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Mediation Analysis with Zero-inflation Using Factored Structural Equation Modeling

Chendong Li

cliatca@gmail.com

James Madison University

Oi-Man Kwok

Texas A&M University

Depeng Jiang

University of Manitoba

Timothy Lawrence

Texas A&M University

Research Article

Keywords:

Posted Date: November 3rd, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11597191/v1

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Additional Declarations: The authors declare no competing interests.

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Keywords: mediation analysis, zero-inflated counts, marginal mean, factored structural equation modeling, latent variables

Abstract

Mediation analysis with zero-inflated count variables often concerns associations with the overall expected count, whereas traditional zero-inflated models yield component-specific interpretations. This article integrates marginalized zero-inflated Poisson (mZIP) modeling with derivative-based mediation within the factored structural equation modeling (FSEM) framework. We formulate conditional indirect effects when either the mediator or the outcome is zero-inflated and incorporate a measurement model for a latent predictor. The measurement and structural submodels are jointly estimated within a Bayesian framework. A simulation study evaluates point estimation and interval coverage with an observed predictor and an mZIP mediator. For nonzero effects, estimation generally became more accurate as sample size increased, with coverage of 95% credible intervals ranging from 91.4% to 95.8%. An application to 952 participants from the Manitoba Follow-up Study examines the indirect association between latent social engagement and functional limitations through mental health. Greater engagement was associated with fewer expected limitations indirectly through better mental health, after adjustment for age. The formulation connects marginal-mean modeling with local, response-scale indirect associations and supports joint analysis of latent constructs and zero-inflated counts.

1. Introduction

Mediation analysis is a useful tool for studying mechanisms in the behavioral, social, educational, and health sciences because it allows researchers not only to answer the question of whether an intervention or predictor is associated with an outcome, but also how that association operates (Baron & Kenny, 1986; Imai et al., 2010; MacKinnon, 2008; VanderWeele, 2016). In a simple mediation model, a predictor X is assumed to affect an outcome Y partly through a mediator M, as shown in Figure 1.

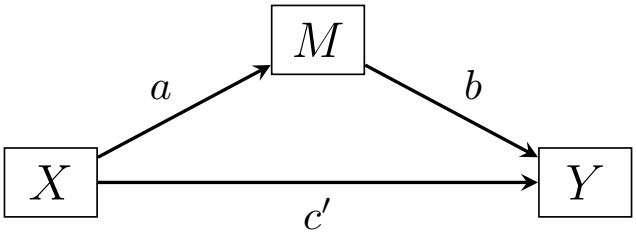


Figure 1. *Path diagram for the simple mediation model.*

This representation allows the total effect of X on Y to be decomposed into a direct effect ($X \rightarrow Y$) and an indirect effect ($X \rightarrow M \rightarrow Y$). The direct effect represents the association between X and Y after accounting for the mediator (i.e., c'), whereas the indirect effect represents the portion of the association that operates through the path from X to M and then from M to Y. Under the linear mediation model, which assumes linear relations among the predictor, mediator, and outcome, the indirect effect is commonly expressed as the product of the a path, linking X to M, and the b path, linking M to Y while controlling for X, yielding ab (MacKinnon, 2008; MacKinnon & Valente, 2014). This decomposition makes mediation analysis appealing to applied researchers because it provides a straightforward way to investigate not only whether an effect exists, but also the mechanism through which the effect may operate (MacKinnon, 2008).

Linear mediation models have been the foundation for much of the literature (e.g., Hughes et al., 2016; Kalsi-Ryan et al., 2014; Lawrence, 2022). Conventional mediation methods have often been developed under the assumption that endogenous variables are continuous (MacKinnon,

2008; MacKinnon & Cox, 2012). This formulation, however, does not apply directly when a count variable is modeled using a nonlinear link function. Count variables record the number of times an event occurs (e.g., the number of drinks consumed, cigarettes smoked, or bullying incidents experienced) (Atkins et al., 2013; O'Rourke & Vazquez, 2019; Sims et al., 2024b). Count data are commonly modeled using Poisson or negative binomial (NB) regression with a log link. Unlike a linear regression coefficient, which represents a constant additive change in the expected outcome, a count regression coefficient under a log link represents a change in the log expected count. It is interpreted as a multiplicative change in the expected count after exponentiation. Thus, the same one-unit change in a predictor may correspond to different absolute changes in the expected count depending on its current predicted level. This distinction is important for mediation analysis. When the outcome Y is a count, the b path on the expected-count scale depends on the predicted outcome mean, and when the mediator is a count, the a path on the expected-count scale similarly depends on the predicted mediator mean. To obtain the mediated effect on the original expected-count scale, the nonlinear path must be transformed through the inverse link function and evaluated at specified values of the variables, resulting in a conditional mediated effect that may vary across participants or covariate values (Cheng et al., 2018; Coxe & MacKinnon, 2010; Geldhof et al., 2018).

The problem becomes more complicated when the count variable contains excess zeros, a phenomenon referred to as zero-inflation. Zero-inflation occurs when the observed number of zeros is greater than would be expected under a standard count distribution such as the Poisson or NB distribution (Lambert, 1992; Mullahy, 1986). This situation is common in many areas of applied research. For example, in substance-use studies, many participants may report zero cigarette use during a given assessment period (O'Rourke & Vazquez, 2019). Standard count models may produce biased estimates and inaccurate statistical inferences in these settings because they do not account for the excess zeros (Lambert, 1992; Mullahy, 1986). The zero-inflated Poisson (ZIP) model addresses this issue by representing the outcome as arising from a mixture of two latent processes, which are an excess-zero process and a count process (Lambert, 1992). The ZIP model therefore has two components corresponding to these processes, which are

a logistic component and a count component. The logistic component models the probability that an observation belongs to the excess-zero process. Observations from this process are interpreted as belonging to a subpopulation that cannot experience the event and therefore necessarily have a value of zero. The count process is modeled using Poisson regression and represents an at-risk subpopulation for which both positive counts and zeros are possible. Zeros generated by the count process are commonly referred to as sampling zeros. For example, in a study of cigarette use, the observed zeros may include both participants who have never smoked, representing excess zeros, and smokers who did not smoke during the observation period, representing sampling zeros (Pittman et al., 2022). The count process can be modeled using NB regression to account for overdispersion when its conditional variance exceeds its conditional mean. The resulting model is referred to as a zero-inflated negative binomial (ZINB) model (Greene, 1994).

Although the traditional ZIP model is useful for describing count variables with excess zeros, its parameters are not always aligned with the substantive questions asked by applied researchers. The count component describes the expected count within the latent count process, whereas the logistic component describes the probability of belonging to the excess-zero process (Lambert, 1992). These latent-class-specific interpretations may be useful when substantive theory explicitly distinguishes the excess-zero process from the count process. In many psychological and educational applications, however, the substantive question concerns the overall expected count in the full population. For example, researchers may want to determine whether an intervention reduces the overall expected number of cigarettes used rather than the expected count only among individuals in a latent count subgroup.

The marginalized zero-inflated modeling framework was developed to address this interpretational problem. The marginalized zero-inflated Poisson (mZIP) model retains the two-component structure of the traditional ZIP model but reparameterizes it so that predictors are linked directly to the overall population mean, while a logistic component is retained to model the excess-zero probability (Long et al., 2014). The coefficients in the marginal mean component can therefore be interpreted with respect to the overall expected count. Although the logistic coefficients can be interpreted when the excess-zero process is of substantive interest,

the excess-zero equation serves as a distributional adjustment when inference focuses on the marginal mean coefficients (Li et al., 2026; Long et al., 2014).

Existing noncausal mediation methods for zero-inflated count variables have been developed primarily within a regression-based framework using traditional ZIP or ZINB models. O'Rourke and Vazquez (2019) described conditional mediated effects for zero-inflated count outcomes using derivatives of the relevant regression equations, and O'Rourke and Han (2023) further distinguished between mediated effects operating through the excess-zero and count components. Accordingly, these methods define and interpret mediated effects separately for the probability of belonging to the excess-zero process and for the expected count within the latent count process. This separation can be substantively useful when theory distinguishes between the two processes. However, it produces latent-class-specific mediated effects rather than a single mediated effect on the overall expected count in the population. A further consideration is that these regression-based implementations typically treat the predictor, mediator, and outcome as observed variables and do not explicitly incorporate measurement models for latent constructs. In behavioral, social, and educational research, where constructs are frequently measured using multiple indicators, an analytical framework is needed that can combine the population-average interpretation of marginalized zero-inflated models with a structural equation modeling (SEM) framework that can be extended to latent variables and more complex mediation systems.

To address these limitations, the present article demonstrates the formulation of mediation with marginalized zero-inflated variables under SEM. SEM represents the mediator and outcome models as parts of a system and summarizes their relations through path diagrams, structural paths, direct effects, and indirect effects (MacKinnon, 2008; MacKinnon & Valente, 2014). It also provides a foundation for incorporating latent variables, measurement error, multiple mediators, and other complex path structures. Classical SEM, however, is commonly formulated using linear relations among continuously and normally distributed variables. Conventional SEM implementations do not readily accommodate customized marginalized zero-inflated distributions while allowing different endogenous variables to follow different distributions. Factored structural equation modeling (FSEM), introduced by Enders and Keller (2026), addresses this

limitation by expressing the joint distribution as a sequence of linked conditional submodels, each of which may have its own distribution and link function. For example, a continuous mediator may be assigned a normal distribution while a zero-inflated count outcome is assigned an mZIP distribution.

