha\lambda}\, s(\theta^e)\big(g_{q,t} - \widetilde{g_q}(\kappa, g_L, \theta)\big) D_t + \frac{\alpha\theta^e}{\overline{g_q}(\kappa, g_L)}\, g_{q,t}. \tag{54}$$

In (53), $D_t + \alpha(1-\theta^e)/\big(\kappa(g_L + \varepsilon)\big)$ is the ratio of aggregate assets to full-production output, K_t/Y_t. In (54), the first term is the negative of the surplus from government debt, $-(g_t - r_t)D_t$, and the second term is government spending on subsidies, $\theta w_t L_t^R/Y_t$. Both are measured relative to full-production output. These equations can be interpreted in the same way as (28) and (29) in the baseline model. As in the baseline model, Proposition 1 holds for $D_0 \geq 0$ with g_t replaced by $g_{q,t}$ (Appendices C.4 and C.5). The economy therefore has an equilibrium path if and only if it has two steady states and the initial ratio of debt to full-production output is below the unsustainable threshold, and the path converges to the long-term BGP. In the following, we examine the long-term BGP.

6.3 Population Growth and the Effect of Research Subsidies

We now examine how population growth changes the answers to the two main questions of this paper: Can the government roll over its debt indefinitely and use the proceeds to subsidize research, and does such a policy raise the long-run growth rate? In the baseline model, the answers depend on the research productivity A, as shown in Figure 6. Figure 10 shows the corresponding results in the second-generation model, where the horizontal axis is the population growth rate g_L. We find that population growth affects the relationship between g and r through different mechanisms depending on the propensity for vertical innovation, $\kappa = a/f$. As the population rises faster, each existing industry receives more new workers and more R&D, raising vertical innovation. This is the main mechanism when κ is larger. However, when κ is small, vertical innovation is dormant. Still, higher population growth creates more new ideas from individuals, enhancing horizontal innovation.

The left column shows the result for the case when $\kappa > \alpha/(1-\alpha)$. We use the value of κ from the U.S. as an example for this case (see Section 6.4 for calibration of κ). When the subsidy rate is lower than $\overline{\theta}(\kappa, g_L)$ shown in the upper panel, the government can finance research subsidies by rolling over its debt. In this case, the economy has a long-term BGP

[54] In this section, $D_t = B_t/Y_t$ denotes the ratio of government debt to the full-production output. When we consider the limit of $f \to 0$, as we do in Section 6.5, the full-production output coincides with full-production GDP, and therefore the definition of D_t becomes the same as in the baseline model.

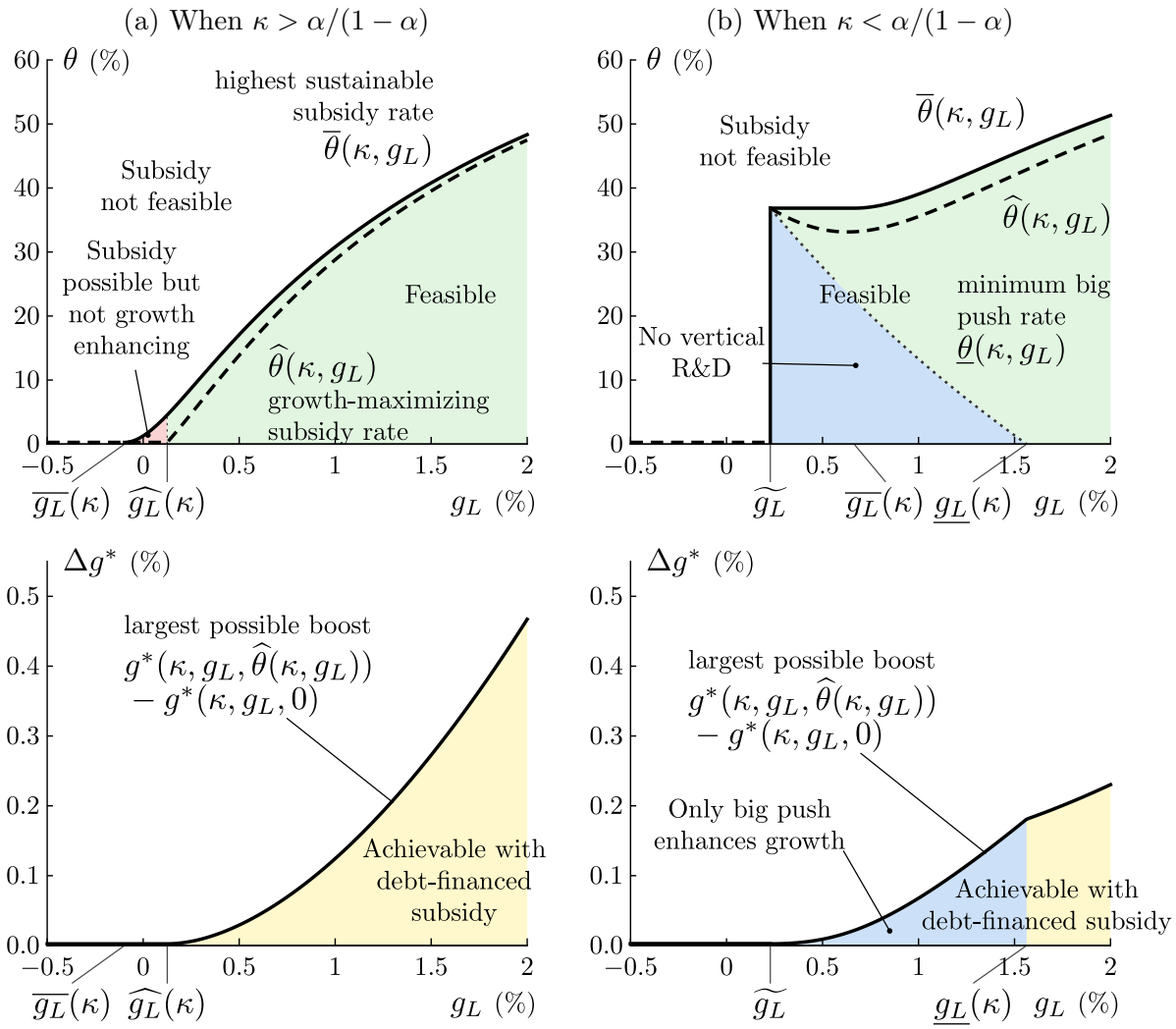


Figure 10: Sustainable and growth-maximizing subsidy rates against population growth. The parameters are those in Table 1 with $\varepsilon = 0.01$ and $\lambda = (1-\alpha)/\alpha$. Column (a) uses $\kappa_{\text{USA}} = 8.60$, and column (b) uses $\kappa_{\text{ITA}} = 2.79$.

with $g > r$, and the equilibrium path is the same as in Panel (a) of Figure 5, where g is replaced by g_q (Appendix C.5). As in the baseline model, if the government tried to set θ above $\overline{\theta}(\kappa, g_L)$, the economy would have no long-term BGP, and there would be no equilibrium path. Lenders would then not lend to the government, and such a subsidy rate cannot be implemented. The upper bound $\overline{\theta}(\kappa, g_L)$ is positive if and only if g_L is higher than the threshold $\overline{g_L}(\kappa)$, which is defined as the larger solution to the quadratic equation in g_L,[55]

$$(1+\kappa)(g_L+\varepsilon)(\delta-\rho+g_L) = (\rho+\mu)(\delta+\mu+g_L). \tag{55}$$

In the absence of subsidy ($\theta = 0$), the growth rate of aggregate output, which includes the effect of population growth, becomes higher than the interest rate, if and only if $g_L > \overline{g_L}(\kappa)$.

[55] See the proof of Proposition 7 in Appendix C.7.

The threshold $\overline{g_L}(\kappa)$ is decreasing in κ, because a higher propensity for vertical innovation helps raise the growth rate, and therefore, an economy can achieve $g > r$ with slower population growth. However, similarly to the case of the baseline model, the policy does not necessarily raise growth. When $\overline{g_L}(\kappa) < g_L < \widehat{g_L}(\kappa)$, any positive subsidy lowers the long-run growth rate. The policy raises the long-run growth rate only when g_L exceeds the second threshold $\widehat{g_L}(\kappa)$. These results are the counterparts of Propositions 2 and 3 in the baseline model, except that the answers depend on κ and g_L, rather than A. The lower panel shows that the largest possible increase in the long-run growth rate, $\Delta g^* \equiv g^*\big(\kappa, g_L, \widehat{\theta}(\kappa, g_L)\big) - g^*(\kappa, g_L, 0)$, rises with g_L.

The right column shows the result for the case when the propensity for vertical innovation is relatively low, $\kappa < \alpha/(1-\alpha)$. We use the value of κ from Italy as an example for this case. In this case, the government can finance research subsidies by rolling over its debt only when g_L is higher than the threshold $\widetilde{g_L}$, which is defined as the larger solution to the quadratic equation in g_L,

$$(g_L + \varepsilon)(\delta - \rho + g_L) = (1-\alpha)(\rho + \mu)(\delta + \mu + g_L). \tag{56}$$

Unlike $\overline{g_L}(\kappa)$, the threshold $\widetilde{g_L}$ does not depend on κ. This is because the propensity for vertical innovation is so low in these economies that they do not conduct any vertical R&D at the threshold level of g_L. In other words, the debt rollover becomes sustainable only when horizontal innovation alone can sustain $g > r$.

This result also has implications for the effectiveness of the subsidies for vertical R&D. Even when $g > r$ holds thanks to horizontal innovation, the subsidies for vertical R&D have a positive effect on growth only when they are big enough. In other words, growth responds only to a big push. Subsidies will not give those economies enough incentives to do vertical R&D when the subsidy rate is smaller than $\underline{\theta}(\kappa, g_L)$. This constraint is not binding when g_L is sufficiently large ($g_L > \underline{g_L}(\kappa)$) because in that case fast population growth gives enough incentives for vertical innovation even with smaller κ.

The following propositions formally establish the results shown in Figure 10.

Proposition 7 (Sustainability with Changing Population) *The economy has a long-term BGP with $g^* > r^*$ if and only if the following two conditions are met.*

(i) When $\kappa > \alpha/(1-\alpha)$, g_L must be higher than $\overline{g_L}(\kappa)$, which is strictly decreasing in κ. When $\kappa \leq \alpha/(1-\alpha)$, g_L must be higher than $\widetilde{g_L}$.

(ii) When $g_L > \overline{g_L}(\kappa)$, θ must be lower than $\overline{\theta}(\kappa, g_L)$, which is strictly increasing in g_L. When $\widetilde{g_L} < g_L \leq \overline{g_L}(\kappa)$, which can occur only if $\kappa < \alpha/(1-\alpha)$, θ must be lower than

$\alpha - (1-\alpha)\kappa$, *which does not depend on* g_L.

The defining equation for $\overline{\theta}(\kappa, g_L)$ *is given in Appendix C.7.*

Proof: See Appendix C.7.

