Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

Optimizing Remanufacturing under Blockchain and Voluntary Carbon Markets: R&D Incentives and Supply Chain Coordination

Mubashir Unnissa M
Vellore Institute of Technology
Shanmuga Sundari G
Vellore Institute of Technology
Jalapathy P
jalapearl36@gmail.com
Chennai Institute of Technology
Prabu D
Chennai Institute of Technology
Bharathy S
Global Institute of Engineering and Technology

Research Article

Keywords: Blockchain technology; voluntary emission reduction markets; remanufacturing; supply chain coordination; emission reduction; carbon credits

Posted Date: November 11th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11532004/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

Optimizing Remanufacturing under Blockchain and Voluntary Carbon Markets: R&D Incentives and Supply Chain Coordination

M Mubashir Unnissa[1†], G. Shanmuga Sundari[2†], P Jalapathy[3*], D Prabu[4†], S Bharathy[5†]

[1]Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, 632 014, Tamil Nadu, India.
[2]Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, 632 014, Tamil Nadu, India.
[3*]Centre for Nonlinear Systems, Chennai Institute of Technology, Chennai, 600 069, Tamil Nadu, India.
[4]Centre for Nonlinear Systems, Chennai Institute of Technology, Chennai, 600 069, Tamil Nadu, India.
[5]Department of Mathematics, Global Institute of Engineering and Technology, Melvisharam, Vellore, 600 069, Tamil Nadu, India.

*Corresponding author(s). E-mail(s): jalapearl36@gmail.com;
Contributing authors: mubashira@vit.ac.in; poonkuzhali1896@gmail.com;
prabumaths12@gmail.com; bharathy81kuttys@gmail.com;
†These authors contributed equally to this work.

Abstract

This paper explores the effect of blockchain technology and voluntary emission reduction markets (VERM) on the remanufacturing supply chain decision, environmental performance, and coordination mechanism. Using a game-theoretic model of a new product manufacturer and an independent remanufacturer, we examine outsourcing and authorization with VERM and analyze the impacts of different research and development leadership types and a coordination mechanism using cost-sharing and two-part tariff contracts. Our results show that while VERM can boost remanufacturing output and benefit both sides, the outcomes really depend on things like governance setup, how much is spent on investments, and how carbon credit revenue is split. In this case, an outsourced remanufacturer can have a higher output than an authorized one and achieve a higher emission reduction effort per unit remanufactured product than an authorized one, but neither has a dominant advantage: consumer acceptance and VER prices may be threshold-dependent factors affecting corporate profits and emission reductions. Counterintuitively, higher VER prices do not necessarily reduce overall emissions, and excessive incentives may lead to off-target over-remanufacturing when remanufacturing is more carbon-intensive. The proposed coordination contracts coordinate the incentives of two decentralized decision makers towards the center target when the decision conditions (the feasibility constraints and the participation constraints) are fulfilled. The results may be used to shed light on links between blockchain-enabled digital certification, carbon credit incentives, and contractual design for reducing economic and environmental costs.

Keywords: Blockchain technology; voluntary emission reduction markets; remanufacturing; supply chain coordination; emission reduction; carbon credits.

Abbreviation	Full Form
BCT	Blockchain Technology
VERM	Voluntary Emission Reduction Market(s)
VER	Voluntary Emission Reduction
OEM	Original Equipment Manufacturer
IR	Independent Remanufacturer
R&D	Research and Development
SC	Supply Chain
CS	Consumer Surplus
OT	Two-Part Tariff (Outsourcing Contract)
BO	Benchmark Outsourcing Model (without VERM)
BA	Benchmark Authorization Model (without VERM)
O	Outsourcing Model (with VERM)
A	Authorization Model (with VERM)
RO	IR-Led R&D Outsourcing Model
RA	IR-Led R&D Authorization Model
C	Centralized Model
CO	Cost-Sharing Outsourcing Model
AE	Aggregate Emissions

1 Introduction

The current and immediate past climate change is drawing more attention and discussion, as environmental damage and carbon emissions have no longer made it possible to avoid pushing businesses toward sustainable, transparent operations. One of the major shifts here is that it has become imperative for the manufacturing industries-major responsible parties for energy usage and emissions- to get to the forefront of sustainability and regulations El Saadany and Jaber (2010) Jaber et al. (2013). A policy mix imposed under environmental policy has included mandated carbon policies such as the carbon tax, quota, and trading schemes, which intend to correct externalities for about one-third of the world's population, according to Xia et al. (2023a). The policies are efficient in reducing emissions, but on the downside have added to the costs of the companies. For small remanufacturing businesses with limited capital, this may lead to difficulties in expanding due to adherence to the environmental policies Alegoz et al. (2021).

Simultaneously, remanufacturing is another main strategy of the circular economy because it can bring costs and emissions down and prolong the product life. According to a study, production costs can be reduced by $40-65\%$, energy consumption by 60%, and emissions by 80% through remanufacturing compared to traditional manufacturing (Yang et al. (2020)). From a cost-saving aspect, it is an attractive approach; nevertheless, the situation poses an obligation for remanufacturers to strengthen product quality assurance and verify the greenness of their products and their adherence to required emission reduction standards. This development is attracting more attention in the aspect of VERM, through which organizations can invest in emission reduction projects and issue VERs, trading them in a voluntary market (Bensoussan and El Ouardighi (2024)). This, compared to stringent laws, relies on voluntary actions of suppliers, which, although effective in encouraging innovative behaviors, has uncertain impacts on the remanufacturing supply chain (SC).

In the developed model, blockchain technology (BCT) is assumed to be an enabling technology that is intended to achieve traceability, transparency, and emission verification in the VERM remanufacturing system. Instead of discussing BCT and non-BCT systems, this study examines how the information transparency of BCT, coupled with the carbon-credit motivation and contractual mechanisms, affects SC members' operational and environmental decisions. In the meantime, BCT has become an important technology for providing enabling capabilities of tracking product movement in the SC, guaranteeing product authenticity, authenticating carbon emission evidence, and resolving information asymmetry problems and enhancing consumers' trust in remanufactured products Saberi et al. (2019). Although BCT can generate these benefits, there are also other potential disadvantages and barriers, such as the implementation cost, data security and privacy issues, and the cost-sharing problem, which may affect

the strategic interdependence between OEM and IR. Therefore, under different contractual approaches, how the implementation of BCT within a VERM-driving SC would balance sustainability, the economic incentive, and the consumer-driven demand effects needs to be considered.

As for governance issues, OEMs are unwilling or may not have the capacity to perform remanufacturing internally. Thus, two governance mechanisms are: "outsourcing" (OEM outsources the remanufacturing process to IR but still owns and controls the sales and distribution process) and "authorization" (IR handles both remanufacturing and the sales process). These types of governance each have key effects on the distribution of profit, on consumer adoption, and on environmental performance. For example, Apple outsources remanufacturing by using contracts with Foxconn, and the whole sales process is under the control of the OEM; on the contrary, Sharp gives authority to its partners, which are experts in a certain technology (e.g., Xin Printing Technology Zou et al. (2016)). When the remanufacturing company becomes an actor within the VER, there will be variations in the carbon credit price as well as the emission reduction. As outsourcing and authorization delegate different components of the remanufacturing profit, environmental benefits, and the operational activities between OEM and IR, the relative dominance of outsourcing or authorization is sensitive to the changes in VER prices.

In practice, the distinction between outsourcing and authorization encompasses not only the division of remanufacturing activities but also property rights, brand liability, quality control decision rights, and allocation of environmental benefits. In outsourcing, OEM retains property rights on remanufactured products and makes the distribution and branding decision. OEM would carry the burden of the quality of the product and of the brand name. IR, on the other hand, is merely an operator. The process of remanufacturing is actually done by IR for OEM. In the authorization case, OEM allows IR the right to manufacture and market the remanufactured products individually; IR can build a direct relationship with consumers, and therefore takes more operational and marketing responsibilities. The difference also causes differences in the incentive structures for environmental benefits. The emission reduction (validated) is achieved through the remanufacturing operations, so it is associated with the one carrying out the operation (IR), while the investment decision in remanufacturing technology could be attributed to OEM. Accordingly, outsourcing and authorization differ with unique incentive allocation rules under profit, quality, environmental investment, and carbon credit revenue, so the choice of governance is important under the circumstances with the BCT-supported VERM.

Recent literature on green SCs and carbon policies has advanced significantly, but there is still scope for more research. For instance, the relationship between VERM and the two remanufacturing modes (outsourcing remanufacturing and authorized remanufacturing) is not thoroughly investigated. The strengths of BCT traceability and privacy concerns haven't been explored much in terms of demand, profits, and emissions aspects. Moreover, although contract coordination mechanisms have been deeply discussed in a mandatory policy environment, a good understanding of how contracts can support remanufacturing with VERM under BCT support for OEMs and IRs' incentive alignment is lacking. Previous research on carbon regulation, BCT, and remanufacturing governance to a certain extent has been considered separately. This study combines VERM incentives, BCT traceability, and remanufacturing governance into one integrated modeling structure and examines the synergies in operational decisions, environmental investment, environmental performance, and SC coordination. Within this integrated framework, the incentive of the economic gain derived from revenue from carbon credits obtained in VERM and the guarantee of authenticity and indelibility of environmental information provided by BCT in transparency thus have combined effects on the selection of remanufacturing governance, operational decisions, and environmental performance under different contract configurations.

Building on the interaction between BCT verification, market incentives for voluntary carbon markets, and governance structures of remanufacturing, this paper asks the following questions:

1. How does VERM influence operational strategies and R&D collaboration in BCT remanufacturing?
2. What is the main reason to prefer outsourcing to authorization models in terms of profitability, consumer surplus, and the environment?
3. How do the trade-offs between transparency and privacy surrounding BCT affect consumer demand and equilibrium prices under VERM?
4. What kind of coordination contracts can be used to effectively align the OEMs' and IRs' interests so that the decentralized decision-making in BCT and VERM can imitate the centralized efficiency?

To answer these questions, a game-theoretic model for BCT remanufacturing SC with consumer heterogeneity, costs of BCT implementation, privacy, and VERM is built. By conducting a comparison analysis on outsourcing and authorization models with/without VERM, equilibrium decisions and environmental influences were tested. In addition, coordinate contracts are proposed to synchronize the interests of OEM and IRs to share costs of BCTs and R&D investment reasonably.

Additionally, this scholarly work offers significant contributions to the body of knowledge in four distinct ways:

1. First, the proposed study presents a combined analytical framework incorporating BCT, VERM, and remanufacturing SC governance. Existing research in the literature has studied only a part of the relationship, such as the choice of BCT adoption or carbon regulation, or the decision of remanufacturing. Here, a comprehensive model combines the interactions of digital traceability, monetization of carbon credits, and operational decisions. By doing so, consumer trust and privacy concerns, emission reduction investment, and remanufacturing strategy are integrated into one game-theoretical framework.
2. Second, the study uncovers a new, unidentified incentive-alignment problem arising in VERM where the investor in emission-reduction investment may not be the one who receives the benefits from carbon-credit revenue arising from remanufacturing. The disconnect between these players (OEMs and IRs) produces strategically incompatible decisions regarding R&D investment and environmental stewardship. The paper shows how this incentive conflict influences the outcomes (equilibrium decision, profit, and environmental impact) for different governance arrangements.
3. Third, this paper introduces the double function of BCT in remanufacturing under market and operational dimensions. It increases remanufactured products' acceptance on the demand side through transparency and credibility of information, and improves the validity and traceability of emissions reduction on the operational side such that the voluntary emission-credit market is able to run effectively. As such, this paper demonstrates how BCT affects equilibrium production, emission-reduction effort, environmental performance, and profit distribution, besides its tracking purpose.
4. Fourth, this paper reveals that carbon-credit prices have a threshold-dependent impact on R&D leadership and governance performance. It indicates that relative dominance between OEM-dominated and IR-dominated emissions-reducing models shifts along VER-price variations. It is found that at low, moderate, and high VER prices, three distinctive equilibria arise for remanufacturing volume, emission-reduction, and environmental performance, respectively. These findings contribute a fresh theoretical perspective on the way the voluntary carbon markets reform technology-investment motivation and governance-selection mechanisms in a circular SC.

Finally, a cost-sharing contract and a two-part tariff coordination contract are constructed to directly take account of BCT implementation costs, investment costs for emission reduction, and carbon credit incentives. The proposed contracts can avoid the inefficiency of decentralization and restore the centralized optimum, implying they are able to contribute to the adjustment of economic and environmental incentives in BCT remanufacturing SC within the voluntary carbon market. From the perspective of practice, there are differences in operational activities, market influence, and environmental inducements of outsourcing and authorization. In the outsourcing mode, ownership and sales channels still belong to the OEM while the IR remanufactures products. The IR has self-operated remanufacturing and sales processes in the authorization mode. According to the differences in governance structure, profit sharing, the emission reduction incentive, and the benefit of carbon credits, VERM varies.

The paper can be structured as follows. Section 2 represents the literature review. Section 3 discusses the model framework, covering the market assumptions, the information systems of BCT, and the incentive mechanisms for emission reduction in a voluntary way. In Section 4, the situations without investments in emission reduction, the effects of BCT on consumer demand and operation decisions are analyzed. Section 5 investigates the contracts on outsourcing and authorization under the backdrop of OEM's emission reduction R&D. With reference to Section 6, the contracts on cost sharing and two-part tariffs are used to coordinate decentralized decisions and centralized efficiency. Finally, the conclusions and further research directions are suggested in section 7. All proofs are supplied in the appendix.

2 Literature Review

Environmental concerns are increasing, and sustainability is becoming the focus; as a result, extensive research is being conducted on various elements of remanufacturing, carbon reduction, and digital technology to improve transparency in the SC. Existing research has concentrated on factors of carbon regulation policies, investment in emission reduction, remanufacturing governance models, and the traceability achieved through the use of blockchain. All these studies appear to have evolved independently. The literature providing theoretical foundations for the framework to be proposed is reviewed through three intertwined processes: (i) the influence of voluntary emission-reduction markets on remanufacturing incentives; (ii) the effect of BCT on consumers' trust and carbon-credit warranty; (iii) the

involvement of outsourcing and licensing governance on profit distribution and investment in emission reduction.

2.1 VERM and remanufacturing incentives

Environmental policies have been a driving force behind remanufacturing. Much of the literature is concerned with compulsory regulatory measures such as carbon taxes, emission caps, and cap-and-trade systems. For instance, Liu et al. (2015) compared the remanufacturing decision-making under different types of regulatory measures (carbon tax, emission cap, cap-and-trade system) and concluded that there are significant differences in the optimal operating decisions across different regimes. Zhu et al. (2022) studied the hybrid carbon-regulation schemes and identified firms' dynamic adjustments of production and emission-reduction when several environmental policies are concurrently enacted. As described in Alegoz et al. (2021), Yenipazarli (2016), and Luo et al. (2022), optimal policy is in the middle since overly strict environmental regulations could discourage remanufacturing activities.

Many papers have focused on environmental regulation and governance in remanufacturing SC in recent years. For example, the problems of outsourcing strategy and authorization strategy in combination with the carbon trading policy were analyzed in Xia et al. (2023) and Xia et al. (2023a), and it demonstrated the influences of emissions reduction on retail price, market demand, and authorization price. As is seen from the analysis of results above, the two things that have been stated to be coupled together are environmental motivation and governance selection.

By contrast with mandatory regulation mechanisms, VERM are less discussed. Most research assesses regional carbon-credit programs, voluntary market efficiency (Gao and Souza (2022); Wei and Xiao (2022); Zheng and Zhang (2023)), and the accounting and investment impact of voluntary carbon credits (Duncan et al. (2022); Schweitzer et al. (2023); Fang et al. (2023b)). The positive externality effects of voluntary carbon credits, as incentives for emissions reduction, have also been addressed by Bensoussan and El Ouardighi (2024) recently. However, there has been little work to study how voluntary emission reduction market participation impacts the operating decisions, governance preferences, and investment incentives of remanufacturing SCs; in particular, the intersection between the income from carbon credits and remanufacturing governance systems has not been investigated.

2.2 BCT, consumer trust, and carbon-credit verification

In improving environmental performance, investment in emission-reduction and green innovation is recognized as an alternative approach. In line with this notion, studies like Wang et al. (2018) and Bai et al. (2019) conclude that clean technology investment and green innovation can effectively alleviate environmental pressures while strengthening competitiveness in the long run. Moreover, collaboration on R&D activities has been identified as another means that contributes to environmental improvement Pun and Ghamat (2016), Li et al. (2021), Chen et al. (2019), Wang et al. (2021), Guo et al. (2022). While collaboration might result in significant positive environmental impact, technology spillovers and shirking may lead to weaker investment incentives Liu and Zhu (2023), Jiang et al. (2024).

BCT appears to be a potential answer to the identified problems, as it can achieve trustworthy and auditable information sharing. The literature thus far has identified that BCT increases transparency and trust and decreases opportunism. The literature found that transparency created by BCT has improved consumer welfare and operational performance for pharmaceutical SCs (Niu et al. (2021)) and prevents counterfeit products (Pun et al. (2021)). Similarly, BCT has also been discussed for the Food SCs (Cao et al. (2023)) and fashion SCs (Badhwar et al. (2023)) as consumer trust in food and fashion SCs depends on the traceability and authenticity of products. BCT is also mentioned in a broader context regarding its use in managing the sustainability of SC. Bibliometric and review studies (Agrawal et al. (2023); Sharabati and Jreisat (2024)) have frequently found that the blockchain benefits, in relation to its application in SCs, include increased transparency, traceability, and security. In carbon market smart contracts on blockchain were proposed to enforce compliance and automated verification of transactions (Merlo et al. (2025)). Later, discussions are carried out about governance, ethics, and regulation challenges associated with massive-scale blockchain implementation (Cheong (2025)).

Despite many research proposals on how applying BCT in SCs can bring advantages, in most of the studies blockchain is applied as an information-sharing or traceability solution, and only limited studies explicitly explore how blockchain can be utilized to generate and certify carbon credits to provide consumers a traceable and understandable framework. Crucially, to what extent BCT, through increased reliability and transparency of carbon credits, influences consumer demand is still unclear. Additionally, while transparency is seen to increase consumer trust, it also potentially compromises consumer privacy due to disclosure of private data. Currently, remanufacturing research has failed to account for these

counteracting impacts in operational decision models, suggesting that there is an area that should be explored further regarding the relationship between consumer privacy choice and commercializing carbon credits under BCT adoption.

Comparing the representative studies for carbon-policy design, the adoption of BCT, governing structure, privacy issues, and coordination contract shown in Table 1, we obtain three interesting observations. Firstly, among the research on remanufacturing, most studies focus on mandatory environmental regulation, but the incentives of VERM are underdeveloped. Secondly, studies related to BCT only concern transparency benefits but rarely involve adoption, privacy issues, and BCT-based carbon credit value. Thirdly, in governance studies, outsourced and authorized SCs have been analyzed, but seldom with BCT verification. Thus, the framework in this research fills these research gaps. Also, from Table 1, the difference with the extant literature is explicitly expressed: rather than analyzing a certain aspect of sustainable remanufacturing, we considered and analyzed integrated issues such as the VERM incentives, the BCT-based emission verification, the consumers' willingness to share data privacy, the selection of governance mode and coordination contract; meanwhile, we also decided to adopt BCT endogenously, depending on the performance of SC.

2.3 Governance structures, profit allocation, and emission-reduction investment

Typically, remanufacturing activities are managed under outsourcing or authorization agreements. The economic implications of these arrangements have been widely studied in the existing literature. The analytical structure developed in this paper adapts the classic game-theoretic literature of remanufacturing in which OEMs and IRs compete in a sequential game structure. Typically, prior works used either a Stackelberg or a Nash game to examine pricing, production, and channel interaction for governance comparison purposes.

By outsourcing to IRs, OEMs can gain more control over their quality and brand, while benefiting from the IRs' expertise in remanufacturing. There has been a lot of research suggesting that authorization contracts can promote profit sharing and enhance OEM-IR coordination (Liu et al. (2018), Zhou et al. (2021), Zhou et al. (2021a)). Jin et al. (2022) also suggests that OEMs tend to adopt an authorization structure when quality control is of strategic importance. Also, some research investigates authorization decisions with respect to the quality of returned products and carbon emission properties (Fang et al. (2023a), Li et al. (2024)). Outsourcing, as another widely used governance strategy, has also received extensive attention. Xia et al. (2021) studied the outsourcing strategy under a carbon trading scheme; Zhou et al. (2023b) and Sun et al. (2024) further investigated the impacts of green design investment and information asymmetry on outsourcing performance. Thus, from all of the above research, it can be summarized that different governing schemes would lead to different profit sharing, pricing, and investment decisions.

Recently, studies have explored the use of BCT in the remanufacturing governance environment. Specifically, Huang et al. (2025) introduced a BCT framework for the reliable and safe sharing of carbon-emission information in the context of outsourcing. In Nan et al. (2025), the application of BCT cooperation in the development of low-carbon technologies with authorization was investigated. These studies have established that BCT can enhance information reliability and minimize opportunism from the SC partners. However, these studies did not explore the interactions between the transparency created by the BCT and the voluntary carbon-credit incentive mechanisms and the governance decisions.

Even though the study-level case comparison is included in Table 1, three kinds of inter-related research gaps are presented: the effect of VERM on remanufacturing incentives, the function of BCT on consumer trust and carbon-credit authentication, and the role of governance mechanisms on profit distribution and emission reduction investment. Generally, the above three mechanisms have not been analyzed together, and their interactions remain underexplored.

The previous research in Table 2 is distributed across the views of environment, technology, and governance. None of them can comprehensively incorporate the participation of VERM, transparency of BCT, and governance in remanufacturing decisions at the same time. Additionally, relationships between consumer privacy concerns and revenue gained from the verified carbon credit, and the coordination contract are scarcely explored. This study attempts to contribute to the body of knowledge by proposing an integrated model that takes BCT-enabled verification, VERM incentives, outsourcing/authorization governance structure, and SC coordination mechanisms simultaneously into account. This model allows us to explore how digital transparency and voluntary carbon markets influence emission reduction investments, price setting, governance choices, and the entire SC performance.

Table 1 Enhanced comparative analysis of related literature and positioning of the present study.

Reference	Carbon Policy	BCT	Governance Mode	BCT Adoption Endogenous	VER Price Modeled	Privacy Preference	Coordination Contract
Chen et al. (2019)	–	–	Joint R&D	–	–	–	✓
Saberi et al. (2019)	–	✓	–	Partial	–	–	–
Alegoz et al. (2021)	Carbon Tax	–	Remanufacturing	–	–	–	–
Li et al. (2021)	–	–	Cooperative R&D	–	–	–	✓
Niu et al. (2021)	–	✓	SC	Partial	–	Partial	Partial
Pun et al. (2021)	–	✓	–	Partial	–	–	–
Yang et al. (2020)	–	–	Remanufacturing	–	–	–	–
Friedman and Ormiston (2022)	–	✓	Food SC	Partial	–	–	–
Guo et al. (2022)	VERM	–	Joint Investment	–	Partial	–	Partial
Luo et al. (2022)	Hybrid Carbon Policy	–	Joint Investment	–	Partial	–	Partial
Agrawal et al. (2023)	–	✓	Industry 4.0	Partial	–	–	–
Badhwar et al. (2023)	–	✓	Fashion/Textile	Partial	–	–	–
Cao et al. (2023)	–	✓	Food SC	Partial	–	–	–
Xia et al. (2023a)	Carbon Trading	–	Outs./Auth.	–	–	–	–
Alotaibi et al. (2024)	ESG/Carbon Accounting	✓	–	Partial	–	–	–
Bensoussan and El Ouardighi (2024)	VERM	–	OEM/Supplier	–	✓	–	–
Sharabati and Jreisat (2024)	–	✓	SC Stakeholders	Partial	–	–	–
Abreu et al. (2025)	–	✓	–	Partial	–	–	–
Merlo et al. (2025)	Carbon Markets	✓	Carbon Trading	Partial	Partial	–	Smart Contracts
This Paper	VERM	✓	Outs./Auth.	✓	✓	✓	✓

Table 2 Research gaps and contributions of the present study.

Research Dimension	Existing Literature	This Paper
VERM in remanufacturing SCs	Limited	✓
Joint analysis of BCT and VERM	Rare	✓
BCT emission verification	Limited	✓
Outs. vs. auth. under VERM	Not addressed	✓
Consumer privacy concerns under BCT	Rare	✓
Explicit modeling of VER prices	Limited	✓
OEM-led versus IR-led emission-reduction R&D	Limited	✓
Coordination contracts under BCT-enabled VERM	Not addressed	✓
Integrated governance, environmental, and economic analysis	Fragmented	✓

3 Model Description and Related Assumptions

This study establishes a BCT-involved remanufacturing SC of OEM and IR in the VERM system. Figure 1 shows the structure and decision flow of the SC. There are two other alternatives to govern structures, namely outsourcing and authorization. The key issue and contribution of this paper address OEM-initiated R&D for emission reductions, given the assumption of a Stackelberg game, while the scenario of IR-initiated R&D is discussed as an extension. BCT traceability and the VERM incentive mechanism are integrated into each model to analyze their impacts on operational decisions, economic and environmental performance. Furthermore, BCT is applied to strengthen transparency of information, consumer trust, and reliability of emission reduction verification, which facilitates VER generation and trading. Thus, BCT impacts market demand as well as environmental incentives; therefore, OEM and IR determine price, remanufacturing quantity, emission-reduction effort, and distribution of BCT costs and benefits.

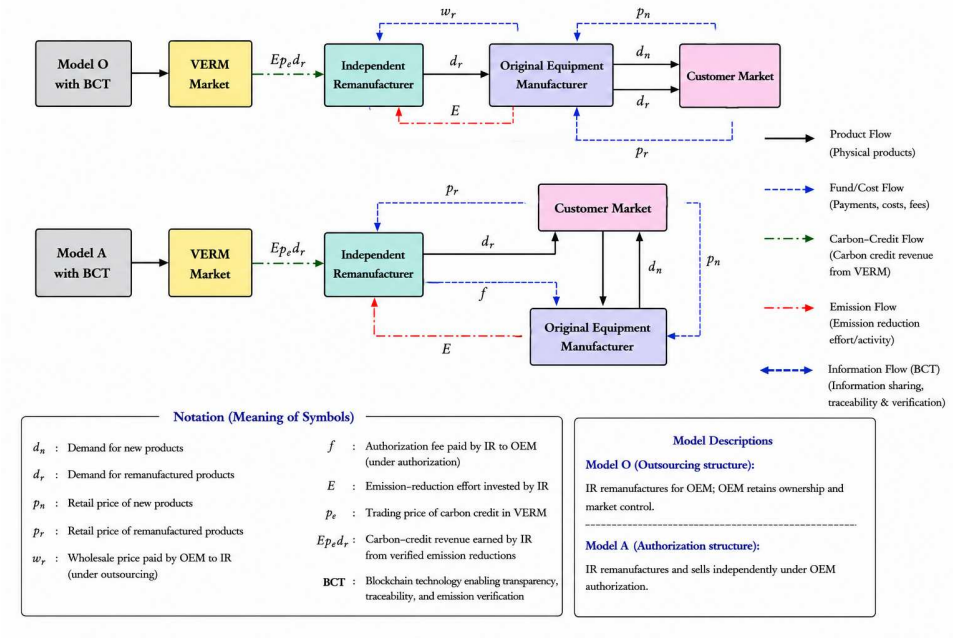


Fig. 1 SC framework with BCT.

The proposed framework includes a model structure consisting of OEM, IR, and heterogeneous consumers. Based on the modeling structure, demands, costs, carbon mechanisms, and BCT-related

variables are described as below; a summary of key model parameters and decision variables is presented in Table 3. The purpose of BCT is to improve consumer trust in the traceability of products and in the credibility of emission reduction verification of VER trading. Thus, BCT affects the demand in the market and the investment in the environment of the remanufacturing SC.

