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A model of non-isothermal solar corona

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Abstract

A model of the non-isothermal solar corona is proposed. This model was obtained as a solution of hydrodynamic equations, in which the density of the solar corona varies with distance following a power-law dependence. In this case the power law index is determined by the polytropic index. The dependences of the polytropic indexes on the distance from the Sun are found for the Baumbach-Allen model. It is shown that the temperature of the coronal plasma in the regions with power-law density dependence is inversely proportional to the distance from the Sun. At the same time a temperature jump takes place at the exit points, where polytropic indexes change. The possibilities for probing the solar corona on an example of a single Type III burst, observed by PSP on March 13, 2023 in a wide frequency range are discussed.

1 Introduction

For a long time, the electron density models for solar corona are discussed by different authors. It is well-known such laws as the Baumbach-Allen dependence (Allen, 1947) $n(r) = 10^8[2.99\left(\frac{R_S}{r}\right)^{16} + 1.55\left(\frac{R_S}{r}\right)^{6}]cm^{-3}$ (R_S is the solar radius), the Newkirk's one (Newkirk, 1958) $n(r) = 4.2 \cdot 10^4 \cdot 10^{4.32\frac{R_S}{r}} cm^{-3}$ or the Saito et al. one (Saito et al., 1977) $n(r) = [1.36 \cdot 10^6 \left(\frac{R_S}{r}\right)^{2.14} + 1.68 \cdot 10^8 \left(\frac{R_S}{r}\right)^{6.13}]cm^{-3}$. All these laws were obtained from optical and radio observations using rather fine statistics. Later other authors using the same methods and sometimes others methods (for example, Leblanc et al., 1998, Vrsnak et al., 2004) got in the most cases different exponential dependences. The existence of different models reflects the fact that these models were obtained at different conditions, which were defined by conditions on the solar surface such as the density of coronal plasma bottom, plasma temperature there and presence of energy sources. In 1999 Mann et al. (Mann et al., 1999) attempted to find a density model for coronal plasma from main hydrodynamic equations in the isothermal case. Close to the Sun they had got that

$$n(r) = n_S \exp[\frac{A}{R_S}(\frac{R_S}{r} - 1)] \quad (1)$$

with $A = \mu G M_S / k_B T$ (here μ is the mean molecular weight, G is the gravitational constant, M_S is the mass of the Sun , k_B is the Boltzmann's constant, T is the temperature and $n_S = n(r = R_S)$ is density of coronal plasma). This law is the Newkirk's model (Newkirk, 1958) at the temperature $T = 1.5 \cdot 10^6 K$. They also concluded that their numerical solution of the corresponding hydrodynamic equations agreed better with Type III bursts observations at the temperature $T = 1 \cdot 10^6 K$ (Mann et al., 1999). But everyone knows (see, for example, Suzuki and Dulk (1985)) that the temperature of coronal plasma slowly, but steadily decreases with distance from the Sun, so it is desirable to consider more realistic case of non-isothermal coronal plasma. In this case it is necessary to add the equation for energy of coronal plasma to main hydrodynamic equations. At that, as shown by Shergelashvili et al., (2020), there can be discontinuous solutions, which can be defined by polytropic exponent in essential way (Westrich et al., 2026). Polytropic exponents are connected also with solar corona temperature and velocity of solar wind (Shi et al., 2022).

In this paper we analyze the case when the density of coronal plasma is continuous dependence on the distance. In particular, we consider a quasi-isothermal solution and compare it with that derived by Mann et al. (1999) for isothermal plasma. We show that the last one is correct in a narrow region near the solar surface. Especial attention we pay to the case when the solar coronal model is presented in the form of sum of power laws with different indexes. We show that in this case the exponent index is defined by polytropic one. Also, we discuss the solar model, which was obtained from the results of Type III observations by PSP in the wide frequency band from 0.3 to 19 MHz (Melnik and Dorovskyy, 2026).

