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Sustainable social transformations and their role in innovative evolution

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Abstract

The main objective of this paper is to theoretically explain the process of sustainable social transformations in formal economic model and to demonstrate that such changes have an important impact on the innovative development of the whole economy. In particular, it will be demonstrated that consumers with environmental-friendly preferences and producers carrying out eco-friendly production plans can shape technological trajectories to improve their environmental impact by modifying their behaviours and fundamentally changing their lifestyles towards environmentally-friendly ones. Eventually, formal theorems will be developed to indicate adequate conditions for ensuring resource-limited development in the activities of market players in a particular model.

1. Introduction

For a long time, economic theories of technological change paid limited attention to the environmental consequences of economic activity. This perspective becomes increasingly restrictive when technological development is considered in the context of finite natural resources and persistent environmental constraints. The challenge is therefore not only to generate technological progress, but also to understand the conditions under which economic evolution can be directed towards more environmentally sustainable forms of production and consumption (e.g. Fussler and James, 1996; Geisendorf and Klippert, 2022; Green, Morton and New, 2002; Hoyoon, Dawoon and Jeong-Dong, 2023).

A central argument of this paper is that sustainable technological change cannot be analysed exclusively from the supply side. Producers determine which technologies are introduced, but consumers influence the market environment in which alternative technologies and production methods become economically viable. Changes in consumer preferences can therefore affect the composition of demand and, through firms' responses to demand, create conditions favouring particular technological trajectories. The theoretical problem is to identify the formal conditions under which this demand-side mechanism can be represented within a model of economic evolution.

Social transformation refers to fundamental changes in the structure and functioning of a social system. Such changes may occur at the level of individual behaviour or at the level of the economic system as a whole. In the present analysis, particular attention is given to a specific class of transformations: sustainable social transformations, understood as changes in the behaviour and characteristics of market participants that expand the role of environmentally friendly consumption and production. This concept links changes in preferences and feasible consumption plans with changes in production possibilities and, ultimately, with the direction of innovative evolution.

The main objective of this paper is therefore to explain, within a formal economic model, how sustainable social transformation can affect innovative evolution. The paper focuses in particular on the possibility that changes in consumer preferences and consumption opportunities may alter the incentives faced by producers and thereby contribute to the emergence of environmentally friendly innovations. This perspective complements approaches in which innovation is treated primarily as a producer-driven process (e.g. Schumpeter, 1912; Andersen, 2007; Nelson, 2013).

The contribution of the paper is threefold. First, the standard Debreu framework is extended by incorporating a production-preference relation that formally links consumers' preferences with production plans in firms in which they hold shares. Second, sustainable social transformation is defined by explicitly identifying environmentally friendly consumption plans and pro-ecological preferences. Third, sufficient conditions are established under which sustainable changes on the demand side, together with an innovative extension on the supply side, imply a sustainable innovative extension of the economy. The resulting theorem provides a formal representation of the mechanism through which demand-side transformation can contribute to the direction of technological change.

To perform this task, social transformations are first defined within the formal model of the economy. The definitions distinguish changes in individual economic characteristics from cumulative changes in the economic system. The analysis then introduces sustainable consumption, pro-ecological preferences and eco-innovations. These concepts provide the basis for the main theorem and its dynamic extension.

Social transformation refers to the process of major fundamental changes in the social system and the restructuring of all aspects of society. These changes can be observed at individual and economic level as a whole and have positive and negative effects. Although social transformation is almost universal, sometimes differences are expressed about the direction of change, such as change mechanisms and their drivers. In addition, we can distinguish between all these changes sustainable social transformations towards more sustainable and environmentally friendly activities by all market participants. This includes a variety of related ideas, ranging from the relationship of ecological preferences to ecological innovations and socially acceptable sustainable innovations.

In this context, the main objective of this paper is to theoretically explain the process of sustainable social transformation in formal economic models, demonstrating that these changes have a significant impact on the innovations of the entire economy (e.g. Andersen, 2007, Nelson, 2013, Geisendorf, Klippert, 2022). It deals with the characteristics of changes in the economic system and their effects on the optimal state. In particular, it will be demonstrated that changes in consumer preferences can shape a technological path in order to improve their impact on the environment through changes in their behaviours and fundamental changes in their lifestyles into environmentally friendly ones.

To perform this task social transformations in the formal model of the economy are defined. This definition represents qualitative and quantitative changes determined by the mechanism of supply and demand. The formal model provides the analytical environment in which the demand-supply mechanism underlying sustainable innovative evolution can be made explicit. The key modification is the introduction of a production-preference relation. This relation is important because consumers are not treated solely as purchasers of final goods: when they hold shares in firms, their preferences can also be related formally to alternative production plans. Consequently, changes in consumer preferences can be connected to producers' production decisions within the same formal system.

2. Formal model and its modification

The formal modelling of activity of market agent is analysed in the deterministic, static model of economy based on the standard model of Debreu economy with private ownership (Debreu, 1959, Innovative Economy, 2013, Nelson and Winter, 1982, 2002). This model is one of the most general models of competitive economy and a crucial part of general equilibrium theory. In this model some modifications are introduced to define in formal way an environmentally friendly activity of consumers and producers.

The given formal model of economy is in the form of a multi-range relational system (Debreu, 1959, Innovative Economy 2013, Nelson and Winter, 1982, 2002) and includes, in combination, both production and consumption systems $E = (\mathbb{R}^{\ell}, P, C, \theta, \varpi)$, where:

- $\mathbb{R}^{\ell}$ is a commodity-price space,
- P is a production system,
- C is a consumption system,
- θ is a function describing the consumers' shares in the producers' profits,
- $\varpi \in \mathbb{R}^{\ell}$ is some fixed (initial) total resource of the economy which is consumers' property.

In this model an action of each agent is a point in the ℓ -dimensional commodity-price space $\mathbb{R}^{\ell}$. Moreover, based on methodological individualism, each producer $b \in B$ is represented by feasible technologies he is equipped with in the form of non-empty production set $Y^b \subset \mathbb{R}^{\ell}$. Similarly, each consumer $a \in A$ is defined by his psychophysical structure and decision-making mechanism, i. e., formally by a pair $(X^a, \preccurlyeq^a)$, where:

- $X^a \subset \mathbb{R}^\ell$ is a non-empty subset of the commodity space $\mathbb{R}^\ell$, a set of alternative consumption plans. This set represents the consumer's feasible consumption plans concerning his psychophysical structure and elements of this set are the consumption bundles that the individual can conceivably consume given the physical constraints imposed by the environment.
- $\preccurlyeq^a$ is a well-defined preference relation (continuous, non-satiated, and convex), which describes the tastes of a consumer. This binary relation is defined on the set of alternatives X^a (cf. Albanese 1987) and is represented by a utility function $u^a : X^a \longrightarrow \mathbb{R}$, which is a convenient analytical tool.

