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Rooted protection: Designing optimal joint contract for realising the complementarity of forest-based solutions and financial insurance

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ABSTRACT

As natural hazards increasingly threaten mountainous areas, Improved Forest Management (IFM) presents a promising Nature-Based Solution. We develop a Principal-Agent model where a houseowner uses both IFM and financial insurance. We demonstrate a perfect complementarity: forests filter low-severity harms, enabling insurers to provide affordable coverage for residual tail risks. This combination resolves the traditional partial insurance puzzle, enabling high or even full coverage despite unfair premiums. However, as this hybrid strategy reduces insurers' profits, private markets fail to incentivize socially optimal levels of protection. Thus, public intervention or public insurers are needed to align private incentives with social welfare.

Keywords: Nature-based solutions; Natural hazards; Financial insurance; Principal-Agent model; Forest management; Moral hazard.

JEL Classifications: G22 (Insurance, Insurance Companies, Actuarial Studies)**,** Q54 (Climate, Natural Disasters and Their Management, Global Warming)**,** D86 (Economics of Contract: Theory).

List of abbreviations

FO: forest owner. HO: house owner. IFM: improved forest management. INS: insurance company.

1. INTRODUCTION

Ecosystems provide a wide range of services. In mountainous areas, forests play a crucial role in protecting against natural hazards, including snow avalanches, rockfalls, and landslides (La Notte & Paletto, 2008; Grêt-Regamey et al., 2013). These hazards have the potential to destroy infrastructure, disrupt vital transportation networks, and cause severe injuries and fatalities (Grêt-Regamey et al., 2013). The attractiveness of the mountainous regions drives up population and infrastructure densities, thereby altering the exposure and vulnerability of elements at risk. Human activities also affect process-controlling factors (e.g., vegetation changes through land-use practices and/or forest management). This anthropization, along with the impact of human activities, may often exceed the impacts of climate change on the natural hazard risk situation in the Alpine region (Slaymaker et al., 2009; Goudie & Viles, 2013), thereby contributing to an increasing number of recorded natural hazard events (Guzzetti et al., 2012; Steger, 2017).

Across the globe, and especially in the Alpine region, forests act as natural barriers and buffers in natural hazard management (i.e., increasing slope stability and preventing soil erosion) that are increasingly considered as effective as technical or civil engineering measures (Dorren, 2003; Berger et al., 2013; Moos et al., 2019). In doing so, they provide substantial protection value (Augeraud-Véron et al., 2019). This protection value of ecosystems stems from the ecosystem's capacity to absorb disturbances while maintaining its basic structures, functions, and feedbacks – a property known as "resilience" (Walker et al., 2004). Thus, the higher the level of resilience, the lower the risk of losses in ecosystem services, income or wealth – and the higher the protection value (Baumgärtner & Strunz, 2014). Since the effectiveness of this protection depends heavily on terrain characteristics and forest structure (Bebi et al., 2009; Dorren et al., 2015; Moos et al., 2016), forest owners are key actors who directly influence the provision of this natural protection.

In densely populated mountainous regions such as the Alps – which are among of the most densely populated mountainous areas in the world (Rudolf-Miklau et al., 2014) – forests cover approximately 52% of the region, making forest management central to local economies and safety (Price et al., 2011). Approximately half of Switzerland's forest area is classified as "protection forest". A protection forest is defined as a one that can prevent recognized potential damage from existing natural hazards or reduce associated risks.

Thus, preserving and enhancing this protective role of forests against natural hazards is key to an efficient strategy for improving the liveability of the Alpine region. In Switzerland, although the protective role of forests has been recognized for a long time – with revised forestry legislation introduced in 1991 requiring cantons to provide for the maintenance of the protection forests (WaG, 1991) - its economic protection value has largely been overlooked to date. Consequently, methodological approaches to quantify this protection value, particularly as a complement to financial insurance, remain scarce[1].

House owners living in hazard-prone areas typically hedge their risk by purchasing insurance contracts. Alternatively, they may pay forest owners for silvicultural practices that maintain forest health, thereby providing natural protection against gravitational hazards (Olschewski et al., 2012). Currently, financial insurance is widely adopted in the housing sector, partly due to legal obligations (FINMA, 2013). In contrast, paying for private natural protection as part of individual risk management is far less common. However, in addition to protecting individual house owners, financing such services also generates positive externalities by promoting other forest ecosystem services, such as carbon storage, recreation, and biodiversity conservation (Buttoud, 2000; Führer, 2000). Since forest owners typically receive no compensation for

[1] To our knowledge, notable exceptions are the studies from Baumgärtner (2007), Quaas & Baumgärtner (2008) and Baumgärtner & Strunz (2014), which assess the protection value of forests (called "insurance value").

providing these ecosystem services, suboptimal provision of natural protection services often results, eventually leading to degraded ecosystem quality (Quaas & Baumgärtner, 2008).

The relationships between insurance, protection, and prevention have been extensively studied in the literature (see Ehrlich & Becker, 1972). While insurance and protection are often viewed as substitutes (see Baumgärtner, 2007[2]), insurance and natural protection can also act as complements: natural protection covers the low-damage, high-probability risks, while insurance covers the high-damage, low-probability risks. Moreover, recent findings call for an optimised risk management framework that includes prevention, post-damage interventions, and insurance in the context of climate and socio-demographic changes (Schwarze et al., 2015). More recently, several studies have highlighted how insurance can provide incentives for prevention or protection. Regarding health risk prevention, Bourlès (2017) shows how revising premiums over time can improve market screening and incentivize prevention, while Gore et al. (2017) study insurer programs designed to motivate policyholders to make preventive efforts. Concerning adaptation efforts in the forest sector (protection), Brunette et al. (2017) show how insurance can incentivize protective measures that reduce the magnitude of harms when they occur. In this paper, we aim to study the extent to which insurance may provide a house owner with incentives to demand a natural protection, which is a source of positive externality, in a frame where this protection is provided by an external operator, the forest owner.

To address this, we develop a theoretical model that combines financial insurance with natural protection provided by a third party. A central and original contribution of our analysis lies in modelling a decision-maker who simultaneously acts as both a Principal and an Agent in two distinct contractual relationships: First, the house owner acts as the Principal by delegating

[2] The paper aims to study the optimal allocation of funds in the trade-off between investing into biodiversity protection and the purchase of insurance, i.e., with natural protection and insurance as substitutes.

natural protection provision to the forest owner (the Agent); second, she acts as the Agent in her relationship with the financial insurer (the Principal). In other words, our study also contributes to the literature on contracting problems involving delegated production to a private agent.

We first present a baseline model with risk-neutral agents, and then introduce a risk-averse house owner. We solve the two contracting problems using backward induction to determine the optimal contract between the house owner (Agent) and the insurance company (Principal) on the one hand, and between the house owner (Principal) and the forest owner (Agent) on the other hand. By relying on a Principal-Agent framework and numerical assessments, this paper models the complex interactions between natural risk mitigation and financial coverage to evaluate the necessity of public intervention.

The remainder of the paper is organized as follows: Section 2 presents the theoretical model, including its basic assumptions and contract definitions. Section 3 presents the results on private equilibrium decisions concerning natural protection and financial insurance. Section 4 extends the analysis to incorporate positive externalities generated by improved forest management (IFM). Section 5 discusses the economic implications and concludes the paper.

2. THEORY

2.1. Basic assumptions

We consider with three agents: a house owner (HO), a forest owner (FO), and an insurer (INS). HO owns a house of value w (wealth) in a mountainous region. The house is exposed to a gravitational hazard (e.g., avalanches, rockfalls, landslides), so that a harm of magnitude h may occur with a probability p, with $0 < p < 1$. The magnitude of harm h follows a random variable, denoted by $\widetilde{R_+}$ and taking value in $[0; H_+]$ with $w > H_+ > 0$ (see Figure 1).

However, it is possible to reduce the impact of gravitational hazards on HO's house by implementing an additional natural protection through an alternative forest management[3], which we call improved forest management (IFM).

FO implements this IFM, which is subject to two sources of risk: a *production risk*, and an *effectiveness risk*.

The production risk relates to the risk of (un)success in setting up the IFM. Succeeding or not succeeding in setting up the IFM depends on an effort made by FO. This effort is denoted by e, with $e \in [0; e_+]$, $e_+ > 0$, and costs $c(e)$, with $c(0) = 0, c(e) \geq 0$ and $c'(e) > 0, c''(e) \geq 0$. Increasing the level of effort increases the probability $q(e)$ that FO to succeeds in setting up the IFM[4], with $0 \leq q(\mathrm{e}) < 1, q'(e) > 0, q''(e) < 0$ and $1 > q(e_+) > q(0) \geq 0$. Not succeeding in setting up the IFM, which occurs with probability $\big(1 - q(e)\big)$, may be the consequence of failure(s) due, for instance, to unsuitable choices in terms of tree species or forest management, and/or to environmental factors (such as climate, soil, or altitude), etc. In case of unsuccess, the risk remains described by the random variable $\widetilde{R_+}$ [5].

In spite of the success in setting up the IFM, the full effectiveness of the IFM in mitigating the gravitational hazard is not certain. This is the effectiveness risk. Conditional to the setting up of the IFM, there is an (exogenous) probability $(1 - s)$ that the IFM is ineffective in mitigating the risk, leaving the risk unchanged at $\widetilde{R_+}$ [6]. With the complementary probability s (with $0 <$

[3] This alternative forest management includes for example standing trees and dead wood lying on the slope surface, with a specific stand density and distances between trees in the fall direction that should not exceed 20 m (Dorren et al., 2005).

[4] Even in the case where FO makes a maximum effort e_+, the success in setting up the IFM is never certain.

[5] Some legal requirements oblige FO to deliver a certain basic level of natural protection. Consequently, risk $\widetilde{R_+}$ is not the real gross risk, but the residual risk that is considered as the baseline. The aim here is to provide an incentive to produce an additional effort by setting up an alternative forest management.

[6] Several factors can explain the ineffectiveness of the IFM in mitigating the gravitational risk, such as stand density, strength of tree's wood or tree's resistance.

$s < 1$), the IFM is effective in mitigating the gravitational hazards. While the implanting of an IFM is observable, its effectiveness in mitigating the risk is not observable by anyone.

An effective IFM makes it possible to mitigate the risk. Risk mitigation operates by eliminating all harms of a magnitude h lower than $\hat{h}$, with $0 < \hat{h} < H_+$. More precisely, an effective IFM makes possible to transform the random variable $\widetilde{R_+}$ into a new random variable $\widetilde{R_-}$ characterized by:

$$\widetilde{R_-} \equiv \begin{cases} 0 \text{ if } h < \hat{h}, \\ h \text{ if } h \geq \hat{h}. \end{cases}$$

with $h \in [0; H_+]$ following $\widetilde{R_+}$. In words, the effective IFM absorbs (and eliminates) all gravitational harms that would otherwise lead to harms of a magnitude lower than $\hat{h}$ (in the absence of such an effective IFM). But the IFM is unable to prevent harms exceeding magnitude $\hat{h}$ [7]. This can be depicted like in Figure 1 below.

