

Comparative Analysis of Viscoelastic Models for Head-Tissue Layers under Step Loading

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Research Article

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Comparative Analysis of Viscoelastic Models for Head-Tissue Layers under Step Loading

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Abstract—This study compares four linear viscoelastic networks for scalp, skull, cerebrospinal fluid, and brain contributions. An eight-element parallel network describes stress relaxation, while an eight-element series network describes creep. Reduced Maxwell and Kelvin models are evaluated alongside these configurations. Closed-form equations connect the spring–dashpot arrangements to their response curves. Under a 5% step strain, the parallel network relaxes from 400.835 to 400.334 MPa, a decrease of about 0.125%. The skull spring accounts for most of the stress. At a held stress of 24.010 kPa, the series network approaches a strain rate of 4802.05 s^{-1} because its free dashpots sustain flow. The reduced Kelvin network gives bounded creep with a 7 ms time constant. Its strain reaches 14.6224 at 0.1 s, exceeding the small-strain range at the selected load. The three-element Maxwell approximation closely follows the total relaxation stress, with a maximum difference of 0.015082 MPa. Comparison of the normalized relaxing components reveals the difference in decay rates. The findings connect network structure, material inputs, and loading to the resulting relaxation and creep behavior, providing a basis for further development of reduced head-tissue models.

Keywords—Linear viscoelasticity, head-tissue modeling, stress relaxation, creep, Maxwell model, Kelvin model, model reduction.

I. Introduction

Understanding how head tissues respond to mechanical loading is relevant to traumatic brain injury research. Injury surveillance through CDC WONDER [1] and studies of concussion in high-school football [2] provide the wider injury context. This study examines how the arrangement of elastic and viscous elements affects the response to a held load.

Head-tissue response depends on stiffness and loading time. A spring represents an immediate, recoverable deformation, while a dashpot represents rate-dependent deformation. Their combinations describe relaxation, delayed strain, and continued flow [3, 4]. These simple mechanical elements provide a starting point for analyzing the contributions of individual tissue layers.

Budday *et al.* measured gray and white matter in bovine brain slices using indentation [5]. The response varied with loading rate, holding time, and location. Mean moduli of 1.389 and 1.895 kPa were reported in the study’s abstract, and two-term relaxation fits captured fast and slow time scales. These findings support time-dependent descriptions of brain tissue.

Lear *et al.* examined in vivo scalp stress relaxation [6]. Lozano-Mínguez *et al.* assessed head-tissue mechanical properties and constitutive descriptions for trauma modelling [7]. Daru *et al.* developed four- and seven-layer electrically equivalent head-tissue phantoms for EEG studies [8]. Their layer organization and fabricated sample thicknesses provide the reference grouping and viscosity-weighting lengths used here.

Network arrangement determines how the element responses combine. Parallel branches share a common strain, and their stresses add. Series components carry a common stress, and their strain contributions add. A stiff branch can dominate a parallel network. In a series arrangement, a compliant component or free dashpot can control the deformation.

Four configurations are analyzed: an eight-element parallel network, an eight-element series network, a three-element

Table 1. Baseline quantities used in the comparison.

Quantity	Symbol	Value
Scalp modulus	E_{sc}	16.7 MPa
Skull modulus	E_{sk}	8000 MPa
Brain modulus	E_{bt}	1.642 kPa
CSF viscosity	η_{csf}	5 Pa s
Scalp time scale	τ_{sc}	270 s
Brain reference time	τ_{bt}	500 s
Scalp reference viscosity	$E_{sc}\tau_{sc}$	$4.509 \times 10^9 \text{ Pa s}$
Brain reference viscosity	$E_{bt}\tau_{bt}$	$8.21 \times 10^5 \text{ Pa s}$

Maxwell approximation, and a five-element reduced Kelvin network. Scalp, skull, cerebrospinal fluid (CSF), and brain labels identify the element groups. The comparison uses one-dimensional linear constitutive models under held strain and stress steps. Analytical limits, response curves, and normalized relaxation quantify the effects of connectivity and parameter choice.

