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Research Article

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Posted Date: November 20th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11612474/v1

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Optimization of Biogas Slurry Concentration in Winter Wheat–Summer Maize Rotation Using Bayesian Uncertainty Quantification and Multi-Objective Trade-Off Analysis

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Abstract

Background and Aims: Winter wheat (*Triticum aestivum* L.) and summer maize (*Zea mays* L.) may differ in their yield responses to a common biogas slurry concentration. We evaluated crop yields and slurry-input trade-offs to identify a candidate concentration for rotation-wide application.

Methods: A field experiment compared a water-only topdressing control, conventional chemical fertilizer topdressing, and four slurry concentrations (25%, 50%, 75%, and 100%), with three independent plots per treatment. Bayesian linear and quadratic models were evaluated by leave-one-concentration-out cross-validation. Slurry inputs, grain quality, and soil metal risk screening informed candidate selection.

Results: The 75% treatment (T3) produced observed mean yields of 9345.50 kg ha^{-1} for winter wheat and 11272.51 kg ha^{-1} for summer maize. Winter wheat yield did not differ significantly between T3 and T4, whereas the summer maize yield difference was significant under Fisher's LSD test (P = 0.046), but not under the Games–Howell procedure (P = 0.167). Rotation-wide slurry input under T3 was 25% lower than under T4. Cross-validation RMSE was 506.11 kg ha^{-1} for the winter wheat linear model and 712.81 kg ha^{-1} for the summer maize quadratic model. From T3 to T4, both wheat models predicted yield increases, whereas maize predictions differed in direction between models. All four slurry concentrations met the applicable soil metal risk screening criteria.

Conclusion: Under the experimental conditions, 75% slurry was the preferred candidate among the tested concentrations for balancing the yields of both crops and slurry inputs. Concentration selection should account for crop-specific responses and predictive uncertainty.

Keywords: Biogas slurry; Winter wheat–summer maize rotation; Bayesian modeling; Crop yield response; Nutrient inputs; Multi-objective optimization

1 Introduction

Anaerobic digestion recovers energy from livestock and poultry manure and produces biogas slurry that can supply water and nutrients to crops. Slurry nutrient composition and fertilizer effectiveness vary with feedstocks, treatment processes, application methods, and soil conditions. Application therefore needs to match nutrient supply with crop requirements. Möller and Müller (2012) reported that changes in nutrient forms during digestion affect digestate utilization, while Smit et al. (2026) found that its nitrogen fertilizer replacement value varies with nutrient characteristics and soil properties. Appropriate application rates are thus needed to balance crop production and resource inputs.

Winter wheat–summer maize double cropping is a predominant production system in the North China Plain. Sun et al. (2007) showed that sowing and harvesting dates affect rotation productivity, while Xin and Tao (2020) highlighted the joint demands of grain production and resource and environmental management. Differences between the two crops in growing periods, nutrient requirements, and fertilization practices complicate water and fertilizer allocation. At the same time, the annual winter wheat–summer maize double-cropping rotation provides opportunities for slurry utilization and nutrient cycling. Evaluating a common slurry concentration therefore requires consideration of yields and inputs in both growing seasons.

Previous studies on biogas slurry application in crop rotation systems have examined crop yields, nutrient utilization, and soil environmental effects. Tang et al. (2022) investigated the effects of biogas slurry application on crop yields and soil physicochemical properties under winter wheat–summer maize rotation in the North China Plain and found that changes in crop yields did not fully correspond to improvements in soil properties. Zhao et al. (2024) evaluated the effects of replacing chemical fertilizers with biogas slurry on crop yields and soil nitrogen losses in a wheat–silage maize rotation, showing that the outcomes depended on the fertilizer substitution ratio and experimental year. Other studies have evaluated biogas slurry topdressing in terms of water and nitrogen use efficiency, grain quality, and economic benefits (Liang et al. 2025; Wang et al. 2025), while also examining nutrient utilization and crop productivity under repeated slurry application and different crop rotation management practices (Nascimento et al. 2023; Winkhart et al. 2024;

Wu et al. 2026). Although previous studies have documented the effects of biogas slurry on crop production and soil properties, the yield–input trade-offs associated with applying a common slurry concentration across winter wheat and summer maize remain insufficiently quantified. In particular, it is unclear whether increasing slurry concentration provides consistent yield benefits for both crops within the rotation.

Crop yield responses to fertilizer inputs are often nonlinear, and comparisons based solely on treatment mean yields may not adequately characterize concentration-dependent responses or the benefits of additional inputs. In recent years, Bayesian statistical methods have increasingly been applied to fertilizer response modeling, providing a means of quantifying uncertainty in model parameters and predictions. Correndo et al. (2021) used a Bayesian approach to analyze maize yield responses to nitrogen inputs and their sources of uncertainty, highlighting the importance of predictive uncertainty in fertilization decisions. Palmero et al. (2026) applied Bayesian model averaging to compare the effects of different response functions on estimates of economically optimal nitrogen rates. Matavel et al. (2025) investigated how the design of fertilizer application gradients influences the precision of optimal application rate estimates using Bayesian experimental design. These findings highlight the influence of model structure and experimental data on fertilizer response estimates. Comparing alternative concentration–response models and evaluating their predictive performance through cross-validation can therefore help characterize uncertainty in the yield benefits of additional biogas slurry inputs (Roberts et al. 2017). In fertilization management, crop models, surrogate models, and multi-objective optimization methods have been applied to agricultural water and fertilizer allocation while considering multiple objectives related to resource utilization and environmental impacts (Cheng et al. 2023; Zhang and Yang 2023). For winter wheat–summer maize rotation systems, previous studies have also evaluated organic material inputs by jointly considering crop production, greenhouse gas emissions, and economic costs (Chen et al. 2025). El-Sharkawy et al. (2024) combined controlled-release fertilizer experiments with modeling in a wheat–maize cropping system and found that fertilization effects were influenced by soil conditions and fertilizer types. Zhang et al. (2024) reported that the combined application of organic amendments and chemical fertilizers improved soil quality in a long-term wheat–sweet potato rotation experiment, indicating that nutrient management in rotation systems should consider both crop production and changes in the soil environment. These studies provide methodological references for optimizing agricultural resource allocation. A rotation-wide fertilization strategy must account for the potentially different yield responses of winter

wheat and summer maize while balancing crop production against slurry inputs. Integrating crop-specific yield predictions, input requirements, and soil environmental screening provides a framework for evaluating common slurry concentrations and their associated trade-offs.

Our previous summer maize field study examined the effects of slurry topdressing concentration on yield, grain quality, and soil properties (Jiang et al. 2025). The present study extends that work to the winter wheat–summer maize rotation by integrating observations from both seasons with Bayesian concentration–response models and uncertainty analysis. It evaluates crop yields, slurry inputs, and soil metal risk screening under a common concentration, with emphasis on rotation-wide yield–input trade-offs.

Accordingly, this study adopts a management strategy in which the same volumetric concentration of biogas slurry is applied during both growing seasons, with three specific objectives: (1) to develop Bayesian concentration–response models for winter wheat and summer maize and evaluate their predictive performance and uncertainty; (2) to compare the yield benefits and input requirements of both crops across slurry concentrations by considering slurry inputs and soil metal risk screening; and (3) to identify candidate concentrations for common application in the rotation system and their applicable conditions through multi-objective trade-off analysis and continuous concentration–response assessment. Together, these analyses establish a quantitative framework for comparing common slurry concentrations in winter wheat–summer maize rotation systems.

2 Materials and Methods

2.1 Experimental Site and Design

The field experiment was conducted from October 2023 to October 2024 near the Wangdu Irrigation Experimental Station in Baoding, Hebei Province, China (38°42′ N, 115°6′ E). The experimental site has a warm temperate continental semi-arid monsoon climate and follows an annual winter wheat–summer maize double-cropping system. The winter wheat (*Triticum aestivum* L.) cultivar was Shimai 30, sown on October 15, 2023, and harvested on June 13, 2024. The summer maize (*Zea mays* L.) cultivar was Keen 702, sown on June 19, 2024, and harvested on October 3, 2024.

Six treatments were established: a water-only topdressing control (CK), conventional chemical fertilizer topdressing (CF), and four biogas slurry concentrations of 25% (T1), 50% (T2), 75% (T3), and 100% (T4). Each treatment was replicated three times, resulting in 18

experimental plots arranged randomly. Each plot covered 144 m² (12 m × 12 m), with a 1 m-wide buffer strip between adjacent plots. After winter wheat harvest, summer maize was planted in the same experimental plots, with treatment assignments maintained across both growing seasons. Observations from the two seasons were matched using the original plot identifiers. The present study used the treatment design and relevant observations from the previously published summer maize experiment (Jiang et al. 2025) and combined them with winter wheat data from the same plots to establish the dataset for analyzing and optimizing a common biogas slurry concentration in the winter wheat–summer maize rotation.

2.2 Fertilization Management and Biogas Slurry Inputs

At sowing, winter wheat received 750 kg ha^{-1} of compound fertilizer with a reported grade of 18:21:12 (N:P_2O_5:K_2O; total nutrient content ≥51%), while summer maize received 900 kg ha^{-1} of compound fertilizer with a reported grade of 28:6:6 (N:P_2O_5:K_2O; total nutrient content ≥40%). Winter wheat was topdressed on November 10, 2023, April 5, 2024, and April 28, 2024, while summer maize was topdressed on July 25 and August 12, 2024. All treatments received the same basal fertilizer application. The CK treatment received only water during topdressing, whereas the CF treatment received 105 kg ha^{-1} of urea per topdressing event, followed by water application. For the biogas slurry treatments, slurry was mixed with water at the designated concentrations and applied by sprinkler irrigation. The total liquid application volume was 60 m^3 ha^{-1} per topdressing event for all treatments. Other field management practices were kept consistent.