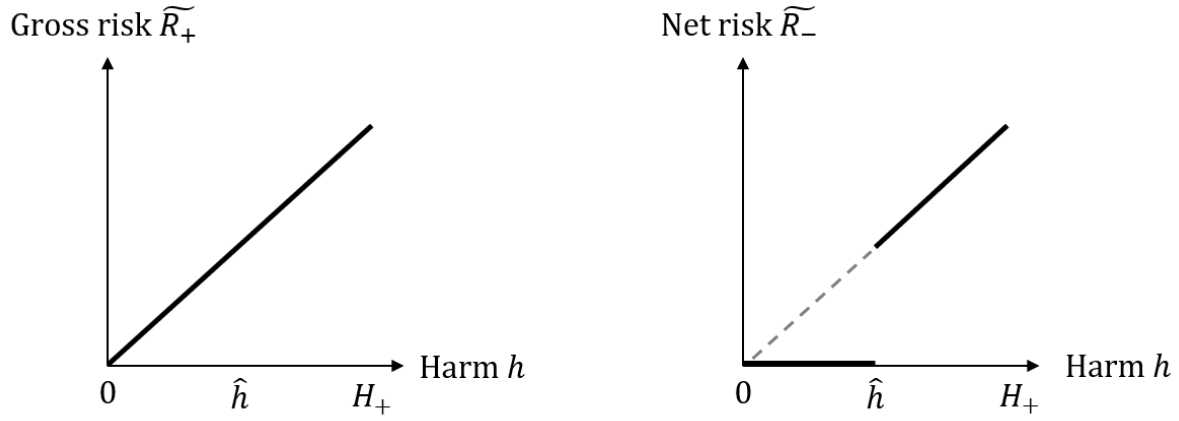


Figure 1. *Representation of the gross risk $\widetilde{R_+}$, and the net risk $\widetilde{R_-}$ in case of success in providing an effective IFM to reduce gravitational hazards.*

All-in-all, uncertainties around the implementation of the IFM can be described by Figure 2 below.

[7] Thanks to an effective IFM, trees act as a barrier that contain all small gravitational damage that would lead to lowest harms. For larger damage, the « barrier » breaks and cannot prevent the harm to occur.

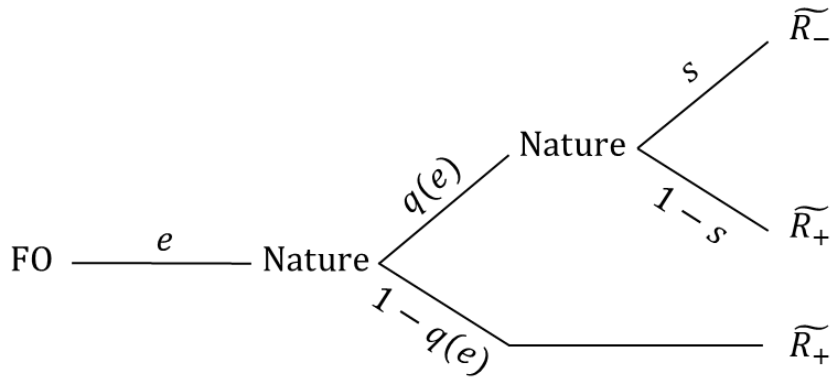


Figure 2. *Representation of the outcome from tempting to implement an IFM (risk $\widetilde{R_+}$ or $\widetilde{R_-}$), depending on the success in setting up the IFM and its effectiveness in reducing the risk.*

Finally, HO can also take out an insurance contract for covering the (residual) risk. The contract is a partial insurance contract that includes a deductible $F \geq 0$, fixed by INS, which is contingent on the observed success of the setting up of the IFM (but independent of its unobservable effectiveness).

As illustrated in Figure 3, HO has several choices, leading to various risk coverage outcomes depending on the chosen strategy (IFM from FO and/or insurance contract from INS) and the successful setting up of an IFM in the case of asking for IFM:

- asking for IFM with the success of its setting up and taking out an insurance contract;
- only asking for IFM with the success of its setting up;
- asking for IFM with the failure of its setting up and taking out an insurance contract;
- asking only for IFM with the failure of its setting up;
- taking out an insurance contract alone;
- do nothing.

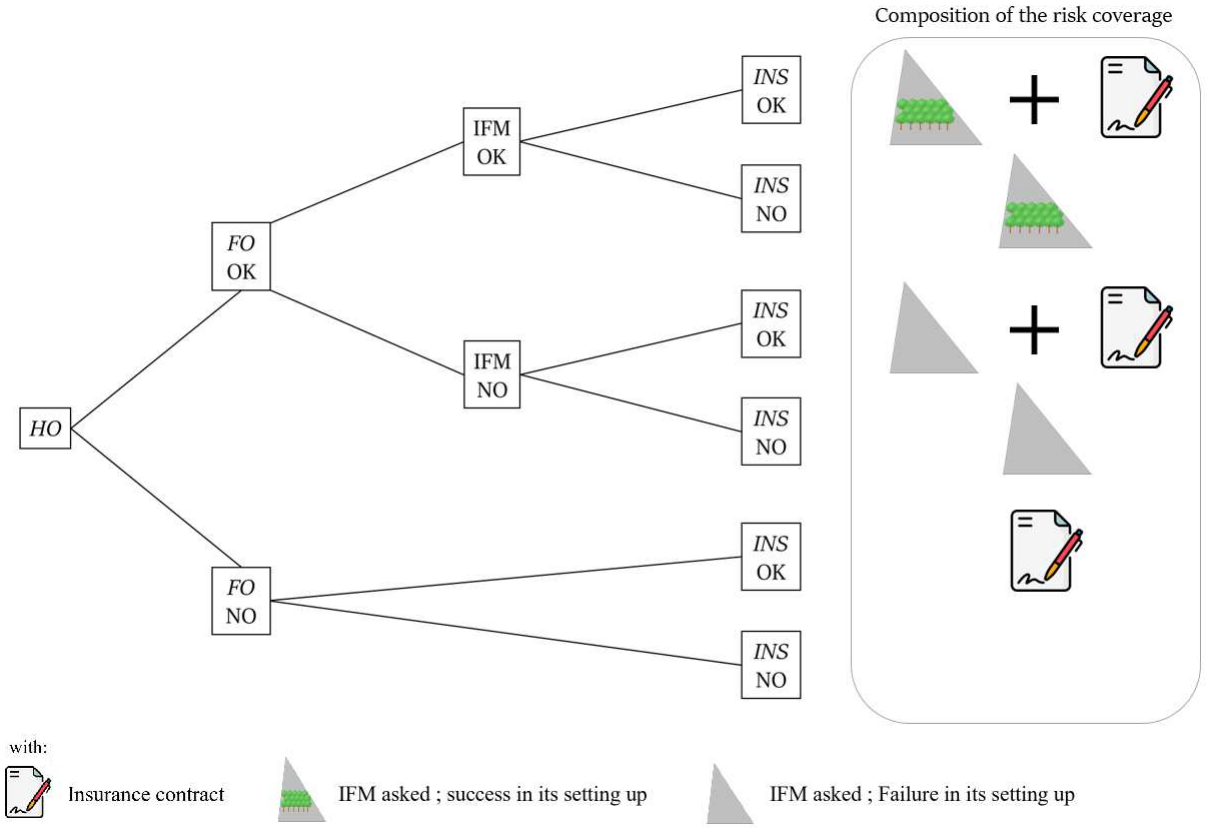


Figure 3. *Representation of HO's choices in terms of IFM and insurance and their outcomes in terms of risk coverage with asking (OK) or not (NO) FO to set up an IFM, the success (OK) or not (NO) in the setting up of the IFM (independent from its effectiveness), and taking out (OK) or not (NO) an insurance contract from INS.*

2.2. Decentralized equilibria

We now determine the private decisions made by all agents. First, we focus on the relationship that links HO and FO regarding natural risk protection (implementation of IFM), and then we address the relationship between HO and INS, regarding covering the remaining risk.

2.2.1. Providing additional natural risk protection: the relation between the house owner and the forest owner

To mitigate the risk, HO asks FO to implement an IFM: a contractual relationship applies, in which HO is the Principal and FO is the Agent. We assume that HO derives a utility $U_{HO}(w_f)$ from a final wealth w_f, while FO is risk-neutral.[8]

Because the effort e provided by FO is unobservable (neither by HO nor INS), HO must design an incentive-compatible contract to induce effort and mitigate moral hazard (e.g., Holmström, 1979; Grossman & Hart, 1983; Holmström & Milgrom, 1991; Mirrlees, 1999 on the Principal-Agent problem in case of moral hazard). Since the actual setting up of the IFM is observable, payments can be made contingent on this success[9].

The monetary transfer to FO is t_+ in case of success in the setting up, and t_- in case of failure. The contract is thus defined as (t_+, t_-). Note that whether the payment is carried out in two parts (one part before the setting up and one part after) or only one part, the entire payment is conditional on this success[10].

An effective IFM shifts HO's risk exposure from $\widetilde{R_+}$ to $\widetilde{R_-}$ when a hazard occurs (probability p).

As a necessary building block to design the optimal joint contract, we first define the HO's expected utility in a baseline scenario where only the natural protection is deployed, without financial insurance. In this standalone IFM configuration, the HO's net expected utility gain from contracting with FO is:

[8] We suppose that FO is a firm, which is able to diversify its risk by engaging in different projects which are not correlated.

[9] Recall that the *effectiveness* of the IFM in mitigating the risk is uncertain, and unobservable by anyone (neither HO, nor FO).

[10] Note that it can be seen as a payment for an ecosystem service, i.e., a voluntary transaction where the forest protection provided by the IFM is being bought by HO from FO.

$$\Delta EU_{HO} = q(e).\begin{bmatrix} (1-p).U_{HO}(w-t_+) + p.s.U_{HO}\left(w - E\left(\widetilde{R_-}\right) - t_+\right) \\ +p.(1-s).U_{HO}\left(w - E\left(\widetilde{R_+}\right) - t_+\right) - \left[(1-p).U_{HO}(w) + p.U_{HO}\left(w - E\left(\widetilde{R_+}\right)\right)\right] \end{bmatrix} \\ +\left(1-q(e)\right).\begin{bmatrix} (1-p).U_{HO}(w-t_+) + p.U_{HO}\left(w - E\left(\widetilde{R_+}\right) - t_-\right) \\ -\left[(1-p).U_{HO}(w) + p.U_{HO}\left(w - E\left(\widetilde{R_+}\right)\right)\right] \end{bmatrix} \quad (1)$$

i.e., the difference between her expected utility with the contract and her reservation utility without it.

To avoid moral hazard, the contract (t_+, t_-) must satisfy the two following constraints:

(i) The FO's participation constraint:

$$q(e).(t_+ - c(e)) + (1-q(e)).(t_- - c(e)) \geq \underline{w^{FO}} \quad (2a)$$

with $\underline{w^{FO}}$ the FO's opportunity cost. In other words, the contract ensures that FO has an interest in making the optimal effort instead of making no effort at all.

(ii) The FO's incentive constraint:

$$q(e^*).t_+ + (1-q(e^*)).t_- - c(e^*) \geq q(e_-).t_+ + (1-q(e_-)).t_- - c(e_-) \quad (2b)$$

i.e., HO prefers to induce an optimal (positive) effort e^*, instead of a lower effort e_- . The positive optimal effort will be numerically determined later on (see Section 3).

2.2.2. Covering the remaining risk by insurance: the relation between the house owner and the insurer

To complement the natural protection, HO can agree with an insurance company in signing an insurance contract for covering the (residual) risk, contingent on the observed success in the setting up of the IFM[11].

We assume INS is risk-neutral, and is able to determine the probability p of the occurrence of harm[12]. The insurance contract specifies a deductible $F \geq 0$ fixed by INS: in case of harm, HO receives *ex-post* compensation. This compensation is partial if the deductible is strictly positive, leaving HO to bear a portion of the losses[13].

The insurance contract parameters depend on the observable setting up of the IFM. The deductible is F_P if the IFM was successfully set up (the subscript P meaning protection), and F_{NP} otherwise (the subscript NP meaning no protection)[14]. Furthermore, the relative position of the deductible F with respect to the IFM effectiveness threshold $\hat{h}$ dictates the net damage borne by HO, as illustrated in Figure 4.

[11] As the *effectiveness* of the IFM is unobservable, the insurance contract is defined on the success (or not) in setting up the IFM, which is the only observable element.

[12] In Switzerland, avalanche run-out zones are officially defined and specified as hazard zones (red, blue, white). The colors correspond to specific intensities and return periods of avalanches and thus indicate the natural hazard risk faced by the residents in these zones. Therefore, INS has a perfect information and can offer the contract associated with the probability of risk according to the location of HO.

[13] We make the assumption of a deductible contract, in so far as deductible contract is the most widespread type of insurance contract. But our results are still valid while assuming a co-insurance contract (details can be provided upon request).

