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Comparing HVAC Energy-Saving Measures in a Saudi Residential Building under a Volume Tariff

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Abstract

Heating, ventilation and air-conditioning (HVAC) dominates Saudi household electricity. We ask, by simulation, which lever lowers it in apartment buildings under the energy-only (volume) tariff households pay: setpoint, operating mode, envelope, equipment or timing. A four-storey building designed to the Saudi Building Code is modelled room by room, with coupled rooms, split heat pumps as installed, typical-year Riyadh and Jeddah weather and per-apartment meters, and compared with EnergyPlus. Every policy keeps each conditioned room's air within 20–24 °C, around the 22–24 °C comfort band. Against the pre-code envelope, the code envelope saves 38–56% of HVAC electricity. Raising the cooling setpoint from 22 to 23.5 °C saves 9.8–12.4%, net of the extra winter heating, and replacing the fixed-speed units with inverter units saves 11–15% on assumed part-load data. None of the 48 thermostat pre-cooling settings saves in the base model; pre-cooling by 2 K before the peak costs 3.4–4.3% (2.0–2.8% in the lowest mode). A perfect-foresight linear program bounds the saving of any periodic schedule of an idealised model on design days at 1.9% of sensible cooling electricity within the band and 3.6% down to 20 °C. Under an illustrative time-of-use tariff, pre-cooling saves 2–27%. A factorial decomposition of four levers ranks the envelope first and timing last, and a global uncertainty analysis of 17 assumed inputs (128 Latin-hypercube samples per city) preserves this ordering. The results assume an energy-only tariff, rooms occupied and conditioned at all hours, a hard ceiling with humidity priced but not constrained, and coverage-sized units.

Keywords: residential air-conditioning; setpoint; building envelope; pre-cooling; volume tariff; Saudi Building Code

1 Introduction

Air-conditioning dominates residential electricity use in Saudi Arabia. It takes about two-thirds (66%) of household consumption in the national stock model [1] and more than 70% in recent building studies [2, 3]. An extensive literature proposes to reduce it by *scheduling* the air-conditioners, that is, by pre-cooling rooms before the afternoon, shifting load to cheaper hours or coordinating the units of neighbouring rooms, and reports large reductions in peak demand or cost [4–6]. These savings, however, are obtained under particular envelopes, particular setpoints and, above all, particular tariffs, most often prices that vary with the time of day.

Motivation. Before households, builders or regulators invest in scheduling, they need to know whether it pays under the conditions that Saudi households actually face and, if it does not, which lever does. Three features of these conditions matter. The residential tariff is a two-tier *volume* tariff, which prices how much energy a household uses but not when it uses it [7]. The summer climate and a hard comfort ceiling leave no room to let occupied rooms warm up, so the only temporal action available is to pre-cool. Many urban households live in apartment buildings, in which every room is coupled to its neighbours through walls, slabs and open doors, part of the floor area (baths, corridors, halls; about a third in the building studied here) has no air-conditioner, and every apartment has its own meter. A saving must therefore be computed room by room and meter by meter. For a single room at constant conditions it is intuitive that pre-cooling cannot pay when only energy is priced. For a building of coupled rooms, part of them without a unit, and with a coefficient of performance (COP) that changes with the hour and the mode, the answer is less obvious. A study of Saudi homes under this tariff reports coordination savings of about 1–4% in a narrow comfort band and more in a wider one [8]. We are not aware of a study that separates the scheduling, setpoint and envelope levers under these three conditions together (Section 2).

This paper asks how much heating, ventilation and air-conditioning (HVAC) electricity can be saved in a Saudi apartment building, with its floor plan, its air-conditioners and the tariff its households pay, in summer and in winter, and through which of five levers: the setpoint, the operating mode of the units, the envelope, the equipment or the timing of operation. We answer with a simulation study, supported by theory, of one building in two climates, hot-dry Riyadh and hot-humid Jeddah, and in three cases: Case 1, one apartment; Case 2, the typical floor of two apartments; and Case 3, the whole four-storey building. The building, designed for this study, meets the residential envelope requirements of the Saudi Building Code (SBC) for climate Zone 1; the same building with the envelope of the existing uninsulated stock serves as the comparison. Throughout, the bill is minimised over the policies that keep every conditioned room within the safe set [20,24] °C, which contains the 22–24 °C comfort band (Section 5.8), and every room is assumed to be occupied and conditioned at all hours (assumption (A4), Section 5.1).

The contributions are:

1. *Model and method.* A room-resolved model of a four-storey building with two-node rooms, wall, slab and open-door coupling, a latent load, catalogue units as installed (thermostats, inverter and fixed-speed compressors, cycling and standby losses, capacity derating) and per-meter billing, compared with EnergyPlus, including its response to setbacks, and with the energy use of the stock (Sections 5 and 8).
2. *Theory.* An energy accounting for multi-mode units under a hard ceiling, which adapts the storage-efficiency view of energy flexibility [9, 10] to the multi-mode framework of Mousa et al. [11, 12]. It gives the cheapest stationary operation of a room (Theorem 1), bounds what

pre-cooling can gain under any tariff that prices energy alone, apart from changes in cycling losses (Propositions 1 and 2) and identifies the only sources of saving (Proposition 3) (Section 6).

3. *Evidence.* A decomposition of the saving by lever across three cases, two cities and both seasons under an energy-only tariff, with a test of every pre-cooling depth pair that a thermostat with 0.5 K steps can realise in two fixed windows, a perfect-foresight bound on any periodic schedule of an idealised model, a time-of-use contrast, a factorial decomposition and a global uncertainty analysis (Section 9).

Section 2 reviews the literature and identifies the gaps, and Section 3 outlines the methodology (Fig. 1). Section 4 describes the case study, Section 5 the model and the problem, and Section 6 the theory. Section 7 sets out the baselines, methods and experiments, Section 8 verifies and validates the model, Section 9 reports the results, Section 10 discusses them and their limits, and Section 11 concludes. Supplementary Sections S1 and S2 give the symbols and the proofs, and Supplementary Sections S3–S9 the supporting analyses.

2 Related Work and Positioning

Residential HVAC control has moved from on–off thermostats, through programmable setbacks, to predictive and learning-based controllers and, in the systems literature, to the scheduling of multi-mode plants. Each step adds sophistication to the controller, but none by itself says how much a household's bill can fall or through which lever. Six strands bear on this question; the gaps they leave are stated at the end of the section.

Table 1 *State-of-the-art comparison: positioning against prior work. ✓: yes; ○: partly; –: no. Param.: envelope parameters taken from the code and cited sources; EUI: energy use checked against the stock or bills; Tariff: the tariff the customers pay is used; Levers: savings compared by lever under a common comfort constraint (○: levers evaluated one at a time); Tariff dep.: dependence of the saving on the tariff (A: one tariff or price signal assumed, not tested; T: compared across tariffs; B: bounded for every energy-priced tariff, with a perfect-foresight benchmark and a time-of-use contrast).*

Ref.	Method (study strand)	Years	Param.	EUI	Tariff	Levers	Tariff dep.
[4–6, 9, 10, 25–31]	Pre-cooling, thermal storage, flexibility	1990–2024	○	–	○	–	T
[19–24, 32]	Building MPC	2010–2026	○	○	○	–	A
[1–3, 35, 37, 38, 40–42]	Saudi envelope, equipment and stock	2015–2025	✓	✓	–	○	–
[44]	Occupancy scheduling (Riyadh homes)	2019	○	○	–	–	–
[8]	Learning-based coordination (Saudi homes)	2026	○	–	✓	○	A
This work	Multi-mode theory, optimal control, room-resolved model	2026	✓	✓	✓	✓	B

Scheduling of multi-mode and hybrid systems. HVAC control is a hybrid system of discrete switching and continuous dynamics [13]. Optimal scheduling has been solved for constant- and linear-rate multi-mode systems [14, 15], where a system is schedulable from the interior of the safe set exactly when there are dwell times whose rate vectors sum to zero, and safe schedulability has been decided for bounded-rate systems [16]; Mousa et al. [11, 12] extended optimal control to simple linear hybrid systems and to discrete switching costs. Green scheduling recasts peak reduction as the scheduling of coupled zones [17, 18]. We adopt this formalism for the plant. Definition 1 specialises the zero-sum condition to a single room, and the coupled-zone dynamics of Nghiem et al. [18] carry the heat exchanged between rooms. The formalism is then embedded in a floor plan, with catalogue equipment, the actual tariff and a latent load, which is where the economic content of a saving lies.

Predictive and learning-based control. Model predictive control (MPC), reviewed by Drgoňa et al. [19] and, with the building models it relies on, by Kim et al. [20], is the most studied advanced approach; recent studies address forecast uncertainty [21] and the tuning of MPC for residential HVAC [22]. Zheng et al. (2024) [23] compare economic MPC based on a resistance–capacitance (RC) model with data-driven variants in the Building Optimization Testing (BOPTEST) framework, and Wang et al. (2024) [24] examine how the accuracy of machine-learning predictions affects MPC for demand response. For Saudi homes under this two-tier tariff, Faraj (2026) [8] developed a reinforcement-learning coordinator, based on physics-informed proximal policy optimisation (PI-PPO), that learns a stationary scheduling policy for multi-zone split units. That work contributes a controller and complements the present one; Section 10 relates its savings to the levers separated here. Across this literature the reported savings vary widely and are seldom split by lever or tested for their dependence on the tariff.

Pre-cooling and passive thermal storage. A large demand-side-management literature promotes *pre-cooling*: cooling rooms during off-peak hours so that the thermal mass stores "coolth" and less cooling is needed later. It began with the optimal control of building thermal storage by Braun (1990) [4], continued with residential simulations [25] and is reviewed, together with solar pre-cooling, by Naderi et al. [5]; recent work combines pre-cooling with a temperature reset [26]. Reviews and rate-structure studies find that the cost savings come mainly from time-of-use prices and demand charges and that pre-cooling usually raises the energy used [25, 27, 28]; the review of Naderi et al. [5] and the study of Mayes et al. (2024) [6] confirm that its value depends on time-of-use pricing and on the thermal mass. Braun [4] lists four opportunities: reducing coincident *peak demand*, using cheaper *night-time rates*, replacing mechanical cooling with *free night cooling*, and improving the *efficiency of mechanical cooling* through cooler condensing conditions and part-load operation at night. Under a residential volume tariff the first two do not apply, and Saudi summer nights are too warm for free cooling, so only the fourth, a diurnal swing of the COP, remains. Proposition 1 bounds what it can yield, and a perfect-foresight linear program bounds what an idealised schedule can attain (Section 9.3). The mechanism is known; the study quantifies it for the Saudi tariff and building.

Energy flexibility and pre-cooling in hot climates. The energy-flexibility literature treats the thermal mass as a

store and accounts for its losses. Le Dréau and Heiselberg [9] quantify the flexibility that short-term storage in the structure gives residential buildings and the extra energy it costs, Canet and Qadrdan [29] the flexibility of a national housing stock, and Reynders et al. [10] review the definitions and indicators of flexibility, among them the capacity and efficiency of structural storage. In hot climates, Arababadi et al. [30] show that envelope measures make residential pre-cooling more effective in a hot arid climate, and Rama Curiel and Thakur [31] control residential air-conditioners directly for demand-side management; both aim to move load away from the peak. Section 6 adapts this storage-efficiency accounting to multi-mode units under a hard ceiling.

Tariffs and the value of flexibility. The tariff, not the controller, gives load shifting its economic value. Time-of-use, critical-peak and real-time prices and demand charges reward temporal flexibility; a pure volume tariff does not. Price-responsive MPC [32] and peak-reducing green scheduling [17] therefore presuppose a price or peak signal. Saudi residential customers pay a two-tier *volume* tariff with no time signal [7]. Matar [33] models how Saudi households would respond to time-of-use pricing and what this would mean for the wider economy, but no such price applies to the bills studied here.

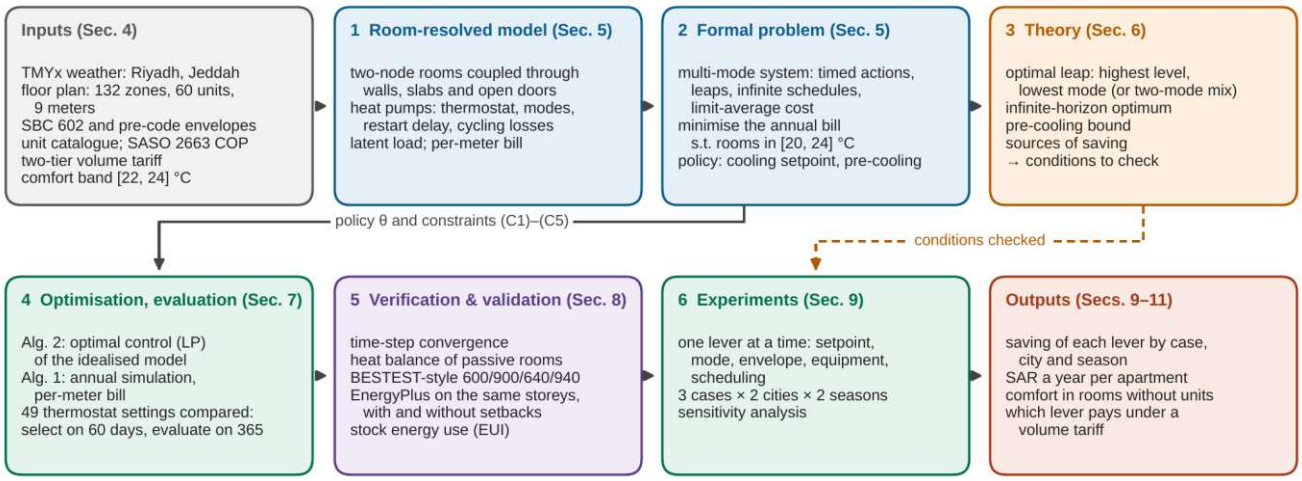


Fig. 1 *Methodology. The inputs (Section 4) define a room-resolved model of the building (1), which is written as a multi-mode system and a bill-minimisation problem within a hard safe set (2, Section 5). The theory (3, Section 6) characterises the optimal schedule and bounds what pre-cooling can gain; the computation (4, Section 7) solves an idealised form of the problem as a perfect-foresight optimal-control program, and evaluates by annual simulation the pre-cooling settings a thermostat can realise. After the model is verified and validated (5, Section 8), the experiments (6, Section 9) change one lever at a time and check the theory's conditions.*

Saudi envelope, equipment and stock studies. Studies of the SBC envelope and the existing stock [34–37] provide the envelope requirements, construction U-values and stock energy use adopted here, and Al-Homoud and Krarti [38] review the state of residential energy efficiency in the Kingdom. The stock model of Krarti et al. (2020) [1] estimates the residential energy use intensity (EUI) of the four regions (Section 8.2) and evaluates national retrofit programmes, among them envelope, setpoint and air-conditioner measures taken one at a time. Building simulations report that combined passive measures cut the cooling demand of a Saudi villa by 68% (IES-VE; Rodrigues et al. (2025) [2]) and that retrofit packages save 25–66% in a low-rise Jeddah apartment building (DesignBuilder/EnergyPlus; Felimban et al. (2023) [3]), with air-conditioning taking more than 70% of residential electricity in both. Krarti and Aldubyan (2022) [39] optimise retrofits for the peak demand of the housing stock, Alaidroos and Krarti [40] optimise residential envelopes for Saudi cities, and Krarti and Howarth [41] assess the costs and benefits of high-efficiency air-conditioners. Measured over 108 summer and autumn days in a Saudi office room, an inverter unit used about 44% less electricity than a fixed-speed unit of the same capacity (46% over an 18-day controlled period) [42]. Operation matters as well. The stay-at-home period of 2020 by itself raised residential demand markedly [43], and Alshahrani and Boait [44], simulating four types of Riyadh dwelling in DesignBuilder with occupancy from a household survey, find that scheduling the units to occupancy saves 15–31% of annual energy and remote setting of the thermostats by the utility 9–13%. Outside the Kingdom, Hoyt et al. [45] find in simulations of US office buildings that widening the range between the cooling and heating setpoints saves a substantial share of HVAC energy. This literature establishes the size of each lever separately. It does not compare the levers under the same comfort ceiling and the actual tariff, nor does it test whether re-timing the cooling of occupied rooms, the lever that dominates the demand-response literature, has any value under the volume tariff.

Research gap and positioning. Three gaps recur. *(G1) Lever conflation*: reported savings mix envelope, setpoint, equipment and schedule, so a headline percentage cannot be attributed to a cause a policymaker can act on. *(G2) Tariff-blindness*: the value of temporal flexibility is assumed rather than tested against the tariff the customer pays, although the tariff creates or removes that value. *(G3) Weak grounding*: envelope parameters and comfort targets are often assumed rather than taken from the governing code and climate. The study addresses (G1) with a decomposition of the levers under one comfort ceiling; (G2) with a test of scheduling under the Saudi tariff, a bound for every energy-only tariff, a perfect-foresight benchmark and a time-of-use contrast; and (G3) with a code-based, room-resolved model of the units as installed, compared with EnergyPlus. Table 1 places the study against representative studies of these strands: its advance

lies in the study design rather than in the optimiser. Section 3 sets out how the design is carried out.

3 Methodology

To address gaps G1–G3, the study proceeds in six stages (Fig. 1), each described in the section named in the figure. The timing lever is pre-cooling, the only re-timing that the hard 24 °C ceiling permits when every room is occupied (A4). One optimisation method is used: a perfect-foresight optimal-control linear program (LP) over an idealised form of the bill-minimisation problem, which bounds what any periodic schedule of that model can save on a design day and, with the engine's COP, finds an achievable saving below that bound (Section 7.2). The thermostat policies a household can choose are not optimised but evaluated: each is simulated for a year with the realistic controller and billed per meter, and the cheapest admissible one is kept (Algorithm 1 and Section 7.2). Each lever (setpoint, mode, envelope, equipment, timing) is then changed on its own against a stated baseline in three cases, two cities and both seasons. A factorial decomposition and a global uncertainty analysis test the interactions between the levers and the robustness of the conclusions (Section 9). One reproducible pipeline carries the inputs, described in Section 4, to every number in the paper.

4 Case Study: Climate, Comfort and the Building

The case study comprises two climates, a comfort band and one building, with its air-conditioners, its two envelopes and the three cases in which it is analysed. The model of Section 5 is built on it, and all results are computed for it.

4.1 Climate of Riyadh and Jeddah

The SBC divides the Kingdom into climate zones, and all Saudi cities are cooling-dominated [1, 34]. Riyadh and Jeddah are the two largest cities and lie in the two main residential climates, interior hot-dry and coastal hot-humid. Riyadh has large daily and seasonal swings, a small winter heating need and shoulder months close to the comfort band. Jeddah is warm and humid all year, so it needs cooling and dehumidification throughout, and its latent load is large where Riyadh's is negligible. Riyadh's cool winter also reveals the winter effect of the envelope, since insulation that helps in summer retains internal gains in winter as well. The pair therefore tests the levers in two different ways and shows whether the results hold across climates.

The model is driven by hourly weather for Riyadh and Jeddah from typical meteorological years (TMYx) assembled from weather-station records and the ERA5 reanalysis of the European Centre for Medium-Range Weather Forecasts (data sources in the Data availability statement). The files give dry-bulb and dew-point temperature, pressure, and global, direct and diffuse solar radiation, and they are the same files read by the EnergyPlus benchmark (Section 8.2). Fig. 2 plots the three components that drive the model: air temperature against the comfort band, relative humidity and solar radiation. The monthly values are summarised in Supplementary Section S5. Riyadh is hot and dry: its monthly mean rises from 14.7 °C in January to 36.4 °C in July, with daily maxima above 43 °C and relative humidity near 12% in summer. Jeddah is milder but humid: its mean never falls below 23.5 °C and its relative humidity stays between 54% and 70% all year. Both receive 4–8 kWh/m^2 of solar radiation per day.

