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Within-Coupon Clean-Price Dynamics: A Three-Region Classification for Fixed-Rate Bonds

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Abstract

Clean bond prices are conventionally obtained by subtracting accrued interest from full (dirty) prices. Even when yield is constant, this quotation convention can generate within-coupon price dynamics that differ from the familiar coupon-date convergence-to-par pattern. This paper derives a three-region classification based on the ratio of the coupon payment to the bond value immediately after a coupon date. For sufficiently deep discount bonds, the clean price increases monotonically during the coupon interval; for sufficiently deep premium bonds, it decreases monotonically; and for an intermediate near-par region—including mildly discounted, par, and mildly premium bonds—the clean price is U-shaped with a unique interior minimum. The classification provides closed-form thresholds and a closed-form turning point. The results matter for interpretation of quoted price changes, portfolio valuation, return attribution, and P&L analysis because deterministic passage-of-time effects can produce clean-price reversals without any change in yield or credit quality.

1 Introduction

A standard fixed-income result is that, holding yield constant, premium bond values decline toward par and discount bond values rise toward par as maturity approaches. That statement is naturally expressed at coupon dates, when accrued interest is zero and clean and dirty prices coincide. In trading and portfolio valuation, however, bonds are observed on dates between coupon payments. On those dates, the quoted clean price is obtained by removing accrued interest from the full economic value, and the two components evolve according to different accumulation rules.

The distinction is practically relevant because a clean-price change is not, by itself, evidence of a change in required yield, credit quality, or market conditions. With yield held fixed, the dirty price compounds as the next coupon payment approaches, while conventional accrued interest is allocated linearly under the applicable day-count convention. Their difference can therefore move in a direction that differs from the coupon-to-coupon convergence pattern. This deterministic component is relevant whenever clean prices are used in valuation reports, performance measurement, market-data reconciliation, or P&L attribution.

Prior work has examined bond valuation between coupon dates and the clean/dirty-price distinction (Martin and Wood 1997; Boyles et al. 2005; Vicknair 2012; Gottschalk 2018; Crack 2024). The contribution here is to organize the within-coupon clean-price path into three analytically defined regions and to identify exact threshold conditions separating them. The resulting classification answers a narrower question than convergence to par: when does the coupon-date direction persist throughout the coupon interval, and when must the quoted clean price reverse direction before the next coupon date?

The main result is simple. Let q denote the coupon payment divided by the bond value immediately after a coupon payment, and let $y > 0$ be the effective yield per coupon period. If q is sufficiently low, the clean price rises monotonically; if q is sufficiently high, it falls monotonically; and for an intermediate range around par, it is U-shaped. The intermediate region is nonempty for every positive yield and contains

mildly discounted, exactly par, and mildly premium bonds. The framework thus refines the conventional premium/par/discount taxonomy without altering the standard coupon-date convergence result.

The remainder of the paper derives the clean-price function and its convexity, establishes the three-region classification, relates it to the conventional premium/par/discount categories, and discusses implications for fixed-income valuation and portfolio management.

2 Setup and clean-price function

Consider a fixed-rate coupon bond immediately after a coupon payment. Let P_0 denote the bond value at that instant, C the coupon payment per period, $y > 0$ the constant effective yield per coupon period, and $t \in [0,1]$ the fraction of the current coupon period that has elapsed. Because accrued interest is zero immediately after the coupon payment, P_0 is also the clean price at $t = 0$.

Figure 1. *Timing of coupon payments within the current coupon period and the remaining life of the bond.*

With yield held constant, the full or dirty value accumulates during the coupon period at the required rate. Hence

$$Pd(t) = P_0(1 + y)^t. \quad (1)$$

Under the linear accrued-interest approximation, accrued interest is

$$AI(t) = Ct. \quad (2)$$

The quoted clean price is therefore

$$Pc(t) = P_0(1 + y)^t - Ct. \quad (3)$$

Equation (3) isolates the deterministic passage-of-time component of the clean price. The first term is exponential in t, whereas the accrued-interest term is linear. No yield change is required for the clean price to exhibit curvature or a reversal within the coupon period.

3 Convexity and the three possible path shapes

Differentiating Eq. (3) with respect to t gives

$$Pc'(t) = P_0(1 + y)^t \ln(1 + y) - C, \quad (4)$$

and

$$Pc''(t) = P_0(1 + y)^t[\ln(1 + y)]^2 > 0. \quad (5)$$

Proposition 1

For y > 0 and linear accrued interest, the clean-price function is strictly convex between consecutive coupon dates.