Table 3 Summary of Notation

Symbol	Description	Type	Unit / Notes
d_n	Quantity of new products sold by OEM	Decision Variable	Units of product
d_r	Quantity of remanufactured products by IR	Decision Variable	Units of product
w_r	Outsourcing fee per remanufactured unit	Decision Variable	Monetary (per unit)
f	Authorization (license) fee per unit	Decision Variable	Monetary (per unit)
E	Emission reduction per remanufactured unit	Decision Variable	Emission units (e.g., kg CO_2)
p_n	Retail price of new products	Market Parameter	Monetary (per unit)
p_r	Retail price of remanufactured products	Market Parameter	Monetary (per unit)
Q	Maximum market demand	Demand Parameter	Units of consumers
θ	Willingness-to-pay for new products	Consumer Parameter	Monetary
δ	Discount factor for remanufactured products	Consumer Parameter	$0 \leq \delta \leq 1$
c	Unit cost of new product	Cost Parameter	Monetary
s	Cost saving from remanufacturing	Cost Parameter	Monetary
g	Core collection cost coefficient	Cost Parameter	Quadratic
ξ	R&D difficulty of emission reduction	Environmental Parameter	Positive scalar
E^c	Emissions from new product	Emission Parameter	CO_2 units
β	Emission intensity ratio (rem. vs. new)	Emission Parameter	$0 < \beta < 1$
p_e	VER price in carbon market	Market Parameter	Monetary (per unit)
κ	Cost-sharing function	Contractual Parameter	$0 \leq \kappa \leq 1$
VER	Total verified emission reductions	Derived Variable	$E \cdot d_r$
c_a	BCT cost per remanufactured unit	BCT Parameter	Monetary
F_c	Fixed BCT infrastructure cost	BCT Parameter	Monetary
F^{OT}	Fixed fee in two-part tariff contract	Contractual Parameter	Monetary
ρ	Cost-sharing ratio borne by OEM	Contractual Parameter	$0 \leq \rho \leq 1$
λ	Trust benefit from BCT	Consumer Parameter	Positive scalar
t	Transparency level of BCT	Consumer Parameter	$t > 0$
p_0	Base consumer privacy cost	Consumer Parameter	Monetary
p_c	Adjusted privacy cost under BCT	Consumer Parameter	$p_c = p_0 - \delta\lambda t$
CS	Consumer surplus	Outcome Variable	Monetary
Π_k	SC Members Profit i.e, $k = \{M, R\}$	Objective Variable	Monetary
SW	Social welfare	Objective Variable	$\Pi_M + \Pi_R + CS$

Assumption 1: Consumer Willingness-to-Pay and BCT Enhanced Demand. This analysis is based on the idea that the market is made up of an almost infinite number of consumers, and they are arranged according to their willingness to pay (WTP), represented by θ, which is uniformly distributed on the interval $[0, Q]$, where Q is the maximum market size. A buyer whose WTP is θ sets the price of a remanufactured product equal to $\delta\theta$, where $\delta \in (0, 1]$ is a factor that depicts the perception of quality loss in remanufactured products. This aspect of vertical differentiation is well-established in the remanufactured products literature (Subramanian and Subramanyam (2012); Zhou et al. (2023a)).

To include the trust-boosting effect of BCT, we set an information transparency level t between 0 and 1 that helps increase consumer trust in remanufactured products. This level also takes a sensitivity parameter $\lambda > 0$ into consideration, and adjusts the utility of remanufactured products. Thus, the inverse demand functions are given by:

$$p_n = Q - (d_n + \delta d_r)$$
$$p_r = \delta\,(Q - (d_n + d_r)) + (1 + \delta)\,\lambda t - p_0$$

These demand functions reflect the market competition, and the positive influence of BCT transparency in demand for remanufactured products. This case can be seen as combining the ideas of vertical differentiation and the usefulness of BCT information, as introduced in Francisco and Swanson (2018) and Saberi et al. (2019) respectively. Full derivation can be seen in A.1.

Assumption 2: Production Costs and BCT Cost Adjustment.

Let c signify the unit production cost for new products. The remanufacturing method has some advantages, and one of them is the cost savings due to the reuse of necessary parts like engines and motherboards. This leads to the manufacturing cost being expressed as $c - s$, with s representing the average cost savings per unit, which is a positive number ($s > 0$). Research shows that remanufacturing can produce a reduction of $40 - 65\%$ in production costs in the automotive and electronics industries, for instance, Yang et al. (2020).

On top of that, a per-unit BCT implementation cost (c_a), which covers the certification, carbon tagging, and the execution of smart contracts expenses, is to be added. Thus, the remanufactured product's effective cost is expressed as:

$$\text{Effective Remanufacturing Cost} = c + c_a - s.$$

With this model, the technological efficiency was clearly separated from the costs of BCT traceability. Moreover, empirical findings refer to the work of Xu and Wang Xu and Wang (2017) and the studies of Wu et al. (2025) that pointed out the same trend of cost reductions in the automotive and electronics channels.

Assumption 3: Carbon Emission Reduction R&D and Innovation Costs Cao and Shao (2024).

OEMs could direct funds to the development of carbon-reducing technologies such as remanufacturing that are cleaner and therefore less polluting. The expense linked to the investment is depicted as a convex function that incorporates diminishing marginal returns:

$$\text{Emission R\&D Cost} = \frac{\xi E^2}{2}.$$

In this formulation, $E \in [0, E^c]$ denotes the decrease in carbon emissions per unit, whereas $\xi > 0$ indicates the level of technological difficulty or capital requirement for the corresponding R&D. The research and development (R&D) effort is a one-time investment that does not influence the marginal cost of production (Ghosh and Shah (2012)). Furthermore, E^c and βE^c denote the unit carbon emissions for new and remanufactured products, with β taking a value within the interval $(0, 1)$.

Assumption 4: End-of-Life (EOL) Product Collection and Recycling Costs.

IRs bear the responsibility for the collection of EOL products that are suitable for remanufacturing. As the scale of collection expands, the associated costs increase in a non-linear manner due to challenges related to quality screening and logistics. This relationship is represented by a quadratic cost function:

$$\text{Collection Cost} = g\,d_r^2.$$

where $g > \frac{1}{2}$ is a scaling parameter greater than $\frac{1}{2}$, ensuring the concavity of the profit function. To ensure the solvability of the model and to maintain concavity of the profit function in relation to all decision variables, certain conditions on parameters must be satisfied, specifically $\xi > \frac{p_e^2}{2(\delta(1-\delta)+g)}$. These conditions have been discussed in similar CLSC studies (Yang et al. (2020); Xu et al. (2023); Xia et al. (2023a)). Additionally, the collection cost is not directly influenced by BCT; however, the implementation of traceability features may mitigate fraud and enhance recovery rates. This aspect could be modeled in future developments.

Assumption 5: VERs and BCT Verified Carbon Credits.

Using green technology in remanufacturing, VERs are produced at a rate that depends on both the total production capacity and how much emissions are reduced. The total VER, shown as Ed_r, can be sold in the voluntary carbon market at the price p_e (Sabzevar et al. (2017); Gao and Souza (2022)). Thus the revenue is given by,

$$\text{VER Revenue} = p_e E d_r$$

It is important to note that the price p_e is considered to be exogenous and is projected to increase over time, reflecting a rise in carbon trading activities and the implementation of more stringent offset requirements (Gao and Souza (2022); Times (2023)). Moreover, BCT supports the verification and traceability of VER transactions.

Assumption 6: Consumer Perception and Viability of Remanufacturing.

For consumers to adopt remanufactured products, it has to be demonstrated that the perceived benefits to the customers will outweigh the costs. This can be expressed through the following equation: $c - s < \delta c$, with c being the cost, and s being the perceived value of the remanufactured products.For this circumstance, it appears that remanufactured products are appealing, even for common market conditions. Furthermore, provided that the information disclosure is better, consumers' acceptance of remanufactured products (δ) increases.

Assumption 7: BCT Deployment and Cost Sharing.

There are two main cost components in the BCT: a fixed capital cost for constructing the infrastructure, which is indicated by F_c (including the ledgers, tokens, and smart contracts) and a variable per-unit-cost of $c_a d_r$ for the operation and validation. These expenses are shared between the OEM and the IR based on a cost-sharing Parameter ρ that ranges from 0 to 1, indicating the fraction of the OEM's part. As a result, the cost allocations are characterized by:

$$\begin{aligned} \text{OEM's Share} &= \rho\left(c_a d_r + F_c\right) \text{and} \\ \text{IR's Share} &= (1-\rho)\left(c_a d_r + F_c\right). \end{aligned}$$

This distribution method corresponds to what is commonly seen in clean technology contracts and BCT consortia, as per the studies of Swami and Shah (2013) and Adhikari and Bisi (2020).

Assumption 8: Privacy Costs and Trade-offs Associated with BCT.

Along with public debate of consumer privacy as a key detraction of the BCT, the BCT will potentially have the ability to appeal to new market groups due to its features concerning privacy. The baseline privacy inconvenience will be represented by p_0. The privacy price that a consumer has to pay if they decide to accept the technology is pc which is calculated as $p_c = p_0 - \delta\lambda t$. $\delta\lambda$ represents the benefits gained from trust and information that BCT transparency brings, while p_c represents the net result from the consumer's perspective, suggesting that traceability, product authenticity assurance, and reliable carbon information could outweigh the privacy issues of information disclosure. In essence, it's about risk balancing, not risk elimination. Hence, the total consumer surplus (CS) can be written as follows:

$$CS = \frac{d_n^2 + 2\delta d_n d_r + \delta d_r^2}{2Q} + d_r\left(\lambda t\left(1+\delta\right) - p_0\right)$$

The above model considers the differences in willingness to pay the price, the utility coming from the openness of the market, and the disutility caused by the privacy issues (Malik et al. (2022); Yadav et al. (2023); Manzoor et al. (2025)).

Assumption 9: R&D responsibility falls to the OEM because they own the basic low-carbon technology itself, while ownership of the VER goes to the remanufacturer, because carbon credits will be allocated on the true emissions savings of the remanufacturing process.

The following proposed assumptions take care of the heterogeneity of consumers, remanufacturing economics, environmental factors, and technology aspects of using BCT. Combined together, these assumptions formulate a rather strong framework to simulate both strategic choice on outsourcing and lawful remanufacturing under a carbon-regulated market and BCT capabilities.

4 Model Formulation and Solution

Four hierarchical models were constructed from two types of remanufacturing governance strategies (outsourced and franchised) and the presence of VERM. Two models have no VERM (benchmark model), while two models include incentives to participate in VERM (extended model). Each of these four models runs under a BCT-facilitated environment in a BCT-enabled OEM-led Stackelberg game, where the OEM commits on a decision variable (e.g., price, fee, and emission-reduction effort) and IR chooses his/her decision variable (remanufacturing volume) next. As it assumes the BCT is adopted, this paper focuses on the implications of the VERM and governance structures but not on the issues regarding adoption of BCT. The central framework in the analysis adopts the OEM-led emission-reduction R&D and then, in the extension section, examines IR-led innovation to illustrate how the choice of who leads the technological investment changes with varying VER prices.

4.1 BCT: Benchmark models without VERM (Model *BO* and Model *BA*)

In the outsourcing benchmark model (Model BO), the OEM assigns the complete remanufacturing process to the IR but keeps the responsibility for selling the new and remanufactured products. The first step of the process is the determination by the OEM of the new product quantity (d_n^{BO}) and the setting of the outsourcing fee (w_r^{BO}), which is the payment made to the IR for each remanufactured product. Then, the IR will lay down the quantity of remanufactured products (d_r^{BO}). The BCT application guarantees not only the origin but also the environmental quality of remanufactured products, with cost sharing governed by the parameter $\rho \in [0, 1]$, indicating the OEM's share of the financial responsibility.

Sequence: OEM sets d_n^{BO} and w_r^{BO} (unit fee). IR chooses d_r^{BO}.

The OEM's advantage of the Model BO is expressed in terms of profit margin as follows:

$$\Pi_M^{BO} = \left(p_n^{BO} - c\right) d_n^{BO} + \left(p_r^{BO} - w_r^{BO}\right) d_r^{BO} - \rho \left(c_a d_r^{BO} + F_c\right) \tag{1}$$

In the first part, $(p_n^{BO} - c)d_n^{BO}$ is the profit that comes from the sale of new products, where p_n^{BO} is the price at which the product is sold and c is the cost of production per unit. The next part, $(p_r^{BO} - w_r^{BO})d_r^{BO}$, indicates the OEM's profit on remanufactured products, since it gets the price p_r^{BO} and, at the same time, incurs a cost of w_r^{BO} to the IR. The last term, $\rho(c_a d_r^{BO} + F_c)$, portrays the OEM's sharing of BCT costs, where c_a denotes the variable BCT cost per remanufactured product and F_c is the fixed BCT development cost.

The equation below illustrates the profit of the IR:

$$\Pi_R^{BO} = \left(w_r^{BO} - c + s\right) d_r^{BO} - g \left(d_r^{BO}\right)^2 - (1 - \rho) \left(c_a d_r^{BO} + F_c\right) \tag{2}$$

Here, the part $\left(w_r^{BO} - c + s\right) d_r^{BO}$ indicates the unit margin for the IR, where s stands for the cost savings coming from remanufacturing instead of producing new products. The second component, $g \left(d_r^{BO}\right)^2$, reflects the escalating collection costs for cores, with g representing the non-linearity parameter. The final term indicates the IR's contribution to the costs incurred by the BCT.

In the authorization framework referred to as Model BA, the OEM grants the IR the authority to remanufacture and independently market remanufactured products, contingent upon the payment of a per-unit authorization fee, denoted as f^{BA}. An OEM is only involved in the production and sale of new products. First, it determines d_n^{BA} and f^{BA}, and then the IR determines the remanufacturing quantity d_r^{BA}.

Sequence: OEM sets d_n^{BA} and f^{BA} (license fee). IR chooses d_r^{BA}.

The profit of the OEM in this instance is as follows:

$$\Pi_M^{BA} = \left(p_n^{BA} - c\right) d_n^{BA} + f^{BA} d_r^{BA} - \rho \left(c_a d_r^{BA} + F_c\right) \tag{3}$$

In this context, the OEM generates revenue through the sales of new products and the authorization fees associated with the IR. The cost-sharing model for business collaboration and BCT is structured in a manner identical to that of the outsourcing scenario.

The profit of the IR is as follows:

$$\Pi_R^{BA} = \left(p_r^{BA} + s - \left(c + f^{BA}\right)\right) d_r^{BA} - g \left(d_r^{BA}\right)^2 - (1 - \rho) \left(c_a d_r^{BA} + F_c\right) \tag{4}$$

The IR accounts for all revenue generated from the sales of remanufactured products and remits the authorization fee for each unit to the OEM. Running the cost structure, including core collection and BCT implementation, remained unchanged.

4.2 BCT: Models with VERM (Model *O* and Model *A*)

This study extends the benchmark models by incorporating VERM into the remanufacturing framework. The OEM, acting as the Stackelberg leader, determines the emission-reduction level E, while the IR subsequently decides the remanufacturing quantity. The cost of emission-reduction R&D is modeled as $\frac{\xi E^2}{2}$, where ξ denotes the technological difficulty of innovation. Total verified emission reductions are given by Ed_r and generate carbon-credit revenue at an exogenous VER price p_e. BCT supports the verification and traceability of emission reductions, facilitating credible VER transactions within the remanufacturing SC.

In the outsourcing model incorporating VERM (Model O), the OEM determines the emission reduction level E^O, the quantity of new products d_n^O, and the outsourcing fee w_r^O. Subsequently, the IR ascertains the remanufacturing quantity d_r^O. The profit attained by the OEM is expressed as follows:

Sequence: OEM chooses E^O, d_n^O, w_r^O. IR chooses d_r^O.

$$\Pi_M^O = \left(p_n^O - c\right) d_n^O + \left(p_r^O - w_r^O\right) d_r^O - \frac{\xi \left(E^O\right)^2}{2} - \rho \left(c_a d_r^O + F_c\right) \tag{5}$$

In contrast to Model BO, the new term $\frac{\xi (E^O)^2}{2}$ signifies the OEM's investment in R&D aimed at emission reduction. All other components of the model continue to exhibit structural similarities. The IR generates supplemental revenue through the sale of carbon credits, and its profit function is defined as follows:

$$\Pi_R^O = \left(w_r^O - c + s\right) d_r^O + \left(p_e E^O\right) d_r^O - g \left(d_r^O\right)^2 - (1-\rho) \left(c_a d_r^O + F_c\right) \tag{6}$$

The second term, $p_e E^O d_r^O$, means the value of the VERs that IR buys from the OEM, which come from the OEM's low-carbon remanufacturing technology.

In the authorization model with VERM (Model A), the OEM authorizes the license for both product and emission-reduction technology to the IR. The OEM specifies the emission reduction level E^A, the number of new products d_n^A, and the charge f^A. Then the IR decides the remanufacturing amount d_r^A.

Sequence: OEM sets E^A, d_n^A, f^A. IR chooses d_r^A. The profit of the OEM in this instance is as follows:

$$\Pi_M^A = \left(p_n^A - c\right) d_n^A + f^A d_r^A - \frac{\xi \left(E^A\right)^2}{2} - \rho \left(c_a d_r^A + F_c\right) \tag{7}$$

The profit of the IR is as follows:

$$\Pi_R^A = \left(p_r^A + s - \left(c + f^A\right)\right) d_r^A + \left(p_e E^A\right) d_r^A - \left(g \left(d_r^A\right)^2 + (1-\rho) \left(c_a d_r^A + F_c\right)\right) \tag{8}$$

In this scenario, IR generates income from product sale and carbon-credit sales and has to pay an authorization fee to OEM. OEM can cover the costs of reduction investment via licensing revenues. Moreover, BCT affects the model not only through the implementation cost $(c_a d_r + F_c)$ but also by affecting demand for remanufactured products and VER generation.

5 Comparative Analysis of Models

Based on the results shown in Section 4, we now outline the main benefits of using the VERM for both outsourcing and authorization remanufacturing models supported by BCT. Comprehensive derivations and proofs are included in B.

5.1 Comparison and analysis of equilibrium outcomes

Proposition 1 *In Model O and BO,*

1. *The optimal quantities of two products satisfy* $d_n^{BO*} > d_n^{O*}$ *and* $d_r^{O*} > d_r^{BO*}$.
2. *The optimal outsourcing fees and retail prices satisfy* $p_n^{BO*} = p_n^{O*}$, $p_r^{O*} < p_r^{BO*}$, *and* $w_r^{O*} < w_r^{BO*}$.
3. *The optimal profits of the OEMs and IRs satisfy* $\Pi_M^{O*} > \Pi_M^{BO*}$ *and* $\Pi_R^{O*} > \Pi_R^{BO*}$.

Proposition 1 implies that the implementation of VERM in the outsourcing model has the effect of remanufactured products over new ones: Model O generates a larger output of remanufactured products and a smaller output of new products in comparison to Model BO, mainly because the IRs get extra income from the VER credits. This encouragement also brings about a reduction in the outsourcing fees and the prices for remanufactured products, while the price for the new product remains constant as it continues to be the OEM's main source of profit. Even though new product sales are falling, OEM and IR profits are rising; IRs are directly profiting from the carbon-credit revenue, and OEMs are benefiting from better cost allocation and more remanufacturing activity. Therefore, VERM has opened up the door to a win-win situation and a stronger perception of remanufacturing under BCT.

Proposition 2 *In Model A and BA,*

1. *The optimal consumption quantities of the two products demand relate as* $d_n^{BA*} > d_n^{A*}$ *and* $d_r^{A*} > d_r^{BA*}$.
2. *The optimal prices of the authorization fee and retail prices coincide in* $p_n^{BA*} = p_n^{A*}$, $p_r^{A*} < p_r^{BA*}$, *and* $f^{A*} < f^{BA*}$.
3. *The optimal profits of the OEMs and retailers are such that* $\Pi_M^{A*} > \Pi_M^{BA*}$ *and* $\Pi_R^{A*} > \Pi_R^{BA*}$.

Proposition 2 reveals that the adoption of VERM in the authorization model has the effect of increasing the demand for remanufactured products because the IRs would have increased their remanufacturing activities in order to obtain VER credits. In contrast to the case of outsourcing, the OEM increases the authorization fee to recoup its costs of emission-reduction research and development and to take part in the VERM value. The prices of new products stay the same for branding reasons, whereas the prices of remanufactured products decrease because of the increased competition and the division of costs under the authorization contract. Consequently, both OEMs and IRs realize higher profits: the former through increased authorization fees and higher efficiency, and the latter through the sale of carbon credits; thus, a win-win situation arises, which is in line with the findings of Cao and Shao (2025).

5.2 Sensitivity analysis

This section examines the equilibrium implications of VERM in BCT-enabled remanufacturing SCs. Building on the equilibrium solutions derived in Section 4, we investigate how the introduction of VERM affects firms' operational decisions, emission-reduction efforts, and profitability under the outsourcing and authorization models. We also examine the sensitivity of equilibrium outcomes to the VER price, the R&D difficulty coefficient, and consumer acceptance of remanufactured products.

The analysis is organized into three parts. First, we investigate the effects of the VER price and R&D difficulty coefficient on equilibrium decisions and firm profitability, followed by an examination of how consumer acceptance influences the IR's profit and the OEM's emission-reduction effort. Second, we evaluate the implications of VERM for consumer surplus under the two governance structures. Third, we compare the environmental performance of the VERM-enabled models with their respective benchmark models without VERM. This analysis identifies the conditions under which VERM can improve economic performance and reduce environmental emissions, highlighting the roles of market incentives, consumer acceptance, and SC governance.

Proposition 3 *For* $l \in \{O, A\}$, *the effects of the VER price* p_e *and the R&D difficulty coefficient* ξ *on the equilibrium outcomes of the BCT-enabled remanufacturing SC are characterized as follows:*

1. *An increase in the VER price* p_e *strengthens the economic incentive for emission reduction. Provided that the relevant equilibrium conditions hold, the equilibrium emission-reduction effort, remanufacturing quantity, and profits of both firms increase with* p_e. *Specifically,*

$$\frac{\partial E^{l*}}{\partial p_e} > 0, \qquad \frac{\partial d_r^{l*}}{\partial p_e} > 0, \qquad \frac{\partial \Pi_M^{l*}}{\partial p_e} > 0, \qquad \frac{\partial \Pi_R^{l*}}{\partial p_e} > 0.$$

2. *An increase in the R&D difficulty coefficient ξ raises the cost of achieving emission reduction and consequently decreases the equilibrium emission-reduction effort:* $\dfrac{\partial E^{l*}}{\partial \xi} < 0$.
3. *The effects of p_e and ξ on other equilibrium outcomes, including the outsourcing fee, the remanufactured-product price, the new-product quantity, and the IR's profit under the authorization model, depend on the corresponding equilibrium expressions and parameter conditions. Their comparative-static effects must therefore be established separately by evaluating the relevant partial derivatives.*

Proposition 3 distinguishes the market incentive for emission reduction from the technological difficulty of achieving it. A higher VER price increases the economic return from verified emission reductions and may encourage greater abatement effort and remanufacturing activity. Conversely, a higher R&D difficulty coefficient increases the cost of emission-reduction investment, thereby discouraging abatement effort. The effects on other prices, quantities, and individual firms' profits depend on the strategic interactions between the OEM and the IR and must be determined from the equilibrium solutions of the outsourcing and authorization models.

Theorem 1 *Under the BCT-enabled remanufacturing SC, the effects of the parameter δ on the IR's equilibrium profit and the equilibrium emission-reduction effort are characterized as follows for $l \in \{O, A\}$, where O and A denote the outsourcing and authorization models, respectively.*

1. *The effect of δ on the IR's equilibrium profit, Π_R^{l*}, depends on the value of δ, the VER price p_e, and the initial VER price p_0. Specifically,*

$$\frac{\partial \Pi_R^{l*}}{\partial \delta} = \begin{cases} > 0, & \text{if } \delta > \dfrac{1}{2}, \\ > 0, & \text{if } \delta < \dfrac{1}{2}, \quad p_e < \overline{p_e}^{\,l}, \quad p_0 < \overline{p_0}^{\,l}, \\ < 0, & \text{if } \delta < \dfrac{1}{2} \text{ and at least one of the threshold conditions is violated.} \end{cases}$$

   Here, $\overline{p_e}^{\,l}$ and $\overline{p_0}^{\,l}$ denote the corresponding threshold values in model l.
2. *The effect of δ on the equilibrium emission-reduction effort, E^{l*}, is non-monotonic and is characterized by*

$$\frac{\partial E^{l*}}{\partial \delta} = \begin{cases} > 0, & \text{if } \underline{\delta_l} < \delta < \overline{\delta_l}, \\ < 0, & \text{if } \delta < \underline{\delta_l} \text{ or } \delta > \overline{\delta_l}, \end{cases}$$

   where $\underline{\delta_l}$ and $\overline{\delta_l}$ denote the lower and upper threshold values of δ in model l, respectively.

Theorem 1 demonstrates that the effect of δ on the IR's profitability depends on consumer acceptance and the prevailing VER-market conditions. Accordingly, when $\delta > \dfrac{1}{2}$, raising δ will lead to a higher equilibrium profit of the IR; When *delta* $< \dfrac{1}{2}$, such a positive effect still exists if both the initial VER price and the VER price are higher than their respective thresholds. If either of the conditions is not met, then the equilibrium profit of the IR is decreasing in delta, also under the equilibrium conditions. With the emission-reduction effort, the impact of δ is also non-monotonic. Specifically, when δ is greater than the lowest threshold level, $\underline{\delta_l}$, and lower than the upper threshold level, $\overline{\delta_l}$, the level of effort is higher; otherwise, the level of effort is lower. This finding suggests that the environmental benefits associated with changes in consumer acceptance are not necessarily monotonic and depend on the prevailing market conditions and the strategic decisions of the SC members.

5.2.1 Threshold-driven behavioral regions in BCT-enabled VERM systems

Figure 2 illustrates the threshold-dependent effects of consumer acceptance of remanufactured products, δ, and the voluntary emission reduction market (VERM) price, p_e, on the equilibrium emission-reduction effort and the IR's profitability under the outsourcing and authorization models. The numerical analysis uses the parameter values $Q = 1000$, $c = 900$, $s = 700$, $g = 4$, $\beta = 0.6$, $\lambda = 0.9$, $t = 1.5$, $p_0 = 2$, $c_a = 60$, $F_c = 100$, $\rho = 0.8$, $\kappa = 0.98$, $\xi = 200$, and $F^{OT} = 80$. These values are adopted for numerical

illustration, following the parameterization approach used in related studies Huang and Wang (2017); Li et al. (2023); Jalapathy and Unnissa (2025).

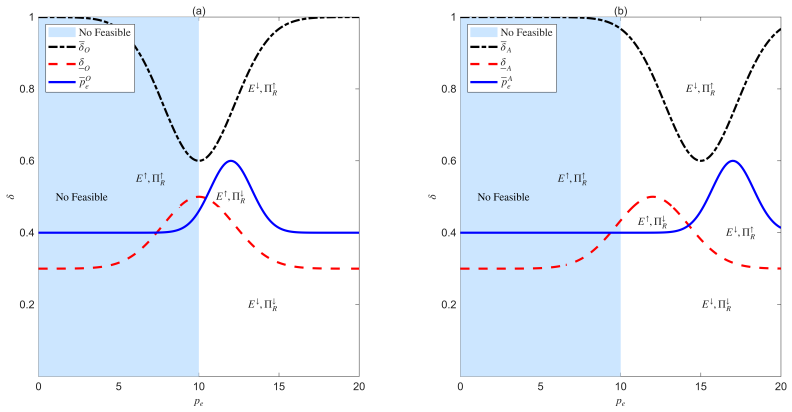


Fig. 2 Sensitivity of equilibrium emission-reduction effort and IR profit to δ and p_e.

Figure 2 illustrates how the interaction between consumer acceptance and the VER price influences the equilibrium outcomes of Models O and A. Theorem 1 illustrates that the influence of the variable δ on the profitability of the IR is contingent upon consumer acceptance as well as the existing conditions within the VER market. Specifically, when δ exceeds $\frac{1}{2}$, an increase in δ will result in enhanced equilibrium profits for the IR. Conversely, when δ is less than $\frac{1}{2}$, a positive effect on profitability may still occur, provided that both the initial VER price and the current VER price are above their respective thresholds. Should either of these conditions fail to be satisfied, the equilibrium profit of the IR will decline as δ increases, even within the established equilibrium framework. Furthermore, in the context of emission-reduction efforts, the relationship between δ and the level of effort exhibited is characterized by a non-monotonic pattern. Specifically, when δ is situated between the lower threshold level, $\underline{\delta_l}$, and the upper threshold level, $\overline{\delta_l}$, the level of effort is elevated; otherwise, the level of effort decreases. This observation underscores that the environmental advantages derived from variations in consumer acceptance are not necessarily consistent and are influenced by prevailing market conditions as well as strategic decisions made by members of the SC.