2 Main hydrodynamic equations and their solutions in non-isothermal case

Plasma of solar corona in the one-dimensional approximation is described by the following main hydrodynamic

equations

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\rho v r^2 = 0 \tag{2}$$

$$\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial}{\partial r}v\right] = -\frac{\partial}{\partial r}p - \rho\frac{G M_S}{r^2} \tag{3}$$

$$\frac{\partial}{\partial t}\left(1 + v\frac{\partial}{\partial r}\right)\left(\frac{T}{\rho^{\alpha-1}}\right) = 0 \tag{4}$$

where $\rho = \mu m_p n$, v, $p = 1.92 nkT$, T, α are the plasma mass density, the solar wind velocity, the (thermal) pressure of the plasma, the plasma temperature and the polytropic index, respectively.

In the stationary case from equation (2) we have the law of mass conservation after integration from $r = R_S$ to r

$$r^2 n(r) v(r) = {R_S}^2 n(R_S) v(R_S) \tag{5}$$

where $v(R_S)$ is the velocity of solar wind at $r = R_S$.

The solution of the second equation (3) for the isothermal plasma ($T(r) = \text{const} = T(R_S), \alpha = 1$) is the solution by Mann et al. (Mann et al., 1999) near the Sun (equation (1)), where the velocity of solar wind $v(r)$ is small compared to the sound velocity $v(r) \ll v_s(R_S) = (\frac{k_B T(R_S)}{\mu m_p})^{1/2}$.

In the non-isothermal case ($\alpha \neq 1$) after integration of equation (4) we have such connection of the density and the temperature of solar corona

$$T(r) = T(R_S) n(r)^{\alpha-1} / n(R_S)^{\alpha-1} \tag{6}$$

Then equation (3) can be rewritten as

$$v(r)\frac{d}{dr}v(r) = -\frac{v_s(R_S)^2}{n(r)n(R_S)^{\alpha-1}}\frac{d}{dr}n(r)^{\alpha} - \frac{v_g(R_S)^2 R_S^2}{r^2}, \tag{7}$$

where $v_g(R_S) = (\frac{GM_S}{R_S})^{1/2}$ and can be transformed to the such form

$$v(r)\frac{d}{dr}v(r) = -\frac{\alpha v_s(R_S)^2}{(\alpha-1)n(R_S)^{\alpha-1}}\frac{d}{dr}n(r)^{\alpha-1} - \frac{v_g(R_S)^2 R_S}{r^2} \tag{8}$$

It can be integrated from $r = R_S$ to some distance r

$$\frac{1}{2}(v^2(r) - v^2(R_S)) = -\frac{\alpha {v_s}^2(R_S)}{(\alpha-1)n^{\alpha-1}(R_S)}[n^{\alpha-1}(r) - n^{\alpha-1}(R_S)] + {v_g}^2(R_S)(\frac{R_S}{r} - 1) \tag{9}$$

Near the Sun, where the speed of the solar wind is small $v(r) \ll v_s(R_S)$, we have for the plasma density

$$n^{\alpha-1}(r) = n^{\alpha-1}(R_S)[1 + \frac{(\alpha-1)\cdot {v_g}^2(R_S)}{\alpha \cdot {v_s}^2(R_S)} \cdot (\frac{R_S}{r} - 1)] \tag{10}$$

Note that at the standard temperature of solar corona near the Sun $T = 1 \div 2 \cdot 10^6 K$, $v_s(R_S) = 1.14 \div 1.6 \cdot 10^7 cm/s$ and $v_g(R_S) = (\frac{GM_S}{R_S})^{1/2} = 4.37 \cdot 10^7 cm/s$, i. e. $\frac{v_g(R_S)}{v_s(R_S)} > 1$.