The biogas slurry was obtained from a local pig farm. Its total nitrogen, total phosphorus, total potassium, and ammonium nitrogen concentrations were 1032.26, 132.70, 812.28, and 77.12 mg L^{-1}, respectively, with a pH of 7.35. Slurry concentration was defined as the percentage of biogas slurry in the total volume of the slurry–water mixture. The same treatment concentration was maintained during the winter wheat and summer maize growing seasons. The volumes of biogas slurry and water applied under each treatment are presented in Table 1.

Table 1 Topdressing liquid inputs under different treatments in the winter wheat–summer maize rotation

Treatment	Wheat-season slurry (m^3 ha^{-1})	Wheat-season water (m^3 ha^{-1})	Maize-season slurry (m^3 ha^{-1})	Maize-season water (m^3 ha^{-1})	Total slurry input (m^3 ha^{-1})
CK	0	180	0	120	0
CF	0	180	0	120	0
T1	45	135	30	90	75
T2	90	90	60	60	150

Treatment	Wheat-season slurry (m^3 ha^{-1})	Wheat-season water (m^3 ha^{-1})	Maize-season slurry (m^3 ha^{-1})	Maize-season water (m^3 ha^{-1})	Total slurry input (m^3 ha^{-1})
T3	135	45	90	30	225
T4	180	0	120	0	300

To quantify nutrient inputs under different slurry concentration treatments, the total amounts of nitrogen, phosphorus, and potassium supplied by biogas slurry during the winter wheat season, summer maize season, and entire rotation cycle were calculated using the measured nutrient concentrations of the undiluted slurry and the corresponding volumes of slurry applied. Nutrient inputs were calculated as the product of the applied slurry volume and the mass concentration of each nutrient and were expressed in kg ha^{-1}. These calculations represent the total nutrient inputs supplied by biogas slurry, excluding the identical basal fertilizer inputs across treatments, rather than the amounts of nutrients available to or absorbed by the crops.

2.3 Crop Yield and Grain Quality Measurements

Grain yields of winter wheat and summer maize were determined by harvesting areas of 1 m^2 and 1.2 m^2, respectively, within each experimental plot. Grain yield was calculated after harvesting, threshing, drying, and weighing, and was expressed in kg ha^{-1}. Yield components were also measured, including the number of productive spikes, grains per spike, and thousand-grain weight for winter wheat, and grains per ear and hundred-grain weight for summer maize. Measured grain yields from independent plots in both growing seasons were used for yield evaluation and model analysis. Some of the summer maize yield and yield-component data were consistent with those reported in the previously published study (Jiang et al. 2025).

Winter wheat quality parameters included crude protein content, wet gluten content, sedimentation value, and flour water absorption. Samples from three independent plots were analyzed for each treatment. Crude protein content was determined according to GB 5009.5—2016, gluten content was determined by the manual washing method according to GB/T 5506.1—2008, and sedimentation value was determined using the Zeleny sedimentation test according to GB/T 21119—2007. Flour water absorption was measured using the farinograph method according to GB/T 14614—2019. Crude protein and wet gluten contents were used for Bayesian concentration–response modeling, whereas sedimentation value and flour water absorption were used for comparisons among treatments.

Summer maize grain quality parameters included crude starch, crude protein, and

crude fat contents. Samples from three independent plots were analyzed for each treatment, using data obtained from the previous field experiment (Jiang et al. 2025). Crude starch content was determined using the iodine colorimetric method, crude protein content using the semi-micro Kjeldahl method, and crude fat content using petroleum ether extraction. Grain quality results for each treatment were expressed as means and standard deviations.

Experimental data were organized using Microsoft Excel 2021. Treatment differences in grain yield were assessed using one-way ANOVA followed by Fisher's least significant difference (LSD) test at $P < 0.05$. For summer maize yield, Welch's ANOVA and Games–Howell multiple comparisons were additionally performed to assess the sensitivity of the results to variance heterogeneity. Statistical analyses were conducted using IBM SPSS Statistics 26.0. Graphs were prepared using Origin 2021.

2.4 Determination and Assessment of Soil Physicochemical Properties and Grain Elemental Composition

Soil samples were collected before the experiment and after the harvests of winter wheat and summer maize. Within each experimental plot, soil samples were collected from the 0–20 cm layer using a five-point composite sampling method, thoroughly mixed, and subsampled by quartering. The measured parameters included soil pH, total nitrogen, alkali-hydrolyzable nitrogen, available phosphorus, available potassium, organic matter, cation exchange capacity, and the concentrations of Fe, Mn, Ca, Mg, Cu, Zn, Pb, Cd, Cr, Ni, As, and Hg. Soil nutrient and metal concentrations after the harvests of winter wheat and summer maize were summarized as means and standard deviations for each treatment, with three independent plot replicates. Soil metal concentrations were screened against risk thresholds, and differences in Ni concentrations among treatments were further assessed for both post-harvest sampling periods.

For soil Ni concentrations, Welch's one-way analysis of variance (ANOVA) was performed separately after winter wheat and summer maize harvests to test for overall differences among the six treatment means. The resulting two P-values were adjusted using the Holm procedure. An additional six soil samples collected after summer maize harvest, one per treatment, were used to present soil metal concentrations for that sampling period and were excluded from treatment-level statistical analyses and environmental risk screening.

Soil concentrations of Cd, Hg, As, Pb, Cr, Cu, Ni, and Zn after winter wheat and summer maize harvests were screened according to the Soil Environmental Quality—Risk

Control Standard for Soil Contamination of Agricultural Land (GB 15618—2018) (Ministry of Ecology and Environment and State Administration for Market Regulation 2018). Risk screening thresholds were determined according to soil pH and agricultural land type. The maximum ratio of soil metal concentration to its corresponding screening threshold was used as the environmental assessment indicator. The indicators based on treatment means and individual plot measurements, $F_2^{(A)}$ and $F_2^{(B)}$, respectively, were calculated as follows:

$$
\begin{aligned}
F_2^{(\mathrm{A})}(c) &= \max_{\tau\in T,\, k\in K} \frac{\bar{x}_{c\tau k}}{S_k\left(\overline{\mathrm{pH}}_{c\tau}\right)} \\
F_2^{(\mathrm{B})}(c) &= \max_{\tau\in T,\, r=1,2,3,\, k\in K} \frac{x_{c\tau rk}}{S_k\left(\mathrm{pH}_{c\tau r}\right)} \\
C_{env} &= \left\{c\in C : F_2^{(\mathrm{A})}(c)\le 1\right\}
\end{aligned}
\tag{1}
$$

where c denotes the volumetric concentration of biogas slurry; $C=\{0.25,0.50,0.75,1.00\}$ is the set of experimental concentrations; $\tau\in T$ indexes the two sampling periods after winter wheat and summer maize harvests; and $k\in K$ indexes the eight metal elements Cd, Hg, As, Pb, Cr, Cu, Ni, and Zn. The variables $x_{c\tau rk}$ and $\bar{x}_{c\tau k}$ represent the measured metal concentration in replicate plot r and the corresponding treatment mean, respectively, both expressed in mg kg^{-1}. Individual plot pH and the corresponding treatment mean pH are denoted by $\mathrm{pH}_{c\tau r}$ and $\overline{\mathrm{pH}}_{c\tau}$, respectively. $S_k(\mathrm{pH})$ denotes the applicable risk screening threshold for element k at the specified soil pH, and $r=1,2,3$ identifies the independent plot replicates. $F_2^{(\mathrm{A})}(c)$ and $F_2^{(\mathrm{B})}(c)$ represent the maximum risk screening ratios calculated from treatment means and individual plot measurements, respectively. The condition $F_2^{(\mathrm{A})}(c)\le 1$ was used for environmental screening, whereas $F_2^{(\mathrm{B})}(c)$ was used for supplementary evaluation of individual plot measurements.

Eight wheat grain samples were submitted for third-party elemental analysis. Eleven elements were measured, including Cd, total As, Pb, Cr, total Hg, Fe, Zn, Ca, Mg, Mn, and Cu, according to GB 5009.268—2016. The concentrations of Cd, total As, Pb, Cr, and total Hg in wheat grain samples were compared with the corresponding reference limits specified in GB 2762—2022 (National Health Commission and State Administration for Market Regulation 2022), which was applicable during the 2024 experiment. The corresponding limits were also checked against GB 2762—2025 (National Health Commission and State Administration for Market Regulation 2025), which came into effect on September 2, 2026. The results were expressed in mg kg^{-1}, and nondetected elements

were recorded according to the original analytical reports. Grain contaminant concentrations were evaluated against the corresponding limits for the analyzed samples, while the remaining mineral elements were used to compare elemental concentrations among samples.

2.5 Development of Bayesian Concentration–Response Models

To investigate the effects of biogas slurry concentration on crop yield and grain quality, Bayesian linear and quadratic regression models were developed separately for winter wheat yield, summer maize yield, winter wheat crude protein content, and winter wheat wet gluten content. Each model was fitted using 12 independent plot observations from four biogas slurry concentration treatments (25%, 50%, 75%, and 100%). The water-only control (CK) and conventional chemical fertilization treatment (CF) were excluded from concentration–response model fitting. Linear and quadratic models were compared to characterize crop responses to different slurry concentrations and differences in model predictions.