[14] Segmenting the market with two contracts allows INS to broaden the market, thus increasing her profit. Indeed, recall that HO has several choices (Fig. 3), and insurance is not mandatory.

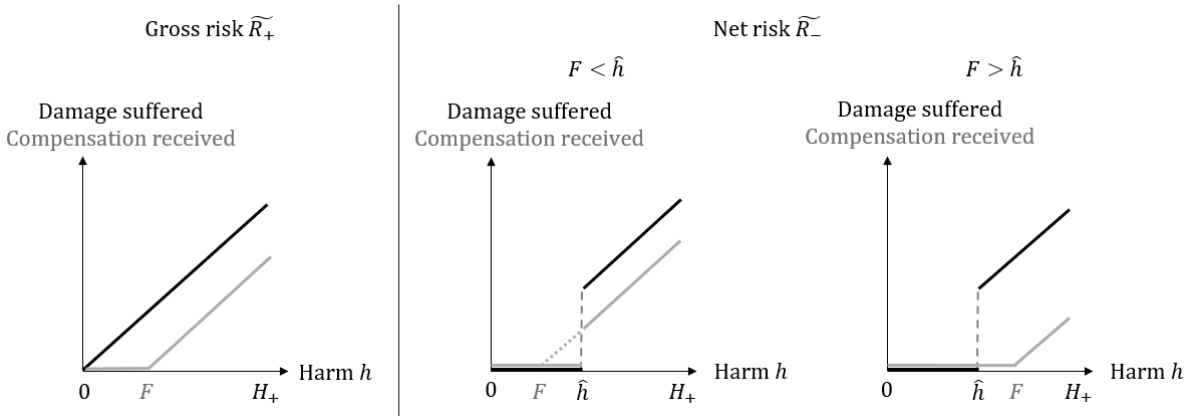


Figure 4. *Representation of the damage suffered and the compensation received when including a deductible, in case of risks* $\widetilde{R_+}$ *and* $\widetilde{R_-}$.

Based on Figure 4, the compensation I in the case of risk $\widetilde{R_+}$ can be defined as:

$$I = \begin{cases} h - F & \text{if } h > F, \\ 0 & \text{if } h < F. \end{cases}$$

As a result, the expected compensation conditional on a harm occurring is:

$$E\left(I|\widetilde{R_+}\right) = \int_0^F 0\, f(h)dh + \int_F^{H_+} (h - F) f(h)dh \tag{3}$$

In the case of risk $\widetilde{R_-}$, the compensation I can be defined as:

$$I = \begin{cases} 0 & if\ h < \max\left(F, \hat{h}\right), \\ h - F & if\ h \geq \max\left(F, \hat{h}\right). \end{cases}$$

As a result, the expected compensation conditional on a harm occurring is:

$$E\left(I|\widetilde{R_-}\right) = \int_0^{\max\left(F,\hat{h}\right)} 0\ f(h)dh + \int_{\max\left(F,\hat{h}\right)}^{H_+} (h - F) f(h)dh \tag{4}$$

INS maximizes her expected profit subject to HO's participation constraint. The expected profit is the premium[15] revenue minus the expected compensation costs. As a result, the expected profit can be written as follows:

$$E(\Pi_{INS}) = p.E(I).(1+\lambda) - p.E(I) = p.E(I).\lambda. \quad (5)$$

with $E(I)$ is the expected conditional compensation, and λ is the loading factor covering INS's management costs with $\lambda > 0$.

However, the INS's problem has to be distinguished depending on the success or the failure in setting up the IFM. In the case of failure in the setting up of the IFM, the INS's problem is written as follows:

$$\max_{F_{NP}} E(\Pi_{INS}) = p.E\left(I|\widetilde{R_+}\right).\lambda. \quad (6)$$

given the participation constraint (i.e., HO has a higher utility with insurance than without insurance):

$$\begin{gathered}(1-p).U_{HO}\left(w - p.E\left(I|\widetilde{R_+}\right).(1+\lambda) - t^-\right) \\ +p.U_{HO}\left(w - E\left(\widetilde{R_+}\right) + E\left(I|\widetilde{R_+}\right) - p.E\left(I|\widetilde{R_+}\right).(1+\lambda) - t^-\right) \geq \\ (1-p).U_{HO}(w - t^-) + p.U_{HO}\left(w - E\left(\widetilde{R_+}\right) - t^-\right). \end{gathered} \quad (7)$$

In case of success in the setting up of the IFM, there is a probability s that HO faces risk $\widetilde{R_-}$ rather than $\widetilde{R_+}$ in case of hazard occurring (p). As a consequence, the INS's problem is:

$$\max_{F_P} E(\Pi_{INS}) = p.\left[s.E\left(I|\widetilde{R_-}\right) + (1-s).E\left(I|\widetilde{R_+}\right)\right].\lambda. \quad (8)$$

given the participation constraint:

$$\begin{gathered}(1-p).U_{HO}(w - \pi_P - t^+) + p.s.U_{HO}\left(w - \pi_P - \left[E\left(\widetilde{R_-}\right) - E\left(I|\widetilde{R_-}\right)\right] - t^+\right) \\ + p.(1-s).U_{HO}\left(w - \pi_P - \left[E\left(\widetilde{R_+}\right) - E\left(I|\widetilde{R_+}\right)\right] - t^+\right) \geq \\ (1-p).U_{HO}(w - t^+) + p.U_{HO}\left(w - \left[s.E\left(\widetilde{R_-}\right) + (1-s).E\left(\widetilde{R_+}\right)\right] - t^+\right). \end{gathered} \quad (9)$$

[15] From an actuarial point of view, the pure premium, which leads to the actuarial balance, is equal to the expected compensation. Then, the insurer's management costs are paid thanks to a loading factor, which is applied to the pure premium.

where $\pi_P = p.\left[s.E\left(I|\widetilde{R_-}\right) + (1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda)$ is the insurance premium.

Note that the value of $E\left(I|\widetilde{R_-}\right)$ depends on the position of F_P relative to $\hat{h}$ [16].

Finally, the HO's expected utility when looking at asking both for IFM and insurance is:

$$E(U_{HO}) =$$

$$q(e).\begin{bmatrix}(1-p).U_{HO}(w-\pi_P-t^+) + p.s.U_{HO}\left(w-\pi_P-\left[E\left(\widetilde{R_-}\right)-E\left(I|\widetilde{R_-}\right)\right]-t^+\right)\\ +p.(1-s).U_{HO}\left(w-\pi_P-\left[E\left(\widetilde{R_+}\right)-E\left(I|\widetilde{R_+}\right)\right]-t^+\right)\end{bmatrix}$$

$$+\left(1-q(e)\right).\begin{bmatrix}(1-p).U_{HO}(w-\pi_{NP}-t^-)\\ +p.U_{HO}\left(w-\pi_{NP}-\left[E\left(\widetilde{R_+}\right)-E\left(I|\widetilde{R_+}\right)\right]-t^-\right)\end{bmatrix} \qquad (10)$$

where $\pi_{NP} = p.E\left(I|\widetilde{R_+}\right).(1+\lambda)$.

In the next section, we first resolve this problem analytically for the case of a risk-neutral HO (i.e., linear U_{HO}), and then numerically for a risk-averse HO (i.e., concave U_{HO}).

3. RESULTS

In order to find analytical solutions to our problem, we have to specify the effort and probability functions. For the FO, we assume a linear cost function $c(e) = c.e$ and an exponential success probability function $q(e) = 1 - exp^{-e}$.

Considering the contract linking HO to FO, taking into account for the participation constraint (2a), the incentive constraint (2b), and applying the specifications defined above, we obtain an equilibrium effort for FO which results in (proof in the Supplementary material, Section B)[17]:

$$e^* = \ln\left(\frac{\Delta t}{c}\right) \qquad (11)$$

with $\Delta t = t_+ - t_-$.

[16] We will distinguish the two cases in the numerical application (see section 3).

[17] Note that in the case of a unique payment (i.e., $t_+ = t_-$) that is the current situation in Switzerland, the resulting FO's effort e^* is null. In other words, when FO receives a payment to implement an IFM regardless the success or not of this IFM (i.e., without monetary incentive), FO will not implement this IFM.

Consequently, the difference between payments Δt needs to be higher than the FO's marginal cost to induce a strictly positive effort.

We then solve the two contracting problems by backward induction to determine first the equilibrium contract between HO and INS, and then between HO and FO. This allows us to determine the private equilibrium for a risk-neutral HO, and then for a risk-averse HO.

3.1. Risk-neutral HO

First, we consider the case of a risk-neutral HO, having a linear utility function given by: $U_{HO}(w_f) = w_f$, with w_f being the level of final wealth ($w_f \geq 0$).

Proposition 1: In the case of a risk-neutral HO, HO does not take out an insurance contract from INS. The differential in the FO's payment is directly equal to the HO's net benefit from an effective IFM (see proofs in Supplementary material, Section D).

The backward resolution of INS's problem results in:

$$F_P = F_{NP} = H_+.$$

The deductible in case of success (F_P) or failure (F_{NP}) of the setting up of the IFM are both equal to the maximum value of the damage H_+. This means that HO does not take out an insurance contract from INS, which is a standard result with non-actuarial insurance and a risk-neutral agent.[18]

[18] HO being risk-neutral (in this case), she has no interest in subscribing a non-actuarial insurance. Because of her participation constraint, the insurer tends to reduce the premium through a decrease in the coverage (increase in the deductible). Here is the limit case where the deductible rises up to the maximum value of harm, which is equivalent to the absence of coverage: no transaction is made between HO and INS.

Then, the optimal FO's payment results in:

$$\Delta t = s.p.\left(E\left(\widetilde{R_+}\right) - E\left(\widetilde{R_-}\right)\right) = s.p.\,\Delta E\left(\tilde{R}\right).$$

In other words, for a risk-neutral HO, the risk coverage against gravitational hazards is solely based on IFM. In the absence of insurance, the difference between the optimal FO's payments t_+ and t_- (in case of success and failure, respectively) is directly equal to the HO's net benefit from an effective IFM.

3.2. Risk-averse HO

Unterberger & Olschewski (2021) show that when HOs are risk-averse individuals, they have a positive willingness to pay for natural protection services provided by ecosystems. We determine the optimal contracts in this case using a DARA (and CRRA) function where $U_{HO} = \frac{w^{(1-\sigma)}}{(1-\sigma)}$ with σ representing the relative risk aversion coefficient (Arrow-Pratt).

This non-linear utility function does not permit us to solve the different programs analytically, requiring us to carry out numerical simulations. To perform these simulations, we first define values for the model's parameters (see Table 1 below and Supplementary material, Section A).

Parameters	Values
$\boldsymbol{s}$	0.5
$\boldsymbol{p}$	0.1
$\boldsymbol{\sigma}$	0.5
$\boldsymbol{\lambda}$	0.05
$\boldsymbol{w}$	300,000
$\boldsymbol{H_+}$	285,000
$\boldsymbol{\hat{h}}$	142,500
$\boldsymbol{c}$	1
$\boldsymbol{t_-}$	0

Table 1. *Values attributed to the different parameters for the numerical simulations.*

Starting from an initial wealth of $w = 300{,}000$ (the HO's house value), we define a maximum value of harm H_+ equal to 95% of the house value, i.e., 285,000. The IFM is able to protect from the 50% lowest-severity harms (when effective), i.e., the IFM protects against harms equal to or lower than $\hat{h} = 142{,}500$. Setting $\sigma = 0.5$ corresponds to a risk-averse individual, having preferences similar to an individual endowed with a square root utility function (DARA, and prudent individual - $U''' > 0$). We also choose a conservative value for the effectiveness of the IFM ($s = 0.5$). Finally, for our baseline scenario, we choose a roughly competitive insurance loading factor: $\lambda = 0.05$. Given these values, we obtain the following result.

Proposition 2: In the case of a risk-averse HO, HO always asks for IFM and takes out a partial insurance contract from INS. Furthermore, the combination of a successfully implemented IFM and partial insurance allows HO to potentially benefit from full coverage (if the IFM is effective), which provides her with the highest level of utility.