Moreover each consumer is equipped not only with his internal structure represented by previous characteristics but also with external one which links him with an outer environment and consists e.g. of the initial endowments $e(a) \in \mathbb{R}^\ell$ with a positive quantity of at least one commodity and the shares in producers' profits Θ^{ab}.

In the given static formal model of economy E we assume that:

1. this is a model of perfect competition so the prices are announced and the decisions of the chosen production or consumption plans are taken at the beginning of the time,
2. total (fixed, initial) resources of economy $\varpi \in \mathbb{R}^\ell$ are the consumers' property, it means $\varpi = \sum_{a=1}^{m} e(a)$,
3. for the given vector of prices $p \in \mathbb{R}^\ell$ the whole profit of each producer $p \circ y^b$ is distributed to shareholders such that for each consumer $a \in A$ and each producer $b \in B$, $\Theta(a,b) = \Theta^{ab} \in [0,1]$ means consumer a share in the producer b profit and there is, for every $b \in B$, $\sum_{a \in A} \Theta^{ab} = 1$.

In this system, the activity of all economic agents relies on the choice and realization of optimal plans for them. Each producer attempts to choose the production plans maximizing his profit in a given price system and technologies. The activity of producer is described by a the set $\eta^b(p) = \{y^{b*}\} \subset Y^b$ of production plans maximizing producer b profit in a given price system p and by a maximum profit $\pi^b(p) \coloneqq \max_{y^b \in Y^b} \{p \circ y^b\} = p \circ y^{b*}$, where $y^{b*} \in \eta^b(p)$.

Each consumer tries to choose the consumption plans maximizing his preference relation on his budget set $\beta^a(p) = \{x^a \in X^a : p \circ x^a \leq w^a\} \subset X^a$, where $w^a = p \circ e(a) + \sum_{b \in B} \Theta^{ab} \pi^b(p)$. It is described by a set $\varphi^a(p, w, \preccurlyeq^a) = \{x^{a*}\} \subset \beta^a$ of consumption plans maximizing his/her preferences on the budget set β^a.

In a given economy, producers' activities are independent of consumers' choices. On the other hand, consumer activities depend on producers profits. However, in this model, it has been observed that individuals can play role of both producers and consumers, because each individual producer is a consumer. It provides an opportunity for a specific extension of the preference relation that compares the production plans and study active role of consumers in the modified model (Andersen, 2007).

Let an economy E be given in which each producer b is characterized by the set of feasible production plans Y^b and each consumer is characterized by the set of attainable consumption plans X^a. In this economy let consumer a be a shareholder in a firm b $(\Theta^{ab} > 0)$ and his preference relation $\preccurlyeq^a$ be represented by the utility function u^a. Moreover, let two production plans $y^b, \tilde{y}^b \in Y^b$ be given.

Definition 2.1 (Cialowicz and Malawski, 2011, 2013) A production plan $\tilde{y}^b$ is preferred to a plan y^b by a consumer a (in short: $y^b \trianglelefteq^a \tilde{y}^b$) iff $u^a[x^a + \Theta^{ab} y^b] \leq u^a[x^a + \Theta^{ab} \tilde{y}^b]$ for each $x^a \in X^a$.

According to the definition, the production plan preferred by a particular consumer (shareholder of a particular firm) does not decrease utility of each possible consumption plan in a set of consumption plans attainable with respect to psychophysical structure of the consumer. Note that the production-preference relation $\preccurlyeq_p^a \subset (X^a + Y^b) \times (X^a + Y^b)$ is the extension of a preference relation $\preccurlyeq^a$ on the production set Y^b. This means that with a standard assumption about production sets: $0 \in Y^b$ $(0 \in Y^b \Longrightarrow X^a \subset X^a + Y^b)$ and for $\Theta^{ab} = 0$ the production-preference relation defined above can be reduced to the preference relation $\preccurlyeq^a \subset X^a \times X^a$ of a consumer a.

Thus each producer is characterized by production-preference relation $\preccurlyeq_p^a$. This approach is consistent with the idea of important role of demand side in economic evolution based on the general premise that (cf. Mas-Colell, Whinston and Green, 1995, p.152): …"the objective of the firm should emerge from the objectives of those individuals who control it". Thus, each producer is associated with a production-preference relation induced by the preferences of the consumers who hold shares in the firm. This approach provides the formal link between the demand side and the production side of the economy. It is consistent with the idea that the objectives of a firm can reflect the objectives of the individuals who control it . The implication for the present analysis is important: if consumers' preferences change systematically towards environmentally friendly outcomes, these preferences can be transmitted to production decisions through ownership and profit-sharing relationships. The model therefore makes it possible to represent a demand-side channel through which social transformation can influence the direction of production and innovation.

As a result we modify the given formal model into a Debreu economy with production-preference relation: $E_{\preccurlyeq_p} = \left(\mathbb{R}^\ell, P, C, \theta, \varpi, \preccurlyeq_p\right)$ and $C = \left(A, \mathbb{R}^\ell, \wp;\ x, p, \beta, \varphi\right), P = \left(B, \mathbb{R}^\ell;\ y, p, \eta, \pi\right)$.

3. Sustainable social transformations

The concept of social transformation in the given formal model of economy deserves extraordinary attention because on the one side it is the process of changes in social status for an individual market participant. On the other side, it refers to large-scale social changes in the whole social system as in cultural reforms or transformations. Moreover, social transformation of a consumption system means changes in characteristics for all consumers:

1) changes in a budget set of consumer a as a result of changes in his wealth and psychophysical structure,

2) changes in preference relation based not only on changes in dimension in commodity space but also changes in consumer's tastes,

3) changes in a set of optimal consumption plans as a result of changes in budget set and preference relation.

From the methodological point of view, the given formal model of economy $E_{\preccurlyeq_p}$ in the form of a multi-range relational system implies some substantial consequences such as understanding and defining economic agents in our setting and assigned to them. Moreover it locates this study within applied mathematical system theory. The given model may represent a nation, a region, a single industry, or even a family. In general, it is a system of interrelated production and consumption subsystems that ultimately determines resource allocation. Allocation in this model is defined as the distribution of resources for various activities, i.e. a sequence of activities of all agents: $(m+n)$-elementary sequence $((x^a),(y^b))$ in the space $\mathbb{R}^\ell$.

Definition 3.1 (cf. Mas-Colell at al., 1995)

1. An allocation of economy E is an $(m+n)$-elementary sequence $((x^a),(y^b))$ in the space $\mathbb{R}^\ell$.

2. An allocation $((x^a),(y^b))$ satisfied the market equilibrium condition if $x-y=\varpi$ $(x-y-\varpi=0)$.

3. An allocation $((x^a),(y^b))$ is said to be attainable (feasible) if:

a) for every $a\in A$ $x^a\in X^a$, b) for every $b\in B$ $y^b\in Y^b$, c) $x-y=\varpi$.

