Supplementary information

Frequency band choice and the incremental value of joint EEG representations for cognitive task classification

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The tables provide numerical values corresponding to the main-text comparisons. SC denotes spatial cognition and EM denotes episodic memory. Accuracy is expressed as a percentage. Each frequency band was evaluated separately.

Supplementary Table S1. Spatial-cognition accuracy by frequency band

Method	Delta	Theta1	Theta2	Alpha1	Alpha2	Beta	Gamma
PCMI	81.2	81.8	82.2	83.5	83.2	82.8	82.0
MFDFA	82.6	83.2	83.8	84.5	84.2	84.0	83.5
WA-MFDFA	86.8	87.5	88.0	89.2	89.0	88.7	88.2
SNW-PCMI	87.2	87.8	88.3	89.4	89.2	88.9	88.5
DWT-Entropy	79.8	80.5	81.2	82.5	82.2	81.8	81.0
CSP	86.0	86.8	87.2	88.2	88.0	87.7	87.0
ICA-WT	85.5	86.2	86.8	87.8	87.5	87.2	86.5
MFCSync	**90.8**	**91.5**	**92.0**	**92.8**	**92.6**	**92.3**	**91.8**

Entries are classification-accuracy point estimates. They underlie the spatial-cognition curves and ranges in Figs. 2 and 3 and the derived comparison in Supplementary Table S5.

Supplementary Table S2. Episodic-memory accuracy by frequency band

Method	Delta	Theta1	Theta2	Alpha1	Alpha2	Beta	Gamma
PCMI	79.2	79.8	80.3	81.2	80.8	80.4	79.6
MFDFA	80.8	81.4	82.0	82.9	82.5	82.2	81.4
WA-MFDFA	87.8	88.5	89.0	90.0	89.7	89.4	88.8
SNW-PCMI	88.8	89.4	89.9	90.8	90.5	90.2	89.6
DWT-Entropy	76.2	77.0	77.6	78.6	78.2	77.8	77.0
CSP	83.0	83.8	84.3	85.2	84.8	84.5	83.7
ICA-WT	81.5	82.2	82.8	83.7	83.3	83.0	82.2
MFCSync	**91.3**	**92.0**	**92.5**	**93.2**	**92.9**	**92.6**	**92.1**

Entries are classification-accuracy point estimates. Mean accuracies and mean gains in the main text use the separately reported mean columns, retaining their reporting precision.

Composite classification indicators

F1 and AUC are presented under their composite evaluation basis. They are not substituted for accuracy averaged across the seven separately evaluated frequency bands. The reporting limits for their class definitions and aggregation conventions are recorded in Supplementary Table S4.

Supplementary Table S3. Composite F1 and AUC values

Method	SC F1	SC AUC	EM F1	EM AUC
PCMI	0.8383	0.9158	0.8019	0.8516
MFDFA	0.8513	0.9307	0.8168	0.8626
WA-MFDFA	0.8892	0.9573	0.8868	0.9165
SNW-PCMI	0.8930	0.9598	0.8969	0.9330
DWT-Entropy	0.8365	0.8925	0.7701	0.8152
CSP	0.8795	0.9315	0.8375	0.8724
ICA-WT	0.8740	0.9260	0.8191	0.8528
MFCSync	**0.9241**	**0.9826**	**0.9213**	**0.9568**

AUC, area under the receiver operating characteristic curve. Entries are aggregate point estimates.

Supplementary Data

Supplementary Data 1 preserves the four source tables: Band_Accuracy (112 rows), Mean_Accuracy_and_Gains (16 rows), Fusion_Gain_by_Band (14 rows), and Supplementary_F1_AUC (16 rows). Additional sheets provide configuration ranges, best-reference comparisons, relative error reductions, indicator contrasts, mean-gap decomposition, and bounded-score sensitivity using auditable formulas. The data dictionary specifies source cells, units, and aggregation levels. These are existing aggregate values and deterministic derived summaries.

Supplementary Data 2 contains SC_S001_original.jpeg and EM_S001_original.jpeg, the unmodified source panels used in Fig. 5. Figure_sources.txt identifies the source figures, native image dimensions, and preparation of the composite. The original voltage-field panels, time labels, and separate color scales were preserved; the amplitude-axis labels were translated into English. These source images document the existing visualizations and are not raw EEG time series.