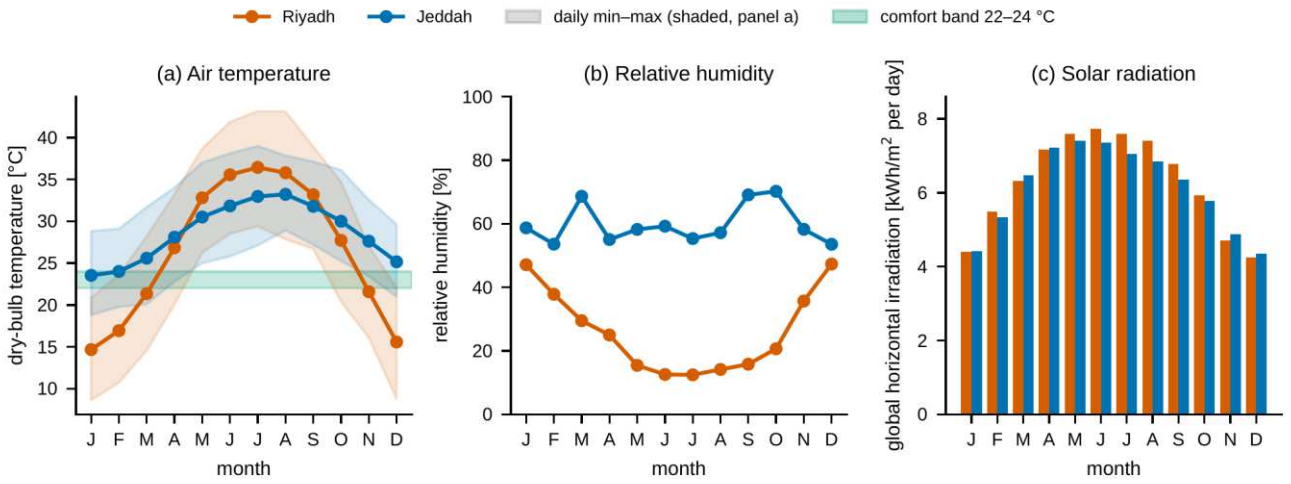


Fig. 2 *Annual weather of Riyadh and Jeddah (TMYx typical years). (a) Monthly mean air temperature and mean daily range against the comfort band: Riyadh crosses the band seasonally and exceeds 43 °C on summer afternoons, while Jeddah's monthly mean never falls below it. (b) Relative humidity: Riyadh is dry in summer and Jeddah humid all year, which drives its latent load. (c) Daily solar irradiation, 4–8 kWh/m^2 in both cities, which drives the solar gains.*

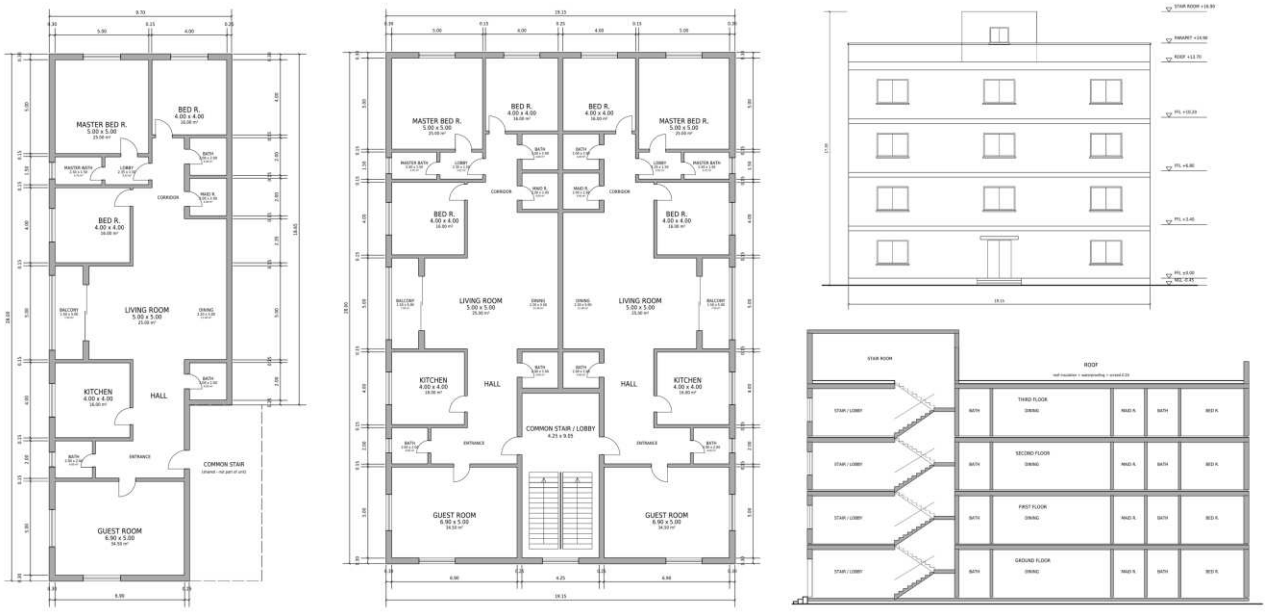
4.2 Comfort range: ASHRAE, SBC and the one-sided ceiling

ASHRAE 55 defines comfort through the predicted mean vote (PMV): a space complies when the PMV stays within ±0.5, that is, at most 10% predicted percentage dissatisfied (PPD) [46, 47]. For summer clothing (0.5 clo), light activity (1.1–1.2 met), still air and 50% relative humidity, this spans roughly 23–27 °C of *operative* temperature, and about 19–24 °C in winter clothing (1.0 clo). The SBC regulates the building fabric (Section 4.3) rather than the

setpoint, which is left to the occupant and to efficiency guidance.

We adopt a comfort band $[V_{\text{low}}, V_{\text{high}}] = [22,24]\,^{\circ}\text{C}$ of air temperature and treat its upper bound as a *hard* ceiling rather than the centre of a symmetric deadband. There are three reasons for this choice in Saudi homes. First, in sunlit construction with warm inner surfaces the operative temperature exceeds the air temperature on summer afternoons, most in rooms under an uninsulated roof. In the insulated building studied here the excess is small (Section 9.1), so a 24 °C ceiling sits at or below the middle of the ASHRAE summer zone. In the uninsulated stock and under a roof, however, the excess moves a 24 °C room towards the upper part of that zone, and letting rooms coast above the ceiling during the peak would take them towards its edge. The hard ceiling is therefore a conservative choice for the insulated building and a protective one for the stock. Second, at Jeddah's 54–70% relative humidity (Fig. 2b) evaporative cooling of the body is suppressed and the comfortable dry-bulb ceiling is lower still. Third, in the summer heat, conditioned rooms are refuges for vulnerable occupants. The ceiling also matches the lower end of the 24–26 °C range recommended by efficiency guidance in the neighbouring Gulf state of Qatar [48]. With this choice no saving is gained at the expense of comfort as measured by the air temperature of the conditioned rooms; the operative temperature it implies is checked in Section 9.1. Humidity is priced through the latent load but not constrained (Section 5.3). The safe set is imposed on the conditioned rooms, where occupants stay; the band is their comfort target. The passive zones, the rooms without a unit (Section 4.3), are compared with the band but not constrained by the safe set, and Supplementary Section S3 reports how the controller brings them close to it.

Flexibility is therefore *one-sided*. Both bounds of the safe set $[V_{\min}, V_{\max}] = [20,24]\,^{\circ}\text{C}$ are hard (Section 5.8), so a controller may pre-cool into $[20,22)\,^{\circ}\text{C}$, below the band, but never let an occupied room rise above the ceiling. Every re-timing of cooling in this paper is thus a pre-cooling action (cool into the headroom, then return to the setpoint) and never an upward coast (illustrated in Section 5.5). Since every room is taken to be occupied at all hours (A4), no room is allowed to float.



(a) Case 1: apartment A (b) Case 2: typical floor (c) Case 3: elevation and section

Fig. 3 *The building analysed in the three cases (drawings generated as layered DXF files; clear dimensions in metres). (a) Apartment A, analysed alone in Case 1. (b) The typical floor: two mirror-image apartments around the common stair; north is up. (c) South elevation (top) and section A–A through the stair (bottom) of the four-storey building (floor-to-floor height 3.40 m, 3.00 m clear; windows from 0.90 to 2.40 m above each floor). The zone rectangles of the model follow the wall centrelines of (b).*

4.3 The building: layout, air-conditioners, envelope and analysis cases

The building studied is a four-storey apartment building with two apartments per floor, a common housing type in Saudi cities (Fig. 3). The building was designed for this study as a set of layered drawings in Drawing Exchange Format (DXF) (plans, section and elevation), with room sizes, wall types, storey height and stair dimensions that follow the SBC (Fig. 3a). Each apartment has a master bedroom with its own bath, two further bedrooms (Bed 2 and Bed 3), a living room open to a dining area and an enclosed west balcony, a kitchen, a guest room, three further baths, a maid room and circulation spaces (Supplementary Section S5 lists every room). The two apartments of a floor mirror each other about a common stair (4.25×9.05 m) on the south side, on a 19.15×28.00 m footprint (Fig. 3b). Four floors of 3.40 m (3.00 m clear) form the building, entered from the south (Fig. 3c). The top of the plan faces north. Apartments are numbered by storey, from 1 (ground) to 4 (top), with A on the west and B on the east.

Each room is one thermal zone of the model (Section 5.2), a rectangle (or union of rectangles) on the wall centrelines of the drawing, so that each zone's clear size is that of the drawing; the enclosed balcony is an unconditioned zone. Each window enters with its own width and orientation, which gives a window-to-wall ratio of 17.3% for the

building. The model derives each zone's floor area, its exterior walls and their orientations, and every shared wall and slab from the rectangles, so no adjacency is entered by hand. An apartment covers 231 m^2 on wall centrelines (without the balcony), a floor has 33 zones and the building has 132.

Every bedroom has its own split unit, as do the living room, the dining area, the kitchen and the guest room; each unit is chosen from a four-model product catalogue by rated coverage (Table 2; 1 ton = 3.517 kW of cooling). The drawing gives the living room and the dining area a unit each; together they cover 36 m^2, the upper limit of the largest unit's coverage, so one unit would have no margin. They are modelled as two zones joined by a 5.00 m opening. The baths, the maid room, the lobby, the corridor, the hall and the balcony have no unit. Interior doors are open, with the widths of the drawing (0.70–0.90 m) and the 2.032 m minimum clear height of egress doors in SBC 201 (Section 1010) [49]; the wall-free openings of the open plan (corridor and hall to the living and dining area, living to dining) are 2.20–5.00 m wide and 3.00 m high. The apartment entrance and balcony doors are closed. The stair is conditioned and billed on a common meter, and each apartment has its own meter, to which the tariff applies. Each apartment therefore has seven units (14.0 tons), and the building has 60 units on nine meters.

The 1.5-, 2.0- and 2.5-ton units are inverter split heat pumps, modelled with four operating modes numbered 0–3: off and three on modes (low, medium and high). The 3-ton unit is a fixed-speed rotary-compressor heat pump; its three fan speeds do not modulate the compressor, so it is modelled as an on/off unit that runs at full capacity whenever it runs. Sizing by rated coverage is how such units are chosen in practice; with the SBC envelope it leaves them several times larger than the peak load, and a right-sized alternative is tested in Section 9.7. Their efficiency is anchored to the air-conditioner standard of the Saudi Standards, Metrology and Quality Organization (SASO), SASO 2663, whose 2014 edition set a minimum split-unit energy efficiency ratio (EER) of 11.5 (Btu/h)/W at test condition T1 [50]; the 2021 edition moved to seasonal ratings [51]. At full load and the T1 rating condition (35 °C outdoors, 27 °C indoors) the modelled COP is 3.37, which equals an EER of 11.5; at the 23.5 °C reference setpoint it is 3.13. The higher efficiency of the low and medium modes follows the assumed part-load ratios of Eq. (1) (Section 5.2). The heating capacity of each unit is taken as 1.14 times its cooling capacity (6.0 kW for the 1.5-ton unit), an assumed value.

Passive zones. About a third of each apartment's floor area has no unit: the four baths, the maid room, the lobby, the corridor, the entrance hall and the enclosed balcony. These *passive zones* are cooled in summer and heated in winter only by the thermal shift from their neighbours (Section 5.3). Their temperatures are therefore outputs of the model, not controlled quantities. Section 6.1 states what they can reach, and Section 9.1 reports them.

Table 2 *Air-conditioner catalogue and the rooms each unit serves. All units are split heat pumps for heating and cooling. The 1.5- to 2.5-ton units are inverter units with four operating modes (off, low, medium, high; capacity fractions 0, 0.35, 0.65 and 1); the 3-ton unit has a fixed-speed rotary compressor and runs on/off at full capacity. Capacity: cooling/heating; coverage: rated floor area. The 2.0-ton unit serves no room of the plan. The stair (38.5 m^2) lies just above the rated coverage of its unit, and the dining area (11.0 m^2) below that of the smallest unit.*

Unit	Capacity [kW]	Coverage	Rooms served
1.5 ton	5.28/6.0	16–18 m^2	Bed 2, Bed 3, kitchen, dining area
2.0 ton	7.03/8.0	19–22 m^2	none
2.5 ton	8.79/10.0	22–30 m^2	master bedroom, living room
3.0 ton	10.55/12.0	30–36 m^2	guest room, stair

Envelope. SBC 602 is the Kingdom's energy-conservation code for low-rise residential buildings [52]. It sets the envelope performance (wall, roof and glazing U-values and the glazing solar heat gain) for three climate zones [1, 34]. The walls, roof and glazing of the building meet the residential requirements for climate Zone 1 (SBC 602, as tabulated by Krarti et al. [1]), the hottest zone, which includes Riyadh. We apply the same requirements in both cities as a common reference: a wall U-value of 0.34 W/m^2K [1, 34, 52], roof and window U-values of 0.20 and 2.67 W/m^2K and a glazing solar heat-gain coefficient (SHGC) of 0.25 [1], with the windows of the drawing. Partitions and floor slabs keep the uninsulated U-values of the existing stock, and the ground slab is uninsulated (Table 3). This SBC-compliant building is the main case of the study. To quantify the value of the code, the same building is also simulated with the envelope of the existing, largely uninsulated stock: wall, roof and window U-values of 2.146, 2.123 and 5.778 W/m^2K and an SHGC of 0.62, taken from the calibrated model of an existing Riyadh residence [36]. Nothing else is changed, so the difference isolates the envelope.

Analysis cases. The same building is analysed as three cases of increasing extent (Fig. 3). *Case 1* is one apartment on its own, apartment A of an intermediate storey (Fig. 3a). This is the dwelling that receives a household bill. Its walls and slabs shared with the other apartment, the core and the storeys above and below face conditioned space and are treated as adiabatic. *Case 2* is the typical floor (Fig. 3b): both apartments side by side and the core, with the heat exchange between them. Its slabs remain adiabatic. *Case 3* is the whole building (Fig. 3c): four storeys of two apartments, with ground contact on the lowest storey, the roof above the highest, and heat exchange through every slab. Where Case 3 results are given for one apartment, it is 2A, on the first storey above the ground, which is neither in contact with the ground nor under the roof. Each case is run in both cities, for both envelopes (SBC-compliant and pre-code, Section 4.3), over the whole TMYx year, so both summer cooling and winter heating are simulated.

5 Model and Problem Formulation

Here the building of Section 4 is cast as a formal model, and the optimisation problem is stated. The formulation uses the multi-mode formalism of Alur et al. [14], Wojtczak [15] and Mousa et al. [12] and the coupled-zone model of Nghiem et al. [18]. Only the parts the theory needs are stated here, and the full formal definitions (zone

network, multi-mode system, timed actions, schedules, runs and their costs, schedulability) with the continuations of the running example are in Supplementary Section S6.

5.1 Modelling assumptions

The theory of Section 6 rests on five assumptions, of which the simulation relaxes the first three. **(A1)** Each zone is well mixed; the theory reads it as one lumped temperature V_i, the simulation as two nodes, air and structure (Section 5.3). **(A2)** The COP is quasi-static, a function of mode and ambient temperature (Eq. (1)) with no cycling loss; the simulation adds its dependence on the indoor air, cycling losses, standby power and capacity derating (Section 5.4). **(A3)** Within one leap cycle the heat gain L and the ambient temperature are constant. This holds only roughly for a lightly loaded room, whose cycle can last hours (about 4.6 h for the running example), so applying the cycle results hour by hour is an approximation; the simulation does not use (A3). **(A4)** Every room of every apartment is occupied and conditioned at all hours, with every conditioned zone following the same policy, so adjacent conditioned zones operate at the same level; the conclusions inherit this idealisation (Section 10.4). **(A5)** Building properties that are uncertain in practice (window sizes and U-values, solar absorptance, internal gains, interior doors all open) are held at the values of the drawing and of Table 3; the absorptance, internal gains and door schedules are varied in Section 9.7, and the windows only through the two envelopes compared.

5.2 Zones, units and the multi-mode system

The building is a finite set of zones $\mathcal{Z} = \{1, \dots, N\}$ (rooms). The *conditioned* zones $\mathcal{Z}_c$ each have their own unit; the *passive* zones $\mathcal{Z}_u$ (baths, maid room, lobby, corridor, hall and enclosed balcony) have none. Zone i has a thermal capacitance C_i and a temperature $V_i(t)$; adjacent zones $i \neq j$, sharing a partition or slab, are coupled by the conductance $K_{ij} = K_{ji}$ (U-value times area, zero otherwise), and each zone exchanges heat with the outdoors through K_i^{env} (infiltration alone for interior zones).

Each conditioned zone is served by a split heat-pump unit with four modes $M = \{0,1,2,3\}$ (off, low, medium, high; a fixed-speed unit has only 0 and 3). In mode m it removes heat at the rate $f_m \psi(T_a) Q_i$ when cooling and adds $f_m Q_i^{\mathrm{h}}$ when heating, where Q_i is its rated cooling capacity, $\psi(T_a) = [1 - 0.01(T_a - 35)]_{0.8}^{1.1}$ derates it with the outdoor temperature, $Q_i^{\mathrm{h}} = 1.14\, Q_i$, and $0 = f_0 < f_1 < f_2 < f_3 = 1$ are the capacity fractions (0.35, 0.65, 1). It draws $f_m \psi Q_i / \mathrm{COP}_m$ when cooling and $f_m Q_i^{\mathrm{h}} / \mathrm{COP}_m$ when heating, with

$$\mathrm{COP}_m^{\mathrm{cool}}(T_a, V_i) = c_m\, \varphi(T_a)\, g(V_i), \quad \mathrm{COP}_m^{\mathrm{heat}} = h_m, \qquad (1a)$$

$$\varphi(T_a) = [1 - 0.02\,(T_a - 27)]_{0.60}^{1.15}, \qquad (1b)$$

$$g(V_i) = [1 - \gamma\,(27 - V_i)]_{0.8}^{1.1}, \quad \gamma = 0.02\,\mathrm{K}^{-1}, \qquad (1c)$$

where $[x]_a^b = \min\{b, \max\{a, x\}\}$, $c_1 = 5.81 > c_2 = 4.98 > c_3 = 4.01$ and $h_1 = 4.6 > h_2 = 4.0 > h_3 = 3.2$: a unit is most efficient at low output. The cooling coefficients are anchored to the Saudi efficiency minimum (Section 4.3); φ lowers the COP as the outdoor air warms and g as the room's air falls below the 27 °C rating condition ($g = 0.93$ at 23.5 °C; γ is assumed and varied in Section 9.7). The building is then a multi-mode system [12] with one mode per unit, the heat balance of Eq. (2) as rate function, a cost $\pi_i(m)$ per unit time for each unit and mode, the safe set $\mathcal{S} = [V_{\min}, V_{\max}]$ and an initial state V_0.