Strict convexity implies that Pc'(t) is strictly increasing. Consequently, only three within-period shapes are possible: the clean price can increase throughout, decrease throughout, or decrease initially and subsequently increase. An interior maximum is impossible. This shape restriction is the basis for the classification below.

4 A three-region classification

The endpoint slopes determine the entire path. At the beginning of the coupon interval,

$Pc'(0) = P_0 \ln(1 + y) - C$, (6)

while immediately before the next coupon date,

$Pc'(1-) = P_0(1 + y) \ln(1 + y) - C$. (7)

Normalize the coupon payment by the beginning-of-period bond value:

$q = C / P_0$. (8)

The signs of the two endpoint slopes then depend only on q and y.

4.1 Region I: deep discount - monotonic increase

If the slope is nonnegative at t = 0, strict convexity ensures that it is nonnegative thereafter. Thus the clean price is monotonically increasing when

$q \le \ln(1 + y)$. (9)

Here the coupon is sufficiently small relative to the beginning-of-period bond value that the marginal compound growth in the dirty price dominates the linear accrual term throughout the interval. In this region, the direction implied by coupon-to-coupon convergence also holds at every point within the coupon period.

4.2 Region II: near par - U-shaped path

If the slope is negative at the beginning and positive at the end, the strictly increasing derivative crosses zero exactly once. This occurs when

$\ln(1 + y) < q < (1 + y) \ln(1 + y)$. (10)

The clean price then falls initially, reaches a unique interior minimum, and rises thereafter. The term near par is used analytically: it denotes the intermediate q-region in which neither compound dirty-price

growth nor linear accrued interest dominates over the entire coupon interval. Since $\ln(1 + y) < y < (1 + y)\ln(1 + y)$ for $y > 0$, an exact par bond with $q = y$ lies strictly inside this region; the region also contains mildly discounted and mildly premium bonds.

The turning point satisfies $Pc'(t^*) = 0$, giving

$$t^* = \ln[q / \ln(1 + y)] / \ln(1 + y). \quad (11)$$

Condition (10) is exactly the condition ensuring $0 < t^* < 1$.

4.3 Region III: deep premium - monotonic decrease

If the slope is nonpositive immediately before the next coupon date, it is nonpositive throughout the interval. Hence the clean price is monotonically decreasing when

$$q \geq (1 + y) \ln(1 + y). \quad (12)$$

In this region, the coupon is sufficiently large relative to the beginning-of-period value that linear accrued interest dominates the marginal compound-growth component throughout the period. The clean-price direction therefore agrees continuously with the conventional premium-bond convergence intuition.

Table 1
Three-region classification of within-coupon clean-price dynamics

Region	Condition	Clean-price path	Economic interpretation
Deep discount	$q \leq \ln(1 + y)$	Monotonically increasing	Dirty-price growth dominates linear accrual throughout
Near par	$\ln(1 + y) < q < (1 + y)\ln(1 + y)$	U-shaped; unique minimum	Dominance changes within the coupon period
Deep premium	$q \geq (1 + y)\ln(1 + y)$	Monotonically decreasing	Linear accrual dominates dirty-price growth throughout

5 Relation to the conventional premium/par/discount classification

The conventional and three-region classifications answer different questions. The conventional classification determines whether a bond is priced above, at, or below par and, under a constant yield, the direction in which values observed at successive coupon dates converge toward par. The three-region classification determines the sign pattern of the clean-price derivative inside one coupon interval.

The two classifications can be linked through q. Let the coupon rate per period be $c = C/F$. For a fixed-rate coupon bond immediately after a coupon payment, the sign of $q - y$ is the same as the sign of $c - y$, and therefore also the same as the sign of $P_0 - F$, where F denotes face value. This equivalence follows directly from

$$q - y = (C - yP_0) / P_0,$$

To determine the sign of the numerator, recall that immediately after a coupon payment, with n coupon payments remaining, the bond value equals the present value of the remaining coupon annuity plus the present value of the face value. That is,

$P_0 = C\,[1-(1+y)^{-n}] / y + F\,(1+y)^{-n}$.

Multiplying both sides by y gives

$yP_0 = C[1-(1+y)^{-n}] + yF(1+y)^{-n}$.

Therefore,

$C - yP_0 = (C - yF)(1+y)^{-n}$.

