Additional file 1: Supplementary tables

Quantum amplitude amplification of improbable futures · Peter Maurice Catt

Table S1. Rare sets for horizons 2 to 6 ($\delta = 1.5$, start state 1)

H	Futures	Rare	p_{rare}	r^*	Ideal success	Nearest future to threshold (bits)	b for zero mismatches
2	25	21	0.1227	2	0.953	6.4×10^{-2}	1
3	125	109	0.0860	2	0.993	2.0×10^{-2}	2
4	625	527	0.0453	3	0.995	9.6×10^{-3}	3
5	3125	2731	0.0375	4	0.967	1.4×10^{-4}	13
6	15625	13888	0.0250	4	0.980	5.7×10^{-4}	7

$r^* = \lfloor \pi/(4\theta) \rfloor$ and ideal success is $\sin^2[(2r^* + 1)\theta]$. The last column is the lowest b found by checking all futures. Rare futures are the majority by count at every horizon but carry little of the probability.

Table S2. Fixed-point precision at $H = 6$

b	Threshold T_b	Accumulator qubits m	Offset c	Total qubits	Mismatched futures	Misclassified mass
1	32	6	0	24	144	2.3×10^{-3}
2	64	7	0	25	60	9.4×10^{-4}
3	128	8	0	26	82	1.3×10^{-3}
4	255	9	1	27	9	1.4×10^{-4}
5	510	10	2	28	12	2.0×10^{-4}
6	1019	11	5	29	37	6.0×10^{-4}
7	2038	12	10	30	0	0
8	4076	13	20	31	0	0
9	8152	14	40	32	2	3.2×10^{-5}
10	16304	15	80	33	0	0

Mismatches compare the fixed-point test with the real-valued rare-set definition over all 15,625 futures.

Table S3. Logical cost of the general circuit (all-to-all connectivity)

H	Precision	b	Qubits	Preparation 2q	Oracle 2q	Reflection 2q	Iteration 2q	Iterations	Total 2q
2	fixed	4	14	62	2441	152	2593	2	5248
2	exact	1	11	62	1453	152	1605	2	3272
3	fixed	4	17	118	4437	288	4725	2	9568
3	exact	2	15	118	3309	288	3597	2	7312
4	fixed	4	21	174	7281	424	7705	3	23289
4	exact	3	20	174	6457	424	6881	3	20817
5	fixed	4	24	230	9529	560	10089	4	40586
5	exact	13	33	230	19725	558	20283	4	81362
6	fixed	4	27	286	11777	696	12473	4	50178
6	exact	7	30	286	15933	696	16629	4	66802

Basis $\{R_z, \sqrt{X}, X, \text{CNOT}\}$, optimisation level 3. Fixed precision is $b = 4$; exact precision is the lowest b with zero mismatches. Preparation is the state-preparation circuit A; reflection is $AS_0A^\dagger$, which holds two preparations and the reflection about zero; an iteration is the oracle plus the reflection. Total counts one preparation plus the stated number of iterations.

Table S4. Predicted oracle-minus-control signal (noise models from IBM calibration snapshots retrieved on 29 October 2026)

H	r	Backend	Routed 2q, oracle / control	Ideal $P(\text{rare})$	Noisy, oracle	Noisy, control	Signal	z
2	1	Fez	228 / 228	0.773	0.700	0.344	+0.356	71.3
2	1	Kingston	228 / 228	0.773	0.708	0.300	+0.408	81.7
2	1	Marrakesh	228 / 228	0.773	0.698	0.346	+0.352	70.4
2	2	Fez	440 / 440	0.953	0.763	0.467	+0.296	60.9
2	2	Kingston	440 / 440	0.953	0.787	0.431	+0.356	73.0
2	2	Marrakesh	440 / 440	0.953	0.758	0.468	+0.290	59.5
3	1	Fez †	1168 / 1114	0.607	0.719	0.673	+0.046	10.0
3	1	Kingston	1168 / 1166	0.607	0.667	0.607	+0.060	12.5
3	1	Marrakesh †	1168 / 1114	0.607	0.717	0.678	+0.039	8.6
3	2	Fez †	2122 / 2020	0.993	0.804	0.785	+0.019	4.7
3	2	Kingston †	2122 / 2020	0.993	0.783	0.738	+0.045	10.7
3	2	Marrakesh †	2122 / 2020	0.993	0.805	0.786	+0.019	4.6

† Control and oracle differ by more than 2% in routed two-qubit count. Means over 3 repeats of 20,000 shots per arm; z is the pooled two-proportion statistic of the stated test. The ideal control probability equals p_{rare}: 0.1227 at $H = 2$ and 0.0860 at $H = 3$. At $r = 0$ the two arms are identical circuits; the largest mean difference, 3.3×10^{-3}, is consistent with sampling error.

Table S5. Sensitivity of the six-step rare set to the rarity offset δ

δ (bits/step)	Rare futures	p_{rare}	I (bits/step)	r^*	Ideal success	Gap g (bits)	Mismatches at $b = 4$	Lowest exact b
1.00	15207	0.0910	0.180	2	0.999	1.4×10^{-2}	0	4
1.25	14586	0.0443	0.293	3	0.993	2.9×10^{-3}	12	9
1.50	13888	0.0250	0.439	4	0.980	5.7×10^{-4}	9	7
1.75	12353	0.0107	0.618	7	1.000	4.3×10^{-3}	19	7
2.00	10575	0.0045	0.834	11	0.999	5.7×10^{-4}	12	7

$H = 6$, start state 1, 15,625 futures throughout. I is the large-deviation rate of equation (5); r^* is the first amplification maximum and ideal success its $\sin^2[(2r^* + 1)\theta]$; the gap is the distance between the threshold and the nearest future. The row for $\delta = 1.5$ is the setting of the paper.

Table S6. Sensitivity of the predicted signal to the matched control's random phases

H	Unused indices	Seeds	Kept	Mean signal	SD	Range	95% interval	SD / shot-noise SE	Lowest z	Sign changes
2	7 of 32	24	24	+0.4093	0.0041	+0.4009 to +0.4185	[+0.4018, +0.4145]	1.29	80	0
3	3 of 128	24	24	+0.0508	0.0029	+0.0434 to +0.0553	[+0.0467, +0.0551]	0.90	9	0

One amplification iteration under the Qiskit FakeKingston noise model, 20,000 shots per arm, each control given independent uniform random phases on the unused indices, compiled to the oracle's layout and kept only if its routed two-qubit count lay within 2% of the oracle's. The oracle arm is the mean of three repeats. Criteria were fixed and hashed before the results were seen; the specification and the per-seed outputs are deposited with the data. The model reproduces the two-step routed count of the hardware run exactly (228) and the three-step count to within 4% (1,127 against 1,168); like the models of Table S4 it omits leakage, so it populates the unused indices by depolarising error alone.