Table 3 *Parameters and their provenance. Compliant envelope: Saudi residential requirements for climate Zone 1 [1, 34, 52]; pre-code: calibrated model of an existing Saudi residential building [36]. Each conductance is $K = UA/1000$ [kW/K] with A from the floor plan.*

Quantity	SBC	Pre-code	Source / justification
Wall / roof / window U [W/m²K]	0.34 / 0.20 / 2.67	2.146 / 2.123 / 5.778	SBC Zone 1 [1]; existing stock [36]
Glazing SHGC	0.25	0.62	SBC Zone 1 [1]; existing stock [36]
Exterior wall incl. 17.3% glazing [W/m²K]	0.74	2.78	derived (area-weighted, building mean)
Partition / slab U [W/m²K]	1.728 / 2.047		existing stock [36]
Ground slab U [W/m²K]; ground temperature	0.40; monthly, 2 m		ISO 13370 [53], uninsulated slab; TMYx ground record
Capacitance C [kWh/K per 20 m²]	1.5 (270 kJ/(m²K))		heavy class of ISO 13790 [54]; block/concrete construction
Air-node share of C; air–structure coupling	5%; 9.9 W/m²K of floor		ISO 13790 two-node simplification, heavy class [54]
Infiltration n_{ach} [ACH]	0.4		nominal, new construction (stock: 0.8 [1]); sensible and latent
Interior doors / openings $W_d \times H_d$ [m]; C_{dis}	0.70–0.90×2.032 / 2.20–5.00×3.00; 0.6		drawing; height SBC 201 [49]; Brown and Solvason [55]
Internal gains [W/m²]: continuous + daytime half-sine; mean	3.5 + 0–6.0 (08:00–20:00, peak 14:00); 5.4		end-use split [1]; times a room factor of 0–1.5 by use, assumed (Supplementary Section S5)
Opaque solar absorptance	0.7		nominal (idealised, A5)
Exterior film / ground reflectance	17 W/m²K / 0.2		nominal (sol-air temperature and diffuse gains)
Sky long-wave: ε; F_{sky} roof / walls, windows	0.9; 1 / 0.5		ISO 13790 [54]; hourly infrared radiation of the TMYx file
Window solar	angle-dependent		normalised SHGC curve of double glazing [56]
Unit sizes [ton]	1.5 / 2.0 / 2.5 / 3.0		catalogue, by coverage (Table 2)
Mode capacity fractions $f_1/f_2/f_3$	0.35 / 0.65 / 1.0		inverter units (Eq. (1)); fixed-speed: f_3 only
Cycling degradation C_d: inverter / fixed-speed	0.15 / 0.25		part-load factor per hour; inverter assumed, fixed-speed Air-Conditioning, Heating, and Refrigeration Institute [57]
Standby power; capacity derating	3 W per unit; ∓1%/K about 35 °C, [0.8,1.1]		nominal

Quantity	SBC Pre-code	Source / justification
Restart delay; mode escalation	3 min; 0.4/1.0 K, full if lagging	anti-short-cycle timer; nominal
Thermostat differential; setpoint step	1.0 K; 0.5 K	typical room thermostat; state updated every 3 min
Cooling COP (modes 1/2/3)	$5.81/4.98/4.01 \times \varphi(T_a)\, g(V)$	SASO 2663 minimum at full load [50]; ratios and γ assumed, Eq. (1)
Heating COP (modes 1/2/3)	4.6 / 4.0 / 3.2	nominal heat-pump values
Heating / cooling capacity Q_i^{h}/Q_i	1.14	nominal inverter heat pump (6.0 vs 5.28 kW at 1.5 ton)
Latent (dehumidification) COP, $\mathrm{COP}_{\mathrm{lat}}$	3.4	nominal, all modes (Eq. (4))
Reference setpoints: cooling / heating [°C]	23.5 / 22.5	ceiling and band floor $\mp$ half the differential
Indoor humidity target ω^*	9.6 g/kg	55% relative humidity at 23 °C; within the 12 g/kg applicability limit of the ASHRAE 55 graphic method [46]
Weather	TMYx hourly	typical year from station records and ERA5 (Fig. 2; Supplementary Section S5)

5.3 Thermal dynamics: inter-zone coupling and latent load

Following Nghiem et al. [18] and the two-node simplification of ISO 13790 [54], each zone has an air node V_i, which the thermostat senses and the unit acts on, and a structure node W_i, the inner layers of its walls and slabs:

$$C_i^{\mathrm{a}}\dot{V}_i = K_i^{\mathrm{w}}(T_a - V_i) + K_i^{\mathrm{am}}(W_i - V_i) + \sum_{j\in\mathcal{D}_i} q_{ji} + (1-f_r)G_i - S_i^{\mathrm{sky}} - s_i\,\beta_i\,f_{u_i}\hat{Q}_i, \qquad (2a)$$

$$C_i^{\mathrm{m}}\dot{W}_i = q_i^{\mathrm{env}} + \sum_{j\neq i} K_{ij}\left(W_j - W_i\right) + K_i^{\mathrm{am}}(V_i - W_i) + S_i^{\mathrm{w}} + f_r G_i, \qquad (2b)$$

$$q_i^{\mathrm{env}} = K_i^{\mathrm{o}}(T_a - W_i) + S_i^{\mathrm{o}} + K_i^{\mathrm{grd}}\left(T_g - W_i\right). \qquad (2c)$$

Here K_i^{w} covers windows and infiltration ($n_{\mathrm{ach}} = 0.4$ air changes per hour, ACH), K_i^{am} joins air and structure, $\mathcal{D}_i$ are the rooms joined by an open door (Eq. (3)), G_i are the internal gains with radiant share f_r, S_i^{sky} is the long-wave loss of the windows and S_i^{w} the window solar gain (SHGC with angle-dependent transmission [56]). S_i^{o} is the sol-air excess of walls and roof, net of the sky loss $\varepsilon F_{\mathrm{sky}}(\sigma_{\mathrm{SB}}T_a^4 - I_{\mathrm{IR}})$ with the hourly infrared sky radiation I_{IR} of the weather file, the Stefan–Boltzmann constant σ_{SB}, $\varepsilon = 0.9$ and $F_{\mathrm{sky}} = 1$ (roof) or 0.5 (walls, windows) [54]. T_a and T_g are the ambient and ground temperatures ($K_i^{\mathrm{grd}} \neq 0$ on the ground storey only), $u_i(t) \in M$ is the unit's mode ($u_i \equiv 0$ in passive zones), $s_i = \pm 1$ for cooling or heating, $\beta_i \in [0,1]$ the fraction of the step the unit runs, and $\hat{Q}_i$ is $\psi(T_a)Q_i$ in cooling and Q_i^{h} in heating. Apart from the door exchange the rates are affine in the temperatures, so the building is close to a linear-rate multi-mode system [15].

Through a door of width W_d and height H_d, buoyancy carries the heat [55]

$$q_{ji} = \rho c_p\; C_{\mathrm{dis}}/3\; W_d\sqrt{g_0 H_d^3/\bar{T}}\;\left(V_j - V_i\right)\left|V_j - V_i\right|^{1/2} \qquad (3)$$

from room j into room i ($C_{\mathrm{dis}} = 0.6$, $\bar{T} = 297$ K, ρ and c_p the density and specific heat of air, g_0 the gravitational acceleration); at 1 K a 0.90 m door carries about 110 W, several times more than a partition wall. We call the exchange through walls, slabs and doors the *thermal shift* between rooms; because $K_{ij} = K_{ji}$ and $q_{ji} = -q_{ij}$, it moves heat between rooms without creating or removing it.

Humid infiltration air imposes a latent load on each conditioned zone whenever the outdoor humidity ratio ω_a exceeds the indoor target $\omega^* = 9.6$ g/kg (55% relative humidity at 23 °C, below 60% everywhere in the band, and independent of the setpoint):

$$L_i^{\mathrm{lat}} = n_{\mathrm{ach}}/3600\;\rho\,\mathrm{Vol}_i\,\max(\omega_a - \omega^*,\,0)\,h_{fg}, \qquad (4)$$

with the zone's air volume Vol_i and the latent heat h_{fg}. The target is assumed met, the latent electricity is priced at a fixed latent COP, internal moisture is omitted and passive zones are not dehumidified; these assumptions are tested in Section 9.7.

5.4 Parameters and their provenance

Every parameter is fixed a priori from the code, the product catalogue, the floor plan, the weather file, cited statistics or a stated nominal value (Table 3); none is fitted to the quantities against which the model is later checked. Each conductance is a U-value times an area from the plan (storey height 3.40 m, clear height 3.00 m), and capacitance, internal gains and latent air mass scale with floor area. The capacitance, 270 kJ/(m^{2}K) of floor, lies close to the heavy-class default of ISO 13790 (260) [54], whose reduced-order room model remains the usual choice at this scale although the standard has been superseded by ISO 52016-1. A share of 5% of the capacitance sits on the air node (our choice, since the standard's air node is massless), the air–structure coupling is given by the ISO surface coefficients in series, and half of the internal gains are radiant. Internal gains average 5.4 W/m^2 with an afternoon peak, derived from the Saudi end-use split [1], raised in the kitchen and reduced in baths and circulation. The ground slab follows ISO 13370 [53] against the monthly ground temperature of the weather file. Each unit draws 3 W on standby, and its hourly electricity is divided by the part-load factor $\mathrm{PLF} = 1 - C_d(1 - \mathrm{RTF})$, where RTF is the fraction of the hour it runs. The coefficient is $C_d = 0.25$ for the fixed-speed units, the rating default for single-speed equipment [57], and an assumed $C_d = 0.15$ for the inverter units, which cycle at or below their minimum speed; both are varied in the sensitivity and uncertainty analyses (Section 9.7). The rated COPs include the indoor fan, which runs only with the compressor.

5.5 Leap cycles, levels and the comfort frontier

A unit's operation is a *schedule* of *timed actions* (m, t), each keeping the unit in mode m for a time t; it is *safe* if its run, the sequence of states it produces from the initial state, stays in $\mathcal{S}$, and its cost is the energy price times its

long-run mean electrical power J_∞ [11, 12, 14]. The theory needs one kind of schedule (Fig. 4).

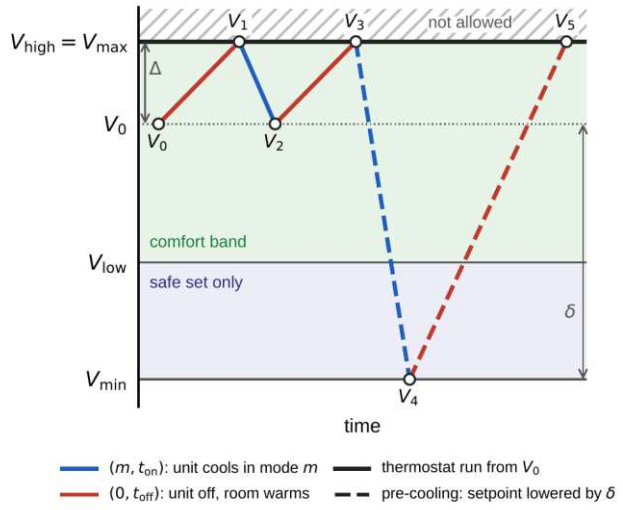


Fig. 4 *Leap cycles (schematic). The comfort band $[V_{low}, V_{high}]$ lies inside the safe set $[V_{min}, V_{max}]$ with a hard ceiling, $V_{high} = V_{max}$. The labels $V_0, \dots, V_5$ are successive states of one zone's run, starting from the initial state V_0, the thermostat's lower switching point: drifts $(0, t_{off})$ (red) and cooling leaps (m, t_{on}) (blue) form a leap cycle of amplitude Δ under the ceiling. Pre-cooling lowers the setpoint by δ (dashed), at most to $V_{min} + 0.5\,K$, so that the thermostat's lower switching point stays at or above the floor V_{min}.*

Definition 1 (Leap cycle, level, schedulability and comfort frontier)**.** Let a conditioned zone in the cooling season have heat gain $L(V)$, the net heat entering it per unit time with its unit off. A *leap cycle* of amplitude $\Delta > 0$ under a top temperature $V^{\top}$ is a drift $(0, t_{\text{off}})$ from $V^{\top} - \Delta$ up to $V^{\top}$ followed by a cooling leap (m, t_{on}) back down; repeated for ever, it is an infinite schedule. Its *level* $\bar{V}$ is its time-mean temperature, about $V^{\top} - \Delta/2$. The zone is *schedulable* in mode m if $f_m \psi Q_i > L(\bar{V})$, so that the mode can bring it back; this is the zero-sum dwell-time condition of Alur et al. [14] for one zone, with *duty* $d_m = L/(f_m \psi Q_i) < 1$ and utilisation $U_i = L/Q_i$. The *comfort frontier* $\bar{V}^*$ is the highest level whose cycle stays at or below V_{high}. In the heating season drift and leap exchange roles.

Running example. Bed 2 of apartment 2A (Section 4.3, Fig. 3a), 17.7 m^2 on the north side next to the party wall, has one exterior wall and a 1.5-ton unit ($Q = 5.28$ kW). With the SBC envelope, $C = 1.33$ kWh/K and $K^{\text{env}} = 19.0$ W/K (wall and window 11.8, infiltration 7.1), while its six neighbours are coupled to it by 138 W/K, about 7 times its envelope conductance. In mean July conditions in Riyadh ($T_a = 36.4$ °C), read as one lumped room at a level of 23.5 °C, it gains $L = 0.363$ kW (conduction and infiltration 0.245, internal gains 0.096, solar minus sky loss 0.022; the conditioned neighbours, at the same level, add no net heat). With its unit off, it warms at 0.27 K/h; mode 1 removes 1.82 kW and cools it at 1.10 K/h, with duty $d_1 = 0.20$. For the 1 K differential one leap cycle is $\langle (0{,}220\text{ min}), (1{,}55\text{ min}) \rangle$.

5.6 The Saudi two-tier tariff

The electrical power of unit i is $P_i(t) = f_{u_i(t)} \hat{Q}_i / \text{COP}_{u_i(t)}$ (Eq. (1)) plus its latent share. A metering map μ assigns every unit to the meter that bills it, one per apartment and one common meter for the stair, and meter r uses $E_{r,k} = \int_{\text{month } k} \sum_{i:\mu(i)=r} P_i(t)\, dt$ in month k. The residential tariff, set by the Saudi Electricity Regulatory Authority and billed by the Saudi Electricity Company, is a two-tier *volume* tariff applied to each meter separately [7]:

$$\Pi^{\text{vol}} = 1.15 \sum_{r=1}^{R} \sum_{k=1}^{12} b\left(E_{r,k}\right), \qquad (5a)$$

$$b(E) = 0.18 \min(E, 6000) + 0.30\,(E - 6000)^{+}, \qquad (5b)$$

in Saudi riyals (SAR) a year, where $(x)^{+} = \max(x, 0)$ and 1.15 adds the 15% value-added tax (VAT). The tariff prices energy alone: it has no time-of-use component, no demand charge and, unlike the multi-mode costs of Mousa et al. [12], no charge per switch; the energy a unit loses by cycling is counted through the part-load factor. Fixed meter charges are excluded. From 1 January 2027 the tier prices rise to 0.21 and 0.33 SAR/kWh, with the 6,000 kWh threshold unchanged, and the meter fee becomes a fixed monthly charge of 70–100 SAR by breaker capacity [58]; that charge, too, does not depend on the policy. The results are reported under the tariff in force since 2018 and re-evaluated under the revised one in Section 9.5. For a meter in tier 1, as every meter of the SBC-compliant building is (Section 9.1), the price is $\lambda = 0.207$ SAR/kWh (US$0.055/kWh at the pegged rate of SAR 3.75 per US$, used throughout), the cost of unit i in mode m is $\pi_i(m) = \lambda f_m \psi(T_a) Q_i / \text{COP}_m(T_a)$, and the bill is the sum of the costs of the units' schedules.

Running example (continued). Apartment 2A of the SBC-compliant building in Riyadh (Case 3) uses 5,527 kWh of HVAC electricity a year at the reference setpoints, never more than 935 kWh in a month, so even if HVAC were only a third of its use, the meter would stay in tier 1. Its HVAC bill is 5,527×0.207≈1,144 SAR, and any policy's bill is proportional to its energy.

5.7 Controller

Choosing every unit's timed actions directly would be a combinatorial problem over $|M|^{|\mathcal{Z}_c|}$ mode vectors per step. Instead, every unit applies one compact *policy* $\theta = \left(V^{\text{cool}}, V^{\text{heat}}, \delta_{\text{night}}, \delta_{\text{peak}}\right)$ to its zone: cooling and heating setpoints and the depth pair $\delta = \left(\delta_{\text{night}}, \delta_{\text{peak}}\right)$ by which the cooling setpoint is lowered from 22:00 to 06:00 (night pre-cooling) and from 10:00 to 13:00 (pre-peak pre-cooling). Setpoints and depths move in 0.5 K steps. Each unit is controlled by an on/off thermostat with a 1 K differential centred on its active setpoint (in cooling: on at the setpoint plus 0.5 K, off at the setpoint minus 0.5 K), with the switching instant resolved within the 3-minute step and at most one change of state per step, which also gives the usual 3-minute restart delay. While on, an inverter unit chooses its mode from the rise its room's air would make within the step without cooling. This rule idealises the units' own logic, which manufacturers do not publish: low by default, medium if that rise would reach 0.4 K above the switch-on point, high if it would reach 1.0 K, and full capacity if its room is still above the switch-on point at the

start of a step; a fixed-speed unit runs at full capacity. Heating works in the same way about the heating setpoint.

The units follow the usual seasonal changeover. In months whose mean air temperature is below the band floor V_{low} (November to March in Riyadh; none in Jeddah), they heat and cool automatically, with the active heating setpoint at least 1 K below the active cooling setpoint. In the other months they cool, and heat only as a *floor guard*: a low-limit heating setpoint of 21 °C (on at 20.5, off at 21.5 °C), capped at 1 K below the active cooling setpoint. Pre-cooling acts only in cooling mode, with depths capped so that the cooling setpoint never falls below $V_{\min} + 0.5$ K; the controller therefore never cools a room below $V_{\min}$. Under the reference and ordinary policies, the guard holds the floor: without it, rooms of the uninsulated pre-code building fall below $V_{\min}$ (0.12 K·h a year in Riyadh; 41 K·h and down to 18.2 °C in Jeddah); it never acts in the SBC-compliant building at the reference policy. Its electricity is booked as heating energy of the month in which it acts and is part of the HVAC bill; in the pre-code building in Jeddah, which has no heating months, it adds 14 kWh of heating a year (about 0.01% of the building's HVAC electricity), and with the small change in cooling that follows it raises the annual bill by 0.02% (full-year runs). During deep night pre-cooling (cooling setpoint 20.5–21.5 °C), however, the cap moves the guard's switch-on point below 20.5 °C, and a room can still drift below 20 °C on a cool night, as it does under some deep night pre-cooling policies of the pre-code building in Jeddah. Other pre-cooling policies of the pre-code building, in both cities, let a room pass the ceiling. All such policies are inadmissible and are excluded. Each unit needs only its own zone temperature and the clock, and the policy induces for every unit an infinite schedule of leap cycles. With $\delta = (0,0)$ the controller is an ordinary thermostat.