Let us consider three different cases

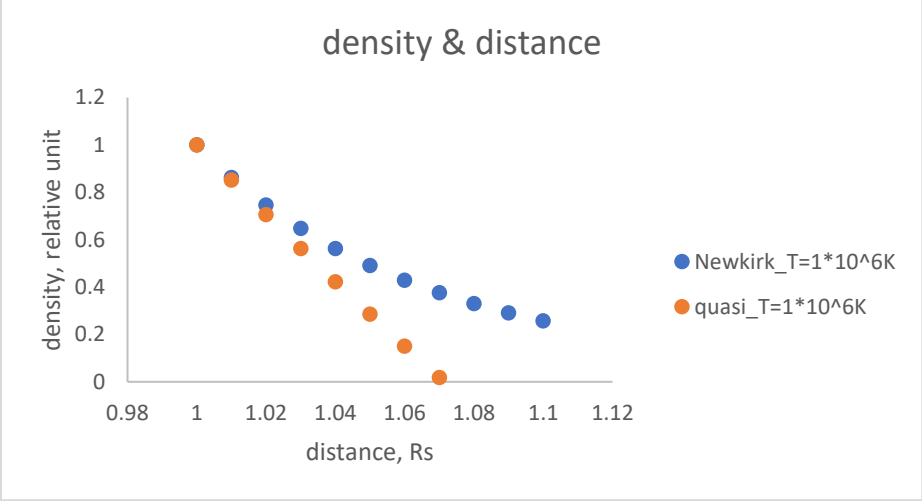
1. $\frac{(\alpha-1)\cdot {v_g}^2(R_S)}{\alpha \cdot {v_s}^2(R_S)} \ll 1$ (11)
2. $\frac{(\alpha-1)\cdot {v_g}^2(R_S)}{\alpha \cdot {v_s}^2(R_S)} \geq 1$ (12)
3. $\frac{(\alpha-1)\cdot {v_g}^2(R_S)}{\alpha \cdot {v_s}^2(R_S)} = 1$ (13)

1. If the polytropic index $\alpha \to 1$ this is the quasi-isothermal case because when $\alpha = 1$, we have the isothermal plasma of Mann et al. (Mann et al., 1999). Let's compare density dependences in the cases of quasi-isothermal and isothermal plasmas (Mann et al., 1999). If $\frac{(\alpha-1)\cdot {v_g}^2(R_S)}{\alpha\cdot {v_s}^2(R_S)} \ll 1$ then from (10) we have

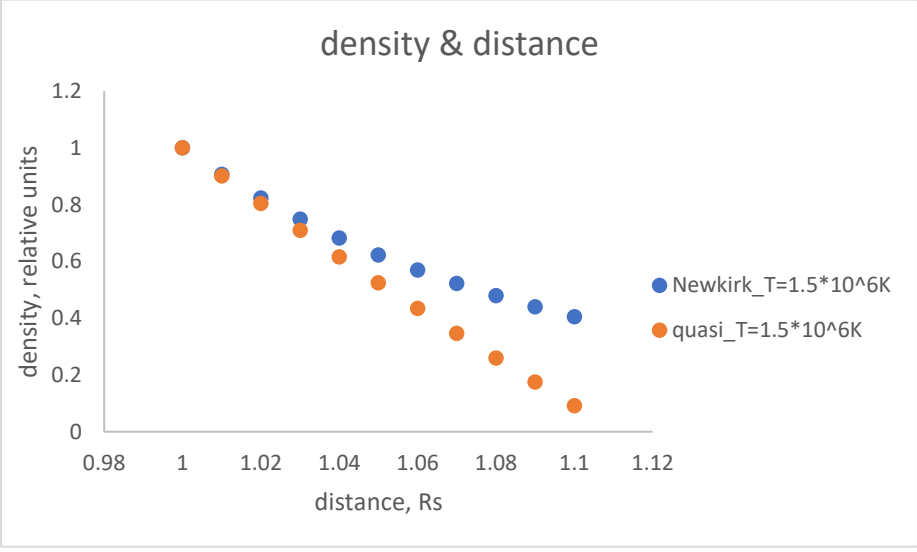
$$n^{\alpha-1}(r) \approx n^{\alpha-1}(R_S)[1 + \frac{{v_g}^2(R_S)}{\alpha\cdot {v_s}^2(R_S)} \cdot (\frac{R_S}{r} - 1)] \quad (14)$$