Proposition 8 (Long-Run Growth Effects with Changing Population)

(i) Debt-financed research subsidies can raise the long-run growth rate if and only if

$$g_L > \begin{cases} \widehat{g_L}(\kappa) & \text{if } \kappa > \alpha/(1-\alpha), \\ \widetilde{g_L} & \text{if } \kappa \leq \alpha/(1-\alpha). \end{cases}$$

Here, $\widehat{g_L}(\kappa)$ *is strictly decreasing in* κ*, and* $\widetilde{g_L}$ *is the same threshold as in Proposition 7.*

(ii) When this condition is met, the growth-maximizing subsidy rate $\widehat{\theta}(\kappa, g_L)$ *lies in the interval* $\big(0, \overline{\theta}(\kappa, g_L)\big)$.

(iii) When $\widetilde{g_L} < g_L < \underline{g_L}(\kappa)$*, which can occur only if* $\kappa < \alpha/(1-\alpha)$*, the long-run growth rate does not increase if* $\theta \in \big(0, \underline{\theta}(\kappa, g_L)\big)$.

The defining equation for $\widehat{g_L}(\kappa)$ *is given in Appendix C.8, and those for* $\underline{g_L}(\kappa)$ *and* $\underline{\theta}(\kappa, g_L)$ *are given in Appendix C.6.*

Proof: See Appendix C.8.

Propositions 7 and 8 divide the (g_L, κ) plane into four regions, as shown in Figure 11. When g_L is low, the government cannot sustain the subsidies by rolling over its debt. For higher g_L, the subsidies are sustainable, and they raise the long-run growth rate if g_L is high enough. When $\kappa > \alpha/(1-\alpha)$, there is an intermediate range of g_L in which the subsidies are sustainable but do not raise growth. When $\kappa < \alpha/(1-\alpha)$, there is instead a range in which only a big push enhances growth.

6.4 Cross-Country Evidence

To see in which of the four regions in Figure 11 each country lies, we determine the values of κ and g_L for each of the 24 countries used in Section 4.4. For ρ, μ, δ, and α, we use the same values as in the baseline model (Table 1). We set $\varepsilon = 0.01$, which means that an industry becomes obsolete after 100 years on average.[56] We set $\lambda = (1-\alpha)/\alpha$, at which

[56]This is in line with studies of product life cycles and technology diffusion. For 25 new product markets in the United States, Agarwal and Gort (1996) find that the stages from the first commercial introduction to maturity last about 50 years in total on average. Grübler (1996) reports that the time required for a diffusion process to unfold from 10 to 90 percent of its extent is between 40 and 60 years on average, and Grübler and Nakićenović (1991) find that it is about 90 years for the substitution of primary energy sources in the United States. If we instead set $\varepsilon = 1/150$, the classification of the 24 countries in Figure 11 does not

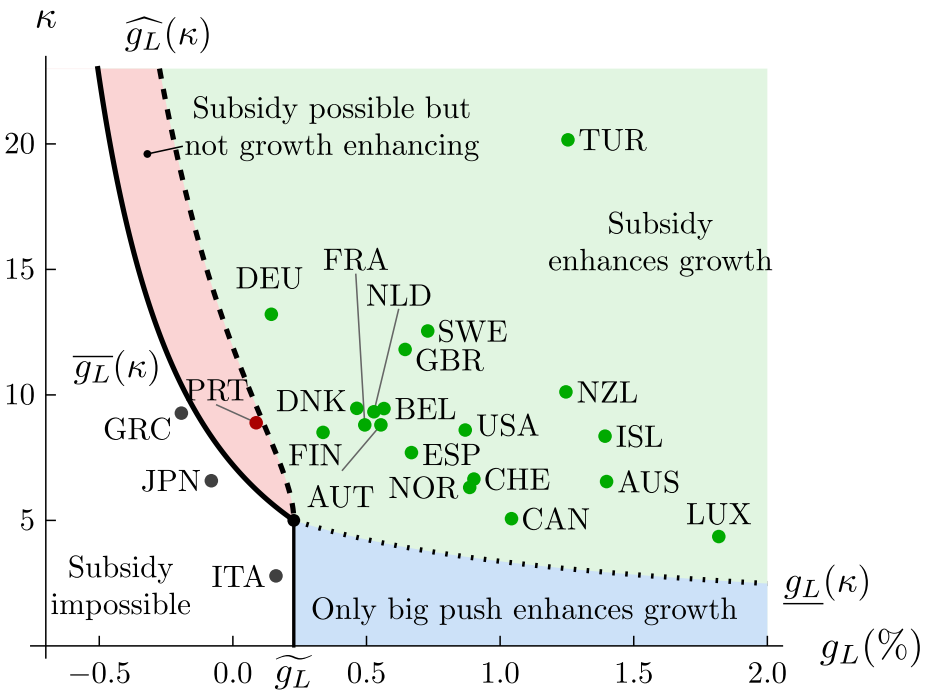


Figure 11: Growth-promoting conditions and calibrated country values in the (g_L, κ) plane.

the step size approximately equals the markup, as in the standard quality ladder model.[57]

The population of each country affects the results in two ways. First, the past population affects the current strength of vertical innovation. Since ideas for new industries occur to individuals, the number of people in the past determines the stock of these ideas, which determines the number of industries today. Therefore, if an economy had a large population relative to today, that economy now has relatively many industries when compared to the current population, which means that the number of workers per industry is small. This weakens the speed of vertical innovation because it depends on the number of researchers, as well as the demand for the goods the industry produces, both of which will be small in a country with low population growth. For this reason, when we estimate κ from the average GDP growth rate and the average investment-GDP ratio of each country over 2000–2023, we take into account its population since 1820.[58] Second, the future population growth affects whether the government can sustain the subsidies by rolling over its debt. We assume that the average population growth rate of each country over 2000–2023

change. If we set $\varepsilon = 1/50$, Japan and Italy lie in the region where only a big push enhances growth, and the other 22 countries lie in the region where the subsidies raise growth. When industries become obsolete faster, the number of workers per industry in (40) depends less on the population growth rate. Therefore, countries with slow population growth become less likely to fail the conditions in Propositions 7 and 8.

[57] If we instead set $\lambda = \ln(1/\alpha)$, at which the step size e^{λ} exactly equals the markup $1/\alpha$, the estimates of κ rise by about 10%, and only Portugal changes its classification. With $\lambda = (1-\alpha)/\alpha$, the assumption $\omega_{\text{in}} < a(1-\alpha)/\alpha$ in Lemma 1 holds whenever $\omega_{\text{out}} > 0$.

[58] We use the population since 1820 or the earliest available year. Appendix C.9 describes the details of the estimation.

continues in the future, and we use it as g_L.

The dots in Figure 11 represent the pairs of g_L and κ for the 24 countries.[59] If their current population growth rates continue, Japan, Italy, and Greece lie in the region where the government cannot sustain the subsidies by rolling over its debt.[60] In the baseline model, these three countries can roll over their debt, although the subsidies do not raise their long-run growth rates. Once population growth is taken into account, the result for them becomes more pessimistic. Portugal lies in the region where the subsidies are sustainable but do not raise growth. The other 20 countries lie in the region where the subsidies raise growth. No country lies in the region where only a big push enhances growth.

The second-generation model also provides a more structural interpretation of the cross-country differences in the effects of the subsidies. The estimates of κ for Japan and Greece are not particularly low compared with those of the other countries. Hence, these two countries fail to sustain the subsidies not because of their low propensity for vertical innovation, but because of their low population growth. When the population grows slowly, or even shrinks, the number of workers becomes small relative to the number of industries created from the ideas of past generations. When each industry has fewer workers, the R&D within the individual industries becomes less intensive, which reduces overall growth. If their population grew as fast as that of the United States, they would lie in the region where the subsidies raise growth. In contrast, the estimate of κ for Italy is the lowest among the 24 countries, and it is below $\alpha/(1-\alpha)$. As shown in the right column of Figure 10, which uses the value of κ from Italy, a slightly higher population growth rate would make the subsidies sustainable in Italy. However, even with the population growth rate of the United States, the subsidies for vertical R&D would raise growth in Italy only if the subsidy rate is high enough.

6.5 Nesting the Baseline Model

The model presented in this section is more general than the baseline model in that it includes population growth and two directions of innovation. In this subsection, we show that the baseline model, which focuses on variety-expansion innovation, as well as another widely used setting of quality-ladder innovation, can be interpreted as special cases of the current model.

[59] Ireland ($g_L = 1.36\%$ and $\kappa = 44.6$) lies above the vertical range of the figure.

[60] This result concerns the long run, after the number of workers per industry has adjusted to the steady-state value given the current population growth rate.

Note that, using Equation (26) from the baseline model, the growth rate of the current model can be written as

$$g_t = \overline{g}(A_t)\frac{L_t^R}{L_t} + \frac{g_L}{\alpha}, \tag{57}$$

if we define $A_t \equiv \big(\alpha\lambda/(1-\alpha)\big)aL_t/n_t$. The first term is the contribution of vertical innovation, and it has the same form as the growth rate (25) of the baseline model. Here, A_t represents the productivity of vertical R&D, which is determined by a and λ, as well as the effective labor per industry, L_t/n_t. A higher a raises A_t because leaders are replaced more frequently, and a larger λ raises it because each replacement brings a larger improvement in quality. A larger L_t/n_t raises A_t because, for a given share of R&D labor, more research is conducted in each industry. In the long run, L_t/n_t converges to $(g_L+\varepsilon)/f$ by (40), and A_t becomes

$$A^* \equiv \frac{\alpha\lambda}{1-\alpha}\,\kappa(g_L+\varepsilon). \tag{58}$$

Hence, A^* is lower when f is higher, because new ideas create more industries and R&D is spread over them. It is higher when ε is higher, because more industries become obsolete, and when g_L is higher, because many new workers enter existing industries.

We first show that this model nests the standard quality-ladder model where the population and the number of industries are constant. In the standard quality-ladder model, the labor supply is constant, $L_t = L$ for all t, and the number of industries is one, $n_t = 1$ for all t. The former requires $g_L = 0$, that is, $\beta = \mu$. Given $g_L = 0$, the latter requires $\varepsilon/f = L$ by (40). Moreover, no industry is created or becomes obsolete in the standard quality-ladder model. Therefore, we take the limit of $f \to 0$ and $\varepsilon \to 0$ while keeping $\varepsilon/f = L$. We also set $\lambda = (1-\alpha)/\alpha$ so that the step size approximately equals the markup, as in Grossman and Helpman (1991a). In this limit, the wage and the cost of R&D no longer include the value of new businesses. Hence, the leader sets the usual monopoly price, and the effective subsidy rate θ^e equals θ. Since industries no longer become obsolete, the leader loses its position only when an outsider succeeds in R&D. Leaders are replaced more frequently when more labor is used for R&D, and each replacement raises the quality by the step size λ. The economy keeps growing through vertical innovation alone, under constant population. Therefore, the main parts of the model coincide with those of the standard quality-ladder model.[61] Appendix C.10 shows the coincidence formally.