5.8 The optimisation problem, feasibility and admissibility

The problem, in the form of Mousa et al. [12], is to choose the policy that minimises the annual bill:

$$\min_{\theta} \quad \Pi(\theta) = \Pi^{\text{vol}}\big(\{E_{r,k}(\theta)\}\big) \qquad (6)$$

s.t.
(C0) $u_i(t) = \kappa_\theta(V_i, t) \;\; \forall i \in \mathcal{Z}_c$,
(C1) Eqs. (2)–(4) hold $\forall i \in \mathcal{Z}$,
(C2) $V_i(t) \le V_{\text{high}} = 24\,^\circ\text{C} \;\; \forall i \in \mathcal{Z}_c$,
(C3) $V_i(t) \ge V_{\min} = 20\,^\circ\text{C} \;\; \forall i \in \mathcal{Z}_c$,
(C4) $u_i(t) \in M \;\forall i \in \mathcal{Z}_c;\;\; u_i \equiv 0 \;\forall i \in \mathcal{Z}_u$,
(C5) $V^{\text{heat}} = V_{\text{low}} + \Delta_{\text{th}}/2 = 22.5\,^\circ\text{C}$,

for all times t. (C0) states that each unit's mode is set by the controller κ_θ of Section 5.7 under policy θ from its own zone temperature and the clock, (C1) the dynamics, (C2) the comfort ceiling, (C3) the safe floor, (C4) the finite modes and (C5) the heating setpoint, fixed at the comfort target of the heating months (the band floor plus half the differential $\Delta_{\text{th}} = 1$ K). The heating setpoint is therefore not a decision variable: without (C5) a colder heating setpoint, which keeps rooms within the safe set but below the band in winter, would be counted as a saving. The decision variables are the cooling setpoint and the depth pair.

Admissibility. The safe set $[V_{\min}, V_{\max}] = [20,24]\,^\circ$C is hard for every policy, at its ceiling and at its floor. A policy is *admissible* if every conditioned zone stays within it at every step of the simulated year: the excursions above the ceiling, $\sum_i \int \max\big(V_i - V_{\text{high}}, 0\big)\, dt$, and below the floor, $\sum_i \int \max(V_{\min} - V_i, 0)\, dt$, must both be zero up to floating-point round-off. Every other policy is excluded, and problem (6) asks for the cheapest admissible policy. The band floor $V_{\text{low}} = 22\,^\circ$C is the comfort target rather than a constraint on the air temperature: in the heating months it is held by the heating setpoint of (C5), and in the cooling months by the building's own gains, while pre-cooling may use $[V_{\min}, V_{\text{low}})$, an allowance that can only favour scheduling. Admissibility is checked on the simulated days: the 60 sampled days during selection and all 365 days in the full-year re-evaluation (Section 7.3).

Feasibility. Each conditioned zone is a *task* demanding cooling at the rate of its heat gain $L_i(t)$ [14, 18]; each unit serves one zone and no power budget is shared, so schedulability (Definition 1) applies zone by zone. A zone is *feasible* if its unit has the hourly capacity to hold it at the ceiling whatever the weather; a sufficient condition is

$$\max_t L_i(t) \;<\; \psi(T_a)\, Q_i \qquad \text{at } V_i = V_{\text{high}}, \qquad (7)$$

a condition that is not necessary, since the capacitance can carry a zone through a short peak. A building is feasible if every conditioned zone is. Feasibility is a property of the envelope and the unit sizes; admissibility is a property of a policy's simulated run. An infeasible building saturates on the hottest afternoons and has no admissible policy; such an instance is reported as infeasible. A feasible building can still have no admissible policy at a given setpoint, when the controller's escalation lag lets a room pass the ceiling within a step (Section 9.5). Section 9.1 evaluates both. Section 6 states what follows from this structure.

6 Theoretical Results

The structure of problem (6) has consequences that can be stated before any computation. They adapt the storage-efficiency accounting of the energy-flexibility literature (Section 2) to multi-mode units under a hard comfort ceiling: they follow from the conservation of energy and the ordering of the mode efficiencies, and they reduce the computation of Section 7 to checking the conditions they state. The proofs read a zone as one lumped node with constant rates over a cycle, as (A1)–(A3) state; the simulation does not (two nodes, nonlinear door flows, a COP that follows the outdoor and indoor air hour by hour, cycling with a differential, a restart delay and cycling losses), so Section 9 tests the theory beyond the conditions under which it is proved. Proofs are in Supplementary Section S2.

6.1 The thermal shift between rooms

Coupled neighbours. Under (A4) the net heat a conditioned zone exchanges with its conditioned neighbours vanishes in the mean, up to the difference in their thermostat cycles, whatever the coupling conductances. The exchange $K_{ij}\big(V_j - V_i\big)$ has zero mean when both zones are held at a common level, and so, to first order in the temperature difference, has the door

exchange of Eq. (3), which grows faster than linearly with it. The thermal shift therefore changes a zone's load only through neighbours at a different temperature: passive zones, the roof, the ground and the outdoors.

Passive zones. Averaging its heat balance over a month gives the heat budget of a passive zone (Supplementary Section S6). In the cooling season such a zone is warmer than the conditioned rooms that take its heat. Its temperature depends on the setpoints those rooms hold, not on unit capacity, and the more heat it sheds to them through walls or open doors, the closer it stays to their setpoint. Section 9.1 and Supplementary Section S3 test these statements.

6.2 The cheapest stationary operation of a room

Theorem 1 (Optimal leap and level)**.** *Under (A1)–(A4), let a conditioned zone run a periodic leap cycle in mode m at level $\bar{V}$ (Definition 1). Then* $\bar{P}(\bar{V}, m) = \frac{L(\bar{V})}{\mathrm{COP}_m(T_a)}$. (8)

(i) The amplitude Δ enters only through the level; for a cycle that reaches the ceiling, $\bar{V} = V_{\text{high}} - \Delta/2$ to first order. Hence a single-mode leap $(\bar{V}^*, m^*)$ is optimal if and only if $\bar{V}^*$ is the comfort frontier and $m^* = \min\{m: f_m \psi Q_i > L(\bar{V}^*)\}$ is the lowest mode that holds the room; for a thermostat whose differential Δ_{th} is centred on its setpoint, $\bar{V}^* = V_{\text{high}} - \Delta_{\text{th}}/2$, the highest admissible setpoint. (ii) If the ambient conditions are also constant over the horizon, let $\check{P}$ be the lower convex envelope of the unit's mode power curve, the points (0,0) and $(f_m \psi Q_i,\ f_m \psi Q_i / \mathrm{COP}_m)$, $m \in M$. Every safe infinite schedule σ satisfies $J_\infty(\sigma) \geq \check{P}\left(L\left(V_{\text{high}}\right)\right)$. Because the COP falls with the mode, the first piece of $\check{P}$ joins off to mode 1, so when mode 1 holds the room at the ceiling the bound is $L\left(V_{\text{high}}\right)/\mathrm{COP}_1$, approached by off/mode-1 cycles of vanishing amplitude at the ceiling; with a given differential, the optimal leap of (i), repeated for ever, attains the least long-run power. When mode 1 cannot hold the room the bound is approached by alternating, at the ceiling, the two vertices of $\check{P}$ that bracket the load (adjacent modes when the mode points are convex), which costs less than cycling m^* on and off; the leap of (i) is then optimal only among single-mode cycles. In both cases the optimal infinite-horizon control is stationary. With a diurnal swing of the COP the bound no longer holds, and Proposition 1 bounds what re-timing can gain instead.

The identity is exact when L is affine in V, because the mean gain over a cycle is then the gain at the mean temperature. The door exchange with a warmer passive zone (Eq. (3)) is convex in V; it makes Eq. (8) a lower bound on the mean power, exact to first order in the amplitude (Supplementary Section S2). The indoor factor g of Eq. (1), which the theorem omits, makes a warmer level cheaper still and so strengthens part (i). The theorem covers the sensible load only: the latent load does not depend on the level or the mode in the model (Eq. (4)). The perfect-foresight program of Section 7.2 prices each unit on $\check{P}$, so it can use the mode alternation of part (ii); the simulated inverter controller instead escalates from mode 1 when a room lags, and the controller variant that holds mode 1 throughout costs almost the same (Section 9.4).

Corollary 1 (What a cycle costs)**.** *Under (A2), which has no cycling loss, and with no charge per switch (Section 5.6), the cost of a safe schedule depends only on the level at which it holds each zone and on the modes it uses: the number of cycles does not enter, and reassigning heat removal between units ("coordination") lowers the cost only if it moves a unit to a cheaper point of its envelope $\check{P}$, a lower mode or a cheaper mix of two modes. With cycling losses, $\bar{P} = L/(\mathrm{COP}_m\,\mathrm{PLF})$ with $\mathrm{PLF} = 1 - C_d\left(1 - L/(f_m \psi Q_i)\right)$, which also favours the lowest mode that holds the room, since it runs longest.*

With the 1 K differential and 0.5 K steps of the modelled thermostats, Theorem 1 places the frontier at 23.5 °C, the reference setpoint; a finer thermostat would move it towards the ceiling (Section 9.1).

Running example (continued). For Bed 2 in July ($L = 0.363$ kW at 23.5 °C, $\varphi = 0.81$; evaluated without the indoor factor g, i.e. $\gamma = 0$, which narrows every difference below), Eq. (8) gives 0.077 kW in mode 1, 0.090 kW in mode 2 and 0.112 kW in mode 3: the same comfort costs 45% more in the highest mode, and $m^* = 1$. The cycle durations give the same value, $(1.82\,\text{kW} \times 0.913\,\text{h}/4.72)/4.580\,\text{h} = 0.077\,\text{kW}$. At the 22 °C of current practice the gain rises to 0.391 kW and the mean power to 0.083 kW. A cycle across the whole comfort band averages 23.01 °C and draws 2.6% more than the thermostat cycle; one across the whole safe set, as the deepest pre-cooling does, averages 22.05 °C and draws 7.7% more, although it switches far less often.

6.3 What re-timing can gain under a volume tariff

Under a tariff of monthly energy alone, the timing of the leaps within a month affects the cost only through the heat removed and the mean efficiency at which it is removed (Eq. (5)). Pre-cooling trades one of these terms against the other.

Proposition 1 (Pre-cooling bound)**.** *Let a conditioned zone be pre-cooled by $\delta > 0$ in the cooling season. In the window its unit removes the extra stored heat $C_i\delta$ and the extra heat $\Delta H_w \geq 0$ that the colder zone admits there, at the incremental COP COP_w (extra heat removed in the window divided by the extra electricity used there). After the window it removes $C_i\delta - \Delta H_r$ less, where $\Delta H_r \geq 0$ is the extra heat admitted during the recovery and $\Delta H_r \leq C_i\delta$, and the incremental COP of the removal it no longer makes (heat no longer removed divided by electricity no longer used) is at least COP_p. Let $\Delta H = \Delta H_w + \Delta H_r$. Under a hard comfort ceiling the change in electrical energy satisfies*

$$\Delta E \geq C_i\delta\left(\eta_w - \eta_p\right) + \Delta H \min\left(\eta_w, \eta_p\right), \quad (9)$$

with $\eta_w = 1/\mathrm{COP}_w$ and $\eta_p = 1/\mathrm{COP}_p$. Pre-cooling cannot save if it is done less efficiently than the removal it displaces; otherwise it saves at most $C_i\delta\left(\eta_p - \eta_w\right)$, and nothing when the fraction of stored coolth lost as extra heat, $\Delta H/(C_i\delta)$, is at least the relative COP gain $\mathrm{COP}_w/\mathrm{COP}_p - 1$.

Proposition 2 (Optimal level and conditional optimality of no pre-cooling)**.** *Under (A1)–(A5), let the tariff be any non-decreasing function of the meter energies alone (the Saudi two-tier tariff, a linear price or any tiered price). (i) Among the policies of problem (6) without pre-cooling,*

and with Theorem 1 applied hour by hour, which requires (A3) to hold within each hour, the cheapest is the frontier policy $(\bar{V}^*, V^{\text{heat}}, 0, 0)$*, with every unit at the comfort frontier and the heating setpoint fixed by (C5). (ii) A pre-cooling policy costs at least as much as the frontier policy wherever its pre-cooling satisfies the no-gain condition of Proposition 1, strictly so if the tariff is strictly increasing and* $\Delta E > 0$*; elsewhere its saving is at most the bound of Proposition 1. Tariffs that weight energy by the time of use or by a coincident peak lie outside the proposition; neither applies to Saudi households.*

Part (ii) is an accounting statement, not a prediction: it excludes a saving where pre-cooling admits more heat than it gains in efficiency and bounds it elsewhere, so whether a saving exists is decided by measuring the two terms. The condition fails mainly when pre-cooling stays in the lowest mode at an hour whose COP is well above the afternoon COP. For the whole building in July, ignoring the extra heat, the bound for a 2 K pre-cool is 3.8% of the day's HVAC electricity in the night window, 4.7% in the four coolest hours around dawn (04:00–08:00) and 0.8% before the peak in Riyadh, and 2.4, 3.0 and -0.3% in Jeddah. Section 9.3 measures the two terms for the thermostat and computes, with perfect foresight, how much of the bound an idealised schedule attains.

6.4 Where the savings come from

Proposition 3 (Sources of saving)**.** *Under (A1)–(A5) and a tariff of Proposition 2, the annual HVAC energy is* $E = \sum_{i \in \mathcal{Z}_c} \int L_i(t)/\mathrm{COP}_{u_i(t)}(T_a)\, dt$ *plus the latent and standby loads, and the bill is non-decreasing in the energy of every meter. A policy therefore saves only if, for at least one meter, it (a) lowers the heat to be removed,* $\sum_i \int L_i\, dt$*, or (b) removes it at a higher COP. A warmer level and a better envelope do (a); a lower mode does (b); with cycling losses, (c) a higher part-load factor, from longer runs in the lowest mode that holds the room, is a third source (Corollary 1). Moving heat removal in time can raise (b) only through the ambient swing of* φ*, lowers it through* g *because the room is colder, and raises the heat in (a); Proposition 1 bounds the net effect.*

Each statement of this section rests on (A1)–(A5); Section 7 sets out the computations that test them where the assumptions do not hold.

7 Experimental Setup

The results of Sections 8 and 9 rest on the baselines and test instances, the policy evaluation and the single optimisation method, the selection and full-year re-evaluation, and the experiments described below.

Algorithm 1 *EVALUATEPOLICY*$(\theta, \mathcal{D}_s, \Delta t)$ *for a given case, city, envelope and tariff*

Input: policy $\theta = \left(V^{\text{cool}}, V^{\text{heat}}, \delta_{\text{night}}, \delta_{\text{peak}}\right)$; case (zones $\mathcal{Z}$, meters r); climate; envelope; tariff; sampled days $\mathcal{D}_s$; step $\Delta t = 3$ min

Output: annual bill Π (SAR), energy E (kWh), excursions above V_{high} and below $V_{\min}$ (K·h; admissible iff both are zero)

initialise $V, W \leftarrow 23$ °C in every zone; all units off; $E_{r,k} \leftarrow 0$

for each hour h of the days in $\mathcal{D}_s$ **do**

set $T_a,\ T_g,\ G,\ S^{\text{o}},\ S^{\text{w}},\ S^{\text{sky}},\ \omega_a$ from the climate; add each unit's standby energy

if seasonal changeover: the month's mean $T_a < V_{\text{low}}$ **then**

automatic heating and cooling: $V^{\text{c}} \leftarrow V^{\text{cool}}$, $V^{\text{h}} \leftarrow \min\left(V^{\text{heat}}, V^{\text{c}} - 1\,\text{K}\right)$; no pre-cooling

else

$$V^{\text{c}} \leftarrow \max\left(V^{\text{cool}} - \delta_{\text{night}}\mathbb{1}_{22\text{–}06}(h) - \delta_{\text{peak}}\mathbb{1}_{10\text{–}13}(h),\ V_{\min} + 0.5\,\text{K}\right)$$

floor guard: $V^{\text{h}} \leftarrow \min(21\,°\text{C},\ V^{\text{c}} - 1\,\text{K})$

for each substep of length Δt **do**

compute the door exchange (Eq. (3)) and the air temperature each zone would reach with its unit off

apply each thermostat's 1 K differential: cooling on at $V^{\text{c}} + 0.5$ K and off at $V^{\text{c}} - 0.5$ K; heating on at $V^{\text{h}} - 0.5$ K and off at $V^{\text{h}} + 0.5$ K; resolve the switching instant $\beta_i \Delta t$ within the step, at most one change of state per unit and step (the 3-min restart delay)

choose the mode from the rise the step would cause without the unit (inverter: low by default; medium from 0.4 K and high from 1.0 K above the switch-on point; full capacity if a running unit's room starts the step above its switch-on point; fixed-speed: full)

update V (Eq. (2a)), then W (Eq. (2b)); accumulate electricity with the cooling COP $c_m \varphi(T_a)\, g(V_i)$ or heating COP h_m (Eq. (1)) and each unit's runtime; accrue the excursions of the conditioned zones above V_{high} and below $V_{\min}$

divide each unit's energy of the hour by $1 - C_d(1 - \text{RTF})$; add the latent energy $\sum_i L_i^{\text{lat}}/\mathrm{COP}_{\text{lat}}$ (Eq. (4)); accrue $E_{r,k}$, scaled from the sampled days to the month

$\Pi \leftarrow$ tariff over $\{E_{r,k}\}$ (Eq. (5))

return $\Pi,\ E,$ excursions

7.1 Baselines and test instances

Every saving is measured against a stated baseline, with everything else held equal:

1. *current practice*: cooling at 22 °C and heating at 20.5 °C without scheduling, the baseline of the setpoint lever. In a survey of Jeddah households, 45% preferred room temperatures of 22–24 °C and about 43% 21 °C or less [37]; the survey asked for preferred temperatures rather than thermostat settings, but it suggests that 22 °C is typical rather than extreme. Its cycle takes rooms below the band floor in summer and lets them cool to 20 °C in winter, so it is a practice reference, not a policy that meets the band. The setpoint lever therefore moves the cooling setpoint from 22 to 23.5 °C, which saves, and the heating setpoint from 20.5 to the comfort target of 22.5 °C (C5), which costs; Section 9.2 reports the two parts, and the ablation of Section 9.4 changes the cooling setpoint, with heating capped 1 K below it;
2. the *reference policy*: cooling at 23.5 °C and heating at 22.5 °C without scheduling, whose 1 K

cycle just fills the band; it is the baseline of every scheduling feature;

3. the *pre-code envelope* at the reference policy, the baseline of the envelope lever;
4. each scheduling feature switched on alone, for the decomposition by lever;
5. *isolated rooms*, with no walls, slabs or doors between them, the baseline of the thermal shift;
6. *EnergyPlus* 26.1 [59] on the same inputs, the external benchmark (Section 8.2).

For the timing lever, the perfect-foresight benchmark (Section 7.2) gives what an idealised schedule could achieve beyond the thermostat policies.

Test instances. The problem is tested on instances of two kinds, both for the whole building (Case 3). An instance is *normal* if every unit holds the ceiling with capacity to spare, its peak hourly sensible load staying below 85% of its rated capacity, and the reference policy is admissible (Section 5.8); otherwise it is *hard*. The instances cross the two envelopes and the two cities with two weather years: the TMYx year, and a heat-wave year with the air temperature of the TMYx year raised by 3 K in every hour (humidity, radiation and ground unchanged; the seasonal changeover follows the warmer months, so Riyadh's heating season shrinks to December–February). Four further instances (TMYx year) replace the catalogue units by units each sized to its room's own peak hourly sensible load with a 15% margin. In each of the twelve instances the thermostat policies are compared as described below.