Participant and implementation specifications

This table records the available descriptions of the source SC and EM evaluations. “Not specified” identifies details not established by that documentation; it does not imply that a procedure was omitted. “Unresolved” identifies inconsistent or ambiguous descriptions.

Supplementary Table S4. Participant and implementation specifications

Item	Specification
Participant characteristics	**Reported:** SC comprised 25 and EM 22 healthy young adults, with 16 EEG channels in each dataset. Participants were primarily faculty members and students associated with the research group who took part as volunteers. **Not specified:** Age range and mean with standard deviation, sex counts, eligibility criteria, and participant overlap between the two cohorts.
Retained trials and windows	**Reported:** SC used 2-s windows from stimulus onset and EM 3-s windows from memory-cue onset, with 50% overlap. At 125 Hz, these correspond to 250 and 375 samples per window. SC and EM classes were pre-training and post-training assessments. **Not specified:** Original and retained trial/window counts by participant and class, exclusions and reasons, trial duration, and the complete window-start schedule. Samples per window are not counts of trials or windows.
Acquisition and preprocessing	**Reported:** Both datasets used 16 channels with task-specific electrode lists given in the main Methods. Processing comprised Chebyshev type-I bandpass filtering at 1-100 Hz, 50-Hz notch filtering, FastICA for ocular, muscle and cardiac artifacts, and mean-based downsampling from 1000 to 125 Hz. Python/MNE-Python supported loading, filtering and segmentation. **Not specified:** Device manufacturer/model, reference and ground, impedance criterion, bad-channel handling, filter order and ripple, notch settings beyond the center frequency, ICA settings and component-rejection criteria, baseline correction, and software versions.
Frequency-band limits	**Reported:** The preprocessing description lists Delta 1-2, Theta 2-8, Alpha1 8-10.5, Alpha2 10.5-13, Beta1 13-20, Beta2 20-30 and Gamma 30-50 Hz. **Unresolved:** The SC and EM accuracy tables instead label their columns Delta, Theta1, Theta2, Alpha1, Alpha2, Beta and Gamma. The available documentation does not establish a one-to-one mapping between these sets. Consequently, the exact Hz boundaries of the classification columns remain unconfirmed; the preprocessing boundaries cannot be assigned by column position.
WA-MFDFA configuration	**Reported:** A cumulative-deviation profile is divided into non-overlapping segments of length s. Polynomial trends are fitted by least squares, and amplitude weights are applied to residual fluctuations. The method evaluates q-order fluctuation functions and derives generalized Hurst and multifractal descriptors, with a separate expression for $q = 0$. **Not specified:** The implemented scale set and scaling-fit interval, q grid, polynomial degree, exact amplitude-weight definition and normalization, remainder handling, and retained descriptor list and dimension.

Item	Specification
SNW-PCMI configuration	**Reported:** The method uses m-dimensional ordinal embeddings, index-based tie breaking, and amplitude standard deviations and their products as weights. Bidirectional delayed conditional-information measures are normalized and used with an eigenvalue-based global synchronization descriptor. A separate delay range of 3-15 is reported; it does not specify the embedding spacing tau. **Not specified:** The values of m and tau; delay units and increment; conditioning and temporal alignment; standard-deviation denominators; delay aggregation; full normalization and zero-value handling; synchronization-matrix construction; and retained feature dimensions.
Feature projection and classifiers	**Reported:** The two feature vectors are concatenated and reduced using covariance-based PCA. The performance-evaluation and results sections identify SVM classifiers for SC and EM. **Unresolved:** An earlier evaluation paragraph describes a CNN for SC. The available descriptions do not resolve which classifier generated the SC accuracy table. **Not specified:** Branch and concatenated dimensions, retained PCA dimension or selection rule, feature scaling, SVM kernel and hyperparameters, class weights, and tuning procedures. The comparison-method settings and whether all methods shared projection and classifier configurations are also unspecified.
Fold construction and fitting scope	**Reported:** Within-participant five-fold cross-validation grouped all overlapping windows from each original trial into the same fold. For each participant, four folds were used for training and hyperparameter optimization and one for validation, rotating through all five folds. **Not specified:** Trial allocation, stratification, randomization, seed, consistency of splits across methods and bands, the internal hyperparameter-selection procedure, and the data used to fit scaling, PCA, CSP or other learned transformations. Trial grouping alone does not establish that these operations were fitted exclusively within training folds.
Metric aggregation and variability	**Reported:** Five validation results were averaged. Frequency-band accuracies and separate composite F1/AUC values are provided in the source tables. **Not specified:** Aggregation across participants, weighting, and the order of averaging; the frequency conditions, prediction outputs, positive-class definitions and averaging conventions underlying F1/AUC. **Unresolved:** The statistical definition of the tabulated plus/minus values is unclear. Although the narrative calls them confidence intervals, it provides no confidence level, calculation method or aggregation unit. No SD, SE or confidence-interval interpretation is assigned to these values here.