Because $P_0 > 0$, $F > 0$, and $(1+y)^{-n} > 0$, the sign of $C - yP_0$ is the same as the sign of $C - yF$. Since $c = C/F$, it follows that

$\operatorname{sgn}(q - y) = \operatorname{sgn}(C - yF) = \operatorname{sgn}(c - y)$.

Hence, $q < y$ corresponds to a discount bond, $q = y$ to a par bond, and $q > y$ to a premium bond. Because the near-par interval in (10) contains y, it necessarily includes portions of both the discount and premium categories.

This observation clarifies why the usual labels do not determine within-period monotonicity. A mildly discounted bond can begin the coupon interval with a falling clean price before reaching a minimum and then rising. Similarly, a mildly premium bond can also decline initially, reach an interior minimum, and then rise before the next coupon date. Thus, a discount bond need not rise at every instant, and a premium bond need not fall at every instant. The standard convergence result remains correct for the discrete sequence of coupon-date values.

Figure 2 provides a visual comparison of the three within-coupon clean-price regimes—deep discount, near par, and deep premium—across successive coupon intervals. It highlights how these paths relate to, but differ from, the conventional coupon-date convergence-to-par pattern.

6 Numerical illustration

Let the effective yield per coupon period be y = 2%. The two thresholds are ln(1.02) = 1.9803% and 1.02 ln(1.02) = 2.0199%. The near-par region is therefore narrow but nonzero. Table 2 uses normalized beginning-of-period value $P_0 = 1$ and three q values to illustrate the classification.

Table 2
Numerical illustration for y = 2% per coupon period

Case	$q = C/P_0$	Initial slope	End slope	Path
Deep discount	1.950%	+ 3.03 bp/period	+ 7.00 bp/period	Increasing
Near par	2.000%	−1.97 bp/period	+ 2.00 bp/period	U-shaped
Deep premium	2.050%	−6.97 bp/period	−3.00 bp/period	Decreasing

Table 2 illustrates how the signs of the endpoint slopes determine the within-coupon clean-price path. For the deep-discount case, both the initial and end-of-period slopes are positive, so the clean price rises throughout the coupon interval. For the near-par case, the initial slope is negative while the end-of-period slope is positive; because the clean-price function is strictly convex, the slope crosses zero exactly once, producing a U-shaped path with a unique interior minimum. For the deep-premium case, both endpoint slopes are negative, so the clean price declines throughout the interval. Thus, the numerical examples provide direct illustrations of the three analytical regions defined in equations (9)–(12). (Note: Slopes are expressed in basis points of beginning-of-period bond value per coupon period; 1 basis point = 0.01%.)

Note

All three paths are normalized to zero at t = 0; the figure compares within-period price changes rather than absolute bond prices.

The figure reports clean-price changes relative to the beginning of the coupon interval in basis points. The near-par path exhibits a small but deterministic decline and reversal even though yield is unchanged. The size of the effect is modest for ordinary yields, but the sign reversal is systematic and can matter for the interpretation of short-horizon quoted price changes.

6.1 Market illustration

A market quotation provides a useful illustration of the near-par case. The Marshall County Alabama Board of Education Special bond (CUSIP 572024AU1), with a 4.000% coupon rate and a March 1, 2041 maturity date, traded several times on June 18, 2026 at a quoted clean price of 99.99 while the reported yield was 4.000%. Thus, based on the published quotation, the reported coupon rate and reported yield were equal even though the quoted clean price was slightly below the conventional par value of 100.

Because published yields are rounded, the underlying unrounded yield may differ slightly from exactly 4.000%. Accordingly, the observation should be interpreted as a market illustration rather than an exact empirical test of the constant-yield par-bond result. Table 3 reports the relevant trading history.

Table 3
Trading history for Marshall County Alabama Board of Education Special bond (CUSIP 572024AU1)

Trade date	Trade time	Maturity date	Coupon	Price	Yield	Trade amount	Trade type
2026-06-18	11:45am	2041 March	4.000%	99.99	4.000	$25,000	Investor bought
2026-06-18	11:43am	2041 March	4.000%	99.99	4.000	$25,000	Investor bought
2026-06-18	11:41am	2041 March	4.000%	99.99	4.000	$50,000	Investor bought
2026-06-18	11:35am	2041 March	4.000%	99.031	4.088	$100,000	Inter-dealer
2026-06-18	11:35am	2041 March	4.000%	98.535	4.134	$100,000	Investor sold

Source: MunicipalBonds.com (2026*), Marshall County Alabama Board of Education Special, CUSIP 572024AU1, trading history.*

7 Practical implications for fixed-income analysis

7.1 Interpretation of quoted price changes

Clean prices are designed to remove accrued interest from quoted bond prices, but they do not remove all deterministic passage-of-time effects. In the near-par region, a small negative clean-price change early in the coupon period can occur with an unchanged yield and unchanged expected cash flows. Conversely, a positive clean-price change later in the same period need not signal improving credit or falling required yields. The three-region classification provides a benchmark for distinguishing this mechanical component from economically informative price changes.