The baseline model and the limit of the current model are based on two different prototypes of first-generation endogenous growth models: variety expansion and quality ladders. Nevertheless, the growth properties of the two settings are almost identical. Note

[61] The markup differs because of the form of the final goods production function.

that, without population growth, $g_t = g_{q,t}$ holds. Then, the dynamic equations for the second-generation model, (54) and (53), become exactly the same as those for the baseline model, (28) and (29), once we replace A by $A^* = aL$. Therefore, the macroeconomic implications are identical between the two settings. Indeed, Propositions 2 and 3 in the baseline model are a special case of Propositions 7 and 8 (see Appendix C.10). In addition, by substituting $A = A^* = aL$, the R&D equation for the baseline setting (15) can be written as $\dot{n}_t/n_t = aL_t^R$, which is standard in the variety-expansion model.[62]

7 Extensions

In this section, we consider four extensions of the baseline model. In the first subsection, we consider the setting in which research productivity A can be enhanced by publicly funded basic research, which is also financed by perpetually rolled-over debt. In the second subsection, we explore the possibility that the government may adopt a policy that cannot be sustained in the long run. The third subsection examines the possibility that the government can levy taxes on capital income, while it cannot resort to a labor tax or other non-distortional taxation. In the final subsection, we confirm that the main properties of our analysis continue to hold in a small-country open-economy model, once we explicitly introduce country premiums.

7.1 Basic Research

The baseline model has shown that the growth-promoting effects of debt-financed R&D subsidies critically depend on the research productivity of private firms, A. Countries with low R&D productivity typically struggle with low growth, but our baseline results suggest that subsidizing R&D will not solve the problem. What alternative policies should we consider? Is the problem of low R&D productivity unsolvable?

Recent studies suggest that the government can influence this parameter by funding basic research, which is typically conducted in government agencies and universities. For example, Akcigit, Hanley and Serrano-Velarde (2020) considered spillovers across different types of research activities and concluded that the policies geared toward basic public research improve welfare. Gersbach, Sorger and Amon (2018) reported that basic research extends the knowledge base for applied research.[63] Given these studies, this section con-

[62] This explains why we assumed (15) in the baseline model.

[63] They considered a variety-expansion model where private applied R&D and publicly funded basic research are complementary in creating new goods. In a related study, Gersbach, Schneider and Schneller (2013) introduced publicly funded basic research into a quality-ladder endogenous growth model, where

siders a simple extension of our model, where the government can improve the research productivity of private firms by investing more resources into publicly funded basic research. We consider two sets of problems: (i) whether debt-financed basic research can raise the long-term growth in countries with low R&D productivity, and (ii) how the funds obtained by rolling over debts should be allocated between funding public research and subsidizing private R&D.

In this extension, the government chooses the number of basic researchers, L^B, in addition to the private R&D subsidy rate $\theta \geq 0$.[64] When the government increases L^B from L_0^B, the productivity of private R&D responds with a constant elasticity of $\phi > 0$. This means[65]

$$A = A_0 \left(\frac{L^B}{L_0^B} \right)^{\phi}, \quad L^B \geq L_0^B, \tag{59}$$

where A_0 is private R&D productivity before the policy change.[66] Basic researchers are employed by the government at the market wage w_t. The total labor supply, L^{Total}, is constant, and we write $L \equiv L^{\text{Total}} - L_0^B$ for the labor available to the private sector before the policy change, so that the labor available to private production and private R&D is $L^{\text{Total}} - L^B$. The wage payments for the initial level of basic research employment, $w_t L_0^B$, are financed by an existing labor income tax at the rate of L_0^B / L^{Total}.[67] We maintain our assumption that the tax rates cannot be raised. Therefore, the additional expenditure associated with increasing L^B, $w_t(L^B - L_0^B)$, must be financed by issuing government bonds. The other parts of the model are the same as those in Section 3.

We define L^{Total} and L, as well as the existing tax, in this way so that the extended model exactly coincides with the baseline model when we set $L^B = L_0^B$. In other words,

---

public funding increases the probability of success in quality-upgrading innovation. Leon-Ledesma and Shibayama (2023) extended Romer (1990) by introducing the basic research sector subsidized by the government. Parui and Prettner (2025) combined a variety-expansion model with human capital and analyzed the tradeoff between spending for basic research and healthcare.

[64] We omit the time subscript in L^B because it is a policy variable.

[65] For the same political reason, we assume that the government cannot reduce basic research employment below its initial level L_0^B.

[66] In our analysis, basic research conducted in other countries and the knowledge that spills over from it are taken as given and included in A_0. The elasticity ϕ captures only the effect of a country's basic research on the productivity of its own private R&D. In reality, basic research in one country also raises A_0 in other countries. If this externality is taken into account, the amount of basic research that maximizes growth in the world as a whole would be even larger than our analysis suggests. Since basic research and subsidies for private R&D are financed from the same limited funds that a sustainable level of government debt can provide, the growth-maximizing combination of the two policies would also change. In particular, it may involve smaller subsidies for private R&D. Gersbach, Schetter and Schmassmann (2023) show that basic research is underprovided in an equilibrium without international coordination, and a similar consideration would apply to our model.

[67] Since the tax lowers the after-tax wage of every individual by the same factor, it does not alter the behavior of individuals, as shown in Appendix D.

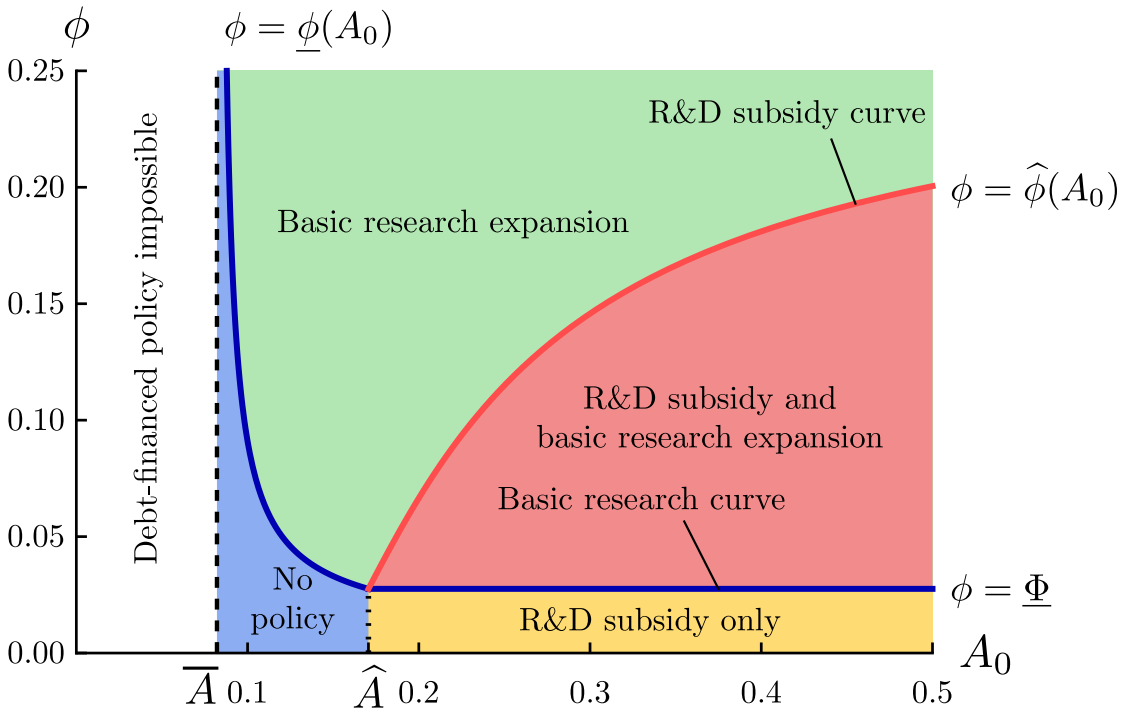


Figure 12: Growth-maximizing policy in the (A_0, ϕ) plane under the U.S. calibration.

the result from the baseline model can be seen as the special case where we impose the restriction $L^B = L_0^B$ in this extended model. In Appendix D, we derive the resulting dynamic system. Similarly to the baseline model, for a given policy pair (θ, L^B), the economy is described by a two-dimensional system in the debt-GDP ratio D_t and the growth rate g_t. As in the baseline model, Proposition 1 holds for $D_0 \geq 0$ (Appendix D.3). The economy therefore has an equilibrium path if and only if it has two steady states and the initial debt-GDP ratio is below the unsustainable threshold, and the path converges to the long-term BGP. We focus on the long-run growth rate on the long-term BGP.

When compared to the baseline model, the government now has another policy instrument. This addition affects the desirable policy through various channels. First, an increase in L^B raises private R&D productivity, A, through (59). The strength of this productivity effect depends on the effectiveness of basic research ϕ. Then, the resulting level of A affects the effectiveness of research subsidies, as we have confirmed in the baseline model. Second, both additional basic research and research subsidies are financed by government debt; therefore, their relative effectiveness matters when allocating funds across policies. Third, both policies increase the debt-GDP ratio and crowd out private R&D. Finally, allocating more labor to basic research reduces the amount of labor available for private production and R&D.

With all these considerations, Figure 12 graphically categorizes the growth-maximizing

policy in the (A_0, ϕ) plane. In the figure, we numerically calculate the location of boundary curves using the parameters from the U.S. calibration, described below. Observe that there are two curves that divide the (A_0, ϕ) plane. One is the basic research curve, which slopes downward in the range of $A_0 \in (\overline{A}, \widehat{A})$ and becomes flat for $A_0 \geq \widehat{A}$. Above this curve, if the government is to maximize the long-term growth rate, it should increase funding for basic research.[68] Intuitively, the basic research should be included in the growth-maximizing policy when its effectiveness, ϕ, is larger. In particular, observe that the curve includes the region of $A_0 \in (\overline{A}, \widehat{A})$, where R&D subsidies are not effective for promoting growth (See Proposition 3). This means that even when the economy's initial research productivity is so low that direct subsidies to firms will not improve growth performance, funding basic research could still improve the situation.

In the region of $A_0 > \widehat{A}$, there is another upward-sloping curve, which we call the R&D subsidy curve. Below this curve, the growth-maximizing policy includes a positive R&D subsidy. Therefore, when the economy lies between the two curves, the growth-maximizing policy is a mix of basic research enhancement and R&D subsidies for private firms. If ϕ is above the R&D subsidy curve, however, focusing only on basic research will eventually maximize the long-term growth.[69] Therefore, contrary to the finding in Proposition 3, the fact of $A_0 \geq \widehat{A}$ does not automatically justify the use of R&D subsidies for private firms. Because the funding pool is common and limited, there is a case for prioritizing basic research, even if it seems less directly connected to economic growth.

The following proposition formally establishes these findings.

Proposition 9 (Basic Research)

(a) Suppose that $\overline{A} < A_0 \leq \widehat{A}$. If $\phi > \underline{\phi}(A_0)$, long-run growth is maximized with additional basic research, that is, at some $L^B > L_0^B$. The threshold $\underline{\phi}(A_0)$ is decreasing in A_0 and satisfies $\underline{\phi}(A_0) \to \infty$ as $A_0 \downarrow \overline{A}$.

(b) Suppose that $A_0 > \widehat{A}$. Define $\underline{\Phi} \equiv (L_0^B/L)\widehat{A}/(\widehat{A} - 2\alpha\overline{A})$.

(i) If $\phi \leq \underline{\Phi}$, long-run growth is maximized by a positive R&D subsidy with $L^B = L_0^B$.

