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Research Article

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Rotating Channel Heat and Mass Transfer with Dufour Diffusion, Chemical Reaction, and Internal Heat Source: Exact Closed-Form Wall Quantities

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Abstract

The study of rotating channels that carry heat and mass transfer underpins turbine cooling, centrifugal separation, and reactor design. Existing treatments of the combined problem are numerical or perturbative, so no closed-form route to the wall shear, heat flux, or mass flux has been available once Dufour diffusion, a chemical reaction, and an internal heat source act simultaneously. However, this paper supplies that route. The two momentum equations are combined into a single complex-velocity equation that integrates exactly, the species equation is independent of temperature, and the energy equation becomes linear once Dufour diffusion is treated as known forcing. The system is therefore solved in sequence, without a perturbation parameter, discretization, or iteration. A unified thermal boundary condition recovers isothermal, constant-flux, and convective walls as special cases. Rotation transfers momentum from the streamwise to the cross-stream direction and lowers the skin friction. Dufour diffusion raises the wall heat flux, and the reaction rate controls the concentration layer and the mass-transfer rate almost entirely. Moreover, the classical vertical-channel limit is recovered exactly, and every wall quantity is confirmed against an independent derivation by undetermined coefficients. Because the final expressions are short and explicit, they can be used directly in channel sizing and as benchmarks for numerical solvers.

Keywords: Rotating channel flow, Dufour effect, Chemical reaction, Thermal source, Complex velocity, Skin friction, Nusselt number, Sherwood number, Exact solution

Nomenclature

Symbol	Definition	SI unit
$A = u + iv$	Complex velocity	$\mathrm{m\,s^{-1}}$
A_1, A_2, A_3	Thermal boundary-condition constants	—
c_p	Specific heat at constant pressure	$\mathrm{J\,kg^{-1}\,K^{-1}}$

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Symbol	Definition	SI unit
c_s	Concentration susceptibility	$\mathrm{m^3\,kg^{-1}}$
C_f	Skin-friction coefficient	—
D_m	Mass diffusivity	$\mathrm{m^2\,s^{-1}}$
Df	Dufour parameter	—
E	Rotation parameter	—
g	Gravitational acceleration	$\mathrm{m\,s^{-2}}$
Gc	Solutal Grashof number	—
Gr	Thermal Grashof number	—
H	Channel width	m
h	Convective heat-transfer coefficient	$\mathrm{W\,m^{-2}\,K^{-1}}$
k	Thermal conductivity	$\mathrm{W\,m^{-1}\,K^{-1}}$
k_T	Thermal diffusion ratio	—
k_1	Chemical reaction rate	$\mathrm{s^{-1}}$
K	Chemical reaction parameter	—
N	Buoyancy ratio	—
Nu	Nusselt number	—
Pr	Prandtl number	—
Q_0	Thermal source strength	$\mathrm{W\,m^{-3}\,K^{-1}}$
ρ	Fluid density	$\mathrm{kg\,m^{-3}}$
S	Thermal source parameter	—
Sc	Schmidt number	—
Sh	Sherwood number	—
U_{ref}	Buoyancy velocity scale, $g\beta_T\Delta T H^2/\nu$	$\mathrm{m\,s^{-1}}$
u, v	Streamwise and cross-stream velocities	$\mathrm{m\,s^{-1}}$
V_0	Suction/injection velocity (denoted W_0 in Fig. 1)	$\mathrm{m\,s^{-1}}$
z	Wall-normal coordinate	m
α	Thermal diffusivity	$\mathrm{m^2\,s^{-1}}$
β_T, β_C	Thermal and solutal expansion coefficients	$\mathrm{K^{-1}}$, $\mathrm{m^3\,kg^{-1}}$
$\Delta T, \Delta\phi$	Reference temperature and concentration differences	K, $\mathrm{kg\,m^{-3}}$
θ	Dimensionless temperature	—
$\theta_r, \theta_0, \theta_\infty$	Reference, wall, and ambient temperatures	K
λ	Suction/injection parameter, $V_0 H/\nu$	—
ν	Kinematic viscosity	$\mathrm{m^2\,s^{-1}}$
ϕ	Dimensionless concentration	—
ϕ_r, ϕ_0	Reference and wall concentrations	$\mathrm{kg\,m^{-3}}$
Ω	Angular velocity of rotation	$\mathrm{rad\,s^{-1}}$
τ	Complex wall shear, $A'(0)$	—

Sign convention. $\lambda > 0$ denotes suction; $\lambda < 0$ denotes injection. The complex wall shear is $\tau = A'(0) = \tau_x + i\,\tau_y$. For $E > 0$ and upward buoyancy, the transverse Coriolis term $-2Eu$ in the v-equation drives $v(z) \neq 0$, so $\tau_y \neq 0$.

1. Introduction

Generally, the rotating channels with simultaneous heat and mass transfer arise in turbine blade cooling, centrifugal separation, geothermal circulation, and chemical reactor design. Rotation introduces Coriolis forces that reshape the flow [5]; cross-diffusion effects such as Dufour [6] become important whenever temperature and concentration gradients coexist. Each effect has been studied in isolation for decades [4]. Combining them with a first-order reaction, an internal heat source, and suction, and solving the resulting problem in closed form, has not been done. First, four recent studies frame the field. Lakshmikaanth et al. [10] examined Hall and heat-source effects on flow past a vertically accelerating plate. Gitte et al. [11] used a regular perturbation method for chemically reacting micro-rotational flow in an open-ended vertical channel. Raizah et al. [12] treated coupled Dufour and Soret effects on a hybrid nanofluid in a gyrating channel. A recent Crank–Nicolson study [13] examined diffusion-thermo and thermo-diffusion effects in a rotating channel. Each relies on either a numerical scheme or a perturbation expansion, and none gives closed-form expressions for skin friction, Nusselt number, or Sherwood number across the parameter range. A designer sizing a channel, or a numerical analyst checking a solver, needs a formula rather than a curve. However, that is the gap this paper fills. The problem is a rotating vertical channel with Dufour cross-diffusion, a first-order chemical reaction, an internal thermal source, and constant suction or injection. Sections 2–4 set up the governing equations, introduce a general thermal boundary condition, reduce the coupled momentum equations using the complex velocity $A = u + iv$, and derive the exact solution together with the closed-form expressions for τ, Nu, and Sh. Section 5 reports the parametric study.