At the temperature $T(R_S) = 1 \div 2 \cdot 10^6 K$, $V_s^2 \ll V_g^2$ and the plasma density decreases with distance very quickly. Figure 1 shows the plasma density dependences on distance according to equations (14) and (1) in the case of temperatures $T = 1 \cdot 10^6 K$, $T = 1.5 \cdot 10^6 K$ and $T = 2 \cdot 10^6 K$. We see that these dependences are close only at small distances and the region of closeness is extending with increasing temperature.

a)



b)



c)

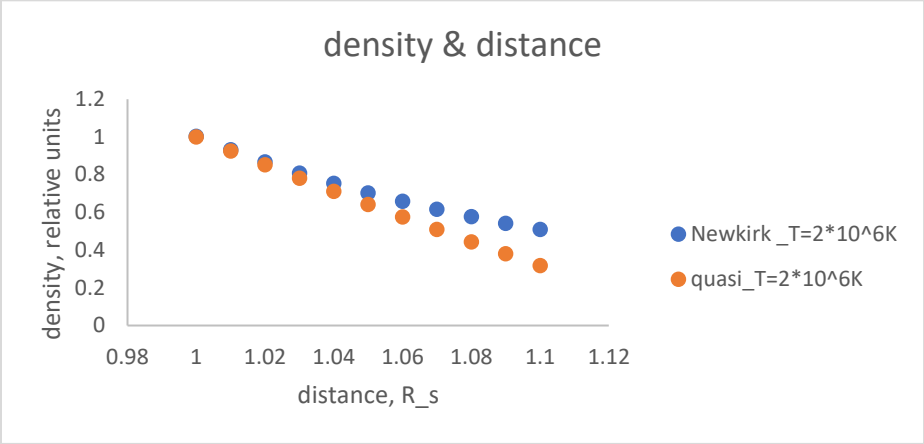


Figure 1. Dependences of coronal plasma density on distance for isothermal (1) and quasi-isothermal (11) cases.

Starting from $r \approx (1 + \frac{{v_s}^2(R_S)}{{v_g}^2(R_S)})R_S$ the plasma density decreases very quickly. At the same time according to (5) the velocity of solar wind increases very quickly also and we need to take into account the term with this velocity in equation (6) when $v(r) \approx v_s(R_S)$ and then for the density we have such expression

$$n(r) = n^{\alpha-1}(R_S)[1 + \frac{{v_g}^2(R_S)}{\alpha \cdot {v_s}^2(R_S)} \cdot (\frac{R_S}{r} - 1) - \frac{1}{2\alpha {v_s}^2(R_S)}\left(v^2(r) - v^2(R_S)\right)] \quad (15)$$

We see that adding of this term leads to decreasing the density. So we can account that the limitary value is about

$$r = R_S/(1 - \frac{\alpha {v_s}^2(R_S)}{{v_g}^2(R_S)}). \quad (16)$$

Physically it means that the coronal plasma with temperatures $T = 1 \div 2 \cdot 10^6 K$ cannot overcome gravitational attraction.

2. When $\frac{(\alpha-1)\cdot {v_g}^2(R_S)}{\alpha \cdot {v_s}^2(R_S)} \geq 1$ we have the equation (10) for the density of coronal plasma near the Sun when the speed of the solar wind is small and the equation with velocity term

$$n^{\alpha-1}(r) = n^{\alpha-1}(R_S)[1 + \frac{(\alpha-1)\cdot {v_g}^2(R_S)}{\alpha \cdot {v_s}^2(R_S)} \cdot (\frac{R_S}{r} - 1) - \frac{(\alpha-1)}{2\alpha {v_s}^2(R_S)}\left(v^2(r) - v^2(R_S)\right)] \quad (17)$$

Again, the coefficient $\frac{V_g^2}{V_s^2}$ is a high value for temperatures $T = 1 \div 2 \cdot 10^6 K$ and as a consequence the density decreases very quickly at distances $r = R_S/(1 - \frac{\alpha v_s(R_S)^2}{(\alpha-1)v_g(R_S)^2})$. According to the equation (6) the temperature of the solar wind $T(r)$ decreases also very quickly. So, we see that in both cases, 1 and 2, the plasma density of solar wind becomes small close to the Sun. This is natural result because the average energy of plasma particles is not enough to get away from the Sun ($V_s^2 \ll V_g^2$).