(ii) If $\phi > \underline{\Phi}$, long-run growth is maximized with additional basic research, that is, at some $L^B > L_0^B$.

(iii) If, in addition, $\underline{\Phi} < \phi < \widehat{\phi}(A_0)$, long-run growth is maximized by a combination

[68] In the region marked as "No policy", we confirmed that a marginal increase in basic research or R&D subsidy from the no-policy BGP will not increase the long-run growth rate (Appendix D.4).

[69] Strictly speaking, the proposition does not formally establish that the growth-maximizing policy never includes an R&D subsidy whenever $\phi > \widehat{\phi}(A_0)$. Still, we numerically confirmed that this statement is true for numerous combinations of parameters, finding no exceptions.

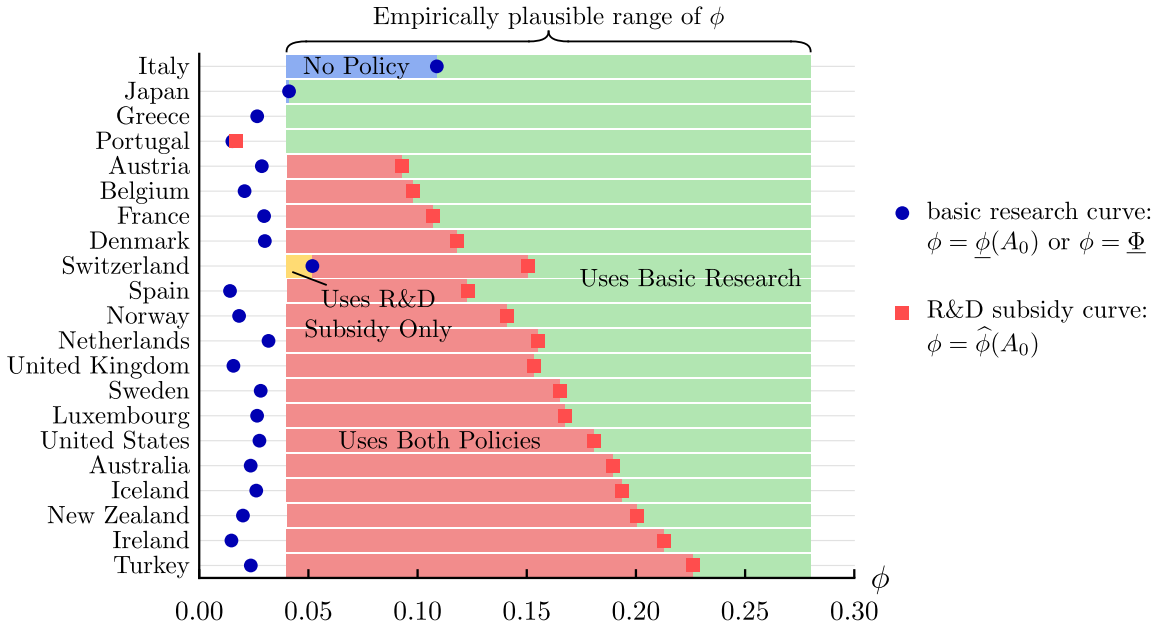


Figure 13: Growth-maximizing policy by country

of additional basic research and a positive R&D subsidy.
The definitions of $\underline{\phi}(A_0)$ *and* $\widehat{\phi}(A_0)$ *are given in Appendix D.4.*
Proof: See Appendix D.4.

In Section 4.4 (see Figure 7), we presented which countries can enhance long-term growth by debt-financed subsidies and which cannot. Now, with an additional policy instrument, we examine how the result changes for each country. There are two new parameters that potentially influence the result. The first is the initial share of labor employed in public basic research, L_0^B/L. For each country, this value is calibrated from basic research expenditure as a share of GDP over the period 2000–2023. As explained in Appendix D.5, this yields $L_0^B/L = 0.545\%$ for the United States and 0.361% for Italy. Another crucial parameter is the elasticity of private R&D productivity with respect to public basic research, ϕ. Jaffe (1989) reports estimates ranging from 0.04 to 0.28 across industries for the elasticity of corporate patenting with respect to university research expenditures in the same state. Using innovation counts rather than patents, Acs, Audretsch and Feldman (1992) obtain an estimate of 0.241. Since the country-specific value for this parameter is not available, we present the result for the plausible range of $\phi \in [0.04, 0.28]$. The remaining parameters are the same as those in Table 1.

Figure 13 shows the result for the 21 countries.[70] Recall that, in Section 4.4, we concluded that Japan, Italy and Greece cannot enhance long-term growth when they are not

[70] Among the 24 countries we studied at baseline, we needed to exclude Canada, Germany, and Finland because their basic research expenditure is not available in the MSTI database.

able to increase the tax rate. Now we have a different picture. Japan and Greece can enhance growth by financing additional basic research expenditures through debt issuance.[71] The result for Italy is mixed. Only when ϕ is larger than 0.109 does our theory imply that such a policy enhances growth, while the plausible range for this parameter is $[0.04, 0.28]$. Therefore, for Italy, further studies to estimate the parameter ϕ will help determine whether it can enhance growth through this route.

The baseline model implies that all the other countries can maximize their growth by relying on debt-financed R&D subsidies. Now, the extended model has a different implication. With the exception of Switzerland, using only R&D subsidies is never growth-maximizing. Instead, it is desirable to use at least a part of the debt-financed funds to expand the basic research. In particular, if the value of ϕ is close to the upper end of the plausible range (for example, in the case of the U.S., $\phi > 0.181$), allocating all the funds to public basic research, rather than R&D subsidies for private firms, leads to the highest growth.

Figure 14 shows the long-term growth rate for different combinations of basic research expansion and R&D subsidy in the United States and Italy, under three values of ϕ in the plausible range. The growth-maximizing policies, shown by the black squares, are as predicted by Figure 13. For the United States, the growth-maximizing policy uses both instruments when $\phi = 0.04$ or 0.13, and only basic research when $\phi = 0.28$. For Italy, it uses neither instrument when $\phi = 0.04$, and only basic research when $\phi = 0.13$ or 0.28. The figure also shows that a policy pair whose total expenses for the two policies are excessive cannot be implemented. If the government tried to choose such a pair, there would be no equilibrium path (Appendix D.3), and lenders would not lend to the government. This illustrates the trade-off between the instruments.

7.2 Unsustainable Subsidies

The benchmark analysis shows that there is a maximum sustainable subsidy rate $\overline{\theta}(A)$, above which a permanent subsidy cannot be implemented. In practice, however, governments often allow fiscal deficits to grow beyond what is consistent with long-run debt sustainability.[72] This section examines what happens to the economy when the government implements such a policy. Such a policy necessarily entails a regime change, because the

[71] Japan's threshold is 0.041, which almost coincides with Jaffe (1989)'s lower bound.

[72] The political economy literature attributes this tendency to a war of attrition among socio-economic groups that disagree over how the burden of fiscal adjustment should be shared, thereby systematically delaying necessary stabilizations (Alesina and Drazen, 1991; Alesina and Passalacqua, 2016).

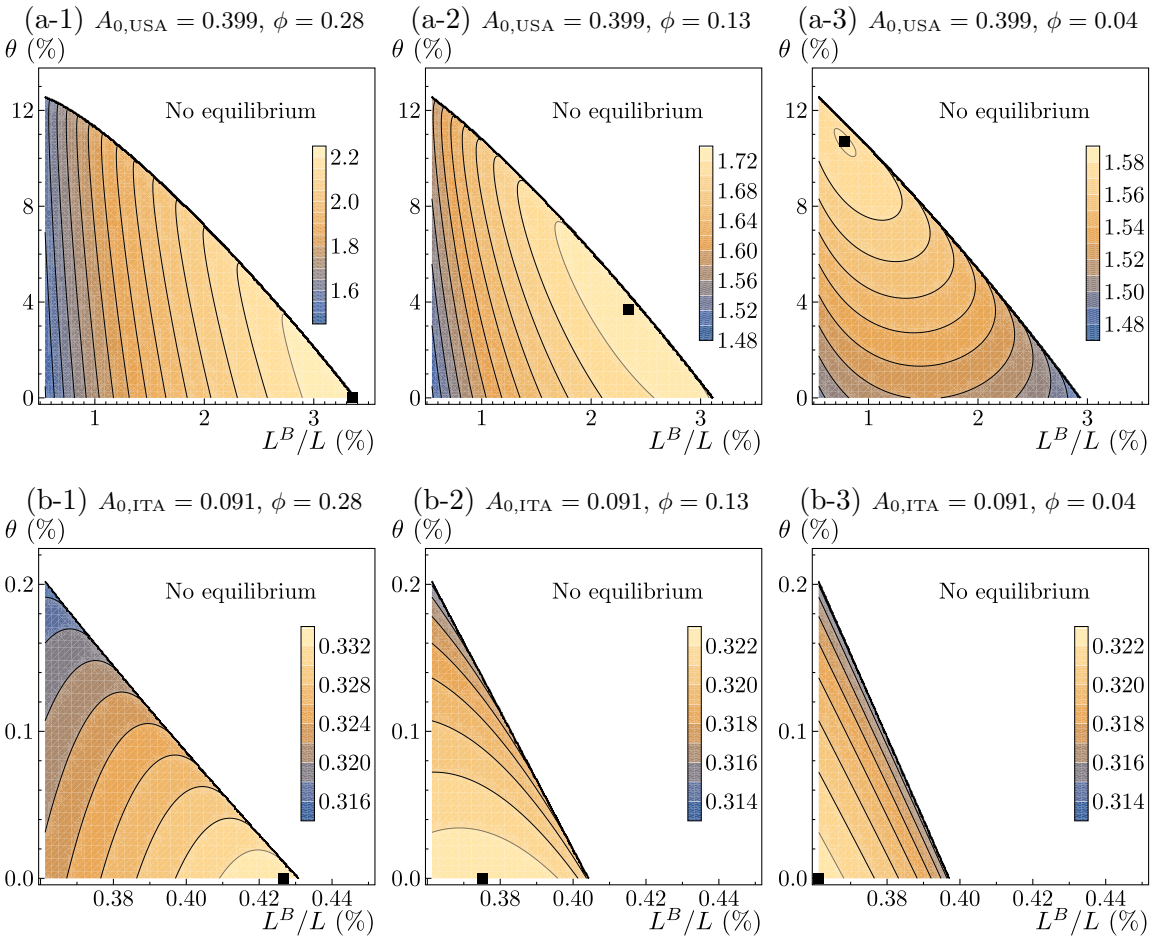


Figure 14: Long-run growth effects of basic research expansion and the R&D subsidy. The colored area shows the long-term growth rate in % as indicated by the color chart. The black square in each panel shows the growth-maximizing policy combination. The economy has no equilibrium path when the policy pair is in the white area (i.e., the expense cannot be financed). The parameters other than A_0 and ϕ are the same as in Figure 13.

"splurge" regime can only be sustained for a finite duration. We assume that when the debt-GDP ratio eventually becomes too high, fiscal consolidation is imposed on the government, either as self-correction to avoid sovereign default or through external pressure from creditors and international institutions such as the IMF.[73] We compare the outcome from such a policy with a sustainable policy that maximizes the long-term growth rate. The following analysis shows whether such an unsustainable policy is actually beneficial to the economy, and if so, how long the benefit lasts.