Additional descriptive comparisons

JCD-EEG characterizes the incremental value of joint EEG representations under frequency-band, reference, and aggregation choices. The comparisons below use the established MFCSync evaluation table. Frequency bands share participants and recordings and are not independent replicates.

Supplementary Table S5. Fusion compared with the best nonfused configuration

Quantity	Spatial cognition	Episodic memory
Nonfused methods × bands	7 × 7 = 49	7 × 7 = 49
Best nonfused configuration	SNW-PCMI / Alpha1	SNW-PCMI / Alpha1
Best nonfused accuracy (%)	89.4	90.8
Lowest fusion accuracy (%)	90.8	91.3
Band of lowest fusion accuracy	Delta	Delta
Minimum separation (percentage points)	1.4	0.5
Highest fusion accuracy (%)	92.8	93.2

Minimum separation equals the lowest MFCSync band accuracy minus the highest accuracy among all nonfused methods and bands. The counts describe configurations rather than independent experiments. The separation is descriptive and has no attached confidence interval.

Supplementary Table S6. Relative error reduction from reported mean accuracies

Quantity	Spatial cognition	Episodic memory
SNW-PCMI reported mean accuracy (%)	88.47	89.90
MFCSync reported mean accuracy (%)	91.97	92.37
Accuracy gain (percentage points)	3.50	2.47
Implied SNW-PCMI mean error rate (%)	11.53	10.10
Implied MFCSync mean error rate (%)	8.03	7.63
Relative mean error reduction (%)	30.36	24.46

Relative reduction is 100 times the accuracy gain divided by the implied SNW-PCMI mean error rate. Calculations retain the original reported mean columns; they do not reaverage rounded band entries. These percentages are not participant-level averages of relative reductions or fractions of identifiable errors corrected.

Reported means and conditional score sensitivity

The source means and the sensitivity calculations complement the decomposition in main Table 2. All differences are in percentage points unless indicated otherwise.

Supplementary Table S7. Reported mean accuracies and fusion gains

Method	SC accuracy (%)	EM accuracy (%)	SC gain (pp)	EM gain (pp)
PCMI	82.39	80.20	9.58	12.17
MFDFA	83.69	81.90	8.28	10.47
WA-MFDFA	88.17	89.03	3.80	3.34
SNW-PCMI	88.47	89.90	3.50	2.47
DWT-Entropy	81.29	77.49	10.68	14.88
CSP	87.27	84.19	4.70	8.18
ICA-WT	86.79	82.67	5.18	9.70
MFCSync	**91.97**	**92.37**	**—**	**—**

Accuracies are reported means across seven separately evaluated bands. Each gain is the MFCSync mean minus the method mean in that row. The original mean precision is retained; the means are not reconstructed from the rounded entries in Tables S1 and S2. SC, spatial cognition; EM, episodic memory; pp, percentage points.

Supplementary Table S8. Conditional sensitivity of the configuration separation

Quantity	Spatial cognition	Episodic memory
Reported separation G (pp)	1.40	0.50
Equal per-entry bound guaranteeing $\tilde{G} > 0$ (pp)	$\varepsilon < 0.70$	$\varepsilon < 0.25$
Reported band display step (pp)	0.10	0.10
Maximum rounding discrepancy per entry, if rounded to nearest step (pp)	0.05	0.05
Conditional unrounded separation lower bound (pp)	**1.30**	**0.40**

The per-entry bound applies to each reported accuracy with potentially different directions of change. At $\varepsilon = G/2$, a tie is possible. The rounding bounds assume rounding to the nearest 0.1 percentage point; the original rounding rule was not specified. These bounds concern changes in point estimates and are not confidence bounds or EEG-noise tolerances.

Supplementary Note 1. Exact relationships for configuration comparisons

For one task, f contains the seven fusion accuracies and c contains the 49 nonfused method–band accuracies. Let L = min(f), C = max(c), and G = L − C. All statements below concern this finite configuration table.