The results are reported in Figures 5 and 6.

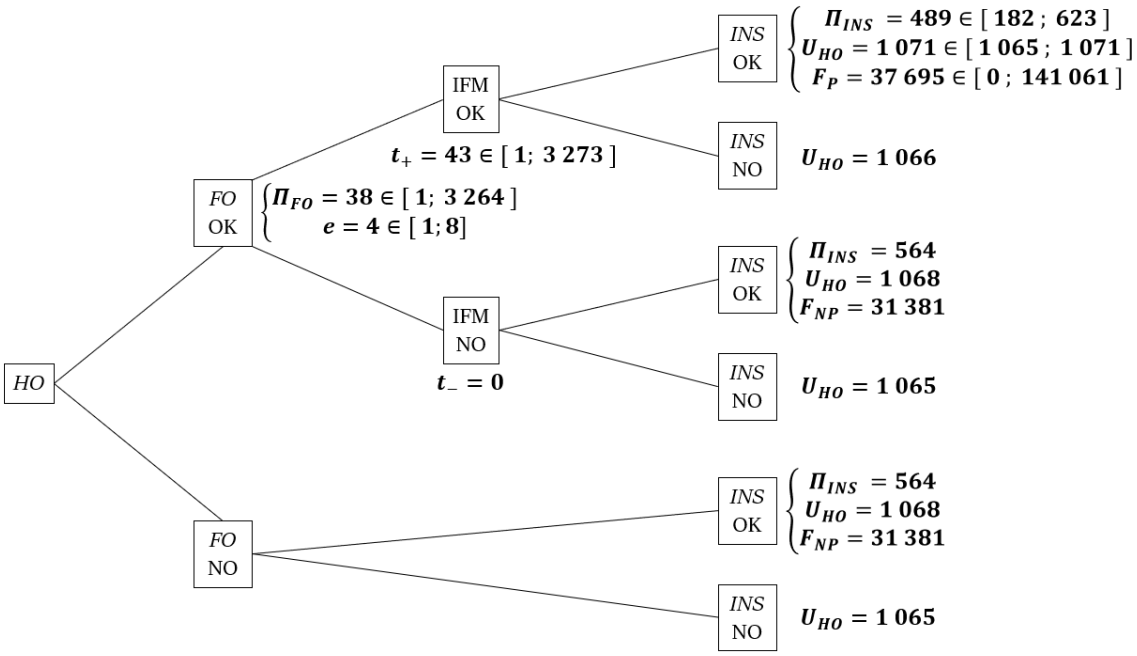


Figure 5. *Equilibrium values of the numerical simulations based on a representation of HO's choices in terms of IFM and insurance with asking (OK) or not (NO) FO to get an IFM, the success (OK) or not (NO) of the setting up of the IFM, and taking out (OK) or not (NO) an insurance contract from INS. Viability ranges are also indicated in the case of global contract (IFM and insurance).*

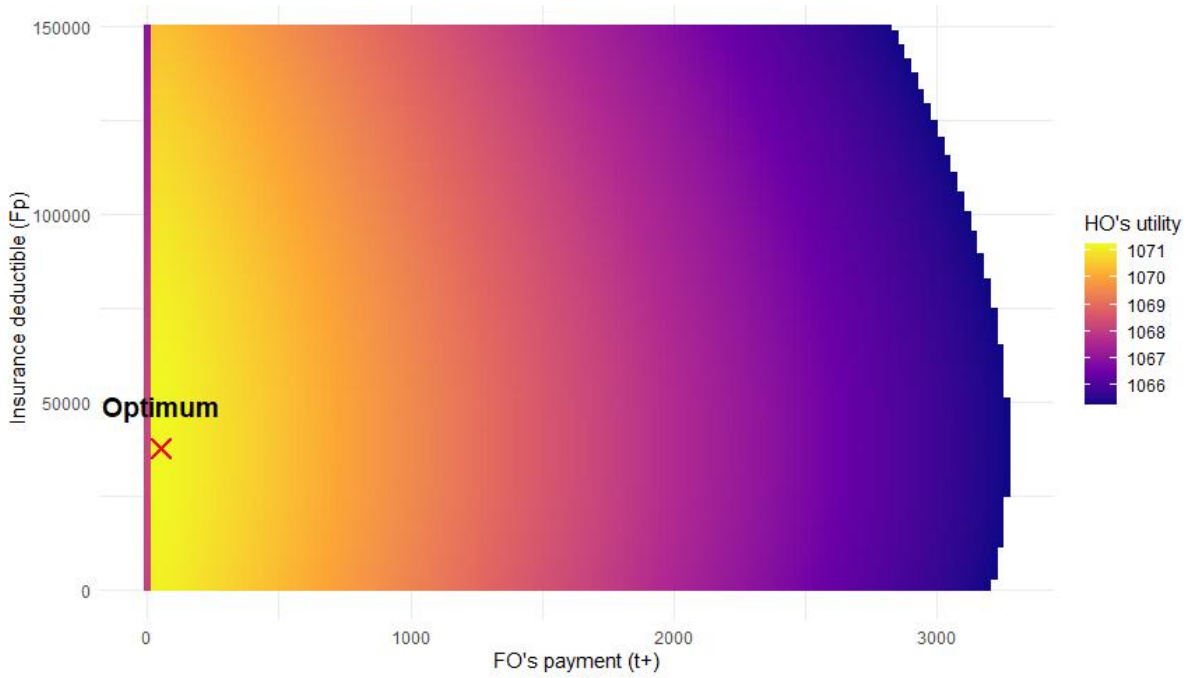


Figure 6. *Viable contractual space for the joint IFM-insurance contract and optimal equilibrium point maximizing utility subject to agents' participation constraints. The colored area represents the set of* (t_+, F_P) *combinations for which the participation constraints of all three agents (HO, INS, FO) are strictly satisfied. The red cross indicates the first-best optimal contract maximizing HO's expected utility (parameters:* $\sigma = 0.5$, $\lambda = 0.05$*).*

We ran a sensitivity analysis (see Supplementary material, Section E), particularly testing a scenario with a non-actuarial insurance loading factor of $\lambda = 0.25$, which reflects standard home insurances[19]. Results are depicted in Figure 7.

[19] See France Assureurs (2024), p. 15. Between 2020 and 2024, loading factors for home insurance lie, on average, between 0.241 and 0.255. We chose a conservative value of 0.25.

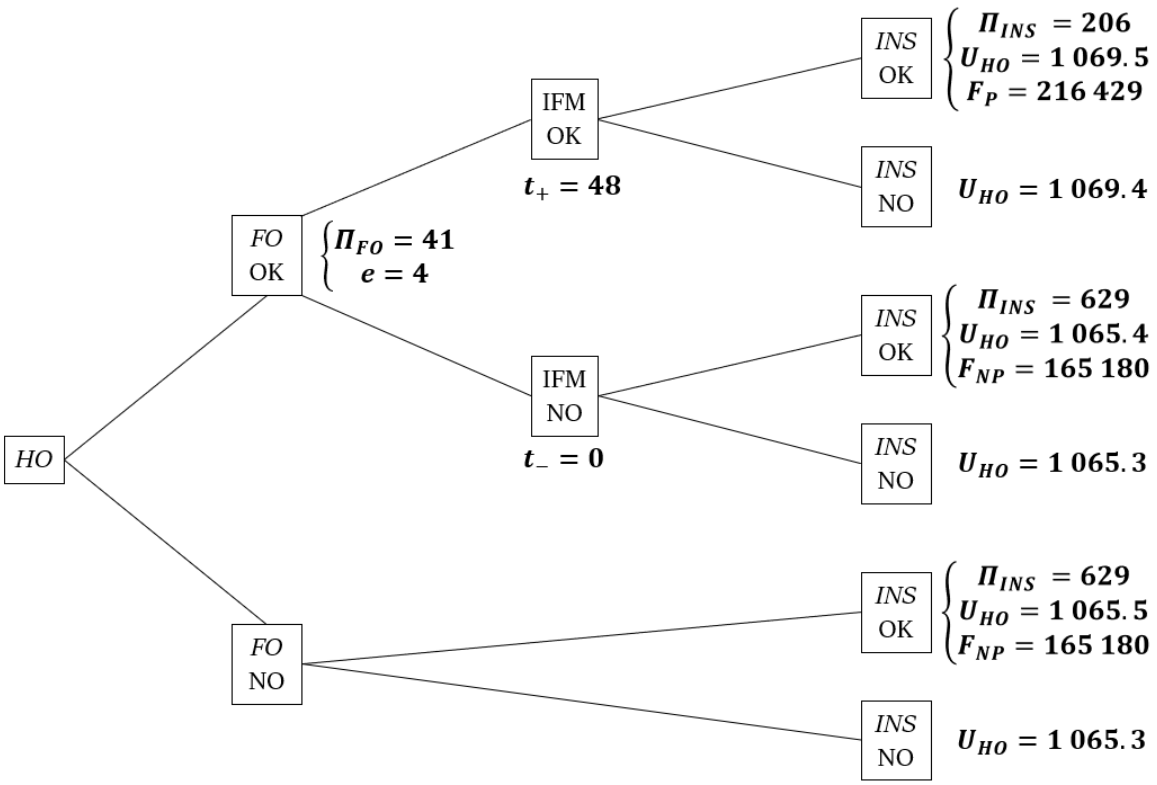


Figure 7. *Equilibrium values of the numerical simulations based on a representation of HO's choices in terms of IFM and insurance with asking (OK) or not (NO) FO to get an IFM, the success (OK) or not (NO) of the setting up of the IFM, and taking out (OK) or not (NO) an insurance contract from INS (λ =0.25)*[20].

Despite the increase in the loading factor, insurance remains appealing to HO. Moreover, the complementarity between IFM and insurance still holds, providing HO with the highest utility level. However, full coverage is no longer achieved: the IFM protects HO against harms equal to or lower than 142,500 (i.e., the value of $\hat{h}$), while insurance covers harms higher than 216,429. Thus, harms with magnitudes lying in the range [142,500; 216,429] remain borne by HO.

[20] Note that in the decision tree figures (Figures 5 and 7), the optimal effort *e* is rounded to the nearest integer for readability. Following Eq. 11, the exact mathematical values are $e = 3.76$ for $t_+ = 43$ ($\lambda = 0.05$) and $e = 3.87$ for $t_+ = 48$ ($\lambda = 0.25$). This confirms that as financial insurance becomes more expensive, the homeowner marginally increases the payment t_+ to induce a higher effort in implementing the natural protection.

We observe that Proposition 2 is robust to non-actuarial insurance: for a risk-averse HO, the joint provision of Nature-Based Solutions (NBS/IFM) and financial insurance yields a strict Pareto improvement over standalone strategies, regardless of whether the insurance is actuarial.

The perfect complementarity of risk coverage

Our results show that natural insurance (IFM) and financial insurance are complementary, thereby updating classical insurance theory (e.g., Mossin, 1968) in the context of ecosystem services. Without IFM, an agent facing an actuarially unfair premium ($\lambda > 0$) would naturally opt for partial insurance. However, introducing IFM acts as a primary filter, eliminating low-severity losses (harms from 0 to $\hat{h}$).

Consequently, the risk pool transferred to INS is substantially "cleaned", by eliminating these events. This risk reduction reduces the expected harm and, consequently, the insurance premium. INS can then offer coverage for the residual risk (the "tail" of the distribution, from F_P to H_+) at a more competitive price. Ultimately, since IFM prevents small harms and the insurance covers large harms, HO achieved full coverage (when the IFM is effective) at the cost of partial insurance. This combination successfully resolves the traditional trade-off highlighted by Mossin (1968) between risk smoothing and loading factor costs, leaving HO better off than with standalone partial insurance.

Optimal contract specifications and wealth distribution

The optimal contract design demonstrates a clear synergy between natural and financial insurances to maximize HO's utility. Figures 5 and 6 detail the optimal values for the decision variables and the resulting welfare distribution (for $\lambda = 0.05$).