2. Mathematical formulation

2.1. Channel geometry and coordinate system

Figure 1 shows the configuration. Two infinite parallel plates are separated by H. The axis normal to the plates is z, with $z = 0$ at the left plate and $z = H$ at the right plate; after non-dimensionalization the walls sit at $z = 0$ and $z = 1$. The x-axis (labelled U) is vertical and points upward so that buoyancy acts along $+x$. The y-axis (labelled V) is horizontal and spanwise. The velocity components are $u = u(z)$ along x and $v = v(z)$ along y.

The channel rotates as a rigid body with constant angular velocity Ω about the z-axis. In the rotating frame the Coriolis acceleration is $-2\boldsymbol{\Omega} \times \mathbf{V}$. With $\boldsymbol{\Omega} = \Omega\,\hat{\mathbf{e}}_z$ and $\mathbf{V} = u\,\hat{\mathbf{e}}_x + v\,\hat{\mathbf{e}}_y$,

$$-2\boldsymbol{\Omega} \times \mathbf{V} = -2\Omega\,\hat{\mathbf{e}}_z \times (u\,\hat{\mathbf{e}}_x + v\,\hat{\mathbf{e}}_y) = 2\Omega v\,\hat{\mathbf{e}}_x - 2\Omega u\,\hat{\mathbf{e}}_y, \tag{1}$$

which yields $+2\Omega v$ in the u-equation and $-2\Omega u$ in the v-equation.

A constant suction/injection velocity, denoted W_0 in Fig. 1 and V_0 in the text, is applied at both plates. With the sign convention of Eq. (2), $V_0 > 0$ corresponds to suction ($\lambda > 0$) and $V_0 < 0$ to injection ($\lambda < 0$). The plates are impermeable to the velocity components, so $u = v = 0$ at $z = 0$ and $z = 1$.

2.2. Governing equations

Under the Boussinesq approximation, the dimensional governing equations for a fully developed, laminar, incompressible, electrically non-conducting fluid are

$$\begin{aligned} \nu\frac{d^2u}{dz^2} + V_0\frac{du}{dz} + 2\Omega v &= -g\beta_T(\theta-\theta_r) - g\beta_C(\phi-\phi_r),\\ \nu\frac{d^2v}{dz^2} + V_0\frac{dv}{dz} - 2\Omega u &= 0,\\ \alpha\frac{d^2\theta}{dz^2} + V_0\frac{d\theta}{dz} + \frac{D_m k_T}{c_s c_p}\frac{d^2\phi}{dz^2} + \frac{Q_0}{\rho c_p}(\theta-\theta_r) &= 0,\\ D_m\frac{d^2\phi}{dz^2} + V_0\frac{d\phi}{dz} - k_1(\phi-\phi_r) &= 0. \end{aligned} \tag{2}$$

2.3. Non-dimensionalization

Introduce the buoyancy velocity scale and reference differences

$$U_{\mathrm{ref}} = \frac{g\beta_T(\theta_0-\theta_r)H^2}{\nu}, \quad \Delta T = \theta_0-\theta_r, \quad \Delta\phi = \phi_0-\phi_r, \tag{3}$$

and

$$z^* = \frac{z}{H}, \quad u^* = \frac{u}{U_{\mathrm{ref}}}, \quad v^* = \frac{v}{U_{\mathrm{ref}}}, \quad \theta^* = \frac{\theta-\theta_r}{\theta_0-\theta_r}, \quad \phi^* = \frac{\phi-\phi_r}{\phi_0-\phi_r}. \tag{4}$$

Dropping asterisks, the dimensionless equations become

$$\begin{aligned} \frac{d^2u}{dz^2} + \lambda\frac{du}{dz} + 2Ev &= -(\theta+N\phi),\\ \frac{d^2v}{dz^2} + \lambda\frac{dv}{dz} - 2Eu &= 0,\\ \frac{d^2\theta}{dz^2} + \lambda Pr\frac{d\theta}{dz} + S\theta + Df\frac{d^2\phi}{dz^2} &= 0,\\ \frac{d^2\phi}{dz^2} + \lambda Sc\frac{d\phi}{dz} - K\phi &= 0, \end{aligned} \tag{5}$$

with

$$\lambda = \frac{V_0H}{\nu}, \quad E = \frac{\Omega H^2}{\nu}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D_m}, \quad S = \frac{Q_0H^2}{\rho c_p\alpha}, \tag{6}$$

$$Df = \frac{D_m k_T\,\Delta\phi}{c_s c_p\alpha\,\Delta T}, \quad K = \frac{k_1H^2}{D_m}, \quad N = \frac{Gc}{Gr}, \quad Gr = \frac{g\beta_T\,\Delta T\,H^3}{\nu^2}, \quad Gc = \frac{g\beta_C\,\Delta\phi\,H^3}{\nu^2}. \tag{7}$$

The Grashof numbers Gr and Gc do not appear explicitly in Eq. (5); they are absorbed into U_{ref} and N.

