Supplementary Material: Neural Control Model Personalization using an Improved Synergy Optimization Framework

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Activation-tracking formulation

The primary analysis tracked joint moments and minimized squared activation with two through six synergies. We compared its six-synergy solution with a combined fit that tracked joint moments and all 45 available activation trajectories from the EMG-driven model. Both cases used the same gait cycle, 21 synergy command nodes, nonnegative synergy commands and synergy vectors, unit activation bounds, initialization settings, and convergence tolerance. The activation targets included estimates for muscles without recorded EMG, so 45 activation trajectories did not imply 45 independently recorded EMG signals. The combined case did not include squared-activation minimization because every modeled muscle had an activation target.

The objective for joint moment tracking with full or partial availability of activation targets is

$$J|_{\{c_{\ell p}\},\{V_{ip}\}} = \frac{1}{N_t N_j} \sum_{k=1}^{N_t} \sum_{j=1}^{N_j} \left(\frac{M_{jk} - M_{jk}^{\star}}{\sigma_M} \right)^2 + \frac{1}{N_t N_m} \sum_{k=1}^{N_t} \sum_{i=1}^{N_m} \left(\frac{a_{ik} - a_{ik}^{\star}}{\sigma_a} \right)^2, \tag{1}$$

where $a_{ik} = \sum_{p=1}^{n} V_{ip} C_{pk}$ and $C_{pk} = \sum_{\ell=1}^{N_b} B_{\ell}(t_k) c_{\ell p}$. The activation target $a_{ik}^{\star}$ was the EMG-driven value when available and zero otherwise. The design variables were synergy command nodes $c_{\ell p}$ and synergy vector weights V_{ip}. The maximum allowable errors were $\sigma_M = 1$ N m and $\sigma_a = 0.05$, as in the primary objective. These constants scaled residuals; they were not hard error bounds. Here $N_t = 101$, $N_j = 6$, and $N_m = 45$. The activation term used the full number of modeled muscles in its denominator, including when only a subset of targets was available.

The primary case set all activation targets to zero, reducing the activation term to squared-activation minimization. The combined case used all 45 EMG-driven activation targets without a separate activation-minimization term. If only partial activation targets were available in a combined fit, the objective would track those samples and minimize activation for the remaining samples. Thus the formulation can use either full or partial availability of EMG-driven activations. The supplementary presents results for the combined fit with full EMG activation availability.

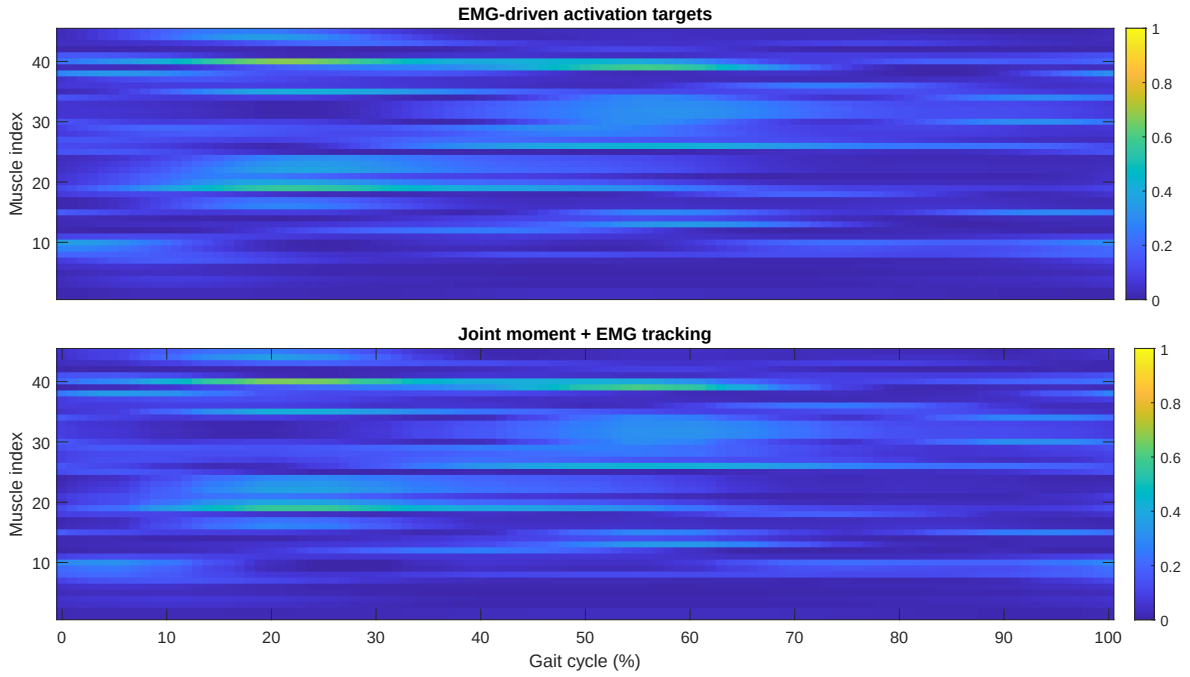


Figure 1: Reference and reconstructed activations over one normalized gait cycle. Rows correspond to the 45 modeled muscles. The upper panel shows activation targets from the EMG-driven model. The lower panel shows six-synergy estimates from joint moment plus EMG-driven activation tracking.