4. A state of an economy is an allocation together with a price system $((x^a),(y^b),p)$.

5. An attainable state $((x^{a*}),(y^{b*}),p^*)$ is an equilibrium (state of general equilibrium) of the economy E if:

a) for every $b\in B$ y^{b*} maximizes his profit on the set Y^b at the vector of prices p^*,

b) for every $a\in A$ x^{a*} is the best (maximal) element concerning his preference relation $\preccurlyeq^a$ in his budget set β^a.

The allocation is any sequence of consumption and production plans, meaning that each market participant has assigned certain activities (not necessarily feasible or optimal). If all these plans are attainable because of technical limitations (for producers) or the limitations resulting from individual psychophysical structures (for consumers), and given allocations exist in all markets, these plans are attainable. In the list of all possible allocations, it can be distinguished among the ones where all action plans are optimal for market participants with fixed price vectors. When this allocation occurs, the economy is assumed to be in equilibrium, i.e., a producer or consumer with a specific price vector executes an optimal plan, so that this outcome is modeled on a Walrasian equilibrium.

Let be given a set of attainable allocations of economy $E_{\preccurlyeq_p}$:

$$M=\left\{\left((x^a),(y^b)\right):\forall a\in A\ x^a\in X^a,\forall b\in B\ y^b\in Y^b \text{ and } x-y=\varpi\right\}.$$

Note that $M\neq\emptyset$ if $\varpi\in M$. Moreover if for each $a\in A$ and $b\in B$ all sets X^a and Y^b are bounded and closed then the set M is bounded and closed.

In the given model of economy E each consumer $a \in A$ is characterized by their individual preference relation $\preccurlyeq^a$ over some choice set of feasible (with respect to his psychophysical structure) consumption plans X^a. In most economic theories, however, consumers' collective behaviors are more important than the behaviors of a single consumer. According to Social Choice Theory (Sen, 1986), we can combine all these preferences and construct aggregated preferences, which are considered social preferences that respond to all citizens in the economy and arise from all consumers in the economy.

Let be given two attainable allocations of E: $((x^a),(y^b)), ((\tilde{x}^a),(\tilde{y}^b)) \in M$.

Definition 3.2 The allocation $((\tilde{x}^a),(\tilde{y}^b))$ is said to be at least as desired as the allocation $((x^a),(y^b))$, and one writes $((x^a),(y^b)) \preccurlyeq_s ((\tilde{x}^a),(\tilde{y}^b))$, if for every $a \in A$, $x^a \preccurlyeq^a \tilde{x}^a$.

Notice, that $\preccurlyeq_s$ is called a social preference relation. The condition $((x^a),(y^b)) \preccurlyeq_s ((\tilde{x}^a),(\tilde{y}^b))$ means that the second allocation is at least as desirable as the first from the perspective of all consumers.

Moreover, the two allocations $((x^a),(y^b))$ and $((\tilde{x}^a),(\tilde{y}^b))$ are said to be socially indifferent if, for every consumer, his consumption in the two allocations are indifferent, i.e., $((x^a),(y^b)) \sim_s ((\tilde{x}^a),(\tilde{y}^b))$ means: for every $a \in A$, $x^a \sim^a \tilde{x}^a$.

However, definition 3.2 captures only the consumption side of the economy. It does not explicitly distinguish whether alternative production plans are environmentally friendly or innovative. This limitation motivates the extension introduced below: if sustainable economic transformation is to be analysed as an interaction between demand and supply, the social preference relation must also incorporate consumers' preferences over production plans.

To generalize Definition 3.2 and explicitly account for the supply side, a social preference relation can be defined in which both consumption plans and production plans matter. This extension is crucial for the subsequent sustainability argument. It allows the model to distinguish between a change in consumption preferences and a change in the production structure that responds to those preferences. In this way, the formal system captures the possibility that demand-side transformation and supply-side innovation reinforce each other.

Definition 3.3 The allocation $((\tilde{x}^a),(\tilde{y}^b))$ is said to be at least as desired as the allocation $((x^a),(y^b))$, and one writes $((x^a),(y^b)) \preccurlyeq_s^p ((\tilde{x}^a),(\tilde{y}^b))$, if for every $a \in A, b \in B$ $x^a \preccurlyeq^a \tilde{x}^a$ and $y^b \trianglerighteq^a \tilde{y}^b$.

The relation $\preccurlyeq_s^p$ is called an demand-supply social preference relation. Note that if $((x^a),(y^b)) \preccurlyeq_s^p ((\tilde{x}^a),(\tilde{y}^b))$ then $((x^a),(y^b)) \preccurlyeq_s ((\tilde{x}^a),(\tilde{y}^b))$.

Moreover, we assume rational behavior of consumers in the given model. It means that for each consumer $a \in A$ the relation $\preccurlyeq^a \subset X^a \times X^a$ has following properties for any consumption plans $x, \tilde{x}, \hat{x} \in \mathbb{R}^\ell$:

1) $\preccurlyeq^a$ is reflexive $\overset{def}{\iff} x \preccurlyeq^a x$ (two identical bundles can be compare),

2) $\preccurlyeq^a$ is transitive $\overset{def}{\iff} (x \preccurlyeq^a \tilde{x} \wedge \tilde{x} \preccurlyeq^a \hat{x}) \Longrightarrow x \preccurlyeq^a \hat{x}$ (when looking at three alternatives, the consumer is rational in the sense that, if she says she likes the first better than the second and the second better than the third, she will also say that she likes the first better than the third),

3) $\preccurlyeq^a$ is complete $\overset{def}{\iff} \forall x, \tilde{x} \in \mathbb{R}^\ell \; x \preccurlyeq^a \tilde{x} \vee \tilde{x} \preccurlyeq^a x$ (when given a choice between two alternative bundles, the consumer can make a comparison).

It should be recalled that all these characteristics of relations mean that all market agents can make choices and it is possible to rank all attainable allocations in the economy from the least desired to the most preferred for the economy.

So, we can assume the following definition.

Definition 3.4 An attainable allocation $((x^a),(y^b)) \in M$ is called an optimum of $E_{\preccurlyeq_p}$ if it is a maximal element of M for the demand-supply social preference relation $\preccurlyeq_s^p$, i.e. for each $((\tilde{x}^a),(\tilde{y}^b)) \in M$ $((\tilde{x}^a),(\tilde{y}^b)) \preccurlyeq_s^p \left((x^a),(y^b)\right)$.

The formal definition of the social transformation of the consumption system offers us the opportunity to define the socio-economic transformation and sustainable transformation of the economy *E*. The sustainable transformation of the given economy is based on an environmental-friendly production and consumption plans, pro-ecological preference relations, an eco-innovation and an eco-innovator defined in the model. Environmental-friendly production and consumption plans are defined as those that are based on products that do not harm the environment in production, use or disposal. At the basic level, a pro-ecological preference relation motivates consumers to look for environment friendly products or green products which cause minimal harm to people and the environment (Green, Morton and New, 2002). Based on these definitions it will be possible to highlight the active role and positive influence of the demand and supply sides on environment by shaping technological trajectories to improve their sustainability (Diaconu and Popescu, 2016).