It should be noted that population-average mediation methods using marginalized zero-inflated models have recently been developed within the counterfactual mediation framework. Sims et al. (2024a) proposed a marginalized zero-inflated mediation approach when the mediator has excess zeros, and Sims et al. (2024b) developed population-average mediation methods for zero-inflated count outcomes. These studies show how marginalized zero-inflated models can be used to define causal direct and indirect effects for the overall population mean. More recently, Cui et al. (2026) developed a Bayesian framework for causal mediation with zero-inflated mediators. These counterfactual approaches define natural direct and indirect effects through contrasts between potential outcomes. In contrast, the present study defines derivative-based conditional indirect associations, which describe local rates of change along the mediated pathway at specified values of the variables.

The purpose of the present article is to formulate and evaluate derivative-based mediation with mZIP models within the FSEM framework. The present article contributes to the literature in three ways. First, it connects mZIP marginal-mean modeling with derivative-based mediation to define conditional indirect associations through the overall expected count when either the mediator or the outcome is zero-inflated. Second, it incorporates a latent predictor measured by multiple indicators within the FSEM framework of Enders and Keller (2026), allowing the measurement and structural submodels to be estimated jointly. Third, it applies this formulation to examine the indirect association between social engagement and functional limitations through mental health in older adults, linking the statistical formulation to a substantive question about social and psychological aspects of aging.

A simulation study evaluates the point estimation and inferential properties of conditional indirect effects with an observed predictor and an mZIP mediator. The empirical application uses data from the Manitoba Follow-up Study (MFUS; Tate et al., 2015) to examine whether

greater latent social engagement is associated with fewer limitations in instrumental activities of daily living indirectly through better mental health, after adjustment for age.

The remainder of the article is organized as follows. We first introduce the traditional ZIP and marginalized ZIP models. We then present mediation with zero-inflated variables under the FSEM framework. We next report the simulation study and demonstrate the approach using an empirical example. Finally, we discuss the benefits, limitations, and directions for future research.

2. Statistical Model

This section introduces the statistical framework for mediation analysis when a mediator or outcome is a zero-inflated count variable. We first review the traditional ZIP model. We then introduce the mZIP model, which directly models the marginal mean instead of the latent mean. Building on the mediation framework for nonlinear outcomes, we consider two common mediation settings: a zero-inflated outcome and a zero-inflated mediator. Finally, we describe how these models can be organized using the FSEM framework.

2.1. Traditional ZIP Model

Let V_i denote a count variable for participant i. The ZIP model assumes that V_i arises from two latent processes (Lambert, 1992). With probability ψ_i, the observation belongs to an excess-zero process and necessarily takes the value zero. With probability $1-\psi_i$, the observation arises from a count process with mean μ_i, which can generate both zero and positive counts.

Let $g(\cdot \mid \mu_i)$ denote the probability mass function of the count process, parameterized by its mean μ_i. The mixture probability mass function is

$$P(V_i = v_i) = \begin{cases} \psi_i + (1-\psi_i)g(0 \mid \mu_i), & v_i = 0, \\ (1-\psi_i)g(v_i \mid \mu_i), & v_i > 0. \end{cases} \tag{2.1}$$

When $g(\cdot \mid \mu_i)$ is Poisson, Equation (2.1) defines the ZIP model. While the present article focuses on ZIP in the derivation, other count distributions, such as the NB or generalized

Poisson distribution, may replace the Poisson distribution when additional dispersion needs to be accommodated.

The excess-zero probability and count-process mean are commonly related to covariates through

$$\begin{aligned} \text{logit}(\psi_i) &= \mathbf{z}_i^\top \boldsymbol{\gamma}, \\ \log(\mu_i) &= \mathbf{x}_i^\top \boldsymbol{\beta}, \end{aligned} \tag{2.2}$$

where $\mathbf{z}_i$ and $\mathbf{x}_i$ are vectors of covariates for the excess-zero and count processes, respectively. The two vectors may contain the same covariates or may differ. Each element γ_j represents the change in the log odds of belonging to the excess-zero process associated with a one-unit increase in the corresponding covariate. Each element β_j represents the change in the log expected count within the latent count process. Accordingly, $\exp(\gamma_j)$ is an odds ratio for membership in the excess-zero process, whereas $\exp(\beta_j)$ is an incidence rate ratio for the expected count within the latent count process.

In the traditional ZIP specification, μ_i is not the expected count for the full population. Averaging over the two latent processes gives

$$\begin{aligned} \nu_i &= E(V_i \mid \mathbf{x}_i, \mathbf{z}_i) \\ &= (1 - \psi_i)\mu_i \\ &= \frac{\exp(\mathbf{x}_i^\top \boldsymbol{\beta})}{1 + \exp(\mathbf{z}_i^\top \boldsymbol{\gamma})}. \end{aligned} \tag{2.3}$$

Here, ν_i is the marginal mean count of V_i with respect to the full population. Equation (2.3) shows that the overall expected count depends on parameters from both the excess-zero and count components and the association is a nonlinear function of both $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ (Long et al., 2014; Preisser et al., 2016). This distinction is important for mediation analysis because the pathways of interest generally concern the observed mediator or outcome in the full population instead of the mean of an unobserved latent count-process subgroup.

2.2. mZIP Model

The mZIP model addresses the interpretational difficulty of the traditional ZIP model by directly modeling the marginal mean ν_i, while retaining the excess-zero process (Long et al., 2014). Specifically, instead of relating covariates to the latent count-process mean μ_i, the mZIP model relates them directly to the overall expected count:

$$\begin{aligned} \text{logit}(\psi_i) &= \mathbf{z}_i^\top \boldsymbol{\gamma}, \\ \log(\nu_i) &= \mathbf{x}_i^\top \boldsymbol{\alpha}, \end{aligned} \tag{2.4}$$

where $\nu_i = E(V_i \mid \mathbf{x}_i, \mathbf{z}_i)$ is the marginal mean count for the full population. The excess-zero equation is unchanged from the traditional ZIP model, so $\boldsymbol{\gamma}$ retains the same interpretation. In contrast, each element α_j represents the change in the log overall expected count associated with a one-unit increase in the corresponding covariate. Accordingly, $\exp(\alpha_j)$ is an incidence rate ratio for the expected count in the full population rather than within the latent count-process subgroup.

The mZIP model remains a zero-inflated mixture model. Therefore, the mean μ_i of the latent count process must satisfy the relationship $\nu_i = (1 - \psi_i)\mu_i$. Solving for μ_i gives

$$\begin{aligned} \mu_i &= \frac{\nu_i}{1 - \psi_i} \\ &= \exp(\mathbf{x}_i^\top \boldsymbol{\alpha}) \left\{1 + \exp(\mathbf{z}_i^\top \boldsymbol{\gamma})\right\}. \end{aligned} \tag{2.5}$$

Substituting the implied value of μ_i from Equation (2.5) into the mixture probability mass function in Equation (2.1) gives the mZIP model. The latent count process remains Poisson, and the logistic component continues to account for excess zeros.

Unlike the traditional ZIP model, the regression equation for ν_i does not combine parameters from the excess-zero and count components. Consequently, $\boldsymbol{\alpha}$ directly describes the association between the covariates and the overall expected count. This parameterization is particularly useful for mediation analysis because it allows the pathways of interest to be defined with respect to the marginal mean of the observed mediator or outcome. Their corresponding effects on the expected-count scale are developed in the following sections.

2.3. Mediation With Zero-Inflation

Let X_i, M_i, and Y_i denote the predictor, mediator, and outcome, respectively. A simple mediation model can be written generally in terms of the conditional means of the mediator and outcome:

$$\begin{aligned} E(M_i \mid X_i = x) &= f_M(i_M + ax)\,, \\ E(Y_i \mid X_i = x, M_i = m) &= f_Y(i_Y + c'x + bm)\,, \end{aligned} \tag{2.6}$$

where $f_M(\cdot)$ and $f_Y(\cdot)$ are inverse-link functions for the mediator and outcome models, respectively. This formulation includes both linear and nonlinear mediation models. For a continuous variable modeled using linear regression, the inverse-link function is the identity function, $f(u) = u$. For a count variable modeled using a log link, the inverse-link function is exponential, $f(u) = \exp(u)$.

Following the derivative-based approach to nonlinear mediation, the conditional indirect effect is defined as the product of the response-scale a and b paths (Geldhof et al., 2018; O'Rourke & Vazquez, 2019):

$$\begin{aligned} IE(x, m) &= \left[\frac{\partial E(M_i \mid X_i = x)}{\partial x}\right]\left[\frac{\partial E(Y_i \mid X_i = x, M_i = m)}{\partial m}\right] \\ &= a\,b\,f'_M(i_M + ax)\,f'_Y(i_Y + c'x + bm)\,, \end{aligned} \tag{2.7}$$

where $f'_M(\cdot)$ and $f'_Y(\cdot)$ denote the derivatives of the inverse-link functions. When both models are linear, $f'_M(u) = f'_Y(u) = 1$, and Equation (2.7) reduces to the familiar indirect effect ab. With a log link, however, $f'(u) = \exp(u)$, so the response-scale path depends on the expected count. Consequently, the indirect effect is generally conditional on the values at which the model is evaluated.

For simplicity, covariates are omitted from the following models. Covariates may be added to either equation and held fixed when evaluating the indirect effect. The term marginal in the mZIP model refers to marginalization over the excess-zero and count processes; the marginal mean remains conditional on any observed covariates included in the model.