In both cases, the economy starts from the no-policy BGP with $D_0 = 0$ at $t = 0$. The sustainable policy sets a permanent subsidy rate that maximizes the long-term growth rate, $\theta_t = \theta^s \equiv \widehat{\theta}(A)$ for all t. Under this policy, the economy converges to the long-term BGP, on which the debt-GDP ratio is $D^*(A, \theta^s)$. The unsustainable policy initially sets the

[73] In an R&D-based growth model, Ferraro and Peretto (2020) examine the macroeconomic consequences of consolidations imposed after a period of unsustainable deficit accumulation.

subsidy rate higher than the highest sustainable rate, $\theta^u > \overline{\theta}(A)$, in what we call the splurge regime, and then switches to the austere regime, where the subsidy is permanently canceled ($\theta = 0$).[74] We assume that the regime change occurs when the post-switch debt-GDP ratio is 0.9 times the unsustainable threshold without subsidies, $D_U(A, 0) = \alpha(1/\overline{A} - 1/A)$, beyond which the economy has no equilibrium path (Proposition 1(b-ii)).[75] The timing of the regime change depends on the subsidy rate θ^u. In our calculations, the duration of the splurge regime, denoted by T, is shorter when θ^u is higher. In the examples below, we use the same parameters as the baseline model (see Table 1 in Section 4.4), again considering two values of A that are calibrated to the U.S. and Japan. For each A, we set θ^u so that the duration of the splurge regime is $T = 100$. This pins down the subsidy rate in the splurge regime as $\theta^u = 37.0\%$ in the high-research-productivity country and 16.4% in the low-A country.

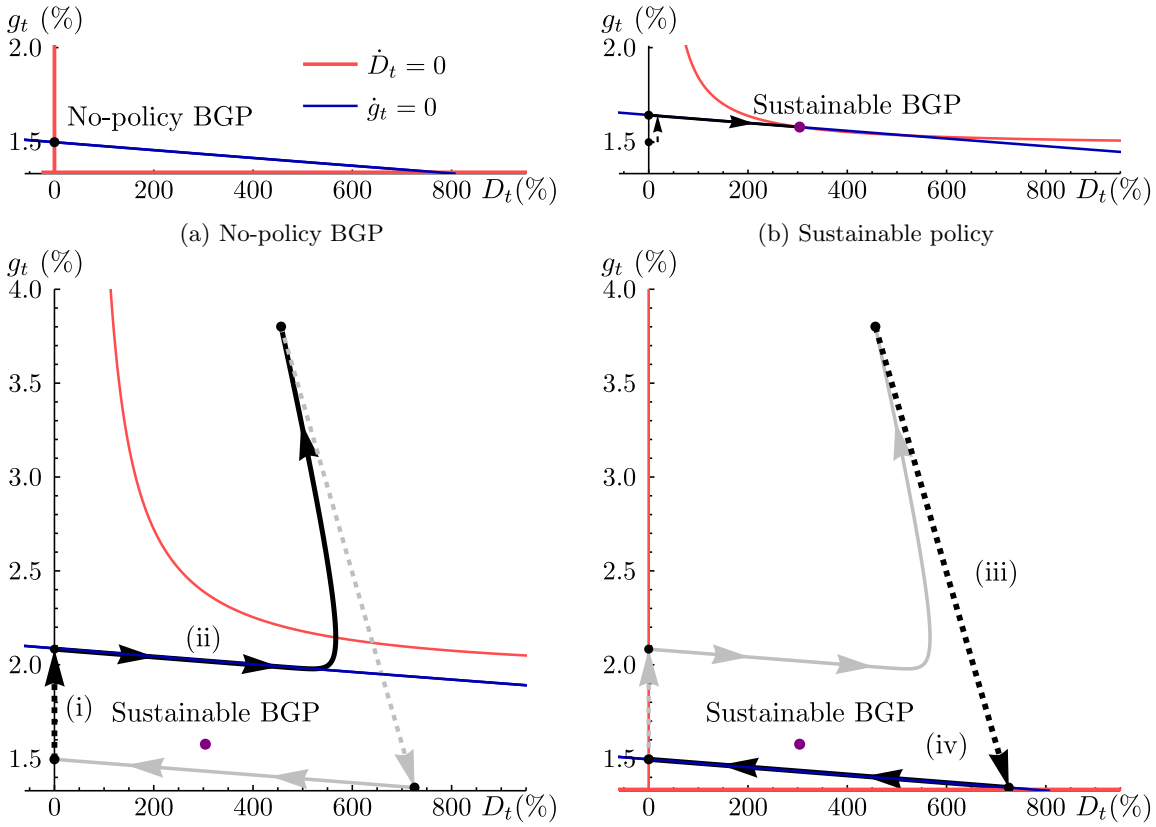
The four panels in Figure 15 show the phase diagram and the transitional dynamics under two policies, numerically calculated using the parameter values for the high-A economy and shown on the same scale. Panel (a) shows the BGP of the no-policy economy. Both policies start from this point. Panel (b) shows the dynamics under the sustainable policy. At the time of the introduction of the policy, the phase diagram changes from (a) to (b), and the growth rate jumps up to the equilibrium path under the new policy. After the jump, the economy evolves along this path, on which the debt-GDP ratio gradually increases and the growth rate declines. In the long run, the economy converges to the sustainable BGP, on which the growth rate is higher than that on the no-policy BGP.

Panels (c) and (d) explain the dynamics of the economy under the unsustainable policy. At the time of the introduction of the unsustainable subsidy, the phase diagram changes from (a) to (c). Since the subsidy rate is higher than the case of panel (b), the shift of the $\dot{g}_t = 0$ locus is more significant. Given this, the size of the jump in the growth rate, as shown by (i) and representing the short-run growth-stimulating effect, is larger under the unsustainable policy than under the sustainable policy.

---

[74] In the main text, we keep the assumption that the government cannot raise taxes even in the future. Therefore, the austere regime only means that the subsidy is terminated. In a separate simulation, however, we computed the case in which the government, after terminating the subsidy, taxes all asset income at the rate of 20% until the debt-GDP ratio returns to zero. We set the subsidy rate and the debt-GDP ratio just after the termination the same as below. Even though the taxes on dividends and capital gains are anticipated, the effect of the last-minute R&D investment is larger, and R&D and the growth rate still rise toward the termination. Due to the opposing effects, the rise is smaller. In the high-A country, the growth rate just before the termination is 3.43%, compared with 3.80% reported in the main text.

[75] The value of 0.9 is illustrative. If we instead set it to 0.8, the subsidy rate that can be maintained for $T = 100$ years becomes slightly lower (from 37.0% to 34.9% in the high-A country and from 16.4% to 15.0% in the low-A country), and the period in which full-production GDP is higher under the unsustainable policy than under the sustainable policy becomes slightly longer (from 293 to 297 years and from 228 to 234 years, respectively).



(a) No-policy BGP

(b) Sustainable policy

(c) Splurge regime under unsustainable policy

(d) Austere regime under unsustainable policy

Figure 15: Phase diagram under sustainable and unsustainable policy ($A = 0.399$). In the case of unsustainable policy, the economy transitions from panel (c) to panel (d), following arrows (i) to (iv).

Technically speaking, the size of the jump is determined not by an equilibrium path that converges to a long-term BGP, which does not exist under the splurge regime, but by the path shown by (ii) in panel (c). In 100 years, this path reaches a point where the post-regime-change value of the debt-GDP ratio is exactly 0.9 times $D_U(A, 0)$. Appendix E.2 explains how this path is computed. It is notable that the growth rate just before the regime change is quite high at 3.80% due to strong last-minute investment demand as the splurge (high-subsidy) regime is about to end. To establish the financial market equilibrium before and after the regime change, the interest rate needs to spike at the time of the regime change. This spike induces consumption to jump up (due to individuals' dynamic optimization) and investment to fall, which creates a downward jump in the growth rate, as represented by (iii) in Panel (d). Also, the debt-GDP ratio jumps up due to this spike in the interest rate. After the jump, the economy gradually converges to the no-policy BGP, as shown by (iv). Appendix E.1 derives the conditions that connect the two regimes at the time of the regime change.

This analysis suggests that a limited-time R&D subsidy could stimulate economic

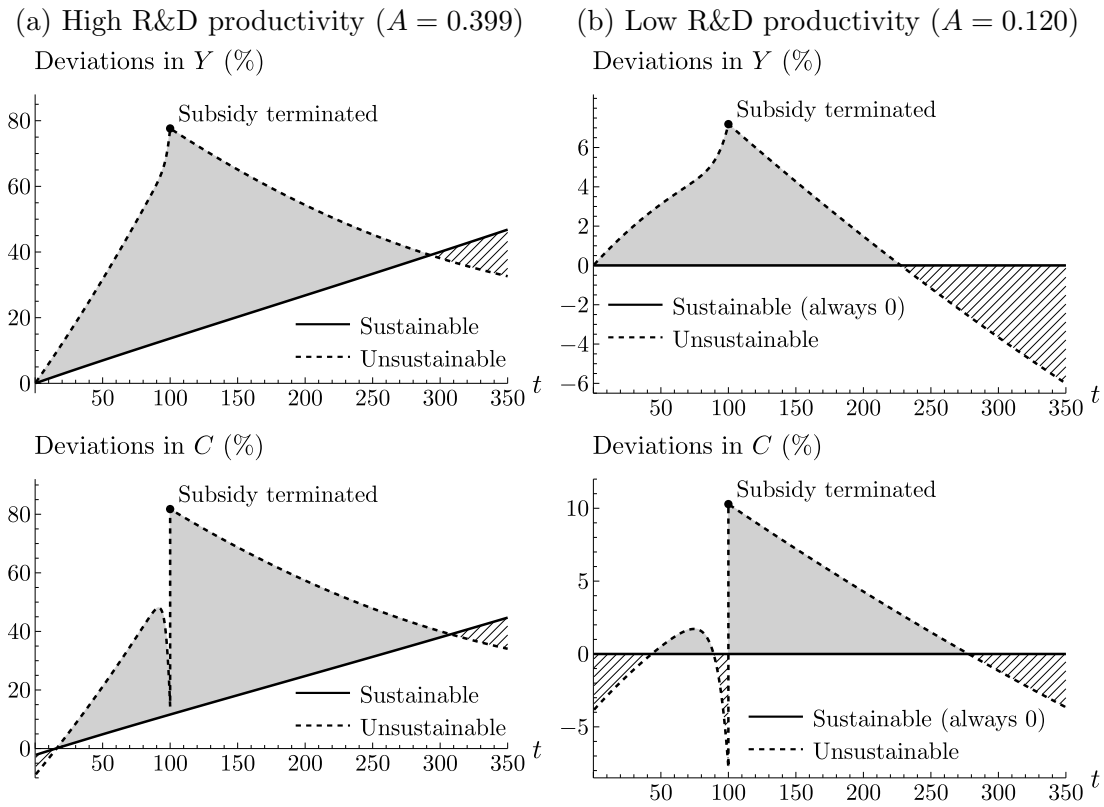


Figure 16: The time path of the full-production GDP (top) and the aggregate consumption (bottom) relative to the no-policy BGP. The gray area indicates the difference between the unsustainable and sustainable subsidies when the former is higher. The hatched area shows the difference when the latter is higher.

growth more than a permanent one. Not only is it possible to dole out a higher subsidy rate, but the fact that the subsidy will be terminated in a finite time also creates a last-minute surge in investment. Anticipating the high return available at the time of the regime change, individuals postpone consumption, which frees labor for R&D before the termination. However, in our numerical example, path (ii) in panel (c) is quite close to the $\dot{g}_t = 0$ locus, which suggests that this anticipation effect is minimal at the time of introduction, given that the termination is 100 years away. As the time nears the point of termination, the anticipation effect becomes larger, and the growth rate rises.[76]

Figure 16 shows the time path of full-production GDP Y_t and aggregate consumption C_t under two policies, as percentage deviations from the no-policy baseline.[77] Under the

[76]In the high-A country with $T = 100$, the growth rate exceeds the value on the $\dot{g}_t = 0$ locus by one percentage point only within the last two years before the termination, and the gap reaches 1.8 percentage points just before it. We also conducted numerical simulations with $T = 10$ and $T = 30$. The gap then reaches one percentage point four years and three years before the termination, and 2.7 and 2.6 percentage points just before it.