II. Model Scope and Parameter Selection

A. Reference Material Quantities

Table 1 lists the material inputs for the comparison. Scalp-relaxation and head-tissue-property studies provide the background for the layer descriptions [6, 7]. The scalp and skull moduli and the CSF viscosity are assigned the values shown in the table. The lumped brain modulus is calculated from the mean gray- and white-matter values reported in the abstract of Budday *et al.* [5]:

$$E_{bt} = \frac{1.389 + 1.895}{2} = 1.642 \text{ kPa}. \quad (1)$$

The average represents the brain contribution as a single modulus.

A 500 s reference time is assigned to the brain model. Together with $E_{bt} = 1.642 \text{ kPa}$, it gives a reference viscosity of $8.21 \times 10^5 \text{ Pa s}$. The scalp reference time is 270 s.

Calculations use pascals, pascal-seconds, and seconds. Stress is displayed in megapascals, strain is dimensionless, and strain rate is expressed in s^{-1} . Table 2 lists the parameters for each network.

B. Network Variables and Physical Geometry

The networks use a one-dimensional linear formulation. Positive stress and strain denote loading magnitudes. The parallel output σ_Σ is the sum of branch stresses, and the series output ε_Σ is the sum of component strains.

For equal-area physical layers with total thickness H , the nominal stack strain is

$$\varepsilon_H(t) = \sum_i \frac{h_i}{H} \varepsilon_i(t), \quad H = \sum_i h_i. \quad (2)$$

Side-by-side material paths use area fractions to obtain nominal stress. The present calculations retain the additive network outputs; Eq. (2) provides the connection to a thickness-weighted layer description. Figure 1 shows the equal-strain parallel and equal-stress series arrangements.

III. Constitutive Formulation

A. Spring, Dashpot, and Step-Response Relations

The elementary constitutive relations are [3, 4]

$$\sigma_s = E\varepsilon_s, \quad \sigma_d = \eta\dot{\varepsilon}_d. \quad (3)$$

A Maxwell branch places a spring and dashpot in series. Eliminating the two component strains gives

$$\dot{\sigma}_M + \frac{E}{\eta}\sigma_M = E\dot{\varepsilon}, \quad \tau_M = \frac{\eta}{E}. \quad (4)$$

Following a step strain of amplitude ε_0 , its regular response is $\sigma_M(t) = E\varepsilon_0 e^{-t/\tau_M}$ for $t > 0$.

A Kelvin–Voigt branch places the elements in parallel:

$$\sigma_K = E\varepsilon_K + \eta\dot{\varepsilon}_K. \quad (5)$$

For a held step stress σ_0 and zero initial strain,

$$\varepsilon_K(t) = \frac{\sigma_0}{E} \left(1 - e^{-t/\tau_K}\right), \quad \tau_K = \frac{\eta}{E}. \quad (6)$$

The ratio η/E governs stress relaxation in a Maxwell branch and creep retardation in a Kelvin branch. The corresponding times depend on the branch arrangement and its parameters.

More generally, a finite linear network admits a differential form

$$\sum_{k=0}^m p_k \frac{d^k \sigma}{dt^k} = \sum_{k=0}^n q_k \frac{d^k \varepsilon}{dt^k}, \quad (7)$$

with coefficients determined by the connections and element values. The following analysis uses the explicit branch solutions.

B. Eight-Element Parallel Network

The scalp and brain groups each contain a Maxwell branch in parallel with a separate spring. The skull is a spring, and the

CSF is a parallel dashpot. The moduli are split 60:40 between the relaxing and equilibrium springs:

$$E_{i,m} = 0.6E_i, \quad E_{i,e} = 0.4E_i, \quad i \in \{\text{sc}, \text{bt}\}. \quad (8)$$

This selected split assigns 60% of each modulus to relaxation and 40% to the equilibrium response.