Although partial EMG activation availability is a capability of the formulation, results for this formulation are not presented for comparison.

Supplementary results

Joint moment plus EMG-driven activation tracking reached joint moment quality 99.88% and activation quality 99.33%. The solver objective decreased from 10.3057 to 4.7974. The primary six-synergy joint-moment-only benchmark reached joint moment quality 99.68%.. Figure 2 compares the joint moments reconstructed by the two fits with the reference.

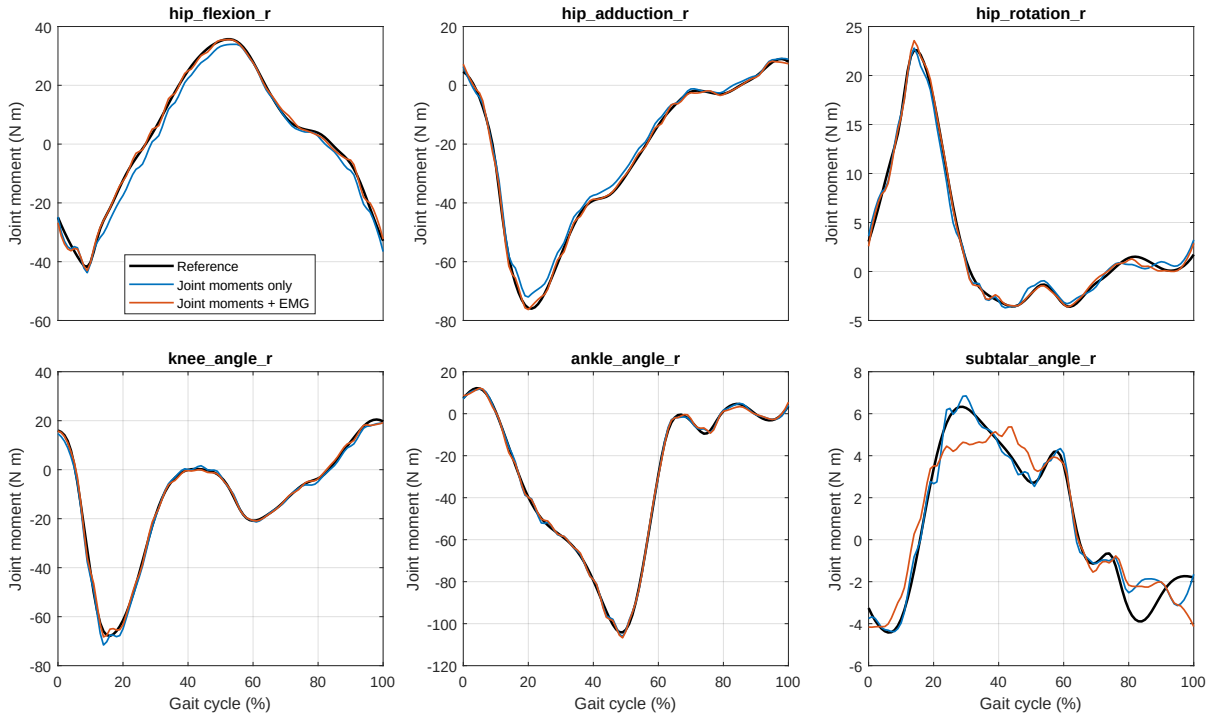


Figure 2: Reference and reconstructed joint moments over one normalized gait cycle. Black denotes the EMG-driven reference. Blue and orange denote six-synergy estimates obtained by tracking joint moments only and joint moments plus EMG-driven activations, respectively. Moments are reported in N m.