2.4. Boundary conditions

At $z = 0$:

$$u = 0, \quad v = 0, \quad \phi = 1, \quad A_1\frac{d\theta}{dz} + A_2\theta = A_3, \tag{8}$$

and at $z = 1$:

$$u = 0, \quad v = 0, \quad \theta = 0, \quad \phi = 0. \tag{9}$$

The unified thermal condition at $z = 0$ recovers three common walls:

- isothermal: $(A_1, A_2, A_3) = (0, 1, 1)$, giving $\theta(0) = 1$;
- constant flux: $(A_1, A_2, A_3) = (1, 0, -1)$, giving $\theta'(0) = -1$;
- convective: $(A_1, A_2, A_3) = (-k, h, h\theta_\infty)$, giving $-k\,\theta'(0) = h(\theta(0) - \theta_\infty)$.

2.5. Structure of the solution

Because the geometry is one-dimensional in z, System (5) is linear and triangular. The species equation does not contain θ. The energy equation is linear in θ with the Dufour term acting as known forcing from ϕ. The momentum equations are forced by both θ and ϕ as inhomogeneous terms. Each equation is solved in turn, with no iteration, perturbation, or discretization.

3. Method of solution

3.1. Complex velocity formulation

Define $A = u + iv$. Multiplying the v-equation by i and adding it to the u-equation gives

$$\frac{d^2A}{dz^2} + \lambda\frac{dA}{dz} - 2iEA = F(z), \tag{10}$$

where

$$F(z) = -(\theta + N\phi), \qquad A(0) = A(1) = 0. \tag{11}$$

3.2. Species equation

With $\phi(0) = 1$, $\phi(1) = 0$,

$$r^2 + \lambda Sc\, r - K = 0, \tag{12}$$

$$r_{1,2} = \frac{-\lambda Sc \pm \sqrt{(\lambda Sc)^2 + 4K}}{2}. \tag{13}$$

Thus

$$\phi(z) = C_1 e^{r_1 z} + C_2 e^{r_2 z}, \qquad C_1 = \frac{-e^{r_2}}{e^{r_1} - e^{r_2}}, \quad C_2 = \frac{e^{r_1}}{e^{r_1} - e^{r_2}}. \tag{14}$$

3.3. Energy equation

The energy equation becomes

$$\frac{d^2\theta}{dz^2} + \lambda Pr\frac{d\theta}{dz} + S\theta = -Df\frac{d^2\phi}{dz^2}. \tag{15}$$

The homogeneous roots are

$$m_{1,2} = \frac{-\lambda Pr \pm \sqrt{(\lambda Pr)^2 - 4S}}{2}. \tag{16}$$

A particular solution is

$$\theta_p = \Theta_1 e^{r_1 z} + \Theta_2 e^{r_2 z}, \qquad \Theta_1 = \frac{-Df C_1 r_1^2}{r_1^2 + \lambda Pr\, r_1 + S}, \quad \Theta_2 = \frac{-Df C_2 r_2^2}{r_2^2 + \lambda Pr\, r_2 + S}. \tag{17}$$

Hence

$$\theta(z) = D_1 e^{m_1 z} + D_2 e^{m_2 z} + \Theta_1 e^{r_1 z} + \Theta_2 e^{r_2 z}. \tag{18}$$

Applying the boundary conditions yields a 2×2 system for D_1, D_2:

$$\begin{bmatrix} A_1 m_1 + A_2 & A_1 m_2 + A_2 \\ e^{m_1} & e^{m_2} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} A_3 - A_1(\Theta_1 r_1 + \Theta_2 r_2) - A_2(\Theta_1 + \Theta_2) \\ -\Theta_1 e^{r_1} - \Theta_2 e^{r_2} \end{bmatrix}, \tag{19}$$

with unique solution whenever

$$\Delta = (A_1 m_1 + A_2) e^{m_2} - (A_1 m_2 + A_2) e^{m_1} \neq 0. \tag{20}$$

3.4. Complex momentum equation

The homogeneous roots of (10) are

$$s_{1,2} = \frac{-\lambda \pm \sqrt{\lambda^2 + 8iE}}{2}, \qquad s_1 + s_2 = -\lambda. \tag{21}$$

Write

$$F(z) = -\sum_{j=1}^{4} k_j e^{p_j z}, \tag{22}$$

where

$$\begin{aligned} &p_1 = m_1, \quad p_2 = m_2, \quad p_3 = r_1, \quad p_4 = r_2, \\ &k_1 = D_1, \quad k_2 = D_2, \quad k_3 = \Theta_1 + NC_1, \quad k_4 = \Theta_2 + NC_2. \end{aligned} \tag{23}$$

The Wronskian of $\{e^{s_1 z}, e^{s_2 z}\}$ is

$$W(\xi) = (s_2 - s_1) e^{-\lambda \xi}. \tag{24}$$

Variation of parameters gives

$$A_p(z) = \frac{1}{s_2 - s_1} \sum_{j=1}^{4} k_j \left[\frac{e^{p_j z} - e^{s_1 z}}{p_j - s_1} - \frac{e^{p_j z} - e^{s_2 z}}{p_j - s_2} \right]. \tag{25}$$

This form satisfies two identities:

$$A_p(0) = 0, \qquad A_p'(0) = 0. \tag{26}$$

The complete solution is

$$A(z) = E_1 e^{s_1 z} + E_2 e^{s_2 z} + A_p(z), \tag{27}$$

with

$$E_1 = \frac{-A_p(1)}{e^{s_1} - e^{s_2}}, \qquad E_2 = \frac{A_p(1)}{e^{s_1} - e^{s_2}}. \tag{28}$$

The roots $s_{1,2}$ are distinct for $E \neq 0$. The denominators $p_j - s_{1,2}$ are nonzero provided $p_j \notin \{s_1, s_2\}$.