Let $H(t)$ denote the unit step and $\delta(t)$ its distributional derivative. For $\varepsilon(t) = \varepsilon_0 H(t)$, define

$$G_\Sigma(t) = E_{\text{sc},m} e^{-t/\tau_{\text{sc}}} + E_{\text{sc},e} + E_{\text{sk}} + E_{\text{bt},m} e^{-t/\tau_{\text{bt}}} + E_{\text{bt},e}. \quad (9)$$

The full idealized step response is

$$\sigma_\Sigma(t) = \varepsilon_0 G_\Sigma(t) H(t) + \eta_{\text{csf}} \varepsilon_0 \delta(t). \quad (10)$$

The CSF dashpot responds to the strain rate. An ideal strain step produces an impulse at onset, followed by zero dashpot stress during the hold. A finite ramp produces the stress $\eta_{\text{csf}} \dot{\varepsilon}$ while the strain changes.

The plotted curve is the regular part of Eq. (10). Its limits are

$$\begin{aligned} \sigma_\Sigma(0^+) &= \varepsilon_0 (E_{\text{sc}} + E_{\text{sk}} + E_{\text{bt}}), \\ \sigma_\Sigma(\infty) &= \varepsilon_0 (E_{\text{sc},e} + E_{\text{sk}} + E_{\text{bt},e}). \end{aligned} \quad (11)$$

The 0^+ value excludes the ideal impulse.

C. Eight-Element Series Network

The scalp consists of an isolated spring in series with a Kelvin branch. The skull is a spring. The CSF is a free dashpot, and the brain group is a Kelvin branch in series with another free dashpot. The creep compliance of this additive network is

$$J_\Sigma(t) = \frac{1}{E_{\text{sc},e}} + \frac{1 - e^{-t/\tau_{\text{sc}}}}{E_{\text{sc},k}} + \frac{1}{E_{\text{sk}}} + \frac{t}{\eta_{\text{csf}}} + \frac{t}{\eta_{\text{bt},f}} + \frac{1 - e^{-t/\tau_{\text{bt},k}}}{E_{\text{bt},k}}, \quad (12)$$

where $\tau_{\text{bt},k} = 200$ s. Under a held step stress,

$$\varepsilon_\Sigma(t) = \sigma_0 J_\Sigma(t), \quad t \geq 0^+. \quad (13)$$

The two isolated springs produce an immediate strain. The Kelvin branches contribute delayed strain, and the free dashpots contribute terms that grow linearly with time.

Differentiating for $t > 0$ gives

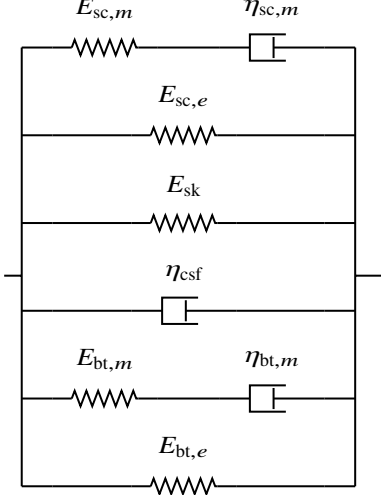
$$\dot{\varepsilon}_\Sigma(t) = \sigma_0 \left[\frac{e^{-t/\tau_{\text{sc}}}}{E_{\text{sc},k} \tau_{\text{sc}}} + \frac{1}{\eta_{\text{csf}}} + \frac{1}{\eta_{\text{bt},f}} + \frac{e^{-t/\tau_{\text{bt},k}}}{E_{\text{bt},k} \tau_{\text{bt},k}} \right]. \quad (14)$$

Consequently,

$$\lim_{t \rightarrow \infty} \dot{\varepsilon}_\Sigma(t) = \sigma_0 \left(\eta_{\text{csf}}^{-1} + \eta_{\text{bt},f}^{-1} \right) > 0. \quad (15)$$

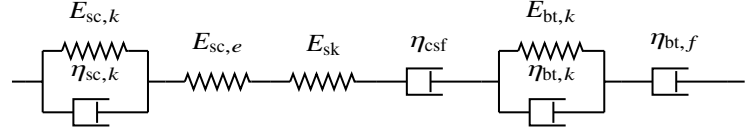
The positive limiting rate produces continued creep under a sustained stress.