To improve the stability of parameter estimation, slurry concentration and response variables were rescaled. The volumetric concentration of biogas slurry was denoted by c, with values of 0.25, 0.50, 0.75, and 1.00. The centered concentration variable and quadratic basis function were defined as follows:

$$z(c) = \frac{c - 0.625}{0.375}, \quad q(c) = z(c)^2 - \frac{5}{9} \tag{2}$$

The two concentration–response models were formulated as follows:

Linear model:

$$y_i^* \sim N\left(\alpha + \beta z_i, \sigma^2\right) \tag{3}$$

Quadratic model:

$$y_i^* \sim N\left(\alpha + \beta z_i + \gamma q_i, \sigma^2\right) \tag{4}$$

where y_i^* is the rescaled observation; c represents the volumetric concentration of biogas slurry; α, β and γ are the intercept, linear coefficient, and quadratic coefficient, respectively; and σ is the residual standard deviation. Each response variable was modeled separately, assuming constant residual variance across slurry concentrations.

The response variables were rescaled as $y_i^* = \frac{y_i - a}{b}$, where y_i is the original observation, and a and b are the location and scaling parameters, respectively. The predicted mean on the model scale was denoted by $\eta(c)$. For the winter wheat and summer maize yield

models, a = 0 and b = 1000. For the winter wheat crude protein model, a = 14 and b = 1, whereas for the wet gluten model, a = 30 and b = 3. Model predictions were transformed back to their original scales using $\mu(c) = a + b\eta(c)$. Grain yield was expressed in kg ha^{-1}, and grain quality contents were expressed as percentages.

Independent prior distributions were assigned to the model parameters. For the yield models, the priors were specified as follows:

$$\alpha \sim N\left(9, 3^2\right), \quad \beta \sim N\left(0, 2^2\right), \quad \gamma \sim N\left(0, 0.75^2\right), \quad \sigma \sim HN(1) \tag{5}$$

For the winter wheat crude protein and wet gluten models, the priors were specified as follows:

$$\alpha \sim N\left(0, 2^2\right), \quad \beta \sim N\left(0, 1^2\right), \quad \gamma \sim N\left(0, 0.5^2\right), \quad \sigma \sim HN(1) \tag{6}$$

where N and HN denote the normal and half-normal distributions, respectively. All parameter priors were mutually independent. The linear model did not include the quadratic coefficient γ. In the quadratic model, the zero-centered prior for the quadratic coefficient constrained excessively large curvature estimates under limited sample sizes without specifying the direction of curvature. The prior scales for the quadratic coefficient and residual standard deviation were further adjusted to evaluate their effects on model predictions and concentration–response estimates.

2.6 Bayesian Parameter Estimation and Model Validation

The posterior distributions of model parameters were estimated using Markov chain Monte Carlo (MCMC) methods. Regression coefficients were updated using Gibbs sampling, while the residual standard deviation was updated using slice sampling after logarithmic transformation. Four independent sampling chains were run for each model. For full-data fitting, each chain included 1,500 warm-up iterations followed by 3,000 retained samples. For cross-validation, each chain included 1,000 warm-up iterations followed by 2,000 retained samples. Sampling convergence was assessed using the rank-normalized potential scale reduction factor (R-hat) and effective sample size (ESS). Convergence was considered satisfactory when R-hat was below 1.01 and both bulk ESS and tail ESS were at least 400 (Vehtari et al. 2021).

Posterior samples were used to calculate predicted means and their 95% credible intervals at different biogas slurry concentrations. Residual variation was incorporated to construct 95% prediction intervals. Model fit was evaluated by comparing the means, standard deviations, extreme values, and residual sums of squares of simulated and

observed data.

Four-fold leave-one-concentration-out cross-validation was used to evaluate model predictive performance. In each fold, the three plot observations from one concentration treatment were used as the validation set, while the nine observations from the remaining three concentrations were used to train the model. Root mean square error (RMSE) and 95% prediction interval coverage were used to evaluate predictive accuracy and interval reliability. RMSE quantified the deviations between predicted and observed values, whereas prediction interval coverage represented the proportion of observed values falling within the 95% prediction intervals. Model validation was guided by the approaches described by Roberts et al. (2017) and Vehtari et al. (2017).

2.7 Biogas Slurry Concentration Optimization and Multi-Objective Decision-Making

To comprehensively evaluate crop yields and resource inputs at different biogas slurry concentrations, the four concentration treatments (25%, 50%, 75%, and 100%) were first screened for soil heavy metal risks according to Eq. (1), and application options satisfying the environmental criteria were identified. On this basis, the relationships between crop yields and slurry inputs were analyzed, with the objectives of maximizing the predicted yields of winter wheat and summer maize while minimizing biogas slurry inputs.

Based on the posterior predictions of the Bayesian models, yield changes between adjacent concentration treatments and their 95% credible intervals were calculated, together with the posterior probabilities of yield increases. Marginal yield return was defined as the change in predicted yield per unit of additional biogas slurry input and calculated as follows:

$$G_s\left(c_2,c_1\right)=\frac{\mu_s\left(c_2\right)-\mu_s\left(c_1\right)}{300\left(c_2-c_1\right)}$$
$$G_R\left(c_2,c_1\right)=\frac{\Delta\mu_w+\Delta\mu_m}{300\left(c_2-c_1\right)} \quad (7)$$

where c_1 and c_2 denote adjacent biogas slurry concentrations; $\mu_s(c)$ represents the predicted yield of crop s at concentration c, with s referring to winter wheat or summer maize; and $\Delta\mu w$ and $\Delta\mu m$ represent the predicted yield changes of winter wheat and summer maize, respectively. G_s and G_r represent the marginal yield returns for an individual crop and the combined two-season rotation, respectively, expressed in kg m^{-3}. The denominator represents the additional biogas slurry input over the rotation cycle,

expressed in m^3 ha^{-1}. The combined two-season value was used to evaluate the overall change in yield, while yield uncertainty was reported separately for each crop.

Pareto non-dominated sorting was used to compare concentration options satisfying the environmental criteria. The optimization objectives were to maximize the predicted yields of winter wheat and summer maize and minimize biogas slurry inputs. An option was considered to dominate another if it was no worse in all three objectives and strictly better in at least one. Non-dominated treatments were retained, and candidate solutions were further compared across different response models and prior settings. Grain quality indicators of winter wheat and summer maize were used to evaluate the applicability of the candidate options.

The predicted yield benefits of additional slurry inputs were further compared across models to determine whether increasing slurry concentration resulted in higher predicted yields for both crops. The four observed concentrations were arranged in ascending order, and an indicator of agreement in predicted yield response direction across models was defined as follows:

$$H_j = \mathbf{1}\left\{\mu_s^{(M)}\left(a_{j+1}\right) - \mu_s^{(M)}\left(a_j\right) > 0, \forall s, \forall M \in \mathrm{M}_s\right\} \tag{8}$$

where a_j and a_{j+1} denote adjacent biogas slurry concentrations; $\mu_s^{(M)}$ represents the predicted yield of crop s under a given response model and prior configuration; and $\mathbf{1}\{\ \}$ denotes the indicator function. When all models predicted yield increases for both crops, $H_j = \mathbf{1}$; otherwise, $H_j = 0$. Adjacent concentrations were compared sequentially from low to high, and the concentration immediately preceding the first occurrence of $H_j = 0$ was defined as the boundary of agreement in predicted yield benefits across models. This boundary was used to assist in identifying candidate biogas slurry concentrations.

To further characterize yield responses within the experimental concentration range, continuous concentration predictions were generated using the Bayesian models over the range of 25%–100%, at intervals of one percentage point. Predicted means and 95% credible intervals were calculated. The first derivatives of the response curves were used to analyze trends in predicted yield across concentrations, and the following set was defined:

$$\mathrm{C}_{\mathrm{mean}} = \left\{c \in [0.25,1] : \mu_s'(c)\big|_M \geq 0, \forall s, \forall M \in \mathrm{M}_s\right\} \tag{9}$$

where $\mu_s'(c)$ denotes the first derivative of the predicted mean yield of crop s with respect to biogas slurry concentration under model M. The set $\mathrm{C}_{\mathrm{mean}}$ contains

concentrations at which the predicted mean yields of both crops do not decrease with increasing concentration across all retained models. The posterior distributions of model parameters and different prior settings were used to evaluate the stability of continuous concentration responses and predicted yield trends, providing a basis for selecting biogas slurry application concentrations.

2.8 Yield Sensitivity Analysis under Biogas Slurry Concentration Deviations

Previous studies have shown that pipeline layout and fertilizer solution concentration affect fertigation uniformity (Tang et al. 2018), while fertilizer solution viscosity, pipe diameter, and flow rate influence mixing within pipelines (Zhang et al. 2022). To evaluate changes in predicted crop yields caused by deviations in biogas slurry concentration, hypothetical concentration deviations of ±1, ±2.5, ±5, and ±10 percentage points (pp) were considered. Yield responses under these scenarios were analyzed for different crops and models within the experimental concentration range of 25%–100%.