2.3.1. Scenario I: Zero-Inflated Outcome

Consider a continuous mediator M_i and a zero-inflated count outcome Y_i. The mediator is modeled using an identity link, $f_M(u) = u$, so the response-scale a path is simply a. The mZIP outcome model is

$$\begin{aligned} \text{logit}(\psi_i^Y) &= \gamma_{0Y} + c'_\psi X_i + b_\psi M_i, \\ \log(\nu_i^Y) &= \alpha_{0Y} + c'_\nu X_i + b_\nu M_i. \end{aligned} \tag{2.8}$$

The first equation models the excess-zero probability, whereas the second models the marginal expected outcome count. On the original count scale,

$$\nu^Y(x, m) = \exp\left\{\alpha_{0Y} + c'_\nu x + b_\nu m\right\}.$$

Because the inverse-link function for the marginal mean is exponential,

$$\frac{\partial \nu^Y(x, m)}{\partial m} = b_\nu \nu^Y(x, m).$$

Applying Equation (2.7), the conditional indirect effect is

$$\begin{aligned} IE_Y(x, m) &= a\, b_\nu \nu^Y(x, m) \\ &= a\, b_\nu \exp\left\{\alpha_{0Y} + c'_\nu x + b_\nu m\right\}. \end{aligned} \tag{2.9}$$

Thus, the indirect effect is defined on the scale of the overall expected outcome count and varies with the values of X and M. The effect may be evaluated at substantively meaningful values of the mediator or at its model-implied conditional mean, $m = E(M_i \mid X_i = x)$.

2.3.2. Scenario II: Zero-Inflated Mediator

Consider a zero-inflated count mediator M_i and a continuous outcome Y_i. The mZIP mediator model is

$$\begin{aligned} \operatorname{logit}(\psi_i^M) &= \gamma_{0M} + a_\psi X_i, \\ \log(\nu_i^M) &= \alpha_{0M} + a_\nu X_i. \end{aligned} \tag{2.10}$$

The marginal expected mediator count is

$$\nu^M(x) = \exp\{\alpha_{0M} + a_\nu x\},$$

and its derivative with respect to X is

$$\frac{\partial \nu^M(x)}{\partial x} = a_\nu \nu^M(x).$$

The continuous outcome is modeled using an identity link, $f_Y(u) = u$, so the response-scale b path remains b. The conditional indirect effect is therefore

$$\begin{aligned} IE_M(x) &= a_\nu b\, \nu^M(x) \\ &= a_\nu b \exp\{\alpha_{0M} + a_\nu x\}. \end{aligned} \tag{2.11}$$

In this scenario, the nonlinear transformation occurs on the $X \to M$ path. The indirect effect therefore varies with X through the overall expected mediator count.

Table 1 summarizes the linear reference model and the two marginalized zero-inflated mediation scenarios.

Table 1. *Conditional indirect effects under linear and mZIP mediation models.*

Model	**ZI variable**	**Indirect effect**	**Interpretation**
Linear	—	ab	Constant across x and m
ZI outcome	Y	$a\, b_\nu \nu^Y(x, m)$	Depends on the overall expected outcome count
ZI mediator	M	$a_\nu b\, \nu^M(x)$	Depends on the overall expected mediator count

The logistic excess-zero equation remains part of each mZIP model but does not enter the indirect effect defined through the marginal mean. Equations (2.9) and (2.11) are local,

derivative-based indirect effects for a continuous predictor. For a binary or categorical predictor, the derivatives should be replaced by finite contrasts between selected values of X. Without the assumptions required for causal identification, these quantities should be interpreted as conditional indirect associations rather than causal mediation effects.

The marginalized ZINB (mZINB) model of Preisser et al. (2016) provides a natural extension when the count process requires additional dispersion. Specifically, the Poisson mass function in Equation (2.1) can be replaced by an NB mass function with count-process variance $\mu_i + \kappa\mu_i^2$, where $\kappa > 0$ is a dispersion parameter. The relationship $\mu_i = \nu_i/(1 - \psi_i)$ in Equation (2.5) is retained. If the same log link is used for ν_i, its derivative is unchanged. Thus, the indirect effects in Equations (2.9) and (2.11) retain their algebraic form under mZINB, while estimation also accounts for κ.

2.4. Factored Structural Equation Modeling Framework

The mediation models described above can be organized within the FSEM framework by representing the joint distribution as a set of linked conditional submodels (Enders & Keller, 2026). Let $p(\cdot)$ denote a probability density or probability mass function. For an observed predictor, mediator, and outcome, the simple mediation model can be factored as

$$\begin{aligned} p(Y_i, M_i \mid X_i, \mathbf{C}_i) = p_Y(Y_i \mid M_i, X_i, \mathbf{C}_i) \\ \times p_M(M_i \mid X_i, \mathbf{C}_i), \end{aligned} \tag{2.12}$$

where $\mathbf{C}_i$ denotes an optional vector of covariates. The first submodel describes the $M \rightarrow Y$ and direct $X \rightarrow Y$ pathways, whereas the second describes the $X \rightarrow M$ pathway. Each submodel may be assigned a distribution and link function appropriate for its dependent variable. For example, a continuous variable may follow a normal distribution with an identity link, whereas a zero-inflated count variable may follow an mZIP distribution with separate excess-zero and marginal mean equations.

The factorization in Equation (2.12) provides the general organization of the mediation model. When the outcome is zero-inflated, $p_Y(\cdot)$ is specified as mZIP. When the mediator is zero-inflated, $p_M(\cdot)$ is specified as mZIP.

2.4.1. Incorporating Latent Variables

A principal advantage of the SEM framework is that the predictor or another construct may be represented as a latent variable measured by multiple indicators. Let ξ_i denote a latent predictor and let

$$\mathbf{R}_i = (R_{1i}, R_{2i}, \ldots, R_{Ji})^\top$$

denote its J observed indicators. A general measurement model can be written as

$$h_j\{E(R_{ji} \mid \xi_i)\} = \tau_j + \lambda_j \xi_i, \qquad j = 1, \ldots, J, \tag{2.13}$$

where τ_j and λ_j are the indicator intercept and factor loading, respectively, and $h_j(\cdot)$ is an appropriate link function. This formulation permits the indicators to be continuous, binary, ordinal, or otherwise nonnormal.

The structural model for the observed continuous mediator may be written as

$$E(M_i \mid \xi_i, \mathbf{C}_i) = a_0 + a\xi_i + \mathbf{a}_C^\top \mathbf{C}_i. \tag{2.14}$$

The coefficient a represents the $\xi \rightarrow M$ pathway. The latent predictor may be assigned a normal conditional distribution, $\xi_i \mid \mathbf{C}_i \sim N(\boldsymbol{\delta}_C^\top \mathbf{C}_i, 1)$, with its intercept fixed to zero and residual variance fixed to one to establish its location and scale. Under conditional independence of the indicators, the factored model becomes

$$
\begin{aligned}
&p(Y_i, M_i, \mathbf{R}_i, \xi_i \mid \mathbf{C}_i) \\
&\quad = p_Y(Y_i \mid M_i, \xi_i, \mathbf{C}_i)\, p_M(M_i \mid \xi_i, \mathbf{C}_i) \\
&\qquad \times \left\{ \prod_{j=1}^{J} p_j(R_{ji} \mid \xi_i) \right\} p_\xi(\xi_i \mid \mathbf{C}_i).
\end{aligned}
\tag{2.15}
$$

Equation (2.15) contains four parts, which are an outcome model, a mediator model, a measurement model, and a conditional distribution for the latent predictor. If conditional independence among the indicators is not assumed, the product of univariate indicator distributions can be replaced by a multivariate measurement distribution $p_R(\mathbf{R}_i \mid \xi_i)$.

For example, when Y_i is a zero-inflated count outcome, its conditional distribution in Equation (2.15) may be specified using the mZIP model.

$$
\begin{aligned}
\operatorname{logit}(\psi_i^Y) &= \gamma_{0Y} + c'_\psi \xi_i + b_\psi M_i + \boldsymbol{\gamma}_C^\top \mathbf{C}_i, \\
\log(\nu_i^Y) &= \alpha_{0Y} + c'_\nu \xi_i + b_\nu M_i + \boldsymbol{\alpha}_C^\top \mathbf{C}_i.
\end{aligned}
\tag{2.16}
$$

Here, b_ν represents the association between the observed mediator and the marginal expected outcome count. The corresponding conditional indirect effect follows the same derivative-based definition introduced previously.

$$
\begin{aligned}
IE_Y(\xi, m, \mathbf{c}) &= \frac{\partial E(M_i \mid \xi_i = \xi, \mathbf{C}_i = \mathbf{c})}{\partial \xi} \frac{\partial \nu^Y(\xi, m, \mathbf{c})}{\partial m} \\
&= a\, b_\nu \nu^Y(\xi, m, \mathbf{c}).
\end{aligned}
\tag{2.17}
$$

Thus, replacing an observed predictor with a latent predictor does not change the mathematical form of the conditional indirect effect. The primary difference is that ξ_i is not directly observed. The effect may be evaluated at specified values of ξ and at the model-implied mediator mean,

$$
m = \overline{m}(\xi, \mathbf{c}) = E(M_i \mid \xi_i = \xi, \mathbf{C}_i = \mathbf{c}),
$$

or averaged over the conditional distribution of the latent predictor while evaluating the mediator at this conditional mean.

$$\overline{IE}_Y(\mathbf{c}) = \int IE_Y\{\xi, \overline{m}(\xi, \mathbf{c}), \mathbf{c}\}\ p_\xi(\xi \mid \mathbf{c})\, d\xi. \tag{2.18}$$

The second approach introduces an additional marginalization over the latent predictor distribution. This should be distinguished from the mZIP marginal mean ν_i^Y, which is marginalized over the excess-zero and count processes but remains conditional on the predictor, mediator, and observed covariates.