For illustrative purposes, consider the case of $\lambda = 0.05$.[21] The optimization yields an expected utility for HO of 1071, which strictly dominates both the "do nothing" scenario (U_{HO}=1065), the "insurance alone" scenario (U_{HO}= 1068) and the “IFM alone” scenario (U_{HO}= 1066). This result mathematically demonstrates that IFM and insurance are complementary tools: their combined use provides the HO with higher utility than either alone.

FO extracts a positive expected profit ($\pi_{FO} = 38$) driven by a conditional payment ($t_+ = 43$) which leads her to induce a strictly positive effort ($e = 4$). Although INS still earns a positive expected profit ($\pi_{INS} = 489$), this profit is lower than in a scenario where insurance is used alone. In the combined framework, the decrease in expected harm - thanks to the IFM - leads to a lower demand for insurance from HO, pushing the insurer to reduce the level of premium[22] through an increase in the deductible ($F_{NP} > F_P$).

Consequently, while HO and FO clearly benefit from the combined use of IFM and insurance, INS may lack incentives to proactively promote such a hybrid risk management strategy, as it reduces its expected profit.

The safety net mechanism: what happens when IFM fails?

A critical feature of our model is the explicit integration of the IFM failure state (probability 1 - *q(e)*). Contrary to concerns that a failure in natural protection might leave the HO entirely exposed, our results show that the insurance market adapts robustly.

In the event of IFM failure, HO reverts to a fall-back insurance contract with a specific deductible ($F_P = 31{,}381$). Since the natural risk mitigation is not working, INS faces the full magnitude of the gross risk ($\widetilde{R_+}$). Given that the INS's profit is a proportional margin (λ) of the

[21] Results are qualitatively the same for $\lambda = 0.25$.

[22] In the case where insurance (alone) is demanded, the premium rises up to 11849. In the case where HO asks for IFM, the insurance premium equals 11266 when the IFM fails and 10255 when IFM is successfully set up.

expected indemnification, its profit structurally increases in this failure state (reaching $\pi_{INS} = 564$).

From an economic perspective, this fall-back mechanism acts as a crucial safety net for HO. Although HO experiences a utility drop compared to the successful IFM state (since the transfer t_- was a sunk cost that failed to provide physical protection), the utility remains bounded and exceeds the "do nothing" baseline. The certainty of this *ex-post* financial safety net is precisely what makes the initial *ex-ante* investment in the IFM rational for a risk-averse HO.

4. Extension: implementation of a first-best prevention policy

In this section, we propose to develop a more normative analysis, by considering our problem from a social point of view.

Since a gravitational harm can be managed by the (costly) implementation of an IFM by a FO, it is possible to define a socially optimal level of effort, denoted e^{sw}, that optimizes this social problem. Moreover, to the extent that implementing an IFM may also benefit other ecosystem users, social optimality requires considering the positive externalities provided by the IFM.

As usually done in the literature, the socially optimal level of effort aims at maximizing a social welfare function, which gathers all costs and benefits from IFM for society as a whole. Acknowledging that forests also provide other services to third parties (e.g., carbon storage, recreation, conservation of biodiversity (Buttoud, 2000; Führer, 2000)), we define $\tilde{A}$ as the collateral harms hurting third parties/other ecosystem users when a gravitational hazard occurs. As the magnitude of harm suffered by HO depends on the (in)effectiveness of the IFM in set up ($\widetilde{R_+}$ or $\widetilde{R_-}$), the same applies to collateral harms: let $\widetilde{A_+}$ be the random variable representing the magnitude of collateral harms when the IFM is ineffective, and $\widetilde{A_-}$ be the random variable

representing the magnitude of collateral harms when the IFM is effective, with $0 < E(\widetilde{A_-}) < E(\widetilde{A_+})$.

As a result, the social planner derives e^{sw} to maximize the social welfare (SW) which is:

$$SW(e) = q(e).\begin{bmatrix} s.\left[(1-p).U_{HO}(w) + p.U_{HO}\left(w - E\left(\widetilde{R_-}\right)\right) - p.E\left(\widetilde{A_-}\right)\right] \\ +(1-s).\left[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+})) - p.E(\widetilde{A_+})\right] \end{bmatrix} + \left(1-q(e)\right).\left[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+})) - p.E(\widetilde{A_+})\right] - c(e). \quad (12)$$

With $c(e)$ the cost of making effort e, and U_{HO} the HO's utility function (with $U'_{HO} > 0, U''_{HO} \leq 0$).

The socially optimal level of effort e^{sw} satisfies:

$$\frac{\delta SW(e)}{\delta e} = 0$$

$$\Leftrightarrow q'(e^{sw}).s.\begin{bmatrix}\left[(1-p).U_{HO}(w) + p.U_{HO}\left(w - E\left(\widetilde{R_-}\right)\right) - p.E\left(\widetilde{A_-}\right)\right] \\ -\left[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+}) - p.E(\widetilde{A_+})\right]\end{bmatrix} = c'(e^{sw}). \quad (13)$$

In other words, the socially optimal effort is the effort whose marginal social benefit equals the marginal FO's cost. The marginal social benefit lies in the increased probability of benefiting from a reduced risk ($\widetilde{R_-}$ and $\widetilde{A_-}$) instead of facing a high risk ($\widetilde{R_+}$ and $\widetilde{A_+}$) in case of a gravitational hazard occurring (p).

Keeping the specifications defined above (i.e., $c(e) = c.e$ and $q(e) = 1 - exp^{-e}$), the socially optimal level of effort in implementing an IFM is (proof in the Supplementary material, Section F):

$$e^{sw} = \ln\left(\frac{p.s.(\Delta U + \Delta A)}{c}\right) \quad (14)$$

with $\Delta U = U_{HO}\left(w - E(\widetilde{R_-})\right) - U_{HO}\left(w - E(\widetilde{R_+})\right) > 0$ and $\Delta A = E(\widetilde{A_+}) - E(\widetilde{A_-}) > 0$.

Note that the higher the cost, the lower e^{sw}, and the higher the social benefit (i.e., a private benefit ΔU_{HO} and a positive externality ΔA), the higher e^{sw}.

Comparing e^{sw}(defined by (14)) to e^* (defined by (11)), we can see that the social and private benefits differ in two ways: the private benefit driving e^* equals the differential in payments ($\Delta t = t_+ - t_-$) defined in the private contract linking HO to FO, while the socially optimal level e^{sw} is driven by the utility perceived by HO in reducing the level of risk (ΔU_{HO}) and the decrease in the risk of collateral harms (ΔA).

Many public policy tools could facilitate the implementation of this socially optimal level e^{sw}, such as stringent standards or subsidies. Before resorting to such tools, we examine whether insurance contracts could fulfill this task. Said differently, could the public agent ask INS to design a contract that ultimately induces the socially optimal level of effort e^{sw}?

Again, we have to run numerical simulations. Starting from the case of an almost actuarial insurance developed above (i.e., $\lambda = 0.05$) and considering no externalities in a first step (i.e. $E(\widetilde{A_+}) = E(\widetilde{A_-}) = 0$), we find:

$$e^{sw} = \ln\left(\frac{p.s.(\Delta U)}{c}\right) = 7{,}5.$$

This socially optimal level of effort can be implemented through offering HO an insurance contract characterized by $(F_{NP} = 62{,}255;\ F_P = 31{,}381)$, i.e., by increasing the level of deductible in a way to increase the HO's benefit from an effective IFM.

In that case, the combined use of insurance and IFM is the best situation for HO, and e^{sw} is implemented while satisfying all participation constraints. However, while HO expected utility is unchanged, profits for both INS and FO are lower than in the case of the free-market outcome developed above.[23] Moreover, the nature of the risk has changed: the implementation of a

[23] Profit of INS fall to 407 in case of success in implementing the IFM, and 435 in case of failure. Profit of FO falls to 41, despite a higher payment in case of success in implementing the IFM ($t_+ = 46.2$). This conditional payment provides him with (high) incentives to exert the effort e^{sw}, which is costlier to implement than e^*.

higher level of effort e can be seen as a higher effort in prevention, increasing the likelihood of implementing an IFM and, thus, reducing the expected harm. Because of the decrease in INS's profit relative to the free-market contract, it may thus be necessary to subsidize INS for the loss suffered while offering this contract implementing e^{sw}.

However, we can also observe that the level of e^{sw} is relatively high, nearly binding the limit of the feasible contracting space (the maximum value of e which can be implemented by contract between HO and FO is equal to 8 in this case; see Figure 5). As a consequence, trying to implement a socially optimal level of effort that considers the decrease in negative externalities (i.e., $\Delta A = E(\widetilde{A_+}) - E(\widetilde{A_-}) > 0$) seems impossible through the sole use of private contracts. Considering externalities would raise e^{sw} to an even higher level, thereby moving outside from our feasible contracting space. It results that the implementation of such a socially optimal effort, considering both the risk reduction for HO and also for collateral harms, would require the use of subsidies to help HO in defining the adequate incentive contract.

Proposition 3: Inducing a socially optimal level of effort (in trying to set up the IFM) cannot be achieved without public subsidies.

4. DISCUSSION AND CONCLUSION

In this paper, we develop a principal-agent model to explore how a house owner can optimally combine a nature-based solution (NBS) and financial insurance to protect against gravitational hazards threatening her house. The nature-based solution consists in delegating the implementation of an Improved Forest Management (IFM) to a forest owner, enabling the forest to mitigate some gravitational hazards. In this regard, the forest increases local resilience

to natural disasters. However, the effectiveness of such a solution is inherently uncertain, depending on forest's ecological functioning and resilience to natural hazards (Olschewski et al., 2012).

We show that, despite the uncertainty surrounding the effectiveness of natural protection, the optimal private risk management strategy requires combining financial insurance with natural protection. This result refines the standard literature (e.g., Baumgärtner, 2007), which often emphasizes the substitutability between biodiversity protection and insurance rather than their complementarity. The combined use of natural protection and insurance provides the individual with the highest level of utility: the natural protection filters low-severity harms, enabling the house owner to obtain insurance that covers only the highest levels of harms at a lower cost. However, compared to the "insurance-only" solution, the insurer earns lower profits and is thus unlikely to promote such a hybrid solution naturally.

Recognizing that natural protection also generates positive externalities—by reducing the risk of gravitational hazards that cause collateral harms to biodiversity and third parties—we extend our analysis to a normative framework. Accounting for these externalities requires greater efforts in trying to implement an IFM. However, we show that these socially optimal efforts cannot be achieved through private contracts alone between economic agents, thereby necessitating public support, such as subsidies for the insurance sector.

In the Swiss context, cantons and the federal government currently pay forest owners directly for basic protection measures (Krumm et al., 2011; FOEN, 2021). Forest owners receive a 'payment for action' to implement natural protection, regardless the actual success of this setting up. In contrast, our results highlight the importance of making the payment dependent on the effective setting up of the natural protection to provide optimal incentives for forest owners, emphasizing that insurance contracts are a powerful incentive tool to trigger these private payments.

Unterberger & Olschewski (2021) show through a choice experiment in seven Swiss municipalities, that most inhabitants are willing to pay for natural protection services provided by ecosystems, that is, to reduce hazard risks and extend protection to traffic infrastructure through an annual payment. Our results suggest that this preference for natural protection could be fostered by combining it with financial insurance, pushing house owners to design incentive contracts with forest owners that offer high payoffs for successful IFM implementation. This result aligns with Brunette et al. (2017), who show that forest insurance can incentivize adaptation efforts to climate change (and/or the implementation of IFM in our case).

Several extensions could enrich our analysis. First, in this paper, we deal with private insurers, although some Swiss insurers are public companies. It could be interesting to investigate how optimal contracts might differ for a public insurer that aims to cover its costs (or has a low loading rate) rather than maximize profit. Furthermore, given our normative finding that reaching the socially optimal level of natural protection requires subsidizing the insurer, implementing such a policy would be structurally and politically much easier with a public insurer, as its mandate aligns more closely with maximizing social welfare rather than extracting private profit. Integrating the various cantonal insurance systems in the housing sector (obligatory vs. non-obligatory) and the different ownership structures in the forest sector (public vs. private) could further extend our analysis.