Separately chosen convex summaries

For nonnegative weights α and β with each weight vector summing to one, $\alpha^{T}f \geq L$ and $\beta^{T}c \leq C$. Their difference is therefore at least G. Choosing all fusion weight on an entry attaining L and all nonfused weight on an entry attaining C gives equality. The minimum in main equation (3) is consequently G. Strictly positive separation is equivalent to a strictly positive difference for every pair of these convex summaries. Allowing zero weights is necessary for the extrema to be attained and for this exact equivalence as stated.

This is a property of arithmetic summaries of existing accuracy entries. It neither trains an ensemble nor estimates the accuracy of a rule that chooses a classifier for each trial. No assumption of statistical independence among entries is used.

Decomposition relative to reported means

Expanding the right-hand side of main equation (4) cancels $\bar{f}$ and $\bar{s}$, leaving L − C. Thus, the identity holds for the reported mean anchors without requiring equality between those means and a fresh average of the displayed band values. The numerical decompositions are 3.50 − 0.93 − 1.17 = 1.40 pp for spatial cognition and 2.47 − 0.90 − 1.07 = 0.50 pp for episodic memory. The adjustments change the reference comparison and do not partition a causal effect.

Bounded score changes and boundary cases

If every fusion entry differs from its reported value by at most ε_F and every nonfused entry by at most ε_C, the perturbed fusion minimum is at least L − ε_F and the perturbed nonfused maximum is at most C + ε_C. Subtracting gives main equation (5), even when the identities of the extreme entries change. For the ranges considered here, shifting all fusion entries down by ε_F and all nonfused entries up by ε_C attains the bound. With $\varepsilon_F = \varepsilon_C = \varepsilon$, strict positivity is guaranteed for ε < G/2. At equality, only nonnegative separation is guaranteed and a tie is possible.

If display values were rounded to the nearest 0.1 pp, each unrounded point estimate differs by at most 0.05 pp. Applying equation (5) gives lower bounds of 1.30 pp and 0.40 pp. The rule is an explicit condition rather than an assertion about the undocumented original rounding procedure. If an additional per-entry bound δ is considered after reserving that rounding budget, the combined lower bound is G − 0.10 − 2δ. Additional strict-positivity thresholds would then be δ < 0.65 pp and δ < 0.20 pp; the same rounding budget cannot be counted twice.

Reproducible calculations

Supplementary Data 3 contains JCD_EEG_Analysis.py, README.txt, and the four source CSV tables. The script uses only the Python standard library and Decimal arithmetic. It reconstructs all derived CSV tables from the reported values, preserving the distinction between band entries, reported means, and composite indicators. It performs no EEG preprocessing, fitting, or experimental evaluation.

Supplementary Note 2. Feature expressions and fixed-projection interpretation

This note records the feature construction of the established MFCSync workflow [1], whose reported results supply the inputs to JCD-EEG. Equations (S1)–(S10) describe that feature and classifier representation; equation (S11) states an algebraic identity for its fixed PCA projection. The analytical comparisons of the present study are defined separately in main equations (1)–(6).

Weighted multifractal complexity features

The MFCSync framework [1] combines two feature branches. The complexity branch uses weighted-amplitude multifractal detrended fluctuation analysis (WA-MFDFA). For a channel sequence x(t), t = 1, …, N, the mean-centered cumulative profile is

$$y(i) = \sum_{t=1}^{i} [x(t) - \bar{x}], \quad \bar{x} = \frac{\sum_{t=1}^{N} x(t)}{N} \tag{S1}$$

At scale s, the profile is divided into equal-length non-overlapping segments. A local polynomial trend is fitted by least squares in segment v. With amplitude-related weight W_v(j), the detrended fluctuation is

$$F_v^2(s) = \frac{1}{s} \sum_{j=1}^{s} W_v(j)[y((v-1)s+j) - pv(j)]^2 \tag{S2}$$

Here p_v(j) is the local polynomial trend. For a nonzero moment order q, the fluctuation values from N_s valid segments are aggregated as

$$F_q(s) = \left[\frac{1}{N_s} \sum_{v=1}^{N_s} [F_v^2(s)]^{q/2}\right]^{1/q} \propto s^{h(q)} \tag{S3}$$

The generalized Hurst exponent h(q) is estimated from the slope of log F_q(s) against log s over the selected scale interval. Different moment orders change the relative contribution of larger and smaller fluctuations. For a one-dimensional sequence, the mass exponent and generalized dimension are related by

$$\tau(q) = qh(q) - 1, D_q = \frac{\tau(q)}{q-1}, \quad q \neq 1 \tag{S4}$$

The retained scale descriptors from all channels form the complexity vector f_{WA}. These descriptors characterize the temporal organization available within the selected windows and scales. The available documentation does not fully specify the implemented moment-order grid, scale set, polynomial order, amplitude-weight rule or retained descriptors; these reporting gaps are identified in Supplementary Table S4.