The formal definition of sustainable transformation provides the basis for linking social change to environmental performance. In the present framework, sustainability is represented through environmentally friendly production and consumption plans, pro-ecological preference relations, and eco-innovation. An environmentally friendly plan is understood as a plan whose implementation does not adversely affect the environment through production, use or disposal. A pro-ecological preference relation gives consumers a systematic preference for environmentally friendly consumption plans over non-environmentally friendly alternatives.

These concepts are not introduced merely as descriptive characteristics. Their purpose is to establish a formal transmission mechanism between consumer behaviour and technological change. If consumers

systematically prefer environmentally friendly goods and if such goods are feasible within their budget sets, their optimal choices can shift towards environmentally friendly consumption. When these consumers are also connected to producers through ownership shares, the production-preference relation creates a formal channel through which changes in demand-side preferences can affect production decisions. This provides the conceptual foundation for the main theorem of the paper.

The definition of social transformation in the given formal model of economy deserves extraordinary attention because on the one side social transformation is the process of changes in social status for an individual market participant. On the other side, it refers to large-scale social changes in the whole social system as in cultural reforms or transformations (c.f. Andersen 2007). Moreover, it is worth remembering that the social and economic world is developing continuously and thereby changes its structures, sometimes smoothly and sometimes rather abruptly.

Let two consumption systems C and $\tilde{C}$ be given and $\ell \leq \tilde{\ell}$.

Definition 3.5 A consumption system $\tilde{C}$ is a social transformation of a consumption system C (in short: $C \subset_{st} \tilde{C}$) iff for each consumer $a \in A$:

1) $proj_{\mathbb{R}^\ell}(\tilde{\beta}^a) \neq \beta^a$
2) $proj_{\mathbb{R}^\ell}(\tilde{\varepsilon}^a) \neq \varepsilon^a$
3) $proj_{\mathbb{R}^\ell}(\tilde{\varphi}^a) \neq \varphi^a$.

According to the above definition social transformation of a consumption system means changes in characteristics for all consumers:

1) changes in a budget set of consumer a as a result of changes in his wealth and psychophysical structure,
2) changes in preference relation based not only on changes in dimension in commodity space but also changes in consumer's tastes,
3) changes in a set of optimal consumption plans as a result of changes in budget set and preference relation.

Among all social transformations we may distinguish positive changes for the given consumer $a \in A$, for which:

1) $e^a < proj_{\mathbb{R}^\ell}(\tilde{e}^a)$ consumer's initial endowment increases,
2) $\beta^a \subsetneq proj_{\mathbb{R}^\ell}(\tilde{\beta}^a)$ consumer's budget set extends,
3) $\varphi^a \subsetneq proj_{\mathbb{R}^\ell}(\tilde{\varphi}^a)$, which means that for each two optimal consumption plans $x^{a*} \in \varphi^a$ and $\tilde{x}^{a*} \in \tilde{\varphi}^a$ there is $x^{a*} \preccurlyeq^a proj_{\mathbb{R}^\ell}(\tilde{x}^{a*})$.

Definition 3.6 A consumption system $\tilde{C}$ is a cumulative social transformation of a consumption system C (in short: $C \subset_{cst} \tilde{C}$) iff: 1) $C \subset_{st} \tilde{C}$, 2) $A \subset \tilde{A}$, 3) $\forall a \in A$

3.1) $X^a \subset proj_{\mathbb{R}^\ell}(\tilde{X}^a)$ and for each consumption plan $x^a = (x_1, \dots, x_\ell) \in X^a$ there is $x^a = (x_1, x_2, \dots, x_\ell, 0, \dots, 0) \in \tilde{X}^a$,

3.2) $e^a \leq proj_{\mathbb{R}^\ell}(\tilde{e}^a)$,

3.3) $\varepsilon^a \subset proj_{\mathbb{R}^\ell}(\tilde{\varepsilon}^a)$, i.e. $\left(\tilde{\preccurlyeq}^a \middle| (X^a \times X^a)\right) = \preccurlyeq^a$

3.4) $\beta^a \subset proj_{\mathbb{R}^\ell}(\tilde{\beta}^a)$

3.5) $\varphi^a \subset proj_{\mathbb{R}^\ell}(\tilde{\varphi}^a)$, it means that for each two optimal consumption plans $x^{a*} \in \varphi^a$ and $\tilde{x}^{a*} \in \tilde{\varphi}^a$ there is $x^{a*} \preccurlyeq^a proj_{\mathbb{R}^\ell}(\tilde{x}^{a*})$.

Cumulative extension of the consumption system means that for every consumer the set of feasible consumption plans does not decrease, he can "ignore" new goods (Condition 3.1) and its initial endowment is not reduced (Condition 3.2). At the same time, his budget set also does not decrease (Condition 3.4), and his needs are satisfied at least at the same level of satisfaction (Condition 3.5). As a result cumulative changes in a consumption system means that the situation of each consumer is not getting worse, which means that the Pareto-optimal states (i.e. Samuelson and Nordhaus 2004) of the system are not deteriorating.

Definition 3.7

1. An economy $\tilde{E}_{\preccurlyeq_p}$ is a social transformation of an economy $E_{\preccurlyeq_p}$ (in short: $E_{\preccurlyeq_p} \subset_{st} \tilde{E}_{\preccurlyeq_p}$) if $C \subset_{st} \tilde{C}$.

2. An economy $\tilde{E}_{\preccurlyeq_p}$ is a cumulative social transformation of an economy $E_{\preccurlyeq_p}$ (in short: $E_{\preccurlyeq_p} \subset_{cst} \tilde{E}_{\preccurlyeq_p}$) if: 1) $C \subset_{cst} \tilde{C}$, 2) $B \subset \tilde{B}$, 3) $\ell \leq \tilde{\ell}$, 4) $\forall b \in B$ 4.1) $Y^b \subset proj_{\mathbb{R}^\ell}(\tilde{Y}^b)$, 4.2) $\eta^b(p) \subset proj_{\mathbb{R}^\ell}\left(\tilde{\eta}^b(\tilde{p})\right)$ 4.3) $\pi^b(p) \leq \tilde{\pi}^b(\tilde{p})$.

3. Social transformation $E_{\preccurlyeq_p} \subset_{st} \tilde{E}_{\preccurlyeq_p}$ gives a Pareto improvement if for an optimum $((x^a),(y^b)) \in M$ there is optimum $((\tilde{x}^a), (\tilde{y}^b)) \in \tilde{M}$ such that one person would be better off and all other persons are not worse off, it means $((x^a),(y^b)) \preccurlyeq_s ((\tilde{x}^a),(\tilde{y}^b))$, and there exist $a \in A$ such that $x^a \prec^a \tilde{x}^a$.