The profitability of the IR sector is significantly influenced by the interplay between the parameter δ and the conditions prevailing in the VER market. Specifically, when δ exceeds $\frac{1}{2}$, the equilibrium profit of the IR sector exhibits an increase in relation to δ, contingent upon the conditions outlined in Theorem 1. Conversely, when δ is less than $1/2$, the nature of this relationship is influenced by whether the VER price p_e and the initial price parameter p_0 remain beneath their respective thresholds, denoted as $\overline{p_e}^{\,l}$ and $\overline{p_0}^{\,l}$. These thresholds delineate parameter regions in which enhanced consumer acceptance yields benefits for the IR sector, as opposed to scenarios where profitability may diminish. Overall, Figure 2 underscores the necessity of concurrently considering consumer acceptance and VER-market conditions when assessing incentives for emission reduction in BCT-enabled remanufacturing SCs. The findings suggest that policies aimed at fostering consumer confidence and validated emission reductions should recognize the threshold-dependent responses of firms, rather than presupposing that heightened consumer acceptance will invariably result in improved profitability or environmental performance.

5.3 Consumer surplus

Consumer surplus (CS) is the difference between the willingness-to-pay and the actual prices paid for new and remanufactured products. When the remanufacturing SC adopts BCT, the consumer surplus may also include the value of traceability and transparency of the remanufactured product and the disutility of privacy concerns.

Based on the specified demand system, consumer surplus can be expressed as

$$CS = \int_0^{d_n} \left(p_n(x) - p_n\right) dx + \int_0^{d_r} \left(p_r(x) - p_r\right) dx + \lambda t(1+\delta) d_r - p_0 d_r.$$

The first two terms represent the conventional surplus associated with purchases of new and remanufactured products, respectively. The remaining terms capture the additional utility associated with BCT-enabled transparency and the disutility associated with privacy concerns.

Using the inverse-demand functions, the closed-form expression for consumer surplus is reported as

$$CS = \frac{d_n^2 + 2\delta d_n d_r + \delta d_r^2}{2Q} + d_r\big[\lambda t(1+\delta) - p_0\big].$$

This expression should be used only after verifying that it follows from the inverse-demand functions and the underlying consumer-utility specification.

Proposition 4 *The changes in consumer surplus associated with the introduction of VERM satisfy*

$$CS^{O*} - CS^{BO*} < 0, \qquad CS^{A*} - CS^{BA*} > 0.$$

Proposition 4 indicates that the effect of VERM on consumer surplus differs between the outsourcing and authorization models. Under outsourcing, the introduction of VERM is associated with a reduction in consumer surplus, whereas under authorization it is associated with an increase. These results can be interpreted as variations in the equilibrium quantities and prices that would be achieved under each of the two structures of governance. In the outsourcing model, changes in the VER price affect the IR's incentives to remanufacture and the OEM's incentives to adjust the outsourcing fee. By comparing the resulting equilibrium of quantities and prices for new and remanufactured products, it can be shown whether the increased gains from transparency are offset by decreases in price or privacy. In the authorization model, adjustments to the authorization fee influence the IR's remanufacturing decisions and the resulting market outcomes. The combined effects of these strategic responses and the BCT-related utility terms determine the change in consumer surplus.

As a result, it is insufficient to draw conclusions on consumer-welfare effects of VERM from differences in contractual fees. Instead, the equilibrium price and quantity for both models need to be compared, and the same expression should be used for consumer surplus when comparing the welfare consequences of each model. Proposition 4 emphasizes the relevance of governance structure for interpreting consumer-welfare effects of VERM in BCT-enabled remanufacturing SCs.

5.4 Analysis of environmental impact

The environmental performance of the SC is dependent on remanufacturing emissions, participation of VERM and demand effects of BCT. Let E^c denote the emissions from manufacturing a new product and $\beta \in (0,1)$ represent the relative emission intensity of remanufacturing. Here, β captures environmental efficiency, whereas δ reflects consumer acceptance of remanufactured products. Therefore, without emission reduction, each remanufactured unit generates βE^c emissions. Under VERM, the OEM invests in emission-reduction technology, achieving a per-unit reduction E^l under model $l \in \{BO, BA, O, A\}$. In the benchmark models (BO and BA), $E^{BO} = E^{BA} = 0$, whereas in Models O and A, E^O and E^A are determined endogenously by the OEM.

Thus, the total amount of emissions coming from model m can be written as follows:

$$AE^l = E^c d_n^l + \left(\beta E^c - E^l\right) d_r^l$$

Here, d_n^l and d_r^l represent the new and remanufactured products' equilibrium quantities, respectively. For the sake of comparison, if the total output of products $(d_n^l + d_r^l)$ were purely made of new products, the baseline emissions would be equal to $E^c(d_n^l + d_r^l)$. The consequent reduction of environmental impact in relation to the baseline is then represented by:

$$\Delta^l = E^c(1-\beta)d_r^l + E^l d_r^l$$

The initial term illustrates the structural gains that accrue from remanufacturing, and the subsequent term indicates the technologically caused pollution reduction benefits that are only paired with VERM. Additionally, for Models O and A, the carbon credits produced are measured as $V^l = E^l d_r^l$, and these can be sold in the carbon market for a price of p_e.

To assess the environmental advantages linked to the implementation of VERM, the difference in total emissions amongst every VERM model and its related benchmark is expressed as follows:

$$\Delta AE^O = AE^{O*} - AE^{BO*} \text{ and } \Delta AE^A = AE^{A*} - AE^{BA*}$$

The superscript $*$ indicates the equilibrium values. The total expression of emissions can be derived based on the definitions laid down before, as shown in:

$$\begin{aligned}\Delta AE^O &= E^c\left[\left(d_n^O + d_r^O\right) - \left(d_n^{BO} + d_r^{BO}\right)\right] - \left(\Delta^O - \Delta^{BO}\right)\\ \Delta AE^A &= E^c\left[\left(d_n^A + d_r^A\right) - \left(d_n^{BA} + d_r^{BA}\right)\right] - \left(\Delta^A - \Delta^{BA}\right).\end{aligned}$$

When the total output is kept unchanged in the different scenarios, the first terms become insignificant, and the changes can be reduced to

$$\Delta AE^O = \Delta^{BO} - \Delta^O \text{ and } \Delta AE^A = \Delta^{BA} - \Delta^A.$$

A negative value of ΔAE indicates that the VERM model generates lower total emissions than its benchmark model and therefore represents an environmental improvement, whereas a positive value indicates environmental deterioration. This is precisely the case, as $E^O, E^A > 0$ and the combined effect of consumer acceptance and BCT transparency benefits increases demand for remanufactured products d_r; therefore, $\Delta AE^O < 0$ and $\Delta AE^A < 0$, which means that both outsourcing and authorization under VERM have better environmental performance than their respective benchmarks.

Finally, BCT is of great importance in the whole process of transformation, and thus the impact of transparency and trust $(1+\delta)\lambda t$ is not only that consumer acceptance of remanufactured products is increased, but privacy concerns are also addressed through the adjusted cost $p_c = p_0 - \delta\lambda t$. Consequently, the equilibrium demand for remanufacturing d_r starts to rise, which not only results in a structural reduction of emissions but also expands the VER credits in Models O and A. The best thing is that even though BCT causes an increase in operation and maintenance costs (c_a, F_c), the environmental benefits still come out on the positive side as long as the trust effect does not fade out.

Theorem 2 *The environmental impact of VERM relative to the corresponding benchmark models is characterized as follows:*

1. *If $\beta < \delta$, then $\Delta AE^{O*} < \Delta AE^{A*} < 0$.*
2. *If $\beta > \delta$, the signs of ΔAE^{O*} and ΔAE^{A*} depend on the VER price p_e and the corresponding environmental thresholds.*
3. *If $\beta > \delta$ and $p_e > p_{e_1}^O$, then $\Delta AE^{O*} < 0$. Similarly, if $\beta > \delta$ and $p_e > p_{e_2}^A$, then $\Delta AE^{A*} < 0$.*

Theorem 2 points out that the VERM's environmental footprint in BCT remanufacturing SCs is contingent upon three major factors: the carbon intensity of remanufacturing, demand from consumers, and the price of VERs. Similar to what happens with authorized and outsourced channels, in the case of remanufacturing being carbon-intensive relative to consumer preference states ($\beta < \delta$), total emissions are reduced since VERM pushes the output of high-quality recycled products even though every single unit has weak carbon savings; the suppressed emissions from new products are not sufficient to counterbalance this expansion, hence reaching $\Delta AE^{O*} < 0$ and $\Delta AE^{A*} < 0$. When remanufacturing is carbon-efficient ($\beta > \delta$), VERM only leads to a decrease in emissions when VER prices are moderate, i.e., when $p_e < p_{e_1}^O$ (for outsourcing) or $p_e < p_{e_2}^A$ (for authorization), because the companies perform remanufacturing at a level of production such that the carbon credits correspond to no emissions plus the actual savings. Nevertheless, once VER prices go beyond these limits, both ΔAE^{O*} and ΔAE^{A*} will be negative again: very high prices of VERs lead to over-remanufacturing and hence, emissions grow even in highly inefficient systems. Collectively, Theorem 2 demonstrates that VERM is neither good nor bad for the environment by definition; its success is totally in the alignment of VER prices with real carbon inefficiency differences, realistic consumer acceptance, and avoiding the scenario of over-remanufacturing driven by incentives.

5.4.1 Comparison analysis of environmental impact

The environmental implications that were discussed through analysis in Theorem 2 are visually supported by Figure 3, which illustrates the dependence of total emissions in both the outsourcing and authorization structures on the relative emission intensities of new and remanufactured products (β, δ) as well as the VER price (p_e). The comparison between β and δ reflects the balance between the environmental-efficiency effect and the demand-expansion effect. A lower β indicates greater emission savings from remanufacturing, whereas a higher δ implies stronger consumer acceptance and market expansion of remanufactured products.

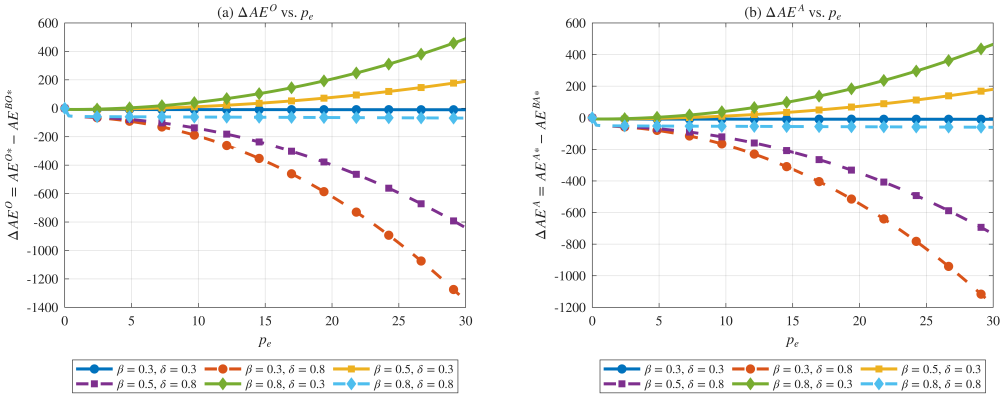


Fig. 3 Comparison analysis of environmental impact.

In the case where remanufacturing is cleaner than new production $(\beta < \delta)$, all curves lie below the axis, which means that VERM always leads to lower emissions regardless of p_e. On the other hand, when remanufacturing is more emission-intensive $(\beta > \delta)$, the sign of ΔAE starts to depend on the price of the VER: emissions are reduced only at very low VER prices, but they increase once p_e goes beyond the threshold set by the particular model, which means that excessively high VER prices can lead to high-emission remanufacturing being more lucrative than new production and the benefits of reduced new production being offset. The authorization model has slightly higher thresholds and a milder slope, which can be interpreted as a sign of better environmental control than outsourcing. To sum up, the figure reveals that the environmental impact of VERM is very much determined by the context; it is advantageous when remanufacturing is really cleaner, but it can be detrimental when emission intensities are not properly aligned with strong VER incentives.

6 Comparative analysis of two models under VERM

This section compares outsourcing with the authorization structure of the VERM. The analysis emphasizes equilibrium decisions, profitability, consumer welfare, and environmental outcomes. Both models operate within a BCT remanufacturing SC, where BCT enhances transparency, verifies emission reductions, and impacts consumer trust as well as perceptions of privacy. Comprehensive proofs for this section are located in C.

Proposition 5 *In Model O and A:*

1. *The optimal quantities of new products and remanufactured products face the following relationships: $d_n^{O*} < d_n^{A*}$ and $d_r^{O*} > d_r^{A*}$.*
2. *The optimal unit carbon emission reduction of remanufactured products faces the following relationships: $E^{O*} > E^{A*}$.*

Proposition 5 demonstrates significant structural contrasts between Models O and A under VERM. The OEM in Model O contracts out remanufacturing to an IR, which is also a competitor in the same market, resulting in the OEM limiting the production of new products so as not to cause an overlap

in the market, while the IR is increasing its remanufacturing output because of the higher profitability driven by VERM; thus $d_n^{O*} < d_n^{A*}$ and $d_r^{O*} > d_r^{A*}$. On the other hand, Model A puts the OEM in charge, who can only permit remanufacturing to the point that it does not affect new product sales, hence leading to higher new product output and lower remanufacturing volumes. The proposition also indicates that the unit emission reduction is larger in Model O, as the process of remanufacturing done by the IR allows the OEM to take advantage of the IR's efficiency and also the outsourcing fees, thus making it more attractive for the OEM to invest in abatement. In the case of authorization, the OEM gets most of its income from new products and only a small portion from remanufactured ones, which lessens the incentive to enhance environmental performance. Therefore, outsourcing, by its very nature, leads to greater abatement and more environmentally friendly production practices under VERM.

Proposition 6 *In Models O and A:*

1. *The highest possible profits of the OEM have the following relationship:* $\Pi_M^{O*} > \Pi_M^{A*}$.
2. *The impact of price and investment coefficient on the OEM's profit in Model O is larger than in Model A, i.e.,* $\left|\frac{\partial \Pi_M^{O*}}{\partial p_e}\right| - \left|\frac{\partial \Pi_M^{A*}}{\partial p_e}\right| > 0$ *and* $\left|\frac{\partial \Pi_M^{O*}}{\partial \xi}\right| - \left|\frac{\partial \Pi_M^{A*}}{\partial \xi}\right| > 0$.

Proposition 6 illustrates that the scenario Model O is generally more profitable for the OEM than the Model A scenario. The remanufactured products are not only the source of the OEM's higher profits in Model O but also the challenge of coordinating investments in emission reduction and product pricing, as well as the area of fees and total profit sharing among the parties involved. On the contrary, Model A does not allow the OEM to benefit directly from selling remanufactured products apart from the minimal authorization fee being paid to the OEM, which thus reduces the OEM's financial gain from VERM. In addition, the OEM's profits through Model O are more sensitive to changes in VER price and R&D investment (ξ) because these factors influence the remanufactured product directly. Model A is not so sensitive because the revenues come mainly from new products. In general, outsourcing gives higher profits but at the same time greater exposure to the market-based VERM dynamics, which makes Model O more profitable and more dependent on the environmental incentives.

It is worth emphasizing that, although the VER revenue is directly earned by the IR, the OEM still has economic incentives to invest in emission-reduction technologies. First, higher emission reductions increase the attractiveness of remanufactured products under the BCT-enabled VERM framework, thereby stimulating remanufacturing demand and enlarging the OEM's outsourcing-related revenues. Second, improved environmental performance enhances the OEM's sustainability reputation and strengthens consumer trust in the product ecosystem. Third, the OEM can strategically appropriate at least some of the environmental value resulting from its technological investments through the outsourcing fee and contractual strategies. Hence, the separation between R&D investment and VER revenue does not necessarily correspond to the separate costs of the investment and the economic benefits.

Unlike conventional outsourcing versus authorization comparisons, the profitability differences identified in Proposition 6 are additionally influenced by the interaction between emission-reduction investments, carbon-credit revenues, and BCT verification. Under VERM, emission reductions create tradable environmental value through VER credits, while BCT provides the transparency and traceability necessary to verify and monetize these reductions. Therefore, the economic feasibility of outsourcing and authorization schemes will depend on the sharing of environmental benefits, emission-reduction costs, and carbon-credit incentives among SC members, as well as on the production and remanufacturing decisions. This is an added feature in the incentive allocation mode that regular remanufacturing models don't have unless BCT and VERM are clearly specified.

6.1 Coordination contracts

In the proposed framework, BCT performs a verification function that is essential for the operation of VERM. Voluntary carbon markets require reliable evidence that reported emission reductions have actually been achieved. By providing tamper-resistant records of production, remanufacturing, and emission-reduction activities, BCT improves the credibility and auditability of VER generation. Therefore, BCT affects not only product-market decisions through consumer trust but also environmental-investment decisions through the verification and monetization of carbon reductions. Consistent with prior research by Swami and Shah (2013) and Adhikari and Bisi (2020), which underscores the importance of centralized decision-making as a standard for designing coordinating contracts, this study first evaluates the centralized model of our BCT-enabled remanufacturing SC. Under centralized

control, the OEM and the IR operate as a single entity, striving to maximize overall system profitability rather than their individual gains. The profit function under centralized management is expressed as follows:

$$\Pi_k^C = \left(p_n^C - c\right) d_n^C + \left(p_r^C - c + s\right) d_r^C - \frac{\xi (E^C)^2}{2} + \left(p_e E^C\right) d_r^C - \left(c_a d_r^C + F_c\right) - g\left(d_r^C\right)^2 \tag{9}$$

This equation combines the revenues from the sales of new products, remanufactured products, and carbon credits and deducts the expenses, which include production, collection, emission-reduction research and development, and BCT implementation (Sharma et al. (2024)). The current model differs from the one presented by Cao and Shao (2024) in that it also takes into account the adoption and use of BCT-related costs ($c_a d_r^C + F_c$) that not only support traceability but also create consumer trust and strengthen the credibility of VERs.

Proposition 7 delves into the equilibrium results that pertain to decentralized decision-making and contrasts them with the outcomes of centralized decision-making processes.

Proposition 7 *In the remanufacturing model of three types:*

1. $E^{C*} > E^{O*} > E^{A*}$, $d_n^{A*} > d_n^{O*} > d_n^{C*}$, $d_r^{C*} > d_r^{O*} > d_r^{A*}$, *and* $\Pi_k^{A*} > \Pi_k^{O*} > \Pi_k^{C*}$.
2. E^{C*}, d_r^{C*} *increase with* p_e, *but decrease with* ξ *and* d_n^{C*}, Π_k^{C*} *is the opposite case.*

Proposition 7 discusses the comparison between the centralized (Model C) and decentralized (Models O and A) BCT remanufacturing structures under the VERM case. The results indicate that centralized decision-making achieves the highest emission-reduction effort and remanufactured output, while minimizing new-product output. Among the decentralized structures, Model O performs better than Model A in terms of emission-reduction effort and remanufactured output. However, the reported profit ranking, $\Pi_k^{A*} > \Pi_k^{O*} > \Pi_k^{C*}$, suggests that the authorization model yields the highest profitability, followed by the outsourcing model, while the centralized model generates the lowest profit. This finding contradicts the conventional expectation that centralized decision-making maximizes overall supply chain profitability. Therefore, the interpretation of the results should be revised to ensure consistency with the equilibrium outcomes, or the corresponding proposition and profit comparisons should be mathematically re-examined. Furthermore, the sensitivity analysis reveals that an increase in the VER price strengthens incentives for emission-reduction investment and remanufacturing activities, whereas higher R&D costs discourage such investments and reduce emission-reduction efforts.

6.2 Numerical analysis of centralized decisions and BCT cost sharing

This subsection numerically examines the effects of the VER price (p_e) and the BCT cost-sharing parameter (ρ) on centralized SC decisions. Figure 4 illustrates how changes in the VER price affect product prices, production quantities, centralized SC profit, and emission-reduction effort. Figure 5 analyzes the sensitivity of these equilibrium outcomes to variations in the BCT cost-sharing parameter. Together, these figures demonstrate the relationships among carbon-credit incentives, remanufacturing decisions, environmental investment, and SC profitability. The numerical findings also emphasize the importance of economic feasibility and consistency between the centralized optimization framework and its implementation.

Figure 4 presents the sensitivity of centralized equilibrium outcomes to changes in the VER price (p_e). As p_e increases, both the price and production quantity of remanufactured products increase, while the quantity of new products decreases. Meanwhile, the price of new products remains constant throughout the considered range. An increase in the VER price provides stronger financial incentives for generating verified emission reductions, thereby increasing the optimal emission-reduction effort E^{C*} within the economically feasible region. Centralized profit initially declines before increasing significantly, suggesting that the profitability of carbon-credit incentives depends on the trade-off between R&D investment costs and additional revenues from remanufacturing and VERs. However, the negative remanufacturing quantities and emission-reduction effort observed at lower VER prices are economically infeasible. Therefore, these regions should be excluded or corrected by enforcing the non-negativity constraints $d_r^{C*} \geq 0$ and $E^{C*} \geq 0$ before interpreting the numerical results.

Figure 5 demonstrates the influence of the BCT cost-sharing parameter (ρ) on centralized equilibrium decisions. As ρ increases, the price of new products remains unchanged, whereas the price and production quantity of remanufactured products increase. In contrast, the quantity of new products

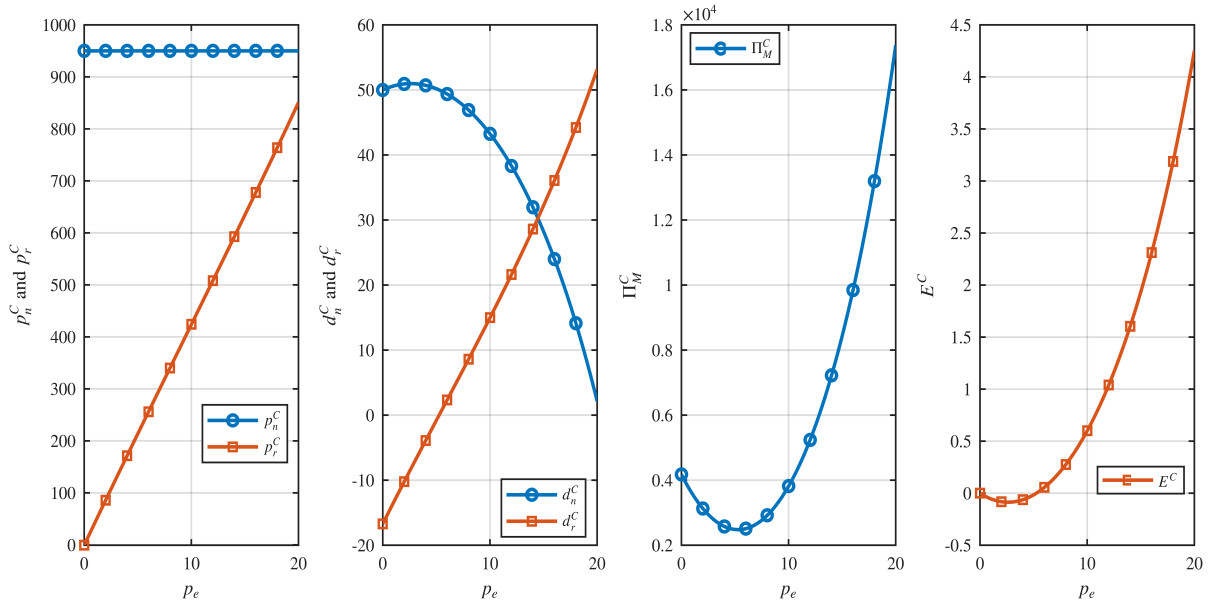


Fig. 4 Sensitivity of centralized equilibrium prices, product quantities, profit, and emission-reduction effort to the VER price p_e.

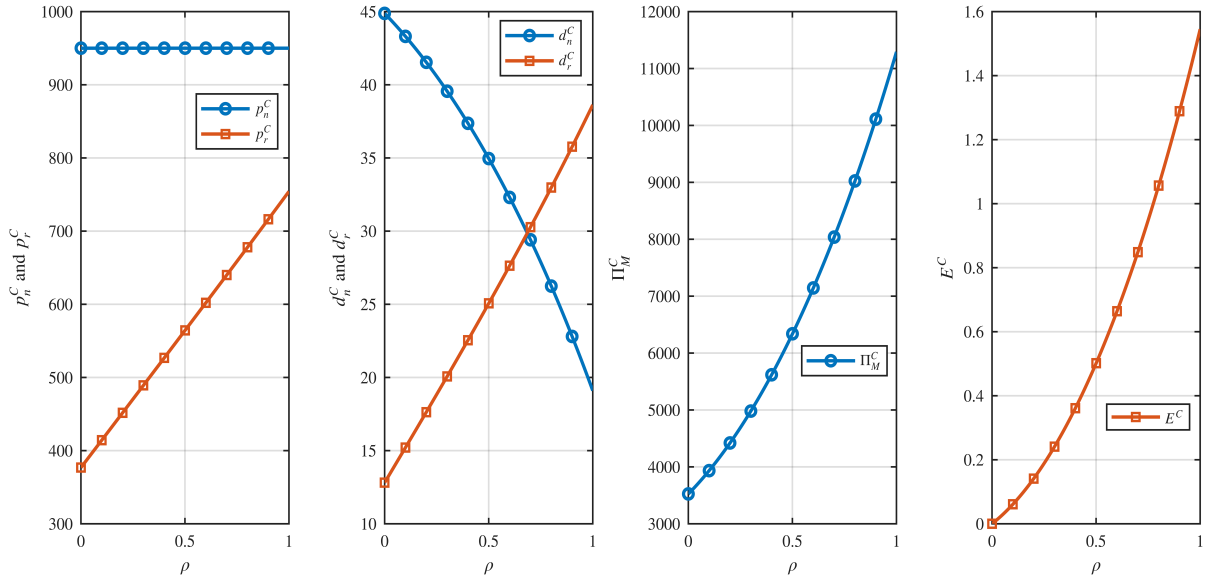


Fig. 5 Sensitivity of centralized equilibrium prices, product quantities, profit, and emission-reduction effort to the BCT cost-sharing parameter ρ.

decreases, indicating a shift in the centralized production strategy toward remanufacturing. The numerical results further indicate that both centralized profit and emission-reduction effort increase with ρ. However, since ρ denotes the OEM's proportion of BCT costs in the decentralized models, its role in centralized decision-making must be interpreted carefully. In the centralized profit formulation, the total BCT cost is given by $c_a d_r^C + F_c$, which is independent of ρ. Consequently, the observed variations in centralized equilibrium outcomes with respect to ρ should be verified using the analytical equilibrium expressions and MATLAB implementation to ensure that the centralized formulation does not unintentionally incorporate decentralized cost-sharing terms.

6.3 Coordination through cost-sharing contracts

Extending the existing studies, Swami and Shah (2013) and Adhikari and Bisi (2020), the authors of this paper suggest a cost-sharing method as the most suitable way to handle BCT remanufacturing SCs. The IR sharing with the OEM of a part of the R&D (research and development) expenses for

reducing emissions is the main idea behind it. Thus, the OEM is encouraged to further work on its emission reductions. The OEM, in turn, agrees to reduce its payment to the IR for outsourcing. The semi-liberalization of the parties' interests through the contractual framework works well because it incorporates the marginal benefits of voluntary emission reductions and it takes care of double marginalization in remanufactured products. In addition, the costs of BCT implementation are shared between the parties, with the OEM and IR taking both the fixed and variable costs according to the mutually agreed ratio ρ. In addition, the need for such coordination arises directly from the BCT-VERM interaction, because the party undertaking emission-reduction investment is not necessarily the same party receiving the resulting carbon-credit revenues.

The proposed cost-sharing contract also serves as a mechanism to mitigate the incentive misalignment inherent in decentralized VERM systems. While the OEM initially bears the responsibility for emission-reduction R&D and the IR receives the VER revenues generated through remanufacturing, the contract allows both parties to redistribute costs and benefits through the R&D sharing parameter (κ) and the outsourcing fee (w_r^{CO}). Consequently, the environmental value generated by emission reductions is partially internalized by both parties, thereby restoring investment incentives and enabling coordinated decision-making.

The OEM, under the cost-sharing contract, has the formal duty to set the new product output (d_n), the outsourcing fee (w_r^{CS}), and also the emission reduction level (E). After that, the IR will decide on the remanufactured products quantity (d_r). The OEM profit function can be defined as below:

$$\Pi_M^{CO} = (p_n^{CO} - c)d_n^{CO} + \left(p_r^{CO} - w_r^{CO}\right) d_r^{CO} - \left(\frac{\xi\,(1-\kappa)\left(E^{CO}\right)^2}{2} + \rho\left(c_a d_r^{CO} + F_c\right)\right). \tag{10}$$

In this equation, the first two terms represent the OEM's profit margins from the sales of new and remanufactured products, respectively. The third term accounts for the OEM's share of R&D expenditure associated with emission-reduction activities, while the final term represents the OEM's allocated share of BCT implementation costs.