7.2 Portfolio valuation and P&L attribution

For portfolio managers and risk systems, the result reinforces the need to separate clean-price return, accrued carry, and yield-driven mark-to-market effects. A valuation or attribution process that implicitly expects clean prices to move monotonically toward the next coupon-date value can misclassify deterministic within-period movements. The threshold test in equations (9)–(12) is inexpensive to implement because it requires only the current coupon payment, clean value immediately after the coupon date, and periodic yield.

7.3 Benchmarking and data-quality controls

The classification can also serve as a diagnostic check for pricing and market-data systems. Under the maintained constant-yield benchmark, the sign pattern of the clean-price slope is fully determined by q and y. Observed movements that differ materially from the benchmark can then be attributed to yield

changes, credit-spread changes, liquidity effects, day-count details, or data issues rather than to the mechanical clean-price convention itself.

8 Robustness, limitations, and extensions

The analysis deliberately uses a constant effective yield per coupon period and linear accrued interest. These assumptions isolate the deterministic mechanism. Actual bond markets apply instrument-specific day-count conventions such as Actual/Actual and 30/360. Within each segment over which accrued interest is linear in calendar time, the same logic applies, but exact thresholds in clock time can vary with the accrual convention and coupon schedule.

If required yield changes during the coupon period, observed clean-price movements combine the deterministic passage-of-time effect derived here with the usual duration/convexity response to yield changes. A natural extension is therefore to decompose short-horizon clean-price changes into carry/accrual, roll-down, and yield or spread components. Another extension would replace a single yield to maturity with a term structure of spot rates and examine whether analogous region boundaries can be expressed in terms of instantaneous forward-value growth.

The labels deep discount, near par, and deep premium are analytical rather than arbitrary percentage bands around face value. They are defined by the slope thresholds in equations (9)–(12). Empirical work could measure how frequently traded bonds fall into each region across coupon frequencies, yield environments, and market conventions. The present results also assume $y > 0$; zero or negative periodic yields require separate boundary analysis.

9 Conclusion

Coupon-date convergence to par is a discrete-time statement. It does not, by itself, determine the path of quoted clean prices between coupon dates. Under constant positive yield and linear accrued interest, the within-coupon clean-price function is strictly convex and admits a complete three-region classification. Deep discount bonds increase monotonically, deep premium bonds decrease monotonically, and bonds in an intermediate near-par region follow a U-shaped path with a unique interior minimum.

The result provides a compact benchmark for fixed-income valuation and portfolio monitoring. It identifies when coupon-to-coupon convergence and within-period movement point in the same direction and when they do not. Because the reversal can occur with no change in yield or credit quality, the distinction is relevant to the interpretation of clean-price returns, P&L attribution, and pricing-system controls as well as to the analytical understanding of bond quotation conventions.

Declarations

Declarations

Conflict of interest:

The author declares no conflict of interest.

Author Contribution

Guan Jun Wang is the sole author of this manuscript and was responsible for the conceptualization, methodology, formal analysis, writing of the original draft, revision and editing, visualization, and overall preparation of the manuscript.

Data availability:

The market illustration uses publicly available trading-history data from MunicipalBonds.com; no proprietary data were used.

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Figures

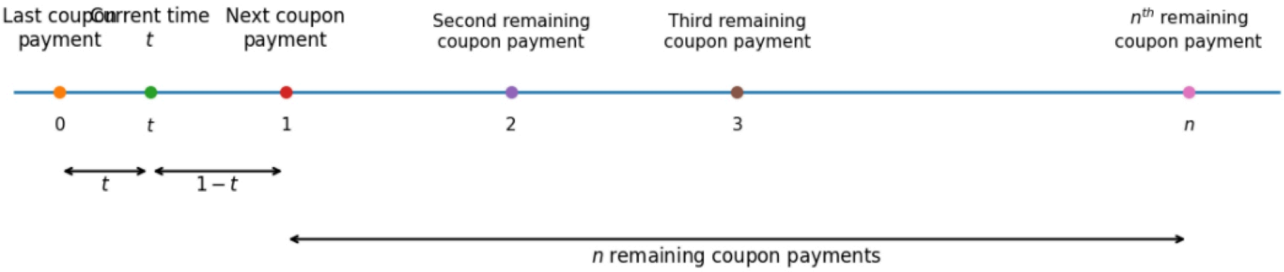


Figure 1

Timing of coupon payments within the current coupon period and the remaining life of the bond.