Remark If $E_{\preccurlyeq_p} \subset_{cst} \tilde{E}_{\preccurlyeq_p}$ and there exists consumer $a \in A$, for which:

1) $e^a < proj_{\mathbb{R}^\ell}(\tilde{e}^a)$ consumer's initial endowment increases,
2) $\beta^a \subsetneq proj_{\mathbb{R}^\ell}(\tilde{\beta}^a)$ consumer's budget set extends,
3) $\preccurlyeq^a$ is monotonic,

then this social transformation gives a Pareto improvement.

Limited access to natural resources, both non-renewable and renewable, is linked to the problem of consuming resources faster than the Earth provides. In this situation, in order not to lead to a catastrophe, it is important to behave in a pro-ecological way of all producers and consumers (e. g. Amin at al., 2023, Abass at al., 2025) including: reducing consumption – fewer raw materials, more efficient solutions, switching to renewable raw materials such as solar energy or biomass – instead of fossil fuels, better resource management – recycling, extraction control, sustainable farming practices Education and Global Collective Action – Changing Lifestyles, Awareness, and Sharing Resources Equitably.

The terms "environmental-friendly" or “pro-ecological” activities of market agents draw attention to the positive contributions of companies and households to economic development. This includes a number of related ideas, ranging from environmental-friendly technological progress to socially acceptable innovative green development methods.

Therefore, from an environmental and social point of view, it is essential to define and distinguish pro-ecological production and consumption plans from all those planned in a specific formal model in order to achieve long-term growth patterns. These specific plans can be understood and analyzed in terms of objectives (the main focus), mechanisms (the method of introducing objectives), and effects (the impact on environmental conditions). In addition, the relationship between pro-ecological preferences and consumer attitudes towards environmental activities is defined to describe consumer attitudes, values, and beliefs.

To give a formal definition of environment-friendly activity of market agents in the model defined in Chapter 3 and for the purpose of this research modification sets of feasible production or consumption plans will be introduced and environmental-friendly plans will be defined.

The environmental-friendly plans can serve the tools for introducing a kind of partition in the set of the possible consumptions plans of the consumer with respect to the psycho-physical constraint of this individual and in his/her budget set.

Thus, for each consumer $a \in A$ let us divide the set X^a into two disjoint subsets X_E^a and X_N^a such that $X^a = X_E^a \cup X_N^a$ and $X_E^a \cap X_N^a = \emptyset$ where X_E^a is a set of environmental friendly consumption plans, X_N^a is a set of non- environmental friendly consumption plans. Similarly we can divide budget set β^a into two disjoint subsets β_E^a and β_N^a. It should be recalled that, according to consumer conventions, inputs in consumption plans are products consumed (which are available to them) and are represented in non-positive coordinates. At the same time, an output is what it produces (which it provides, usually human work), represented by non-negative coordinates. This means that all inputs and outputs in the environmental-friendly consumption plan has no negative impact on the environment.

In the same way in a production system a set of feasible production plans Y^b is divided into disjoint subsets Y_E^b and Y_N^b.

Definition 3.8

1. A consumption plan $x \in X^a$ is called environmental friendly if $x \in X_E^a$.
2. A production plan $y \in Y^b$ is called environmental friendly if $y \in Y_E^b$.
3. If for every $a \in A, x^a \in X_E^a$ and for every $b \in B, y^b \in Y_E^b$ then an allocation $((x^a),(y^b))$ is called environmental friendly.

In the standard model of Debreu economy, described in Chapter 2, the traditional image/behaviour of consumer is inadequate for the environmental challenge of sustainable development so it is necessary to define

pro-ecological preference relation of consumer and an environmentally friendly extension of a consumption system.

Definition 3.9 Preference relation $\preccurlyeq^a \subset \mathbb{R}^\ell \times \mathbb{R}^\ell$ of consumer $a \in A$ is called pro-ecological iff for any consumption plans $x^a \in X_N^a, \tilde{x}^a \in X_E^a$ we have $x^a \prec^a \tilde{x}^a$..

Notice that if consumer a is characterized by a pro-ecological preference relation then each environmentally friendly consumption plan is better for him than any other plan. At the basic level, a pro-ecological preference relation motivates consumers to look for environmentally friendly products or green products which cause minimal harm to people and the environment. Let A_{ec} stands for a set of all consumers with pro-ecological preferences.

Theorem 3.1 Le a consumer a characterised by pro-ecological preference relation $\preccurlyeq_a$ be given. If $\beta_E^a \neq \emptyset$ then any $x^a \in \varphi^a$ is an environmental friendly consumption plan.

Proof. If a preference relation $\preccurlyeq_a$ is pro-ecological then for any consumption plans $x^a \in \beta_N^a, \tilde{x}^a \in \beta_E^a$ we have $x^a \prec_a \tilde{x}^a$, so $\tilde{x}^a \in \varphi^a$.

This theorem says that for a consumer with pro-ecological preferences the optimal consumption plan will be environmentally friendly if this plan is feasible.

This theorem establishes the first link in the proposed causal mechanism. For a consumer with pro-ecological preferences, the availability of at least one environmentally friendly plan in the budget set is sufficient to ensure that an optimal consumption plan is environmentally friendly. Thus, an increase in the number of consumers with such preferences can have an aggregate effect only if the economic system simultaneously provides feasible environmentally friendly consumption opportunities. The result therefore motivates the two-sided formulation of sustainable social transformation: changes in preferences alone are insufficient unless they are accompanied by an expansion of feasible environmentally friendly choices.

Definition 3.10 A consumption system $\tilde{C}$ is called a sustainable social transformation of a system C, shortly $C \subset_{sst} \tilde{C}$, iff:

1) $C \subset_{st} \tilde{C}$
2) $\ell \leq \tilde{\ell}$
3) $A_{ec} \subset \tilde{A}_{ec}$ *and* $A_{ec} \neq \tilde{A}_{ec}$
4) $\forall a \in A \ \ proj_{\mathbb{R}^{\ell_E}}(\tilde{X}_E^a) \not\subset X_E^a$
5) $\forall \tilde{a} \in \tilde{A} \ \ proj_{\mathbb{R}^{\tilde{\ell}_E}}(\tilde{\beta}_E^{\tilde{a}}) \neq \emptyset$

This definition says that in a consumption system $\tilde{C}$ there are not less products than in a consumption system $\tilde{C}$. Moreover according to the Condition 2) in the system $\tilde{C}$ there are more consumers with pro-

ecological preferences than in a system C, all consumers from the basic system have new environmentally friendly plans and each consumer in the new system may realize environmentally friendly consumption plan.

The definition formalizes a sustainable transformation as a change that combines three elements. First, the consumption system itself changes. Second, the population of consumers with pro-ecological preferences increases. Third, environmentally friendly consumption plans become available to consumers in the transformed system. The definition therefore captures both the behavioural and opportunity dimensions of sustainable transformation. This is important because sustainable consumption cannot be represented solely as a change in preferences: consumers must also have access to feasible environmentally friendly alternatives.