4. Engineering quantities

4.1. Skin friction

By (26), $A_p'(0) = 0$, so the complex wall shear is

$$\tau = \left.\frac{dA}{dz}\right|_{z=0} = E_1 s_1 + E_2 s_2, \qquad C_f = \sqrt{\tau_x^2 + \tau_y^2}. \tag{29}$$

4.2. Nusselt number

$$Nu = -\left.\frac{d\theta}{dz}\right|_{z=0} = -(D_1 m_1 + D_2 m_2 + \Theta_1 r_1 + \Theta_2 r_2). \tag{30}$$

4.3. Sherwood number

$$Sh = -\left.\frac{d\phi}{dz}\right|_{z=0} = -(C_1 r_1 + C_2 r_2). \tag{31}$$

5. Results and discussion

The baseline parameters are

$$Pr = 0.71,\ Sc = 0.22,\ N = 0.5,\ E = 2.5,\ Df = 0.5,\ K = 1.0,\ S = 1.5,\ \lambda = 0.5, \tag{32}$$

with isothermal conditions $(A_1, A_2, A_3) = (0, 1, 1)$. The value $Df = 0.5$ is a parametric choice for illustration.

Table 2: Baseline engineering quantities at $z = 0$ from (29)–(31). $Nu_{\text{isoflux}} = 1$ is fixed by construction of the constant-flux wall.

Quantity	Expression	Isothermal	Isoflux
τ_x	$\mathrm{Re}[A'(0)]$	+0.842	+0.710
τ_y	$\mathrm{Im}[A'(0)]$	+0.217	+0.010
C_f	$\sqrt{\tau_x^2 + \tau_y^2}$	+0.870	+0.710
Nu	$-\theta'(0)$	+0.630	+1.000
Sh	$-\phi'(0)$	+1.369	+1.369

Setting $E = Df = K = S = 0$ and $\lambda = 0$ recovers the textbook vertical-channel result: $\theta = \phi = 1 - z$, so $Nu = Sh = 1$; $u'' = -(1 + N)(1 - z)$ with $u'(0) = (1 + N)/3$; $v \equiv 0$ and $C_f = (1 + N)/3$.

Increasing E suppresses u and generates v (Figs. 2, 3), reducing C_f. Increasing Df raises θ and Nu while slightly reducing C_f (Figs. 4, 5). Increasing K sharpens the concentration layer and increases Sh (Figs. 6, 7). Suction ($\lambda > 0$) steepens $\theta'(0)$ and raises Nu (Figs. 8, 9). Higher Pr and Sc thin the respective boundary layers and increase Nu and Sh. Table 3 compares the present work with the four cited studies; none of them provides closed-form C_f, Nu, or Sh.

Table 3: Qualitative comparison with recent studies.

Study	Configuration	Method	Dufour	Chem. rxn	Source	Closed form
Lakshmikaanth *et al.* [10]	Vertical plate, Hall	Parametric	Yes	Yes	Yes	No
Gitte *et al.* [11]	Vertical channel	Perturbation	No	Yes	Yes	No
Raizah *et al.* [12]	Gyrating channel	Numerical	Yes	Yes	No	No
Crank–Nicolson [13]	Rotating channel	Numerical	Yes	No	No	No
Present work	Rotating vertical channel	Exact	Yes	Yes	Yes	Yes

6. Conclusions

The species equation does not feel the temperature; the energy equation is linear in θ once Dufour is known forcing; and the momentum equations collapse into one complex ODE. The system therefore integrates in sequence, without perturbation parameter or discretization. The wall quantities τ, Nu, and Sh are short closed-form expressions, verified against the classical limit and against an independent undetermined-coefficients derivation. Rotation transfers momentum out of the streamwise direction; Dufour diffusion raises wall heat flux; and the reaction rate sets the concentration-layer thickness.