In a Bayesian FSEM implementation, latent variables are treated as unobserved data and sampled using data augmentation (Enders & Keller, 2026). The measurement, mediator, and outcome submodels are therefore estimated jointly rather than by first estimating factor scores and subsequently treating those scores as observed predictors. Joint estimation propagates measurement uncertainty into the structural coefficients and the resulting indirect effects. At each posterior iteration, the conditional or averaged indirect effect can be calculated from the jointly sampled model parameters and latent-variable values.

The same construction extends to other mediation configurations. If the predictor is observed and no other construct is latent, the measurement-model component and latent-predictor distribution are omitted. If the mediator is an observed zero-inflated count, its structural distribution is specified as mZIP. If the mediator or outcome is latent, a corresponding measurement model is added. Multiple latent variables or mediators similarly add measurement and structural submodels to the factorization.

3. Simulation Study

The simulation study evaluated the point estimation and inferential properties of the conditional indirect effect with an observed continuous predictor, an mZIP mediator, and a continuous outcome. The mediator and outcome submodels were estimated jointly within the FSEM framework. We examined how sample size, the excess-zero mixing probability, and the magnitude of the indirect effect were associated with estimation performance.

3.1. Simulation Design

The design crossed three factors, which were (1) sample size, $N = 200$, 400, or 800, (2) the excess-zero mixing probability at the population mean of the predictor, $q = 0.40$, 0.60, or 0.80, and (3) the conditional indirect effect at that predictor value, $\theta = 0$, 0.5, or 2.0. This resulted in 27 conditions, with 1,000 replications specified for each condition. Variables were generated and analyzed on their original scales without standardization.

The predictor was generated as $X_i \sim N(1.75, 2^2)$. The mediator and outcome submodels were

$$\begin{aligned} \operatorname{logit}(\psi_i^M) &= \gamma_{0M} - 0.30X_i, \\ \log(\nu_i^M) &= -0.75 + 0.30X_i, \\ \mu_i^M &= \frac{\nu_i^M}{1 - \psi_i^M}, \\ Y_i \mid M_i, X_i &\sim N(0.75 - 0.70X_i + bM_i, 1). \end{aligned} \tag{3.1}$$

The mediator was set to zero with probability ψ_i^M and otherwise generated from a Poisson distribution with mean μ_i^M. To obtain the selected excess-zero mixing probability at $X = 1.75$, the intercept was set to $\gamma_{0M} = \operatorname{logit}(q) + 0.30(1.75)$. Thus, q refers to the probability of the excess-zero process at the reference predictor value.

The coefficient b was selected so that Equation (2.11) equaled θ at $X = 1.75$.

$$b = \frac{\theta}{0.30 \exp\{-0.75 + 0.30(1.75)\}}. \tag{3.2}$$

The resulting values of b were approximately 0, 2.087, and 8.349. The null condition therefore set the mediator-to-outcome coefficient to zero while retaining the predictor-to-mediator coefficient $a_\nu = 0.30$. Conditional indirect effects were evaluated at the population mean of X and one population standard deviation below and above that mean, which corresponded to $x = -0.25$, 1.75, and 3.75. The true effects at these values were $\theta \exp(-0.60)$, θ, and $\theta \exp(0.60)$, respectively.

3.2. Estimation and Performance Measures

The mediator and outcome submodels were estimated jointly using the observed-variable factorization in Equation (2.12), with an mZIP mediator distribution and a normal outcome distribution as specified in Equation (3.1). The conditional indirect effect was calculated as $a_v b \exp(\alpha_{0M} + a_v x)$ at each evaluation point.

Bayesian estimation was conducted in Stan (Carpenter et al., 2017) through `rstan`. All regression coefficients received independent $N(0, 10^2)$ priors, and the outcome residual standard deviation received a half-normal prior with scale 10. Four chains each included 5,000 iterations, of which the first 1,000 were discarded as warm-up, leaving 16,000 posterior draws per fitted model. The indirect effect was calculated within each posterior draw and summarized using its posterior mean and an equal-tailed 95% credible interval (CrI).

Performance was evaluated using bias, root mean square error (RMSE), interval coverage, Type I error, and power (Morris et al., 2019). Bias was the average estimated effect minus the true effect at each evaluation point. RMSE was the square root of the average squared estimation error. Coverage was the proportion of intervals containing the true effect. Type I error and power were the proportions of intervals excluding zero under the null and nonzero-effect conditions, respectively.

3.3. Simulation Results

Table 2 summarizes mZIP performance by sample size and indirect-effect condition. Each range summarizes the three excess-zero mixing probabilities and three predictor evaluation points. For nonzero effects, bias ranged from −0.1348 to 0.0030 across all conditions and evaluation points. At $N = 800$, this range narrowed to −0.0290 to 0.0005. Under the null condition, bias ranged from −0.0011 to 0.0014.

Table 2. *Summary of mZIP simulation performance.*

N	θ	Bias range	RMSE range	Coverage range
200	0.0	[−0.0011, 0.0014]	[0.0042, 0.0270]	[0.922, 0.957]
200	0.5	[−0.0340, 0.0030]	[0.0282, 0.2863]	[0.937, 0.954]
200	2.0	[−0.1348, −0.0399]	[0.1094, 1.1543]	[0.914, 0.958]
400	0.0	[−0.0009, 0.0001]	[0.0031, 0.0172]	[0.948, 0.955]
400	0.5	[−0.0145, −0.0038]	[0.0184, 0.2001]	[0.944, 0.958]
400	2.0	[−0.0662, −0.0172]	[0.0766, 0.8092]	[0.924, 0.950]
800	0.0	[−0.0001, 0.0004]	[0.0022, 0.0119]	[0.952, 0.966]
800	0.5	[−0.0063, 0.0005]	[0.0130, 0.1482]	[0.943, 0.958]
800	2.0	[−0.0290, −0.0013]	[0.0512, 0.5685]	[0.937, 0.958]

Note. Each entry gives the minimum and maximum across $q = 0.40$, 0.60, and 0.80 and $x = -0.25$, 1.75, and 3.75. θ is the true conditional indirect effect at $x = 1.75$. Bias is the average estimated effect minus the true effect at the corresponding evaluation point. Bias and root mean square error (RMSE) are on the original outcome scale per unit of the predictor. Coverage is the proportion of equal-tailed 95% credible intervals containing the true effect.

Figures 2, 3, and 4 present bias, RMSE, and coverage, respectively, at the population mean of the predictor ($x = 1.75$). For nonzero effects, estimates showed downward bias, which decreased in magnitude as sample size increased (Figure 2). This pattern was most evident when $\theta = 2.0$ and $q = 0.80$, for which bias decreased in magnitude from −0.1348 at $N = 200$ to −0.0290 at $N = 800$. For nonzero effects, RMSE also decreased as sample size increased and increased with the excess-zero mixing probability (Figure 3).

Across all conditions and evaluation points, coverage of the 95% CrIs for nonzero effects ranged from 0.914 to 0.958. The lowest coverage, 0.914, occurred at $x = -0.25$ when $N = 200$, $q = 0.80$, and $\theta = 2.0$. At $x = 1.75$, coverage ranged from 0.933 to 0.958 for nonzero effects (Figure 4). Under the null condition, Type I error ranged from 0.034 to 0.078 across all conditions and evaluation points. Under the nonzero-effect conditions, power ranged from 0.956 to 1.000.