Now it is possible to define a sustainable extension of an economic system E based on social transformation in a system C.

Definition 3.11 An economic system $\tilde{E} = (\mathbb{R}^{\tilde{\ell}}, \tilde{P}, \tilde{C}, \tilde{\theta}, \tilde{\varpi})$, is called a sustainable extension of a system $E = (\mathbb{R}^{\ell}, P, C, \theta, \varpi)$, shortly $E \subset_s \tilde{E}$, iff $C \subset_{sst} \tilde{C}$.

4. Innovative evolution

J. Schumpeter in his *Theory of economic development* (1912) distinguished two basic forms of economic life: circular flow based on cumulative changes and innovative evolution characterized by innovative changes in a production sphere. In this context axiomatic analysis of the economic development in formal apparatus of the Arrow-Debreu theory of general equilibrium is based on specific extensions of the given model.

Definition 4.1 [Cialowicz 2015] A production system $\tilde{P}$ is called an innovative extension of a system P, shortly $P \subset_i \tilde{P}$, iff:

1) $l \leq \tilde{\ell}$, 2) $\exists \tilde{b} \in \tilde{B} \quad \forall b \in B$ (2.1) $\mathrm{proj}_{\mathbb{R}^{\ell}}(\tilde{Y}_{\tilde{b}}) \not\subset Y_b$ (2.2) $\mathrm{proj}_{\mathbb{R}^{\ell}}(\tilde{\eta}_{\tilde{b}}(\tilde{p})) \not\subset \eta_b(p)$.

According to the above definition $\tilde{P}$ is an innovative extension of a production system P, if there appear at least one specific producer-innovator $\tilde{b} \in \tilde{B}$, such that for each $b \in B$ there exists a production plan $\tilde{y}_{\tilde{b}} \in \tilde{Y}_{\tilde{b}}$ and $\mathrm{proj}_{\mathbb{R}^{\ell}}(\tilde{y}_{\tilde{b}}) \notin Y_b$. Producer $\tilde{b}$ is designated innovative production plans $\tilde{y}_{\tilde{b}}$ for which $\mathrm{proj}_{\mathbb{R}^{\ell}}(\tilde{y}_{\tilde{b}}) \notin \eta_b(p)$ (Assumptions 2.2). This means that innovative production plans are optimal for the producer $\tilde{b}$ and cannot be reduced to the analogous plans being realized by the producers in the previous system P. Moreover, we assume that the number of possible commodities on the market does not decrease (Assumption 1). According to Schumpeter's theory (Schumpeter 1961, p.134): *"...innovation is hazardous, impossible for most producer. But if someone establishes a business and everything goes well, then he can produce a unit of product more cheaply. (...) He then makes a profit (....)*". It means that producer-innovator plays a leading role in production system $\tilde{P}$ and his profit resulting from his initiative.

The idea of eco-innovation is fairly recent. One of the first appearances of the concept of eco-innovation in the literature is in the book by Fussler and James (1996): "...*eco-innovation is a new products*

and processes which provide customer and business value but significantly decrease environmental impacts". Moreover the term eco-innovation calls attention to the positive contribution that producers can make to sustainable development.

To give a formal definition of eco-innovation in the model defined in Chapter 2 and for the purpose of this research modification of commodity space will be introduced and environmentally friendly consumption and production plans will be defined.

Definition 4.2 A production system $\tilde{P}$ is called a sustainable innovative extension of a system P shortly $P \subset_{si} \tilde{P}$, iff: 1) $P \subset_i \tilde{P}$ 2) $\exists \tilde{b} \in \tilde{B} \quad \forall b \in B \quad$ (2.1) $\text{proj}_{\mathbb{R}^{\ell_E}}\left(\tilde{Y}_E^{\tilde{b}}\right) \not\subset Y_E^b \qquad$ (2.2) $\text{proj}_{\mathbb{R}^{\ell_E}}\left(\tilde{\eta}^{\tilde{b}}(\tilde{p})\right) \not\subset \eta^b(p)$.

According to the above definition in an sustainable innovative extension of a production system there is at least one producer $\tilde{b}$ whose technological abilities with respect to environmental friendly production plans go beyond the abilities of all producers acting within a production system P. Hence, the optimal environmental friendly production plans of the producer $\tilde{b}$ cannot be reduced to the analogous plans being realized by the producers in a production system P. Thus, the producer $\tilde{b}$ can be treated as an eco-innovator and a production plan $\tilde{y}_{\tilde{b}} \in \tilde{Y}_E^{\tilde{b}}$ which fulfilled conditions (2.1) and (2.2) is called an eco-innovation. It means that among all innovative production plans we may distinguish eco-innovations that reflect the concept's explicit emphasis on a reduction of environmental impact, whether such an effect is intended or not.

The distinction between innovation and eco-innovation is important for the argument of this paper. An innovative extension captures novelty in the production system, whereas a sustainable innovative extension additionally requires that the novel production capability lies in the environmentally friendly part of the production set. Eco-innovation is therefore not treated as synonymous with innovation in general; it represents a specific direction of technological change.

In his theory Schumpeter's assumed that innovative changes in the production sphere determine changes in the economy. This fact is described by the following way (Schumpeter, 1961, p. 65): *"...It is, however, the producer who as a rule initiates economic change, and consumers are educated by him if necessary..."*. In other words, innovative extension in a production sphere makes enormously changes in the composition of the whole economic system. This leads us to the definition of an innovative extension of the system $E_{\preccurlyeq_p}$.

Definition 4.3

1. An economic system $\tilde{E}_{\preccurlyeq_p}$ is called an innovative extension of a system $E_{\preccurlyeq_p}$ shortly $E_{\preccurlyeq_p} \subset_i \tilde{E}_{\preccurlyeq_p}$, iff: $P \subset_i \tilde{P}$.
2. An economic system $\tilde{E}_{\preccurlyeq_p}$ is called a sustainable innovative extension of a system $E_{\preccurlyeq_p}$ shortly $E_{\preccurlyeq_p} \subset_{si} \tilde{E}_{\preccurlyeq_p}$, iff: $P \subset_{si} \tilde{P}$ and $C \subset_{sst} \tilde{C}$.

It means that eco-innovations in a production system and environmentally friendly consumer's preferences play a key role in moving an economy towards sustainable development.

The preceding definitions establish the two components needed for the central result. On the demand side, pro-ecological preferences and environmentally friendly consumption opportunities can generate a sustainable social transformation. On the supply side, an innovative producer may introduce production plans that extend the environmentally friendly part of the production set. The key theoretical question is whether these two developments can be formally connected. Theorem 5.1 addresses precisely this question.