For each deviation range, the minimum yield predicted by the mean response curve and its change relative to the predicted yield at the target concentration were calculated to assess the maximum potential yield loss associated with concentration deviations. In addition, posterior samples from the Bayesian models were used to calculate the minimum predicted yield under each sampled parameter configuration. The resulting means and 95% credible intervals were summarized to quantify uncertainty in estimated yield losses. These two approaches characterized yield changes based on the model mean response and posterior parameter uncertainty, respectively.

For the two boundary concentrations of 25% and 100%, yield responses to increases and decreases in concentration were analyzed separately. The sensitivity of each candidate concentration to application deviations was compared across crops, response models, and prior settings.

3 Results

3.1 Yield and Quality Responses of Winter Wheat and Summer Maize to Biogas Slurry Concentration

Different topdressing treatments significantly affected the grain yields of both winter wheat and summer maize (Table 2). Compared with CK, CF and T1–T4 significantly increased the yields of both crops ($P < 0.05$), with CF increasing winter wheat and summer

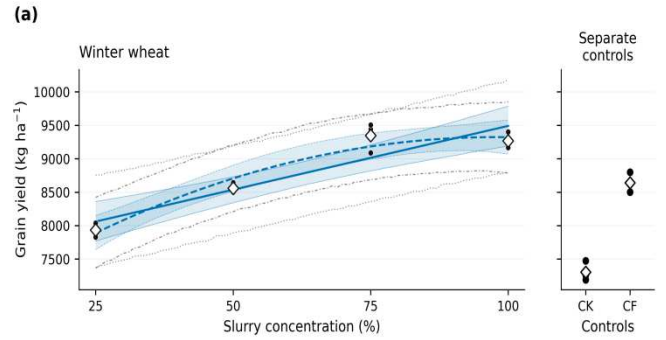
maize yields by 18.27% and 12.91%, respectively. Winter wheat yield initially increased and then decreased slightly with increasing biogas slurry concentration. Yields under T3 and T4 were significantly higher than those under the other treatments ($P < 0.05$), but did not differ significantly from each other. The yield under T2 did not differ significantly from that under CF, and both were significantly higher than that under T1. Summer maize yield also reached its highest value under T3, at 11272.51 kg ha^{-1}, representing increases of 22.52% and 8.51% compared with CK and CF, respectively ($P < 0.05$). According to Fisher's LSD test, the yield under T3 was significantly higher than those under T1, CF, and T4, but did not differ significantly from that under T2. The yield under T2 was also significantly higher than that under T1. The yield difference between T3 and T4 was significant under Fisher's LSD test ($P = 0.046$), but not under the Games–Howell procedure ($P = 0.167$). Overall, T3 produced the highest observed mean yields of winter wheat and summer maize, reaching 9345.50 and 11272.51 kg ha^{-1}, respectively, corresponding to increases of 27.92% and 22.52% compared with CK.

Table 2 Grain yields of winter wheat and summer maize under different topdressing treatments

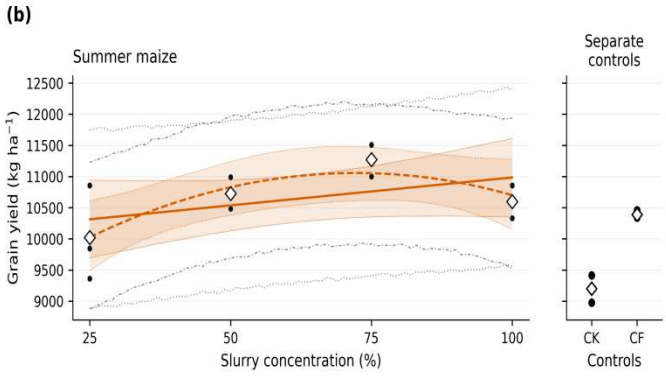
Treatment	Winter wheat grain yield (kg ha^{-1})	Summer maize grain yield (kg ha^{-1})
CK	7305.50 ± 148.60^{d}	9200.83 ± 219.52^{d}
CF	8640.50 ± 150.11^{b}	10388.83 ± 62.33^{bc}
T1	7933.00 ± 105.00^{c}	10021.55 ± 761.55^{c}
T2	8557.50 ± 76.54^{b}	10726.21 ± 253.72^{ab}
T3	9345.50 ± 225.97^{a}	11272.51 ± 256.18^{a}
T4	9266.50 ± 120.25^{a}	10597.16 ± 262.95^{bc}

Note: Values are presented as means ± standard deviations. Different lowercase letters within the same column indicate significant differences among treatments according to Fisher's least significant difference (LSD) test ($P < 0.05$).

Bayesian model predictions revealed differences in yield responses to biogas slurry concentration between winter wheat and summer maize (Fig. 1). For winter wheat, the predicted mean yields from both the linear and quadratic models increased with slurry concentration, reaching 9491.19 and 9321.97 kg ha^{-1} under T4, respectively. This differed from the observed yield pattern, in which the highest mean yield occurred under T3. For summer maize, the linear model also predicted a continuous increase in yield, whereas the quadratic model predicted the highest yield of 11053.38 kg ha^{-1} under T3, followed by a decrease to 10700.65 kg ha^{-1} under T4. Thus, the models consistently predicted increasing winter wheat yields at higher slurry concentrations, whereas summer maize showed model-dependent responses, with a declining yield predicted by the quadratic model beyond T3.



(a) Winter wheat



(b) Summer maize

Fig. 1 Bayesian yield responses of winter wheat and summer maize to biogas slurry concentration

Note: Points represent individual plot observations, and diamonds indicate treatment means. Solid and dashed lines represent linear and quadratic model predictions, respectively. Shaded bands indicate 95% credible intervals, whereas gray dotted and dash-dotted lines indicate 95% prediction interval bounds. CK and CF were excluded from model fitting.

Grain quality responses to biogas slurry concentration differed between the two crops. Crude protein and wet gluten contents of winter wheat generally increased with slurry concentration, reaching 14.80% and 31.73%, respectively, under T4 (Fig. 2a,b). Winter wheat sedimentation value and flour water absorption were also highest under T4, reaching 40.46 mL and 58.20%, respectively (Table 3). Compared with CF, the sedimentation value increased significantly by 6.3% ($P < 0.05$), whereas flour water absorption did not differ significantly. For summer maize, crude starch and crude protein contents were highest under T3, reaching 68.89% and 7.91%, respectively, while crude fat content was highest under T2, at 3.09% (Fig. 2c–e). The highest values of the measured grain quality indicators therefore occurred at different slurry concentrations in winter wheat and summer maize.

Table 3 Sedimentation value and flour water absorption of winter wheat under different topdressing treatments

Treatment	Sedimentation value (mL)	Flour water absorption (%)
CK	35.54 ± 0.62^{c}	55.90 ± 0.36^{d}
CF	38.06 ± 1.14^{b}	57.67 ± 0.25^{abc}
T1	37.60 ± 1.00^{b}	57.53 ± 0.38^{bc}
T2	37.46 ± 0.80^{b}	57.17 ± 0.32^{c}
T3	39.06 ± 0.80^{ab}	57.80 ± 0.30^{ab}

Treatment	Sedimentation value (mL)	Flour water absorption (%)
T4	40.46 ± 0.80[a]	58.20 ± 0.30[a]

Note: Values are presented as means ± standard deviations. Treatments sharing no lowercase letters within the same column differ significantly according to Fisher's LSD test ($P < 0.05$).

3.2 Bayesian Model Predictive Performance and Uncertainty Analysis

Posterior sampling for all Bayesian models met the predefined convergence criteria. For the yield models, the maximum R-hat was 1.00183, while the minimum bulk ESS and tail ESS were 2885 and 2748, respectively. The corresponding values for the winter wheat quality models were 1.00182, 2722, and 3202, respectively, indicating satisfactory convergence for posterior parameter estimation.

Four-fold leave-one-concentration-out cross-validation revealed differences in predictive performance between the linear and quadratic models for the two crops (Table 4). For winter wheat, the linear model achieved a root mean square error (RMSE) of 506.11 kg ha^{-1}, lower than the 672.20 kg ha^{-1} obtained with the quadratic model. However, its 95% prediction interval coverage was only 58.3%, compared with 75.0% for the quadratic model. For summer maize, the quadratic model achieved an RMSE of 712.81 kg ha^{-1}, representing a 25.4% reduction compared with the linear model, while prediction interval coverage increased from 83.3% to 100%. The quadratic model showed lower cross-validation RMSE and higher prediction interval coverage for summer maize, whereas the two winter wheat models differed in predictive accuracy and interval coverage.