In contrast, the IR's profit in the same contract setting is represented by:

$$\Pi_R^{CO} = \left(w_r^{CO} - c + s\right) d_r^{CO} + \left(p_e E^{CO}\right) d_r^{CO} - g\left(d_r^{CO}\right)^2 - \left((1-\rho)\left(c_a d_r^{CO} + F_c\right) + \frac{\kappa\xi\left(E^{CO}\right)^2}{2}\right). \tag{11}$$

In this setup, the first term indicates the IR's remanufacturing profit margin after deducting the outsourcing fee. The second term indicates the increase in revenue coming from the selling of VERs, whereas the rest of the terms consist of the costs of collection represented by a quadratic function, the IR's share of BCT costs, and its proportional share of R&D expenses.

Equations (10) and (11) together portray the decentralized optimization issues that were faced by the OEM and IR during the cost-sharing contract. The contract parameters κ (R&D cost-sharing ratio), w_r^{CO} (adjusted outsourcing fee), and ρ (BCT cost-sharing ratio) can be set to suitable values that will facilitate the SC coordination to replicate the desired centralized outcome. Furthermore, the IRs are allowed to accept a contract only if the expected profit from the contract is greater than the profit that was already there before their acceptance of the contract.

Proposition 8 *The SC can be coordinated when* $d_n^{CO*} = d_n^{C*}$, $d_r^{CO*} = d_r^{C*}$, $E^{CO*} = E^{C*}$, w_r^{CO*}, *and* $\kappa \in (\underline{\kappa}, min\,(\overline{\kappa}, 1))$. *Where*

$$w_r^{CO*} = \frac{{p_e}^2\left((1-2\rho)\,c_a + t\,(1+\delta)\,\lambda + \delta c - p_0\right)}{p_e^2 - \xi A_4} - \frac{\xi A_4\left((1-\rho)\,c_a + c - s\right)}{p_e^2 - \xi A_4}$$

Proposition 8 demonstrates that the BCT remanufacturing SC can be totally synchronized by a well-structured cost-sharing contract. Under it, the contract decisions independently made are considered the same as the ones made at the centralized position, namely, the OEM and the IR would use the optimal new product quantity, remanufactured quantity, and the level of emissions reduction for their respective products. This synchronization is achieved by the correct setting of the outsourcing fee w_r^{CO*} and taking the R&D sharing ratio κ within the feasible range $(\underline{\kappa}, \min(\overline{\kappa}, 1))$. The formulation of w_r^{CO*} not only accounts for emission-reduction incentives but also BCT-related costs, which will, consequently,

aid in the proper division of the marginal advantages of VER technologies and shared costs between the OEM and IR. Therefore, the contract will be successful in eliminating the double marginalization occurring between the two camps, ensuring proper incentive alignment across tiers, and operating the decentralized SC just as the centralized one.

Although cost-sharing contracts are widely used in remanufacturing SCs, the coordination problem examined here is fundamentally broader than that considered in conventional models. Beyond coordinating production and remanufacturing decisions, the proposed contract must also align emission-reduction efforts, carbon-credit revenues generated through VERs, and BCT implementation costs. While BCT enhances consumer acceptance and introduces additional traceability costs, VERM provides economic incentives through revenues obtained from verified emission reductions. Consequently, the resulting incentive structure differs from that of conventional remanufacturing systems. The coordinating outsourcing fee w_r^{CO*} and the R&D cost-sharing ratio κ serve two important purposes: mitigating double marginalization and aligning environmental incentives by appropriately distributing emission-reduction costs, carbon-credit benefits, and digital traceability expenditures among SC members. Therefore, the proposed coordination mechanism addresses a multidimensional incentive-alignment problem arising from the simultaneous integration of BCT and VERM into the remanufacturing SC.

Table 4 The optimal solutions for Model O, C, and CO, where $l \in \{O, C, CO\}$.

	Model O	Model C	Model CO				
	$\kappa = 0.90$	$\kappa = 0.90$	$\kappa = 0.90$	$\kappa = 0.92$	$\kappa = 0.94$	$\kappa = 0.96$	$\kappa = 0.98$
E^{l*}	0.0169	0.0580	0.0580	0.0580	0.0580	0.0580	0.0580
p_n^{l*}	925	925	925	925	925	925	925
p_r^{l*}	270.995	263.137	270.993	270.993	270.992	270.990	270.985
w_r^{l*}	140.542	N/A	60.483	60.483	60.483	60.483	60.483
d_n^{l*}	62.993	51.767	51.766	51.766	51.766	51.766	51.766
d_r^{l*}	40.024	77.445	77.445	77.445	77.445	77.445	77.445
Π_M^{l*}	11.6980 (Lakhs)	12855.617	18040.744	18040.771	18040.798	18040.825	18040.852
Π_R^{l*}	25406.438	N/A	1580.513	1575.870	1565.624	1535.721	1370.583
Π_T^{l*}	11.9521 (Lakhs)	12855.617	19621.256	19616.640	19606.421	19576.545	19411.434
AE^{l*}	1535.843	2353.107	2082.226	2080.741	2078.265	2073.310	2058.4200

The numerical results presented in Table 4 confirm the coordination effect of the suggested contract. The contract, irrespective of the value of κ, actually reproduces the results of the centralized system: the values of $E^{CO*} = E^{C*} = 0.0580$, $d_n^{CO*} = d_n^{C*} = 51.766$, and $d_r^{CO*} = d_r^{C*} = 77.445$ are constant across the board. This confirms that the cost-sharing contract has not only made production decision-making but also emission reductions under the VERM coordinated. It is quite surprising that the coordination outsourcing fee of $w_r^{CO*} = 60.483$ is significantly lower than the fee under Model O, which indicates that the OEM is actually the one reducing the fee as part of the incentive-alignment strategy. The OEM is also winning big from a profitability point of view, as mentioned, since its profit increases from 11.698 lakhs in Model O to around 0.1804 lakhs under the coordinated structure. An increase in κ causes the OEM's profit to rise even more, which means that the R&D expense has become less of a burden.

On the other hand, the IR's profit gets lower as κ increases, as it covers a bigger part of the abatement cost. But still, the IR management contract is enjoying a profit that is higher than the one that would have been obtained through outsourcing of the baseline, which is the precondition for volunteering participation. This is a classic case of cost-sharing mechanisms and is in line with the results found in the literature. The total SC profit under the contract is the same as the centralized benchmark ($\Pi_T^{CO*} = \Pi_T^{C*}$), which indicates that the efficiency is completely restored. Moreover, the SC's environmental performance has become significantly better. The total abatement rises from 0.0169 in Model O to 0.0580 under coordination, which, in turn, causes a great increase in the total environmental impact reduction. The findings definitely point out that cost-sharing agreements not only solve the issue of economic incentives but also contribute to the environmental sustainability of the remanufacturing system with VERM by strengthening it.

Proposition 8 proves that cost-sharing contracts act as a reliable and efficient tool for managing BCT remanufacturing SCs. The OEM and the IR can achieve centralized efficiency, fairly share the

advantages, and improve economic and environmental outcomes to a great extent by sharing the costs of both BCT and the reduction of emissions.

6.4 Two-part tariff contract under BCT and VERM

Under the two-part tariff contract, the OEM charges the IR a per-unit outsourcing fee, denoted by w_r^{OT}, along with a fixed fee F^{OT}. The per-unit fee enables the OEM to capture a portion of the marginal revenue generated through remanufacturing activities, whereas the fixed fee allows the OEM to obtain additional surplus without affecting the IR's production decisions. Furthermore, within the BCT-enabled framework, the contract incorporates a cost-sharing arrangement for blockchain implementation. Specifically, the OEM bears a proportion ρ of the total fixed and variable BCT costs, while the IR is responsible for the remaining proportion $(1-\rho)$.

Formally, the OEM's optimization problem under Model OT, representing the outsourcing arrangement with a two-part tariff contract, is formulated as follows:

$$\max_{d_n^{OT}, w_r^{OT}, F^{OT}, E^{OT}} \Pi_M^{OT} \tag{12}$$

The profit function for the OEM is expressed as:

$$\Pi_M^{OT} = \left(p_n^{OT} - c\right) d_n^{OT} + \left(p_r^{OT} - w_r^{OT}\right) d_r^{OT} + F^{OT} - \left(\frac{\xi \left(E^{OT}\right)^2}{2} + \rho\left(c_a d_r^{OT} + F_c\right)\right) \tag{13}$$

In parallel, the profit function for the IR is defined as:

$$\Pi_R^{OT} = \left(w_r^{OT} - c + s\right) d_r^{OT} + \left(p_e E^{OT}\right) d_r^{OT} - g\left(d_r^{OT}\right)^2 - \left((1-\rho)\left(c_a d_r^{OT} + F_c\right) + F^{OT}\right) \tag{14}$$

In this formulation, the fixed fee F^{OT} is borne entirely by the IR, while both parties share the costs associated with BCT implementation in accordance with the pre-established ratio ρ. Additionally, the OEM internalizes the expenses related to emission-reduction R&D, while the IR continues to profit from the sale of VERs.

Proposition 9 *The SC can be coordinated when* $d_n^{OT*} = d_n^{C*}$, $d_r^{OT*} = d_r^{C*}$, $E^{OT*} = E^{C*}$, w_r^{OT*}, *and* $F^{OT} \in \left[\underline{F^{OT}}, \overline{F^{OT}}\right]$. *Where*

$$w_r^{OT*} = \frac{{p_e}^2\left((1-2\rho)c_a + t(1+\delta)\lambda + \delta c - p_0\right)}{p_e^2 - \xi A_4} - \frac{\xi A_4\left((1-\rho)c_a + c - s\right)}{p_e^2 - \xi A_4}$$

Proposition 9 demonstrates that a two-part tariff contract can fully coordinate the BCT-enabled VERM remanufacturing SC by aligning decentralized decisions with the centralized optimum. Coordination is achieved when the equilibrium decisions satisfy $(d_n^{OT*} = d_n^{C*})$, $(d_r^{OT*} = d_r^{C*})$, and $(E^{OT*} = E^{C*})$. The coordinating outsourcing fee (w_r^{OT*}) aligns the marginal incentives of the OEM and the IR by internalizing the effects of remanufacturing profits, BCT implementation costs, and VER-related revenues, thereby eliminating inefficiencies arising from decentralized decision-making and restoring centralized performance.

The fixed fee F^{OT} serves a distinct purpose in the coordination mechanism. Since it does not affect marginal operational decisions, it is primarily used to distribute the additional SC surplus generated through coordination. The feasible range $[\underline{F}^{OT}, \overline{F}^{OT}]$ is established by the individual rationality constraints of both SC members. Specifically, the IR must earn a profit no lower than its decentralized-equilibrium profit, $\Pi_r^{OT} \geq \Pi_r^D$, while the OEM must also obtain at least its decentralized-equilibrium profit, $\Pi_M^{OT} \geq \Pi_M^D$. These participation constraints jointly determine the admissible interval of F^{OT}, ensuring that both parties voluntarily accept the contract without altering the coordinated operational decisions. Within this feasible range, the contract separates operational coordination from surplus allocation. The optimal outsourcing fee w_r^{OT*} coordinates production, remanufacturing, and emission-reduction decisions, whereas the fixed fee F^{OT} determines the distribution of coordination benefits

between the OEM and IR. Consequently, the total SC profit satisfies $\Pi_S^{OT} = \Pi_S^C$, demonstrating that the decentralized SC achieves the centralized outcome under the proposed contract.

From an economic perspective, the proposed contract not only mitigates double marginalization but also aligns environmental incentives associated with VERM and the verification benefits provided by BCT. Verified emission reductions create additional sources of economic value, making coordination dependent on both operational efficiency and the allocation of environmental benefits. Accordingly, the fixed fee acts as a flexible mechanism for distributing the coordination surplus. When $F^{OT} = \underline{F}^{OT}$, the IR receives a larger proportion of the additional surplus, whereas $F^{OT} = \overline{F}^{OT}$ allows the OEM to capture a greater share. Intermediate values of F^{OT} represent alternative bargaining outcomes between the two parties while maintaining the efficiency of the coordinated SC.

6.5 Numerical and qualitative analysis of the two-part tariff contract

This subsection numerically investigates the performance of the two-part tariff (OT) contract with respect to variations in the remanufacturing acceptance parameter δ, the BCT cost-sharing ratio ρ, the VER price p_e, and the fixed fee F^{OT}. Figures 6–9 illustrate the corresponding effects on product prices, outsourcing fees, production quantities, individual firms' profits, and emission-reduction effort. The results reveal that these parameters affect the decentralized SC through different channels: δ changes the relative market attractiveness of remanufactured products, ρ redistributes BCT costs between the OEM and IR, p_e changes the economic return from verified emission reductions, and F^{OT} determines the transfer between the two parties within the contract.

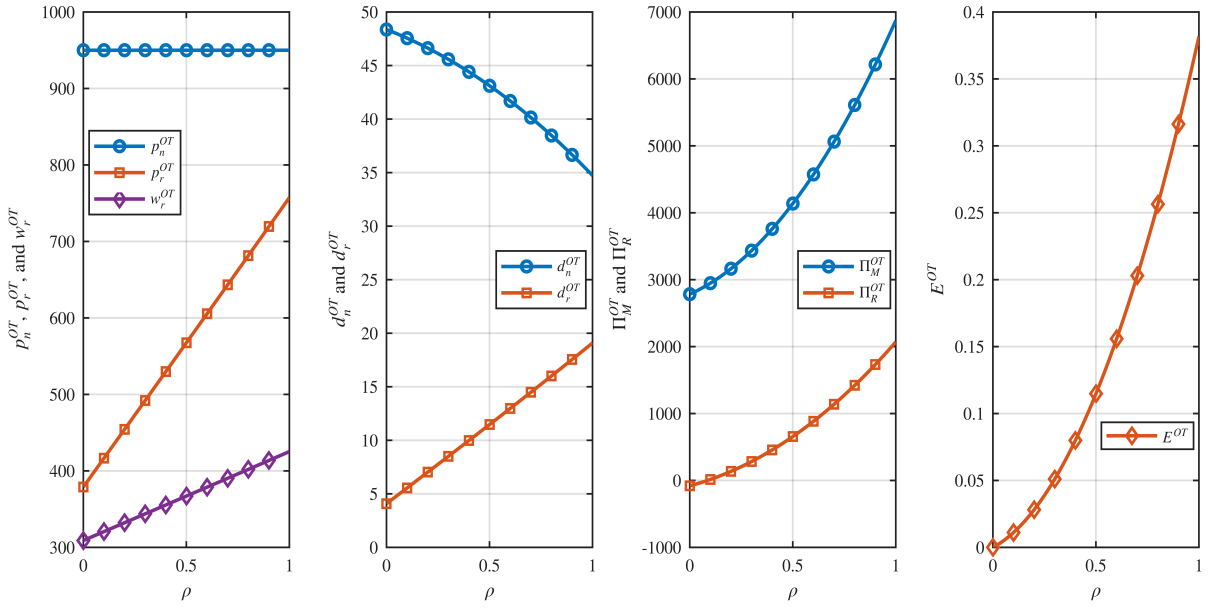


Fig. 6 Effects of the remanufacturing acceptance parameter δ.

Figure 6 illustrates the effects of δ on the equilibrium outcomes. The new-product price p_n^{OT} remains constant over the parameter range, whereas both the remanufactured-product price p_r^{OT} and the per-unit outsourcing fee w_r^{OT} increase with δ. The quantity of new products, d_n^{OT}, initially changes only slightly and subsequently declines as δ increases. In contrast, the remanufacturing quantity d_r^{OT} increases throughout the plotted range, indicating a shift in the product mix toward remanufacturing as consumer acceptance improves.

The profit and emission-reduction responses are nonlinear. Both Π_M^{OT} and Π_R^{OT} initially decline, reach a minimum, and then increase as δ becomes larger. Similarly, E^{OT} initially decreases before turning upward and increasing over the higher-δ range. These patterns suggest that improvements in consumer acceptance do not necessarily generate immediate gains for both parties; the benefits become more pronounced once the market and production incentives sufficiently favor remanufacturing. An important qualification is that the low-δ portion of Figure 6 includes negative values of d_r^{OT} and E^{OT}. Such values are not economically feasible when remanufacturing quantity and emission-reduction effort are constrained to be non-negative. Accordingly, the corresponding portion of the curves should be interpreted as an algebraic continuation of the equilibrium expressions rather than as a feasible equilibrium. The

economic interpretation should therefore focus on the parameter range satisfying the relevant feasibility conditions.

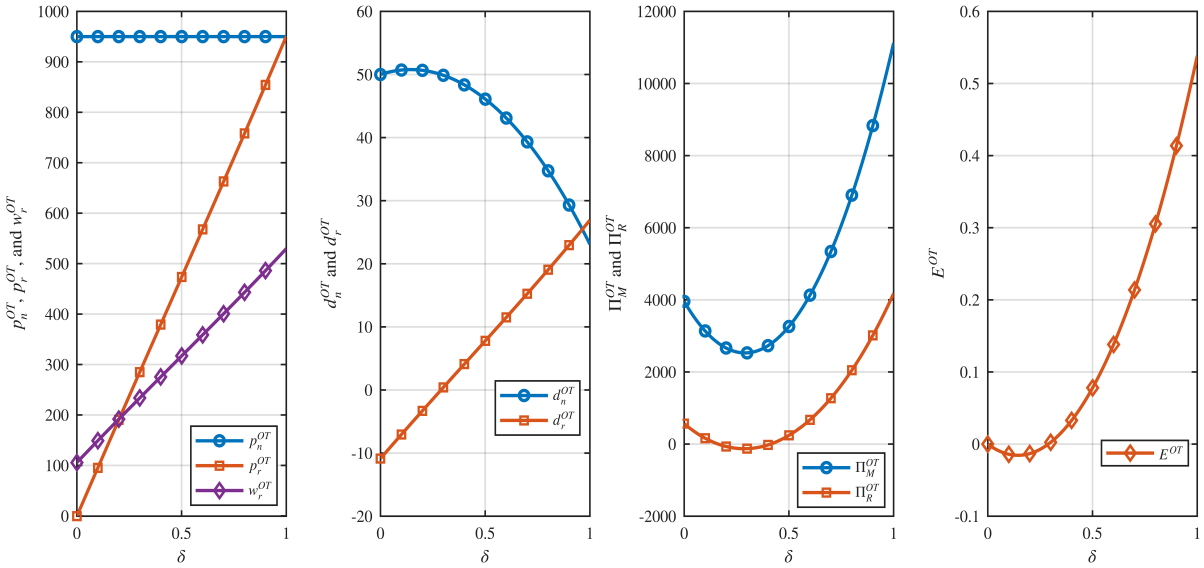


Fig. 7 Effects of the BCT cost-sharing ratio ρ.

Figure 7 examines the role of the BCT cost-sharing ratio ρ, which represents the proportion of BCT costs borne by the OEM. As ρ increases, the new-product price p_n^{OT} remains unchanged, while the remanufactured-product price p_r^{OT} and outsourcing fee w_r^{OT} increase. The production quantities move in opposite directions: d_n^{OT} decreases, whereas d_r^{OT} increases. This pattern suggests that, under the considered parameter setting, a higher OEM contribution to BCT implementation costs is associated with a shift in the product mix from new products toward remanufactured products. Both the OEM's and IR's profits increase throughout the examined range of ρ, accompanied by an increase in emission-reduction effort E^{OT}. These findings indicate that the distribution of BCT costs can affect operational decisions and environmental incentives within the decentralized SC. Specifically, a greater proportion of BCT expenditures borne by the OEM may be associated with increased remanufacturing activity and stronger emission-reduction efforts. However, these numerical observations alone do not establish complete SC coordination or an improvement in total SC profitability. Such conclusions require an independent evaluation of aggregate SC profit against the centralized benchmark.

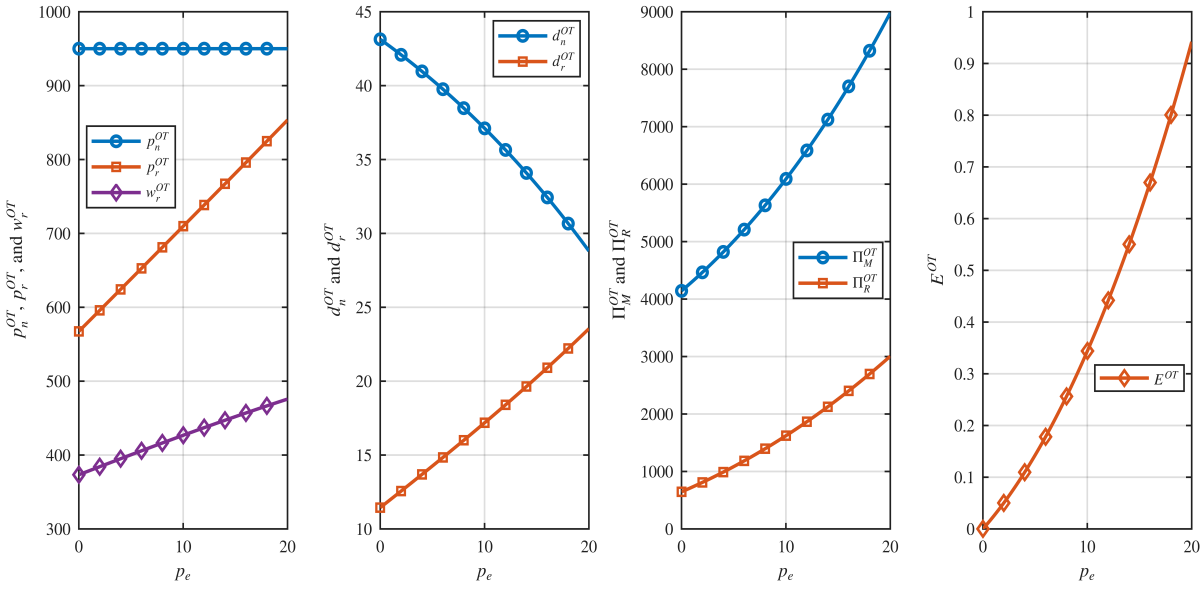


Fig. 8 Effects of the VER price p_e.

Figure 8 illustrates the sensitivity of equilibrium outcomes to variations in the VER price p_e. As p_e increases, the new-product price p_n^{OT} remains unchanged, whereas the remanufactured-product price p_r^{OT} and outsourcing fee w_r^{OT} increase. Meanwhile, the production quantity of new products d_n^{OT} decreases, while the remanufacturing quantity d_r^{OT} increases. These results indicate that higher VER prices encourage remanufacturing activities and shift the product mix away from new-product sales.

Under the specified numerical parameters, the profits of both the OEM and IR increase with p_e. Furthermore, emission-reduction effort E^{OT} rises considerably, from approximately 0.23 to 0.94 across the examined range. This trend reflects the additional economic benefits associated with VER revenues, as higher carbon-credit prices provide stronger incentives for investment in emission-reduction activities. The concurrent increase in remanufacturing quantity further suggests that improved returns from emission reduction promote remanufacturing activities under the considered parameter setting. Nevertheless, these findings represent numerical comparative-static observations, and their general validity should be confirmed through analytical examination of the equilibrium expressions.

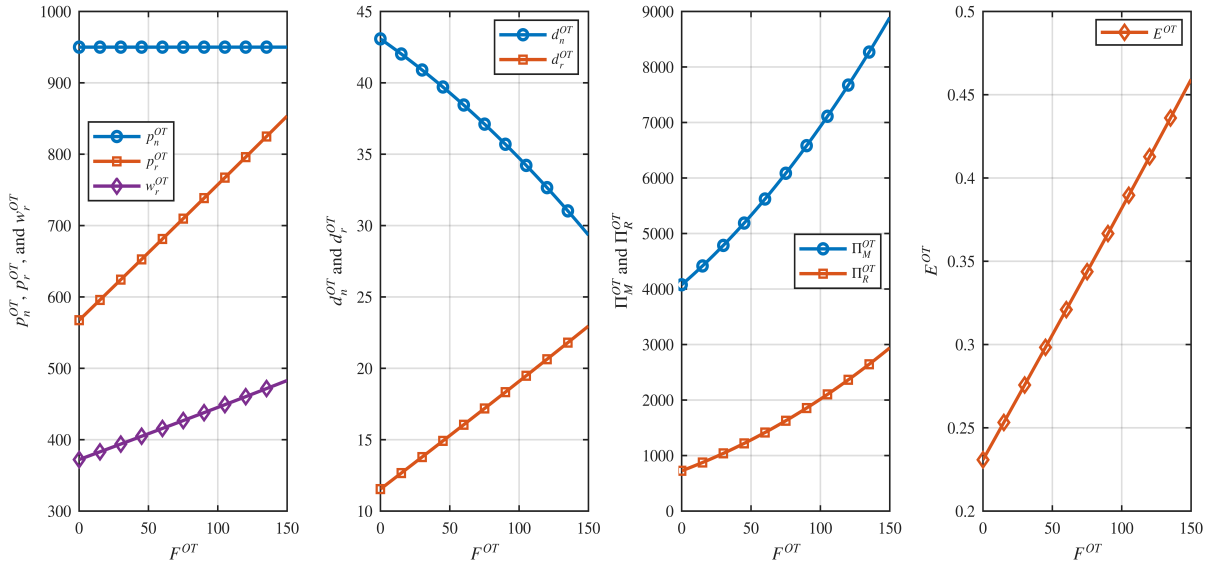


Fig. 9 Effects of the fixed fee F^{OT} under the two-part tariff contract.

Figure 9 demonstrates the sensitivity of equilibrium outcomes to changes in the fixed fee F^{OT}. Within the illustrated range, the new-product price p_n^{OT} remains constant, while the remanufactured-product price p_r^{OT} and per-unit outsourcing fee w_r^{OT} increase. In contrast, the quantity of new products d_n^{OT} declines, whereas the remanufacturing quantity d_r^{OT} increases. Similarly, the numerical results indicate an increase in emission-reduction effort E^{OT} as the fixed fee rises. The two profit curves, Π_M^{OT} and Π_R^{OT}, both increase over the displayed range. These curves therefore indicate that the numerical outcomes along the plotted contract path involve changes in operational decisions as well as in the allocation of contract payments.

This result requires careful interpretation in light of the role of a fixed fee in a two-part tariff. If F^{OT} alone changes while the per-unit fee and all other marginal decision terms remain fixed, the fixed fee does not affect the marginal incentives governing prices, quantities, or emission-reduction effort. Its direct effect is to increase the OEM's profit and decrease the IR's profit by the same amount. Consequently, the variations in w_r^{OT}, product quantities, emission-reduction effort, and both firms' profits shown in Figure 9 should be understood as responses along a re-optimized contract path, in which the per-unit fee or other equilibrium decisions also adjust with F^{OT}, rather than as the isolated effect of changing a lump-sum transfer. Under this interpretation, the figure illustrates how changes in the overall contract configuration can alter both operational outcomes and profit allocation.

Taken together, Figures 6–9 demonstrate that the performance of the OT contract depends on the interaction between market acceptance, BCT cost allocation, VER incentives, and contract design. The VER price has a particularly clear positive association with both firms' profits and emission-reduction effort in the numerical example, while the BCT cost-sharing ratio is associated with greater remanufacturing activity. The effects of δ are nonlinear, and the results for F^{OT} depend on whether the contract is re-optimized as the fixed fee changes. Accordingly, the numerical findings complement

Proposition 9, but full coordination must ultimately be verified by comparing the coordinated decisions and total SC profit with their centralized counterparts.

6.6 Extension: IR-led R&D under BCT-enabled VERM

The present section investigates a case where IR entities are involved in investing in emission-reduction technologies in an SC framework, enabled by BCT. This analysis considers that the IRs have solid distribution networks, sophisticated remanufacturing capabilities, and exceptional skills in low-carbon technology development, contrary to baseline models where OEMs are the major players. BCT assures accurate reporting of emissions reduction results, guarantees quality verification that can be reliable, and supports carbon credit transactions. In addition, BCT builds up the consumer acceptance of remanufactured products by providing them with transparent information about the reductions in carbon emissions, but it also brings about extra costs which need to be fairly shared between OEMs and IRs.