Algorithm 2 *PERFECTFORESIGHTBENCHMARK*$(case, H, v_{min}, \gamma)$

Input: case (zones, units); design day or horizon H with its weather; pre-cooling floor $v_{\min}$; indoor COP slope γ

Output: gap $= (E_{\text{hold}} - E_{\text{best}})/E_{\text{hold}}$

build the linear two-node model of Eq. (2): implicit Euler, 15-min steps, periodic horizon ($x_0 = x_T$), door exchange linearised about a temperature difference $\Delta T_0 = 0.5$ K

build the lower-convex-envelope cost $P_i(q, T_a)$ of each unit

$g_{it} \leftarrow g(V_{\text{high}})$ for all i, t; solve the LP (10) by interior point $\rightarrow (V, q, E)$; $E_{\text{best}} \leftarrow$ cost of (V, q) priced with $g(V)$

if $\gamma > 0$ **then**

$g_{it} \leftarrow g(V_{it})$; re-solve the LP; keep the cheaper of the two schedules priced with $g(V)$

for start in {that schedule, the LP with every $V_{it} \geq V_{\text{high}} - 0.01$ K} **do**

$(V^0, q^0) \leftarrow$ start; $\varrho \leftarrow 1$ K

repeat

solve the LP with cost $\sum P_i(q)/g(V^0) - \sum(\partial g/\partial V)\, P_i(q^0)/g(V^0)^2\, V$ and $|V - V^0| \leq \varrho \rightarrow (V, q)$

if cost of (V, q) priced with $g(V) <$ that of (V^0, q^0) **then**

$(V^0, q^0) \leftarrow (V, q)$; $\varrho \leftarrow 2\varrho$

else

$$\varrho \leftarrow \varrho/2$$

until $\varrho < 0.02$ K, or 8 steps

$$E_{\text{best}} \leftarrow \min\left(E_{\text{best}},\ \text{cost of } (V^0, q^0)\right)$$

$E_{\text{hold}} \leftarrow$ same linear model, each room held at V_{high} whenever it needs cooling (active-set solve per step, iterated over the horizon until periodic), cost divided by $g(V_{\text{high}})$

return gap

7.2 Policy evaluation and optimisation

Algorithm 1 evaluates a thermostat policy; the theory of Section 6 locates the optimum of these policies beforehand, and Algorithm 2 computes the perfect-foresight benchmark. EVALUATEPOLICY integrates the coupled sensible dynamics and the latent load over five days per month sampled evenly from the TMYx year (3-minute steps, monthly energies scaled), starting from 23 °C in January and carrying the state from one sampled day to the next. Each step is semi-implicit: every node is updated implicitly in its own conductances and explicitly in its neighbours' temperatures, which is unconditionally stable. The algorithm also accrues the cost per meter through the tariff and records the excursions that decide admissibility. Many policies are evaluated in one vectorised run.

Thermostat policies. The setpoints are fixed at the reference values and the pre-cooling windows at 22:00–06:00 and 10:00–13:00, and every unit applies the same depth pair $\delta = (\delta_{\text{night}}, \delta_{\text{peak}})$. Because the thermostat moves in 0.5 K steps, the realisable pairs form the lattice $\Theta = \{0{,}0.5, \ldots, 3\}^2$ K of 49 settings (48 pre-cooling pairs and $\delta = (0,0)$, no pre-cooling), to which the rest of the paper refers. All 49 settings are evaluated by Algorithm 1 in one vectorised run, only the admissible ones are kept, and the cheapest of them wins (none, if no setting is admissible). This exhaustive comparison is exact, deterministic and free of tuning parameters, so a search heuristic would add nothing at this size. The setpoint is not part of it: by Theorem 1 its optimum is the comfort frontier, found by the sweep of Section 7.4. The *selected policy* θ^* combines the frontier setpoint with the winning depth pair. Per-room depths and other windows are not compared, apart from the top-floor and west-room subsets of Supplementary Section S8; because this class could hide a better schedule, the optimisation is posed over every periodic schedule of the idealised model on the design days.

Optimisation: perfect-foresight optimal control. To find what *any* periodic schedule could save on a design day, an idealised form of problem (6) is posed as the linear program

$$\min_{q,x} \sum_t \Delta t_{\text{LP}} \sum_{i \in \mathcal{Z}_c} P_i\left(q_{it}, T_a(t)\right)/g_{it} \qquad (10)$$

$$\text{s.t. } \mathbf{C}\frac{x_t - x_{t-1}}{\Delta t_{\text{LP}}} = b_t - \mathbf{K}x_t - \mathbf{B}q_t, \quad x_0 = x_T,$$

$$v_{\min} \leq V_{it} \leq V_{\text{high}}, \quad 0 \leq q_{it} \leq \psi(T_a)Q_i \quad (i \in \mathcal{Z}_c),$$

$$q_{it} = 0 \quad (i \in \mathcal{Z}_u),$$

over the heat removal q_{it} of every unit and the air and structure temperatures $x_t = (V_t, W_t)$ of every zone, with the weather known in advance (Algorithm 2); the structure temperatures W_t and the air temperatures of the passive zones are free.

This bound differs from that of Proposition 1, which limits the gain of a given pre-cool from its two energy terms: the program computes the least cost over all periodic schedules of the idealised model. $\mathbf{K}$, $\mathbf{C}$, b and $\mathbf{B}$ are the conductance and capacitance matrices, the drives and the unit inputs of Eq. (2), with the step $\Delta t_{\text{LP}} = 15$ min; P_i, the lower convex envelope of the four mode points ($\check{P}$ of

Theorem 1), is piecewise linear with slopes rising with the mode; and g_{it} is the indoor COP factor of Eq. (1). The program may run any mode at any time, pre-cool any room down to $v_{\min}$ (22 °C, the band floor, or 20 °C, the floor of the safe set) at the hours of highest COP, and coordinate the rooms. Its units, like those of the reference that holds the ceiling, modulate continuously, without cycling losses, standby or a differential, so a gain from bunching runtime to cut cycling losses is not represented (the simulated thermostat policies include it). The schedule-independent latent load is excluded. With $\gamma = 0$ the gap is a rigorous upper bound on the saving that any periodic schedule of the idealised model on that horizon can achieve at any $\gamma \geq 0$: since $g(V) \leq g(V_{\text{high}})$ for $V \leq V_{\text{high}}$ and the reference holds V_{high}, scaling the cost by $1/g$ raises every schedule's electricity at least as much as the reference's.

With $\gamma = 0.02$ the program is not linear. It is searched by sequential linear programming from two starts: the cheaper of the program's first solution and its re-solve with the indoor factor of that solution, and the program held within 0.01 K of the ceiling, which does not pre-cool. From each start the cost is linearised about the incumbent schedule in both q and V, a step is taken within a trust region on V, and the step is kept only if the schedule, priced exactly with its own indoor factor, is cheaper. The region widens after a kept step and shrinks otherwise, for at most eight steps from each start. Every schedule kept is one the idealised units can follow, so the best one found gives an achievable saving. On every periodic horizon solved, the optimal saving at $\gamma = 0.02$ lies between that achievable saving and the $\gamma = 0$ bound; the achievable saving is at least zero (the reference itself) and, down to 20 °C, at least the saving of the schedule found within the band.

Case 1 is solved for the mean day of every month from May to September, the hottest July day and the 31 days of July as one horizon (heat may be stored across days), and Case 2, which adds the coordination between apartments and the core, for the mean and hottest July days and the mean August day. The top floor under the roof is solved as the typical floor with the roof above it and an adiabatic slab below, for the same design days. The whole building (Case 3, 264 nodes) is solved for the same days, for the bound ($\gamma = 0$) only: the interior-point solver used elsewhere, IPX [60] within HiGHS [61], is slow on its program, so the Clarabel interior-point solver [62] is used for it and for the 31-day horizon. On the typical floor, Clarabel reproduces the optimum of HiGHS. Both solvers run at their default tolerances (relative optimality gap 10^{-8}; primal and dual feasibility 10^{-7} in HiGHS and 10^{-8} in Clarabel), and a solve ends only when the relative gap between the primal and dual objectives is below 10^{-8}, so the bound, taken from the primal objective, is accurate far beyond the two decimals reported. The bound and the achievable saving are percentages of the sensible cooling electricity of the reference that holds the ceiling, not of the bill.

7.3 Policy selection, full-year re-evaluation and cost

No surrogate or learned model is trained: every policy is evaluated by the annual physics simulation. The only fitted quantity is the policy itself. *Selection*: the policy is selected on 60 days, five per month sampled evenly from the TMYx year. *Full-year re-evaluation*: the selected policy and all its competitors are re-simulated on all 365 days of the year, which include the 60 selection days; we report whether the optimum and the ranking survive and give the errors of the sampled-year estimates (Section 9.6).

Computational complexity. One annual evaluation costs $O\left(n_p(N^2 + D)\, T_{\text{sim}}/\Delta t\right)$ for n_p policies, N zones, D doors and the simulated time T_{sim} (28,800 steps), so comparing the 49 policies costs one batch run. The optimal-control program is a sparse linear program with $O(N\, T_{\text{LP}}/\Delta t_{\text{LP}})$ variables for a horizon T_{LP}, solved to optimality in polynomial time at $\gamma = 0$, and the search at $\gamma = 0.02$ solves at most 19 such programs. The controller in the building needs $O(N)$ comparisons per step. Measured times are in Section 9.6.

7.4 Experimental design

Besides the three cases of Section 4.3 and the test instances, the experiments comprise the following.

- *The comfort frontier* is found by raising the cooling setpoint in 0.5 K steps and keeping the highest admissible value; a *fine thermostat* (0.1 K differential, 0.05 K steps) shows how it depends on the control precision.
- *A ramped pre-peak setback* lowers the setpoint by 0.5 K per hour up to 13:00, sparing the higher modes; *a pre-specified dawn setback* applies the timing the benchmark selects under the realistic controller (2 K at 03:00, returned in 0.5 K steps by 09:00, every cooling-month day, Case 3).
- *The passive zones* are compared with and without the controller: all units off, interior doors closed, the reference setpoint, current practice, and a 1-ton unit, a size below the catalogue range, in every bath (Supplementary Section S3).
- *Sizing*: units are sized by load at the smallest of the margins 1.15, 1.5, 2.0 and 2.5 times each zone's peak hourly load at which no room leaves the safe set (0.25-ton steps from 0.75 ton); a finer sweep adds 1.25, 1.75 and 3.0, also with a controller that starts each unit at full capacity.
- *One-at-a-time sensitivity runs* vary the zone model (nodes, air share and air–structure coupling, capacitance), the controller and COP model (differential, escalation thresholds, modes, the COP's indoor slope γ), the equipment (cycling loss, unit type, standby, derating, part-load COP, indoor fan, sizing), the envelope and air (thermal bridges, absorptance, infiltration, interior doors, internal gains, ground model, latent and humidity models) and the weather (a wider daily swing, a heat wave, an imposed day–night COP swing), as well as the tariff. The full list, apart from the capacitance and imposed COP-swing runs reported in the text (Section 9.7), is summarised in Supplementary Section S8. The setpoint lever and the 2 K pre-peak cost are also recomputed at 6- and 1-minute steps. Three further sets test the pre-cooling result where it is most likely to fail (Supplementary Section S8): the choices most favourable to pre-cooling

combined, the lattice applied to the top-floor or west-facing units only, and a finer controller resolution (1-minute step, minimum on- and off-times, a start-based cycling loss calibrated to the part-load factor).

- *A global uncertainty analysis* varies 17 assumed inputs jointly over uniform ranges by a Latin hypercube [63] of 128 samples per city (fixed seed). Its outputs (the setpoint, envelope and inverter levers, the best admissible pre-cool and the cost of 2 K of pre-cooling) are summarised by the median, the 5–95% range, the share of samples in which any of the pre-cooling settings tested saves, and partial rank correlation coefficients (PRCC) [64]. Coefficients with $|\mathrm{PRCC}| > 0.19$ differ from zero at the 5% level for 128 samples with the other 16 inputs as covariates (ranges in Supplementary Section S9). The two designs have different roles [65]. One-at-a-time runs isolate each assumption against the base case, and many of them are alternative models rather than values on a range (one or two nodes, unit type, controller resolution, ground and humidity models, heat wave, tariff), which cannot be sampled. However, they explore only the neighbourhood of the base case and miss interactions [66]. The joint analysis covers the continuous inputs over their whole ranges, with their interactions.
- *A* 2^4 *factorial lever decomposition* crosses envelope, equipment (the 3-ton units as inverter units), setpoint and pre-cooling (2 K pre-peak or none) and attributes the bill saving to the four levers by Shapley values.
- *A tariff revision*: the three cases, the lattice and the apartment meters are billed again under the tariff in force from 1 January 2027 (Section 9.5).
- *External checks*: the operative temperature is estimated from the inner-surface temperatures, and a *stock-equivalent* run gives the pre-code building the equipment efficiency and air-tightness of the Saudi stock model [1] (rated EER 8.5, latent COP 2.49, 0.8 ACH), with nothing fitted.

7.5 Reproducibility

Every parameter, the floor plan and the run matrix are read from configuration files, and every result is written by resumable analysis blocks from which the figures, tables and every number in the text are generated. The pipeline is deterministic: rerunning it reproduces every stored value exactly, except the measured calculation times. A test suite checks the geometry, the engine, the optimal-leap identity, the thermostat result, the passive-zone heat budget, both seasons, the test instances, the full-year re-evaluation, the benchmark and the headline results, and a single command (`make reproduce`) regenerates everything, including the PDF. The results of Sections 8 and 9, their tables and the numbers quoted in the abstract are written from the stored results by the generator of the manuscript, which also asserts the statements made about them (for example, that no pre-cooling setting saves). Claude (Anthropic), a generative AI model, assisted with writing and debugging the simulation and analysis code, with structuring and drafting the text, with language editing and with checking the references against their DOI records; the study, the modelling choices and the check of every result against the stored outputs and the tests were not delegated to it (see also the statement at the end of the paper). Before the results, Section 8 checks the model itself.

8 Model Verification and Validation

The building was designed for this study, so no metered data exist against which to calibrate the model. It is checked instead by the three approaches used to test building simulation programs [67], on which Standard 140 of the American National Standards Institute (ANSI) and ASHRAE is built [68]. These are analytical verification, comparative testing against EnergyPlus [59] on identical inputs (the storey geometry and the cases of the Building Energy Simulation Test (BESTEST) of Standard 140), and empirical comparison with the energy use of the Saudi housing stock.

8.1 Verification

Physical consistency. With every unit switched off, the top-storey master bedroom of the pre-code Jeddah building averages 38.3 °C in July and peaks at 43.6 °C, so active cooling is required, the premise of Definition 1.

Numerical convergence. All results use a 3-minute step, which also sets how often a unit can switch, so changing it changes the controller as well as the integration. A 1-minute step, at which the 3-minute restart delay spans three steps and binds, changes the annual HVAC energy by at most 2.9%, and a 6-minute step by at most 2.6%. At a 1-minute step the setpoint lever changes by at most 0.6 and the cost of a 2 K pre-peak pre-cool by at most 1.5 percentage points, but the pre-cool costs at least 2.1% at every step tested.

Heat balance. The engine conserves the inter-zone exchanges of Section 5.3 to rounding: the July heat budget of every bath of the representative apartment closes to within 0.6 W (1.3% of its inflow; Section 6.1).

8.2 Validation

Three comparative tests against EnergyPlus 26.1 and a comparison with the housing stock test the model's physics and magnitudes; the savings are differences between runs of the same model, in which a common bias largely cancels.

BESTEST-style comparative test. On the single-zone cases 600 (lightweight) and 900 (heavyweight) of Standard 140, run in both programs on the Riyadh weather, the engine's annual cooling differs from that of EnergyPlus by −0.2% and +4.3% and its peak hourly cooling by −11.3% and −10.4%; free-floating, its annual maximum temperature is 5.5 and 4.5 K lower. The engine thus reproduces annual loads closely but damps the sharpest peaks, because its structure is a single node and its exterior film coefficient is fixed (Supplementary Section S7).

Like-for-like comparison with EnergyPlus. The second test isolates the engine from the zoning. EnergyPlus and the engine are run on the building as one well-mixed zone per storey (the 18.85×27.70 m centreline body with the drawing's window-to-wall ratios and the storey's capacitance, gains, infiltration and ground slab), both holding the reference setpoints with ideal equipment (Fig. 5; simulation settings in Supplementary Section S7). On the same geometry the two programs differ by −6.0 to +1.3% in delivered sensible cooling in the four configurations, partly through compensating differences in the heat-gain components (Supplementary Section S7). The room-resolved model delivers 5.8–12.3% *less* cooling than the engine's storey box, because a third of the floor area is passive and floats above the setpoint, admitting less heat than if it were held there. The envelope effect agrees closely: SBC compliance cuts delivered cooling by 56.1% in Riyadh and 54.1% in Jeddah in EnergyPlus, and by 50.3% in Riyadh and 49.6% in Jeddah in the room-resolved model.

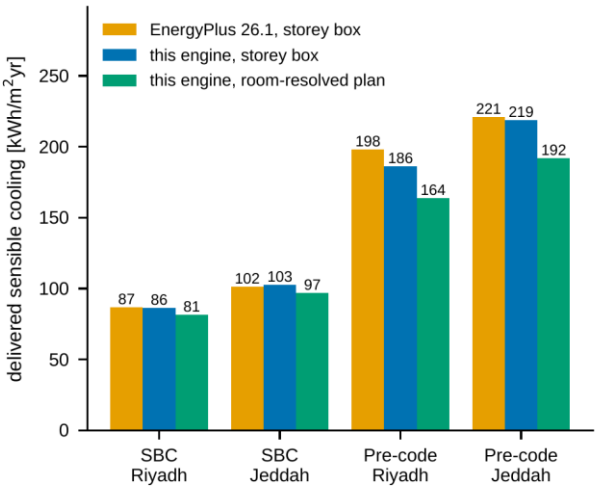


Fig. 5 *Like-for-like comparison with EnergyPlus 26.1: delivered sensible cooling under ideal control at the reference setpoints. On the same storey-box geometry the two programs agree within a few per cent; the room-resolved plan delivers less, because its passive rooms float above the setpoint.*

Table 4 *Response to setbacks on the storey box (ideal control): extra sensible cooling delivered by a 2 K setback relative to the reference setpoint [%]. E+: EnergyPlus 26.1; this: the engine of the model.*

		Pre-peak, 2 K		Night, 2 K	
Envelope	City	E+	this	E+	this
SBC	Riyadh	+1.2	+1.5	+3.3	+4.4
SBC	Jeddah	+1.9	+2.2	+5.0	+6.6
Pre-code	Riyadh	+1.6	+1.7	+3.1	+4.7
Pre-code	Jeddah	+2.4	+2.5	+4.5	+7.0

Table 5 *Absolute and relative errors of the model's estimates against their references (pairs of values: Riyadh / Jeddah, or case 600 / 900 and 640 / 940; the BESTEST-style cases are detailed in Supplementary Section S7).*

Quantity	Reference	Absolute error	Relative error
Annual energy, 60 sampled days	full year (365 days)	−1953 / +400 kWh	−3.7% / +0.5%
Annual energy, 3-min step	1-min step	–	≤2.89%
Passive-zone heat budget	exact balance	≤0.6 W	≤1.3%
Annual cooling, BESTEST-style 600 / 900	EnergyPlus	−0.02 / +0.41 MWh	−0.2% / +4.3%
Annual cooling, BESTEST-style 640 / 940	EnergyPlus	−0.01 / +0.41 MWh	−0.1% / +4.3%
Extra heat removed by a 2 K setback, storey box	EnergyPlus	0.1–2.5 percentage points	4–56%
Delivered cooling, storey box	EnergyPlus, same geometry	−11.8 to +1.3 kWh/m²yr	−6.0 to +1.3%
Stock-equivalent EUI	regional average [1]	−15 / +1 kWh/m²yr	−11% / +1%

Response to setbacks. The scheduling result rests on how much extra heat a colder room admits. The third test therefore applies the 2 K pre-peak and night setbacks of the thermostat lattice to the storey box in both programs, under ideal control and with the seasonal changeover of the model (Table 4). Both find that every setback removes more heat than holding the reference setpoint, EnergyPlus by 1.2–5.0% and the engine by 1.5–7.0%. The engine thus overstates this extra heat; EnergyPlus finds 64–96% of it. Section 9.3 tests whether this could change the sign of the scheduling result.