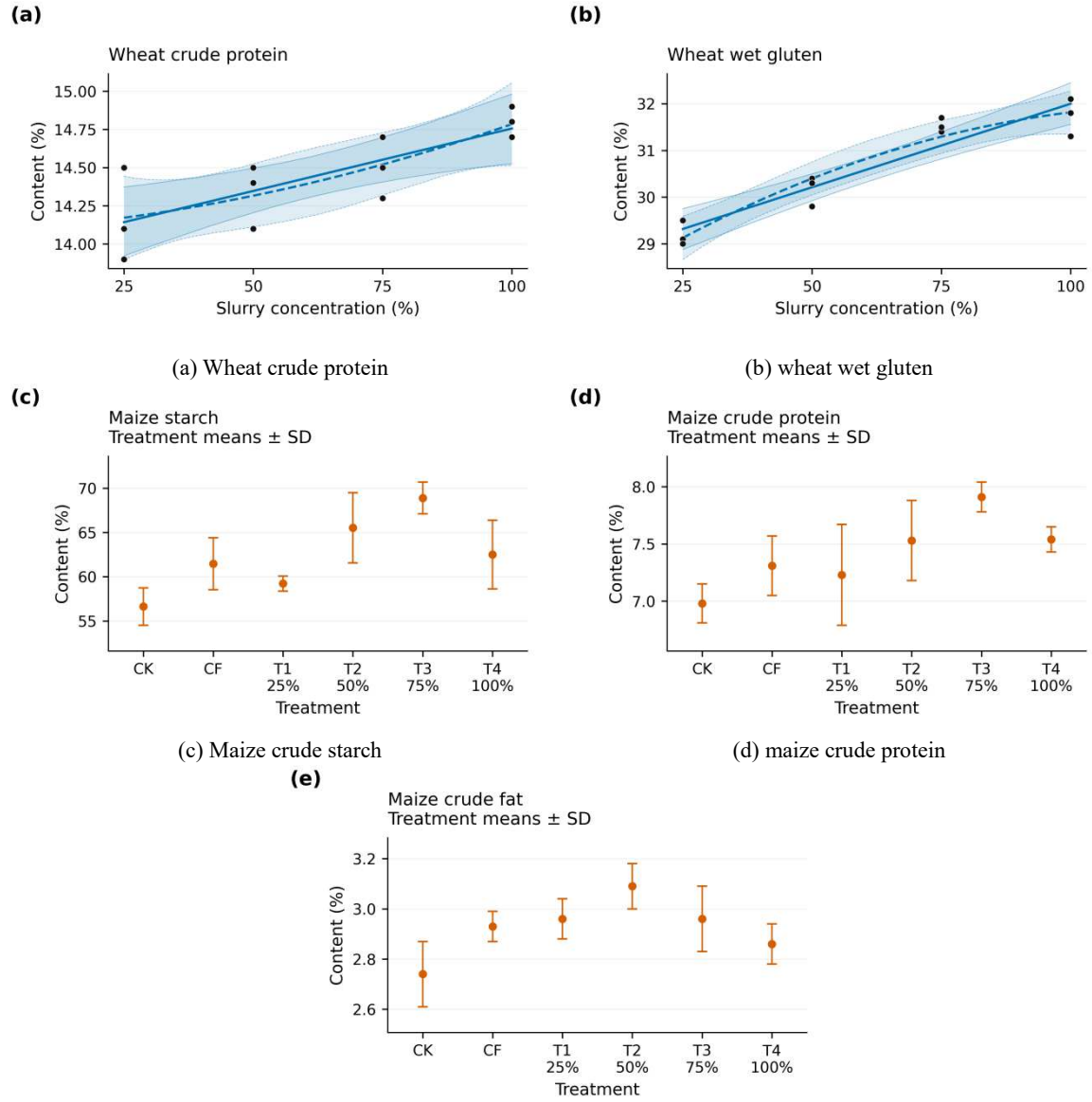
Table 4 Cross-validation results for winter wheat and summer maize yield and wheat grain quality models

Crop	Response variable	Model	RMSE	95% prediction interval coverage
Winter wheat	Grain yield	Linear	506.11	58.3%
Winter wheat	Grain yield	Quadratic	672.20	75.0%
Summer maize	Grain yield	Linear	955.65	83.3%
Summer maize	Grain yield	Quadratic	712.81	100.0%
Winter wheat	Crude protein content	Linear	0.199	100.0%
Winter wheat	Crude protein content	Quadratic	0.203	100.0%
Winter wheat	Wet gluten content	Linear	0.626	91.7%
Winter wheat	Wet gluten content	Quadratic	1.022	91.7%

Note: RMSE is expressed in kg ha^{-1} for grain yield and percentage points for grain quality. Prediction interval coverage represents the proportion of validation observations falling within the 95% prediction intervals.

Cross-validation results for winter wheat grain quality showed that the linear models for crude protein and wet gluten contents achieved RMSE values of 0.199 and 0.626 percentage points, respectively, both lower than the corresponding quadratic model values of 0.203 and 1.022 percentage points (Table 4). The 95% prediction interval coverage rates for crude protein and wet gluten were 100% and 91.7%, respectively, for both model types.

Figure 2 presents the Bayesian concentration responses of winter wheat quality indicators and the differences in summer maize grain quality among treatments.



(a) Wheat crude protein

(b) wheat wet gluten

(c) Maize crude starch

(d) maize crude protein

(e) maize crude fat

Fig. 2 Bayesian responses of winter wheat grain quality and treatment differences in summer maize grain quality

Note: In panels (a) and (b), solid and dashed lines represent the predicted means of the Bayesian linear and quadratic models, respectively, and shaded areas indicate 95% credible intervals for the predicted means. In panels (c)–(e), points represent observed treatment means, and error bars indicate standard deviations.

Prior sensitivity analysis showed that the maximum changes in predicted mean yield across different prior settings were 2.5 and 22.1 kg ha^{-1} for the linear and quadratic winter wheat models, respectively, and 4.8 and 109.6 kg ha^{-1} for the corresponding summer maize models. These results indicate that the quadratic models were more sensitive to prior settings. Based on the cross-validation and prior sensitivity results, both linear and quadratic models were retained for subsequent concentration optimization to evaluate the

influence of model structure on yield predictions and slurry concentration selection.

3.3 Biogas Slurry Inputs and Marginal Yield Returns under a Common Application Concentration in the Rotation System

Under the winter wheat–summer maize rotation system, each 25-percentage-point increase in biogas slurry concentration resulted in an additional 75 m^3 ha^{-1} of undiluted slurry input over the rotation cycle. Based on the measured nutrient concentrations of the undiluted slurry, the total nitrogen, phosphorus, and potassium inputs supplied by slurry under T3 were 232.26, 29.86, and 182.76 kg ha^{-1}, respectively, compared with 309.68, 39.81, and 243.68 kg ha^{-1} under T4. Increasing the concentration from T3 to T4 therefore increased the total nitrogen, phosphorus, and potassium inputs from slurry by 77.42, 9.95, and 60.92 kg ha^{-1}, respectively. Across different model combinations, the combined predicted yield increments of the two crops ranged from 700.46 to 1619.77 kg ha^{-1} from T1 to T2 and from 700.46 to 702.67 kg ha^{-1} from T2 to T3 (Fig. 3). All models predicted positive yield increments for both crops across these two transitions. However, the 95% credible intervals for some summer maize yield differences included zero, indicating uncertainty in the magnitude of yield increases.

When slurry concentration increased from T3 to T4, the linear and quadratic models predicted winter wheat yield increases of 476.94 and 138.30 kg ha^{-1}, respectively. In contrast, the linear model predicted a summer maize yield increase of 223.51 kg ha^{-1}, whereas the quadratic model predicted a decrease of 352.73 kg ha^{-1}. Across the four model combinations, the combined predicted yield increment of the two crops ranged from −214.43 to 700.46 kg ha^{-1} (Table 5), corresponding to marginal yield returns of −2.86 to 9.34 kg m^{-3} per unit of additional slurry input (Fig. 3). The combination of two linear models produced the highest marginal yield return, whereas the combination of two quadratic models resulted in a negative return.

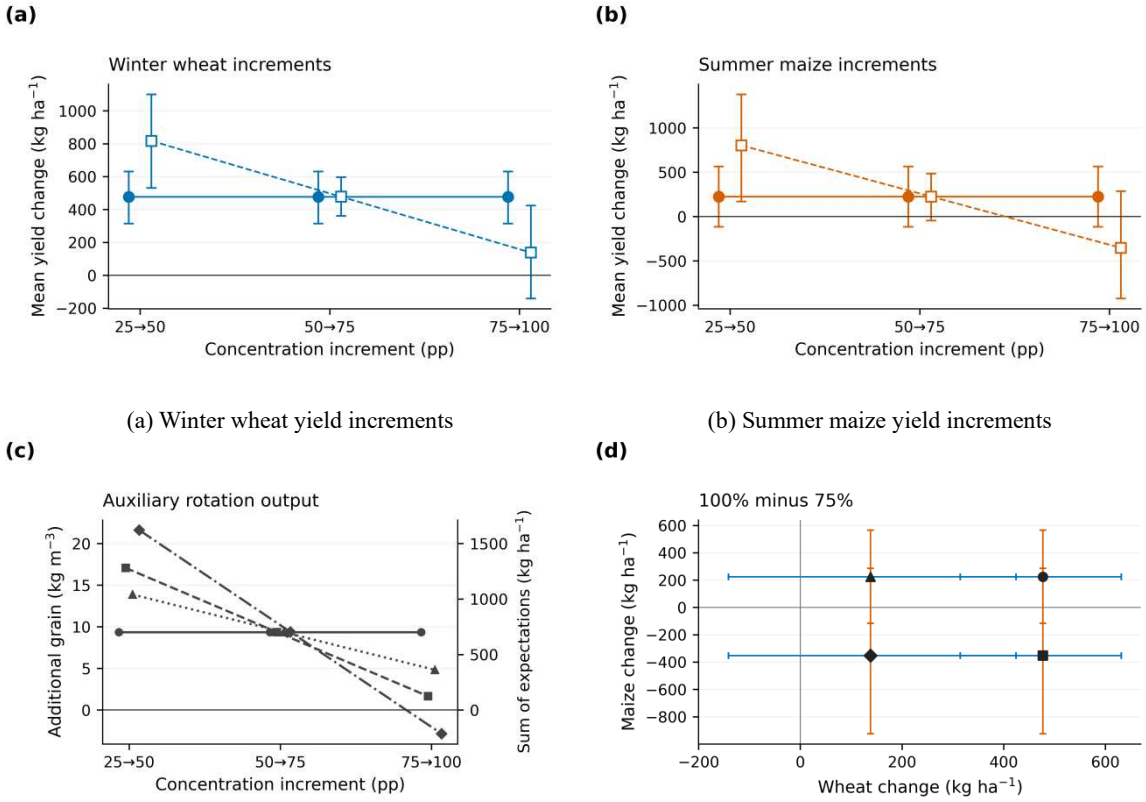
Table 5 Predicted yield increments and marginal returns from T3 to T4