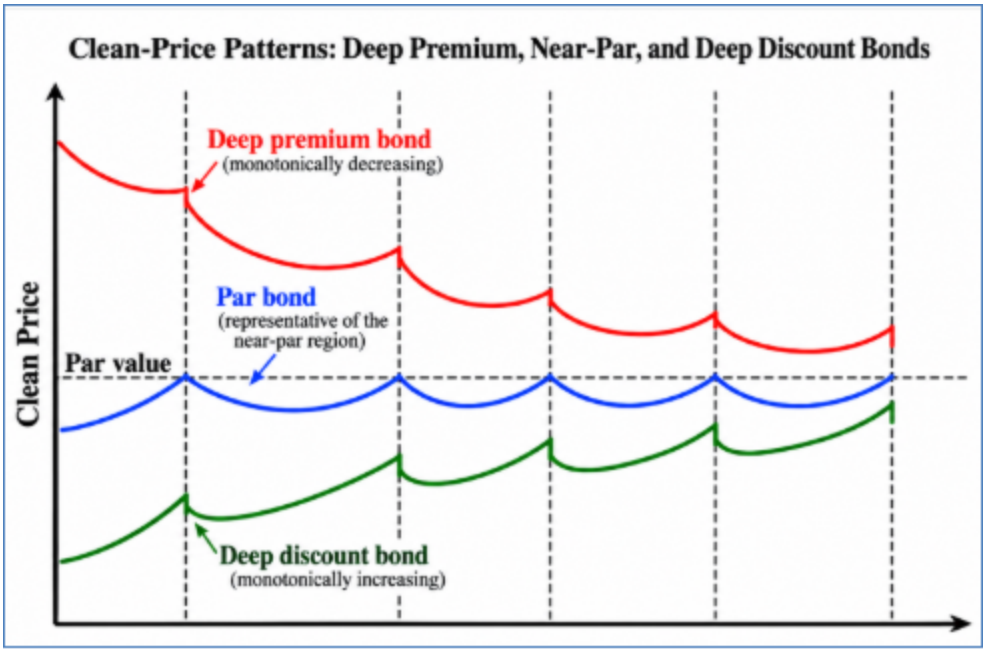


Figure 2

Clean-price patterns for premium, par, and discount bonds under a constant yield.

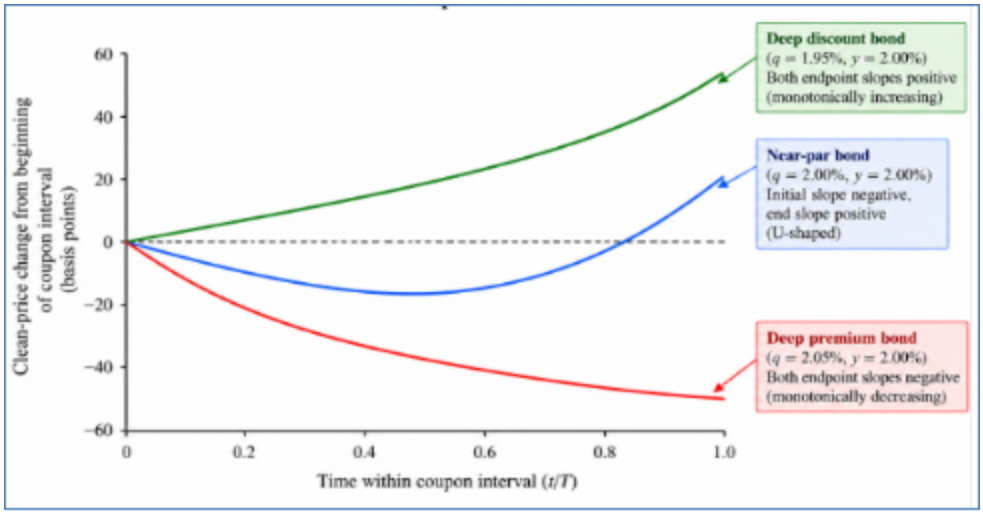


Figure 3

Illustrative within-coupon clean-price paths for y = 2% per coupon period.