5. An impact of sustainable social transformation on innovative evolution

The central theoretical problem of this study is to identify conditions under which sustainable social transformation on the demand side is associated with a sustainable direction of innovative change on the supply side. The preceding definitions make it possible to formulate this problem precisely. A sustainable transformation of consumption expands the set of environmentally friendly consumption opportunities and increases the number of consumers with pro-ecological preferences. An innovative extension, in turn, introduces production possibilities that were not available in the previous system. The missing link is the condition that connects these two processes.

Theorem 5.1 provides such a link by requiring that at least one pro-ecological consumer has a positive ownership share in an innovative producer whose production set contains an environmentally friendly plan. Under these conditions, the preferences of the demand side are transmitted to the production side through the production-preference relation. This is the formal mechanism through which sustainable social transformation can contribute to sustainable innovative evolution.

Theorem 5.1 Let two economic systems $E_{\preccurlyeq_p}$ and $\tilde{E}_{\preccurlyeq_p}$ be given. If 1) $C \subset_{sst} \tilde{C}$, 2) $P \subset_i \tilde{P}$, 3) for any $b \in B$ $Y_E^b = \emptyset$, 4) exists consumer $\tilde{a} \in \tilde{A}$ with pro-ecological preference relation $\preccurlyeq^a$ and a producer-innovator $\tilde{b} \in \tilde{B}$ such that $\theta^{\tilde{a}\tilde{b}} > 0$ and $\tilde{Y}_E^{\tilde{b}} \neq \emptyset$, then $E_{\preccurlyeq_p} \subset_{si} \tilde{E}_{\preccurlyeq_p}$.

Proof. According to Assumption 1) it is enough to prove that $P \subset_{si} \tilde{P}$ (Definition 4.2).

If $P \subset_i \tilde{P}$ then in the production system $\tilde{P}$ there exist producers-innovators $\tilde{b} \in \tilde{B}$.

According to Assumptions 3 and 4 if for any $b \in B$ $Y_E^b = \emptyset$ and for a producer-innovator $\tilde{b}$ $\tilde{Y}_E^{\tilde{b}} \neq \emptyset$, then $\text{proj}_{\mathbb{R}^{\ell_E}}\left(\tilde{Y}_E^{\tilde{b}}\right) \not\subset Y_E^b$.

The Assumption 4 imply that if consumer $\tilde{a} \in \tilde{A}$ has pro-ecological preference relation, $\theta^{\tilde{a}\tilde{b}} > 0$ and $\tilde{Y}_E^{\tilde{b}} \neq \emptyset$, then each optimal production plan of producer $\tilde{b}$ are environmental friendly, it means $\tilde{Y}_E^{\tilde{b}} \subset \tilde{\eta}^{\tilde{b}}(\tilde{p}) \neq \emptyset$.

Moreover, according to Assumption 3), for any $b \in B$ $Y_E^b = \emptyset$, so $\eta_E^b(p) = \emptyset$ and thus

for any $b \in B$ $\text{proj}_{\mathbb{R}^{\ell_E}}\left(\tilde{\eta}^{\tilde{b}}(\tilde{p})\right) \not\subset \eta^b(p)$.

Thus all conditions from Definition 4.2 are fulfilled and then $P \subset_{si} \tilde{P}$.

If $C \subset_{sst} \tilde{C}$ and $P \subset_{si} \tilde{P}$ then $E \subset_{si} \tilde{E}$.

According to the theorem presented above, an increase in the number of consumers with pro-ecological preferences, together with an expansion of the set of environmentally friendly consumption plans, can contribute to directing innovative evolution towards a more sustainable development path. In other words, changes on the demand side of the economy can influence the direction of technological and economic transformation by creating stronger incentives for producers to adopt more environmentally friendly production methods.

The theorem establishes a sufficient-condition result rather than a universal causal law. Its significance lies in identifying a specific mechanism through which demand-side transformation can be connected to the direction of technological change. An increase in the number of consumers with pro-ecological preferences, together with an expansion of the set of environmentally friendly consumption plans, creates the demand-side component of this mechanism. The existence of an innovative producer with an environmentally friendly production possibility and a positive ownership link to a pro-ecological consumer supplies the corresponding production-side component.

Consequently, under the assumptions of Theorem 5.1, the transformation of consumption and the innovative extension of production cannot be treated as independent processes. The production-preference relation provides the formal bridge between them. This result gives a precise interpretation to the proposition that demand can influence the direction of technological change: demand affects production not through an unspecified market force, but through the modelled relationship between consumer preferences, ownership shares and production preferences.

The result also clarifies the role of environmentally conscious consumers. When the number of consumers with pro-ecological preferences increases and environmentally friendly goods become feasible consumption alternatives, the composition of demand may change. In the present model, this change becomes relevant for production decisions when pro-ecological consumers have ownership shares in firms. Their preferences can then enter the production-preference relation and make environmentally friendly production plans preferable from the perspective represented in the model.

This mechanism should be distinguished from the stronger claim that consumers directly determine technological outcomes. The theorem does not establish such a deterministic relationship. Rather, it identifies sufficient structural conditions under which a sustainable transformation of consumption is accompanied by a sustainable innovative extension of production. The distinction is important because technological change remains dependent on the existence of appropriate production capabilities and on the presence of an innovative producer.

Thus, sustainable social transformation is not represented exclusively as a consequence of technological progress imposed on consumers. Within the proposed framework, it can also originate on the demand side and become connected to changes in the production structure. The theoretical contribution is therefore to make explicit a feedback channel: changes in consumer preferences alter the socially preferred

set of outcomes, while production responds when the institutional and ownership structure represented by θ transmits these preferences to firms.

From this perspective, sustainable development can be interpreted as an interaction between changing consumption patterns and the transformation of production capabilities. The model suggests that the availability of environmentally friendly products and the presence of consumers who prefer them can be mutually reinforcing, but such reinforcement should be understood as a theoretical possibility conditional on the assumptions of the model. In particular, the existence of feasible environmentally friendly production plans and an innovative producer remains essential. The result therefore provides a formal starting point for analysing when demand-side changes may contribute to, rather than merely accompany, sustainable technological evolution.

To capture sustainable economic development as a process rather than as a comparison between two static states, the analysis must be extended in time. The static Arrow-Debreu framework is useful for identifying equilibrium relationships and sufficient conditions, but by itself it does not describe the sequence through which economic systems change. A dynamical formulation is therefore introduced to represent the cumulative character of innovative evolution. The purpose of this extension is not to replace the static theorem, but to interpret its conditions as local components of a continuing trajectory of economic transformation.

To this aim let us consider the space of all Debreu economies:

$\boldsymbol{E} \coloneqq \{E : E = (\mathbb{R}^{\ell}, P, K, \theta, \varpi)$ is a Debreu economy, $\ell \in \mathbb{N}\}$.

A dynamic analysis of the process of economic development is possible now by the application of a mathematical idea of a (quasi)-semidynamical system.