Weighted ordinal dependence and global synchronization

For channel sequences X and Y, delay vectors are constructed with embedding dimension m and spacing τ. For X,

$$X_t = [X(t), X(t+\tau), \ldots, X(t+(m-1)\tau)] \tag{S5}$$

The entries are ranked in ascending order to obtain an ordinal pattern; equal values are ordered by their original indices. There are at most m! patterns. Vectors sharing a pattern are pooled, and the standard deviation of the associated amplitudes supplies its weight. With π_X denoting the ordinal pattern and w_i its amplitude weight, the weighted entropy is

$$H_w(X) = -\sum_{i=1}^{m!} w_i p(\pi X = i) \log p(\pi X = i) \tag{S6}$$

Marginal, joint, and conditional probabilities are obtained from pattern counts. Corresponding joint and conditional entropy descriptors incorporate products of the relevant amplitude standard deviations. Terms with zero probability follow the convention 0 log 0 = 0. Let X_δ denote the delayed state of X. A weighted conditional-information descriptor is defined by

$$I_w(Y; X_\delta \mid X) = H_w(Y \mid X) + H_w(X_\delta \mid X) - H_w(Y, X_\delta \mid X) \tag{S7}$$

When its denominator is positive, normalization by the associated conditional-entropy sum gives

$$C_{Y \to X}(\delta) = \frac{2 I_w(Y; X_\delta \mid X)}{H_w(Y \mid X) + H_w(X_\delta \mid X)} \tag{S8}$$

The reverse-direction score is obtained by exchanging X and Y. The source reports delay indices from 3 to 15; their units and the aggregation rule are not established in the available documentation (Supplementary Table S4). Pairwise scores form a 16 × 16 synchronization matrix with a unit diagonal. Let λ_i denote its eigenvalues and p_i their values normalized by the eigenvalue sum. When this spectrum is nonnegative and sums to one, the global synchronization descriptor is

$$S = 1 + \frac{\sum_{i=1}^{C} p_i \log p_i}{\log C} \tag{S9}$$

C is the number of channels. The delayed dependence descriptors and global synchronization score form f_{SNW}. Supplementary Table S4 identifies the reported conventions and remaining gaps in embedding, weighting, delay aggregation, matrix construction and spectral handling. These amplitude-weighted expressions are used as coupling descriptors; their numerical properties depend on those implementation choices and are not assumed to inherit every property of Shannon mutual information.

Feature projection and classification

The two feature vectors are concatenated and centered before PCA projection. With μ denoting the centering vector and P the retained principal directions, the SVM [2] receives the projected input z:

$$f = [f_{WA}{}^T, f_{SNW}{}^T]^T, \qquad z = P^T(f - \mu), \qquad \hat{y} = f_{SVM}(z) \tag{S10}$$

To characterize the existing projection, let A(x) and B(x) denote the complexity and dependence vectors supplied to PCA for EEG window x, with all feature mappings and the fitted projection held fixed. Let P_A contain the rows of P corresponding to A. For two windows with identical dependence features,

$$B(x) = B(x') \;\Rightarrow\; z(x) - z(x') = P_A^T[A(x) - A(x')] \tag{S11}$$

The additional distinction supplied by A survives in the classifier input precisely when the right-hand side is nonzero. This identity characterizes the fixed projection; survival alone establishes neither label relevance nor improved classification. No frequency of such window pairs or empirical projection-retention statistic was estimated. The source descriptions and unresolved settings for feature dimensions, projection, scaling, classifiers and comparator implementations are recorded in Supplementary Table S4.

References for Supplementary Note 2

[1] Wang, S. et al. MFCSync: A multifractal-causal synchronization framework for spatiotemporal EEG feature extraction in cognitive assessment. Expert Syst. Appl. 314, 131598 (2026). https://doi.org/10.1016/j.eswa.2026.131598

[2] Cortes, C. & Vapnik, V. Support-vector networks. Mach. Learn. 20, 273–297 (1995). https://doi.org/10.1007/BF00994018