[77]For the case of the low-A country, the growth-maximizing sustainable policy is actually $\theta = 0$, given that a tax on R&D is not available (See Proposition 3). Therefore, the deviation of the sustainable policy from the no-policy BGP is zero.

unsustainable policy, full-production GDP is higher than under the sustainable policy for about 290 years in the high-A country and about 230 years in the low-A country. This result holds even in the low-A country, where $\theta = 0$ is recommended when only the long-term growth is considered.[78] Aggregate consumption is initially lower than the no-policy BGP because the high R&D subsidy encourages R&D rather than consumption, but it surpasses the no-policy path in a few decades thanks to higher productivity, and then peaks just after the subsidy is terminated. There is another dip in aggregate consumption just before the subsidy's termination due to a surge in last-minute R&D investment, which crowds out consumption.

We also computed the expected lifetime utility of each cohort alive at the introduction of the policy, under both the sustainable and the unsustainable policies (Figure E.1 in Appendix E.3). If each individual supports the policy that gives the higher expected utility, virtually all individuals (younger than 426 years old) support the unsustainable policy under the U.S. parameters. Under the Japanese parameters, in contrast, only individuals younger than 45.8 support it. The elderly in Japan do not support it because the subsidy makes entry easier, and the expected increase in future competition lowers the value of the shares of existing firms.

The analysis in this section has shown that unsustainable policies can be beneficial to the economy and individuals for extended periods, and that the population at the time of voting may support such policies if they decide based on their own benefits. However, this does not necessarily mean that these policies are recommendable. After the finite beneficial period, both output and consumption fall below the sustainable policy path, and the gap widens over time. Therefore, the question of whether subsidy policies should be sustainable boils down to the intergenerational trade-off.[79]

7.3 Asset Taxation

Thus far, we have assumed that the government cannot raise any tax. This assumption allows us to examine the implications of the political constraint in a simple way, but it may be too strong. The literature on taxes and political economy has suggested that opposition

[78] The right bottom panel of Figure 9 in Section 4.5 shows that a sustainable positive subsidy can also temporarily raise Y and C relative to the no policy case. However, the magnitude of deviation is much smaller (less than 1%) than the unsustainable subsidy considered in this section (up to around 7% in Y_t and 10% in C_t).

[79] Actual politics may not weigh the welfare of future generations, but it might still prevent such a policy. The elderly in Japan would oppose such a policy not because they value future generations, but because they want to protect their vested interest in the shares of existing firms. The same could happen in the U.S. if wealthy individuals have disproportionately strong political power.

to raising capital income taxes might be weaker than opposition to raising labor income taxes.[80] This section relaxes the assumption of the baseline model and examines how our results change when the government can use an asset income tax.

Allowing asset income taxation raises two questions. First, can the government attain a higher long-term growth rate than in Proposition 3? Second, can the tax revenue eliminate the need for government debt when the government maximizes the growth rate? The answers to both questions are negative. The highest attainable growth rate is unchanged. Moreover, attaining the maximum growth rate requires a higher research subsidy and a higher debt-GDP ratio when the asset income tax rate is higher.

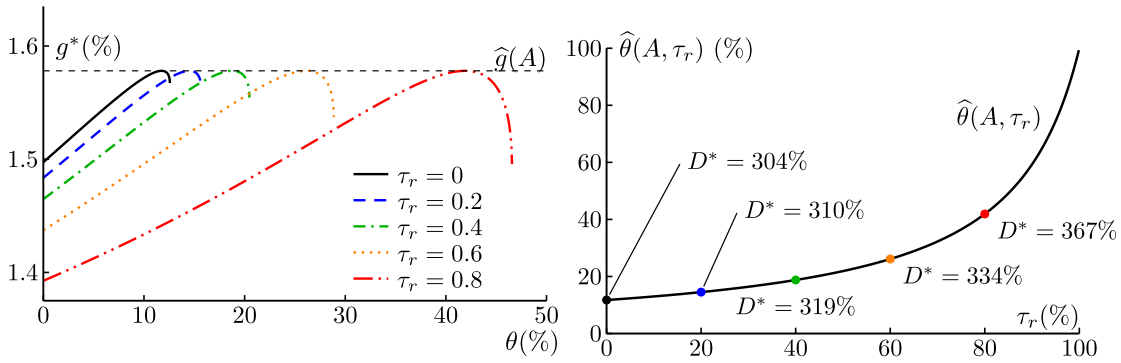
To examine these questions, suppose that the government taxes income from all assets at the uniform rate $\tau_r \in [0, 1)$. We write the values on the long-term BGP with τ_r as an additional argument, such as $g^*(A, \theta, \tau_r)$, so that $g^*(A, \theta, 0)$ equals $g^*(A, \theta)$ in the baseline model. To simplify the analysis, we only consider the case where the research subsidy is growth-promoting ($A > \widehat{A}$) and the interest rate is positive on the long-term BGP without the subsidy and the tax ($\theta = \tau_r = 0$).[81] We treat τ_r as given and examine how the government chooses the research subsidy under this tax. Individuals then receive the after-tax return $(1-\tau_r)r_t$, while the government obtains the tax revenue $\tau_r r_t K_t$. Since the same tax rate applies to government bonds and the shares of intermediate goods firms, the no-arbitrage condition and the free entry condition are unchanged. The other parts of the model are the same as in Section 3. The equilibrium dynamics are derived in Appendix F.

The asset income tax has two opposing effects on the possibility of raising the growth rate by debt-financed research subsidies. First, the additional revenue enables the government to sustain a higher research subsidy. Second, the lower after-tax return weakens the life-cycle saving motive of households. The resulting reduction in the supply of funds to the capital market raises the pre-tax interest rate and lowers the value of firms.[82] Therefore, the asset income tax discourages R&D, for a given rate of research subsidy. Surprisingly,

[80] Statutory tax rates on capital income have been repeatedly revised in OECD countries (Harding and Marten, 2018). This observation suggests that the political constraint on this tax may be weaker than that on labor income taxation. Dynamic optimal-taxation studies also allow capital income taxes to be revised over time (e.g., Farhi, Sleet, Werning and Yeltekin, 2012; Scheuer and Wolitzky, 2016). In an R&D-based growth model, Peretto (2011) considers a debt-financed cut in the tax on dividends.

[81] If the interest rate is negative on the growth-maximizing BGP and the government reimburses a fraction τ_r of the loss from the investment, parts (b)–(d) of Proposition 10 below hold except that the signs in parts (c) and (d) are reversed, as long as $\widehat{\theta}(A, \tau_r) \in [0, 1)$.

[82] The asset income tax itself is neutral to the value of the firm given the pre-tax interest rate, because the tax proportionally affects the required rate of return and the post-tax dividends. The results do not change if government bonds are exempt from the tax. Taxing the interest on government bonds only returns part of the interest to the government itself. If the bonds were exempt, their interest rate would equal the after-tax return $(1-\tau_r)r_t$, and the government would pay the same net interest.



(a) Long-run growth rate under different τ_r and θ

(b) Growth-maximizing subsidy rate and debt-GDP ratio

Figure 17: The long-term growth rate and the debt-GDP ratio with asset income taxation and research subsidy, computed with the U.S. calibration ($A = 0.399$).

the two opposing effects exactly cancel out if the government adjusts the research subsidy rate across different asset income tax rates.

Proposition 10 (Asset Income Taxation) *Suppose $A > \widehat{A}$ and $r^*(A, 0, 0) > 0$. Then, for any $\tau_r \in [0, 1)$, the following results hold.*

(a) $r^*(A, \theta, \tau_r) > 0$ holds whenever the long-term BGP exists.

(b) The highest attainable long-term growth rate is the same as $\widehat{g}(A)$ in Proposition 3.

(c) The subsidy rate $\widehat{\theta}(A, \tau_r)$ that attains this growth rate is positive and strictly increasing in τ_r, with $\widehat{\theta}(A, 0) = \widehat{\theta}(A)$.

(d) The long-term debt-GDP ratio at this subsidy rate is strictly increasing in τ_r.

Proof: See Appendix F.3.

As shown by Figure 17(a), when the government levies a higher asset income tax rate, it can subsidize research at a higher rate to maximize long-term growth. However, the growth-reducing effect of the asset income tax is exactly offset by the higher research subsidy rate made possible by the asset income tax rate. Still, not everything comes down to the same. As shown by Figure 17(b), the debt-GDP ratio is higher when the maximum growth is attained by the pair of a high asset income tax rate and a correspondingly high research subsidy rate, when compared to both being low (in particular, when $\tau_r = 0$).

The intuitive reason is as follows. When the government provides a research subsidy and then levies an asset income tax, it effectively gives money at the firm's founding and then takes it back by taxing the returns from the firm (dividends and capital gains), which continues forever. The fact that the growth rate is unchanged (i.e., the incentives for innovation are unaffected) implies that the present value of the taxes on the returns from the firm, discounted at the after-tax interest rate, equals the increase in the subsidy

compared with the case of $\tau_r = 0$. Still, from the government's perspective, there is a time difference between paying the subsidy and taking it back. The difference shows up as increased government debt. When τ_r is higher, the government has to pay a larger subsidy at the beginning, although it can levy a higher tax rate on the returns from the firm. This leads to a higher debt-GDP ratio.

7.4 Country Premium in a Small Open Economy

In reality, the capital market of a country is not closed to the rest of the world. Government bonds can be absorbed by foreign funds as well as by the savings of domestic households. In our baseline model, the crowding out comes entirely from the competition for those savings between the government and the intermediate goods firms, as shown by (18). Access to foreign funds weakens this competition, and it disappears in a small open economy that takes the world interest rate as given.