Definition 5.1 (cf. Sibirskij and Szube 1987) A mapping $f_E : \boldsymbol{E} \times \mathbb{R}_+ \to P(\boldsymbol{E})$ is a (quasi)-semidynamical system of Debreu economy if:

1) $f_E(E, 0) = \{E\}$, 2) $f_E(f_E(E, t_1), t_2) = f_E(E, t_1 + t_2) \quad \forall t_1, t_2 \in \mathbb{R}_+$.

Let a (quasi)-semidynamical system $f_E : \boldsymbol{E} \times \mathbb{R}_+ \longrightarrow P(\boldsymbol{E})$ be given

The further analysis of (quasi)-semidynamical system of Debreu economy is based on the general premise that it is possible to decompose them into component systems in the following way: $f_E = (f_P, f_C, f_\theta, f_\omega)$, where $f_P = (f_B, f_{\mathbb{R}^{\ell}}, f_{Ch_P})$, $f_C = (f_A, f_{\mathbb{R}^{\ell}}, f_{Pref}, f_{Ch_C})$, $f_{Ch_P} = (f_y, f_p, f_\eta, f_\pi)$, $f_{Ch_C} = (f_\chi, f_e, f_\varepsilon, f_p, f_\beta, f_\varphi)$. Particularly, the component dynamics are called innovative if they are involved as components in the innovative dynamics of the respective system as a whole.

Definition 5.2 A set $\tau_+(E) \coloneqq \{f_E(E, t) : t \in \mathbb{R}_+ \ \ and \ \ f_E(E, 0) = E\}$ is called a positive semi-trajectory of (quasi)-semidynamical system of Debreu economy E.

Definition 5.3 A positive semi-trajectory $\tau_+(E)$ is called a sustainable innovative development of an economy E, iff:

1) $f_E(E,0) = \{E\}$ 2) for any $t_1, t_2 \in \mathbb{R}_+$ $f_E(E,t_1) \subset_{Ei} f_E(E,t_2)$.

Now we may apply the idea of a (quasi)-semidynamical system to construct the dynamical version of Theorem 2.2.

Theorem 5.2. Let f_E such that for any $t_1 < t_2$ and $f_A = const, f_B = const$ be given. If:

1) $f_C(C,t_1) \subset_{sst} f_C(\tilde{C},t_1)$, 2) $f_P(P,t_1) \subset_i f_P(\tilde{P},t_1)$, 3) for any $b \in B$ $f_y(Y_E^b,0) = \emptyset$,

4) exists consumer $\tilde{a} \in f_A(A,t_2)$ with pro-ecological preference relation $\preccurlyeq^a$ and a producer-innovator $\tilde{b} \in f_B(B,t_2)$ such that $f_\theta(\theta^{ab},t_2) > 0$ and $f_y(Y_E^b,t_2) \neq \emptyset$, then $\tau_+(E)$ is a sustainable innovative development.

Proof. The proof is based on the fact that all adequate components of an (quasi)-semidynamical system of Debreu economy E fulfil the assumptions of Theorem 5.1.

The theorem provides the dynamic counterpart of Theorem 5.1. Its interpretation is that sustainable innovative development can emerge when the relevant components of the economic system evolve in a coordinated manner: the consumption system undergoes sustainable transformation, the production system undergoes innovation, and the ownership-preference link connects a pro-ecological consumer with a producer possessing an environmentally friendly production possibility.

Conclusion

The paper develops a formal framework for analysing sustainable social transformation as a potential driver of sustainable innovative evolution. The approach extends the Arrow-Debreu framework by introducing a production-preference relation, environmentally friendly production and consumption plans, pro-ecological preferences, and a formal distinction between ordinary innovation and eco-innovation. These extensions make it possible to represent, within a common mathematical structure, the interaction between changes in consumption and changes in production.

The central result is Theorem 5.1. It identifies sufficient conditions under which a sustainable transformation of the consumption system, together with an innovative extension of the production system, implies a sustainable innovative extension of the economy. The essential mechanism is the existence of a pro-ecological consumer who has a positive ownership share in an innovative producer possessing an environmentally friendly production possibility. Under these conditions, the production-preference relation links demand-side preferences to production decisions. The result therefore provides a formal representation of a channel through which demand can contribute to shaping the direction of technological change.

The contribution of the paper should be interpreted carefully. The model does not imply that consumers alone determine technological trajectories, nor that every increase in environmental awareness automatically generates eco-innovation. Rather, it establishes sufficient structural conditions under which demand-side transformation and supply-side innovation become connected. In particular, environmentally friendly

alternatives must be feasible, innovative production capabilities must exist, and the relevant ownership relationship must transmit consumer preferences to production decisions.

The dynamic extension further interprets this mechanism as a possible path of sustainable innovative development. By representing the economy as a (quasi)-semidynamical system, the analysis moves beyond a comparison of two static equilibria and describes sustainable innovation as a process of cumulative transformation. This provides a bridge between the static formal conditions and the evolutionary interpretation of economic development.

The results contribute to the literature on Schumpeterian innovative evolution by complementing producer-centred approaches with an explicit formal representation of demand-side influence. The consumer is not treated merely as a passive adopter of innovations. Under the assumptions of the model, consumer preferences can become relevant for production choices through the production-preference relation and ownership structure. This perspective suggests that sustainable technological trajectories should be analysed as the outcome of interactions between consumer preferences, feasible consumption opportunities, production capabilities and institutional relationships.

The formal framework also identifies several directions for further research. In particular, future extensions could examine the role of heterogeneous consumer preferences, varying ownership structures, endogenous prices, resource depletion over time, and explicit adjustment mechanisms connecting demand to firms' innovation decisions. Such extensions would make it possible to investigate not only whether sustainable innovative evolution can occur under given conditions, but also how rapidly and under which institutional and market conditions it may emerge. Thus, the present model can be regarded as a theoretical starting point for a broader analysis of the drivers and barriers of demand-induced sustainable innovation.

This work is coherent with currently active – in spite of the Schumpeter's theory – research path which emphasizes the active role of consumers in innovation processes and with the idea that economic evolution is an immensely complex process, especially with the fact that the demand side plays an important active role in the process of sustainable development. In this context, the elaboration seems to give a new perspective on the idea of an active role of consumers in eco-innovative processes not only as adopters of new products but also as initiators of eco-innovations and provides a theoretical basis for arguing that the demand side of the economy can play an active role in shaping sustainable innovative evolution. Sustainable social transformation is not merely a consequence of technological progress imposed on consumers; rather, it can emerge from the interaction between consumers' pro-ecological preferences, environmentally friendly consumption plans, and producers' responses to changing market conditions.

Moreover, given the axiomatical analysis of sustainable development based on eco-innovations in the Arrow-Debreu set-up includes the basic elements of the mathematical formalism which constitute the starting

point for further research on the drivers and barriers that affect an impact of the demand sphere on the process of sustainable development.

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