Stock energy use. The stock model of Krarti et al. [1] puts the whole-building EUI of Saudi residential buildings at 125.6–182.4 kWh/m²yr across the four regions, with 136.0 for the Middle region, which contains Riyadh, and 159.6 for the West, which contains Jeddah. For comparison, the model's HVAC energy is divided by 0.70, a share of residential electricity for air-conditioning between the 66% of the stock model and the more than 70% of recent studies (Section 1). On this basis the pre-code building, with new units at the SASO minimum and 0.4 ACH, uses 82 (Riyadh) and 98 kWh/m²yr; with the equipment and air-tightness of existing homes (Section 7.4) it uses 121 and 161 kWh/m²yr, −11% and +1% against its region's average (−6% and +7% with a share of 0.66). This is agreement in magnitude rather than calibration, since the regional averages mix villas, apartments and traditional houses.

8.3 Absolute and relative errors

Table 5 collects the error of every quantity that the study estimates against a reference; the comparison of the thermostat policies is exhaustive over the lattice, and the optimal-control programs with the indoor COP factor are approximate through their sequential-linear-programming search. None of these errors is large enough to change the ordering of the levers.

9 Results

All results use the room-resolved model of Section 5, the TMYx weather of Riyadh and Jeddah and the per-meter volume tariff. The main case is the SBC-compliant building; the pre-code envelope (Section 4.3) is reported

alongside to show what code compliance is worth. Savings are bill-based and measured against the baselines of Section 7.1. Every optimum, and every saving attributed to a selected policy, compares admissible policies only (Section 5.8); runs that leave the safe set are excluded and shown as "–". Energy is reported as the HVAC energy intensity, the electricity of the units (sensible, latent and standby) per square metre of floor. Section 9.1 checks comfort and feasibility; Sections 9.2–9.4 report the levers, the timing lever and the ablation; and Sections 9.5–9.7 test scale, stability, sensitivity and uncertainty.

Table 6 *The three cases (SBC-compliant building; pre-code value in parentheses). Intensity: HVAC energy intensity, electricity of the units per floor area [kWh/m²yr]; the other columns are savings [%]. Setpoint: saving of the setpoint lever, from current practice (cooling 22, heating 20.5 °C) to the reference setpoints (23.5 and 22.5 °C). Pre-cooling: saving of the best admissible thermostat policy of the lattice. Envelope: SBC versus pre-code at the reference setpoints.*

City	Case	Intensity	Setpoint	Pre-cooling	Envelope
Riyadh	1: apartment	23.8 (48.0)	12.2%	0%	50.5%
	2: floor	23.3 (46.9)	12.4%	0%	50.4%
	3: building	25.1 (57.5)	12.4%	0%	56.4%
Jeddah	1: apartment	34.8 (56.4)	9.8%	0%	38.3%
	2: floor	35.6 (58.4)	9.8%	0%	38.9%
	3: building	37.7 (68.5)	10.5%	0%	45.0%

Table 7 *Annual HVAC bill and savings per apartment in the SBC-compliant building (Case 3; apartments numbered by storey, 1 = ground, 4 = top; A west, B east). Bill: at current practice [SAR/yr]. Setpoint: saving of the setpoint lever [SAR/yr] (% of the bill). Envelope: extra bill of the same apartment in the pre-code building at the reference setpoints [SAR/yr] (% of the pre-code bill). Percentages from unrounded bills.*

	Riyadh			Jeddah		
Apartment	Bill	Setpoint	Envelope	Bill	Setpoint	Envelope
1A	1,433	190 (13.2%)	1,132 (47.7%)	2,028	231 (11.4%)	998 (35.7%)
1B	1,394	181 (13.0%)	1,084 (47.2%)	2,076	234 (11.3%)	1,088 (37.1%)
2A	1,306	162 (12.4%)	1,171 (50.6%)	1,869	189 (10.1%)	1,047 (38.4%)
2B	1,266	156 (12.3%)	1,123 (50.3%)	1,920	191 (9.9%)	1,135 (39.6%)
3A	1,307	159 (12.1%)	1,316 (53.4%)	1,871	186 (9.9%)	1,203 (41.7%)
3B	1,268	152 (12.0%)	1,263 (53.1%)	1,921	187 (9.7%)	1,289 (42.6%)
4A	1,462	168 (11.5%)	2,612 (66.9%)	2,044	212 (10.4%)	2,355 (56.2%)
4B	1,423	162 (11.4%)	2,555 (67.0%)	2,095	214 (10.2%)	2,432 (56.4%)

Table 8 *Savings of the setpoint and envelope levers by case, city and season (SBC-compliant building; % of the HVAC energy of the case in that season; annual savings in Table 6). Summer: June–September; winter: December–February. Reference: HVAC energy at the reference policy [kWh]. Under the tier-1 volume tariff the bill saving equals the energy saving.*

		Annual	Summer			Winter		
City	Case	Reference	Setpoint	Envelope	Reference	Setpoint	Envelope	Reference
Riyadh	1	5,484	9.8%	49.2%	3,313	26.6%	77.4%	222
	2	11,711	10.0%	48.9%	7,126	25.8%	78.5%	459
	3	50,512	10.6%	54.1%	31,239	11.9%	82.8%	1,971
Jeddah	1	8,041	7.9%	40.0%	3,758	16.7%	33.6%	1,011
	2	17,945	7.9%	40.1%	8,352	16.7%	36.5%	2,268
	3	75,912	8.5%	46.3%	35,610	17.6%	41.1%	9,450

9.1 Comfort and feasibility

Conditioned rooms. At the reference policy every conditioned room of the SBC-compliant building stays at or below 24.0 °C all year. The building is feasible with a wide margin: in its peak hour the busiest unit removes 19% of its rated capacity in both cities, because units sized by coverage are several times larger than the loads of an insulated building, and the units are off 77% (Riyadh) and 71% (Jeddah) of the time. Even the pre-code building holds the ceiling with these units, its busiest unit peaking at 53–55% of its capacity, so with coverage-sized units the envelope is an energy lever; the comfort margin it buys appears when the units are sized by load (Section 9.5). No meter reaches the 6,000 kWh tier-2 threshold: the busiest meter of the SBC-compliant building uses at most 1,137 kWh of HVAC electricity in a month, and no apartment meter of either building would exceed 4,579 kWh even if HVAC were only 66% of the household's electricity [1].

Comfort frontier. Raising the cooling setpoint by the thermostat's 0.5 K step to 24 °C lets the rooms rise 0.5 K above the ceiling in every cycle (84,964 and 119,545 K·h a year), so the comfort frontier is the reference setpoint itself, the frontier that Theorem 1 gives for a 1 K differential centred on the setpoint. A finer thermostat (0.1 K differential, 0.05 K steps) at 23.5 °C holds the room lower and costs more, 5.1% in Riyadh and 3.0% in Jeddah; its own frontier is 23.95 °C in both cities, where its bill differs from that of the base thermostat by −0.5%. Finer control moves the setpoint but hardly the time-mean temperature, which is what the theorem prices.

Passive zones. With their doors open, the rooms without a unit stay close to their conditioned neighbours (Section 6.1): in apartment 2A (Riyadh) the lobby, corridor and hall average 23.7–23.8 °C over the year, the interior baths and the maid room 23.9–24.0 °C, and the two baths with an exterior wall 24.2 °C, peaking at 25.8–26.2 °C. Isolated from their neighbours (baseline B5), they would average 33.7 °C instead of 24.7 °C (Jeddah); the thermal shift that cools them raises the building's energy by 13.5%.

Operative temperature. At the reference setpoint the operative temperature of the conditioned rooms, estimated from the inner-surface temperatures of each room, peaks at 25.0–25.1 °C (in the top-floor rooms under the roof) and exceeds 25 °C in fewer than 0.05% of the room-hours,

inside the 23–27 °C operative range of the ASHRAE summer comfort zone (Section 4.2). It lies on average 0.29–0.35 K above the air temperature, so it exceeds the 24 °C air ceiling in 35.3% of the room-hours in Riyadh and 46.9% in Jeddah: the hard ceiling bounds the air temperature, not the operative temperature.

9.2 Savings in the three cases

Cases 1 and 2. For one apartment, modelled as an intermediate apartment whose shared walls and slabs face conditioned space, the HVAC energy intensity is 23.8 kWh/m^{2}yr in Riyadh and 34.8 in Jeddah, and the setpoint lever, from current practice to the reference setpoints, saves 12.2% in Riyadh and 9.8% in Jeddah (Table 6). Adding the neighbouring apartment and the core (Case 2) changes the representative apartment's energy by only 0.9% (Riyadh) and 0.6% (Jeddah), since the conditioned rooms on either side of the party wall differ by a fraction of a kelvin (Section 6.1), and moves the setpoint lever by at most 0.2 percentage points. In both cases the best admissible thermostat policy is no pre-cooling: every pre-cooling policy of the lattice raises the bill, as Proposition 1 implies where its no-gain condition holds.

Case 3: the whole building. The building adds the roof, the ground and the vertical coupling of four storeys. Its HVAC energy intensity is 25.1 (Riyadh) and 37.7 kWh/m^{2}yr (Jeddah), and the setpoint lever saves 12.4% in Riyadh and 10.5% in Jeddah. The lever has two parts of opposite sign (percentages of the HVAC electricity at current practice, each rounded): raising the cooling setpoint from 22 to 23.5 °C saves 14.1% in Riyadh and 10.5% in Jeddah, about 9.4% and 7.0% per kelvin. Raising the heating setpoint from 20.5 °C to the comfort target of 22.5 °C (C5) costs 1.6% in Riyadh (6.0% in the pre-code building) and nothing in Jeddah, which has no heating months. Every step of the cooling setpoint lowers the energy, and every run stays in the safe set. At the reference policy the insulated roof keeps the mean HVAC energy of a top-storey apartment within 13% (Riyadh) and 9% (Jeddah) of that of an apartment on an intermediate storey, whereas in the pre-code building the top apartment uses 1.5–1.7 times as much; this is why the envelope saving of the whole building (56.4%, 45.0%) exceeds that of one apartment (50.5%, 38.3%).

Savings per apartment. Every apartment has its own meter, so the levers can be priced per household (Table 7). The setpoint lever lowers an apartment's annual HVAC bill by 152–190 SAR (US$41–51) in Riyadh and 186–234 SAR (US$50–62) in Jeddah, most on the ground floor, whose rooms at 22 °C also draw heat from the ground. The envelope is worth far more: the same apartment in the pre-code building would cost 1,084–2,612 SAR (US$289–697) more a year in Riyadh and 998–2,432 SAR (US$266–649) more in Jeddah, most of all under the roof. Between the two mirror-image apartments of a storey, the bill at current practice differs by 2.4–3.2% of the cheaper one, the west apartment paying more in Riyadh and the east one in Jeddah, while the setpoint saving differs by at most 9 SAR a year. The lever percentages of the abstract and Table 6 are for whole cases (one apartment, a floor, the building), not for each apartment.

Seasons. Table 8 splits the setpoint and envelope levers by season. In summer (June–September), which carries the largest share of the energy, the setpoint lever saves 9.8–10.6% in Riyadh and 7.9–8.5% in Jeddah. In winter (December–February) it saves 11.9–26.6% (Riyadh) and 16.7–17.6% (Jeddah) of the little energy then used; in Riyadh part of that energy is heating, whose setpoint the lever raises to the comfort target. Winter is a substantial heating season only for the pre-code building in Riyadh (11,116 kWh of heating a year, against 1,367 kWh in the SBC-compliant building). The latent load is negligible in Riyadh (0.4% of the HVAC energy) but reaches 26% in Jeddah, where it dilutes the setpoint lever (10.1% for apartment 2A with it, 13.3% without); the seasonal operation and the modes used are in Supplementary Section S8.

9.3 The timing lever

Table 9 *Why thermostat pre-cooling does not pay (SBC-compliant building, Case 3, volume tariff): change in the annual bill, in the heat removed and in the mean cooling COP when each feature is switched on alone, relative to no scheduling. Default: the controller of Section 5.7; low only: inverter units held in their lowest mode. The heat removed and the mean COP are the aggregate counterparts of the two terms of Proposition 1, not its incremental quantities. Every row is admissible: it stays within the safe set (constraints (C2) and (C3)).*

	Default	Low mode only		
Feature	Bill	Bill	Heat removed	Mean COP
Riyadh				
Pre-peak, 0.5 K	+0.51%	+0.48%	+0.40%	3.44→3.44
Pre-peak, 2 K	+3.40%	+1.95%	+1.96%	3.44→3.44
Night, 0.5 K	+0.95%	+0.97%	+1.32%	3.44→3.45
Night, 2 K	+4.57%	+3.88%	+5.71%	3.44→3.49
Jeddah				
Pre-peak, 0.5 K	+0.74%	+0.65%	+0.61%	3.63→3.62
Pre-peak, 2 K	+4.30%	+2.84%	+3.05%	3.63→3.60
Night, 0.5 K	+1.15%	+1.11%	+1.92%	3.63→3.64
Night, 2 K	+5.27%	+4.72%	+8.67%	3.63→3.70

Table 10 *Perfect-foresight benchmark (SBC-compliant building): saving [%] of the sensible cooling electricity that an ideal schedule achieves over holding the 24 °C ceiling with the same idealised units, with the rooms pre-cooled at most to the band floor (22 °C) or to the floor of the safe set (20 °C), with a COP that depends on the outdoor air only ($\gamma = 0$) or the COP of the engine ($\gamma = 0.02$, falling by 2%/K of indoor air below 27 °C). The $\gamma = 0$ columns are upper bounds on the saving of any periodic schedule of the idealised model on that horizon; the $\gamma = 0.02$ columns are achievable savings, those of the best valid schedule found (Section 7.2). Hold: electricity of the reference at $\gamma = 0.02$ [kWh/day]. Season: May–September mean days weighted by the reference energy of their month (Case 1), a weighted mean of the daily values and not itself a bound for the season; July, 31 days: the whole month as one horizon (Case 1, band floor only). Case 3: the bound only; the search at $\gamma = 0.02$ is not run. Selected design days (all are summarised in Supplementary Section S4).*

		Riyadh					Jeddah				
			$\gamma = 0.02$ (achievable)		$\gamma = 0$ (bound)			$\gamma = 0.02$ (achievable)		$\gamma = 0$ (bound)	
Case	Day	Hold	22 °C	20 °C	22 °C	20 °C	Hold	22 °C	20 °C	22 °C	20 °C
1: apartment	Jul, mean day	22.3	0.28	0.28	1.26	2.41	16.8	0.10	0.10	0.78	1.43
	Jul, hottest day	25.4	0.46	0.47	1.50	2.86	19.3	0.54	0.54	1.79	3.34

	Season	–	0.23	0.23	1.07	2.07	–	0.06	0.06	0.58	1.03
	Jul, 31 days	22.3	0.33	–	1.31	–	16.9	0.23	–	0.96	–
2: floor	Jul, mean day	46.6	0.35	0.35	1.39	2.65	35.9	0.13	0.13	0.86	1.58
	Jul, hottest day	53.8	0.54	0.56	1.63	3.09	41.6	0.63	0.67	1.92	3.57
Top floor	Jul, mean day	55.8	0.12	0.12	0.75	1.42	43.5	0.02	0.02	0.35	0.63
	Jul, hottest day	64.5	0.19	0.19	0.96	1.82	50.1	0.25	0.25	1.08	2.02
3: building	Jul, mean day	204.8	–	–	0.94	1.78	157.3	–	–	0.50	0.91
	Jul, hottest day	235.6	–	–	1.16	2.21	181.1	–	–	1.34	2.50

Thermostat pre-cooling. None of the pre-cooling settings of the lattice saves anything in any case, city or envelope (Table 6). Over all 365 days, on which all of them stay within the safe set in the SBC-compliant building, every one of them raises the bill, by at least 0.52%, and on the sampled days the shallowest step, a 0.5 K pre-peak pre-cool, costs 0.51% in Riyadh and 0.74% in Jeddah. At a depth of 2 K, night pre-cooling costs 4.6% in Riyadh and 5.3% in Jeddah and pre-peak pre-cooling 3.4% in Riyadh and 4.3% in Jeddah. Table 9 splits each feature into aggregate counterparts of the two terms of Proposition 1: every depth run (0.5–2 K, step and ramped setbacks, default controller and low mode only) raises the heat removed by 0.12–8.67% because colder rooms admit more heat, while the mean COP rises slightly at night and falls or stays level before the peak: the heat added outweighs the COP gained. Stress tests (Supplementary Section S8) cover inverter units held in their lowest mode, ramped and dawn setbacks, the extra heat scaled to what EnergyPlus finds, pre-cooling of the top floor or the west rooms alone, and a 1-minute controller with minimum on- and off-times and a cycling loss charged per start (inadmissible in Riyadh, where it is excluded). None of the admissible ones saves, and, scaled to EnergyPlus's extra heat, a 2 K pre-peak pre-cool still costs 2.9–3.8%. Apart from the variant with one node per zone and a COP independent of the indoor air, which changes two assumptions at once (0.08%; Section 9.7), only with every choice most favourable to pre-cooling combined (one node per zone, a COP independent of the indoor air, inverter units held in their lowest mode and no cycling loss) does the best policy save, at most 0.77%. Because every unit pre-cools at the same time, a 2 K pre-peak pre-cool also raises the building's largest hourly electrical demand, from 21.4 to 34.1 kW in Riyadh and from 22.0 to 35.3 kW in Jeddah.

Running example (continued). Apartment 2A of the SBC-compliant building in Riyadh (Case 3) pays 1,144 SAR (US$305) a year at the reference policy. A thermostat that pre-cools by 2 K before the afternoon adds about 39 SAR a year to that bill and nothing to comfort: under the Saudi tariff the right setting is to switch night and pre-peak pre-cooling off.

Perfect-foresight benchmark. With perfect knowledge of the weather, the linear program of Section 7.2 schedules the idealised units of Cases 1 and 2, of the top floor under the roof and, for the bound only, of the whole building (Case 3). It compares the result with holding the ceiling with the same units (Table 10 shows selected design days, Supplementary Section S4 summarises all of them, and the ranges below are over all of them). With a COP that depends on the outdoor air only ($\gamma = 0$), its saving is an upper bound on what any periodic schedule of the idealised model could save: at most 1.9% of the sensible cooling electricity on a design day with the rooms kept in the comfort band and 3.6% down to the 20 °C floor of the safe set. Weighted over the May–September mean days of Case 1, the daily bounds average 0.58–1.07% within the band and 1.03–2.07% down to 20 °C, a weighted mean that is not itself a bound for the season. Under the roof the bound is smaller than on the typical floor: 0.75% in Riyadh and 0.35% in Jeddah on the mean July day within the band, against 1.39% in Riyadh and 0.86% in Jeddah for the typical floor. For the whole building it is 0.13–1.34% within the band and 0.22–2.50% down to 20 °C on the design days of Case 2, below the typical floor on every day.