However, a larger government debt tends to raise the country premium, and the firms in that country then face a higher interest rate.[83] Since a higher interest rate reduces the present value of the future profits realized by R&D, it discourages research even when the competition for domestic savings is absent. The crowding-out channel thus moves from the domestic asset market to the country premium. In the baseline model, the relative size of this channel and the direct effect of the subsidy determined whether the policy can be financed at all (Proposition 2) and whether it raises the long-term growth rate (Proposition 3). This section shows that there are similar conditions in a small open economy once the country premium is taken into account.

We describe this country-premium channel in the simplest possible setting. The capital market is connected to the world market, and the interest rate of each country is determined by

$$r_t = \overline{r} + \eta \max\{D_t, 0\}, \qquad \eta > 0. \tag{60}$$

Here $\overline{r} > 0$ is the world safe interest rate, which is the rate faced by a country that has no government debt and hence no country premium. The parameter η measures how strongly the government debt affects the country premium.[84]

[83] Ağca and Celasun (2012) document that corporate borrowing costs are higher in emerging market economies where the public external debt is higher, and Li, Magud and Werner (2023) find that the long-run pass-through from sovereign to corporate bond yields in these economies is close to one. While the response of the country premium to the debt level could differ among countries, Proposition 11 depends only on the slope of the premium at zero debt.

[84] A debt-elastic country premium is a common specification in small open economy models. See, for example, Schmitt-Grohé and Uribe (2003), where the premium is an exponential function of the debt. We let the interest rate respond linearly to a positive debt-GDP ratio for simplicity.

The market for final goods is also connected to the world market. Intermediate goods, on the other hand, are used in the domestic supply chain and are sold only to the domestic final goods sector, and we do not consider international migration of labor. The R&D sector, the intermediate goods sector, and the allocation of labor are therefore the same as in the baseline model, and the supply-side relations given by (16), (19), and (25) continue to hold. Since the interest rate is given by (60) and final goods are traded in the world market, the consumption and saving of households do not affect the dynamics of the debt-GDP ratio and the growth rate. We therefore do not consider the household side in this section. As shown in Appendix G, combining (60) with the supply-side relation (27) gives the growth rate for a given level of government debt D_t:[85]

$$g_t = \psi_1(A,\theta) - \psi_2(\theta)D_t, \text{ where} \tag{61}$$
$$\psi_1(A,\theta) \equiv \frac{(1-\alpha)\left(\overline{g}(A) - (1-\theta)\overline{r}\right)}{\alpha - (2\alpha-1)\theta}, \quad \psi_2(\theta) \equiv \frac{(1-\alpha)(1-\theta)\eta}{\alpha-(2\alpha-1)\theta}.$$

Equation (61) shows that a larger government debt raises the country premium and the interest rate faced by firms, and that it therefore lowers the equilibrium growth rate.

In the small open economy, r_t and g_t are both determined by (60) and (61) as functions of D_t. Substituting them into the debt dynamics (29) gives a single differential equation in D_t (see Appendix G). In this equation, D_t increases when the subsidy expenditure exceeds the surplus from government debt, $(g_t - r_t)D_t$, and decreases otherwise. Since r_t rises and g_t falls with D_t, the gap $g_t - r_t$ narrows as D_t rises, and the surplus first increases and then decreases with D_t. As long as the subsidy rate is not too large, the surplus equals the subsidy expenditure at two positive levels of the debt-GDP ratio. Below the lower level, the subsidy expenditure exceeds the surplus, and the debt-GDP ratio rises. Between the two levels, the surplus exceeds the subsidy expenditure, and the ratio falls. Hence, the lower level is the long-term BGP of this economy, and the higher level is the unsustainable threshold, $D_U(A,\theta)$. We write the debt-GDP ratio and growth rate on the long-term BGP as $D^*(A,\theta)$ and $g^*(A,\theta)$, and its interest rate as $r^*(A,\theta)$. As in the baseline model, the economy converges to the long-term BGP if the initial debt-GDP ratio is below $D_U(A,\theta)$, and there is no equilibrium path if it is above $D_U(A,\theta)$ (Appendix G).

Proposition 11 (Small Open Economy) *(a) Suppose $\theta = 0$. The economy has a long-term BGP with a nonnegative debt-GDP ratio if and only if $A > \overline{A}^{\text{open}} \equiv \alpha\overline{r}/(1-\alpha)^2$. On this BGP, $D^*(A,0) = 0$ and the growth rate $g^*(A,0)$ is higher than*

[85] For simplicity, we consider only the situation in which R&D is positive in equilibrium.

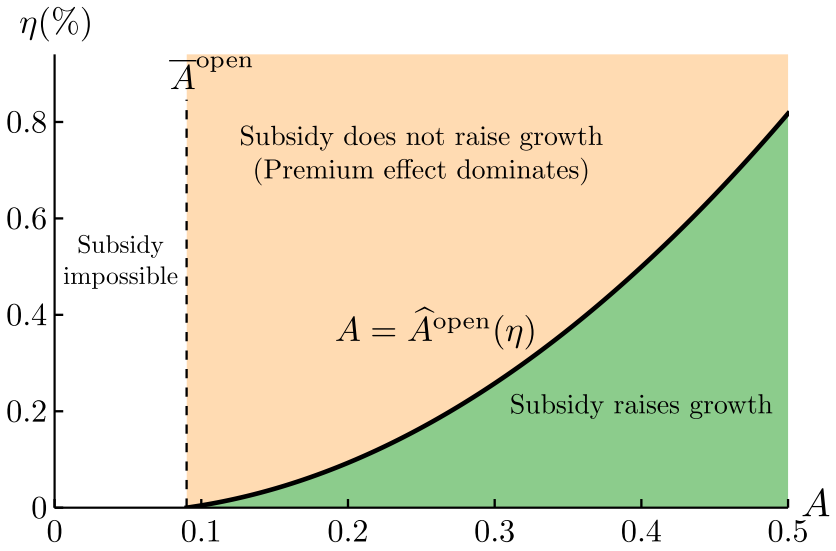


Figure 18: Country premium and the growth effect of debt-financed R&D subsidies. The figure illustrates the qualitative results of Proposition 11, using $\overline{r} = 0.3\%$ as an example (other parameters are the same as in the baseline model). The regions show the effect of the subsidy when its rate is marginally increased from zero.

the interest rate $r^*(A, 0)$.

(b) Suppose $A > \overline{A}^{\text{open}}$ *holds. Then, a marginal increase in the research subsidy rate* θ *from 0 raises the long-term growth rate* $g^*(A, \theta)$ *if and only if* $A > \widehat{A}^{\text{open}}(\eta)$*, where* $\widehat{A}^{\text{open}}(\eta)$ *is the inverse function of*

$$\eta = \frac{(1-\alpha)A\left((1-\alpha)^2 A - \alpha\overline{r}\right)\left((2\alpha-1)A + \alpha\overline{r}\right)}{\alpha^4\left((1-\alpha)A - \alpha\overline{r}\right)}, \tag{62}$$

which is well defined, strictly increasing in η*, and satisfies* $\widehat{A}^{\text{open}}(0) = \overline{A}^{\text{open}}$.

Proof: See Appendix G.

The two parts of Proposition 11 show that the intuition behind Propositions 2 and 3 carries over to the small open economy. As Figure 18 illustrates, the growth rate exceeds the interest rate in the long run only when the research productivity A is above the threshold $\overline{A}^{\text{open}}$. This condition alone does not make a debt-financed research subsidy effective. For a marginal subsidy to raise the growth rate, A must exceed the higher threshold $\widehat{A}^{\text{open}}(\eta)$. The condition becomes more demanding as η becomes larger, because a stronger response of the interest rate to government debt strengthens the crowding out and offsets the direct effect of the subsidy.

8 Conclusion

In the literature on endogenous growth, it is widely agreed that economic growth is driven by R&D, and growth can be promoted by subsidizing R&D. However, it is often difficult to raise tax rates to finance new R&D subsidies. Given this reality, we considered two questions. The first question is whether governments can obtain the funds required to implement R&D subsidies by issuing debt, without raising tax rates now or in the future. Put another way, the question asks whether the government could run a Ponzi scheme, in which it perpetually rolls over its debts. The second question is whether, if the answer to the first question is yes, such a policy can actually promote economic growth.

We find the result depends on each country's characteristics. In particular, feasibility crucially depends on R&D productivity, which measures the relationship between the proportion of R&D labor and the rate of new product development. In the calibration, we find that the answer to the first question is yes for all the countries in our sample. The condition for the second question is tighter, but most countries, except Japan, Italy and Greece, satisfy it. Those countries can indeed promote long-term growth by R&D even when taxes cannot be raised. In addition, we confirmed that the result is not due to the economy being dynamically inefficient. Even though the private return on R&D can be lower than the growth rate, we find that the social rate of return of R&D is higher than the growth rate on any balanced growth path.

We re-evaluated this result in alternative settings. When population growth is explicitly introduced in the model, we find that the condition becomes even tighter for countries with slow population growth. Japan, Italy and Greece cannot now pass the condition for the first question; i.e., they cannot roll over their debts without raising taxes. In addition, Portugal fails to meet the second threshold. It cannot enhance growth by debt-financed subsidies.

The findings so far are ironic, in the sense that countries most in need of reviving growth could not be saved by this scheme. These countries' low growth implies that their R&D productivity is low, which makes it harder for them to meet the conditions. However, another specification gives those slow-growing countries some hope. We extend the baseline model to include public basic research, which the literature finds helps improve the private R&D productivity. Then, all the countries, with the possible exception of Italy, seem to be able to enhance growth by appropriately combining the public basic research expansion and R&D subsidies, using the funds obtained by debt rollover.

It is worth noting that even when countries fail to meet the second condition, they could

enhance growth for a limited time by providing unsustainable levels of subsidies. We find that the benefit could continue for a long time, to the extent that the current generation and their government may well support it, potentially explaining why such policies are implemented in some countries. We also extended the model to the cases where political constraints are relaxed and where the country is open to the international capital market. Both extensions confirm that the main qualitative results of this study continue to hold.

Acknowledgments

The authors are grateful for comments and suggestions from Real Arai, Anton Braun, Carmelo Pierpaolo Parello, Elias Carroni, Robert Chirinko, Ippei Fujiwara, Yuichi Furukawa, Tomohiro Hirano, Maurizio Iacopetta, Keiichi Kishi, Miguel Leon-Ledesma, Jean-Baptiste Michau, Alessio Moro, Pietro Peretto, Akihisa Shibata, Francesco Venturini, and seminar participants at the University of London, CIGS, Università degli Studi di Urbino Carlo Bo, Università degli Studi di Cagliari, and the JEA Annual Meeting. This study was financially supported by the JSPS Grant-in-Aid for Scientific Research (25K00620, 24KK0031, 24KJ1584, 23K20145, 20H05633, 20H05631), the Canon Institute for Global Studies, and the Behavioral Economics Research Center at ISER. Needless to say, all remaining errors are our own.

References

Abel, A. B., N. G. Mankiw, L. Summers, and R. Zeckhauser (1989): "Assessing Dynamic Efficiency: Theory and Evidence," *The Review of Economic Studies*, 56, 1–19.

Acs, Z. J., D. B. Audretsch, and M. P. Feldman (1992): "Real Effects of Academic Research: Comment," *The American Economic Review*, 82, 363–367.