Winter wheat model	Summer maize model	Wheat yield increment (kg ha^{-1})	Maize yield increment (kg ha^{-1})	Combined yield increment (kg ha^{-1})	Marginal yield return (kg m^{-3})
Linear	Linear	476.94	223.51	700.46	9.34
Linear	Quadratic	476.94	-352.73	124.22	1.66
Quadratic	Linear	138.30	223.51	361.81	4.82
Quadratic	Quadratic	138.30	-352.73	-214.43	-2.86

Note: The increase in undiluted biogas slurry input over the rotation cycle from T3 to T4 was 75 m^3 ha^{-1}. Marginal yield returns are expressed in kg m^{-3}.

Compared with the T1-to-T3 concentration transitions, the predicted yield benefits

from T3 to T4 were more sensitive to model structure. Higher slurry inputs could still increase winter wheat yield, whereas the direction of the summer maize yield response differed between models. These results indicate that further increasing the common slurry application concentration may not simultaneously improve the yields of both crops.



(a) Winter wheat yield increments

(b) Summer maize yield increments

(c) Marginal yield returns over the two-season rotation

(d) Yield responses of both crops from T3 to T4

Fig. 3 Predicted yield increments and marginal returns across slurry concentrations

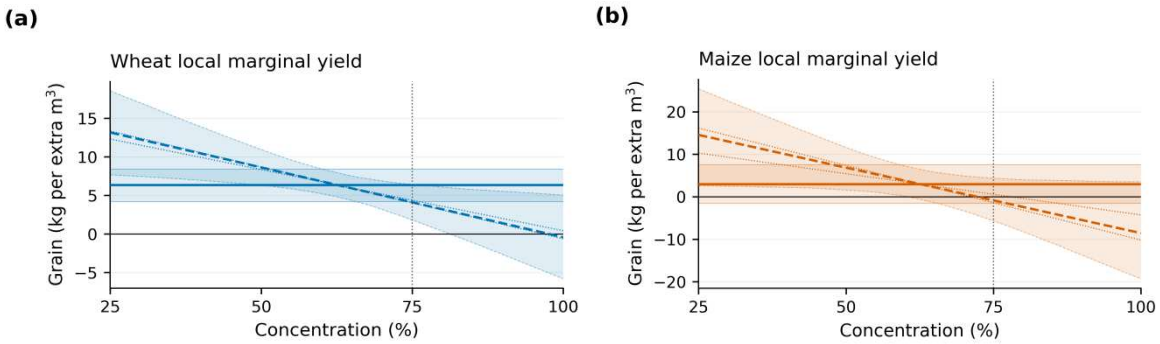
Note: L and Q denote the linear and quadratic models, respectively. Model combinations are ordered as winter wheat followed by summer maize. Error bars in panels (a), (b), and (d) represent the 95% credible intervals of the corresponding crop yield increments. Panel (c) presents rotation-wide marginal yield returns on the left axis (kg m^{-3}) and combined predicted yield increments on the right axis (kg ha^{-1}) under different model combinations

3.4 Candidate Biogas Slurry Concentrations and Continuous Response Analysis

The predicted yield changes from T3 to T4 differed substantially between models (Fig. 3d). For winter wheat, the posterior probabilities of yield increases were 99.98% and 85.04% under the linear and quadratic models, respectively, although the 95% credible interval of the latter included zero. For summer maize, the 95% credible intervals of the predicted yield differences included zero under both models, with the quadratic model indicating an 88.32% probability of yield reduction. These results indicate that further increasing biogas slurry concentration may increase winter wheat yield, whereas the predicted yield response of summer maize is subject to considerable model uncertainty.

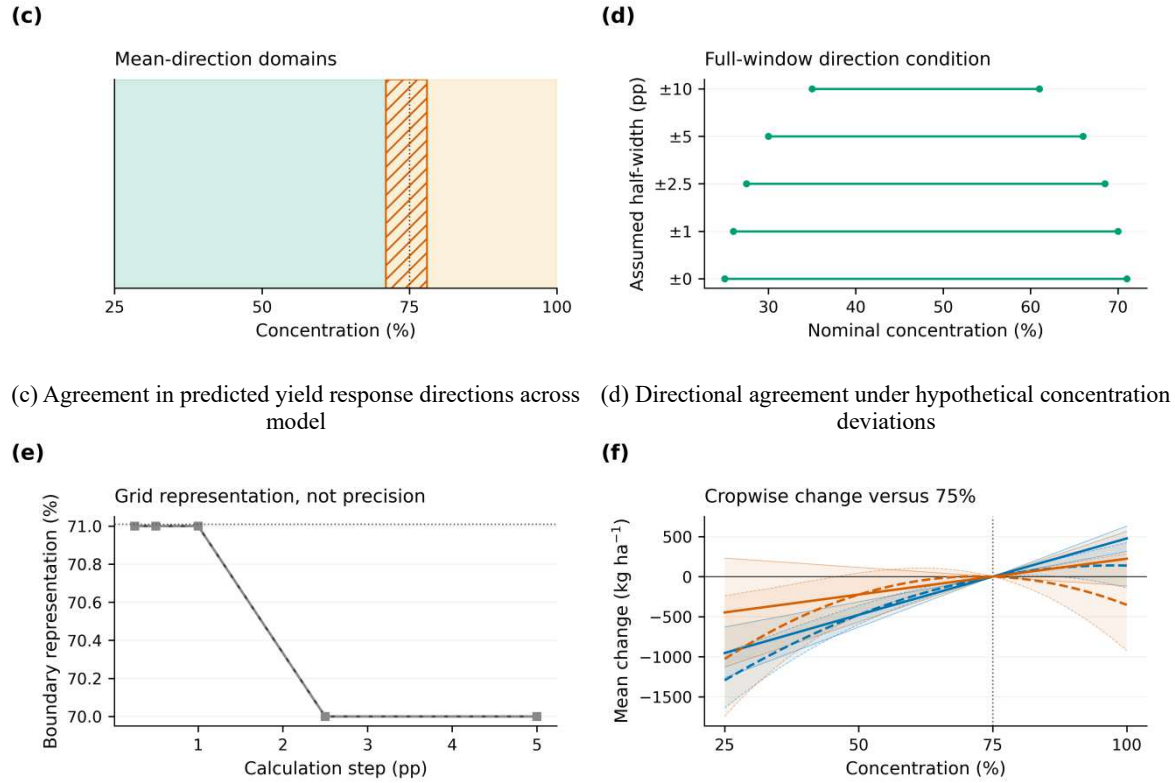
Soil metal risk screening showed that all four slurry concentration treatments (T1–T4) met the applicable screening criteria. Across 28 model and prior configurations, all four observed concentrations were Pareto non-dominated when maximizing the predicted yields of both crops and minimizing slurry inputs. The non-dominated set remained unchanged when the soil metal screening ratio was included as an additional objective to be minimized. The candidate set remained unchanged after including the minimization of soil metal risk ratios as an additional objective. Predicted mean yields increased from T1 to T3 for both crops across all retained models, whereas the direction of yield change from T3 to T4 varied among models. T3 produced the highest observed mean yields of both crops while requiring 25% less slurry than T4, supporting its selection as the preferred concentration among the four treatments evaluated.

Continuous concentration analysis further revealed model differences in the higher concentration range (Fig. 4). Within the range of 25%–71%, the retained models generally agreed on the directions of changes in predicted mean yields for both crops. Beyond approximately 71%, some quadratic models for summer maize predicted declining yields with increasing slurry concentration. Across different prior settings, the turning points of the predicted mean response curves for summer maize under the quadratic models ranged from approximately 71% to 78%, with a value of 72.2% for the baseline model. In contrast, the linear models continued to predict increasing yields with concentration, indicating that marginal yield responses at higher slurry concentrations were strongly influenced by model structure.



(a) Winter wheat marginal yield response

(b) Summer maize marginal yield response



(c) Agreement in predicted yield response directions across model

(d) Directional agreement under hypothetical concentration deviations

(e) Sensitivity of boundary estimates to calculation step size

(f) Predicted yield changes relative to T3

Fig. 4 Marginal yield responses and model differences across continuous biogas slurry concentrations

Note: Shaded bands in (a), (b), and (f) indicate 95% credible intervals.

At the concentration corresponding to T3, the marginal yield returns per unit of additional biogas slurry input were 6.36 and 4.11 kg m^{-3} for winter wheat under the linear and quadratic models, respectively, and 2.98 and −0.85 kg m^{-3} for summer maize (Fig. 4a,b). The approximately 71% boundary marked a change in agreement among model-predicted yield responses rather than a yield optimum. At 71%, predicted winter wheat yields were 76.31 and 53.61 kg ha^{-1} lower than at T3 under the linear and quadratic models, respectively, while rotation-wide slurry input decreased by 12 m^3 ha^{-1}. Together with the concentration deviation analysis, these results favor T3 among the experimentally evaluated concentrations.