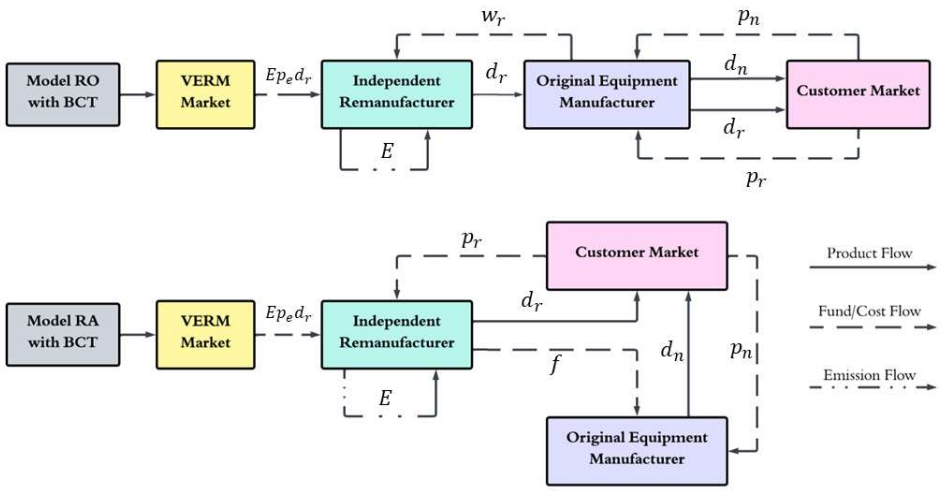


Fig. 10 SC framework of IR-led extension model.

As illustrated in Figure 10, to classify the two IR-led R&D models as *RO* (outsourcing with IR-led R&D) and *RA* (authorization with IR-led R&D). The respective optimization problems are outlined as follows:

Model *RO* (IR-led Outsourcing with R&D and BCT):

$$\Pi_M^{RO} = \left(p_n^{RO} - c\right) d_n^{RO} + \left(p_r^{RO} - w_r^{RO}\right) d_r^{RO} - \rho\left(c_a d_r^{RO} + F_c\right) \text{ and} \tag{15}$$

$$\Pi_R^{RO} = \left(w_r^{RO} - c + s\right) d_r^{RO} + \left(p_e E^{RO}\right) d_r^{RO} - \left(g\left(d_r^{RO}\right)^2 + \frac{\left(E^{RO}\right)^2}{2} + (1-\rho)\left(c_a d_r^{RO} + F_c\right)\right). \tag{16}$$

Model *RA* (IR-led Authorization with R&D and BCT):

$$\Pi_M^{RA} = \left(p_n^{RA} - c\right) d_n^{RA} + f^{RA} d_r^{RA} - \rho\left(c_a d_r^{RA} + F_c\right) \text{ and} \tag{17}$$

$$\Pi_R^{RA} = \left(p_r^{RA} + s - \left(c + f^{RA}\right)\right) d_r^{RA} + \left(p_e E^{RA}\right) d_r^{RA} - \left(g\left(d_r^{RA}\right)^2 + \frac{\left(E^{RA}\right)^2}{2} + (1-\rho)\left(c_a d_r^{RA} + F_c\right)\right). \tag{18}$$

In the above models, Integrated IRs are tasked with covering the research and development costs associated with emission reduction, in addition to managing expenses related to BCT traceability. Specifically, IRs incur both variable costs represented as $c_a d_r$ and a fixed infrastructure investment denoted

as F_c, of which the OEM contributes a fraction ρ within the interval $[0, 1]$. Under Model RO, the OEM initially determines the outsourcing fee and the production quantity of new products, while the IR subsequently decides on the remanufacturing quantity and emission-reduction effort. In Model RA, the OEM first sets the per-unit authorization fee, after which the IR determines the remanufacturing quantity and the corresponding level of emission-reduction effort.

The integration of BCT serves a dual purpose in these extensions: (i) on the consumer side, it enhances confidence in remanufactured products through traceability and carbon disclosure, represented as $(1+\delta)\lambda t - p_0$, thereby increasing the effective demand for remanufactured products; (ii) on the operational side, it guarantees the verifiability of VERs, thus enhancing their credibility and market value. Consequently, the incentives for abatement within the IR are strengthened, as emission reductions can be reliably monetized in voluntary carbon markets.

All equilibrium outcomes for the two models are compiled and presented in Table 7, which can be found in C.

Before presenting the comparative results, it is useful to note that two critical VER-price thresholds emerge from the equilibrium solutions, namely $\sqrt{(\delta + A_1)\xi}$ and $\sqrt{2\xi(\delta + A_1)}$. These thresholds represent the points at which the marginal carbon-credit revenue generated by additional emission reduction becomes comparable to the marginal cost of abatement investment. As the VER price crosses these thresholds, the relative effectiveness of OEM-led and IR-led R&D changes, leading to different equilibrium rankings in emission-reduction effort, remanufacturing output, and contractual decisions.

Proposition 10 *By comparing the equilibrium solutions, we have:*

1. *When* $p_e < \sqrt{(\delta + A_1)\,\xi}$ *the inequality relations are as follows:* $E^A < E^O < E^{RA} < E^{RO}$; $d_r^A < d_r^O < d_r^{RA} < d_r^{RO}$; $f^A < f^{RA}$; $d_n^{RA} < d_n^A < d_n^{RO} < d_n^O$; $w_r^{BO} < w_r^{RO} < w_r^O$; *and* $p_r^{RO} < P_r^A$.
2. *When* $\sqrt{(\delta + A_1)\,\xi} < p_e < \sqrt{2\xi\,(\delta + A_1)}$ *the inequality relations are as follows:* $E^{RA} < E^{RO} < E^A < E^O$; $d_r^{RA} < d_r^{RO} < d_r^A < d_r^O$; $f^{RA} < f^A$; $d_n^A < d_n^{RA} < d_n^O < d_n^{RO}$; $w_r^O < w_r^{RO} = w_r^{BO}$; *and* $p_r^A < P_r^{RO}$.
3. *When* $p_e > \sqrt{2\xi\,(\delta + A_1)}$ *the inequality relations are as follows:* $E^A < E^{RO} < E^O < E^{RA}$; $d_r^A < d_r^{RO} < d_r^O < d_r^{RA}$; $f^{RA} < f^A$; $d_n^{RA} < d_n^A < d_n^O < d_n^{RO}$; $w_r^O < w_r^{RO} = w_r^{BO}$; *and* $p_r^{RO} < P_r^A$.

Proposition 10 analyzes the equilibrium outcomes of four VERM models across three levels of carbon-emission pricing. When the carbon-credit price is comparatively low, $p_e < \sqrt{(\delta + A_1)\xi}$, the IR-led models outperform the OEM-led models in terms of both emission-reduction effort and remanufactured output, as reflected by the rankings $E^A < E^O < E^{RA} < E^{RO}$ and $d_r^A < d_r^O < d_r^{RA} < d_r^{RO}$. This finding indicates that, when carbon-market incentives are relatively weak, IRs may leverage their direct involvement in remanufacturing operations to implement emission-reduction measures more efficiently than OEMs. Consequently, allocating R&D responsibility to IRs can strengthen incentives for emission abatement and remanufacturing expansion under low VER prices. Within this regime, the authorization fee under the IR-led structure exceeds that of the OEM-led authorization model ($f^A < f^{RA}$), reflecting the IR's greater involvement in emission-reduction activities. The new-product quantities exhibit a mixed hierarchy, with the authorized IR-led Model RA generating the fewest new products. Furthermore, the outsourcing-related wholesale prices satisfy ($w_r^{BO} < w_r^{RO} < w_r^O$), while the remanufactured retail price in Model RO remains lower than that in Model A.

Moreover, the four models' hierarchy is reversed when the carbon-emission price climbs to the intermediate range $\sqrt{(\delta + A_1)\xi} < p_e < \sqrt{2\xi(\delta + A_1)}$. In this situation, the IR-led models (RA and RO) have the least emission-reduction effort and remanufactured output, as they experience lower equilibrium abatement efforts and remanufacturing outputs than the OEM-led models. This finding indicates that, within the intermediate VER-price range, the greater responsiveness of OEM-led R&D investment to carbon-market incentives outweighs the operational advantages associated with IR-led structures. Specifically, the new inequalities manifest as $E^{RA} < E^{RO} < E^A < E^O$ and $d_r^{RA} < d_r^{RO} < d_r^A < d_r^O$, illustrating the greater technological aggression of OEM-led models in response to escalating emission prices. The authorization fee now falls below the OEM-set fee ($f^{RA} < f^A$), and the ordering of new product quantities adjusts correspondingly. Within this pricing regime, the wholesale prices for the IR-related models equalize such that $w_r^O < w_r^{RO} = w_r^{BO}$, and the remanufactured retail price in Model A becomes lower than that in Model RO.

When the carbon-emission price escalates to high levels $\left(p_e > \sqrt{2\xi(\delta + A_1)}\right)$, the ordering shifts once more, this time underscoring the superior responsiveness of authorized IR-led R&D (RA) compared to outsourced remanufacturing (RO). The ranks for effort and remanufactured quantities become

$E^A < E^{RO} < E^O < E^{RA}$ and $d_r^A < d_r^{RO} < d_r^O < d_r^{RA}$, indicating that RA emerges as the most technologically proactive model under stringent environmental regulations. Despite authorization remaining less expensive for IRs ($f^{RA} < f^A$), the new product quantities mirror the same hierarchy as in the low-price regime, suggesting that the structure of new product development is more influenced by R&D leadership than by intensifying emission prices. The outsourcing fees follow the relationship $w_r^O < w_r^{RO} = w_r^{BO}$, while Model *RO* continues to maintain a lower retail price for remanufactured products than Model *A*. Overall, Proposition 10 establishes a threshold-dependent relationship between OEM-led and IR-led emission-reduction strategies. When VER prices are relatively low, IR-led outsourcing achieves the highest emission-reduction effort and remanufacturing quantity. At intermediate VER prices, OEM-led outsourcing becomes the preferred structure, as OEM-led R&D investment responds more effectively to increasing carbon-market incentives.

However, when VER prices exceed a sufficiently high threshold, IR-led authorization (Model *RA*) emerges as the most proactive structure in terms of emission-reduction investment, since higher carbon-credit revenues encourage the IR to expand remanufacturing activities while simultaneously increasing emission-reduction effort. These transitions indicate that the appropriate allocation of R&D responsibilities is strongly influenced by the prevailing level of carbon-market incentives. Therefore, the main contribution of Proposition 10 lies not merely in comparing alternative R&D leadership arrangements, but in demonstrating how the relative effectiveness of OEM-led and IR-led emission-reduction strategies varies across different VER-price regimes. The emergence of the threshold values $\sqrt{(\delta + A_1)\xi}$ and $\sqrt{2\xi(\delta + A_1)}$ reveals regime-switching behavior that links carbon-market incentives, consumer acceptance, and emission-reduction investment decisions. Such threshold-dependent transitions are not present in traditional remanufacturing models that do not explicitly incorporate VERM and BCT interactions.

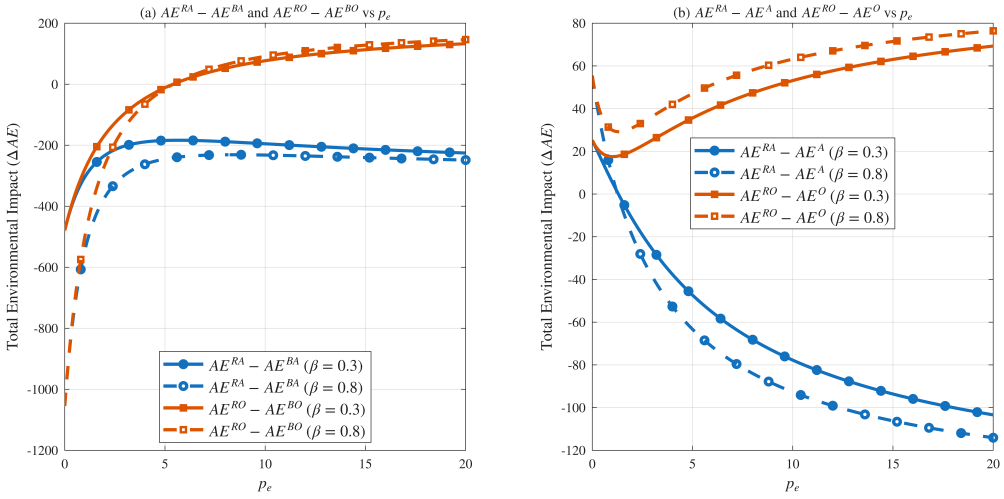


Fig. 11 The changes in atmospheric environmental impact.

Figure 11 illustrates the environmental trade-offs associated with IR-led R&D investment in a BCT-enabled VERM framework. It compares total emissions between IR-led and OEM-led models under varying VER prices p_e. The left panel presents the emission differences $AE^{RA} - AE^{BA}$ (blue) and $AE^{RO} - AE^{BO}$ (orange) for two remanufacturing emission-intensity levels, $\beta = 0.3$ (solid lines) and $\beta = 0.8$ (dashed lines). Similarly, the right panel compares $AE^{RA} - AE^{A}$ and $AE^{RO} - AE^{O}$ under the same parameter settings. All environmental comparisons are expressed in terms of ΔAE, where $\Delta AE < 0$ indicates that the IR-led model produces lower total emissions and therefore achieves better environmental performance, whereas $\Delta AE > 0$ indicates higher emissions and poorer environmental performance.

The economic interpretation of Proposition 10 depends on the interaction between carbon-market incentives and the allocation of emission-reduction responsibilities. At relatively low VER prices, carbon-credit revenues may be insufficient to compensate for R&D expenditures. Under these conditions, IR-led structures primarily benefit from the IR's direct involvement in remanufacturing operations, which facilitates effective emission-reduction implementation and supports greater remanufacturing

output. As p_e increases, carbon-market incentives become stronger. However, IR-led R&D may experience partial incentive misalignment when the investing party does not fully internalize the benefits of emission reduction. Consequently, OEM-led structures may achieve greater emission-reduction effort within the intermediate VER-price range. When p_e becomes sufficiently high, the economic benefits from carbon-credit generation can outweigh investment costs, making IR-led authorization (Model RA) more attractive. In this structure, the IR directly receives remanufacturing profits and VER revenues, thereby strengthening its incentives to expand remanufacturing and invest in emission reduction. These threshold-dependent outcomes reflect changes in the relative importance of R&D costs and carbon-credit revenues.

It is important to distinguish emission-reduction effort E from total atmospheric emissions AE. While E represents the intensity of emission reduction per unit, AE measures the overall environmental impact by accounting for emission intensity and production volume. Therefore, greater emission-reduction effort does not necessarily result in lower aggregate emissions when accompanied by a sufficiently large increase in production. Figure 11 complements Proposition 10 by examining environmental performance through total emissions rather than emission-reduction effort alone.

Several important findings emerge from the numerical analysis. Before interpreting these results, the distinct roles of the parameters β and δ should be clarified. The parameter β represents the relative emission intensity of remanufacturing compared with new-product manufacturing and therefore reflects the environmental efficiency of remanufacturing technology. A higher β indicates a smaller environmental advantage from remanufacturing. In contrast, δ captures the influence of BCT-enabled transparency and consumer trust on remanufactured-product demand through improved information credibility and carbon disclosure. Accordingly, β affects environmental outcomes through production technology, whereas δ influences them indirectly through market demand.

1. In the left panel, the difference $AE^{RA} - AE^{BA}$ is negative at lower VER prices, indicating that IR-led authorization (Model RA) generates fewer emissions than the corresponding benchmark Model BA. As p_e increases, the emission difference gradually decreases in magnitude and approaches zero. This pattern suggests that IR-led R&D provides a greater environmental advantage under weaker carbon-market incentives, potentially because of the IR's operational proximity to remanufacturing activities. However, stronger carbon incentives gradually reduce this relative advantage as emission-reduction incentives become more effective across the alternative structures.
2. The emission difference $AE^{RO} - AE^{BO}$ exhibits a different trend. Although it is negative at very low VER prices, it becomes positive as p_e increases, indicating that IR-led outsourcing (Model RO) eventually produces higher total emissions than benchmark outsourcing (Model BO). This transition suggests that increasing VER prices may encourage IRs to expand remanufacturing activities to obtain additional carbon-credit revenues. When remanufacturing is relatively emission-intensive or the production expansion effect exceeds the benefits of emission reduction, aggregate emissions may increase despite stronger abatement incentives.
3. The right panel demonstrates that the environmental performance of IR-led and OEM-led structures varies with the VER price and governance arrangement. The blue curves representing $AE^{RA} - AE^{A}$ become increasingly negative as p_e rises, indicating an increasing environmental advantage for IR-led authorization under stronger carbon-market incentives. Conversely, the orange curves representing $AE^{RO} - AE^{O}$ become increasingly positive, suggesting that OEM-led outsourcing achieves relatively better environmental performance at moderate and high VER prices. These contrasting trends demonstrate that the environmental effectiveness of R&D leadership depends on both the governance structure and the strength of carbon-credit incentives.
4. The influence of β is evident across the environmental comparisons. Increasing β from 0.3 to 0.8 magnifies both positive and negative emission differences, indicating that the relative emission intensity of remanufacturing affects the environmental performance gap between governance structures. A higher β reduces the environmental advantage of remanufacturing and makes differences in aggregate emissions more pronounced. When remanufacturing is relatively clean, represented by a lower β, the environmental differences between models are smaller. In contrast, when remanufacturing is more emission-intensive, VER-induced production expansion can amplify differences in environmental outcomes.

Three main conclusions can be drawn from these findings.

1. VERM can achieve stronger environmental performance under IR-led authorization (Model RA), particularly when carbon-market incentives are sufficiently high. The negative emission differences associated with Model RA indicate that assigning authorization and R&D responsibilities to the

IR can improve environmental outcomes relative to the corresponding OEM-led structures. This suggests that authorization may provide more effective alignment between operational decisions and emission-reduction incentives in a BCT-enabled VERM framework.

2. IR-led outsourcing (Model *RO*) may create environmental risks at moderate and high VER prices. The positive emission differences and increasing trends shown in Figure 11 suggest that stronger carbon-credit incentives can encourage excessive remanufacturing expansion. When the environmental benefits of emission reduction are outweighed by increased production-related emissions, total emissions may rise. This finding highlights the importance of appropriate carbon-credit verification and production-related environmental safeguards to prevent unintended environmental consequences.
3. Effective environmental policy and SC coordination should jointly account for remanufacturing emission intensity β and BCT-enabled verification mechanisms. Higher values of β increase the differences in environmental performance across governance structures, suggesting that carbon-credit pricing, emission verification, and the selection of remanufacturing arrangements should consider the relative environmental efficiency of remanufacturing. BCT can support these objectives by improving the credibility of emission-reduction verification and facilitating contractual arrangements that align economic incentives with environmental performance.

Collectively, Figure 11 provides additional numerical support for the threshold-dependent outcomes discussed in Proposition 10. It demonstrates that the environmental consequences of IR-led R&D depend not only on emission-reduction effort but also on production scale and the distribution of economic incentives. These findings emphasize that aggregate environmental performance is jointly determined by VER prices, remanufacturing governance structures, and emission intensity. Consequently, appropriate coordination mechanisms and environmental policies are essential for ensuring that carbon-market incentives and BCT-enabled traceability contribute to effective reductions in total emissions.

6.6.1 Numerical and Qualitative Analysis of IR-Led R&D under BCT-Enabled VERM (Model *RO*)

This subsection numerically examines the equilibrium outcomes of Model *RO*, in which the IR undertakes emission-reduction R&D activities within the BCT-enabled VERM framework. Figures 12–14 illustrate the effects of variations in the VER price p_e, BCT cost-sharing ratio ρ, and remanufacturing acceptance parameter δ on equilibrium product prices, demand quantities, individual firms' profits, and emission-reduction effort. The numerical findings demonstrate the significant roles of carbon-market incentives, BCT cost allocation, and consumer acceptance in determining the operational decisions, environmental performance, and economic outcomes of the IR-led remanufacturing structure.

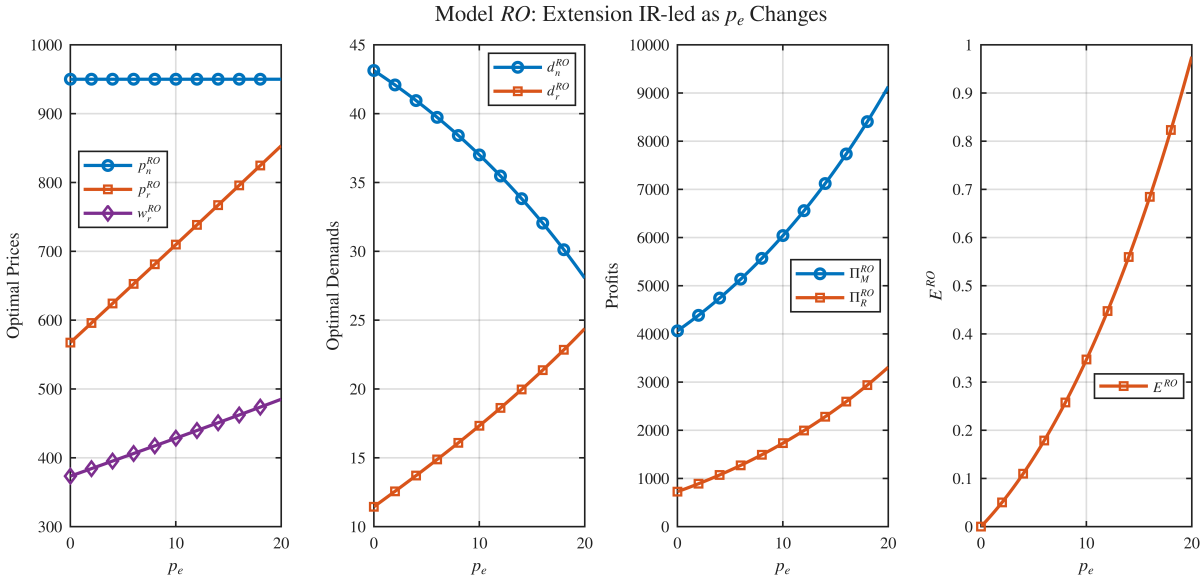


Fig. 12 Effects of the VER price p_e under Model *RO*.

Figure 12 presents the sensitivity of Model *RO* to changes in the VER price p_e. The new-product price p_n^{RO} remains constant over the plotted range, whereas both the remanufactured-product price p_r^{RO}

and the outsourcing fee w_r^{RO} increase with p_e. The demand for new products, d_n^{RO}, decreases, while the remanufacturing demand, d_r^{RO}, increases. These trends indicate a gradual shift in the product mix toward remanufacturing as the market value of verified emission reductions rises. The profits of both the OEM and IR also increase with p_e in the numerical example. In addition, the emission-reduction effort E^{RO} increases markedly, from zero at the lower end of the plotted range to approximately 0.97 at $p_e = 20$. This trend reflects the IR's direct economic incentive to undertake emission-reduction investments as revenues from VER sales become more attractive. The concurrent increase in remanufacturing demand indicates that, under the considered parameter setting, stronger carbon-credit incentives promote both emission-reduction efforts and remanufacturing activities. However, these findings are based on numerical comparative-static analysis and should not be generalized beyond the examined parameter range without further analytical verification.

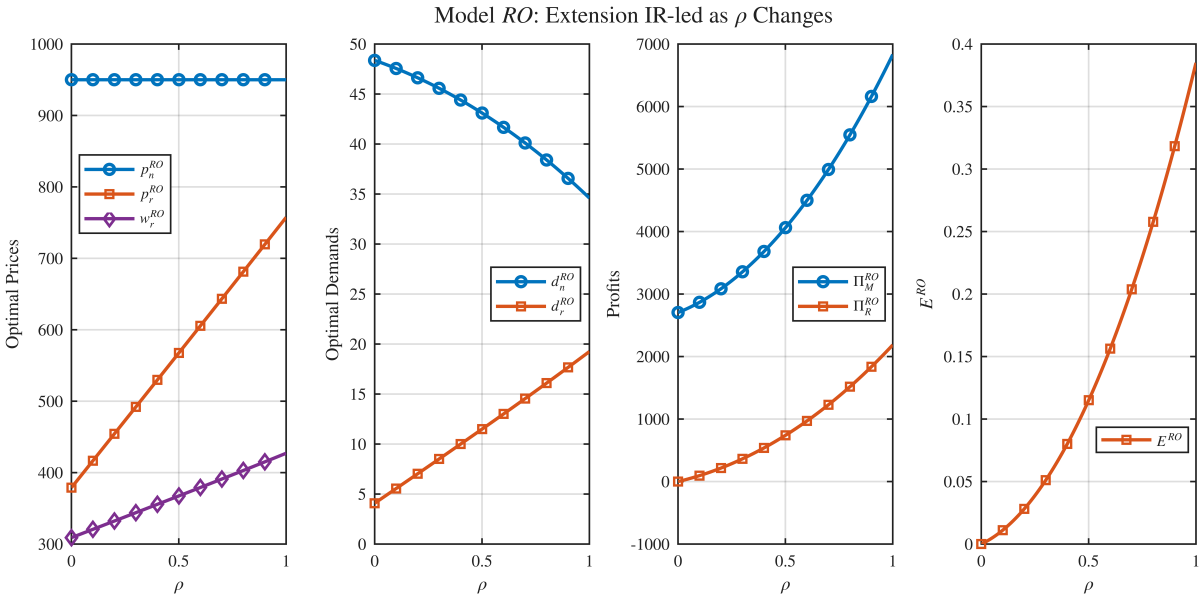


Fig. 13 Effects of the BCT cost-sharing ratio ρ under Model *RO*.

Figure 13 presents the sensitivity of equilibrium outcomes to the BCT cost-sharing ratio ρ, representing the proportion of BCT implementation costs covered by the OEM. As ρ increases, the new-product price p_n^{RO} remains unchanged, whereas the remanufactured-product price p_r^{RO} and outsourcing fee w_r^{RO} increase. Meanwhile, the demand for new products d_n^{RO} decreases, while remanufacturing demand d_r^{RO} increases. These findings indicate that, under the considered parameter setting, a higher OEM contribution to BCT expenditures encourages a shift in demand toward remanufactured products. Furthermore, the profits of both the OEM and IR increase with ρ, accompanied by an increase in the IR's emission-reduction effort E^{RO}. This behavior may be attributed to changes in the allocation of operational costs between the two firms, while the IR continues to benefit directly from VER revenues generated through emission-reduction activities. The increasing emission-reduction effort observed in the figure is consistent with stronger investment incentives under the specified equilibrium conditions. Nevertheless, the relationship between BCT cost sharing and environmental investment depends on the underlying profit functions and equilibrium solutions. Therefore, these numerical findings do not necessarily imply that increasing ρ always improves emission-reduction effort or overall SC profitability.

Figure 14 illustrates the sensitivity of Model *RO* to changes in the remanufacturing acceptance parameter δ. Unlike the effects observed for p_e and ρ, variations in δ produce different patterns in equilibrium decisions. The new-product price p_n^{RO} remains constant, while the remanufactured-product price p_r^{RO} and outsourcing fee w_r^{RO} increase with δ. The demand for new products d_n^{RO} initially exhibits only minor changes before declining, whereas remanufacturing demand d_r^{RO} increases throughout the examined range. At relatively high values of δ, remanufacturing demand approaches and eventually surpasses new-product demand, indicating a substantial shift in consumer preferences toward remanufactured products.