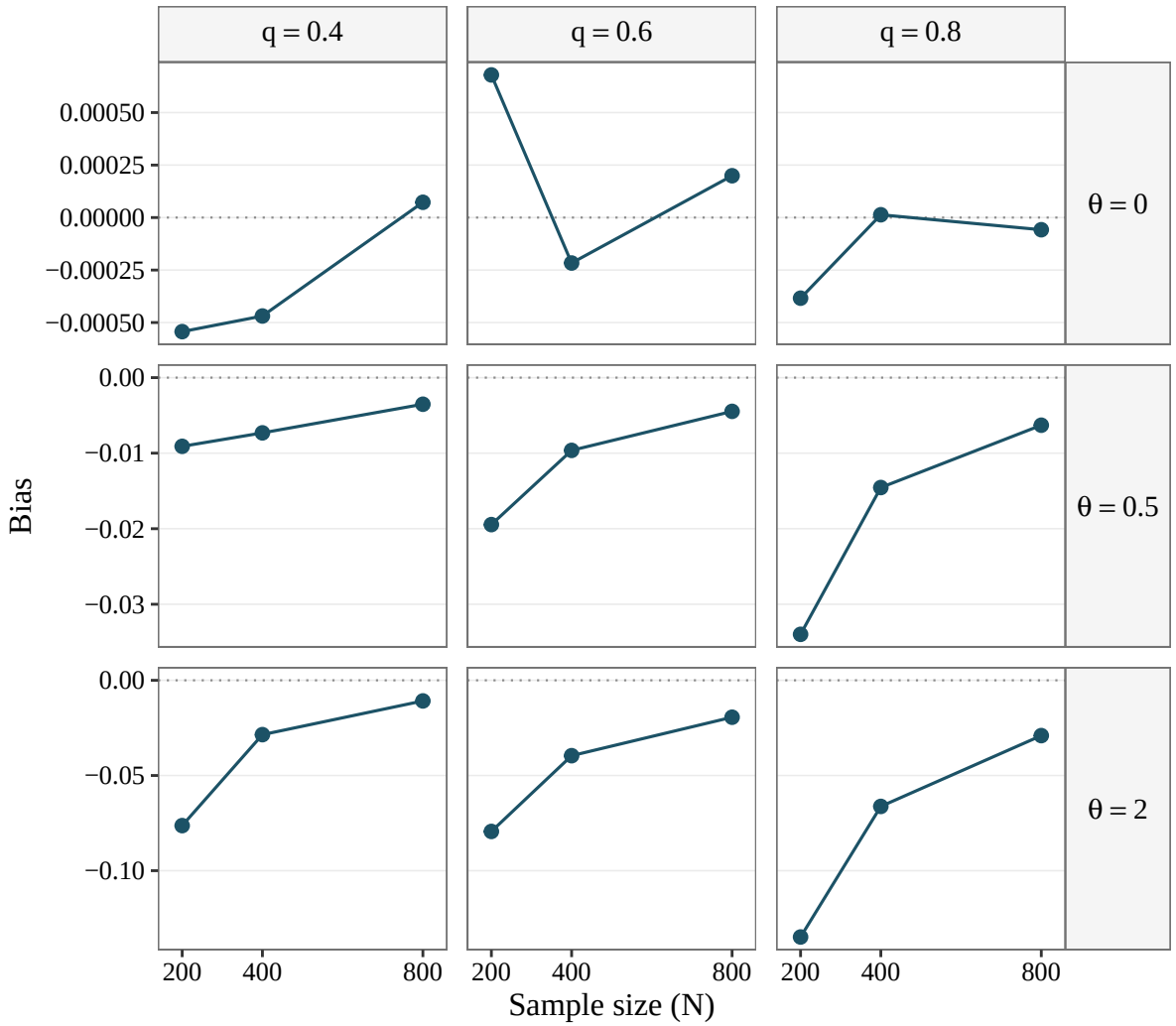


Figure 2. *Bias of the mZIP conditional indirect effect at* $x = 1.75$.

Note. Columns show the excess-zero mixing probability $q = \psi^M(1.75)$, and rows show the true conditional indirect effect $\theta = IE_M(1.75)$. Bias retains the original outcome scale per unit of the predictor. Vertical scales differ across rows but are shared across columns within each row. The dotted line indicates zero bias.

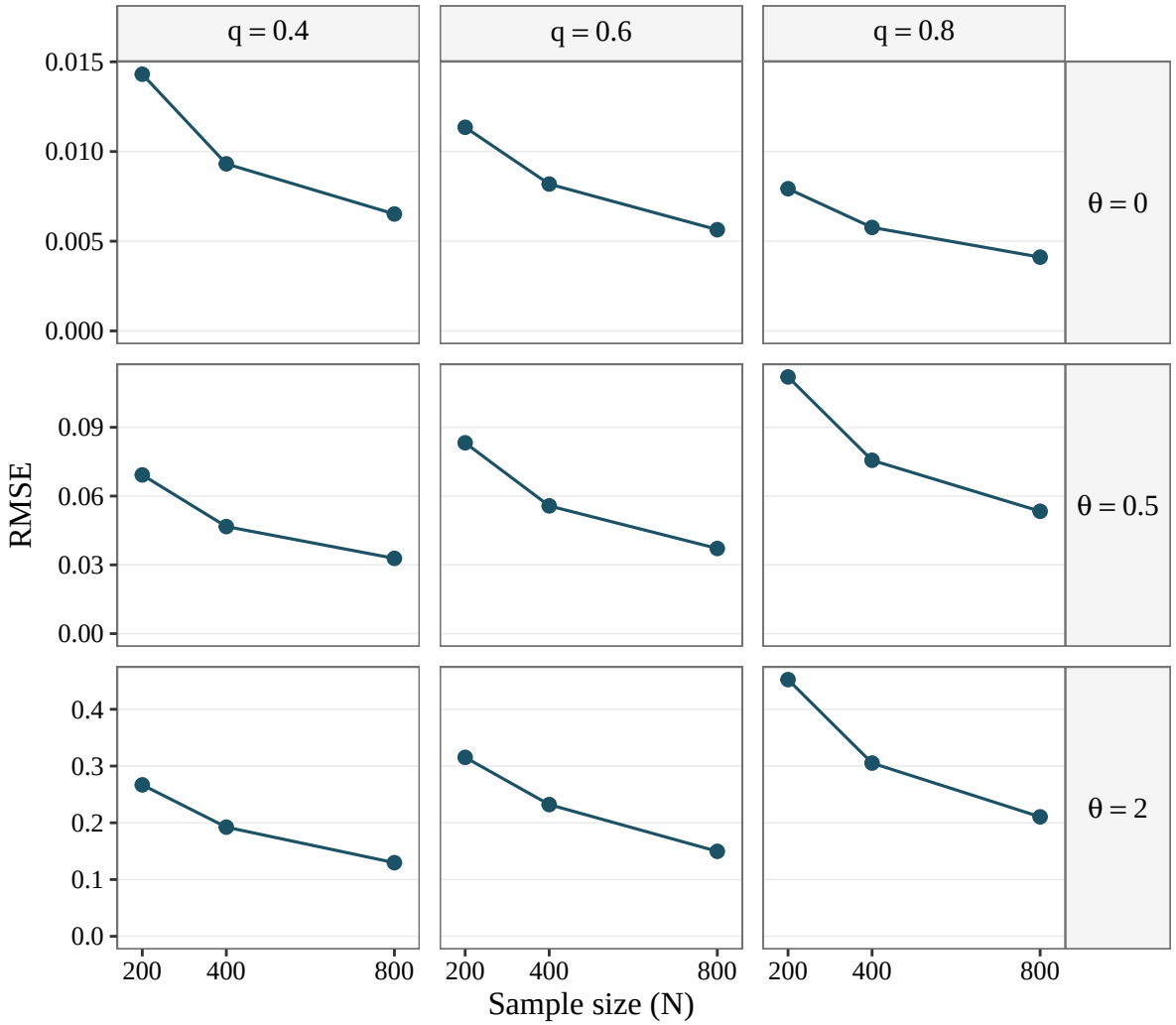


Figure 3. *RMSE of the mZIP conditional indirect effect at* $x = 1.75$.

Note. RMSE = root mean square error. Columns show the excess-zero mixing probability $q = \psi^M(1.75)$, and rows show the true conditional indirect effect $\theta = IE_M(1.75)$. RMSE retains the original outcome scale per unit of the predictor. Vertical scales differ across rows but are shared across columns within each row.

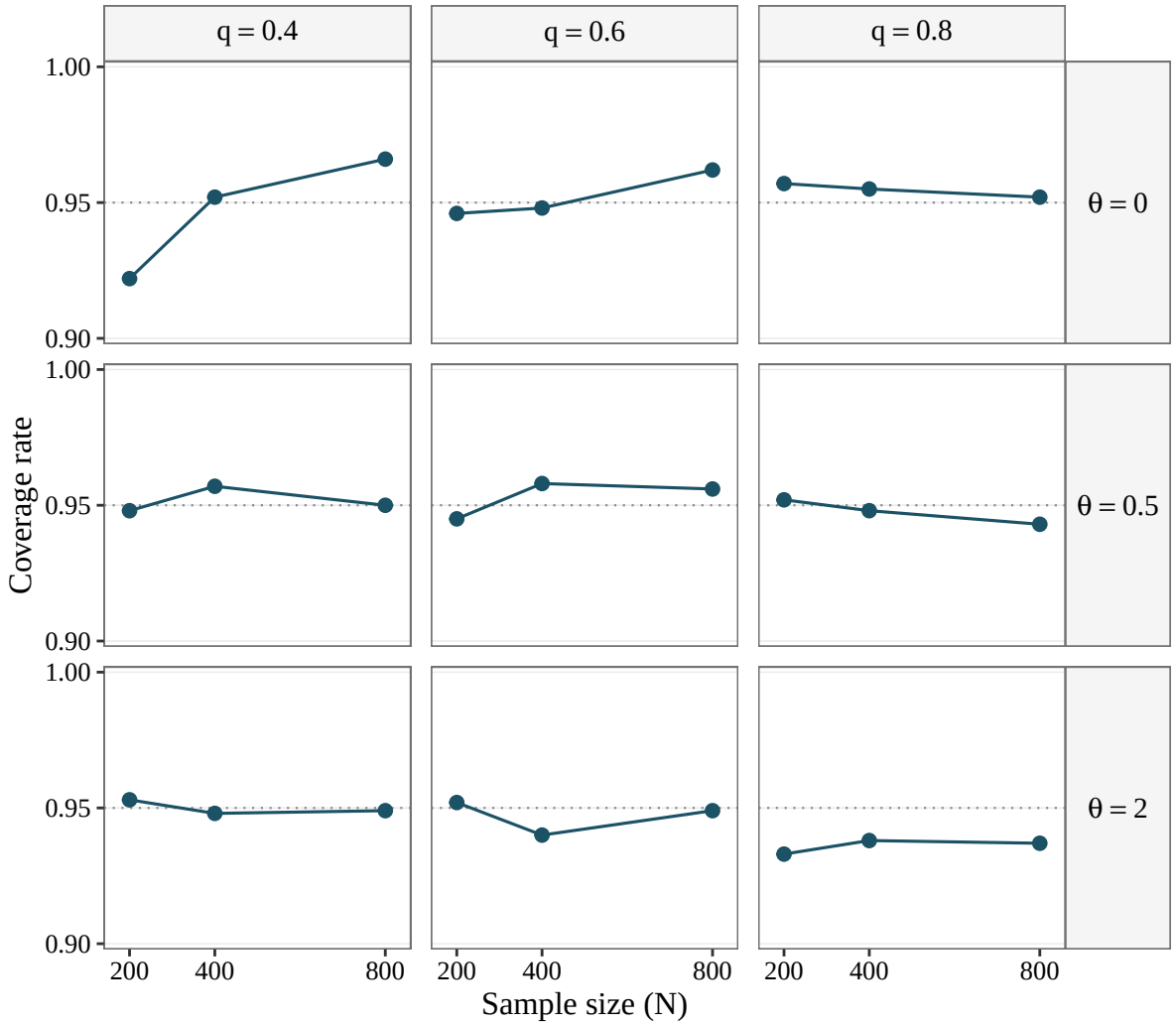


Figure 4. *Coverage of 95% credible intervals for the mZIP conditional indirect effect at* $x = 1.75$.