3.5 Sensitivity of Yield Responses to Biogas Slurry Concentration Deviations

The hypothetical concentration deviation analysis showed that the sensitivity of predicted yields to concentration changes varied among crops and response models. At a target concentration of 75% with a deviation of ±5 percentage points, the maximum predicted mean yield losses for winter wheat were 95.4 and 68.4 kg ha^{-1} under the linear and quadratic models, respectively. Under the same conditions, the maximum yield loss

predicted by the quadratic model for summer maize was 24.3 kg ha^{-1}, lower than the 114.4 kg ha^{-1} predicted at a target concentration of 50%. These results indicate that predicted yield losses depend on the target concentration.

After accounting for posterior parameter uncertainty, the posterior mean of the maximum yield loss predicted by the quadratic summer maize model at 75% concentration was 43.1 kg ha^{-1}, with an upper 95% credible interval bound of 105.9 kg ha^{-1}, exceeding the 24.3 kg ha^{-1} obtained from the predicted mean response curve alone. As the hypothetical concentration deviation increased from ±5 to ±10 percentage points, the upper boundary of the nominal concentration range within which all retained models agreed on the direction of predicted yield changes decreased from approximately 66% to 61% (Fig. 4d). These findings indicate that concentration deviations affect the assessment of yield benefits across continuous concentration levels, and that slurry concentration selection should consider both model responses and the accuracy of concentration control.

3.6 Grain Quality and Soil Environmental Assessment

Grain quality responses differed between winter wheat and summer maize under T3 and T4. Compared with T3, the linear models predicted increases of 0.204 and 0.894 percentage points in winter wheat crude protein and wet gluten contents, respectively, under T4, with neither 95% credible interval including zero. The corresponding increases predicted by the quadratic models were 0.264 and 0.521 percentage points, respectively, although both credible intervals included zero. In contrast, the observed mean crude starch, crude protein, and crude fat contents of summer maize under T4 were 6.39, 0.37, and 0.10 percentage points lower than those under T3, respectively, indicating differences in grain quality responses between the two crops at higher slurry concentrations.

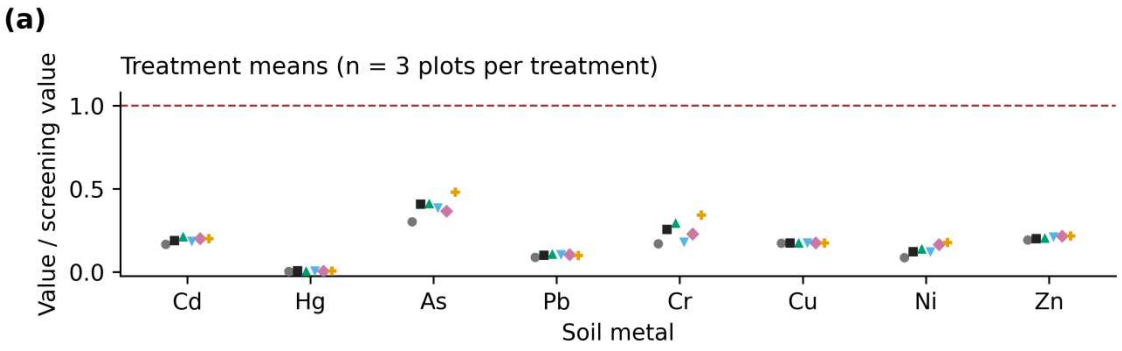
Elemental analysis of eight wheat grain samples showed that the highest quantified concentrations of Cd and total As were 0.0382 and 0.0290 mg kg^{-1}, respectively, while the highest quantified Pb concentration was 0.0770 mg kg^{-1}. All three values were below their corresponding reference limits. Neither Cr nor total Hg was detected (Table 6). Treatment-level elemental results showed that Ca, Mg, Mn, and Cu concentrations were higher under T4 than under T3, whereas Zn and Fe concentrations were slightly lower. Cd and total As concentrations increased from 0.0233 and 0.0263 mg kg^{-1} under T3 to 0.0237 and 0.0277 mg kg^{-1} under T4, respectively.

Table 6 Contaminant concentrations and reference limits in wheat grain samples

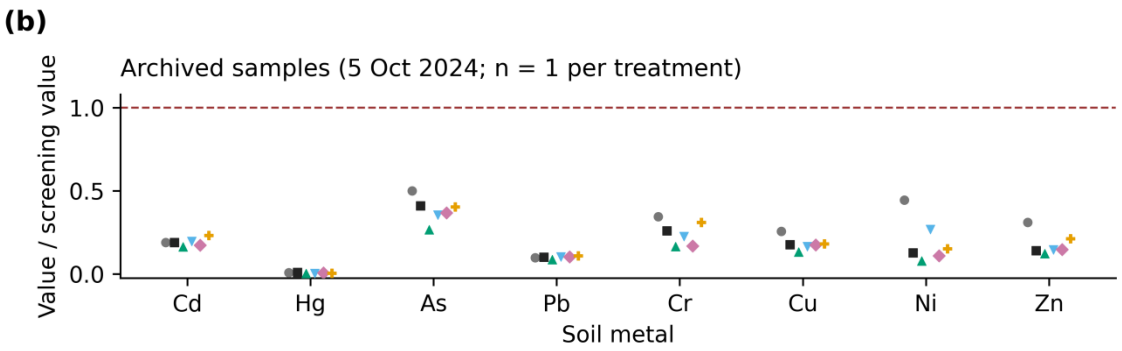
Contaminant	Quantified / nondetected samples	Maximum quantified concentration	Reported limit of quantification	Reference limit	Maximum quantified concentration / reference limit
Cd	8/0	0.0382	0.005	0.1	0.382
Total As	6/2	0.029	0.010	0.5	0.058
Pb	2/6	0.0770	0.05	0.2	0.385
Cr	0/8	Not detected	0.2	1.0	—
Total Hg	0/8	Not detected	0.003	0.02	—

Note: Concentrations are expressed in mg kg^{-1}. A total of eight samples were analyzed. The dash (—) indicates that no quantified result was available.

Soil nutrient and metal concentrations exhibited different patterns across biogas slurry concentrations. After summer maize harvest, alkali-hydrolyzable nitrogen and organic matter contents under T4 reached 126.98 mg kg^{-1} and 20.97 g kg^{-1}, respectively. Soil metal risk screening showed that the $F_2^{(A)}$ values for T1–T4 were 0.412, 0.408, 0.401, and 0.480, respectively, while the corresponding $F_2^{(B)}$ values were 0.418, 0.411, 0.407, and 0.500. All values were below 1, and As contributed the maximum screening ratio under both assessment methods. Soil concentrations of Cd, Hg, As, Pb, Cr, Cu, Ni, and Zn after both crop harvests were below the corresponding risk screening values specified in GB 15618—2018 (Fig. 5).



(a) Mean soil metal concentrations under different treatments after summer maize harvest



(b) Metal concentrations in individual soil samples collected during the same period

Fig. 5 Soil metal concentrations relative to risk screening values under different slurry treatments

Note: The vertical axis represents the ratio of soil metal concentration to the corresponding risk screening value. The

dashed line indicates the threshold of 1. Panel (a) shows treatment means from three independent plots; panel (b) shows results from additional samples collected during the same period, one sample per treatment, which were not used for treatment-level statistical inference

Welch's one-way ANOVA revealed significant overall differences in soil Ni concentrations among the six treatments after both winter wheat and summer maize harvests, with $F(5, 5.22) = 302.84$ and $F(5, 5.41) = 93.90$, respectively. The corresponding Holm-adjusted P-values were 4.2×10^{-6} and 3.23×10^{-5}. After summer maize harvest, the mean soil Ni concentrations under T3 and T4 were 31.06 and 33.70 mg kg^{-1}, respectively. Overall, all four slurry concentrations met the soil metal risk screening criteria, so none was excluded from the subsequent concentration selection on this basis.

4 Discussion

4.1 Trade-Offs between Crop Yields and Biogas Slurry Inputs under a Common Application Concentration

The contrasting yield responses of winter wheat and summer maize highlight a key trade-off in selecting a common biogas slurry concentration for rotation management. Although additional slurry inputs may benefit winter wheat, they do not necessarily provide comparable yield gains in summer maize. Consequently, optimizing slurry concentration for the rotation requires consideration of crop-specific responses rather than maximizing inputs or yield in either season alone. The favorable observed yields of both crops under T3, together with its 25% lower slurry requirement relative to T4, illustrate this trade-off.

Winter wheat and summer maize differ in growing periods, topdressing frequencies, and nutrient requirements. The same slurry concentration therefore does not imply the same seasonal slurry input or yield response. Although a common concentration simplifies application settings, the yield benefits of additional inputs need to be evaluated separately for each crop.