(a) Eight-element parallel network



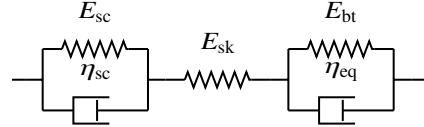
Common strain; branch stresses add.

(b) Eight-element series network



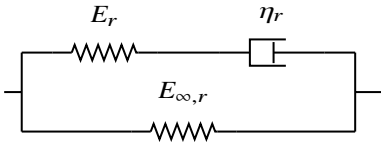
Common stress; component creep responses add.

(c) Five-element reduced Kelvin network



No free series dashpot; creep is bounded.

(d) Three-element Maxwell approximation



Scalp: sc; skull: sk; cerebrospinal fluid: csf; brain: bt.
 m: relaxing Maxwell branch; e: isolated spring;
 k: Kelvin branch; f: free dashpot.
 Element values are specified in Tables 1 and 2.

Figure 1. Spring–dashpot networks used for the constitutive comparison. The eight-element models retain separate scalp, skull, CSF, and brain contributions. The reduced Kelvin network combines the fluid-related contribution into a viscous coefficient in the brain branch. The three-element Maxwell approximation has one relaxing branch and one equilibrium spring.

Table 2. Elastic and viscous parameters for the four network configurations.

Network	Elastic elements	Viscous elements and time constants
Eight-element parallel	$E_{sc,m} = 10.02$ MPa; $E_{sc,e} = 6.68$ MPa; $E_{sk} = 8000$ MPa; $E_{bt,m} = 985.2$ Pa; $E_{bt,e} = 656.8$ Pa	$\eta_{sc,m} = 2.7054 \times 10^9$ Pa s; $\eta_{bt,m} = 4.926 \times 10^5$ Pa s; $\eta_{csf} = 5$ Pa s; relaxing times 270 and 500 s.
Eight-element series	Isolated scalp spring $E_{sc,e} = 6.68$ MPa; scalp Kelvin spring $E_{sc,k} = 10.02$ MPa; $E_{sk} = 8000$ MPa; brain Kelvin spring $E_{bt,k} = 1642$ Pa	$\eta_{sc,k} = 2.7054 \times 10^9$ Pa s; $\eta_{bt,k} = 3.284 \times 10^5$ Pa s; free brain dashpot $\eta_{bt,f} = 4.926 \times 10^5$ Pa s; $\eta_{csf} = 5$ Pa s. Kelvin times 270 and 200 s.
Five-element reduced Kelvin	$E_{sc} = 16.7$ MPa; $E_{sk} = 8000$ MPa; $E_{bt} = 1642$ Pa	$\eta_{sc} = 4.509 \times 10^9$ Pa s; mixed viscosity $\eta_{eq} = 11.5499$ Pa s; rounded brain time $\tau_{eq} = 0.007$ s.
Three-element Maxwell approximation	Relaxing spring $E_r = 10$ MPa; equilibrium spring $E_{\infty,r} = 8006.4$ MPa	$\tau_r = 291.55$ s; $\eta_r = E_r \tau_r = 2.9155 \times 10^9$ Pa s.

D. Five-Element Reduced Kelvin Network

The reduced network contains a scalp Kelvin branch, a skull spring, and a brain Kelvin branch with a combined viscous coefficient. Reference lengths of $h_{csf} = 3.0$ mm and $h_{bt} = 3.93$ mm are adopted from the fabricated four-layer phantom in Table 1 of Daru *et al.* [8]. These sample thicknesses set the weights in the harmonic viscosity rule:

$$\frac{1}{\eta_{eq}} = \frac{h_{csf}}{h_{csf} + h_{bt}} \frac{1}{\eta_{csf}} + \frac{h_{bt}}{h_{csf} + h_{bt}} \frac{1}{\eta_{bt}}. \quad (16)$$

The reference brain viscosity is $\eta_{bt} = 821000$ Pa s from Table 1. Adding the thickness-weighted viscous strain rates at a common

stress gives Eq. (16). The resulting coefficient is assigned to the reduced brain Kelvin branch.