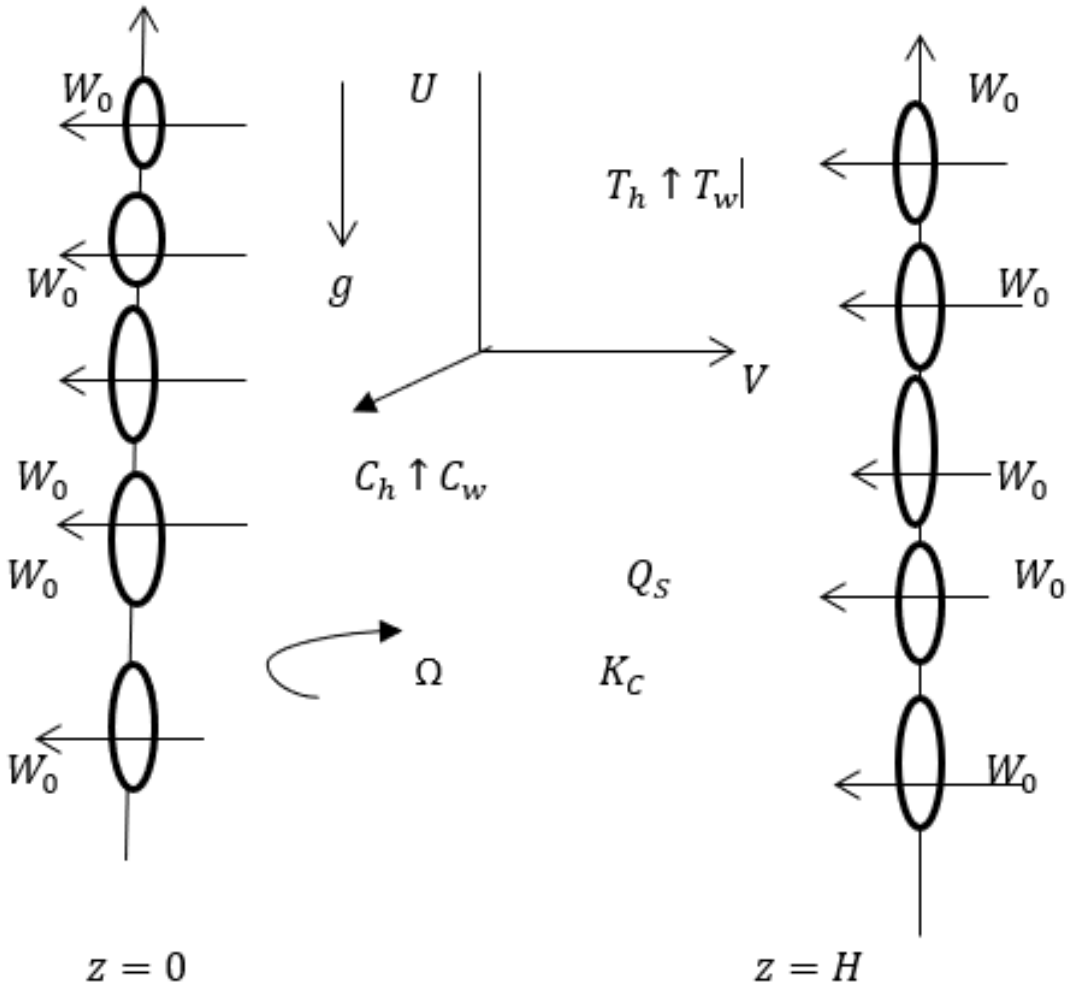


Figure.1 Geometry of the problem

Figure 1: Geometry of the problem. Plates separated by H, with $z = 0$ and $z = H$ denoting the walls. U is the vertical (buoyancy) direction, V the spanwise direction, Ω the rotation, W_0 the suction/injection, Q_S the heat source, K_C the reaction rate.

Appendix A. Numerical verification of the closed-form expressions

For the baseline $(\lambda, Sc, K) = (0.5, 0.22, 1.0)$:

$$\begin{aligned} \lambda Sc &= 0.11, \qquad (\lambda Sc)^2 + 4K = 4.0121, \\ r_1 &= 0.94651, \quad r_2 = -1.05651, \\ C_1 &= -0.15596, \quad C_2 = 1.15596, \\ \phi'(0) &= -1.36893, \quad Sh = 1.369. \end{aligned}$$

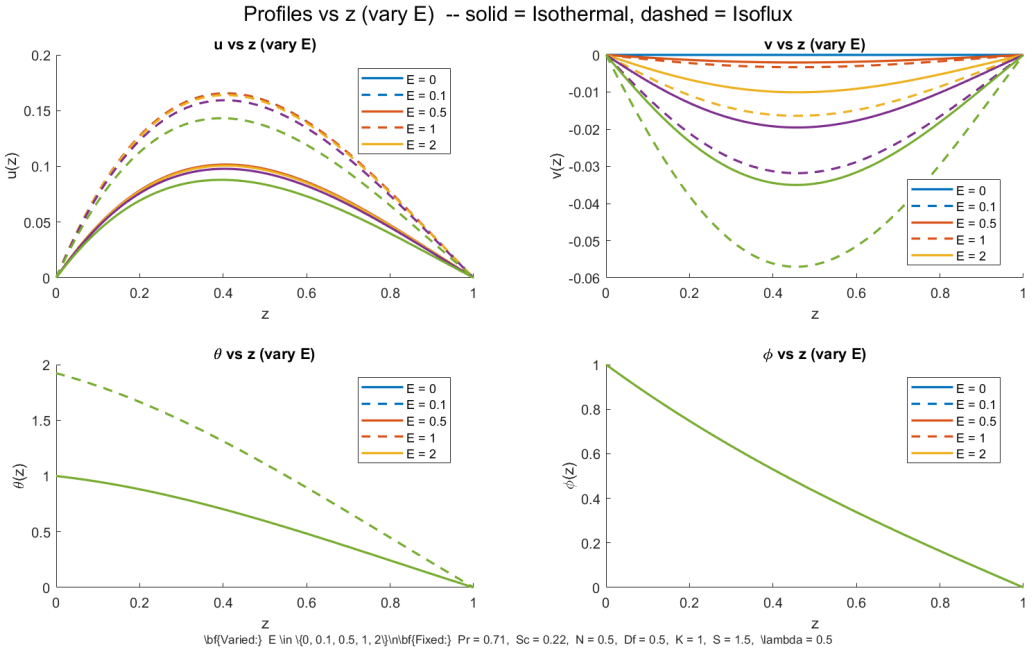


Figure 2: Profiles versus z for $E \in \{0, 0.1, 0.5, 1, 2\}$; solid = isothermal, dashed = isoflux.