The profit responses exhibit non-monotonic behavior. Both the OEM's profit Π_M^{RO} and the IR's profit Π_r^{RO} initially decline, reach their respective minimum values, and subsequently increase as δ rises. Similarly, emission-reduction effort E^{RO} initially decreases into the negative region, reaches a minimum, and

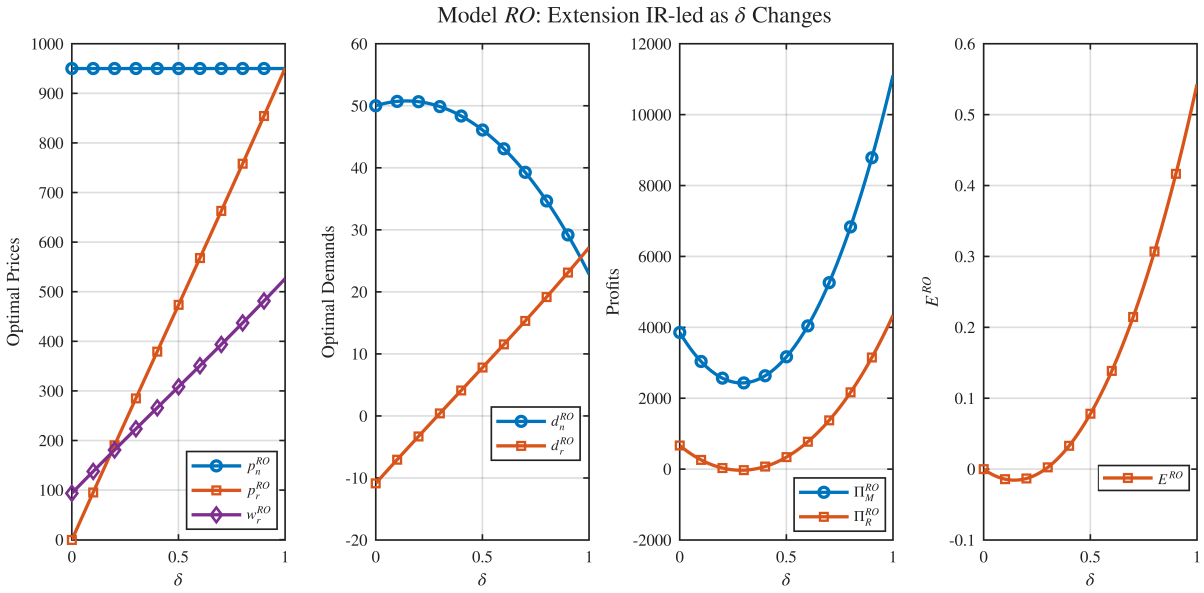


Fig. 14 Effects of the remanufacturing acceptance parameter δ under Model *RO*.

then increases, becoming positive at approximately $\delta = 0.3$ in the numerical example. These results suggest that improvements in consumer acceptance may not immediately translate into greater profitability or environmental investment when δ is relatively low. However, their effects become more favorable at higher acceptance levels under the specified parameter setting. Nevertheless, negative remanufacturing quantities or emission-reduction effort are economically infeasible when the corresponding decision variables are restricted to non-negative values. Accordingly, the low-δ region of Figure 14 should be interpreted as an algebraic extension of the equilibrium expressions rather than an economically feasible solution. Therefore, the numerical conclusions should be restricted to the feasible region satisfying $d_r^{RO} \geq 0$ and $E^{RO} \geq 0$, along with the remaining equilibrium and participation constraints.

Collectively, Figures 12–14 demonstrate that, under the considered numerical parameter setting, an increase in the VER price positively influences firms' profits, remanufacturing demand, and emission-reduction effort in Model *RO*. A higher OEM share of BCT costs is likewise associated with increased remanufacturing demand, profits, and & effort in the plotted range. By contrast, the effects of δ are nonlinear, with the initial decline in profits and emission-reduction effort followed by an improvement at higher acceptance levels. These results illustrate how market incentives and BCT cost allocation interact under IR-led &, while also emphasizing the need to impose feasibility constraints before interpreting the numerical equilibria as economically implementable outcomes.

6.6.2 Numerical and Qualitative Analysis of IR-Led R&D under BCT-Enabled VERM (Model *RA*)

This subsection examines the numerical performance of Model *RA*, in which the IR leads emission-reduction R&D under an authorization arrangement supported by BCT and the VERM. Figures 15–17 illustrate the effects of the VER price p_e, the remanufacturing acceptance parameter δ, and the BCT cost-sharing ratio ρ on equilibrium prices, authorization fees, product demands, firms' profits, and emission-reduction effort. The results demonstrate how environmental incentives, consumer acceptance, and the allocation of BCT costs influence the performance of the IR-led authorization model.

Figure 15 illustrates the sensitivity of equilibrium outcomes under Model *RA* to variations in the VER price p_e. As p_e increases, the new-product price p_n^{RA} remains unchanged, whereas the remanufactured-product price p_r^{RA} and authorization fee f^{RA} increase. Meanwhile, the production quantity of new products d_n^{RA} declines, while remanufacturing demand d_r^{RA} increases. These findings suggest that, under the considered parameter setting, higher VER prices encourage a shift in the product mix toward remanufactured products. Furthermore, the profits of both the OEM and IR increase with p_e, accompanied by a substantial increase in emission-reduction effort E^{RA}, which reaches approximately 0.90 when $p_e = 20$. This behavior reflects the IR's stronger economic incentive to invest in emission-reduction R&D as revenues from verified emission reductions become more attractive in the VER market. The concurrent increase in remanufacturing demand further indicates that stronger carbon-market incentives can promote both remanufacturing activities and environmental investment. Nevertheless, these observations

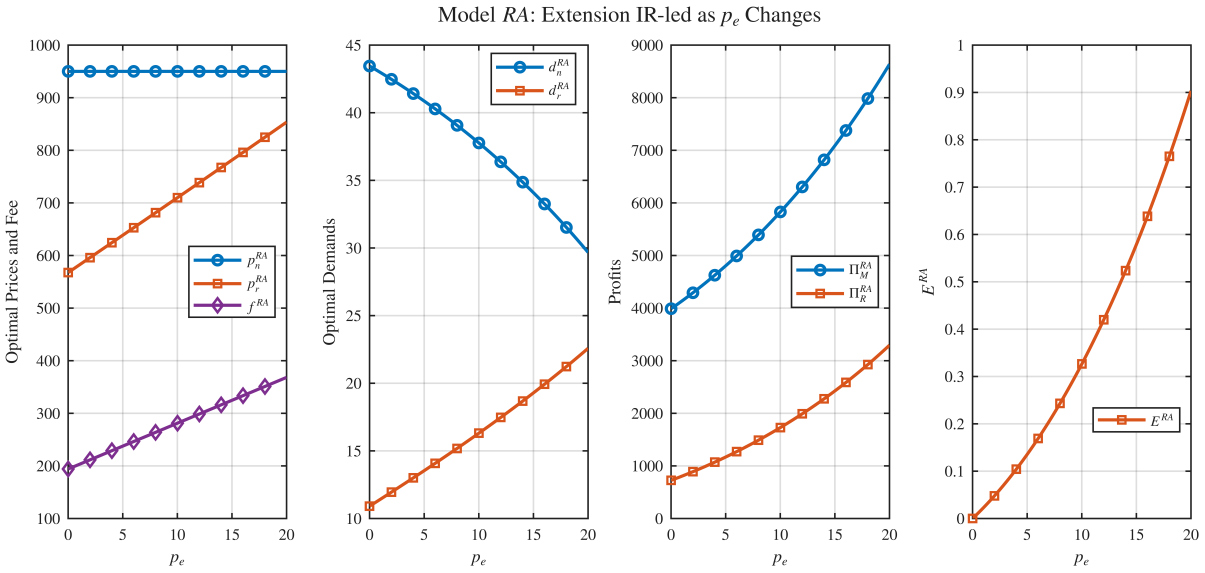


Fig. 15 Effects of the VER price p_e under Model RA.

are based on numerical comparative-static analysis under the selected parameter values, and the general validity of these relationships should be verified through the analytical equilibrium expressions.

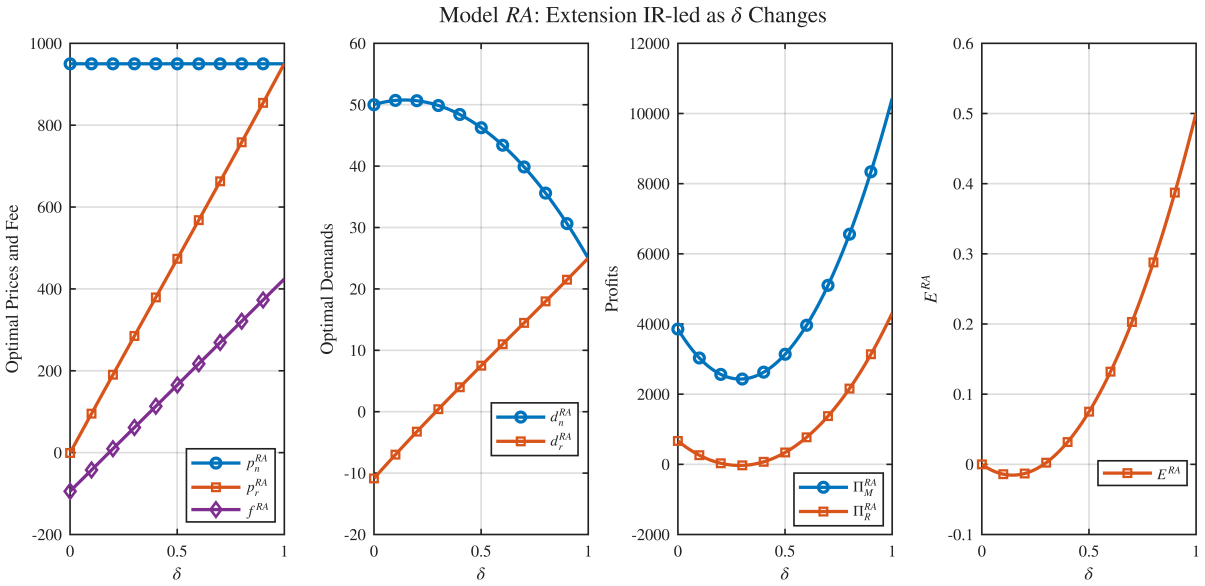


Fig. 16 Effects of the remanufacturing acceptance parameter δ under Model RA.

Figure 16 presents the effects of the remanufacturing acceptance parameter δ. The new-product price p_n^{RA} remains constant, while both the remanufactured-product price p_r^{RA} and authorization fee f^{RA} increase with δ. The quantity of new products, d_n^{RA}, initially changes only slightly and subsequently declines. In contrast, remanufacturing demand, d_r^{RA}, increases throughout the plotted range. This pattern indicates that greater consumer acceptance of remanufactured products is associated with a stronger shift toward remanufacturing. The profit responses are nonlinear. Both Π_M^{RA} and Π_r^{RA} initially decline, reach a minimum, and then increase as δ rises. Emission-reduction effort E^{RA} follows a similar pattern: it initially becomes negative, reaches a minimum, and subsequently increases, becoming positive at approximately $\delta = 0.3$. These results suggest that improvements in consumer acceptance do not necessarily translate into immediate economic gains or greater environmental effort at low acceptance levels. Under the illustrated conditions, the effects become more favorable as δ increases.

However, the negative values of d_r^{RA} and E^{RA} in the low-δ region are not economically feasible if remanufacturing quantity and emission-reduction effort are constrained to be non-negative. Therefore,

this region of the figure should be regarded as an algebraic extension of the equilibrium solutions rather than an economically feasible operating condition. Accordingly, the economic interpretation should be restricted to parameter values satisfying the non-negativity constraints $d_r^{RA} \geq 0$ and $E^{RA} \geq 0$, together with all other equilibrium feasibility and participation requirements.

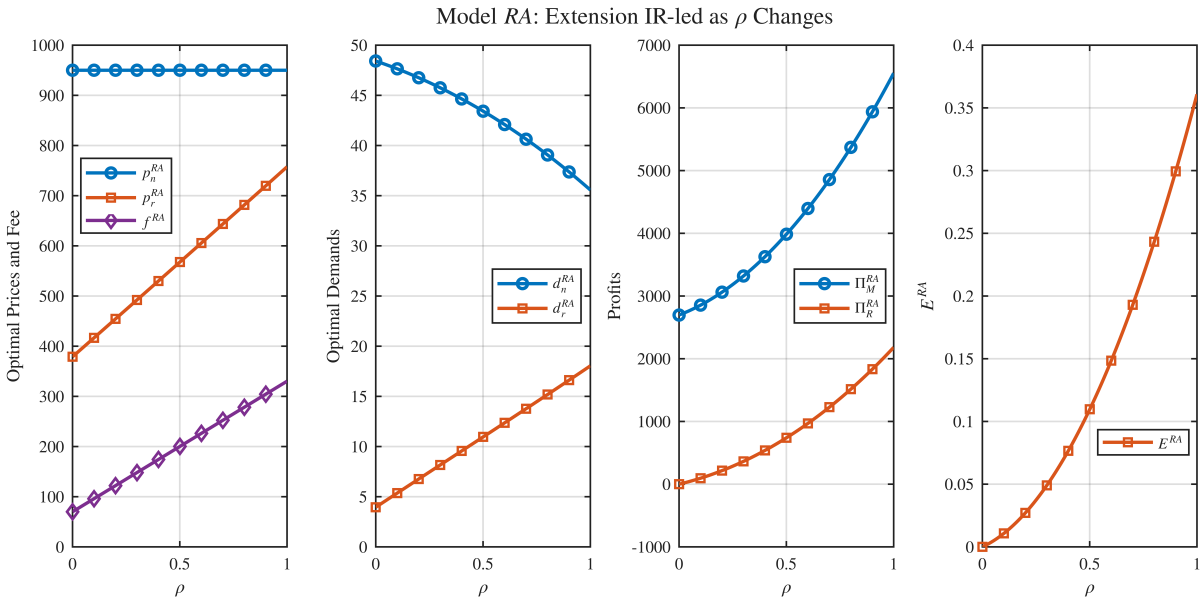


Fig. 17 Effects of the BCT cost-sharing ratio ρ under Model RA.

Figure 17 illustrates the sensitivity of equilibrium outcomes under Model RA to changes in the BCT cost-sharing ratio ρ, which denotes the proportion of BCT implementation expenditures covered by the OEM. As ρ increases, the new-product price p_n^{RA} remains unchanged, while the remanufactured-product price p_r^{RA} and authorization fee f^{RA} increase. Meanwhile, the production quantity of new products d_n^{RA} decreases, whereas remanufacturing demand d_r^{RA} increases. These trends suggest that, under the considered numerical setting, a greater OEM contribution to BCT costs encourages a shift in market demand from new products toward remanufactured products. Furthermore, the profits of both the OEM and IR increase throughout the examined range of ρ, accompanied by an increase in the IR's emission-reduction effort E^{RA}. These findings indicate that BCT cost allocation can influence the economic incentives associated with remanufacturing activities and environmental investment under the authorization arrangement. Specifically, a higher proportion of BCT costs borne by the OEM is associated with greater profitability for both SC members and stronger emission-reduction effort in the illustrated example. However, these relationships represent numerical comparative-static observations rather than general properties of the model. Their broader validity depends on the analytical equilibrium expressions and the parameter conditions required for economically feasible solutions.

Together, Figures 15–17 demonstrate that, under the selected numerical parameters, an increase in the VER price positively affects both firms' profits, remanufacturing demand, and emission-reduction effort in the IR-led authorization model. Similarly, a higher BCT cost-sharing ratio is associated with increased remanufacturing demand and greater environmental investment. In contrast, the influence of consumer acceptance exhibits a nonlinear pattern, with firms' profits and emission-reduction effort initially declining before increasing at higher values of δ. These findings emphasize the combined importance of carbon-market incentives, consumer acceptance, and BCT cost allocation in determining the operational, economic, and environmental performance of the IR-led authorization structure. Nevertheless, meaningful policy implications should be derived only after excluding economically infeasible parameter regions and confirming that all equilibrium outcomes satisfy the required non-negativity, feasibility, and participation constraints.

6.7 Robustness analysis of key managerial insights

To assess the robustness of the managerial implications derived from Models RO and RA, this subsection examines whether the principal comparative-static findings remain consistent under alternative parameter settings. The baseline numerical results illustrate how the voluntary emission reduction (VER) price

p_e, consumer acceptance of remanufactured products δ, and BCT cost-sharing ratio ρ affect product pricing, production quantities, emission-reduction effort, and individual SC members' profits. However, the direction and magnitude of these effects may depend on production costs, remanufacturing efficiency, the difficulty of emission-reduction investment, and the benefits associated with BCT-enabled transparency. Accordingly, the robustness analysis varies these parameters individually while keeping the remaining parameters fixed at their baseline values.

First, the analysis investigates the sensitivity of equilibrium outcomes to the remanufacturing emission-intensity parameter β. A lower value of β indicates that remanufacturing generates fewer emissions per unit relative to new-product manufacturing, whereas a higher value reduces the environmental advantage of remanufacturing. This examination aims to determine whether the economic incentives provided by VERM remain effective under different levels of remanufacturing environmental efficiency. Specifically, variations in remanufacturing quantities d_r^{RO*} and d_r^{RA*}, together with emission-reduction efforts E^{RO*} and E^{RA*}, are evaluated alongside the corresponding changes in total emissions. This approach ensures that increases in remanufacturing output or emission-reduction effort are not automatically interpreted as improvements in overall environmental performance.

Next, the R&D difficulty coefficient ξ is varied to examine the robustness of emission-reduction investment decisions. An increase in ξ raises the expenditure required to achieve a given level of emission-reduction effort, potentially weakening firms' incentives to undertake environmental investment. The resulting changes in equilibrium emission-reduction effort, remanufacturing quantities, and profits are compared across Models *RO* and *RA*. This comparison helps determine whether the positive incentive effects associated with higher VER prices persist when emission-reduction investments become more costly and whether outsourcing and authorization arrangements exhibit different sensitivities to R&D difficulty.

The robustness of BCT-enabled transparency benefits is further investigated by varying the corresponding transparency parameter within its theoretically admissible range. This analysis examines whether the effects of enhanced consumer trust on remanufacturing demand and profitability remain qualitatively consistent under alternative transparency conditions. Since consumer acceptance and BCT transparency represent conceptually different mechanisms, their respective influences are examined separately unless they are explicitly incorporated into a common parameter within the model.

Furthermore, Models *RO* and *RA* are compared under these alternative parameter configurations to evaluate the stability of their relative performance. Particular attention is given to the equilibrium rankings of remanufacturing quantities, emission-reduction efforts, individual firms' profits, and total emissions. The analysis also considers the economic feasibility of the equilibrium solutions by ensuring that production quantities and emission-reduction efforts remain nonnegative and that the relevant second-order optimality and participation conditions are satisfied. Parameter combinations that violate these requirements are excluded from the economically meaningful comparisons.

Overall, the robustness analysis distinguishes findings that remain consistent across different parameter settings from those that depend on specific market and technological conditions. If the main qualitative relationships remain unchanged, the results provide additional support for the managerial implications derived from the baseline analysis. Conversely, changes in equilibrium rankings or comparative-static patterns help identify the conditions under which outsourcing or authorization becomes more advantageous. In particular, the selection of an appropriate remanufacturing governance structure should consider both economic profitability and aggregate environmental performance, since higher remanufacturing quantities and stronger emission-reduction efforts do not necessarily guarantee lower total emissions.

6.8 Managerial implications

This study provides several managerial implications for OEMs, IRs, SC decision-makers, and policymakers involved in BCT-enabled remanufacturing systems operating within a VERM. By incorporating emission-reduction investments, carbon-credit incentives, consumer acceptance, and alternative contractual arrangements, the analysis demonstrates how digital traceability and environmental incentives influence operational decisions and overall SC performance. The findings address four principal research questions concerning the role of VERM in promoting remanufacturing, the influence of contractual governance on economic and environmental outcomes, the effects of transparency and privacy concerns on consumer behavior, and the effectiveness of coordination mechanisms in improving decentralized SC decisions.

1. **How does VERM influence operational strategies and R&D collaboration in BCT remanufacturing?**

The findings indicate that the VER price is an important determinant of the IR's incentives to undertake emission-reduction R&D and expand remanufacturing activities. As the market value of verified emission reductions increases, the potential economic returns from environmental investment become more attractive, encouraging greater emission-reduction effort and remanufacturing output. From a managerial perspective, these results emphasize the importance of integrating production planning with environmental investment decisions rather than treating remanufacturing and emission reduction as independent activities. OEMs and IRs should jointly consider anticipated carbon-credit revenues, R&D expenditures, and market demand when formulating their operational and investment strategies. Moreover, firms should carefully evaluate the marginal benefits of emission-reduction investments, since higher VER prices do not necessarily guarantee improvements in aggregate environmental performance.

The comparison between OEM-led and IR-led R&D structures further highlights the importance of allocating environmental investment responsibilities to the SC member best positioned to implement emission-reduction technologies effectively. When the IR assumes responsibility for R&D, contractual arrangements should provide sufficient incentives for environmental investment while allowing the OEM to benefit from improved traceability, product credibility, and environmental performance. BCT can support this collaboration by improving the verifiability of remanufacturing activities and emission-reduction claims, thereby enhancing the credibility of carbon-credit generation and trading.

2. **What is the main reason to prefer outsourcing to authorization models in terms of profitability, consumer surplus, and environment?**

   The comparison between outsourcing and authorization demonstrates that remanufacturing governance significantly influences pricing strategies, production decisions, and profit allocation between the OEM and IR. Under outsourcing, the OEM determines the outsourcing fee w_r, whereas under authorization, the OEM establishes the authorization fee f associated with the IR's remanufacturing activities. These arrangements provide different mechanisms for distributing remanufacturing-related economic benefits and shaping operational incentives.

   Consequently, managers should select the appropriate governance structure according to their strategic priorities and prevailing market conditions. Outsourcing may be preferable when the OEM seeks greater control over remanufacturing activities through a per-unit outsourcing-fee arrangement. Conversely, authorization may be advantageous when the OEM intends to govern remanufacturing through licensing or authorization rights while allowing the IR to operate more independently. However, neither governance structure is universally superior. The preferred arrangement depends on equilibrium profitability, consumer surplus, emission-reduction investment, and total environmental impact. In particular, greater remanufacturing output or stronger emission-reduction effort should not automatically be regarded as evidence of superior environmental performance. Therefore, managerial decisions should be based on a comprehensive comparison of economic and environmental outcomes under both structures.

3. **How do the trade-offs between transparency and privacy surrounding BCT affect consumer demand and equilibrium prices under VERM?**

   The adoption of BCT-enabled traceability can strengthen consumer confidence by improving the credibility and verifiability of information concerning remanufacturing processes and environmental performance. Nevertheless, increased transparency may also generate privacy concerns and perceived costs associated with information disclosure. Accordingly, the demand effects of BCT depend on the balance between the benefits of greater information credibility and consumers' perceived privacy-related disadvantages.

   From a managerial perspective, greater transparency should not automatically be assumed to increase consumer demand. Its commercial effectiveness depends on consumers' perceptions of information reliability, usefulness, and privacy protection. OEMs and IRs should therefore provide verifiable information regarding product recovery, remanufacturing operations, and environmental performance while limiting unnecessary disclosure of sensitive consumer and transaction data. Privacy-conscious data management, selective information disclosure, and reliable verification procedures can help maintain consumer confidence without compromising legitimate confidentiality requirements.

   Furthermore, the remanufacturing acceptance parameter δ should be distinguished from the parameters representing BCT transparency and privacy unless the model explicitly establishes a functional relationship among them. Maintaining this distinction enables managers to identify whether limited demand arises from low consumer acceptance of remanufactured products, privacy concerns, or insufficient confidence in environmental claims. Such differentiation is important for developing appropriate pricing, information-disclosure, and market-development strategies.

4. **What kind of coordination contracts can be used to effectively align the OEMs' and IRs' interests so that the decentralized decision-making in BCT and VERM can imitate the centralized efficiency?**

   Decentralized SC decision-making may create conflicts between the OEM's profitability objectives and the IR's incentives to expand remanufacturing and undertake emission-reduction investments. Effective coordination therefore requires contractual mechanisms that align the marginal incentives of both parties rather than relying exclusively on conventional outsourcing or authorization fees. Depending on the governance structure, suitable mechanisms may include R&D cost-sharing agreements, carbon-credit revenue-sharing arrangements, performance-based compensation, and contracts linking financial transfers to verified environmental outcomes.

   BCT can facilitate the implementation of these coordination mechanisms by providing traceable records of remanufacturing operations and verifiable information on eligible emission reductions. Such information can reduce asymmetries between SC members and support contractual arrangements based on measurable environmental performance. Nevertheless, improved transparency alone is insufficient to guarantee coordination. An effective contract must satisfy the individual rationality constraints of both the OEM and IR while ensuring that their operational and environmental investment decisions are aligned. To establish centralized efficiency, the proposed coordination mechanism must be evaluated against the centralized SC benchmark and shown to reproduce the relevant optimal decisions and aggregate profit under the model's assumptions.

   From a policy perspective, carbon-market authorities should establish reliable verification standards, transparent carbon-credit issuance procedures, and effective safeguards against double counting. These measures can strengthen confidence in VER revenues and reinforce the relationship between verified emission reductions and environmental investment incentives. For SC practitioners, the findings emphasize that BCT-enabled traceability, carbon-market incentives, and contractual coordination should be implemented as complementary mechanisms rather than independent strategic decisions.

Overall, the managerial implications demonstrate that the effectiveness of BCT-enabled VERM depends on the alignment of three essential elements: credible environmental verification, economically meaningful emission-reduction incentives, and contractual mechanisms that appropriately distribute costs and benefits among SC members. OEMs and IRs should consider these elements jointly when determining production quantities, R&D investment levels, and remanufacturing governance structures. Policymakers can further support these arrangements by strengthening carbon-credit verification procedures and promoting privacy-conscious digital governance. Ultimately, an effective remanufacturing strategy should balance economic profitability with credible environmental improvements while maintaining sufficient participation and investment incentives for both the OEM and IR.

7 Conclusion

This study investigates the integration of BCT and VERM into remanufacturing SCs, emphasizing operational decision-making, emission-reduction investment, contractual governance, and SC coordination. By developing and examining alternative remanufacturing arrangements, including outsourcing, authorization, and IR-led R&D structures, the study explores how carbon-credit incentives, consumer acceptance, technological investment costs, and BCT-enabled transparency influence the economic and environmental performance of SC members. Furthermore, centralized decision-making and contractual coordination mechanisms are examined to identify potential approaches for mitigating inefficiencies associated with decentralized SC operations.

The findings highlight the importance of incorporating environmental incentives into remanufacturing and R&D decisions. The VER price plays a significant role in determining the economic attractiveness of emission-reduction investments and can strengthen incentives for environmental improvement and remanufacturing expansion. However, the magnitude and direction of these effects depend on the equilibrium structure and the allocation of costs and benefits between the OEM and IR. Moreover, the R&D difficulty coefficient affects the economic feasibility of emission-reduction activities, emphasizing the importance of technological investment costs when evaluating carbon-market opportunities. These findings suggest that carbon-credit revenues should be incorporated into production and investment strategies rather than treated as an independent source of additional income.

The comparative analysis of outsourcing and authorization further demonstrates the importance of contractual governance in determining production quantities, emission-reduction efforts, and profit allocation. These arrangements establish different mechanisms for distributing decision-making authority and capturing the economic benefits generated through remanufacturing. Their relative attractiveness

depends on market conditions, environmental incentives, and contractual parameters. Consequently, selecting an appropriate governance structure requires consideration of individual firms' profitability, consumer welfare, and the overall environmental consequences of remanufacturing activities.

The study also reveals that the influence of consumer acceptance associated with BCT-enabled remanufacturing may exhibit nonlinear characteristics. Although greater consumer acceptance can improve the market competitiveness of remanufactured products, its effects on IR profitability and emission-reduction investment depend on the interaction among consumer demand, VER prices, and contractual incentives. In particular, threshold-dependent responses indicate that the benefits of improved consumer acceptance may vary across different market conditions. Therefore, BCT should be regarded as a mechanism for enhancing consumer trust and the credibility of environmental information rather than a technology that automatically improves every aspect of SC performance. Its economic and environmental benefits should be evaluated alongside implementation expenditures and potential privacy-related concerns.

The analysis of centralized decision-making and coordination contracts further emphasizes the need to align operational and environmental incentives across SC members. Centralized optimization provides a benchmark for assessing decentralized performance, while cost-sharing arrangements and two-part tariff contracts offer potential mechanisms for coordinating production, remanufacturing, and emission-reduction decisions. Subject to appropriate feasibility and participation conditions, a properly designed contract can align decentralized equilibrium decisions with the centralized benchmark. Specifically, the per-unit outsourcing fee influences marginal operational incentives, whereas the fixed fee facilitates the distribution of coordination surplus without directly affecting the IR's marginal production decisions. In addition, sharing BCT implementation and R&D expenditures can mitigate incentive conflicts arising when environmental investment costs and carbon-credit revenues are distributed unevenly between the OEM and IR.

Overall, this study contributes to the existing literature by integrating BCT-enabled verification, VERM incentives, remanufacturing governance, and SC coordination within a unified analytical framework. The findings demonstrate that sustainable remanufacturing requires more than technological transparency or participation in voluntary carbon markets. Its effectiveness depends on the interaction among credible emission verification, economically feasible environmental investments, consumer acceptance, and contractual arrangements that appropriately distribute costs, risks, and economic benefits. By examining these interrelated factors, the study provides a theoretical foundation for developing remanufacturing strategies that balance economic profitability, consumer welfare, and environmental sustainability.