Note. Columns show the excess-zero mixing probability $q = \psi^M(1.75)$, and rows show the true conditional indirect effect $\theta = IE_M(1.75)$. Coverage is the proportion of equal-tailed 95% credible intervals containing the true effect. The dotted line indicates the nominal coverage rate of 0.95.

4. Empirical Demonstration

This application illustrates the mZIP mediation parameterization within FSEM using data from MFUS, with a latent predictor, an observed continuous mediator, and a zero-inflated count outcome. Two model specifications are considered. The first combines a linear mediator submodel with an mZIP outcome submodel and expresses the conditional indirect effect on the scale of the overall expected outcome count. The second is a conventional linear SEM fitted to the same participants, providing a familiar reference model. The comparison illustrates how distributional assumptions and link functions shape effect interpretation, particularly the distinction between a constant indirect effect under linear SEM and a conditional indirect effect on the expected-count scale under mZIP.

4.1. MFUS

MFUS is a prospective cohort study that began on July 1, 1948, with 3,983 men recruited from the Royal Canadian Air Force. The study initially focused on cardiovascular disease and later expanded to examine the physical, psychological, and social aspects of aging (Tate et al., 2015). Successful Aging Questionnaires (SAQs) were introduced in 1996 and administered again in 2000 and subsequent years. The present application uses complete-case data from the 2000 SAQ.

The empirical analysis examines whether greater latent social engagement is associated with fewer limitations in instrumental activities of daily living (IADLs) indirectly through better mental health, after adjustment for age. This proposed pathway is motivated by research linking social engagement with depressive symptoms in older adults (Glass et al., 2006) and identifying depressive symptoms as a predictor of subsequent deterioration in physical performance (Penninx et al., 1998). Social activity has also been associated with a lower incidence of disability, including IADL disability (James et al., 2011). Because all variables were measured at the same assessment and adjustment was limited to age, the direct and indirect effects are interpreted as associations.

Social engagement was represented by six binary SAQ items: visiting family or relatives (`ACTIV1`), visiting friends or neighbours (`ACTIV2`), other social group activity (`ACTIV5`),

church-related activities (`ACTIV6`), participation in a service, fraternal, or Legion organization (`ACTIV8`), and community volunteer work (`ACTIV9`). These items were specified as reflective indicators of a common social engagement factor. The observed mediator was the SF-36 mental health index (MHI) provided in the dataset, which ranges from 0 to 100, with higher scores indicating better mental health (Ware & Sherbourne, 1992). The outcome was the supplied `IADLScore`, representing the number of IADL limitations and ranging from 0 to 9, with higher values indicating greater functional limitation. Age at questionnaire completion was included as the covariate.

The complete-case analysis included 952 participants. Of these participants, 346 (36.3%) reported no IADL limitations. Figure 5 displays the observed probability-mass distribution of the outcome. To evaluate the concentration at zero without conditioning on predictors, intercept-only Poisson and ZIP models were fitted to `IADLScore`. A Vuong test conducted using the `pscl` package (Zeileis et al., 2008) favored the intercept-only ZIP model over the intercept-only Poisson model, $z = 6.64$, $p < .001$.

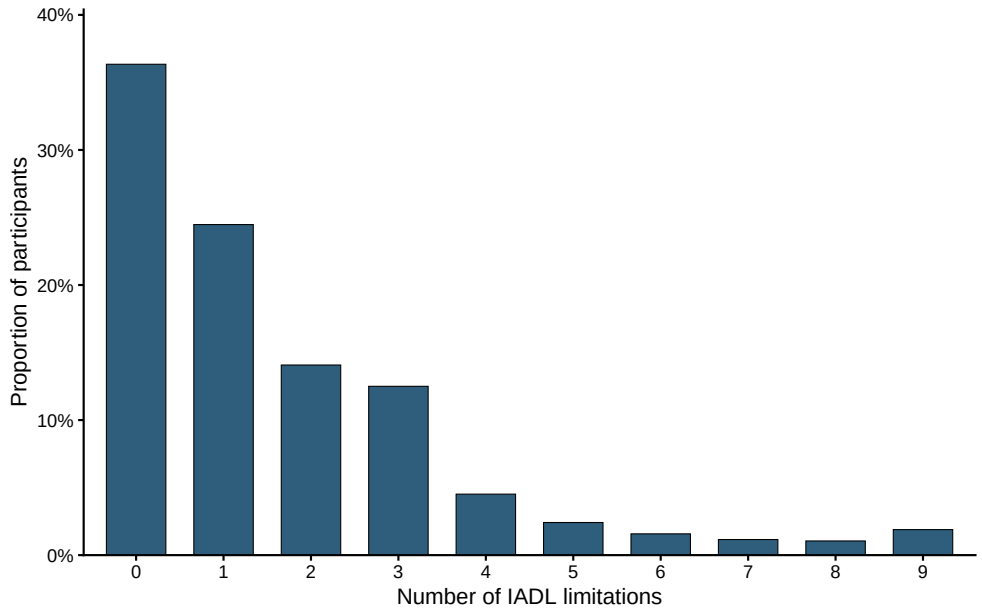


Figure 5. *Observed distribution of IADL limitations in the complete-case analysis sample (N = 952).*

4.2. Model Specification

The empirical model corresponds to Scenario I, with social engagement represented as a latent predictor and mental health retained as an observed continuous mediator. Figure 6 presents the measurement and structural pathways. Let ξ_i denote latent social engagement, $\mathbf{R}_i$ the vector of six activity indicators, M_i mental health, A_i age, and Y_i the IADL count. In both specifications, social engagement was represented by a one-factor categorical confirmatory factor analysis (CFA) model with six binary indicators. Each indicator loaded on the common factor, and no residual correlations among the indicators were specified. The measurement and structural components were estimated jointly. In both models, MHI and age were standardized using their means and standard deviations in the same complete-case sample of 952 participants. The binary indicators and IADL count retained their original scales.

Following the FSEM framework introduced above, the joint distribution of the observed variables and latent predictor, conditional on age, was factored as

$$\begin{aligned} &p(Y_i, M_i, \mathbf{R}_i, \xi_i \mid A_i) \\ &\quad = p_Y(Y_i \mid M_i, \xi_i, A_i)\, p_M(M_i \mid \xi_i, A_i) \left\{ \prod_{j=1}^{6} p_j(R_{ji} \mid \xi_i) \right\} p_\xi(\xi_i \mid A_i). \end{aligned} \tag{4.1}$$

This factorization contains the outcome submodel $p_Y(Y_i \mid M_i, \xi_i, A_i)$, the mediator submodel $p_M(M_i \mid \xi_i, A_i)$, the measurement submodel $\prod_{j=1}^{6} p_j(R_{ji} \mid \xi_i)$, and the conditional latent distribution $p_\xi(\xi_i \mid A_i)$. The measurement submodel links the six binary activity indicators to latent social engagement, whereas $p_\xi(\xi_i \mid A_i)$ describes the distribution of latent engagement conditional on age. The mediator and outcome submodels jointly determine how the marginal expected IADL count changes with social engagement, both directly and indirectly through mental health.

The factor, measurement, and mediator submodels were specified as

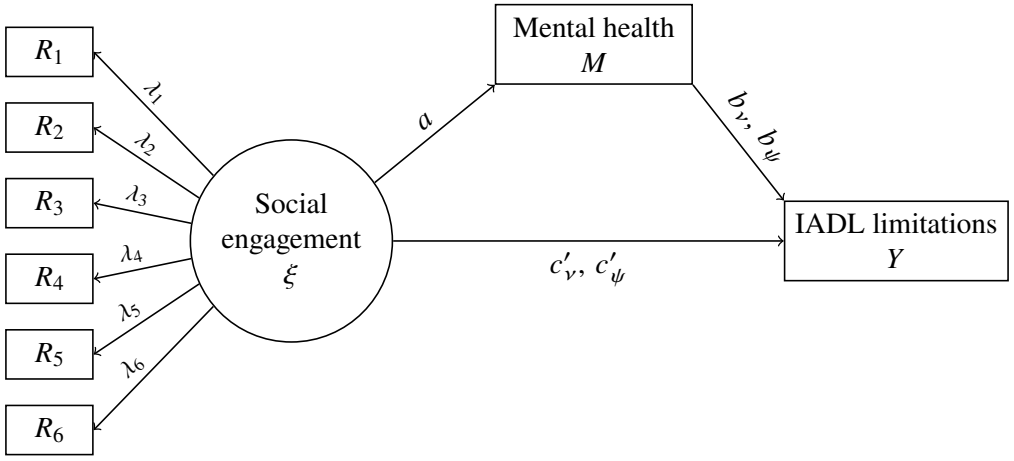


Figure 6. *FSEM mediation model for the MFUS application.*
Note. Standardized age is included in the conditional distribution of engagement, the mental health submodel, and both mZIP outcome equations. These adjustment paths are omitted from the diagram.