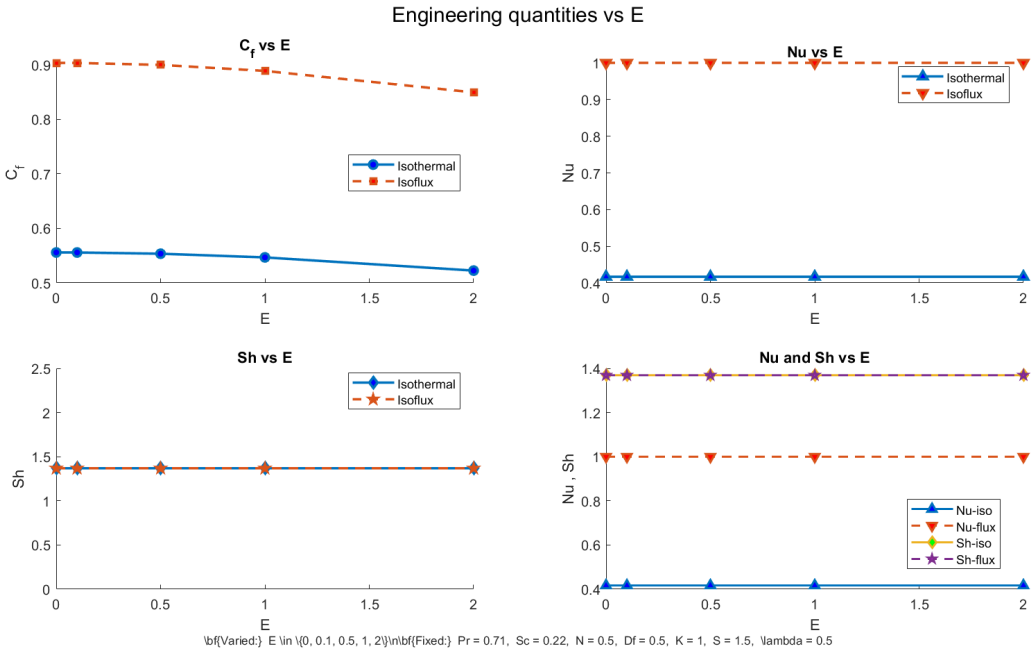


Figure 3: C_f, Nu, Sh versus E.

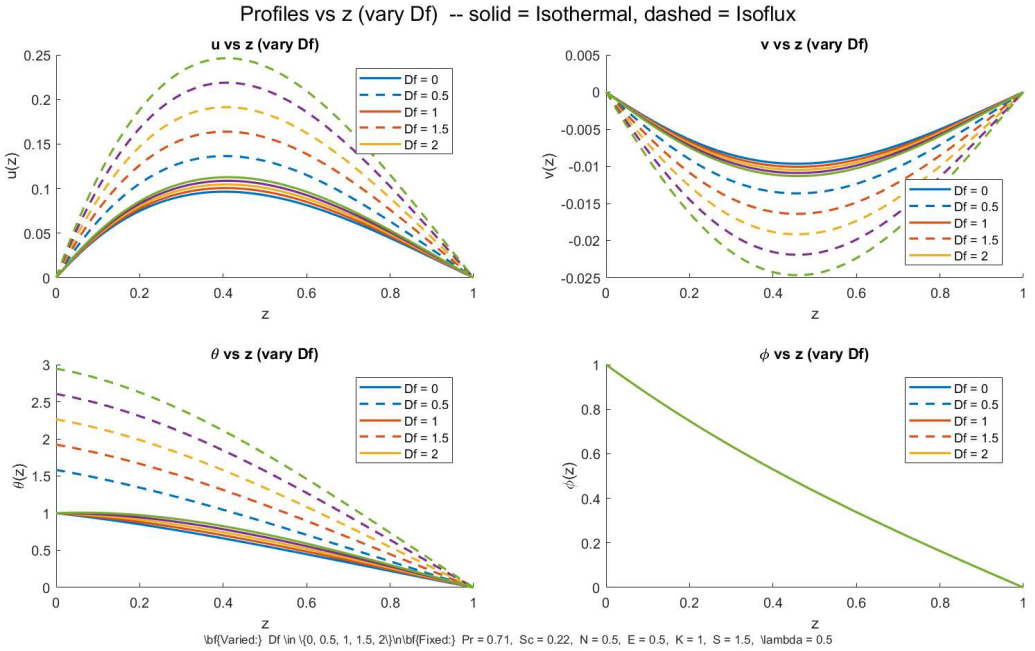


Figure 4: Profiles versus z for $Df \in \{0, 0.5, 1, 1.5, 2\}$.

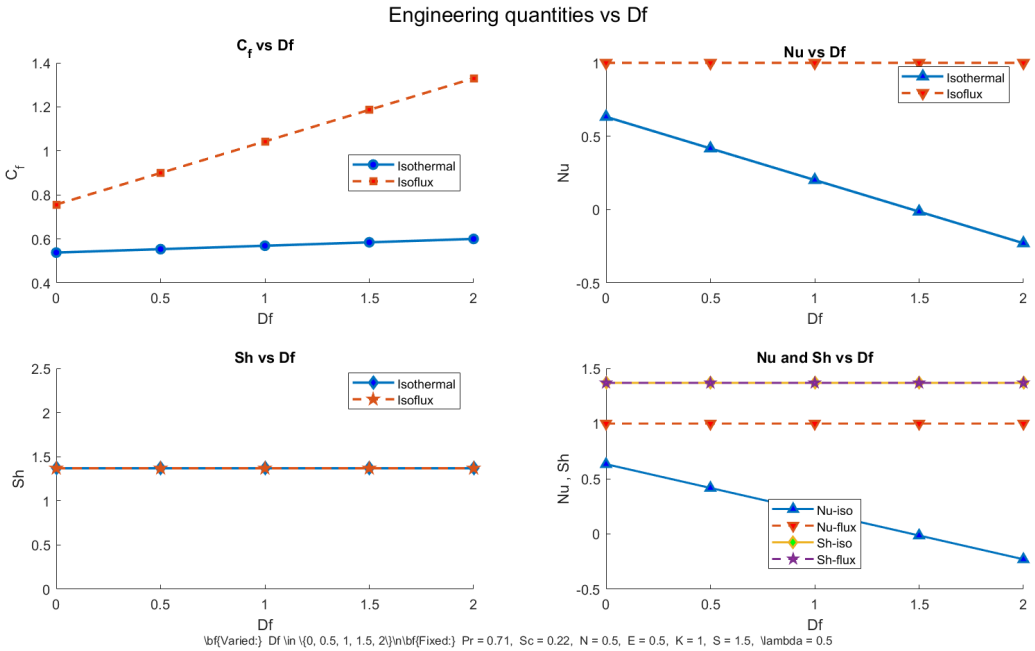


Figure 5: C_f, Nu, Sh versus Df.