Second, we assumed here a uniform probability of harm occurrence p. It would be interesting to test how inhabitants' decision are affected under the more realistic assumption that low-severity harms occur more frequently than high-severity ones. Assuming further that low-severity harms are covered by natural protection, this distribution of p could further encourage inhabitants to adopt natural protection in addition to financial insurance, thereby reinforcing our results.

In conclusion, incorporating natural protection into insurance contracts can encourage the provision of such protection by forest owners, which is beneficial from both a private and societal point of view. Operationalizing the insurance value of ecosystems can therefore improve decision making in natural hazard management and increase adoption rate among inhabitants.

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Author contributions

S.B.A. and J.J. developed the theoretical model and wrote the main manuscript text. S.B.A. performed the analytical and numerical calculations and prepared the figures. R.O., R.Y. and M.H. coordinated the research project, supervised the work, and validated the theoretical results and policy implications. All authors reviewed and approved the final manuscript.

Competing Interests

The authors have no relevant financial or non-financial interests to disclose.

Data Availability

Data sharing is not applicable to this article as no external empirical datasets were generated or analysed during the current study. The R code used for the numerical simulations is available from the corresponding author on reasonable request.

SUPPLEMENTARY MATERIAL

A. List of parameters and variables

$\tilde{A}$	Collateral harms for other ecosystem users than HO.
$\widetilde{A_+}$	Collateral harms in case of risk $\widetilde{R_+}$.
$\widetilde{A_-}$	Collateral harms in case of risk $\widetilde{R_-}$.
$c(e)$	Cost of FO's effort.
e	Effort of FO to provide IFM.
e_+	Maximum of FO's effort.
e_-	Low FO's effort.
$E(\widetilde{R_+})$	Expected harm from risk $\widetilde{R_+}$.
$E(\widetilde{R_-})$	Expected harm from risk $\widetilde{R_-}$.
$E(I)$	Expected compensation from insurance contract.
$E(I\|\widetilde{R_+})$	Expected compensation in case of risk $\widetilde{R_+}$.
$E(I\|\widetilde{R_-})$	Expected compensation in case of risk $\widetilde{R_-}$.
F	Deductible fixed by INS.
F_{NP}	Deductible in case of failure in setting up the IFM.
F_P	Deductible in case of success in setting up the IFM.
h	Magnitude of harm.
$\hat{h}$	Threshold below which magnitude of harm are eliminated by IFM.
H_+	Maximum value of harm.
I	Financial compensation of HO by INS in case of damage.
λ	Loading rate for covering INS's management costs.
p	Probability of harm.
Π_{FO}	FO's profit function.

Π_{INS}	INS's profit function.
$q(e)$	Probability of FO's success in setting up an IFM.
$\widetilde{R_+}$	Gross risk, i.e., magnitude of harm before the implementation of any additional natural protection.
$\widetilde{R_-}$	Net risk in case of success in providing an effective IFM to reduce harm.
s	Probability of success of the IFM to reduce the risk from $\widetilde{R_+}$ to $\widetilde{R_-}$.
σ	Relative risk aversion coefficient.
SW	Social welfare.
t_+	Payment of HO to FO in case of success in setting up the IFM.
t_-	Payment of HO to FO in case of failure in setting up the IFM.
U_{HO}	HO's utility function.
w	HO's wealth, value of HO's house.
$\underline{w^{FO}}$	FO's opportunity cost.

B. The optimal level of FO's effort

From (2.a), we can deduce the FO's profit is:

$$E(\Pi_{FO}) = q(\mathrm{e}).t_+ + \big(1 - q(e)\big).t_- - c(e),$$

$$\text{with } c(e) = c.e \text{ and } q(e) = 1 - exp^{-e}.$$

The first order condition defining the optimal level of FO's effort e^* is:

$$\frac{d\Pi_{FO}}{de} = 0 \Leftrightarrow exp^{-e^*}.[t_+ - t_-] - c = 0 \Leftrightarrow e^* = -\ln\left[\frac{c}{[t_+ - t_-]}\right]$$

$$e^* = \ln\left(\frac{\Delta t}{c}\right) \text{ with } \Delta t = t_+ - t_-.$$

C. Specification of the different parameters and problems

Considering (with no loss of generality) that the level of harm h is uniformly distributed within $[0; H_+]$ and with $H_+ < w$, the expected harm from $\widetilde{R_+}$ suffered by HO is:

$$E\big(\widetilde{R_+}\big) = \int_0^{H_+} h\, f(h)\, dh = \frac{H_+}{2}.$$

Hence the resulting expected damage from $\widetilde{R_-}$ is:

$$E\big(\widetilde{R_-}\big) = \int_0^{\hat{h}} 0\, f(h)dh + \int_{\hat{h}}^{H_+} h\ f(h)dh = \frac{1}{H_+}.\left[\frac{(H_+)^2}{2} - \frac{(\hat{h})^2}{2}\right] = \frac{H_+}{2} - \frac{(\hat{h})^2}{2H_+}.$$

In the case where the IFM is not effective (risk $\widetilde{R_+}$), the compensation I of HO is (Eq. 3):

$$E\big(I|\widetilde{R_+}\big) = \int_0^F 0 + \int_F^{H_+} (h - F)f(h)dh = \frac{1}{H_+}.\left[\left(\frac{(H_+)^2}{2} - F.H_+\right) - \left(\frac{(F)^2}{2} - (F)^2\right)\right]$$

$$= \frac{H_+}{2} - F + \frac{(F)^2}{2H_+},$$

And, in the case where the IFM is effective (risk $\widetilde{R_-}$), the compensation I is (Eq. 4):

$$E\left(I|\widetilde{R_-}\right) = \int_0^{\max(F,\hat{h})} 0 \; f(h)dh + \int_{\max(F,\hat{h})}^{H_+} (h-F)f(h)dh$$

$$= \frac{1}{H_+}.\left[\left(\frac{(H_+)^2}{2} - F.H_+\right) - \left(\frac{(\max(F,\hat{h}))^2}{2} - F.\max(F,\hat{h})\right)\right]$$

$$= \frac{H_+}{2} - F - \max(F,\hat{h}).\left(\frac{\max(F,\hat{h})}{2H_+} - \frac{F}{H_+}\right).$$

More precisely, depending on the position of F relative to $\hat{h}$:

$$E\left(I|\widetilde{R_-}\right) =$$

$$\begin{cases} \frac{H_+}{2} - F - F.\left(\frac{F}{2H_+} - \frac{F}{H_+}\right) = \frac{H_+}{2} - F + \frac{(F)^2}{2H_+} \text{ if } F > \hat{h} \; (i.e. \max(F,\hat{h}) = F), \\ \frac{H_+}{2} - F - \hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F}{H_+}\right) \text{ if } F < \hat{h} \; (i.e. \max(F,\hat{h}) = \hat{h}). \end{cases}$$

Assuming, first, that all agents (HO, FO, INS) are risk neutral, we find the following programs. First, in case of failure in the setting up of the IFM, the expected profit of INS is written as follows (Eq. 6):

$$E(\Pi_{INS}) = p.E\left(I|\widetilde{R_+}\right).\lambda = p.\left[\frac{H_+}{2} - F_{NP} + \frac{(F_{NP})^2}{2H_+}\right].\lambda,$$

given the participation constraint (i.e., HO has a higher utility with insurance than without insurance) (Eq. 7):

$$\begin{gathered} (1-p).U_{HO}\left(w - p.E\left(I|\widetilde{R_+}\right).(1+\lambda) - t^-\right) \\ +p.U_{HO}\left(w - E\left(\widetilde{R_+}\right) + E\left(I|\widetilde{R_+}\right) - p.E\left(I|\widetilde{R_+}\right).(1+\lambda) - t^-\right) \geq \\ (1-p).U_{HO}(w - t^-) + p.U_{HO}\left(w - E\left(\widetilde{R_+}\right) - t^-\right) \end{gathered}$$

$$\Leftrightarrow (1-p).\left(w-p.\left[\frac{H_+}{2}-F_{NP}+\frac{(F_{NP})^2}{2H_+}\right].(1+\lambda)-t^-\right)$$
$$+p.\left(w-\frac{H_+}{2}+\left[\frac{H_+}{2}-F_{NP}+\frac{(F_{NP})^2}{2H_+}\right]-p.\left[\frac{H_+}{2}-F_{NP}+\frac{(F_{NP})^2}{2H_+}\right].(1+\lambda)-t^-\right)\geq$$
$$(1-p).(w-t^-)+p.\left(w-\frac{H_+}{2}-t^-\right)$$

$$\Leftrightarrow w-p.\left[\frac{H_+}{2}-F_{NP}+\frac{(F_{NP})^2}{2H_+}\right].(1+\lambda)-t^-+p.\left(\frac{(F_{NP})^2}{2H_+}-F_{NP}\right)\geq w-t^--p.\frac{H_+}{2}.$$

In the case of success in setting up the IFM, recall that the value of $E\left(I|\widetilde{R_-}\right)$ differs depending on the position of F_P relative to $\hat{h}$. In the case where $F_P > \hat{h}$, the expected profit of INS is (Eq. 8):

$$\mathrm{E}(\Pi_{INS}) = p.\left[s.E\left(I|\widetilde{R_-}\right)+(1-s).E\left(I|\widetilde{R_+}\right)\right].\lambda$$

$$=p.\left[s.\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]\right].\lambda=p.\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].\lambda\,,$$

given the participation constraint (Eq. 9):

$$(1-p).U_{HO}\left(w-p.\left[s.E\left(I|\widetilde{R_-}\right)+(1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda)-t^+\right)$$
$$+p.s.U_{HO}\left(w-p.\left[s.E\left(I|\widetilde{R_-}\right)+(1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda)-\left[E\left(\widetilde{R_-}\right)-E\left(I|\widetilde{R_-}\right)\right]-t^+\right)$$
$$+p.(1-s).U_{HO}\left(w-p.\left[s.E\left(I|\widetilde{R_-}\right)+(1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda)-\left[E\left(\widetilde{R_+}\right)-E\left(I|\widetilde{R_+}\right)\right]-t^+\right)\geq$$
$$(1-p).U_{HO}(w-t^+)+p.U_{HO}\left(w-\left[s.E\left(\widetilde{R_-}\right)+(1-s).E\left(\widetilde{R_+}\right)\right]-t^+\right)$$

$$\Leftrightarrow (1-p).\left(w-p.\begin{bmatrix} s.\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right] \\ +(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]\end{bmatrix}.(1+\lambda)-t^+\right)$$

$$+p.s.\left(\begin{matrix} w-p.\left[s.\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]\right].(1+\lambda) \\ -\left[\left[\frac{H_+}{2}-\frac{(\hat{h})^2}{2H_+}\right]-\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]\right]-t^+\end{matrix}\right)$$

$$+p.(1-s).\left(\begin{matrix} w-p.\left[s.\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]\right].(1+\lambda) \\ -\left[\frac{H_+}{2}-\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]\right]-t^+\end{matrix}\right)\geq$$

$$(1-p).(w-t^+)+p.\left(w-\left[s.\left[\frac{H_+}{2}-\frac{(\hat{h})^2}{2H_+}\right]+(1-s).\frac{H_+}{2}\right]-t^+\right)$$

$$\Leftrightarrow w-p.\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].(1+\lambda)-t^++\frac{(F_P)^2}{2H_+}-F_P+p.s.\frac{(\hat{h})^2}{2H_+}$$

$$\geq w-t^++p.\left(s.\frac{(\hat{h})^2}{2H_+}-\frac{H_+}{2}\right).$$

In case where $F_P < \hat{h}$, the INS's profit is (Eq. 8):

$$\mathrm{E}(\Pi_{INS})=p.\left[s.E\left(I|\widetilde{R_-}\right)+(1-s).E\left(I|\widetilde{R_+}\right)\right].\lambda$$