$$
\begin{aligned}
\xi_i &= \delta_A A_i + u_i, & u_i &\sim N(0,1),\\
R_{ji} \mid \xi_i &\sim \text{Bernoulli}\{\Phi(\tau_j + \lambda_j \xi_i)\}, & j &= 1,\ldots,6,\\
M_i \mid \xi_i, A_i &\sim N\left(a_0 + a\xi_i + a_A A_i, \sigma_M^2\right),
\end{aligned}
\tag{4.2}
$$

where Φ is the standard normal distribution function. The Bayesian measurement component was a categorical CFA model with a probit link. This link was used because it represents each binary response as a thresholded continuous response with a normally distributed residual, which connects the measurement component to the latent-response formulation of categorical CFA (Muthén, 1984). The indicators were assumed conditionally independent given engagement. The factor intercept was fixed to zero, and its residual variance conditional on age was fixed to one. Thus, $\xi = 0$ represents the model-implied engagement mean at average age, and one engagement unit corresponds to one conditional residual standard deviation. The probit link fixes the residual variance of the underlying continuous response for each binary indicator to one, establishing its response scale. All six factor loadings were constrained to be positive and assigned half-normal priors. This specification assumes that greater engagement is associated with a higher probability of participation in each activity.

The IADL outcome was assigned an mZIP distribution. Consistent with the notation used for Scenario I, its excess-zero probability and marginal mean were specified as

$$\begin{aligned}
\text{logit}(\psi_i^Y) &= \gamma_{0Y} + c'_\psi \xi_i + b_\psi M_i + \gamma_A A_i, \\
\log(\nu_i^Y) &= \alpha_{0Y} + c'_\nu \xi_i + b_\nu M_i + \alpha_A A_i, \\
\mu_i^Y &= \frac{\nu_i^Y}{1 - \psi_i^Y}.
\end{aligned} \tag{4.3}$$

Here, $\nu_i^Y = E(Y_i \mid \xi_i, M_i, A_i)$ is the overall expected IADL count after averaging over the excess-zero and count processes, and μ_i^Y is the mean within the Poisson count process (Long et al., 2014). Engagement, mental health, and age were included in both outcome equations. The mediation pathways of interest were defined through the marginal mean equation. The logistic equation remained part of the outcome distribution but served as a distributional adjustment and did not enter the calculation of the indirect effect on the overall expected-count scale.

The conditional indirect effect follows Equation (2.9), with the latent predictor ξ replacing the observed predictor x:

$$\begin{aligned}
IE_Y(\xi, m, A) &= \frac{\partial E(M_i \mid \xi_i = \xi, A_i = A)}{\partial \xi} \frac{\partial \nu^Y(\xi, m, A)}{\partial m} \\
&= a\, b_\nu \nu^Y(\xi, m, A),
\end{aligned} \tag{4.4}$$

where

$$\nu^Y(\xi, m, A) = \exp\left\{\alpha_{0Y} + c'_\nu \xi + b_\nu m + \alpha_A A\right\}.$$

The corresponding conditional direct effect is

$$DE_Y(\xi, m, A) = \frac{\partial \nu^Y(\xi, m, A)}{\partial \xi} = c'_\nu \nu^Y(\xi, m, A), \tag{4.5}$$

with mental health and age held fixed. To define the local total effect, let $\overline{m}(\xi, A) = a_0 + a\xi + a_A A$ denote the model-implied mediator mean. Applying the chain rule along this mean function gives

$$\begin{aligned} TE_Y(\xi, A) &= \frac{d}{d\xi} \nu^Y \{\xi, \overline{m}(\xi, A), A\} \\ &= \left(c'_\nu + a\, b_\nu\right) \nu^Y \{\xi, \overline{m}(\xi, A), A\}\,. \end{aligned} \tag{4.6}$$

The total effect therefore equals the sum of the direct and indirect effects evaluated along the mediator mean function. These quantities are local, derivative-based associations on the expected-count scale.

For the empirical summary, the effects were evaluated at average age ($A = 0$), the corresponding latent engagement mean ($\xi = 0$), and the model-implied mediator mean ($m = a_0$). The reference expected count was therefore

$$\nu^Y_{\text{ref}} = \exp(\alpha_{0Y} + b_\nu a_0)\,. \tag{4.7}$$

This evaluation follows the conditional-effect framework developed above. It describes the associations at a specified reference profile and does not average the effects over the mediator or latent predictor distributions.

The comparison model retained the same six-indicator factor structure and mediation pathways, with linear conditional means for both MHI and the IADL score:

$$\begin{aligned} E(M_i \mid \xi_{Li}, A_i) &= a_{0L} + a_L \xi_{Li} + a_{AL} A_i, \\ E(Y_i \mid M_i, \xi_{Li}, A_i) &= d_{0L} + c'_L \xi_{Li} + b_L M_i + d_{AL} A_i. \end{aligned} \tag{4.8}$$

Here, M_i denotes standardized MHI and A_i denotes standardized age. The subscript L identifies parameters from the linear model. The engagement factor had marginal variance one and a freely estimated covariance with age. Its direct, indirect, and total effects were c'_L, $a_L b_L$, and $c'_L + a_L b_L$, respectively. These effects are constant additive associations under the specified linear model. The IADL score was treated as continuous, without a separate excess-zero process.

4.3. Model Estimation

The FSEM model, including its categorical CFA measurement component, was estimated using Bayesian estimation in Stan (Carpenter et al., 2017) through CmdStanR. The latent engagement values and model parameters were sampled jointly using the No-U-Turn sampler. The binary-item likelihood was evaluated directly through the probit probabilities, and membership in the excess-zero process was marginalized analytically. A noncentered parameterization expressed engagement in terms of the standard normal variables u_i in Equation (4.2). Joint estimation incorporated uncertainty about latent engagement into the structural coefficients and indirect effects, consistent with the FSEM framework (Enders & Keller, 2026).

The prior distributions were

$$\begin{aligned} \tau_j,\ a_0 &\sim N(0,1), & \lambda_j &\sim N^{+}(0,1), \\ \delta_A,\ a_A,\ \alpha_A &\sim N(0,0.5^2), & \sigma_M &\sim t_3^{+}(0,1), \\ a,\ c'_\nu,\ b_\nu,\ \gamma_A &\sim N(0,0.75^2), & \alpha_{0Y} &\sim N(\log \overline{Y},1), \\ c'_\psi,\ b_\psi &\sim N(0,1), & \gamma_{0Y} &\sim N(0,2^2). \end{aligned} \tag{4.9}$$

The second argument of each normal distribution denotes its variance, superscript + denotes restriction to positive values, and $t_3^{+}(0,1)$ denotes a half-Student-t distribution with three degrees of freedom and unit scale. Priors were independent across the listed parameters. The prior for α_{0Y} was centered at the log of the observed sample mean, $\log \overline{Y}$, and therefore included a data-dependent calibration. The remaining priors regularized the measurement and structural parameters on their specified scales.

Four chains each included 2,000 warm-up iterations and 5,000 retained iterations, yielding 20,000 posterior draws without thinning. Estimates were summarized using posterior means and 95% CrIs. The reference expected count and direct, indirect, and total effects were calculated within each joint posterior draw and then summarized. In particular, the mediator reference value a_0 and all coefficients entering an effect were taken from the same draw. Convergence and

sampling efficiency were assessed using rank-normalized $\widehat{R}$ and effective sample sizes (Vehtari et al., 2021).

The linear comparison model was estimated using mean- and variance-adjusted weighted least squares (WLSMV) in `lavaan`, version 0.7-2 (Rosseel, 2012). WLSMV provided robust standard errors and a mean- and variance-adjusted model test. Confidence intervals (CIs) for the indirect effect were obtained using the first-order delta method.

4.4. Model Results

The reported diagnostics indicated adequate sampling for the summarized Bayesian parameters and derived effects. Their $\widehat{R}$ values rounded to 1.00, and bulk effective sample sizes for the model parameters ranged from approximately 4,392 to 22,186. There were no post-warm-up divergences or maximum-tree-depth hits. For the linear comparison model, the adjusted test was $\chi^2(24) = 76.091$, $p < .001$. The scaled CFI and TLI were 0.920 and 0.879, respectively. The scaled RMSEA was 0.048, with a 90% CI of [0.036, 0.060], and SRMR was 0.079.

Table 3 presents the selected measurement and structural coefficients. In the FSEM model, greater engagement was associated with better mental health, $a = 0.305$ (95% CrI [0.213, 0.397]). Because MHI was standardized, this coefficient represents a 0.305-standard-deviation increase in expected MHI per unit of engagement, holding age fixed. Better mental health was associated with a lower overall expected IADL count, $b_\nu = -0.216$ (95% CrI [−0.279, −0.149]), conditional on engagement and age. The direct engagement coefficient in the marginal mean equation was also negative, $c'_\nu = -0.332$ (95% CrI [−0.435, −0.229]). The two outcome coefficients are on the log-mean scale. The linear model yielded the same directions of association. Although both models used standardized MHI and age, their structural coefficients should not be compared directly in magnitude because the engagement factors have different scaling and the outcome coefficients are expressed on additive and log-mean scales, respectively.