3. The case $\frac{(\alpha-1)\cdot {v_g}^2(R_S)}{\alpha\cdot {v_s}^2(R_S)} = 1$ is the special one. In this occasion we have

$$n^{\alpha-1}(r) = n^{\alpha-1}(R_S) \cdot R_S/r \tag{18}$$

and

$$T(r) = T(R_S) \cdot R_S/r \tag{19}$$

from (10) and (6) and as a result

$$n(r) = n(R_S)(\frac{R_S}{r})^{\gamma} = n(R_S)(\frac{R_S}{r})^{1/(\alpha-1)} \tag{20}$$

for density, from where the polytropic index is

$$\alpha = 1 + 1/\gamma. \tag{21}$$

Different authors obtained different density dependences from the observational data in the form of sum of various power-law relations. Let us consider conditions, at which these relations are realized. As an example, let's analyze the well-known Baumbach-Allen law (Allen, 1947) $n(r) = 10^8[2.99(\frac{R_S}{r})^{16} + 1.55(\frac{R_S}{r})^6]cm^{-3}$, which consists of two power-law terms with exponents $\gamma_1 = 16$ and $\gamma_2 = 6$. Transit from one dependence to the another one occurs at the distance $r_2 = 1.07 \cdot R_S$ (let $r_1 = R_S$). According to (21) the polytropic indexes are $\alpha_1 = 1.06$ and $\alpha_2 = 1.17$ in the regions $r_1 < r < r_2$ and $r > r_2 = 1.07R_S$ correspondingly. For the first region $r_1 < r < r_2$ the condition $\frac{(\alpha-1)\cdot {v_g}^2(r_1)}{\alpha\cdot {v_s}^2(r_2)} = 1$ is fulfilled if the sound velocity $v_s(r_1 = R_S) = \sqrt{(\alpha_1 - 1)/\alpha_1} \cdot v_g(r_1 = R_S) = 1.09 \cdot 10^7 cm/s$, and the temperature $T(r_1 = R_S) = 0.93 \cdot 10^6 K$. In the second region $r > r_2$ the equality $\frac{(\alpha_2-1)V_g^2(r_2)}{\alpha_2 V_s^2(r_2)} = 1$ holds true at $v_s(r_2 = 1.07R_S) = \sqrt{(\alpha_2 - 1)/\alpha_2} v_g(r_2 = 1.07R_S) = 1.6 \cdot 10^7 cm/s$ and the temperature $T(r_2 = 1.07R_S) = 2 \cdot 10^6 K$ correspondingly. But according to (19) the temperature decreases as

$$T_1(r) = T(r_1) \cdot R_S/r \tag{22}$$

in the first region and at the distance r_2 it should be $T_1(r_2 = 1.07R_S) = 0.87 \cdot 10^6 K$. The difference between values $T_2(r_2 = 1.07R_S) = 2 \cdot 10^6 K$ and $T_1(r_2 = 1.07R_S) = 0.87 \cdot 10^6 K$ means that some heating occurs at the distance $r_2 = 1.07$. In respect to the second region the temperature decreases there as