Equation (16) gives

$$\eta_{eq} = 11.5499 \text{ Pa s}, \quad \frac{\eta_{eq}}{E_{bt}} = 0.00703405 \text{ s}. \quad (17)$$

The numerical comparison uses the rounded value $\tau_{eq} = 0.007$ s. This corresponds to $E_{bt} \tau_{eq} = 11.494$ Pa s, about 0.484% below the unrounded mixed viscosity.

The reduced creep response is

$$\varepsilon_{eq}(t) = \sigma_0 \left[\frac{1 - e^{-t/\tau_{sc}}}{E_{sc}} + \frac{1}{E_{sk}} + \frac{1 - e^{-t/\tau_{eq}}}{E_{bt}} \right], \quad t \geq 0^+. \quad (18)$$

Its regular strain rate is

$$\dot{\varepsilon}_{eq}(t) = \sigma_0 \left[\frac{e^{-t/\tau_{sc}}}{E_{sc}\tau_{sc}} + \frac{e^{-t/\tau_{eq}}}{E_{bt}\tau_{eq}} \right]. \quad (19)$$

This derivative excludes the instantaneous spring-strain jump at the onset of the applied stress. The regular rate decreases monotonically because all coefficients are positive.

Unlike Eq. (12), the reduced compliance contains no free-flow term. Its limiting strains are

$$\begin{aligned} \varepsilon_{eq}(0^+) &= \frac{\sigma_0}{E_{sk}}, \\ \varepsilon_{eq}(\infty) &= \sigma_0 \left(\frac{1}{E_{sc}} + \frac{1}{E_{sk}} + \frac{1}{E_{bt}} \right). \end{aligned} \quad (20)$$

Replacing the free-flow terms with Kelvin branches changes the long-time response from continued growth to a finite plateau. The reduced network therefore represents a distinct bounded model.

E. Three-Element Maxwell Approximation

A separate reduced relaxation model is evaluated using

$$\sigma_r(t) = \varepsilon_0 \left(E_r e^{-t/\tau_r} + E_{\infty,r} \right), \quad (21)$$

with $E_r = 10$ MPa, $E_{\infty,r} = 8006.4$ MPa, and $\tau_r = 291.55$ s. These selected parameters define a single-mode approximation to the two-mode relaxation response in Eq. (9).

The comparison evaluates the difference relative to both total stress and the relaxation drop. Subtracting each equilibrium stress and dividing by the initial relaxation amplitude gives

$$\begin{aligned} R_8(t) &= \frac{E_{sc,m} e^{-t/\tau_{sc}} + E_{bt,m} e^{-t/\tau_{bt}}}{E_{sc,m} + E_{bt,m}}, \\ R_3(t) &= e^{-t/\tau_r}. \end{aligned}$$

These normalized responses separate the decay rates from the equilibrium stiffness.

F. Force–Displacement Connection

The scalar constitutive model can also be connected to a uniform bar analogy. Let $\tilde{G}(s)$ be the Laplace transform of a relaxation modulus, P the force, u the axial displacement, A the area, and L the reference length. With zero prehistory,

$$\tilde{u}(s) = \frac{L \tilde{P}(s)}{A s \tilde{G}(s)}. \quad (22)$$

Equation (22) relates force and displacement through the relaxation modulus in a quasistatic uniform-bar representation.

IV. Loading Cases and Numerical Procedure

A. Prescribed Steps

The relaxation calculation uses $\varepsilon_0 = 0.05$ over 0–2000 s with 2000 uniformly spaced points. The primary creep load follows the stated illustrative force-to-area calculation:

$$\sigma_0 = \frac{ma}{A} = \frac{(5)(98 \times 9.8)}{0.2} = 24010 \text{ Pa}. \quad (23)$$

The selected mass of 5 kg, acceleration of 98g, and area of 0.2 m² define the stress amplitude. This stress is applied as a held step.