Agarwal, R., and M. Gort (1996): "The Evolution of Markets and Entry, Exit and Survival of Firms," *The Review of Economics and Statistics*, 78, 489–498.

Ağca, Ş., and O. Celasun (2012): "Sovereign debt and corporate borrowing costs in emerging markets," *Journal of International Economics*, 88, 198–208.

Aghion, P., and P. Howitt (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60, 323–351.

Akcigit, U., D. Hanley, and N. Serrano-Velarde (2020): "Back to Basics: Basic

Research Spillovers, Innovation Policy, and Growth," *The Review of Economic Studies*, 88, 1–43.

Akcigit, U., and W. R. Kerr (2018): "Growth through Heterogeneous Innovations," *Journal of Political Economy*, 126, 1374–1443.

Alesina, A., and A. Passalacqua (2016): "The Political Economy of Government Debt," in *Handbook of Macroeconomics* ed. by Taylor, J. B., and Uhlig, H. Volume 2 of Handbook of Macroeconomics: Elsevier, 2599–2651.

Alesina, A., and A. Drazen (1991): "Why are Stabilizations Delayed?" *The American Economic Review*, 81, 1170–1188.

Angeletos, G.-M., C. Lian, and C. K. Wolf (2024): "Can Deficits Finance Themselves?" *Econometrica*, 92, 1351–1390.

Ball, L., and N. G. Mankiw (2023): "Market Power in Neoclassical Growth Models," *The Review of Economic Studies*, 90, 572–596.

Barro, R. J. (2023): "r Minus g," *Review of Economic Dynamics*, 48, 1–17.

Bassetto, M., and D. S. Miller (2025): "A Monetary-Fiscal Theory of Sudden Inflations," *The Quarterly Journal of Economics*, 140, 1959–2000.

Benartzi, S., A. Previtero, and R. H. Thaler (2011): "Annuitization Puzzles," *Journal of Economic Perspectives*, 25, 143–64.

Blanchard, O. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93, 223–247.

——— (2019): "Public Debt and Low Interest Rates," *American Economic Review*, 109, 1197–1229.

Bolt, J., and J. L. van Zanden (2025): "Maddison-Style Estimates of the Evolution of the World Economy: A New 2023 Update," *Journal of Economic Surveys*, 39, 631–671.

Buiter, W. H. (1988): "Death, Birth, Productivity Growth and Debt Neutrality," *The Economic Journal*, 98, 279–293.

Chu, A. C., Y. Furukawa, and L. Ji (2016): "Patents, R&D subsidies, and endogenous market structure in a Schumpeterian economy," *Southern Economic Journal*, 82, 809–825.

Chu, A. C., and X. Wang (2022): "Effects of R&D Subsidies in a Hybrid Model of Endogenous Growth and Semi-Endogenous Growth," *Macroeconomic Dynamics*, 26, 813–832.

Clemens, M. A., and J. G. Williamson (2004): "Wealth bias in the first global capital market boom, 1870-1913," *Economic Journal*, 114, 304–337.

Cozzi, G. (2023): "Semi-endogenous or fully endogenous growth? A unified theory," *Journal of Economic Theory*, 213, 105732.

De Nardi, M., E. French, and J. B. Jones (2010): "Why Do the Elderly Save? The Role of Medical Expenses," *Journal of Political Economy*, 118, 39–75.

Diamond, P. A. (1965): "National Debt in a Neoclassical Growth Model," *The American Economic Review*, 55, 1126–1150.

Farhi, E., C. Sleet, I. Werning, and S. Yeltekin (2012): "Non-linear Capital Taxation Without Commitment," *The Review of Economic Studies*, 79, 1469–1493.

Feenstra, R. C., R. Inklaar, and M. P. Timmer (2015): "The Next Generation of the Penn World Table," *American Economic Review*, 105, 3150–3182.

Ferraro, D., and P. F. Peretto (2020): "Innovation-led growth in a time of debt," *European Economic Review*, 121, 103350.

Gersbach, H., U. Schetter, and S. Schmassmann (2023): "From local to global: A theory of public basic research in a globalized world," *European Economic Review*, 160, 104530.

Gersbach, H., M. Schneider, and O. Schneller (2013): "Basic research, openness, and convergence," *Journal of Economic Growth*, 18, 33–68.

Gersbach, H., G. Sorger, and C. Amon (2018): "Hierarchical growth: Basic and applied research," *Journal of Economic Dynamics and Control*, 90, 434–459.

Grossman, G. M., and E. Helpman (1991a): *Innovation and Growth in the Global Economy*, Cambridge, MA: The MIT Press.

——— (1991b): "Quality Ladders in the Theory of Growth," *The Review of Economic Studies*, 58, 43–61.

Grossmann, V., T. Steger, and T. Trimborn (2013): "Dynamically optimal R&D subsidization," *Journal of Economic Dynamics and Control*, 37, 516–534.

Grübler, A. (1996): "Time for a Change: On the Patterns of Diffusion of Innovation," *Daedalus*, 125, 19–42.

Grübler, A., and N. Nakićenović (1991): "Long Waves, Technology Diffusion, and Substitution," *Review (Fernand Braudel Center)*, 14, 313–342.

Harding, M., and M. Marten (2018): "Statutory Tax Rates on Dividends, Interest and Capital Gains: The Debt-Equity Bias at the Personal Level,"Technical Report 34, OECD Taxation Working Papers.

Hirano, T., and A. A. Toda (2025): "Bubble Necessity Theorem," *Journal of Political Economy*, 133, 111–145.

Horii, R., and T. Iwaisako (2007): "Economic Growth with Imperfect Protection of Intellectual Property Rights," *Journal of Economics*, 90, 45–85.

Iwaisako, T. (2023): "Optimal mix of R&D subsidy and patent protection in a heterogeneous-industry R&D-based growth model," *Journal of Economic Dynamics and Control*, 154, 104723.

Jaffe, A. B. (1989): "Real Effects of Academic Research," *The American Economic Review*, 79, 957–970.

Jiang, W., T. J. Sargent, N. Wang, and J. Yang (2022): "A p Theory of Taxes and Debt Management," NBER Working Papers 29931, National Bureau of Economic Research, Inc.

Jones, C. I., and J. C. Williams (1998): "Measuring the Social Return to R&D," *The Quarterly Journal of Economics*, 113, 1119–1135.

Jordà, Ò., K. Knoll, D. Kuvshinov, M. Schularick, and A. M. Taylor (2019): "The Rate of Return on Everything, 1870–2015," *Quarterly Journal of Economics*, 134, 1225–1298.

Jordà, Ò., M. Schularick, and A. M. Taylor (2017): "Macrofinancial History and the New Business Cycle Facts," in *NBER Macroeconomics Annual 2016* ed. by Eichenbaum, M., and Parker, J. A. Volume 31, Chicago: University of Chicago Press.

Judd, K. L. (1985): "Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model," *Journal of Political Economy*, 93, 298–319.

Keynes, J. (1923): *A Tract on Monetary Reform*: Macmillan.

King, I., and D. Ferguson (1993): "Dynamic inefficiency, endogenous growth, and Ponzi games," *Journal of Monetary Economics*, 32, 79–104.

Kocherlakota, N. R. (2023): "Infinite Debt Rollover in Stochastic Economies," *Econometrica*, 91, 1629–1658.

Kopczuk, W., and J. P. Lupton (2007): "To Leave or Not to Leave: The Distribution of Bequest Motives," *Review of Economic Studies*, 74, 207–235.

Leon-Ledesma, M., and K. Shibayama (2023): "(Endogenous) Growth Slowdowns," Studies in Economics 2303, School of Economics, University of Kent.

Li, D., N. E. Magud, and A. Werner (2023): "The long-run impact of sovereign yields on corporate yields in emerging markets," *Journal of International Money and Finance*, 130, 102748.

Lockwood, L. M. (2018): "Incidental Bequests and the Choice to Self-Insure Late-Life Risks," *American Economic Review*, 108, 2513–2550.

Lucas, R. E. (1988): "On the mechanics of economic development," *Journal of Monetary Economics*, 22, 3–42.

Mauro, P., and J. Zhou (2021): "$r - g < 0$: Can We Sleep More Soundly?," *IMF Economic Review*, 69, 197–229.

Mehrotra, N. R., and D. Sergeyev (2021): "Debt sustainability in a low interest rate world," *Journal of Monetary Economics*, 124, 1–18.

Mincer, J. (1974): *Schooling, Experience, and Earnings*, New York: National Bureau of Economic Research.

Minniti, A., C. Parello, and P. Segerstrom (2013): "A Schumpeterian growth model with random quality improvements," *Economic Theory*, 52, 755–791.

Murphy, K. M., and F. Welch (1990): "Empirical Age-Earnings Profiles," *Journal of Labor Economics*, 8, 202–229.

O'Connell, S. A., and S. P. Zeldes (1988): "Rational Ponzi Games," *International Economic Review*, 29, 431–450.

OECD (2023a): "Current retirement ages," in *Pensions at a Glance 2023: OECD and G20 Indicators*, Paris: OECD Publishing.

——— (2023b): "Life expectancy at birth," in *Health at a Glance 2023: OECD Indicators*, Paris: OECD Publishing.

Olivier, J. (2000): "Growth-Enhancing Bubbles," *International Economic Review*, 41, 133–151.

Parui, P., and K. Prettner (2025): "Public provision of healthcare and basic research: What are the joint effects on economic growth and welfare?" *Journal of Economic Growth*, 30, 383–415.

Peretto, P. F. (1998): "Technological Change and Population Growth," *Journal of Economic Growth*, 3, 283–311.

——— (2011): "The Growth and Welfare Effects of Deficit-Financed Dividend Tax Cuts," *Journal of Money, Credit and Banking*, 43, 835–869.

Peretto, P., and S. Smulders (2002): "Technological Distance, Growth and Scale Effects," *The Economic Journal*, 112, 603–624.

Rachel, Ł., and L. H. Summers (2019): "On Secular Stagnation in the Industrialized World," Working Paper 26198, National Bureau of Economic Research.

Reis, R. (2021): "The constraint on public debt when $r < g$," CEPR Discussion Papers 15950, Center for Economic Policy Research.

Romer, P. M. (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98, S71–S102.

Saint-Paul, G. (1992): "Fiscal Policy in an Endogenous Growth Model," *The Quarterly Journal of Economics*, 107, 1243–1259.

Scheuer, F., and A. Wolitzky (2016): "Capital Taxation under Political Constraints," *American Economic Review*, 106, 2304–2328.

Schmitt-Grohé, S., and M. Uribe (2003): "Closing small open economy models," *Journal of International Economics*, 61, 163–185.

Song, Z., K. Storesletten, and F. Zilibotti (2012): "Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt," *Econometrica*, 80, 2785–2803.

Tirole, J. (1985): "Asset Bubbles and Overlapping Generations," *Econometrica*, 53, 1499–1528.

Wan, J., and J. Zhang (2021): "Optimal growth through innovation, investment, and labor," *European Economic Review*, 132, 103644.

Yared, P. (2019): "Rising Government Debt: Causes and Solutions for a Decades-Old Trend," *Journal of Economic Perspectives*, 33, 115–140.

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- JEGrevisiononlineappendix20261108.pdf