With the engine's indoor COP factor ($\gamma = 0.02$) the program is not linear. The achievable saving, that of the best valid schedule found by the search of Section 7.2, is 0.00–0.63% within the band and 0.00–0.67% down to 20 °C on a design day, and 0.23% in Riyadh and 0.06% in Jeddah within the band over the season in Case 1.

With July treated as one 31-day horizon (Case 1), which lets heat be stored across days, the program achieves 0.33% in Riyadh and 0.23% in Jeddah, against 0.28% in Riyadh and 0.10% in Jeddah for the mean July day, with a bound of 1.31% in Riyadh and 0.96% in Jeddah.

The program saves in the way Proposition 1 allows: it pulls the rooms down around dawn, when the outdoor air is coolest and the COP highest (the rooms are coldest at 06:00 in the Riyadh July profile), never uses more than the lowest mode, and lets the rooms rise to the ceiling through the hottest hours. Its linearisation and its search are checked in Supplementary Section S4. The benchmark's continuous low-mode modulation without cycling losses is part of what it measures: a dawn setback fixed in advance, which imitates its timing on a simulated thermostat, raises the bill of the whole building (Case 3) by 0.99–1.80% (Supplementary Section S8).

9.4 Ablation study

The selected policy θ^* has four components: the frontier cooling setpoint (the 23.5 °C reference; the heating setpoint is fixed by (C5)), no pre-cooling, the controller's mode selection (lowest mode first, escalating when a room lags) and the SBC envelope. Table 11 changes one of them at a time (Case 3; the simulation is deterministic, so each variant is a single annual run). The envelope matters most: the pre-code envelope raises the bill by 129.5% in Riyadh and 81.8% in Jeddah. Lowering the cooling setpoint to 22 °C, with heating then capped at 21 °C, costs 14.5% in Riyadh and 11.7% in Jeddah, which differs from the setpoint lever of Table 6 because that lever starts from heating at 20.5 °C and is expressed relative to current practice. Running every inverter unit in its highest mode costs 22.4% in Riyadh and 17.3% in Jeddah; this largely reproduces the assumed ratio of mode efficiencies (Theorem 1). The pre-cooling rows carry the penalties of Section 9.3. The only variant cheaper than θ^* holds the inverter units in their lowest mode, a controller variant of Section 9.7: it changes the bill by −0.31% in Riyadh and

−0.03% in Jeddah and keeps every room in the safe set. The mode rule is a property of the controller, not of the policy θ that a household sets, so the escalation of θ^* costs at most this margin.

Table 11 *Ablation of the selected policy θ^* (whole building, Case 3, volume tariff). Each variant changes one component and keeps the others, including the heating setpoint (which Algorithm 1 caps at 1 K below the cooling setpoint, so the current-practice row heats to 21 °C). Bill: annual HVAC bill (SAR); Δ: its change relative to θ^* [%]. All variants keep every conditioned room within the safe set (zero excursion above 24 °C and below 20 °C) and are therefore admissible. Lowest bill in bold.*

	Riyadh		Jeddah	
Variant (changed component)	Bill	Δ	Bill	Δ
Selected policy θ^* (SBC)	10,456	–	15,714	–
Current-practice cooling ($23.5 \rightarrow 22$ °C; heating 21 °C)	11,968	+14.5	17,551	+11.7
Inverter units in lowest mode (controller)	**10,424**	−0.31	**15,709**	−0.03
Highest mode (instead of the lowest)	12,801	+22.4	18,440	+17.3
Pre-code envelope (instead of SBC)	23,994	+129.5	28,566	+81.8
Night pre-cooling (2 K)	10,934	+4.6	16,542	+5.3
Pre-peak pre-cooling (2 K)	10,811	+3.4	16,390	+4.3

Table 12 *Factorial (2^4) decomposition of the bill saving (Case 3). Levers, from start to end state: pre-code → SBC envelope; fixed-speed → inverter 3-ton units; setpoints $22/20.5 \rightarrow 23.5/22.5$ °C; 2 K pre-peak pre-cool → none. Shapley value: each lever's marginal saving averaged over all orders (SAR/yr and share of the total); main effect: mean saving of the lever over the eight combinations of the others (% of the bill without it). Values are rounded individually, so the rows may not add exactly to the total.*

	Riyadh			Jeddah		
Lever	SAR	Share	Main	SAR	Share	Main
Envelope	13,346	74.6%	56.0%	13,503	68.6%	45.7%
Inverter 3-ton units	2,395	13.4%	13.5%	2,667	13.5%	11.1%
Setpoints	1,742	9.7%	10.2%	2,774	14.1%	11.0%
No pre-cooling	416	2.3%	2.6%	747	3.8%	3.4%
Total	17,899	100%		19,692	100%	

Table 13 *Normal and hard test instances (Case 3, the whole building). Catalogue: the eight instances with the catalogue units (two envelopes, two cities, TMYx and TMYx +3 K weather) in one row, with ranges over them (all instances are summarised in Supplementary Section S8). Units: catalogue (sized by coverage) or each sized to its room's own peak hourly load with a 15% margin. Peak load ratio: largest hourly sensible load of any unit over its rated capacity; Exc.: excursion of the reference policy above 24 °C and below 20 °C; best policy: the cheapest admissible thermostat policy, night/pre-peak depth (none: no pre-cooling; –: no lattice setting at the reference setpoints is admissible; in brackets, the highest admissible cooling setpoint without pre-cooling, where one exists); admissible policies: number of the 49 lattice policies that keep every conditioned room within the safe set on the sampled days (the others are excluded). No unit reaches a load ratio of 0.85.*

Envelope	City	Units	Weather	Class	Peak load ratio	Exc. above/below [K·h]	Best policy	Admissible policies
SBC, pre-code	both	catalogue	both	normal	0.19–0.62	0/0	none	30–49 of 49
SBC	Riyadh	by load	TMYx	hard	0.58	69/0	– (23 °C)	0 of 49
SBC	Jeddah	by load	TMYx	hard	0.57	18/0	– (23 °C)	0 of 49
Pre-code	Riyadh	by load	TMYx	hard	0.79	81/0	– (23 °C)	0 of 49
Pre-code	Jeddah	by load	TMYx	hard	0.75	29/0	– (23 °C)	0 of 49

Factorial decomposition. The one-at-a-time levers are measured against different baselines. A full 2^4 factorial of four levers (envelope, the 3-ton units as inverter units, the setpoints of current practice or of the reference, and a 2 K pre-peak pre-cool or none; Case 3, all 16 combinations admissible) puts them on one base. Going from the pre-code building with catalogue units, current-practice setpoints and pre-peak pre-cooling to the SBC-compliant building with inverter units, the reference setpoints and no pre-cooling lowers the building's annual HVAC bill from 26,788 to 8,889 SAR (US$7,143 to US$2,370) in Riyadh and from 33,616 to 13,924 SAR (US$8,964 to US$3,713) in Jeddah, savings of 66.8% and 58.6%. Table 12 attributes this saving to the levers by their Shapley values, the average of each lever's marginal saving over every order in which the levers can be applied. The envelope takes 74.6% of it in Riyadh and 68.6% in Jeddah, and dropping pre-cooling takes the least, 2.3% in Riyadh and 3.8% in Jeddah. The levers are sub-additive: each saves less once another has lowered the load, most for the envelope with the equipment in Riyadh and the envelope with the setpoints in Jeddah (an interaction of up to 7.0% of the starting bill), while pre-cooling interacts with the other levers by at most 0.9%. In both cities the envelope comes first and timing last; the equipment saves more than the setpoints in Riyadh, and the two, close to each other, swap places in SAR in Jeddah, as in the one-at-a-time results. The Shapley value of the timing lever is the avoided penalty of its start level, the 2 K pre-peak pre-cool: because no tested pre-cool saves, pre-cooling only costs and timing ranks last; its share would grow with a deeper start level and vanish without one. The ranking also compares steps chosen for the study (the code envelope, a 1.5 K setpoint step, one unit type), so it orders these steps, not the levers in general; the mode lever is not part of the factorial, since the controller sets the mode.

9.5 Generalisation: scale and test instances

Scale and saturation. Across the three cases the model grows from 16 to 33 to 132 zones and from 7 to 15 to 60 units, yet the optimum stays at no pre-cooling in every case, city and envelope: scale changes the load on each unit but not the value of re-timing, and how close the units come to their capacity depends on the envelope and on how they are sized (Table 13).

Tariff revision. To test whether the findings survive the revision of the tariff from 1 January 2027 (Section 5.6), the three cases with both envelopes, the 49 lattice policies of Case 3 and the eight apartment meters were simulated again and billed at the revised prices. Every percentage is unchanged to the reported precision: the setpoint lever saves 12.2–12.4% in Riyadh and 9.8–10.5% in Jeddah across the three cases, the envelope lever 50.4–56.4% and 38.3–45.0%, no pre-cooling remains the cheapest of the 49 policies in both cities, and a 2 K pre-peak pre-cool still costs 3.4% in Riyadh and 4.3% in Jeddah. Every amount in SAR rises by a sixth, the ratio 0.21/0.18: the setpoint lever is worth 178–221 SAR (US$47–59) a year per apartment in

Riyadh and 217–273 SAR (US$58–73) in Jeddah, and the envelope 1,264–3,047 SAR (US$337–813) and 1,165–2,837 SAR (US$311–757). The invariance follows from the structure of the tariff: in no case or envelope does a meter use more than 3,395 kWh of HVAC electricity in a month in Riyadh (3,077 kWh in Jeddah), so every meter stays in tier 1 unless its other uses exceed about 2,600 kWh a month, and a tier-1 bill is proportional to energy. A tariff whose tiers the meters do cross changes the costs of pre-cooling but not the optimum (Section 9.7).

Normal and hard instances. All eight instances with the catalogue units are normal: even in the heat-wave year, 3 K hotter in every hour, the peak load ratio stays at or below 0.26 in the SBC-compliant building and 0.62 in the pre-code building, because units sized by coverage leave a wide margin even for the uninsulated envelope. Hard instances arise when the units are sized tightly, each to 1.15 times its room's largest hourly load, after derating, rounded up to a multiple of 0.25 ton (at least 0.75 ton). All four such instances are hard. Their peak load ratio rises to 0.57–0.79, so they satisfy the feasibility condition (7) on hourly loads, yet the rooms exceed the ceiling by 18–81 K·h a year. These excursions do not reflect a shortage of hourly capacity. Starting each unit at full capacity leaves them unchanged, so they do not arise at start-up: they arise while a running unit escalates from a lower mode, with the switch-on point at the ceiling itself (Supplementary Section S8). At the reference setpoints no lattice setting, with or without pre-cooling, is admissible in any of the four. A lower cooling setpoint, which moves the switch-on point below the ceiling, restores admissibility. The highest admissible setpoint is 23 °C in all of them; there the tightly sized units still save 10.2–16.7% against the catalogue units at the reference policy, so for these units, with no tolerance on the safe set, the optimum moves to a lower setpoint rather than to another lattice setting.

Tolerance of the safe set. Admissibility requires zero excursion, a strict rule. Allowing a tolerance of 0.05, 0.1 or 0.2 K on both bounds leaves the result unchanged with the catalogue units: in the four instances of the TMYx year the reference policy stays admissible and no pre-cooling stays the best policy; only the number of admissible lattice policies of the pre-code building in Jeddah grows, from 36 to 41 at 0.1 K and 42 at 0.2 K. The tightly sized units exceed the ceiling by at most 0.15–0.19 K at the reference policy, so of the tolerances tested only 0.2 K makes their reference admissible, where no pre-cooling is again the best policy. At 0.1 K or less the reference stays inadmissible, and only pre-cooling settings stay within the bounds (at 0.1 K, 7 and 30 of the 49 in the SBC-compliant building in Riyadh and Jeddah, and none and 7 in the pre-code building), so the cheapest admissible setting is then a pre-cooling one.

Table 14 *Sensitivity analysis (SBC-compliant building, Case 3): HVAC energy intensity [kWh/m²yr], setpoint lever, saving of the best thermostat pre-cool (0.5–2 K in either window; positive: saves), and the change in cost of a 2 K pre-peak pre-cool (positive: costs). C_d: cycling degradation coefficient; sized by load: at 2.5 times the peak hourly load, the first tested margin that keeps every room within the safe set, unless stated; excluded: inadmissible, because the reference policy lets a room leave the safe set by the annual excursion shown; nothing else of the run is reported. Selected variants; all 37 are summarised in Supplementary Section S8.*

	Riyadh				Jeddah			
Variant	Int.	Setp.	Pre-cool	2 K	Int.	Setp.	Pre-cool	2 K
Reference (base case)	25.1	12.4%	0.00%	+3.4%	37.7	10.5%	0.00%	+4.3%
One node, COP independent of indoor air	23.8	8.2%	0.08%	+1.8%	36.1	8.0%	0.00%	+5.0%
Escalation at 0.2 / 0.6 K	26.0	12.6%	0.00%	+3.1%	38.5	10.8%	0.00%	+4.4%
COP independent of indoor air	23.6	9.5%	0.00%	+2.2%	36.0	8.1%	0.00%	+3.0%
Indoor COP factor 3%/K	25.9	14.0%	0.00%	+4.1%	38.6	11.8%	0.00%	+5.1%
No cycling loss ($C_d = 0$)	21.3	12.5%	0.00%	+4.0%	33.2	10.1%	0.00%	+4.8%
3-ton units as inverter units	21.3	12.4%	0.00%	+4.5%	33.4	10.0%	0.00%	+5.2%
Sized by load, all inverter	20.9	11.7%	0.00%	+2.9%	33.5	9.6%	0.00%	+3.0%
Sized by load, fixed-speed kept	24.5	11.9%	0.00%	+2.6%	37.6	10.1%	0.00%	+2.9%
Sized by load with a 15% margin	excluded: 68.6 K·h above				excluded: 17.8 K·h above			
Indoor fan on while compressor off	28.0	10.9%	0.00%	+3.3%	43.2	8.8%	0.00%	+4.1%
Infiltration 0.8 ACH	31.5	10.0%	0.00%	+3.4%	52.4	10.2%	0.00%	+3.8%
Heat wave +3 K, swing ×1.3	33.4	12.2%	0.00%	+3.1%	46.7	9.7%	0.00%	+4.1%

9.6 Thermostat comparison: full-year re-evaluation and calculation time

Selection and full-year re-evaluation. The comparison is exhaustive over the 49 realisable thermostat policies, and its best policy, no pre-cooling, is the same in all four envelope–climate configurations and in every case and test instance with an admissible reference policy. Selected on the 60 sampled days and re-evaluated on all 365 days of the year, which include them (Section 7.3), the optimum for the SBC-compliant building is unchanged in both cities, and the policies keep almost the same cost ranking (Spearman rank correlation 0.999 in both cities). The sampled-year estimate of the building's annual HVAC energy at the reference policy differs from the full-year value by −3.7% (Riyadh) and +0.5% (Jeddah), and the relative cost of every pre-cooling policy by at most 0.56 percentage points. Unless labelled as full-year, absolute energies and bills are sampled-year values. Admissibility, checked on the sampled days, also holds on all 365 days for every run the results rely on: the 16 runs of the factorial, the units sized by load at the selected margin and the dawn setback stay within the safe set in both cities. In the pre-code building 29 of the 49 lattice policies stay within the safe set over the full year in each city, fewer than on the sampled days (Table 13); the cheapest of them is still no pre-cooling, and every other one costs more, at least 0.51% in Riyadh and 0.74% in Jeddah.

Calculation time. On a two-core virtual machine one annual evaluation of the whole building (132 zones with two nodes each, 120 open doors, 28,800 steps) takes 13.4 s for one policy and 52.5 s for all 49 policies of the lattice together. The optimal-control program for one day has 5,088 variables in Case 1 and 10,656 in Case 2; its bound ($\gamma = 0$, one solve) takes 1.3 and 18.4 s, and the search for the indoor COP factor (19 solves) 21.2 and 284.3 s, so its

time grows about 13-fold for twice the zones, while the simulation time hardly grows. The bound of the whole building, with Clarabel, takes 233.8 s. Selecting the modes of all 60 units at one control step takes 8.4 μs, and a single unit's thermostat logic is trivial for ordinary hardware.

9.7 Sensitivity analysis

The sensitivity analysis varies, one at a time, the assumed inputs and model choices: the zone model, the controller and COP model, the equipment, the envelope and air, and the weather (37 variants of the SBC-compliant building; Table 14 shows the decisive ones, and Supplementary Section S8 summarises all of them). In 35 of the 36 admissible variants the best thermostat pre-cool saves nothing, and a 2 K pre-peak pre-cool costs 2.0–6.7%; the only exception, a one-node zone whose COP does not depend on the room temperature (two changes at once), saves at most 0.08%, and its 2 K pre-peak pre-cool still costs 1.8%. The variant sized by load with a 15% margin is inadmissible: the reference policy already lets a room pass the ceiling, by 69 K·h a year in Riyadh and 18 K·h a year in Jeddah, so no lever or cost is reported for it ("excluded" in Table 14). The setpoint lever stays between 8.0 and 17.0%.

Equipment. In the SBC-compliant building the installed equipment matters more than its operation. Without cycling losses the building would use 15.2% less energy in Riyadh and 11.8% less in Jeddah, and replacing the twelve fixed-speed 3-ton units by inverter units of the same size would save 15.0% in Riyadh and 11.4% in Jeddah; these guest-room and stair units, the most oversized, use 39–40% of the units' electricity for 33% of the conditioned floor area. Units sized by load keep every room within the safe set only with a margin of 2.5 times each room's peak hourly load (the smallest sufficient margin lies between 2 and 2.5; Supplementary Section S8). Against the catalogue's 124 tons, such units would save 16.5% in Riyadh and 11.2% in Jeddah as inverter units throughout, and 2.1% in Riyadh and 0.2% in Jeddah with the fixed-speed zones keeping fixed-speed units, which isolates the effect of size. An indoor fan of 30 W that runs whenever the compressor is off in the cooling months, as many wall-mounted units do by default, adds 11.8% in Riyadh and 14.7% in Jeddah, comparable to the setpoint lever, although whether the catalogue units do so was not established. None of these changes the ordering of the operational levers or makes pre-cooling pay.

Controller and COP model. With escalation at 0.2/0.6 K, a 2 K pre-peak pre-cool costs 3.1% in Riyadh and 4.4% in Jeddah; with escalation at 0.8/2.0 K it costs 2.3% in Riyadh and 3.2% in Jeddah, and with the inverter units held in their lowest mode 2.0% in Riyadh and 2.8% in Jeddah. Without the indoor-temperature factor of the COP the setpoint lever falls to 9.5% in Riyadh and 8.1% in Jeddah and the 2 K pre-peak cost to 2.2% in Riyadh and 3.0% in Jeddah; with an indoor slope of 3%/K the lever rises to 14.0% in Riyadh and 11.8% in Jeddah and the penalty to 4.1% in Riyadh and 5.1% in Jeddah. The cycle length, set by the time step, moves the absolute bill but not the sign of pre-cooling (Supplementary Section S8).