At the reference profile, the overall expected IADL count was $\nu^Y_{\text{ref}} = 1.38$ (95% CrI [1.28, 1.48]). The conditional indirect effect was −0.090 (95% CrI [−0.125, −0.060]). Its

Table 3. *Selected measurement and structural coefficients for the MFUS application.*

Parameter	**Linear SEM** Estimate [95% CI]	**FSEM (mZIP)** Mean [95% CrI]
Measurement loadings		
Family or relatives	0.370 [0.238, 0.502]	0.309 [0.158, 0.469]
Friends or neighbours	0.763 [0.633, 0.893]	0.763 [0.537, 1.040]
Other social groups	0.466 [0.364, 0.567]	0.524 [0.381, 0.680]
Church activities	0.438 [0.335, 0.541]	0.495 [0.349, 0.656]
Service/fraternal/Legion	0.534 [0.434, 0.634]	0.638 [0.479, 0.818]
Volunteer work	0.602 [0.495, 0.709]	0.784 [0.556, 1.060]
Mediator and outcome-mean pathways		
Engagement → MHI	0.291 [0.201, 0.382]	0.305 [0.213, 0.397]
MHI → IADL	−0.550 [−0.653, −0.447]	−0.216 [−0.279, −0.149]
Engagement → IADL	−0.482 [−0.659, −0.304]	−0.332 [−0.435, −0.229]
Age → MHI	−0.045 [−0.107, 0.017]	−0.041 [−0.106, 0.025]
Age → IADL	0.460 [0.362, 0.559]	0.247 [0.185, 0.309]
Excess-zero equation		
Intercept, γ_{0Y}	—	−1.860 [−2.490, −1.400]
Engagement, c'_{ψ}	—	−0.805 [−1.340, −0.342]
MHI, b_{ψ}	—	0.794 [0.453, 1.200]
Age, γ_A	—	−0.245 [−0.523, 0.019]

Note. MHI and age were standardized within the common complete-case sample (N = 952) using means of 82.35 and 80.07 and standard deviations of 14.34 and 3.38, respectively. Both models use these same standardized variables. The excess-zero coefficients are reported to document the distributional component used to accommodate additional zeros and are not interpreted as substantive pathways.

negative direction reflects the combination of a positive association between engagement and mental health and a negative association between mental health and the overall expected IADL count. Under the derivative-based interpretation, the indirect effect is the slope of the expected count through the mediator pathway, expressed per unit of engagement.

The conditional direct effect was −0.457 (95% CrI [−0.593, −0.317]). The local total effect, evaluated along the mediator mean function, was −0.547 (95% CrI [−0.674, −0.417]). All three intervals excluded zero. These effects describe local changes in the expected number of limitations.

For the linear comparison, the direct effect was −0.482 (95% CI [−0.659, −0.304]), the indirect effect was −0.160 (95% CI [−0.214, −0.107]), and the total effect was −0.642 (95% CI [−0.813, −0.471]). Under that model, a one-standard-deviation increase in engagement

Table 4. *Direct, indirect, and total associations under the two outcome specifications.*

Effect	**Linear SEM** Additive effect [95% CI]	**FSEM (mZIP)** Reference derivative [95% CrI]
Direct	−0.482 [−0.659, −0.304]	−0.457 [−0.593, −0.317]
Indirect	−0.160 [−0.214, −0.107]	−0.090 [−0.125, −0.060]
Total	−0.642 [−0.813, −0.471]	−0.547 [−0.674, −0.417]

was associated with 0.160 fewer expected limitations through mental health, with direct and total associations of 0.482 and 0.642 fewer limitations, respectively. These additive effects are constant across values of engagement, mental health, and age. Under the mZIP specification, the conditional effects on the expected-count scale vary with the predicted overall expected count.

The excess-zero equation was included as an auxiliary distributional component to accommodate zeros beyond those implied by the Poisson count process. Its coefficients are reported in Table 3 to document the fitted model, but they were not targets of substantive interpretation and do not enter the direct, indirect, or total effects defined through the marginal mean.

5. Discussion

This article has integrated mZIP marginal-mean modeling with derivative-based mediation and a measurement model for a latent predictor within the FSEM framework of Enders and Keller (2026). We presented the formulation of conditional indirect effects when either the mediator or the outcome is a zero-inflated count and showed how the measurement and structural submodels can be estimated jointly. The simulation study evaluated conditional indirect-effect estimation with an observed predictor and an mZIP mediator. The empirical application examined the indirect association between latent social engagement and IADL limitations through observed mental health.

A central contribution is connecting marginalized zero-inflated modeling with the derivative-based approach to mediation. O'Rourke and Vazquez (2019) described conditional mediated effects for zero-inflated count outcomes, and O'Rourke and Han (2023) clarified that the excess-zero and count components require different derivatives because they use different link functions.

Building on this work, the present study applies the same response-scale logic to the marginal mean parameterization introduced by Long et al. (2014). The resulting indirect effect concerns the overall expected count across the two latent processes.

The choice among linear SEM, traditional ZIP, and mZIP should be guided by the research question and assumptions about how the outcome arises. In the specified linear model, the outcome is treated as continuous and its conditional mean is additive, giving the constant indirect effect $a_L b_L$ (MacKinnon, 2008). This effect represents the same additive change through the mediator across levels of the predictor and covariates. In contrast, the mZIP model represents the count outcome as a mixture of excess zeros and Poisson counts and uses a log link for the overall expected count. Its response-scale indirect effect, $ab_\gamma \nu^Y(\xi, m, A)$, is a local rate of change evaluated at a specified reference profile. Its magnitude therefore depends on the predicted overall expected count. The linear and mZIP analyses target different forms of the indirect association, reflecting their different assumptions about the outcome and its conditional mean.

Traditional ZIP and mZIP share the same excess-zero and Poisson mixture structure, but their regression coefficients address different quantities (Long et al., 2014). When the research question concerns the excess-zero process and event frequency within the latent count process, the component-specific approach of O'Rourke and Han (2023) provides relevant information. When the question concerns an indirect association with the overall expected count across both processes, the mZIP parameterization provides a direct formulation of that quantity. Researchers interested in both questions may report both types of effects, provided their interpretations are distinguished.

The simulation results showed that estimation of nonzero conditional indirect effects became more accurate as sample size increased. At the population mean of the predictor, downward bias and RMSE were greater at higher excess-zero mixing probabilities, particularly in smaller samples. Coverage of the 95% CrIs ranged from 0.914 to 0.958 for nonzero effects, with the lowest coverage occurring at the smaller sample size and highest excess-zero mixing probability.

These results suggest that larger samples are particularly useful for estimating conditional indirect effects when the mediator has a high excess-zero mixing probability.

The mZIP mediation parameterization also fits within an existing framework that accommodates latent variables. Following Enders and Keller (2026), FSEM links the measurement, mediator, and outcome submodels while allowing each to have an appropriate distribution and link function. In the empirical example, six binary indicators measured social engagement, whereas mental health and IADL limitations followed normal and mZIP submodels, respectively. Joint estimation incorporated uncertainty about engagement into the structural coefficients and indirect effects. This formulation is useful when the constructs involved in mediation cannot be measured directly, and it provides a foundation for models with latent mediators or multiple constructs.

The causal mediation literature provides a complementary interpretation based on exposure contrasts. Natural direct and indirect effects describe changes between selected exposure values, accounting for the corresponding counterfactual mediator distributions (Cheng et al., 2018; Wang & Albert, 2012). This interpretation has been developed for population-average effects using mZIP models (Sims et al., 2024a, 2024b) and for zero-inflated mediators using Bayesian estimation (Cui et al., 2026). In comparison, the effects defined here describe local rates of change along the mediated pathway. The choice between these approaches depends on whether the research question concerns a local indirect association or a natural effect for an exposure contrast. Researchers seeking causal interpretations need to define the relevant causal estimands and justify their identification assumptions, including assumptions about confounding, consistency, and positivity (Imai et al., 2010; VanderWeele, 2016).

There are several avenues for future research. One extension is to replace the Poisson count process with an NB distribution. Although the mZIP mixture already permits the overall variance to exceed the overall mean, its count process retains the Poisson mean–variance relationship. Additional variation within that process may require a more flexible distribution. The mZINB model developed by Preisser et al. (2016) provides a basis for this extension while retaining direct regression on the overall expected count. For the two scenarios considered here, retaining

the same mean functions would preserve the algebraic form of the derivative-based indirect effects, although statistical inference would depend on the expanded distributional model.

A second extension concerns longitudinal mediation. The mZIP models have been extended to correlated observations using random effects, providing a starting point for repeated-measures applications (Li & Luo, 2026; Long et al., 2015). Combining these models with FSEM could allow researchers to examine whether changes in a predictor precede changes in a mediator and subsequent count outcomes. Such models would require explicit temporal ordering, appropriate treatment of dependence, and separation of within-person and between-person associations.

Acknowledgments

OpenAI Codex (https://openai.com/codex/, used in November 2026) assisted with grammar and language editing, LaTeX formatting, drafting and revising portions of the abstract and discussion, and preparation and verification of analysis code. The authors take responsibility for the final manuscript and code.

Funding Statement

No funding was received for this study.

Competing Interests

All authors declare no competing interests.

Data Availability Statement

Simulation and empirical analysis code are provided in Supplemental Material and will be shared in a public repository. The MFUS data are not publicly available because they contain sensitive participant-level health information. Data access may be requested from the corresponding author, subject to participant confidentiality protections.

Ethical Standards

Research ethics approval for the empirical analysis was obtained under REB file number HS25880 (H2023:059).

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