$$=p.\left[s.\left[\frac{H_+}{2}-F_P+\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]\right]+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].\lambda$$

$$=p.\left[\frac{H_+}{2}-F_P+s.\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]+(1-s).\left[\frac{(F_P)^2}{2H_+}\right]\right].\lambda\,.$$

given the participation constraint (Eq. 9):

$$(1-p).U_{HO}\left(w-p.\left[s.E\left(I|\widetilde{R_-}\right)+(1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda)-t^+\right)$$
$$+p.s.U_{HO}\left(w-p.\left[s.E\left(I|\widetilde{R_-}\right)+(1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda)-\left[E\left(\widetilde{R_-}\right)-E\left(I|\widetilde{R_-}\right)\right]-t^+\right)$$
$$+p.(1-s).U_{HO}\left(w-p.\left[s.E\left(I|\widetilde{R_-}\right)+(1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda)-\left[E\left(\widetilde{R_+}\right)-E\left(I|\widetilde{R_+}\right)\right]-t^+\right)\geq$$
$$(1-p).U_{HO}(w-t^+)+p.U_{HO}\left(w-\left[s.E\left(\widetilde{R_-}\right)+(1-s).E\left(\widetilde{R_+}\right)\right]-t^+\right)$$

$$\Leftrightarrow(1-p).\left(w-p.\left[\begin{array}{c}s.\left[\frac{H_+}{2}-F_P+\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]\right]\\+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]\end{array}\right].(1+\lambda)-t^+\right)$$

$$+p.s.\left(\begin{array}{c}w-p.\left[s.\left[\frac{H_+}{2}-F_P+\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]\right]+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].(1+\lambda)\\-\left[\left[\frac{H_+}{2}-\frac{(\hat{h})^2}{2H_+}\right]-\left[\frac{H_+}{2}-F_P+\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]\right]\right]-t^+\end{array}\right)$$

$$+p.(1-s).\left(\begin{array}{c}w-p.\left[s.\left[\frac{H_+}{2}-F_P+\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]\right]+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].(1+\lambda)\\-\left[\frac{H_+}{2}-\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]\right]-t^+\end{array}\right)\geq$$

$$(1-p).(w-t^+)+p.\left(w-\left[s.\left[\frac{H_+}{2}-\frac{(\hat{h})^2}{2H_+}\right]+(1-s).\frac{H_+}{2}\right]-t^+\right)$$

$$\Leftrightarrow w-p.\left[\frac{H_+}{2}-F_P+s.\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]+(1-s).\left[\frac{(F_P)^2}{2H_+}\right]\right].(1+\lambda)-t^+-p.F_P$$

$$+p.s.\left[\frac{(\hat{h})^2}{2H_+}+\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]+p.(1-s).\frac{(F_P)^2}{2H_+}\geq$$

$$w-t^++p.\left(s.\frac{(\hat{h})^2}{2H_+}-\frac{H_+}{2}\right).$$

As a result, the HO's expected utility when looking at asking both for IFM and insurance is written as follows (Eq. 10):

$$E(U_{HO})$$

$$= q(e).\begin{bmatrix} (1-p).U_{HO}\left(w - p.\left[s.E\left(I|\widetilde{R_-}\right) + (1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda) - t^+\right) \\ +p.s.U_{HO}\left(w - p.\left[s.E\left(I|\widetilde{R_-}\right) + (1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda) - \left[E\left(\widetilde{R_-}\right) - E\left(I|\widetilde{R_-}\right)\right] - t^+\right) \\ +p.(1-s).U_{HO}\left(w - p.\left[s.E\left(I|\widetilde{R_-}\right) + (1-s).E\left(I|\widetilde{R_+}\right)\right].(1+\lambda) - \left[E\left(\widetilde{R_+}\right) - E\left(I|\widetilde{R_+}\right)\right] - t^+\right) \end{bmatrix}$$

$$+\left(1-q(e)\right).\begin{bmatrix} (1-p).U_{HO}\left(w - p.E\left(I|\widetilde{R_+}\right).(1+\lambda) - t^-\right) \\ +p.U_{HO}\left(w - E\left(\widetilde{R_+}\right) + E\left(I|\widetilde{R_+}\right) - p.E\left(I|\widetilde{R_+}\right).(1+\lambda) - t^-\right) \end{bmatrix},$$

which results in, in case where $F_P > \hat{h}$:

$$E(U_{HO}) =$$

$$q(e).\begin{bmatrix} (1-p).\left(w - p.\begin{bmatrix} s.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right] \\ +(1-s).\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right] \end{bmatrix}.(1+\lambda) - t^+\right) \\ +p.s.\begin{pmatrix} w - p.\left[s.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right] + (1-s).\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right]\right].(1+\lambda) \\ -\left[\left[\frac{H_+}{2} - \frac{(\hat{h})^2}{2H_+}\right] - \left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right]\right] - t^+ \end{pmatrix} \\ +p.(1-s).\left(w - p.\left[s.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right] + (1-s).\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right]\right].(1+\lambda) \right. \\ \left. -\left[\frac{H_+}{2} - \left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right]\right] - t^+\right) \end{bmatrix}$$

$$+\left(1-q(e)\right).\begin{bmatrix} (1-p).\left(w - p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].(1+\lambda) - t^-\right) \\ +p.\left(w - \frac{H_+}{2} + \left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right] - p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].(1+\lambda) - t^-\right) \end{bmatrix}$$

$$E(U_{HO}) = q(e).\left[w - p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].(1+\lambda) - t^+ + \frac{(F_P)^2}{2H_+} - F_P + p.s.\frac{\left(\hat{h}\right)^2}{2H_+}\right]$$

$$+\left(1-q(e)\right).\left[w - p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].(1+\lambda) - t^- + p.\left[\frac{(F_P)^2}{2H_+} - F_P\right]\right],$$

and in case where $F_P < \hat{h}$:

$$E(U_{HO}) =$$

$$q(e).\left[\begin{array}{l}(1-p).\left(w-p.\left[s.\left[\frac{H_+}{2}-F_P+\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]\right]+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].(1+\lambda)-t^+\right)\right.\\ +p.s.\left(\begin{array}{c}w-p.\left[s.\left[\frac{H_+}{2}-F_P+\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]\right]+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].(1+\lambda)\\ -\left[\left[\frac{H_+}{2}-\frac{(\hat{h})^2}{2H_+}\right]-\left[\frac{H_+}{2}-F_P+\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]\right]\right]-t^+\end{array}\right)\\ \left.+p.(1-s).\left(\begin{array}{c}w-p.\left[s.\left[\frac{H_+}{2}-F_P+\left[\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]\right]+(1-s).\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].(1+\lambda)\\ -\left[\frac{H_+}{2}-\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]\right]-t^+\end{array}\right)\right]\end{array}\right.$$

$$+\big(1-q(e)\big).\left[\begin{array}{c}(1-p).\left(w-p.\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].(1+\lambda)-t^-\right)\\ +p.\left(w-\frac{H_+}{2}+\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right]-p.\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].(1+\lambda)-t^-\right)\end{array}\right]$$

,

$$E(U_{HO}) = q(e).\left[w-p.\left[\frac{H_+}{2}-F_P+s.\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)+(1-s).\left[\frac{(F_P)^2}{2H_+}\right].(1+\lambda)-t^+\right.\right.$$

$$\left.\left.-p.F_P+p.s.\left[\frac{(\hat{h})^2}{2H_+}+\hat{h}.\left(\frac{\hat{h}}{2H_+}-\frac{F_P}{H_+}\right)\right]+p.(1-s).\frac{(F_P)^2}{2H_+}\right]\right.$$

$$+\big(1-q(e)\big).\left[w-p.\left[\frac{H_+}{2}-F_P+\frac{(F_P)^2}{2H_+}\right].(1+\lambda)-t^-+p.\left[\frac{(F_P)^2}{2H_+}-F_P\right]\right].$$

,

D. Results of the backward resolution for a risk-neutral HO

D.1. Resolution of INS's problem (insurance contract)

In case of risk $\widetilde{R_+}$, INS's problem is written as follows:

$$E(\Pi_{INS}) = p.\left[\frac{H_+}{2} - F_{NP} + \frac{(F_{NP})^2}{2H_+}\right].\lambda,$$

given the participation constraint:

$$w - p.\left[\frac{H_+}{2} - F_{NP} + \frac{(F_{NP})^2}{2H_+}\right].(1+\lambda) - t^- + p.\left(\frac{(F_{NP})^2}{2H_+} - F_{NP}\right) \geq w - p.\frac{H_+}{2} - t^-.$$

The Lagrangian is written as follows:

$$\mathcal{L} = p.\left[\frac{H_+}{2} - F_{NP} + \frac{(F_{NP})^2}{2H_+}\right].\lambda - \left[\begin{array}{c} w - p.\left[\frac{H_+}{2} - F_{NP} + \frac{(F_{NP})^2}{2H_+}\right].(1+\lambda) - t^- + p.\left(\frac{(F_{NP})^2}{2H_+} - F_{NP}\right) \\ -w + p.\frac{H_+}{2} + t^- \end{array}\right]$$

$$\mathcal{L} = p.\left[\frac{H_+}{2} - F_{NP} + \frac{(F_{NP})^2}{2H_+}\right].(1+2\lambda) - p.\left(\frac{(F_{NP})^2}{2H_+} - F_{NP}\right) - p.\frac{H_+}{2}$$

$$\mathcal{L} = p.\left[\left[\frac{H_+}{2} - F_{NP} + \frac{(F_{NP})^2}{2H_+}\right].(2\lambda)\right]$$

The first order condition results in:

$$\frac{d\mathcal{L}}{dF_{NP}} = 0 \Leftrightarrow 2\lambda p.\left[\frac{F_{NP}}{H_+} - 1\right] = 0$$

$$F_{NP} = H_+.$$

In case of risk $\widetilde{R_-}$ with $F_P > \hat{h}$, INS's problem is defined such as:

$$\mathrm{E}(\Pi_{INS}) = p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].\lambda\,,$$

given the participation constraint:

$$w - p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].(1+\lambda) - t^+ + \frac{(F_P)^2}{2H_+} - F_P + p.s.\frac{\left(\hat{h}\right)^2}{2H_+} \geq$$

$$w - t^+ + p.\left(s.\frac{\left(\hat{h}\right)^2}{2H_+} - \frac{H_+}{2}\right).$$

The Lagrangian is written as follows:

$$\mathcal{L} = p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].\lambda$$

$$-\left[\begin{array}{c} w - p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].(1+\lambda) - t^+ + \frac{(F_P)^2}{2H_+} - F_P + p.s.\frac{\left(\hat{h}\right)^2}{2H_+} \\ -w + t^+ - p.\left(s.\frac{\left(\hat{h}\right)^2}{2H_+} - \frac{H_+}{2}\right) \end{array}\right]$$

$$\mathcal{L} = p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].(1+2\lambda) - \frac{(F_P)^2}{2H_+} + F_P - p.s.\frac{\left(\hat{h}\right)^2}{2H_+} + p.\left(s.\frac{\left(\hat{h}\right)^2}{2H_+} - \frac{H_+}{2}\right)$$