A second case uses $\sigma_0 = 7000$ Pa. The series model is evaluated over 0–10 μ s at 20 points for this short-time case and at the primary stress for comparison. Its longer response is evaluated over 0–2000 s. The reduced Kelvin model is evaluated over 0–0.1 s at 500 points at both stresses. These time windows resolve the slow relaxation modes, early fluid-related creep, and reduced 7 ms response.

The linear formulation makes each response proportional to load amplitude. Increasing the step strain from 5% to 25% multiplies the relaxation stresses by five.

B. Numerical Evaluation

The step responses are evaluated directly from the closed-form equations using the parameters in Tables 1 and 2 and the time grids specified above. Figures 2–5 show the calculated relaxation, creep, and strain-rate responses. Selected numerical results are summarized in Table 3.

The relaxation, creep, and strain-rate expressions are checked against their analytical initial and long-time values. Equation (19) gives the analytical strain rate. A forward difference provides the sampling comparison in Section V-D.

V. Results

A. Relaxation Is Dominated by the Skull Spring

Figure 2 compares the eight-element parallel network with the three-element approximation. The parallel response starts at 400.835082 MPa, reaches 400.334338 MPa at 2000 s, and approaches 400.334033 MPa. The complete relaxation drop is 0.501049 MPa, only 0.1250% of the initial regular stress.

The small fractional drop follows from the element values. The skull spring alone contributes $0.05 \times 8000 = 400$ MPa. The relaxing scalp branch contributes 0.501 MPa initially, while the corresponding brain branch contributes only 0.00004926 MPa. The model is nearly elastic at the network level because the large skull contribution remains after the viscoelastic branches relax.

The reduced Maxwell approximation starts at 400.820 MPa and approaches 400.320 MPa. Across the sampled 0–2000 s interval, its largest absolute difference from the eight-element result is 0.015082 MPa. This is about 0.00376% of total stress, but 3.01% of the eight-element network's full relaxation drop. The two measures show close agreement in total stress and a larger relative difference in the relaxing component. The normalized responses defined in Section III-E compare their decay rates after removal of the equilibrium contributions.

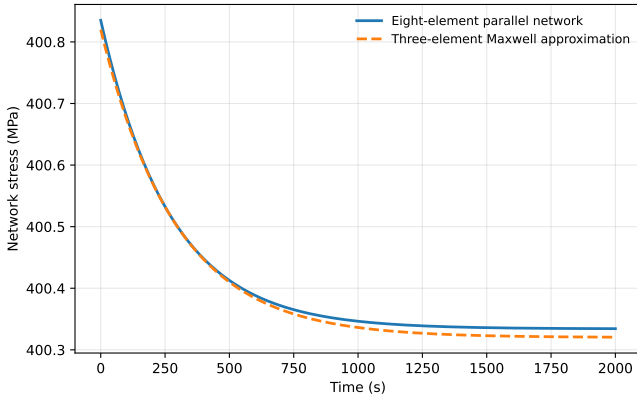


Figure 2. Regular stress relaxation under a 5% strain step for the eight-element parallel network and three-element Maxwell approximation. Parameters are listed in Table 2.

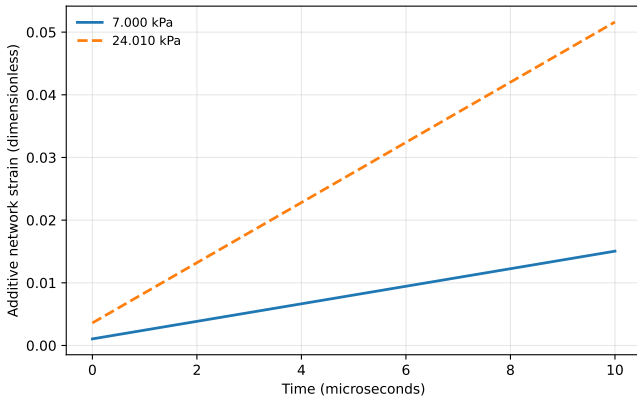


Figure 3. Short-time additive strain of the eight-element series network under 7.000 and 24.010 kPa held stresses. Each curve contains 20 points over 0–10 μ s.