Previous studies have demonstrated clear trade-offs among fertilizer inputs, crop yields, and resource-use efficiency. Zhou et al. (2025) found in a water–nitrogen coupling experiment on spring maize that the highest yield occurred at an intermediate nitrogen application rate, while nitrogen partial factor productivity declined as nitrogen inputs increased. Chauhdary et al. (2025b) combined wheat field experiments with crop modeling and reported that the management practices associated with maximum yield, water productivity, and economic returns were not necessarily identical. Studies on biogas slurry application have also shown that appropriate management of slurry application rates and nutrient inputs can improve crop yields and nutrient-use efficiency in rotation systems

(Tang et al. 2022; Zhao et al. 2024). Consistent with these findings, the present study showed that increasing the common slurry application concentration from 75% to 100% increased total nitrogen, phosphorus, and potassium inputs from slurry over the rotation cycle by 77.42, 9.95, and 60.92 kg ha^{-1}, respectively. However, increasing the slurry concentration from T3 to T4 did not increase the observed mean yields of either crop, and the predicted yield benefits of additional inputs differed between winter wheat and summer maize. The additional nutrient inputs associated with T4 therefore provided no corresponding increase in the observed mean yields of either crop, highlighting the importance of evaluating marginal yield returns when selecting a common slurry concentration.

4.2 Yield Formation and Grain Quality Differences under Different Biogas Slurry Concentrations

Increasing biogas slurry concentration did not produce identical responses in yield and grain quality between winter wheat and summer maize. When the concentration increased from T3 to T4, the number of productive spikes in winter wheat continued to increase, whereas thousand-grain weight decreased from 46.48 to 45.23 g, with little change in the number of grains per spike. These contrasting changes in yield components were consistent with the similar observed yields under the two treatments. Makhdum et al. (2024) also observed that thousand-grain weight did not increase continuously with increasing phosphorus fertilizer application in a wheat experiment. In one cultivar treatment, thousand-grain weight decreased from 45.2 g at 80 kg P_2O_5 ha^{-1} to 41.9 g at 120 kg P_2O_5 ha^{-1}. This pattern was similar to the decline in winter wheat thousand-grain weight under the higher slurry concentration in the present study, indicating that yield components do not necessarily respond consistently to additional fertilizer inputs. For summer maize, the observed mean yield decreased from T3 to T4, accompanied by a reduction in hundred-grain weight from 35.13 to 32.49 g, while the number of grains per ear changed from 661.41 to 658.81 (Jiang et al. 2025). The corresponding changes in hundred-grain weight and mean yield suggest that the yield response of summer maize at the higher concentration may be associated with grain weight.

Grain quality changes at higher slurry concentrations further demonstrated differences in crop responses. Crude protein and wet gluten contents in winter wheat generally increased with slurry concentration, while sedimentation value and flour water absorption reached their highest values under T4. The sedimentation value under T4 was significantly

higher than that under CF, indicating more favorable values for some winter wheat grain and flour processing quality indicators at higher slurry concentrations. In contrast, crude starch and crude protein contents in summer maize were highest under T3, and further increases in slurry concentration did not improve these quality indicators. Therefore, T3 performed favorably in terms of observed yields of both crops and slurry inputs, whereas T4 showed certain advantages in selected winter wheat quality indicators.

Previous studies have shown that crop responses to biogas slurry depend on application rates, nutrient supply, and growing conditions (Liang et al. 2025; Nascimento et al. 2023; Wang et al. 2025; Winkhart et al. 2024; Wu et al. 2026). In the present rotation, T3 offered a favorable balance between crop yields and slurry input, while T4 retained advantages in selected winter wheat quality indicators. These contrasting outcomes suggest that grain quality should be considered alongside yield–input trade-offs when selecting slurry concentrations.

4.3 Uncertainty in Concentration–Response Predictions and Its Implications for Slurry Application Decisions

The influence of model structure on yield predictions differed between winter wheat and summer maize, particularly at higher slurry concentrations. Cross-validation favored the linear model for winter wheat yield and the quadratic model for summer maize yield, while predictions from the two model forms diverged most clearly between T3 and T4. This finding is consistent with previous Bayesian fertilization studies showing that response functions and parameter uncertainty influence estimated fertilizer responses (Correndo et al. 2021; Matavel et al. 2025; Palmero et al. 2026). Retaining both model forms therefore allowed the yield–input trade-offs to be assessed across alternative representations of crop responses.

In the centered quadratic model used in this study, the quadratic terms corresponding to T2 and T3 had identical values, causing them to cancel out when calculating the predicted yield difference between these two treatments. Therefore, the similar predicted yield increments obtained from the linear and quadratic models between T2 and T3 were partly attributable to the mathematical structure of the models. In contrast, the predicted yield increment from T3 to T4 was more sensitive to curvature.

Continuous concentration analysis further showed that the predicted mean yields of summer maize began to decline beyond approximately 71% slurry concentration under some quadratic models, with turning points ranging from approximately 71% to 78% across

different prior settings. Chauhdary et al. (2025a) combined wheat field experiments with the SALTMED model to investigate yield responses under different water quality and nitrogen application conditions, finding that yield benefits at higher input levels were influenced by water quality. This indicates that continuous fertilizer response patterns depend not only on input levels but also on experimental conditions and model prediction scenarios. By comparing different response functions and their posterior predictions, the present study further quantified uncertainty in the direction of yield changes at higher slurry concentrations.

Continuous concentration analysis complemented the comparisons among experimental treatments by identifying changes in predicted marginal yield responses within the tested concentration range. The divergence between linear and quadratic predictions at higher concentrations demonstrates how response-model uncertainty can influence fertilization decisions. Combining these predictions with slurry-input requirements provides a quantitative basis for evaluating common concentration strategies in winter wheat–summer maize rotations.

4.4 Study Limitations and Prospects for Practical Application

The field experiment covered one winter wheat–summer maize rotation cycle at a single site, with four slurry concentration treatments. Leave-one-concentration-out cross-validation and prior sensitivity analysis were used to assess the robustness of concentration–response predictions. Further multi-year and multi-site experiments would help establish the transferability of the identified yield–input trade-offs and the preferred concentration.

Grain quality and environmental assessments provided complementary evidence for evaluating slurry concentration treatments. Winter wheat quality was modeled using plot-level observations, whereas summer maize quality was compared among treatment means. Soil metal concentrations remained below the applicable risk screening thresholds across the tested slurry treatments, indicating no exceedance of these thresholds during the rotation cycle. However, long-term slurry application may alter metal accumulation and transport in soil–plant systems, as highlighted by Kama et al. (2024). Monitoring these processes over successive rotations would provide further evidence of environmental sustainability.

Future research could focus on increasing the number of experimental concentration levels within the higher concentration range, conducting multi-year and multi-site

validation, and comparing the yield and input benefits of a common rotation-wide concentration with independently determined concentrations for each growing season. Regarding equipment applications, Jiang et al. (2024) developed a precision biogas slurry application control system that improved fertilizer solution preparation accuracy through mixing ratio control and electrical conductivity feedback. In equipment tests, the relative errors in individual component concentrations were maintained within 10%, while electrical conductivity fluctuations remained within 5%. Their findings demonstrate that slurry mixing control and online monitoring can provide engineering support for precision application. Future field evaluations could incorporate measured concentration-control errors and slurry application costs to assess the operational and economic performance of candidate slurry concentrations.

5 Conclusions

Based on a winter wheat–summer maize rotation field experiment, this study integrated Bayesian concentration–response models, marginal yield return analysis, and soil metal risk screening to evaluate crop yields, grain quality, and trade-offs among common biogas slurry application concentrations. The main conclusions are as follows.

(1) Biogas slurry topdressing significantly increased winter wheat and summer maize yields. The highest observed yields of both crops were obtained under T3, representing increases of 27.92% and 22.52% compared with CK and 8.16% and 8.51% compared with CF, respectively ($P < 0.05$). Winter wheat yield did not differ significantly between T3 and T4, whereas summer maize yield was significantly higher under T3 according to Fisher's LSD test ($P = 0.046$). However, this difference was not significant under the Games–Howell procedure ($P = 0.167$). As slurry concentration increased, crude protein and wet gluten contents in winter wheat generally increased, whereas crude starch and crude protein contents in summer maize were higher under T3, indicating differences in grain quality responses between the two crops.

(2) Under a common slurry application concentration, increasing the concentration from T3 to T4 increased biogas slurry input over the rotation cycle by 75 $m^3\ ha^{-1}$, but the predicted yield responses differed between the two crops. Across the four model combinations, the combined predicted yield increment ranged from −214.43 to 700.46 kg ha^{-1}, corresponding to marginal yield returns of −2.86 to 9.34 kg m^{-3} per unit of additional slurry input. Both winter wheat models predicted yield increases, whereas the direction of the summer maize yield response depended on model structure. T3 required 25% less slurry

than T4.

(3) Continuous concentration analysis revealed differences among models in the predicted direction of yield changes at higher slurry concentrations, with the predicted turning points influenced by prior settings. Soil metal concentrations under all four experimental slurry treatments met the risk screening criteria adopted in this study, and contaminant concentrations in the analyzed wheat grain samples were below the corresponding reference limits. Integrating observed crop yields, slurry-input requirements, and model predictions identified 75% biogas slurry (T3) as the preferred common concentration among the treatments evaluated in this rotation.

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Statements and Declarations

Funding

The authors received no specific funding for this research.

Competing Interests

The authors declare that they have no competing interests.

Author Contributions

Yue Jiang: Conceptualization, methodology, supervision, and writing – review & editing.

Xuying Wang: Data curation, formal analysis, visualization, and writing – original draft.

Run Xue: Investigation, validation, and writing – review & editing.

All authors read and approved the final manuscript.

Data Availability

The datasets generated and analyzed during the current study are available from the corresponding author upon reasonable request.