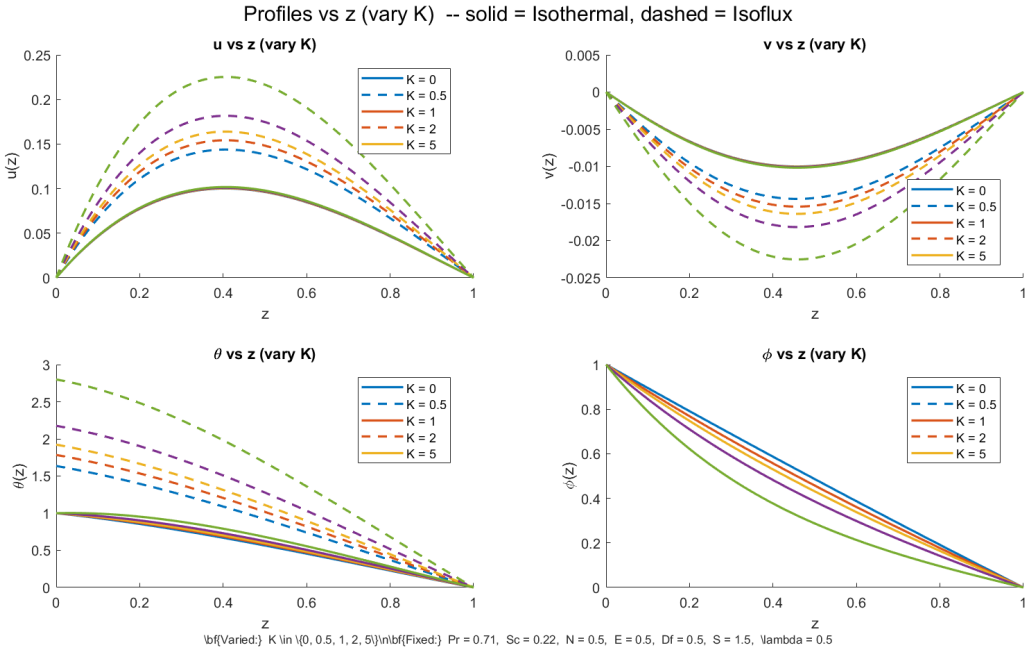


Figure 6: Profiles versus z for $K \in \{0, 0.5, 1, 2, 5\}$.

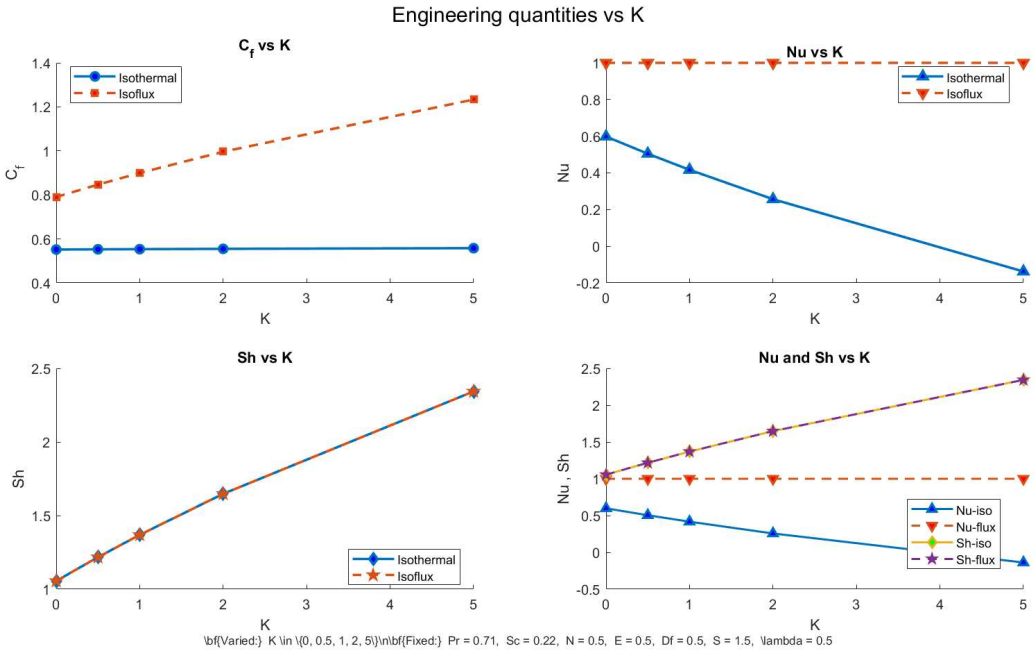


Figure 7: C_f, Nu, Sh versus K.