B. Free Dashpots Control Series Creep

The eight-element series network has an initial additive strain of 0.00359731 under 24.010 kPa. Figure 3 shows its early response at both prescribed stresses. At 10 μ s, the output is 0.0150491 at 7 kPa and 0.0516185 at 24.010 kPa. At these short times, the free CSF dashpot already supplies most of the increase beyond the instantaneous spring response.

Under a held stress of 24.010 kPa, the CSF contribution to the long-time rate is 4802 s^{-1} , whereas the free brain dashpot contributes about 0.0487414 s^{-1} . The total limiting rate is $4802.04874 \text{ s}^{-1}$. At 2000 s, the additive strain is approximately 9.60411×10^6 .

The low CSF viscosity controls the dominant slope over both the short and long time windows.

C. Bounded Creep in the Reduced Kelvin Network

Figure 4 shows the five-element reduced network. At 24.010 kPa, its instantaneous strain is 3.00125×10^{-6} . It then increases rapidly as the brain branch responds. The strain reaches 14.622406 at 0.1 s and has a long-time limit of 14.623852. At 7 kPa, the 0.1 s value is 4.263092.

Elastic compliance sets the plateau, while viscosity controls the approach to it. Under the primary stress, the brain

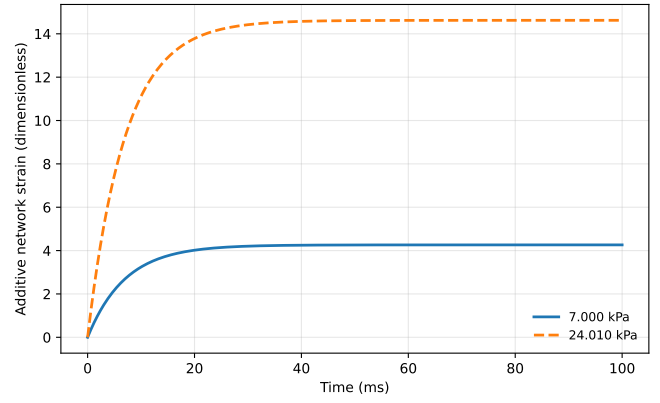


Figure 4. Reduced Kelvin creep under 7.000 and 24.010 kPa held stresses with a 7 ms brain-branch time constant. The curves approach finite plateaus.

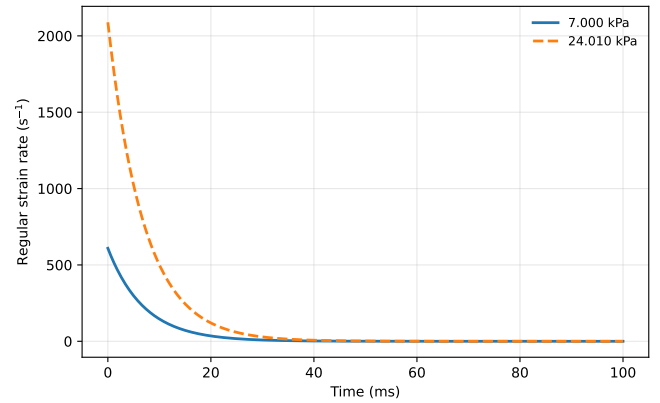


Figure 5. Analytical regular strain rate of the reduced Kelvin network during the stress hold, evaluated from Eq. (19).

contribution tends to $\sigma_0/E_{bt} = 14.622412$. Changing viscosity therefore changes the response time while preserving the limiting strain.

The 7 ms mode reaches 95% of its own asymptotic contribution at $-\tau_{eq} \ln(0.05) = 20.97 \text{ ms}$. The scalp branch has a much slower 270 s time scale, but its strain amplitude is small because its modulus is much larger. The early plateau is dominated by the more compliant brain branch, with a smaller, slower contribution from the scalp.