$$T_2(r) = T(r_2 = 1.07R_S) \cdot R_S/r \quad (23)$$

We analyze variations of polytropic indexes and temperatures in the case of larger number of power-law relations on the example of the coronal model, which was obtained from the analysis of spectral characteristics of powerful Type III burst observed in the wide frequency range from 0.3 to 19 MHz by PSP on 13 March 2023 [Melnik and Dorovskyy, 2026]. This is the plasma, in which the density changed in such way (Figure 2) - $n_1(r) = 1.55 \cdot 10^8 \cdot (R_S/r)^{5.5} cm^{-3}$, $n_2(r) = 5.9 \cdot 10^6 \cdot (R_S/r)^{2.6} cm^{-3}$ and $n_3(r) = 3 \cdot 10^6 \cdot (R_S/r)^{2.3} cm^{-3}$ with exponents $\gamma_1 = 5.5$, $\gamma_2 = 2.6$ and $\gamma_3 = 2.3$ (corresponding polytropic indexes are $\alpha_1 = 1.18$, $\alpha_2 = 1.38$ and $\alpha_3 = 1.43$) in the regions $1.9R_S < r < 3.1R_S$, $3.1R_S < r < 9.3R_S$, $9.3R_S < r < 31R_S$ correspondingly. The generation of radio emission at the first harmonic at frequencies 19 MHz, 5 MHz, 1.2 MHz and 0.3 MHz occurs at distances $r_1 = 1.9R_S$, $r_2 = 3.1R_S$, $r_3 = 9.3R_S$ and $r_4 = 31R_S$, respectively.

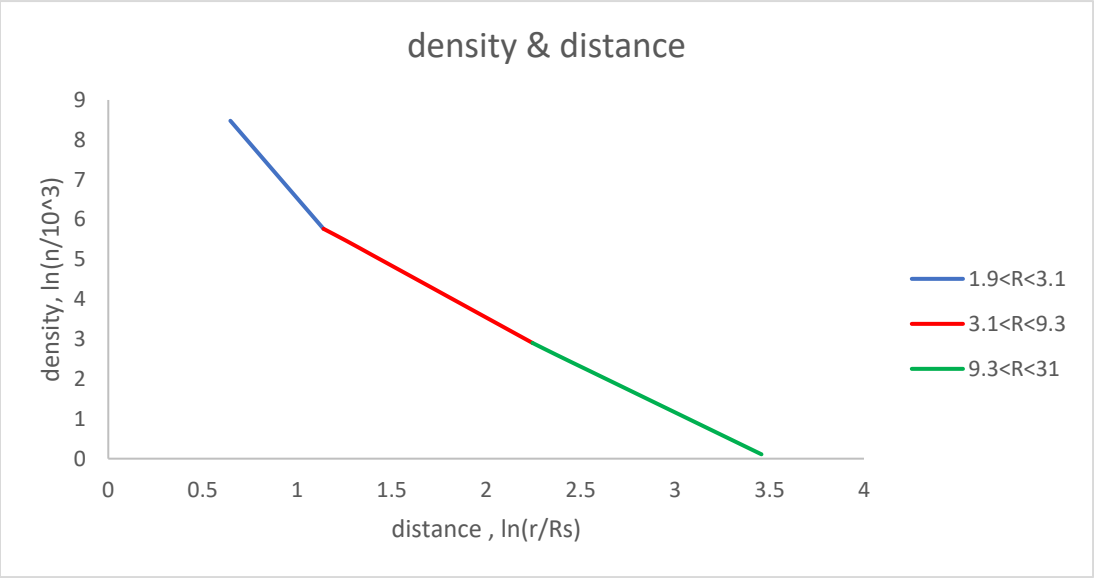


Figure 2. The density of coronal plasma through which the source of Type III burst observed in the frequency range 0.3-19 MHz by PSP on 13 March 2023 moved.