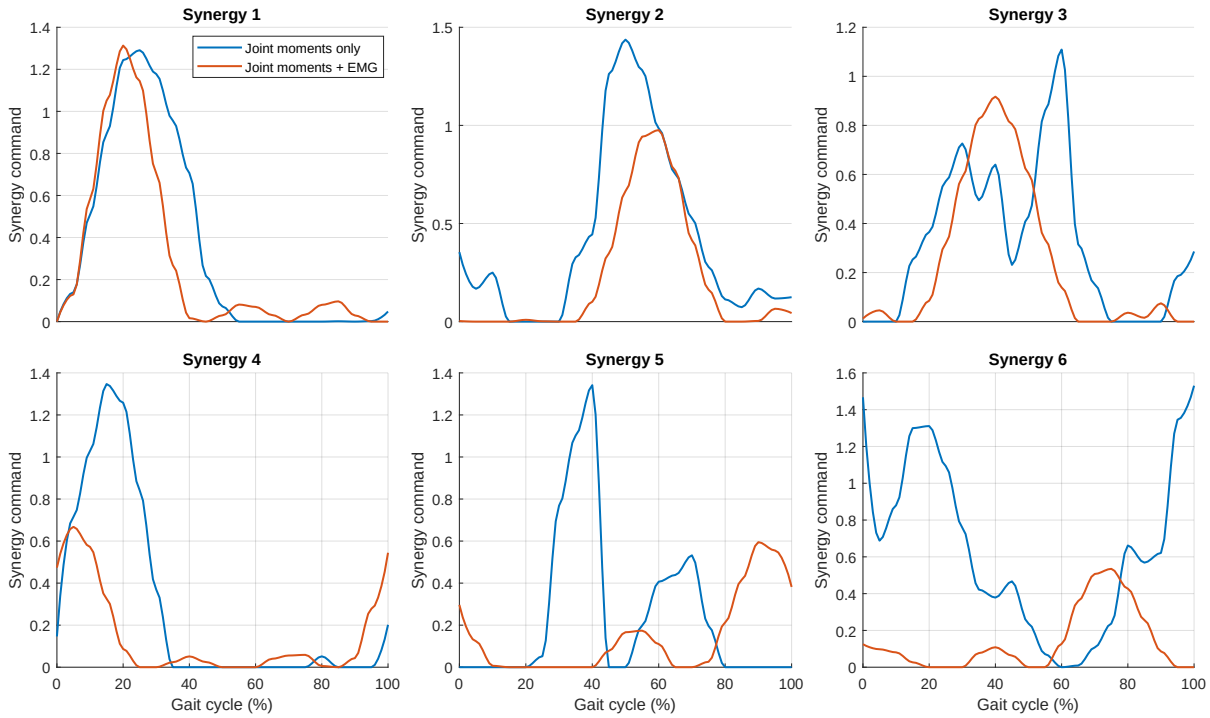


Figure 3: Matched six-synergy commands over one normalized gait cycle. Blue and orange denote joint moment-only and joint moment plus EMG-driven activation tracking, respectively. Synergies from the joint moment-only case were matched to the combined case by maximizing summed synergy-vector cosine similarity.

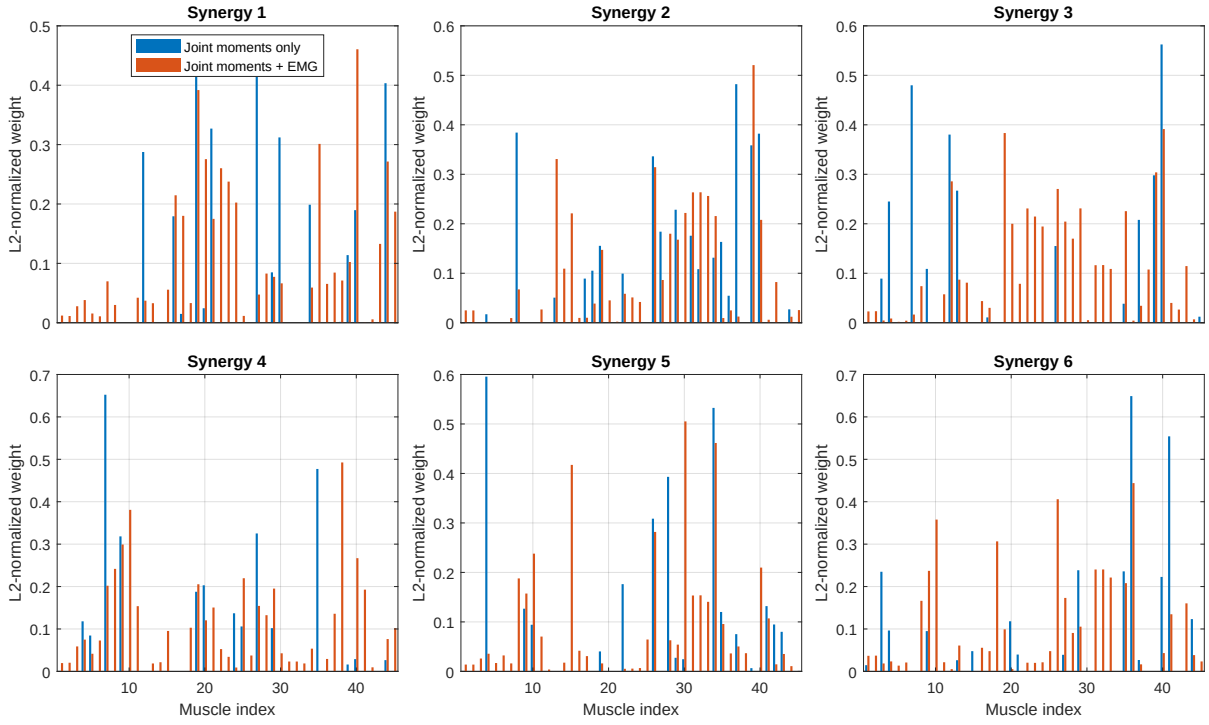


Figure 4: Grouped bar plots of the matched synergy vectors for the two six-synergy fits from one gait cycle. Each vector has unit L_2 norm. Muscle indices follow the model ordering used in the activation heatmaps. Blue and orange denote joint moment-only and joint moment plus EMG-driven activation tracking, respectively.