D. Regular Strain Rate and Sampling

The regular initial strain rate from Eq. (19) is $2088.91596 \text{ s}^{-1}$ at 24.010 kPa. It decreases with time, as shown in Fig. 5. A first forward difference on the 500-point grid gives $2059.29782 \text{ s}^{-1}$ over the first interval. The difference arises because the slope is already decreasing during that interval.

The calculated rate describes the time dependence of the creep response. Its initial magnitude follows from the spring moduli and branch time constants in Eq. (19).

Table 3. Selected numerical results at the primary loads.

Output	Value
Parallel stress, 0 ⁺	400.835082 MPa
Parallel stress, 2000 s	400.334338 MPa
Total relaxation fraction	0.1250%
Series strain, 0 ⁺	0.00359731
Series strain, 10 μ s	0.0516185
Series strain, 2000 s	9.60411×10^6
Series limiting strain rate	$4802.04874 \text{ s}^{-1}$
Reduced Kelvin strain, 0.1 s	14.622406
Reduced Kelvin limiting strain	14.623852
Reduced initial regular rate	$2088.91596 \text{ s}^{-1}$

VI. Discussion

A. Network Structure and Model Reduction

The comparison identifies three distinct response patterns. The parallel network retains a large equilibrium stress because of its skull spring. Free dashpots sustain creep in the series network. Replacing those flow terms with Kelvin branches gives a finite creep limit. These outcomes follow directly from the element connections.

The Maxwell approximation also illustrates the importance of the comparison measure. Its maximum difference is 0.00376% relative to total stress and 3.01% relative to the full relaxation drop. The large elastic contribution accounts for this contrast. Normalized relaxation makes the difference in decay rates visible and provides a useful measure for evaluating the time-dependent part of a reduced model.

B. Viscosity and Geometric Weighting

The CSF coefficient of 5 Pa s controls the dominant creep rate in the series network. In the reduced model, the thickness-weighted viscosity sets the brain-branch response time. The reference dimensions from the fabricated phantom of Daru *et al.* [8] provide the relative weights in Eq. (16).

The additive network outputs describe the selected element arrangements. A physical layered representation connects these outputs to nominal strain through Eq. (2), together with loading areas and interface conditions. Pressure and fluid confinement become relevant when extending the CSF branch to a coupled fluid–solid description.

C. Response Time Scales

The 270 and 500 s reference times set the slow relaxation processes. Combining the effective viscosity with the 1.642 kPa brain spring produces the reduced 7 ms time scale. The latter reaches 95% of its creep contribution in 20.97 ms. These branch times describe constitutive relaxation and retardation. Their relationship to measured tissue response depends on calibration under matching loading conditions.

D. Scope and Further Development

The study provides an analytical comparison of one-dimensional linear networks under sustained steps. At the

selected creep loads, the reduced Kelvin strain exceeds unity and falls outside the small-strain range. The reported network values therefore characterize the behavior of the stated equations. Extension to physical head-impact prediction requires finite loading pulses, inertia, anatomical geometry, fluid confinement, and appropriate tissue and skull constitutive laws. Calibration against measured relaxation and creep would establish the usable loading range.

VII. Conclusion

Four head-tissue-inspired viscoelastic networks were analyzed using explicit step-response equations. The eight-element parallel network shows a 0.125% stress decrease under a 5% strain step, reflecting the dominant skull stiffness. The series network develops continued creep through its free dashpots. A reduced Kelvin arrangement changes this response to a finite plateau, with elastic compliance setting the strain magnitude and viscosity setting the response time. The three-element Maxwell approximation closely follows total stress while retaining a larger relative difference in the relaxing component.

The analysis connects spring–dashpot arrangement, parameter selection, and loading to the resulting response. It provides a basis for choosing reduced models and identifies the constitutive features to address in a calibrated head-tissue representation.

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