7.1 Future Research Directions

Although this study provides valuable insights into BCT-enabled remanufacturing under VERM, several opportunities remain for extending the proposed analytical framework and improving its applicability to practical circular SCs. Future research could investigate endogenous BCT adoption decisions, uncertainty in VER prices and product returns, and competitive interactions among multiple OEMs and IRs. Incorporating life-cycle assessment methods would also allow a more comprehensive evaluation of environmental performance by distinguishing per-unit emission reductions from aggregate emissions across the product life cycle. Furthermore, alternative coordination mechanisms could be developed to address the allocation of R&D expenditures, BCT implementation costs, and carbon-credit revenues under asymmetric information. Future studies may also consider heterogeneous consumer preferences regarding transparency, trust, and privacy to better capture market responses to BCT-enabled remanufactured products. Empirical investigations using industry datasets and practical case studies would strengthen the applicability and validation of the theoretical findings. Finally, integrating digital twins, smart contracts, and real-time life-cycle information could support more adaptive decision-making and facilitate the development of economically efficient and environmentally sustainable BCT-enabled remanufacturing SCs.

Conflict of interest statement

The authors have no competing interests to declare that are relevant to the content of this article.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data availability statement

The data sets used and /or analyzed during the current study available from the corresponding author on reasonable request. We don't used any published data for solving the paper.

Credit authorship contribution details

All authors contributed equally to the research implementation, analysis of the results, and the composition and refinement of the manuscript.

A Proof of Section 4

A.1 Proof of assumption 1 : derivation of the inverse demand functions

Consider a scenario in which a customer is willing to pay (WTP) for a brand new product, referred to as θ, that is distributed uniformly over the range $[0, Q]$. In the case of a remanufactured product, the WTP is simply multiplied by a factor $\delta \in [0, 1]$, representing the common avoidance of remanufactured products. Further, the application of BCT not only secures the consumers' trust but also adds to it by making the information accessible on a transparent basis, which is depicted as λt, in which λ stands for the trust coefficient and t stands for the transparency level. Nevertheless, the increased transparency might lead to people's privacy issues being raised, and this is represented by a standard privacy disutility p_0. Therefore, the privacy cost incurred becomes $p_c = p_0 - \delta\lambda t$, indicating that the privacy issue has been lessened due to the trust that was created.

Assuming the above-stated conditions, the new product's consumer's net utility is calculated as: $U_n(\theta) = \theta - p_n$, in which case p_n denotes the price of the new product. The consumer's net utility for the remanufactured product is given by:

$$U_r(\theta) = \delta\theta - p_r + \lambda t - p_c$$

. In this case, p_r refers to the price of the remanufactured product. The expressions for utilities are the basis for the inverse demand functions of both new and remanufactured products. Upon substituting for p_c, the equation becomes $U_r(\theta) = \delta\theta - p_r + (1+\delta)\lambda t - p_0$. It is generally presumed that consumers will choose the alternative that yields the greatest benefit, provided that the benefit is positive. Let us define θ_1 as the threshold where new and remanufactured products are equally preferred, which is obtained by setting $U_n(\theta) = U_r(\theta)$. This results in the expression $\theta_1 = \dfrac{p_n - p_r + (1+\delta)\lambda t - p_0}{1-\delta}$. In the same way, θ_2 represents the point of indifference between buying a remanufactured product and not buying anything at all, which is derived from the equation $U_r(\theta) = 0$. This calculation yields $\theta_2 = \dfrac{p_r - (1+\delta)\lambda t + p_0}{\delta}$.

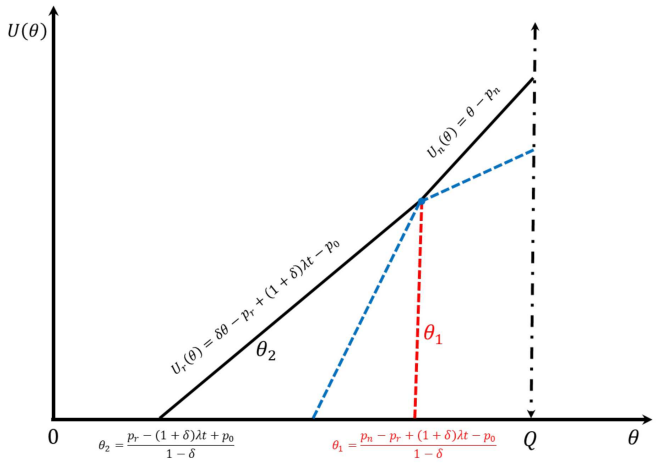


Fig. 18 Net utility functions pertaining to consumer behavior.

As illustrated in Figure 18, consumers whose characteristic is $\theta \in [\theta_1, Q]$ are the most eagerly expected to buy the newest products, whereas those in the interval $[\theta_2, \theta_1]$ prefer the remanufactured ones. Consumers with $\theta < \theta_2$ do not take part in the market. By considering θ to be uniformly distributed, the new products demand can be written as $d_n = Q - \theta_1$, while the remanufactured ones' demand is $d_r = \theta_1 - \theta_2$. By substituting for θ_1 and θ_2, we derive the inverse demand functions for new products:

$$p_n = (1-\delta)(Q - d_n) + p_r - (1+\delta)\lambda t + p_0,$$

and for remanufactured products as

$$p_r = \delta(Q - d_n - d_r) + (1+\delta)\lambda t - p_0.$$

With the help of these functions, the impacts of vertical product differentiation and consumer perception dynamics, which were introduced by BCT, along with the privacy issues related to them, are all encapsulated.

A.2 Proof of Model *O*

As a first step in our investigation, we want to look into the shape of the IR's profit function in terms of the decision variable d_r^O. The second derivative shows that $\dfrac{\partial^2 \Pi_R^O}{\partial \left(d_r^O\right)^2} = -2g < 0$. This implies that the IR's profit function is completely concave in the variable d_r^O. Thus, the first-order condition returns the following optimal response for IR:

$$d_r^O = \frac{(-1+\rho)\, c_a + w_r^O - c + s + p_e E^O}{2g}$$

Thereafter, the OEM's profit function is injected with the optimal response to, in turn, examine its concavity related to the decision variables d_n^O, w_r^O, and E^O. Noting that the inverse demand functions for p_n^O and p_r^O are both linear, the profit function Π_M^O is a second-degree polynomial in d_n^O and w_r^O. The next step involves calculating the Hessian matrix H^O with respect to these variables and we find that the leading principal minor is negative while the determinant is positive under the condition:

$$H^O = \begin{bmatrix} -2 & -\dfrac{\delta}{g} \\ -\dfrac{\delta}{g} & \dfrac{\delta + 2g - \delta^2}{g^2} \end{bmatrix}.$$

From the Hessian matrix, it was concluded that parameter values $H_1^O = -2 < 0$ and $H_2^O = \dfrac{\delta(1-\delta) + 2g}{g^2}$ were determined to be in effect under the condition $\delta(1-\delta) + 2g > 0$ where the latter stands for Π_M^O being jointly concave in d_n^O and w_r^O thus. The best responses of the OEM are as follows which were obtained by solving the first-order conditions:

$$w_r^O = \frac{-\delta^2 c + ((c+\lambda t)\, g + c)\,\delta + g\,(c + \lambda t - p_0)}{\delta + 2g - \delta^2} + \frac{\left((-1+\rho)\,\delta^2 + (1-\rho)\,\delta + (-2\rho+1)\, g\right) c_a}{\delta + 2g - \delta^2} - \frac{A_4\left(p_e E^O + s\right)}{\delta + 2g - \delta^2} \text{ and}$$

$$d_n^O = \frac{\left(Q - \left(\lambda t + p_e E^O + A_3\right)\right)\delta - (Q + \lambda t)\,\delta^2}{2A_1} + \frac{2g\,(Q-c)}{2A_1}$$

In order to assess the concavity with respect to the decision on emission reduction E^O, we first insert d_n^{O*} and w_r^{O*} into the OEM's profit function and then find the second derivative of the profit function with respect to E^O:

$$\frac{\partial^2 \Pi_M^O}{\partial \left(E^O\right)^2} = \frac{2\xi\left(\delta\left(\delta-1\right)-2g\right)+{p_e}^2}{2A_1}.$$

The profit of the OEM demonstrates concavity in E^O when the following condition holds: $2\xi\left(\delta\left(\delta-1\right)-2g\right)+p_e^2>0$. This stipulation ensures that there is only one optimal level of emission reduction E^O. The model of outsourcing remanufacturing that takes into account BCT and the verification of emission reductions among other things (Model O) gives a unique equilibrium for the four variables (d_n^O, d_r^O, w_r^O, and E^O) as shown in Table 5.

A.3 Proof of Model *A*

It is evident that the second derivative of the function Π_R^A with respect to d_r^A satisfies the condition $\frac{\partial^2 \Pi_R^A}{\partial \left(d_r^A\right)^2} = -2\left(g+\delta\right) < 0$. This leads to the conclusion that the Hessian matrix H^A is defined as follows:

$$H^A = \begin{bmatrix} -2+\dfrac{\delta^2}{\delta+g} & 0 \\ 0 & -\left(\delta+g\right)^{-1} \end{bmatrix}.$$

Consequently, it can be determined that Π_M^A is concave in both d_n^A and f^A when $\frac{2\left(\delta+g\right)-\delta^2}{\left(\delta+g\right)^2}>0$. Additionally, the second derivative of the function Π_M^A with respect to E^A is expressed as:

$$\frac{\partial^2 \Pi_M^A}{\partial \left(E^A\right)^2} = \frac{2\left(\delta^2-2\left(\delta+g\right)\right)\xi+{p_e}^2}{2\left(2\left(\delta+g\right)-\delta^2\right)}.$$

This condition holds true under the constraint that $-2\xi\left(\left(\delta+A_1\right)\right)+{p_e}^2>0$. From these analyses, the equilibrium solutions of Model A can be derived and are further summarized in Table 6.

A.4 Proof of Model *BO*

It is evident that the second derivative of the function Π_R^{BO} with respect to d_r^{BO} satisfies the condition $\frac{\partial^2 \Pi_R^{BO}}{\partial \left(d_r^{BO}\right)^2} = -2g < 0$. This leads to the conclusion that the Hessian matrix H^{BO} is defined as follows:

$$H^{BO} = \begin{bmatrix} -2 & -\dfrac{\delta}{g} \\ -\dfrac{\delta}{g} & \dfrac{-\left(\delta+2g\right)}{2g^2} \end{bmatrix}.$$

Consequently, it can be determined that Π_M^{BO} is concave in both d_n^{BO} and w_r^{BO} when $\frac{\delta+2g-\delta^2}{g^2}>0$. From these analyses, the equilibrium solutions of Model BO can be derived and are further summarized in Table 5.

A.5 Proof of Model *BA*

It is evident that the second derivative of the function Π_R^{BA} with respect to d_r^{BA} satisfies the condition $\frac{\partial^2 \Pi_R^{BA}}{\partial \left(d_r^{BA}\right)^2} = -2\left(g+\delta\right) < 0$.. This leads to the conclusion that the Hessian matrix H^{BA} is defined as follows:

$$H^{BA} = \begin{bmatrix} -\dfrac{2\delta + 2g - \delta^2}{\delta + g} & 0 \\ 0 & -(\delta + g)^{-1} \end{bmatrix}.$$

Consequently, it can be determined that Π_M^{BA} is concave in both d_n^{BA} and f^{BA} when $\dfrac{2\delta + 2g - \delta^2}{(\delta + g)^2} > 0$. From these analyses, the equilibrium solutions of Model BA can be derived and are further summarized in Table 6.

B Proof of Section 5

B.1 Proof of Proposition 1

By conducting a comparison of the equilibrium solutions pertaining to Models O and BO, we are able to derive the following insights:

$$d_n^{O*} - d_n^{BO*} = -\frac{(\delta S - 2A_2 + 2(s + \lambda t))\,\delta {p_e}^2}{4(2\xi A_2 - {p_e}^2)A_1} < 0; d_r^{O*} - d_r^{BO*} = \frac{(\delta S - 2A_2 + 2(s + \lambda t))\,{p_e}^2}{4(2\xi A_2 - {p_e}^2)A_1} > 0;$$
$$w_r^{O*} - w_r^{BO*} = -\frac{(\delta S - 2A_2 + 2(s + \lambda t))\,{p_e}^2 A_4}{2(2\xi A_2 - {p_e}^2)A_1} < 0; p_r^{O*} - p_r^{BO*} = \frac{(\delta S - 2A_2 + 2(s + \lambda t))\,{p_e}^2(-1 + \delta)\,\delta}{4(2\xi A_2 - {p_e}^2)A_1} < 0;$$
$\Pi_M^{O*} - \Pi_M^{BO*} > 0$; and $\Pi_R^{O*} - \Pi_R^{BO*} > 0$.

B.2 Proof of Proposition 2

Upon examining the equilibrium solutions associated with Models A and BA, one can ascertain the following:

$$d_n^{A*} - d_n^{BA*} = -\frac{\delta(\delta S - 2A_2 + 2(s + \lambda t))\,{p_e}^2}{8A_2(-{p_e}^2 + 2\xi(\delta + A_2))} < 0; d_r^{A*} - d_r^{BA*} = \frac{(\delta S - 2A_2 + 2(s + \lambda t))\,{p_e}^2}{8A_2(2\xi(\delta + A_2) - {p_e}^2)} > 0;$$
$$f^{A*} - f^{BA*} = \frac{(\delta S - 2A_2 + 2(s + \lambda t))\,{p_e}^2}{4(2\xi(\delta + A_1) - {p_e}^2)} > 0; p_r^{A*} - p_r^{BA*} = \frac{\delta {p_e}^2(-1 + \delta)(\delta S + 2(-A_2 + s + \lambda t))}{4(-{p_e}^2 + 2\xi(\delta + A_2))(\delta + A_2)} < 0;$$
$\Pi_M^{A*} - \Pi_M^{BA*} > 0$; and $\Pi_R^{A*} - \Pi_R^{BA*} > 0$.

B.3 Proof of Proposition 3

By computing the first derivatives of optimal prices concerning p_e and ξ, in a manner similar to Model O, one can derive the following results:

$$\frac{\partial E^{O*}}{\partial p_e} = \frac{\left(2\xi A_2 + {p_e}^2\right)(\lambda t(\delta + 1) - (c_a + p_0))}{(2\xi A_2 - {p_e}^2)^2} + \frac{(\delta(c + Q) + 2(s - c))\left(2\xi A_2 + {p_e}^2\right)}{2(2\xi A_2 - {p_e}^2)^2} > 0;$$
$$\frac{\partial d_n^{O*}}{\partial p_e} = -\frac{(\delta S - 2A_2 + 2(s + \lambda t))\,\delta \xi p_e}{(2\xi A_2 - {p_e}^2)^2} < 0; \frac{\partial d_r^{O*}}{\partial p_e} = \frac{\xi(\delta c + 2\delta\lambda t + \delta Q - 2A_2 + 2(s + \lambda t))\,p_e}{(2\xi A_2 - {p_e}^2)^2} > 0;$$
$$\frac{\partial w_r^{O*}}{\partial p_e} = -\frac{2\left(-\delta^2 + \delta + g\right)(\delta S - 2A_2 + 2(s + \lambda t))\,\xi p_e}{(2\xi A_2 - {p_e}^2)^2} < 0; \frac{\partial p_r^{O*}}{\partial p_e} = \frac{\xi p_e(-1 + \delta)\,\delta(\delta(S) - 2A_2 + 2(s + \lambda t))}{(2\xi A_1 - {p_e}^2)^2} < 0;$$
$$\frac{\partial \Pi_M^{O*}}{\partial p_e} > 0; \text{ and } \frac{\partial \Pi_R^{O*}}{\partial p_e} > 0; \frac{\partial E^{O*}}{\partial \xi} = -\frac{A_1 p_e(\delta S - 2A_2 + 2(s + \lambda t))}{(2\xi A_2 - {p_e}^2)^2} < 0;$$
$$\frac{\partial d_n^{O*}}{\partial \xi} = \frac{\delta(\delta S - 2A_2 + 2(s + \lambda t))\,{p_e}^2}{2(2\xi A_2 - {p_e}^2)^2} > 0; \frac{\partial d_r^{O*}}{\partial \xi} = -\frac{(\delta S - 2A_2 + 2(s + \lambda t))\,{p_e}^2}{2(2\xi A_2 - {p_e}^2)^2} < 0;$$
$$\frac{\partial w_r^{O*}}{\partial \xi} = \frac{{p_e}^2(\delta S - 2A_2 + 2(s + \lambda t))\left(-\delta^2 + \delta + g\right)}{(2\xi A_2 - {p_e}^2)^2} > 0; \frac{\partial p_r^{O*}}{\partial \xi} = -\frac{{p_e}^2(-1 + \delta)\,\delta(\delta S - 2A_2 + 2(s + \lambda t))}{2(2\xi A_2 - {p_e}^2)^2} > 0;$$
$$\frac{\partial \Pi_M^{O*}}{\partial \xi} < 0; \text{ and } \frac{\partial \Pi_R^{O*}}{\partial \xi} < 0.$$

Regarding Model A, the following conclusions can be drawn:

$$\frac{\partial E^{A*}}{\partial p_e} = \frac{(\delta S - 2A_2 + 2(s+\lambda t))\left({p_e}^2 + 2\xi(\delta + A_2)\right)}{2\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} > 0;\ \frac{\partial d_n^{A*}}{\partial p_e} = -\frac{\delta\xi(S\delta - 2A_2 + 2(s+\lambda t))p_e}{\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} < 0;$$
$$\frac{\partial d_r^{A*}}{\partial p_e} = \frac{(\delta S - 2A_2 + 2(s+\lambda t))p_e\xi}{\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} > 0;\ \frac{\partial f^{A*}}{\partial p_e} = \frac{p_e(\delta + A_1)\xi(\delta S - 2A_2 + 2(s+\lambda t))}{2\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} > 0;$$
$$\frac{\partial p_r^{A*}}{\partial p_e} = \frac{\xi\delta(\delta S - 2A_2 + 2(s+\lambda t))(-1+\delta)p_e}{\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} < 0;\ \frac{\partial \Pi_M^{A*}}{\partial p_e} > 0;\text{ and } \frac{\partial \Pi_R^{A*}}{\partial p_e} > 0;$$
$$\frac{\partial E^{A*}}{\partial \xi} = -\frac{p_e(\delta S - 2A_2 + 2(s+\lambda t))(\delta + A_1)}{\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} < 0;\ \frac{\partial d_n^{A*}}{\partial \xi} = \frac{\delta(S\delta - 2A_2 + 2(s+\lambda t)){p_e}^2}{2\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} > 0;$$
$$\frac{\partial d_r^{A*}}{\partial \xi} = -\frac{{p_e}^2(\delta S - 2A_2 + 2(s+\lambda t))}{2\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} < 0;\ \frac{\partial f^{A*}}{\partial \xi} = -\frac{(\delta + A_1){p_e}^2(\delta S - 2A_2 + 2(s+\lambda t))}{2\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} < 0;$$
$$\frac{\partial p_r^{A*}}{\partial \xi} = -\frac{{p_e}^2(\delta S - 2A_2 + 2(s+\lambda t))(\delta - 1)\delta}{2\left(-{p_e}^2 + 2\xi(\delta + A_2)\right)^2} > 0;\ \frac{\partial \Pi_M^{A*}}{\partial \xi} < 0;\text{ and } \frac{\partial \Pi_R^{A*}}{\partial \xi} < 0.$$

B.4 Proof of Proposition 4

In accordance with the definition of consumer surplus, denoted as $CS = \frac{d_n^2 + 2\delta d_n d_r + \delta d_r^2}{2Q} + d_r[\lambda t(1+\delta) - p_0]$, it is possible to derive the following:

$$CS^O - CS^{BO} < 0 \text{ and}$$
$$CS^A - CS^{BA} > 0.$$

B.5 Proof of Proposition 7

Given that $A_4 < A_1 < \delta + A_1$, we can infer the following relationships: $E^{O*} > E^{A*} > E^{C*}$, $d_n^{A*} > d_n^{O*} > d_n^{C*}$, and $d_r^{C*} > d_r^{O*} > d_r^{A*}$. Furthermore, concerning the overall profit of the SC, the conclusion can be drawn that: $\Pi_k^{O*} - \Pi_k^{A*} > 0$. Hence, it follows that $\Pi_k^{A*} < \Pi_k^{O*} < \Pi_k^{C*}$. By taking the first-order derivatives of the equilibrium solutions for Model C with respect to p_e and ξ, we can derive the necessary insights:

$$\frac{\partial E^{C*}}{\partial p_e} = \frac{\left(\xi A_4 + {p_e}^2\right)((c+\lambda t)\delta + \lambda t + s - (p_0 + c + c_a))}{\left(\xi A_4 - {p_e}^2\right)^2} > 0;$$
$$\frac{\partial d_n^{C*}}{\partial p_e} = -\frac{\xi\delta((c+\lambda t)\delta + \lambda t + s - (p_0 + c + c_a))p_e}{\left(\xi A_4 - {p_e}^2\right)^2} < 0;$$
$$\frac{\partial d_r^{C*}}{\partial p_e} = \frac{((c+\lambda t)\delta + \lambda t + s - (p_0 + c + c_a))\xi p_e}{\left(\xi A_4 - {p_e}^2\right)^2} > 0;$$
$$\frac{\partial \Pi_k^{C*}}{\partial p_e} = \frac{\xi((c+\lambda t)\delta + \lambda t + s - (p_0 + c + c_a))^2 p_e}{2\left(\xi A_4 - {p_e}^2\right)^2} > 0;$$
$$\frac{\partial E^{C*}}{\partial \xi} = -\frac{A_4((c+\lambda t)\delta + \lambda t + s - (p_0 + c + c_a))p_e}{\left(\xi A_4 - {p_e}^2\right)^2} < 0;$$
$$\frac{\partial d_n^{C*}}{\partial \xi} = \frac{((c+\lambda t)\delta + \lambda t + s - (p_0 + c + c_a))\delta {p_e}^2}{2\left(\xi A_4 - {p_e}^2\right)^2} > 0;$$
$$\frac{\partial d_r^{C*}}{\partial \xi} = -\frac{((c+\lambda t)\delta + \lambda t + s - (p_0 + c + c_a)){p_e}^2}{2\left(\xi A_4 - {p_e}^2\right)^2} < 0;$$
$$\frac{\partial \Pi_k^{C*}}{\partial \xi} = -\frac{((c+\lambda t)\delta + \lambda t + s - (p_0 + c + c_a))^2 {p_e}^2}{4\left(\xi A_4 - {p_e}^2\right)^2} < 0.$$

B.6 Proof of Proposition 8

According to Eqn. 10, the first-order condition indicates that

$$d_r = \frac{(-1+\rho)\, c_a + w_r - c + s + p_e E}{2g}.$$

In order for the CLSC's profitability to be in line with that of the centralized model, the following conditions must be met: $d_n^{CO*} = d_n^{C*}$, $d_r^{CO*} = d_r^{C*}$, and $E^{CO*} = E^{C*}$. Subsequently, the outsourcing fee is delineated by

$$w_r^{CO*} = \frac{{p_e}^2 \left((1-2\rho)\, c_a + t\,(1+\delta)\,\lambda + \delta c - p_0\right)}{p_e^2 - \xi A_4} - \frac{\xi A_4 \left((1-\rho)\, c_a + c - s\right)}{p_e^2 - \xi A_4}.$$

When a win-win outcome is achieved, the conditions $\Pi_M^{CO*} \geq \Pi_M^{O*}$ and $\Pi_R^{CO*} \geq \Pi_R^{O*}$ are satisfied. By comparing the profits, one can derive the sharing ratio $\kappa \in (\underline{\kappa}, \min(\overline{\kappa}, 1))$ and $\Pi_S^{C*} = \Pi_M^{CO*} + \Pi_R^{CO*}$ where $\kappa = \dfrac{2\,(\rho-1)\left(p_e^2 - \xi A_4\right)^2 F_c}{\xi\left(W_1 + s - (p_0 + \rho c_a)\right)^2 p_e^2} - \dfrac{\xi g}{2p_e^2}$.

B.7 Proof of Proposition 9

In a manner analogous to the cost-sharing contract, the outsourcing fee within a two-part tariff contract is determined by the equation

$$w_r^{OT*} = \frac{{p_e}^2 \left((1-2\rho)\, c_a + t\,(1+\delta)\,\lambda + \delta c - p_0\right)}{p_e^2 - \xi A_4} - \frac{\xi A_4 \left((1-\rho)\, c_a + c - s\right)}{p_e^2 - \xi A_4}.$$

When $d_n^{OT*} = d_n^{C*}$, $d_r^{OT*} = d_r^{C*}$, and $E^{OT*} = E^{C*}$. Under an optimal win-win scenario, the fixed costs must satisfy the condition $F^{OT} \in \left[\underline{F^{OT}}, \overline{F^{OT}}\right]$, where

$$\begin{aligned}
\underline{F}^{OT} &= \frac{\left(-cQt^2\,(1+\delta)^2\,\lambda^2 - 2Q^2 A_1 - 4\left((\delta-1)\,c + A_3\right)(1+\delta)\,t\lambda\right)\xi + Q^2 {p_e}^2}{8\xi A_1 - 4{p_e}^2} \\
&\quad + \frac{\left(-2A_5 c^2 - 4A_3\,(\delta-1)\,c + 8A_1\rho F_c - 2{A_3}^2\right)\xi + {p_e}^2\left(c^2 - 4\rho F_c\right)}{8\xi A_1 - 4{p_e}^2} \\
\overline{F}^{OT} &= \frac{\xi\left(2\xi g - {p_e}^2\right)\left((1+\delta)\,\lambda t + A_3 - c\,(1-\delta)\right)^2}{2\left(2\xi A_1 - {p_e}^2\right)^2} + F_c\,(-1+\rho).
\end{aligned}$$

It is important to note that $\Pi_S^{OT} = \Pi_S^C$. For the coordination mechanism to be implementable, the feasible interval must be non-empty. This requires $\underline{F}^{OT} \leq \overline{F}^{OT}$, which is equivalent to requiring that the total coordinated SC profit exceeds the sum of the reservation profits under decentralized operation. Because $\Pi_S^C > \Pi_S^O$, the coordination contract generates a positive surplus that can be redistributed between the OEM and the IR through the fixed fee. Therefore, a non-empty feasible interval always exists whenever coordination improves total SC profitability, ensuring voluntary participation by both parties.

B.8 Proof of Subsection 6.6

It is evident that the second derivative of the function Π_R^{RO} with respect to d_r^{RO} and E^{RO} satisfies $H_1^{RO} = \begin{bmatrix} -2g & p_e \\ p_e & -\xi \end{bmatrix}$. This indicates that the profit function of the Π_R^{RO} is strictly concave in d_r^{RO} and E^{RO} when $2g\xi - p_e^2 > 0$. Consequently, the first-order condition yields the IR's optimal response:

$$d_r^{RO}\left(w_r^{RO}\right) = \frac{-\xi\left(c_a\,(1-\rho) - w_r^{RO} + c - s\right)}{2g\xi - {p_e}^2}$$

$$E^{RO}\left(w_r^{RO}\right) = \frac{-p_e\left(c_a\left(1-\rho\right) - w_r^{RO} + c - s\right)}{2g\xi - {p_e}^2}$$

Next, to substitute this optimal response into the OEM's profit function to assess its concavity concerning the decision variables d_n^{RO} and w_r^{RO}. Considering the linear nature of the inverse demand functions for p_n^{RO} and p_r^{RO}, the profit function Π_M^{RO} is quadratic in d_n^{RO} and w_r^{RO}. We proceed to compute the Hessian matrix H_2^{RO} with respect to these variables and determine that the leading principal minor is negative while the determinant is positive given the condition:

$$H_2^{RO} = \begin{bmatrix} -2 & -\dfrac{2\delta\xi}{2g\xi - {p_e}^2} \\ -\dfrac{2\delta\xi}{2g\xi - {p_e}^2} & -\dfrac{2\left(\xi\left(\delta+2g\right) - {p_e}^2\right)\xi}{\left(2g\xi - {p_e}^2\right)^2} \end{bmatrix}.$$

According to the Hessian matrix, it can be concluded that values $H_1^{RO} = -2 < 0$ and $H_2^{RO} = \dfrac{4\xi\left(\xi A_1 - {p_e}^2\right)}{\left(2g\xi - {p_e}^2\right)^2}$ are valid when $\xi A_1 - {p_e}^2 > 0$. Therefore, to conclude that Π_M^{RO} is jointly concave in d_n^{RO} and w_r^{RO}. Consequently, the equilibrium solutions for Model RO are provided in a summary format within Table 7. Given that the optimal solution process of Model RA closely resembles that of Model RO, we will limit our discussion to the conditions necessary for the existence of the optimal solution.