The first order condition results in:

$$\frac{d\mathcal{L}}{dF_P} = 0 \Leftrightarrow p.(1+2\lambda).\left(\frac{F_P}{H_+} - 1\right) - \frac{F_P}{H_+} + 1 = 0$$

$$F_P = H_+.$$

$$\Pi_{INS} = E\left(I|\widetilde{R_-}\right).\lambda = \left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].\lambda\,,$$

with $F_P < \hat{h}$, the INS's problem is:

$$E(\Pi_{INS}) = p.\left[\frac{H_+}{2} - F_P + s.\left[\hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right)\right] + (1-s).\left[\frac{(F_P)^2}{2H_+}\right]\right].\lambda\,,$$

given the participation constraint:

$$w - p.\left[\frac{H_+}{2} - F_P + s.\left[\hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right)\right] + (1-s).\left[\frac{(F_P)^2}{2H_+}\right]\right].(1+\lambda) - t^+ - p.F_P$$

$$+ p.s.\left[\frac{(\hat{h})^2}{2H_+} + \hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right)\right] + p.(1-s).\frac{(F_P)^2}{2H_+} \geq$$

$$w - t^+ + p.\left(s.\frac{(\hat{h})^2}{2H_+} - \frac{H_+}{2}\right).$$

The Lagrangian is written as follows:

$$\mathcal{L} = p.\left[\frac{H_+}{2} - F_P + s.\left[\hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right)\right] + (1-s).\left[\frac{(F_P)^2}{2H_+}\right]\right].\lambda$$

$$- \left[w - p.\left[\frac{H_+}{2} - F_P + s.\left[\hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right)\right] + (1-s).\left[\frac{(F_P)^2}{2H_+}\right]\right].(1+\lambda) - t^+ - p.F_P\right.$$

$$+ p.s.\left[\frac{(\hat{h})^2}{2H_+} + \hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right)\right] + p.(1-s).\frac{(F_P)^2}{2H_+} - w + t^+$$

$$\left. - p.\left(s.\frac{(\hat{h})^2}{2H_+} - \frac{H_+}{2}\right)\right]$$

$$\mathcal{L} = p.\left[\frac{H_+}{2} - F_P + s.\left[\hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right)\right] + (1-s).\left[\frac{(F_P)^2}{2H_+}\right]\right].(1+2\lambda) + p.F_P$$

$$- p.s.\left[\frac{(\hat{h})^2}{2H_+} + \hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right)\right] - p.(1-s).\frac{(F_P)^2}{2H_+} + p.\left(s.\frac{(\hat{h})^2}{2H_+} - \frac{H_+}{2}\right)$$

$$\mathcal{L} = p.\left[\left[\frac{H_+}{2} - F_P + s.\hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right) + (1-s).\left[\frac{(F_P)^2}{2H_+}\right]\right].(1+2\lambda) + F_P\right.$$

$$\left. - s.\hat{h}.\left(\frac{\hat{h}}{2H_+} - \frac{F_P}{H_+}\right) - (1-s).\frac{(F_P)^2}{2H_+} - \frac{H_+}{2}\right].$$

The first order condition results in:

$$\frac{d\mathcal{L}}{dF_P} = 0$$

$$\Leftrightarrow p.\left[(1+2\lambda).\left((1-s).\frac{F_P}{H_+} - \frac{s.\hat{h}}{H_+} - 1\right) + 1 + \frac{s.\hat{h}}{H_+} - (1-s).\frac{F_P}{H_+}\right] = 0$$

$$\Leftrightarrow p.\lambda.\left[s\left(-1 + \frac{\hat{h}}{H_+}\right) + (1-s)\left(-1 + \frac{F_P}{H_+}\right)\right] < 0$$

Therefore, $F_P = H_+$ is the result maximising INS's profit.

INS's profit will be:

$$\mathrm{E}(\Pi_{INS}) = p.\left[\frac{H_+}{2} - F_P + \frac{(F_P)^2}{2H_+}\right].\lambda \;\Leftrightarrow \Pi_{INS} = p.\left[\frac{H_+}{2} - H_+ + \frac{(H_+)^2}{2H_+}\right].\lambda$$

$$\Pi_{INS} = 0.$$

The final result is thus:

$$F_P = F_{NP} = H_+ \Leftrightarrow F = H_+.$$

D.2. Results in terms of FO's payment (natural protection contract)

INS does not propose an insurance contract. Therefore, the program between FO and HO can be written as follows:

$$E(U_{HO}) = w + q(e).U_{HO}\left(s.\widetilde{R_-} + (1-s).\widetilde{R_+} - t_+\right) + \left(1-q(e)\right).U_{HO}\left(\widetilde{R_+} - t_-\right),$$

$$E(U_{HO}) = q(e).\begin{bmatrix} s.p.\left[U_{HO}\left(w - E\left(\widetilde{R_-}\right) - t_+\right) - U_{HO}\left(w - E\left(\widetilde{R_+}\right)\right)\right] \\ +(1-s.p).\left[U_{HO}\left(w - E\left(\widetilde{R_+}\right) - t_+\right) - U_{HO}\left(w - E\left(\widetilde{R_+}\right)\right)\right] \end{bmatrix}$$

$$+\left(1-q(e)\right).\left[U_{HO}\left(w - E\left(\widetilde{R_+}\right) - t_-\right) - U_{HO}\left(w - E\left(\widetilde{R_+}\right)\right)\right]$$

,

$$\Leftrightarrow E(U_{HO}) = q(e).\begin{bmatrix} s.p.\left[\left(w - E\left(\widetilde{R_-}\right) - t_+\right) - \left(w - E\left(\widetilde{R_+}\right)\right)\right] \\ +(1-s.p).\left[\left(w - E\left(\widetilde{R_+}\right) - t_+\right) - \left(w - E\left(\widetilde{R_+}\right)\right)\right] \end{bmatrix}$$

$$+\left(1-q(e)\right).\left[\left(w - E\left(\widetilde{R_+}\right) - t_-\right) - \left(w - E\left(\widetilde{R_+}\right)\right)\right]$$

,

$$\Leftrightarrow E(U_{HO}) = q(e).\left[s.p.E\left(\widetilde{R_+}\right) - t_+ - s.p.E\left(\widetilde{R_-}\right)\right]$$

$$-\left(1-q(e)\right).t_-.$$

,

given FO's participation constraint:

$$q(e).(t_+ - c(e)) + (1-q(e)).(t_- - c(e)) - \overline{w} \geq 0$$

$$\Leftrightarrow \left[1 - \frac{c}{\Delta t}\right].t^+ + \left[\frac{c}{\Delta t}\right].t^- + c.\left[-ln\left[\frac{c}{\Delta t}\right]\right] - \overline{w} \geq 0.$$

The Lagrangian is written as follows:

$$\mathcal{L} = \left[1 - \frac{c}{\Delta t}\right].\left[s.p.E\left(\widetilde{R_+}\right) - t_+ - s.p.E\left(\widetilde{R_-}\right)\right] - \left[\frac{c}{\Delta t}\right].t_-$$

$$-\left[\left[1 - \frac{c}{\Delta t}\right].t^+ + \left[\frac{c}{\Delta t}\right].t^- + c.\left[-ln\left[\frac{c}{\Delta t}\right]\right] - \overline{w}\right].$$

The first order condition results in:

$$\frac{d\mathcal{L}}{d\Delta t} = 0$$

$$\Leftrightarrow \frac{c}{\Delta t^2}.\left(s.p.E\left(\widetilde{R_+}\right) - t_+ - s.p.E\left(\widetilde{R_-}\right)\right) + \frac{c}{\Delta t^2}.t_- + \frac{c}{\Delta t^2}.t_+ - \frac{c}{\Delta t^2}.t_- - \frac{c}{\Delta t} = 0$$

$$\Leftrightarrow \Delta t = s.p.\left(E\left(\widetilde{R_+}\right) - E\left(\widetilde{R_-}\right)\right) = s.p.\Delta E\left(\tilde{R}\right).$$

E. Sensitivity analysis of the results for a risk-averse HO

	$\downarrow s$	$\uparrow s$	$\uparrow p$	$\downarrow \sigma$	$\uparrow \sigma$	$\uparrow \lambda$	$\downarrow w$	$\uparrow w$	$\downarrow \hat{h}$	$\uparrow \hat{h}$	$\uparrow c$
Π_{INS}						↑	↑	↓	↑	↓	
U_{HO}	↓	↑	↓	↑	↓	↓	↓	↑	↓	↑	
Π_{FO}											
e											
Δt											↑
F_+							↓	↑			
F_-							↓	↑	↓	↑	

Table E.1. *Summary of the sensitivity analysis of the parameters and their impact on the target variables characterized by arrows (↑ for an increase of the parameter/variable and ↓ for a decrease of the parameter/variable). An empty cell corresponds to an absence of impact (i.e., equal variables in both cases compared). List of the parameters in the Supplementary material, Section A.*

F. The socially optimal level of effort

We assume $c(e) = c.e$ and $q(e) = 1 - exp^{-e}$.

We determine the socially preferable effort e^* such as (Eq. 2):

$$\begin{aligned}\underset{e}{\text{Max}}\, SW &= q(e).\Big[s.\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_-})) - p.E(\widetilde{A_-})\big] \\ &\quad + (1-s).\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+})) - p.E(\widetilde{A_+})\big]\Big] \\ &\quad + \big(1 - q(e)\big).\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+})) - p.E(\widetilde{A_+})\big] - c(e) \\ \Leftrightarrow \underset{e}{\text{Max}}\, SW &= (1 - exp^{-e}).\Big[s.\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_-})) - p.E(\widetilde{A_-})\big] \\ &\quad + (1-s).\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+})) - p.E(\widetilde{A_+})\big]\Big] \\ &\quad + (exp^{-e}).\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+})) - p.E(\widetilde{A_+})\big] - c.e\end{aligned}$$

$$\begin{aligned}\frac{dSW}{de} = 0 \Leftrightarrow q'(e^{**}).s.\Big[&\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_-})) - p.E(\widetilde{A_-})\big] \\ &- \big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+})) - p.E(\widetilde{A_+})\big]\Big] \\ \Leftrightarrow exp^{-e^{**}}.s.\Big[&\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_-})) - p.E(\widetilde{A_-})\big] \\ &- \big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+})) - p.E(\widetilde{A_+})\big]\Big] = c \\ \Leftrightarrow -e^{**}.\Big[s.&\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_-})) - p.E(\widetilde{A_-})\big] \\ &- s.\big[(1-p).U_{HO}(w) + p.U_{HO}(w - E(\widetilde{R_+})) - p.E(\widetilde{A_+})\big]\Big] = \ln c\end{aligned}$$

$$e^{**} = \ln\left(\frac{p.s.(\Delta U + \Delta A)}{c}\right),$$

with $\Delta U = U_{HO}\big(w - E(\widetilde{R_-})\big) - U_{HO}\big(w - E(\widetilde{R_+})\big) > 0$ and $\Delta A = E(\widetilde{A_+}) - E(\widetilde{A_-}) > 0$.

Note that e^{**} is lower than the maximal effort e^+ that can provide FO to implement the IFM:

$$d\Big[q(e^+).\Big[s.\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E(\widetilde{R_-}))-p.E(\widetilde{A_-})\big]$$

$$+(1-s).\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E(\widetilde{R_+}))-\ p.E(\widetilde{A_+})\big]\Big]$$

$$+\big(1-q(e^+)\big).\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E(\widetilde{R_+}))-p.E(\widetilde{A_+})\big]\Big]$$

$$<d[c(e^+)]$$

$$\Leftrightarrow d\Big[\big(1-exp^{-e^+}\big).\Big[s.\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E(\widetilde{R_-}))-p.E(\widetilde{A_-})\big]$$

$$+(1-s).\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E(\widetilde{R_+}))-\ p.E(\widetilde{A_+})\big]\Big]$$

$$+\big(exp^{-e^+}\big).\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E(\widetilde{R_+}))-p.E(\widetilde{A_+})\big]\Big]$$

$$<d[c.e^+]$$

$$\Leftrightarrow exp^{-e^+}.s.\Big[\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E(\widetilde{R_-}))-p.E(\widetilde{A_-})\big]$$

$$-\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E\big(\widetilde{R_+}\big))-\ p.E(\widetilde{A_+})\big]\Big]<c$$

$$\Leftrightarrow -e^+.\Big[s.\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E(\widetilde{R_-}))-p.E(\widetilde{A_-})\big]$$

$$-s.\big[(1-p).U_{HO}(w)+p.U_{HO}(w-E\big(\widetilde{R_+}\big))-\ p.E(\widetilde{A_+})\big]\Big]<\ln c$$

$$e^+>\ln\left(\frac{p.s.(\Delta U+\Delta A)}{c}\right)\Leftrightarrow e^+>e^{**}.$$