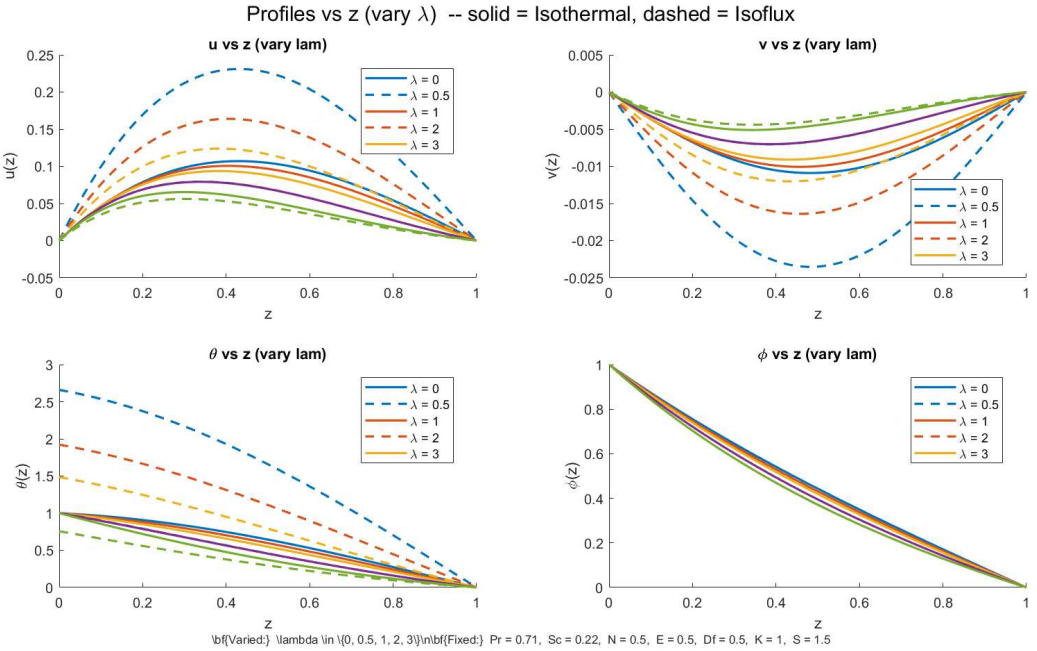


Figure 8: Profiles versus z for $\lambda \in \{0, 0.5, 1, 2, 3\}$.

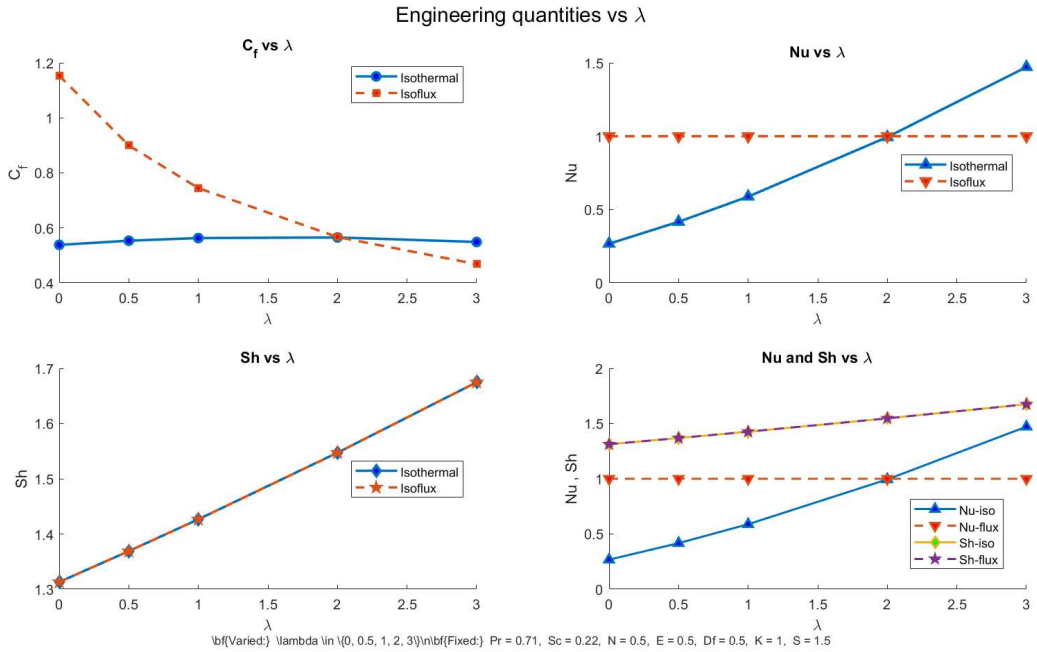


Figure 9: C_f, Nu, Sh versus λ.

Classical limit $E = Df = K = S = \lambda = 0$: $\phi = 1 - z$, $\theta = 1 - z$, $Nu = Sh = 1$; $u'' = -(1+N)(1-z)$ gives $u'(0) = (1+N)/3$, $v \equiv 0$, $C_f = (1+N)/3$; for $N = 0.5$, $C_f = 0.5$.

Appendix B. Independent derivation by undetermined coefficients

Taking $A_p^{\mathrm{UC}}(z) = \sum_{j=1}^{4} B_j e^{p_j z}$ with $B_j = -k_j/[(p_j - s_1)(p_j - s_2)]$, the boundary conditions give

$$E_1^{\mathrm{UC}} = \frac{-\sum_j B_j e^{p_j} + e^{s_2} \sum_j B_j}{e^{s_1} - e^{s_2}}, \quad E_2^{\mathrm{UC}} = \frac{\sum_j B_j e^{p_j} - e^{s_1} \sum_j B_j}{e^{s_1} - e^{s_2}}. \tag{B.1}$$

The resulting wall shear $\tau^{\mathrm{UC}} = E_1^{\mathrm{UC}} s_1 + E_2^{\mathrm{UC}} s_2 + \sum_j B_j p_j$ equals $\tau^{\mathrm{VOP}} = E_1^{\mathrm{VOP}} s_1 + E_2^{\mathrm{VOP}} s_2$ because the correction $c_1 s_1 + c_2 s_2 + \sum_j B_j p_j$ vanishes identically.

CRediT authorship contribution statement

Basant Kumar Jha: Conceptualization, Supervision, Writing – review & editing. **Auwalu Inusa Adamu:** Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing – review & editing. **Ado Balili:** Validation, Writing – review & editing. **Muhammad Yusuf Muhammad:** Software, Visualization, Writing – review & editing.

Declaration of competing interest

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Data availability

The MATLAB code is provided as supplementary material.

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