According to (19) temperatures in these regions change inversely to distance

$$T_i(r) = T_i(r_i) \cdot R_S/r \quad (24)$$

where $T_i(r_i)$ are found from equations

$$\frac{(\alpha_i - 1)V_g^2(R_i)}{\alpha_i V_s^2(R_i)} = 1, \quad (25)$$

here $v_g(r_i) = (\frac{GM_S}{r_i})^{1/2}$ and $v_s(r_i) = (\frac{k_B T_i(r_i)}{\mu m_p})^{1/2}$

At distances $r_2 = 3.1R_S$ and $r_3 = 9.3R_S$ the temperature of solar corona needs to have jumps from $T_1(r_2) = 0.74 \cdot 10^6 K$ to $T_2(r_2) = 1.32 \cdot 10^6 K$ and from $T_2(r_3) = 0.44 \cdot 10^6 K$ to $T_2(r_3) = 0.49 \cdot 10^6 K$, correspondingly (Figure 3) (note, that the value $T_2(r_3)$ and its decreasing according to (22) is close to those obtained by Shi et al, 2022). Such situation can be realized if some kind of heating exists at these distances. Thus, in spite of considering the problem without heating we conclude that some heating is necessary to realize the density dependence, which consists of different power-law dependences. Physically it means that coronal plasma can move away from the Sun overcoming gravitational forces if it receives some energy, for example, in the form of heating.

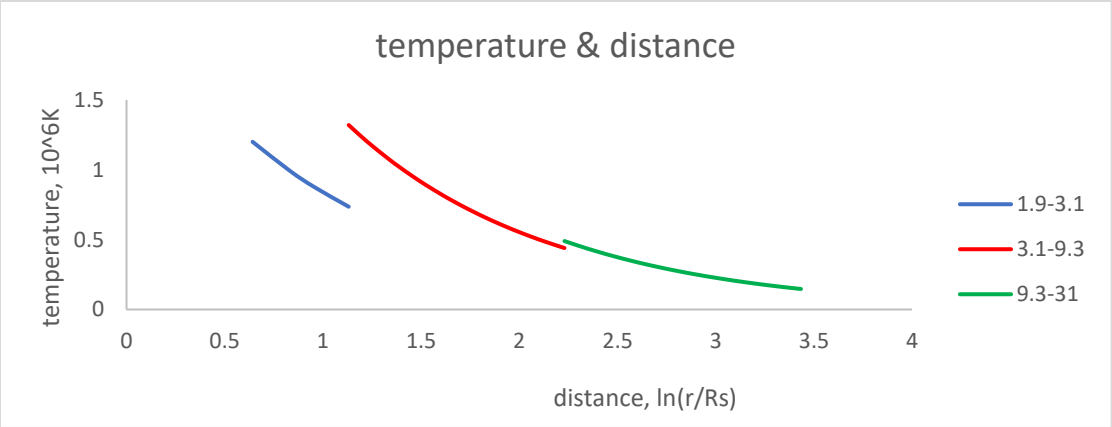


Figure 3. The temperature dependences in the regions from $r_1 = 1.9R_S$ to $r_4 = 31R_S$.

If the third region in this plasma extends to the Earth's orbit ($r = 215R_S$) then the density of solar wind is $n(r = 215R_S) = 13cm^{-3}$ and its temperature was $T(r = 215R_S) = 2.1 \cdot 10^4 K$ at Earth orbit ($r = 215R_S$). Herein the velocity of the solar wind increases by a factor of $\frac{v(215)}{v(1.9)} = \frac{1.9}{215}\frac{n(1.9)}{n(215)} = 3 \cdot 10^3$.

3 Conclusion

We show that power-law dependence of solar wind density on distance can be obtained from a solution of hydrodynamic equations in non-isothermal case in some regions without heating. At that exponential indexes are defined by polytropic indexes. Temperatures of coronal plasma in these regions decreased inversely to distance. But it should be temperature jumps at transit points between regions with different polytropic indexes. Values of these jumps decrease with increasing polytropic indexes and can be achieved by heating of coronal plasma in these transit points. This situation is similar to that considered by Shergelashvili et al. (2020, Westrich et al. (2026). In contrast to the temperature the coronal density is continuous function of distance in these points. Although we analyzed hydrodynamic equations without any heating (the right-hand side of equation (4) equals zero) the power-law density dependences require some heating

at coupling points of plasma states. So, we conclude that it is necessary to consider some processes of heating of coronal plasma.

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Disclosure of Potential Conflicts of Interest

The authors declare that they have no conflicts of interest.

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