Zone model, envelope and weather. With one node per zone, so that a pull-down stores coolth in the whole mass at once, a 2 K pre-peak pre-cool still costs 3.3% in Riyadh and 6.7% in Jeddah, because the colder room also lowers the COP. The air-node share of the capacitance (2 to 30%) and the air–structure coupling (halved or doubled) leave the best pre-cool at 0.00%; thermal bridges and the solar absorptance move the energy by a few per cent; infiltration between 0.2 and 0.8 ACH moves Jeddah's intensity from 30.4 to 52.4 kWh/m^{2}yr through the latent load; and a 30% wider daily swing, alone or in a heat wave, leaves the scheduling result unchanged. A room capacitance of 144 to 450 kJ/(m^{2}K) leaves the best pre-cool at 0% (Jeddah), and an imposed day–night COP swing makes pre-cooling pay only far beyond the physical range (in Jeddah only once the COP at 22 °C is 4.0 times that at 40 °C, and in Riyadh not at all up to a ratio of 6.7, against a ratio of 1.5 for the model's curve).

Humidity. A diagnostic humidity balance per zone (Supplementary Section S8) tests the assumption that the indoor humidity target is met. It holds in Riyadh, where the conditioned rooms exceed 60% relative humidity in 0.8% of the cooling-season hours, but not in Jeddah, where the oversized units run too briefly: at a sensible heat ratio of 0.75 the rooms exceed 60% in 41% of the hours, so holding the target needs dedicated dehumidification, whose electricity the latent term prices. The Jeddah results therefore price humidity but do not constrain it. The setpoint lever hardly moves this diagnostic, and pre-peak pre-cooling worsens it.

Cost model. Under three energy-priced tariffs (a flat price of 0.18 SAR/kWh before VAT, the actual two tiers, and the two tiers with the threshold lowered from 6,000 to 500 kWh a month, which the apartment meters exceed in summer), no pre-cooling remains optimal, with 0% saving, as Proposition 2 implies for any tariff of energy alone once the no-gain condition of Proposition 1 holds. The stricter tier only raises the 2 K pre-peak cost, to 4.4% in Riyadh and 5.3% in Jeddah at the 500 kWh threshold, because part of the extra energy is billed in the higher tier.

Contrast: a time-of-use tariff. To check that the null result is a property of the tariff rather than of the model, the same lattice is priced under an illustrative time-of-use tariff that charges the cooling-month energy used from 13:00 to 17:00 at r times the off-peak price. The window is that of the Saudi time-of-use programme for large industrial and commercial customers, which did not include households [69]. At $r = 1$ the result is that of the volume tariff. At $r = 2$, 3 and 5 the best schedule pre-cools before the peak and saves 2–27% with the base-case zones (13–50% with one node per zone, whose air holds the full capacitance). The same model that finds no value in re-timing under the Saudi tariff therefore finds it as soon as the tariff prices the hour.

Global uncertainty analysis. The one-at-a-time runs leave the joint effect of the assumed inputs open. Seventeen of them (Supplementary Section S9) are varied together over uniform ranges by a Latin hypercube of 128 samples per city. The reference policy is admissible in every sample; the pre-code building, which serves only as the envelope comparison, leaves the safe set in 66–68 of them. As admissibility requires, those comparisons are excluded, and the envelope lever is taken over the other 60–62 samples. Over all samples, including those in which the pre-code building leaves the safe set (and so holds its rooms warmer, which understates its energy and the lever),

the envelope lever has a 5–95% range of 49.0–59.8% (median 53.6%) in Riyadh and 33.9–51.2% (median 41.7%) in Jeddah. Over the admissible comparisons the medians are 54.3% and 42.4%, so the exclusion moves the median by at most 0.7 percentage points. Table 15 gives the median and the 5–95% range of each result. None of the pre-cooling settings tested in each sample (0.5–2 K, night or pre-peak) saves in any of the 256 samples, and a 2 K pre-peak pre-cool costs in every sample. The ordering (envelope, then setpoints, then timing) holds in every sample in which all the compared runs are admissible, and in all 256 samples if the inadmissible pre-code comparisons are included. With none of 128 samples per city saving, the share of the sampled input space in which a tested pre-cooling setting saves is below 3/128≈2.3% per city at 95% confidence (rule of three; approximate here, since a Latin hypercube is not a simple random sample). The analysis varies assumed inputs only, not the ceiling, occupancy, tariff, sizing, controller logic or policy class. The setpoint lever depends most on the indoor COP slope γ and the infiltration, the cost of pre-peak pre-cooling on the indoor COP slope γ and the air-node share of capacitance (partial rank correlations, Riyadh; both cities in Supplementary Section S9).

Table 15 *Global uncertainty analysis (SBC-compliant building, Case 3, 128 Latin-hypercube samples of 17 assumed inputs per city): median and 5–95% range of each result [%], over the samples in which both compared runs are admissible (envelope: 62 and 60 samples in Riyadh and Jeddah; all others: 128). Levers are savings; pre-cooling rows are the saving of the best admissible pre-cool and the cost of a 2 K pre-cool. Ranges of the inputs and the partial rank correlations: Supplementary Section S9.*

	Riyadh		Jeddah	
Result	Median	5–95%	Median	5–95%
Setpoint lever	10.8	7.9–13.4	9.6	7.6–11.7
Envelope lever	54.3	49.5–60.3	42.4	35.6–51.5
3-ton units as inverter units	14.6	8.7–19.9	10.6	6.5–15.2
Best pre-cool (0.5–2 K)	0.0	0.0–0.0	0.0	0.0–0.0
2 K pre-peak pre-cool, cost	2.5	1.7–4.3	3.1	1.9–5.3
2 K night pre-cool, cost	4.5	2.3–6.9	4.4	2.5–6.7

10 Discussion

Read against the three gaps of Section 2, the results separate the levers under one comfort ceiling (G1), test the timing lever against the tariff households pay (G2) and rest on a model built from the code, the catalogue and the plan (G3). They raise three questions: why they differ from the larger scheduling savings in the literature, what the volume tariff rewards and where the findings apply. These are taken in turn, followed by the limits of the study.

10.1 Reconciliation with the larger savings reported in the literature

The apparent conflict with studies that report large scheduling savings disappears once the levers are separated. Where such studies start from a low setpoint or a leaky envelope, part of their "scheduling" saving is the setpoint or envelope saving measured separately here. Where they assume time-of-use or real-time prices, the benefit comes from a tariff Saudi households do not face. Where they let rooms float to the comfort limit through the peak or switch empty rooms off, the saving comes from warmer rooms or vacancy, excluded here by the hard ceiling and (A4), not from moving cooling in time; the 15–31% that Alshahrani and Boait [44] find for scheduling Riyadh dwellings to occupancy is of this kind. The same reading explains the larger coordination savings reported for Saudi homes under the same two-tier tariff [8]: about 4.3% for a five-zone villa and 1.0% for a twenty-zone building with a 23–25 °C band, and 13.4% and 7.8% with a wider band. That study coordinated on/off units against uncoordinated thermostats, with consumption partly near or above the 6,000 kWh threshold, which no apartment meter here reaches. Its wide-band savings come mostly from the level at which rooms are held, and its narrow-band building saving is of the order of our perfect-foresight bound under the COP assumption most favourable to pre-cooling (Table 10; ours is a share of sensible cooling electricity, theirs of the bill).

10.2 What the volume tariff rewards

Under a tariff that prices energy alone, the theory implies wherever the no-gain condition of Proposition 1 holds, and in the base model the simulations confirm, that the best thermostat policy is the fixed frontier policy of Proposition 2: the cooling setpoint whose cycle just fills the band, no pre-cooling, and the lowest mode that holds the room. It needs no forecast, and a thermostat that pre-cools by default, a feature suited to time-varying prices, raises the Saudi bill (Table 11). The tariff decides this: under the illustrative time-of-use tariff of Section 9.7, the same model and lattice make pre-cooling save 2–27%. The household's levers are therefore the cooling setpoint, worth about 9.4% (Riyadh) and 7.0% (Jeddah) of the HVAC electricity per kelvin between 22 and 23.5 °C, and, where the unit offers the setting, an indoor fan that stops with the compressor (Section 9.7). Heating to the comfort target (20.5 → 22.5 °C) is the price of winter comfort, not a saving: it adds 1.6% in Riyadh (Section 9.2). For owners and code bodies the envelope, a one-off decision, saves the most energy, and the equipment comes next: units chosen by rated coverage cycle at a fraction of their capacity, and inverter units sized by load would save more than their size alone accounts for (Section 9.7). The equipment figures rest on assumed part-load ratios, cycling coefficients and escalation thresholds rather than on measured data for the catalogue units; when these inputs are varied jointly in the uncertainty analysis (Table 15), the inverter lever spans 6.5–19.9% (5–95% range over both cities), so its size is uncertain but its sign is not. The measured saving of an inverter over a fixed-speed unit in a Saudi office room, about 44% [42], lies above this range, as expected for a comparison of single units rather than a building in which only the 3-ton units change. Against the setpoints, its rank in Jeddah depends on the measure (percentage of HVAC electricity or SAR). For designers, the heat budget of Section 6.1 explains what passive zones can achieve: a bath or corridor stays close to the band only if most of its heat can pass to conditioned rooms, through its walls or an open door, and little heat enters it from outdoors.

10.3 Where the findings do and do not apply

The optimal policy of Proposition 2 rests on conditions that can be checked for any building:

1. the tariff prices energy alone, as the Saudi residential tariff does in every tier, before and after its 2027 revision (Proposition 2, Sections 9.7 and 9.5);
2. the 24 °C ceiling is hard in rooms that are occupied and conditioned at all hours (A4), so they may not float above the band during the peak;
3. the units' efficiency falls with the mode, and cycling losses fall with the runtime, so the lowest mode that holds the room is the cheapest, or, when mode 1 cannot hold it, a mix of the two modes that bracket the load (Theorem 1, Corollary 1);
4. the neighbouring apartments are occupied and conditioned, so the coupling between them carries little heat (Section 6.1);
5. pre-cooling is done by a thermostat setback; with $\gamma = 0$, the perfect-foresight benchmark bounds what any periodic schedule of the idealised model could gain from the day–night swing of the COP on the horizons solved, and with the engine's indoor COP slope it finds a smaller achievable saving (Section 9.3, Proposition 1); neither that bound nor Proposition 1 covers the cycling-loss source (c) of Proposition 3 (the benchmark's coverage is stated in Section 10.4);
6. the building is feasible (Eq. (7)) and an admissible policy exists at the reference setpoints (Section 5.8).

Condition (vi) combines two distinct properties. The tightly sized units of Section 9.5 are feasible, yet no policy at the reference setpoints is admissible, because the modelled controller switches on at the ceiling itself and a running unit escalates from a lower mode with a lag within a step; their rooms overshoot the ceiling by less than 0.2 K, and a tolerance of 0.2 K would admit them (Section 9.5). The margin of 2.5 times the peak hourly load at which units sized by load first keep every room within the safe set (Section 9.7) is therefore a property of the modelled controller, not of the units, and the study does not recommend oversizing. A 23 °C setpoint, whose switch-on point lies below the ceiling, restores admissibility for the tightly sized units, as would a controller that switches on below the ceiling at 23.5 °C; no lattice setting does. Sizing by load with such a controller is outside the policies evaluated here. The policy must be re-derived where another condition fails: under time-of-use or coincident-peak tariffs, next to vacant apartments or with empty rooms switched off, and for central plants with thermal storage.

10.4 Limitations and future work

Internal validity (whether the model attributes each saving to the right cause). Each room is a two-node zone after ISO 13790; the walls have no layered conduction and the windows no reveal shading or curtains. The sign of the thermostat result rests on the structure being a separate node or on the COP falling in a colder room, both physically founded: only a one-node zone with a COP that ignores the room temperature lets a pre-cool save, by at most 0.08% on its own and 0.77% with every choice most favourable to pre-cooling combined (Sections 9.3 and 9.7). The engine agrees with EnergyPlus in annual cooling on the storey geometry and the BESTEST cases (Section 8.2), partly through compensating differences in the heat-gain components (Supplementary Section S7), but the BESTEST cases run on Riyadh weather, so the acceptance ranges of Standard 140 do not apply. The engine also overstates the extra heat that a setback removes, although, scaled to EnergyPlus, pre-cooling still costs (Supplementary Section S8). The part-load ratios of the modes, the escalation thresholds, the indoor COP slope, the cycling coefficients, standby and derating are assumed rather than measured. The cycling loss is charged through the runtime fraction and the shortest cycle is set by the 3-minute step, so the number of starts per hour is an artefact of the step, and the absolute size of the cycling loss is the least certain equipment input. None of the other one-at-a-time variants changes the ordering of the levers or makes thermostat pre-cooling pay, and the global analysis varies the assumed inputs jointly, over ranges that are themselves judgements. The capacitance, the ground model and the weather year are varied only one at a time or not at all, and rank correlations, unlike variance-based indices, do not apportion the interactions. The latent load is priced at a fixed latent COP with the humidity target assumed met; in Jeddah the units' sensible runtime alone does not meet it, so a higher setpoint is credited with no loss of dehumidification and the Jeddah results hold for sensible comfort. Heating COP and capacity do not depend on the outdoor temperature and there is no defrost, which affects only Riyadh's small heating energy; the timing result is a cooling-season result. The cooling COPs are anchored to the 2014 minimum of SASO 2663, not to the seasonal ratings of its 2021 edition, so units sold today may be more efficient than those modelled.

External validity (whether the result carries over). Every room is occupied and conditioned at all hours (A4); vacant apartments and switching off empty rooms are outside the study, and a fair assessment needs a validated pull-down of single rooms and a moisture model for the hours a unit is off. One building, designed for this study rather than surveyed, is modelled; its stair is conditioned and its guest room is treated like any other room, which households may not do; villas, high-rise blocks and mixed stocks are not covered, although the conditions of Section 10.3 can be checked for each. The thermostat comparison covers the uniform, fixed-window lattice of Section 7.2 with ramped and dawn setbacks, and two spatial subsets. The perfect-foresight benchmark covers the design days of Cases 1 and 2 and of the top floor under the roof, the 31-day July horizon of Case 1 and, for the bound only, Case 3, with idealised units and linearised door flows; its $\gamma = 0$ result bounds the idealised model only, and neither it nor the thermostat comparison establishes what a closed-loop predictive controller on discrete, cycling units would achieve. One typical year per city is used, with a wider swing and a heat wave as weather tests.

Construct validity (whether the measured quantities represent the intended ones). Comfort is the air temperature against a fixed band rather than the predicted mean vote, with the operative temperature estimated separately (Section 9.1). "SBC-compliant" refers to the walls, roof and glazing; slabs and partitions keep the uninsulated construction of the stock (Section 4.3). The model is compared with EnergyPlus and with the stock energy use, not with metered apartments, so the bills in SAR are indicative; the ordering of the levers and the relative savings are the results least affected by this.

Future work follows from these limits, in order of priority: (1) occupancy-based operation with measured routines, vacant apartments and validated room pull-down; (2) field data from monitored apartments or a room-resolved EnergyPlus model; (3) a coil model with mode-dependent dehumidification and a humidity limit; (4) the costs and payback of the envelope and equipment levers; (5) MPC with forecast error on the discrete, cycling units, tested against the small margin the benchmark leaves to re-timing [8]; and (6) multi-year weather.

11 Conclusion

We asked how much the operation of multi-mode heat pumps can save in a Saudi apartment building under the volume tariff, and which lever produces the saving. The theory shows that, without re-timing, the cheapest policy holds every room at the top of the comfort band in the lowest mode that holds the room (or a mix of the two modes that bracket its load), and, apart from changes in cycling losses, bounds what pre-cooling can gain under any energy-only tariff. The room-resolved simulations of one apartment, one floor and the whole building in Riyadh and Jeddah give the following results (Tables 6 and 8):

1. *Setpoint level.* Moving from current practice (cooling at 22, heating at 20.5 °C) to the reference setpoints (23.5 and 22.5 °C) saves 12.2–12.4% of the HVAC energy in Riyadh and 9.8–10.5% in Jeddah across the three cases, 152–234 SAR a year per apartment (178–273 SAR under the tariff revised from January 2027, which leaves every percentage unchanged). The saving comes from the cooling setpoint, about 9.4% (Riyadh) and 7.0% (Jeddah) per kelvin in the whole building, net of the 1.6% that heating to the comfort target adds in Riyadh. With a 1 K thermostat differential and 0.5 K steps, 23.5 °C is also the comfort frontier; only finer control would allow more.
2. *Envelope.* The SBC envelope saves 50.4–56.4% (Riyadh) and 38.3–45.0% (Jeddah) of the HVAC energy of the same building with the pre-code envelope, 998–2,612 SAR a year per apartment.
3. *Pre-cooling.* No thermostat pre-cooling policy pays under the volume tariff in the base model; in the sensitivity analysis the only exception, a one-node zone whose COP does not depend on the room temperature (two changes at once), saves at most 0.08%, and with every choice most favourable to pre-cooling combined the best policy saves 0.77% in Riyadh and 0.03% in Jeddah. Pre-peak pre-cooling by 2 K adds 3.4–4.3%, and still 2.9–3.8% if its extra heat is scaled to EnergyPlus. On the solved periodic design days no schedule of the idealised model could save more than 1.9% of the sensible cooling electricity within the band and 3.6% down to 20 °C (the upper bound with a COP that depends on the outdoor air only); with the engine's indoor COP factor the achievable saving, that of the best valid schedule found, is at most 0.6% within the band and 0.7% down to 20 °C, and a dawn setback fixed in advance on a simulated thermostat costs 0.99–1.80% (Case 3).
4. *Equipment.* In the SBC-compliant building, units sized by load at 2.5 times the peak load, the first of the tested margins that keeps every room within the safe set, would use 11.2–16.5% less energy as inverter units, 0.2–2.1% of it from their size alone; replacing only the fixed-speed 3-ton units by inverter units saves 11.4–15.0%, and an indoor fan left running adds 11.8–14.7%.
5. *Passive zones.* By holding their neighbours at the setpoint, with heat passing through walls and open doors, the controller removes 96% of the excursion above the band that the baths would have with every unit off, although they still exceed 24 °C for 62–91% of the year; a small unit in each bath would remove the rest for 2.8–4.9% more energy (Supplementary Section S3).

The operating mode is set by the controller, not the household, and holding the highest mode costs more (Table 11). The four levers of the factorial together lower the building's annual HVAC bill from 26,788 to 8,889 SAR in Riyadh and from 33,616 to 13,924 SAR in Jeddah; a factorial decomposition on that base places the envelope first and timing last among them (Table 12), where timing's share is the penalty avoided by dropping the pre-cooling of the start state. A global uncertainty analysis of 17 assumed inputs leaves that ordering unchanged in all its 2×128 samples, including those in which the pre-code comparison leaves the safe set, and none of the pre-cooling settings tested saves in any of them (Table 15). For one building designed to the code, the findings separate the levers (G1), test re-timing against the tariff households pay (G2) and rest on a code-based model (G3); they are conditional on an energy-only tariff, every room occupied and conditioned at all hours (occupancy-based operation is outside the study), a hard 24 °C ceiling on the air temperature of the conditioned rooms, humidity priced but not constrained, and catalogue units sized by coverage; the uncertainty analysis varies inputs, not these conditions. Within these conditions, the implications for Saudi households and code bodies are to (1) improve the envelope, above all under the roof, where the top-floor apartments carry the largest envelope penalty; (2) size units by load only with a switch-on point below the ceiling (a 23 °C setpoint was tested; Section 9.5); (3) raise the cooling setpoint from 22 to 23.5 °C, with the heating setpoint held at the comfort target; (4) not pre-cool under the volume tariff; and (5) where the unit allows it, let the indoor fan stop with the compressor. Moving cooling in time pays substantially only under a tariff that prices the time of use or the peak (Section 9.7).

Author contributions

The author(s) carried out the conceptualisation, methodology, software, validation, formal analysis, investigation, data curation, visualisation, writing (original draft) and writing (review and editing), and read and approved the final manuscript.

Declarations

Competing interests. No competing interests are declared.

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