It is evident that the second derivative of the function Π_R^{RA} with respect to d_r^{RA} and E^{RA} satisfies $H_1^{RA} = \begin{bmatrix} -2\left(\delta+g\right) & p_e \\ p_e & -\xi \end{bmatrix}$. This indicates that the profit function of the Π_R^{RA} is strictly concave in d_r^{RA} and E^{RA} when $2\left(\delta+g\right)\xi - {p_e}^2 > 0$. Next, to substitute this optimal response into the OEM's profit function to assess its concavity concerning the decision variables d_n^{RA} and w_r^{RA}. Considering the linear nature of the inverse demand functions for p_n^{RA} and p_r^{RA}, the profit function Π_M^{RA} is quadratic in d_n^{RA} and w_r^{RA}. We proceed to compute the Hessian matrix H_2^{RA} with respect to these variables and determine that the leading principal minor is negative while the determinant is positive given the condition:

$$H_2^{RO} = \begin{bmatrix} -2 + \dfrac{2\xi\delta^2}{2\left(\delta+g\right)\xi - {p_e}^2} & 0 \\ 0 & -\dfrac{2\xi}{2\left(\delta+g\right)\xi - {p_e}^2} \end{bmatrix}.$$

According to the Hessian matrix, it can be concluded that values $H_1^{RA} = \dfrac{-2\left(\xi\left(\delta+A_1\right) - p_e^2\right)}{2\left(\delta+g\right) - p_e^2} < 0$ and $H_2^{RA} = \dfrac{4\xi\left(\left(\delta+A_1\right)\xi - p_e^2\right)}{\left(2\xi\left(\delta+g\right) - p_e^2\right)^2}$ are valid when $\left(\delta+A_1\right)\xi - p_e^2 > 0$. Therefore, to conclude that Π_M^{RA} is jointly concave in d_n^{RA} and w_r^{RA}. Consequently, the equilibrium solutions for Model RA are provided in a summary format within Table 7.

B.9 Proof of Proposition 10

By examining the equilibrium solutions of the aforementioned models, it is possible to derive the following insights:

$$\begin{aligned}
&d_n^{RO*} - d_n^{O*} < 0; d_r^{RO*} - d_r^{O*} > 0; d_n^{RA*} - d_n^{A*} < 0;\ d_r^{RA*} - d_r^{A*} > 0; d_r^{RA*} - d_r^{O*} < 0;\\
&d_r^{RA*} - d_r^{RO*} < 0;\ E^{RA*} - E^{O*} < 0;\ E^{RA*} - E^{A*} < 0;\ E^{RO*} - E^{O*} < 0;\ E^{RA*} - E^{RO*} < 0;\\
&f^{RA*} - f^{A*} < 0; w_r^{RO*} - w_r^{O*} > 0;\ w_r^{RO*} - w_r^{BO*} < 0;\ p_r^{RO*} - p_r^{A*} < 0.
\end{aligned}$$

The interrelationships among the models are articulated in the subsequent three cases:
Case 1: When $p_e < \sqrt{\left(\delta+A_1\right)\xi}$ the inequality relations are as follows:

$$E^A < E^O < E^{RA} < E^{RO}; d_r^A < d_r^O < d_r^{RA} < d_r^{RO};\ f^A < f^{RA}; d_n^{RA} < d_n^A < d_n^{RO} < d_n^O;$$
$$w_r^{BO} < w_r^{RO} < w_r^O; \text{and } p_r^{RO} < P_r^A.$$

Case 2: When $\sqrt{(\delta + A_1)\,\xi} < p_e < \sqrt{2\xi\,(\delta + A_1)}$ the inequality relations are as follows:

$$E^{RA} < E^{RO} < E^A < E^O; d_r^{RA} < d_r^{RO} < d_r^A < d_r^O;\ f^{RA} < f^A; d_n^A < d_n^{RA} < d_n^O < d_n^{RO};$$
$$w_r^O < w_r^{RO} = w_r^{BO}; \text{ and } \quad p_r^A < P_r^{RO}.$$

Case 3: When $p_e > \sqrt{2\xi\,(\delta + A_1)}$ the inequality relations are as follows:

$$E^A < E^{RO} < E^O < E^{RA}; d_r^A < d_r^{RO} < d_r^O < d_r^{RA};\ f^{RA} < f^A; d_n^{RA} < d_n^A < d_n^O < d_n^{RO};$$
$$w_r^O < w_r^{RO} = w_r^{BO}; \text{ and } \quad p_r^{RO} < P_r^A.$$

B.10 Proof of Theorem 1

By computing the first derivatives of Π_R^{O*} and Π_R^{A*} concerning δ, and it is evident that: $\frac{\partial \Pi_R^{A*}}{\partial \delta} > 0$ and $\frac{\partial \Pi_R^{O*}}{\partial \delta} > 0$ when $\delta > \frac{1}{2}$; $\frac{\partial \Pi_R^{A*}}{\partial \delta} < 0$ when $\delta < \frac{1}{2}$ and $p_0 > \overline{p_0^A}$; $\frac{\partial \Pi_R^{O*}}{\partial \delta} < 0$ when $\delta < \frac{1}{2}$ and $p_0 > \overline{p_0^O}$. In this context, the expression for $\overline{p_0^A}$ and $\overline{p_0^O}$ are defined as follows:

$$\overline{p_0^A} = \frac{4\left((\lambda t + c)\,\delta - (c + c_a) + s + \lambda t\right)(\delta + A_1)\,\xi + \delta {p_e}^2\,(Q - c)}{4\xi\,(\delta + A_1)}$$
$$\overline{p_0^O} = \frac{4A_1\left((\lambda t + c)\,\delta - (c + c_a) + s + \lambda t\right)\xi + \delta {p_e}^2\,(Q - c)}{4\xi A_1}.$$

Then, by taking the first derivatives of E^{O*} with respect to δ, one can obtain:

$$\frac{\partial E^{O*}}{\partial \delta} = \frac{S p_e}{2\left(2\xi A_1 - {p_e}^2\right)} + \frac{\left(2\lambda\,(\delta + 1)\,t + (c + Q)\,\delta + 2\,(s - A_2)\right)(-1 + 2\delta)\,\xi p_e}{\left(2\xi A_1 - {p_e}^2\right)^2}.$$

The equation $\frac{\partial E^{O*}}{\partial \delta} = 0$ yields roots $p_{e_1}^O$ and $-p_{e_1}^O$, where

$$p_{e_1}^O = \frac{2\sqrt{-\delta\xi A_1\left((c + \lambda t)\,\delta + A_3 + \lambda t - c\right)(Q - c)}}{\delta\,(Q - c)}.$$

Consequently, the roots of the equation $\frac{\partial E^{O*}}{\partial \delta} = 0$ are $\overline{\delta_O}$ and $\underline{\delta_O}$, and $\frac{\partial E^{O*}}{\partial \delta} < 0$ when $\delta \in \left(\underline{\delta_O}, \overline{\delta_O}\right)$, where

$$\overline{\delta_O} = \frac{2\sqrt{-\left(2\left[\begin{array}{l}(g-1)\left(4\lambda^2 t^2 + c^2\right) + ((5+4g)c + (4g-1)Q - 6A_3)\,\lambda t \\ + ((1+2g)Q + 3A_3)\,c + Q^2 g - A_3(Q - 2A_3)\end{array}\right]\xi - \frac{p_e^2 S^2}{2}\right)\xi} + 4\,(A_2 - (\lambda t + s))\,\xi}{2\xi(c + Q + 2\lambda t)}$$

$$\underline{\delta_O} = \frac{-2\sqrt{-\left(2\left[\begin{array}{l}(g-1)\left(4\lambda^2 t^2 + c^2\right) + ((5+4g)c + (4g-1)Q - 6A_3)\,\lambda t \\ + ((1+2g)Q + 3A_3)\,c + Q^2 g - A_3(Q - 2A_3)\end{array}\right]\xi - \frac{p_e^2 S^2}{2}\right)\xi} + 4\,(A_2 - (\lambda t + s))\,\xi}{2\xi(c + Q + 2\lambda t)}.$$

For Model A, a similar analysis yields:

$$\frac{\partial E^{A*}}{\partial \delta} = \frac{S p_e}{2\left((\delta + A_1)\,\xi - {p_e}^2\right)} + \frac{2\,(-1 + \delta)\,p_e\,(\delta S - 2A_2 + 2\,(s + \lambda t))\,\xi}{\left(2\xi\,(\delta + A_1) - {p_e}^2\right)^2}.$$

The equation $\frac{\partial E^{A*}}{\partial \delta}=0$ yields roots $p_{e_2}^A$ and $-p_{e_2}^A$, where

$$p_{e_2}^A=\frac{2\sqrt{-\xi\left(Q-c\right)\left(\delta+A_1\right)\left(\left(c+\lambda t\right)\delta+A_3+\lambda t-c\right)\delta}}{\delta\left(Q-c\right)}.$$

Consequently, the roots of the equation $\frac{\partial E^{A*}}{\partial \delta}=0$ are $\overline{\delta_A}$ and $\underline{\delta_A}$, and $\frac{\partial E^{A*}}{\partial \delta}<0$ when $\delta\in\left(\underline{\delta_A},\overline{\delta_A}\right)$, where

$$\overline{\delta_A}=\frac{2\sqrt{-\left(2\left[\begin{array}{l}2(2g-3)\lambda^2t^2+2\left(Q(2g-1)+c(2g+3)-4A_3\right)\lambda t\\+\left(Q^2+c^2\right)g+2\left(\left(g+1\right)c-A_3\right)Q+2A_3(c-A_3)\end{array}\right]\xi-\frac{p_e^2S^2}{2}\right)\xi}+4\left(A_2-\left(s+\lambda t\right)\right)\xi}{2\xi S}$$

$$\underline{\delta_A}=\frac{-2\sqrt{-\left(2\left[\begin{array}{l}2(2g-3)\lambda^2t^2+2\left(Q(2g-1)+c(2g+3)-4A_3\right)\lambda t\\+\left(Q^2+c^2\right)g+2\left(\left(g+1\right)c-A_3\right)Q+2A_3(c-A_3)\end{array}\right]\xi-\frac{p_e^2S^2}{2}\right)\xi}+4\left(A_2-\left(s+\lambda t\right)\right)\xi}{2\xi S}.$$

B.11 Proof of Theorem 2

Based on the equation $AE^l=E^cd_n^l+\left(\beta E^c-E^l\right)d_r^l$, it is possible to derive the total carbon emissions for Models O and A as follows:

$$\begin{aligned}AE^{O*}=&\frac{-E^c\left(-Q\xi{A_1}^2+\xi\left(\delta^2\lambda t+\left(A_3+\lambda t\right)\delta+2gc\right)A_1+{p_e}^2\left(Q-c\right)\left(\frac{\delta+A_1}{4}\right)\right)}{4A_1\left(2\xi A_1-{p_e}^2\right)}\\&-\frac{\left(\begin{array}{l}\left(2\beta E^c{p_e}^2+\left(\delta S+2\left(\lambda t+A_3-c\right)\right)p_e-4\xi\beta E^cA_1\right)\times\\\left(4A_1\left(\left(c+\lambda t\right)\delta+A_3+\lambda t-c\right)\xi+{p_e}^2\delta\left(Q-c\right)\right)\end{array}\right)}{8\left(2\xi A_1-{p_e}^2\right)^2A_1};\\AE^{A*}=&\frac{E^c\left(-\left(Q+\lambda t\right)\delta^2+2g\left(Q-c\right)\right)}{2\left(\delta+A_1\right)}+\frac{E^c\left(\left(\delta+A_1\right)\left(2Q-\left(c+\lambda t+A_3\right)\right)\xi-\frac{\left(S\delta+4\left(Q-c\right)\right){p_e}^2}{4}\right)\delta}{\left(\delta+A_1\right)\left(2\xi\left(\delta+A_1\right)-{p_e}^2\right)}\\&-\frac{\left(\begin{array}{l}\left(4\left(\delta+A_1\right)\left(\left(c+\lambda t\right)\delta+A_3+\lambda t-c\right)\xi+{p_e}^2\delta\left(Q-c\right)\right)\times\\\left(-4\beta E^c\left(\delta+A_1\right)\xi+\left(\delta S+2\left(\beta E^cp_e+A_3-c+\lambda t\right)\right)p_e\right)\end{array}\right)}{8\left(2\xi\left(\delta+A_1\right)-{p_e}^2\right)^2\left(\delta+A_1\right)}.\end{aligned}$$

When compared to the baseline model, the alterations in total carbon emissions under the VERM framework can be summarized as follows:

$$\begin{aligned}\Delta AE^{O*}=AE^{O*}-AE^{BO*}=&\frac{\delta p_e\left(4\left(\left(c+\lambda t\right)\delta^2+\left(A_3-2c\right)\delta-\lambda t-A_3+c\right)\xi+{p_e}^2c\right)L}{8\left(2\xi A_1-{p_e}^2\right)^2A_1}\\&-\frac{L\left(\left(-4E^cA_1\left(\beta-\delta\right)p_e+8g\left(\left(c+\lambda t\right)\delta+A_3+\lambda t-c\right)\right)\xi+\left(2E^c\left(\beta-\delta\right)p_e+Q\delta\right){p_e}^2\right)p_e}{8\left(2\xi A_1-{p_e}^2\right)^2A_1};\\\Delta AE^{A*}=AE^{A*}-AE^{BA*}=&-\frac{L\left(4\left(\delta+A_1\right)\left(\left(c+\lambda t\right)\delta+A_3+\lambda t-c\right)\xi+{p_e}^2\delta\left(Q-c\right)\right)p_e}{8\left(2\xi\left(\delta+A_1\right){p_e}^2\right)^2\left(\delta+A_1\right)}\\&+\frac{L\left(\beta-\delta\right)E^c{p_e}^2}{4\left(2\xi\left(\delta+A_1\right)-{p_e}^2\right)\left(\delta+A_1\right)}.\end{aligned}$$

Where $L=\delta S+2\left(\lambda t+A_3-c\right)$. The signs of ΔAE^{O*} and ΔAE^{A*} are unequivocally determined by the magnitude of the numerator. When the condition $\beta<\delta$ is satisfied, it consistently follows that ΔAE^{O*} is less than ΔAE^{A*}, and both are less than zero. In the context where $\beta>\delta$, it follows that ΔAE^{O*} is greater than zero (and ΔAE^{A*} is also greater than zero).

C Proof of Section 6

C.1 Proof of the Propositions 5 and 6

When comparing the equilibrium solutions between Model O and A, it is evident that A_1 is less than $\delta + A_1$, signifying that $\frac{1}{2\xi A_1 - {p_e}^2} > \frac{1}{2\xi(\delta + A_1) - {p_e}^2}$. This relationship allows us to conclude that

$$E^{O*} - E^{A*} = \frac{\delta\xi(\delta S + 2(s + \lambda t - A_1))p_e}{(2\xi A_1 - {p_e}^2)(-{p_e}^2 + 2\xi(\delta + A_1))} > 0;$$

$$d_n^{O*} - d_n^{A*} = \frac{\left(-(Q + \lambda t)\delta^2 + (Q - (\lambda t + A_3))\delta + (Q - c)(2g - A_1)\right)\delta}{2A_1(\delta + A_1)} - \frac{{p_e}^2(\delta S + 2(s + \lambda t - A_2))\delta^2\left(2\xi(\delta + 2A_1) - {p_e}^2\right)}{4(2\xi A_1 - {p_e}^2)A_1(2\xi(\delta + A_1) - {p_e}^2)(\delta + A_1)} < 0;$$

$$d_r^{O*} - d_r^{A*} = \frac{\left[4((c + \lambda t)\delta - A_2 + \lambda t + s)A_1(\delta + A_1)\xi^2 + \delta(Q - c)\left(-0.5p_e^2 + \xi(2A_1 + \delta)\right){p_e}^2\right]\delta}{2\left(2\xi A_1 - p_e^2\right)\left(2\xi(\delta + A_1) - p_e^2\right)A_1(\delta + A_1)} > 0;$$

$$\Pi_M^{O*} - \Pi_M^{A*} = -\frac{\delta\left[-8A_1((c + \lambda t)\delta - A_2 + \lambda t + s)^2(\delta + A_1)\xi^2 + (Q - c)^2{p_e}^2\delta^2\left(-0.5{p_e}^2 + \xi(2A_1 + \delta)\right)\right]}{8(2\xi A_1 - {p_e}^2)A_1(2\xi(\delta + A_1) - {p_e}^2)(\delta + A_1)} > 0.$$

Furthermore, an analysis of the sensitivity of OEM profits reveals that

$$\left|\frac{\partial\Pi_M^{O*}}{\partial p_e}\right| - \left|\frac{\partial\Pi_M^{A*}}{\partial p_e}\right| = \frac{\left[\begin{array}{l}\left(\xi(\delta + 2A_1) - {p_e}^2\right)\xi^2(\delta S + 2(s + \lambda t - A_2))p_e \\ \times \delta S((Q - (2\lambda t + 3c))\delta + 2(A_2 - (\lambda t + s)))\end{array}\right]}{(2\xi A_1 - {p_e}^2)^2(2\xi(\delta + A_1) - {p_e}^2)^2} > 0$$

$$\left|\frac{\partial\Pi_M^{O*}}{\partial \xi}\right| - \left|\frac{\partial\Pi_M^{A*}}{\partial \xi}\right| = \frac{\left[\begin{array}{l}(2(1 - \delta)\lambda t + \delta(Q - 3c) + 2(-s + A_2))\left(2(3A_1 + \delta)\xi - 3{p_e}^2\right) \\ \times (\delta S - 2(A_2 - (s + \lambda t)))\left(2\xi\left(2g - \delta^2\right) - {p_e}^2\right){p_e}^2\end{array}\right]}{8(2\xi A_1 - {p_e}^2)^2(2\xi(\delta + A_1) - {p_e}^2)^2} > 0.$$

C.2 Proof of Subsection 6.1

The Hessian matrix of the profit function Π_k^C, with respect to the variables d_n^C and d_r^C is given by:

$$H^C = \begin{bmatrix} -2 & -2\delta \\ -2\delta & -2(\delta + g) \end{bmatrix}.$$

It can be determined that $H_1^C = -2 < 0$, indicating that the first principal minor is negative. Furthermore, the second principal minor can be expressed as: $H_2^C = 4A_1$. Thus, the first-order conditions yield the optimal reaction functions:

$$d_n^C\left(E^C\right) = \frac{-(\lambda t + Q)\delta^2 + (Q + p_0 - (s + \lambda t + p_e E^c) + \rho c_a)\delta + g(Q - c)}{2A_4}$$
$$d_r^C\left(E^C\right) = \frac{(c + \lambda t)\delta + p_e E^c - (p_0 + c) + \lambda t + s - \rho c_a}{2A_4}.$$

By substituting the best response into the original profit function, it becomes apparent that

$$\Pi_k^C = \frac{\xi(((1 + \delta)\lambda t + s) + 2(\delta c - (c + \rho c_a + p_0)))((1 + \delta)\lambda t + s)}{4(\xi A_4 - {p_e}^2)} + \frac{(Q(Q - 2c) - 4\rho F_c)}{4} + \frac{\left((g + 1 - \delta)c^2 + 2(1 - \delta)(p_0 + \rho c_a)c + (p_0 + \rho c_a)^2\right)\xi - (cp_e)^2}{4(\xi A_4 - {p_e}^2)}.$$

Table 5 The Optimal Solution for the Model O and BO.

	Model O
E^{O*}	$\frac{(\delta S - 2(A_2 - (s+\lambda t)))p_e}{2(2\xi A_1 - {p_e}^2)}$
w_r^{O*}	$\frac{\left(2\xi g - {p_e}^2\right)(\lambda t(\delta+1) - p_0)}{2\xi A_1 - {p_e}^2} + \frac{(s-c)\delta^2 + ((g+1)c - s)\delta + (-s+c)g}{A_2}$ $- \frac{{p_e}^2(\delta(c+Q) + 2(s-c))A_4}{2(2\xi A_1 - {p_e}^2)A_1} + \left(-\rho + \frac{2\xi A_4}{2A_1\xi - {p_e}^2}\right)c_a$
p_n^{O*}	$\frac{c+Q}{2}$
p_r^{O*}	$\frac{\xi(-p_0 + s - c)\delta^2 + \xi(p_0 + c - s + 2gc)\delta}{2A_1\xi - {p_e}^2} - \frac{\delta {p_e}^2\left(\delta(1-\delta)\left(\frac{3Q}{4} + \frac{c}{4}\right) + (c+Q)g\right)}{(2\xi A_1 - {p_e}^2)A_1} - p_0$ $+ \frac{\left(-(\lambda t\delta + c_a)\delta^2 + (t(4g+1)\lambda + c_a)\delta + 4\lambda t g\right)\xi - t\lambda {p_e}^2(\delta+1)}{2A_1\xi - {p_e}^2} + \frac{QA_1\delta\xi}{2\xi A_1 - {p_e}^2}$
d_n^{O*}	$\frac{-Q\delta^2 + (Q - s + p_0)\delta + 2g(Q-c)}{2A_1} - \frac{{p_e}^2((c+Q)\delta + 2s - 2c - 2p_0)\delta}{4(2\xi A_1 - {p_e}^2)A_1} - \frac{((\delta+1)t\lambda - c_a)\delta\xi}{(2A_1)\xi - {p_e}^2}$
d_r^{O*}	$\frac{4A_1((c+\lambda t)\delta - A_2 + \lambda t + s)\xi + \delta {p_e}^2(Q-c)}{4A_1(2\xi A_1 - p_e^2)}$
	Model BO
w_r^{BO*}	$\frac{c_a(-1+\rho)\delta^2 + ((1-\rho)c_a + g\lambda t)\delta + ((1-2\rho)c_a + \lambda t)g}{A_2}$ $+ \frac{(-c+s)\delta^2 + (c(g+1) - s)\delta + (c - (p_0 + s))g}{A_2}$
p_n^{BO*}	$\frac{c+Q}{2}$
p_r^{BO*}	$\frac{-\lambda t\delta^3 + (p_0 - c_a)\delta^2 + ((4tg+t)\lambda - p_0 + c_a)\delta + 4g(\lambda t - p_0)}{2A_2}$ $+ \frac{-\delta^3 Q + (s - c + Q)\delta^2 + (2(Q+c)g - s + c)\delta}{2A_2}$
d_n^{BO*}	$\frac{-(Q+\lambda t)\delta^2 + (Q + c_a + p_0 - (s+\lambda t))\delta + 2g(Q-c)}{2A_2}$
d_r^{BO*}	$\frac{(c+\lambda t)\delta - A_2 + \lambda t + s}{2A_2}$

References

Abreu H, Pereira V, Barata J. Blockchain-based digital product passport: design principles and demonstration. International Journal of Production Research. 2025;p. 1–20. https://doi.org/https://doi.org/10.1080/00207543.2025.2464161.

Adhikari A, Bisi A. Collaboration, bargaining, and fairness concern for a green apparel supply chain: An emerging economy perspective. Transportation Research Part E: Logistics and Transportation Review. 2020;135:101863. https://doi.org/https://doi.org/10.1016/j.tre.2020.101863.

Agrawal R, Yadav A, Garg RK, Sachdeva A. Building a Greener Supply Chain with Blockchain: A Review and Future Research Directions. In: International Conference on Production and Industrial Engineering Springer; 2023. p. 139–152.

Alegoz M, Kaya O, Bayindir ZP. A comparison of pure manufacturing and hybrid manufacturing–remanufacturing systems under carbon tax policy. European Journal of Operational Research. 2021;294(1):161–173. https://doi.org/https://doi.org/10.1016/j.ejor.2021.01.018.

Alotaibi EM, Khallaf A, Abdallah AAN, Zoubi T, Alnesafi A. Blockchain-driven carbon accountability in supply chains. Sustainability. 2024;16(24):10872. https://doi.org/https://doi.org/10.3390/su162410872.

Table 6 The Optimal Solution for the Model A and BA.

	Model A
E^{A*}	$\frac{p_e\left(\delta S-2A_2+2\left(s+\lambda t\right)\right)}{-2{p_e}^2+4\xi\left(\delta+A_1\right)}$
f^{A*}	$\frac{4\left(\delta+A_1\right)\left(\left(\lambda t+Q\right)\delta+\left(2\rho-1\right)c_a+\lambda t-\left(p_0+c\right)+s\right)\xi-{p_e}^2\left(\left(Q-c\right)\delta+4\rho c_a\right)}{8\xi\left(\delta+A_1\right)-4{p_e}^2}$
p_n^{A*}	$\frac{c+Q}{2}$
p_r^{A*}	$\frac{\left(\left(\delta+A_1\right)^2\xi-\left(\frac{aA_1+\delta\left(3-\delta\right)}{4}\right){p_e}^2\right)\delta Q}{\left(-{p_e}^2+2\xi\left(\delta+A_1\right)\right)\left(\delta+A_1\right)}+\frac{\left(\left(2g+1\right)\left(\delta+A_1\right)\xi-\left(\frac{2\left(\delta+g\right)+A_1}{4}\right){p_e}^2\right)\delta c}{\left(-{p_e}^2+2\xi\left(\delta+A_1\right)\right)\left(\delta+A_1\right)}$ $\frac{\left(-p_0+\lambda t\left(1+\delta\right)\right)\left(\left(2\left(\delta+g\right)+A_1\right)\xi-{p_e}^2\right)}{2\left(\delta+A_1\right)\xi-{p_e}^2}+\frac{\delta\xi\left(-1+\delta\right)\left(-c_a+s\right)}{2\left(\delta+A_1\right)\xi-{p_e}^2}$
d_n^{A*}	$\frac{-\delta^2Q+\left(-c+2Q-s\right)\delta+2g\left(Q-c\right)}{2\left(\delta+A_1\right)}-\frac{\delta\xi\left(\left(1+\delta\right)\lambda t-c_a-p_0\right)}{\left(2\left(\delta+A_1\right)\right)\xi-{p_e}^2}-\frac{\delta{p_e}^2\left(\left(c+Q\right)\delta-2c+2s\right)}{4\left(-{p_e}^2+4\delta\xi+4\xi g-2\delta^2\xi\right)\left(2A_2\right)}$
d_r^{A*}	$\frac{4\left(\left(c+\lambda t\right)\delta-A_2+\lambda t+s\right)\left(\delta+A_1\right)\xi+\delta{p_e}^2\left(Q-c\right)}{4\left(2\left(\delta+A_1\right)\xi-{p_e}^2\right)\left(\delta+A_1\right)}$
	Model BA
f^{BA*}	$\frac{\left(\left(\lambda t+Q\right)\delta+\left(2\rho-1\right)c_a-p_0+\lambda t-c+s\right)}{2}$
p_n^{BA*}	$\frac{c+Q}{2}$
p_r^{BA*}	$\frac{-\lambda t\delta^3+\left(2\lambda t-c_a+p_0\right)\delta^2+\left(t\left(4g+3\right)\lambda+c_a-3p_0\right)\delta+4g\left(\lambda t-p_0\right)}{2\left(\delta+A_1\right)}$ $+\frac{-\delta^3Q+\left(s+2Q\right)\delta^2+\left(2\left(Q+c\right)g-s+c\right)\delta}{2\left(\delta+A_1\right)}$
d_n^{BA*}	$\frac{-\left(Q+\lambda t\right)\delta^2+\left(2Q-\left(c+\lambda t+s\right)+c_a+p_0\right)\delta+2g\left(Q-c\right)}{2\left(\delta+A_1\right)}$
d_r^{BA*}	$\frac{\left(c+\lambda t\right)\delta-A_2+\lambda t+s}{2\left(\delta+A_1\right)}$

Badhwar A, Islam S, Tan CSL. Exploring the potential of blockchain technology within the fashion and textile supply chain with a focus on traceability, transparency, and product authenticity: A systematic review. Frontiers in Blockchain. 2023;6:1044723. https://doi.org/https://doi.org/10.3389/fbloc.2023.1044723.

Bai Q, Jin M, Xu X, et al. Effects of carbon emission reduction on supply chain coordination with vendor-managed deteriorating product inventory. International Journal of Production Economics. 2019;208:83–99. https://doi.org/https://doi.org/10.1016/j.ijpe.2018.11.008.

Bensoussan A, El Ouardighi F. Voluntary environmental effort under (s, S) inventory policy. International Journal of Production Research. 2024;62(1-2):522–535. https://doi.org/https://doi.org/10.1080/00207543.2023.2242968.

Cao J, Shao Q. Outsourcing and authorized remanufacturing under voluntary emission reduction mechanism. Flexible Services and Manufacturing Journal. 2024;p. 1–40. https://doi.org/https://doi.org/10.1007/s10696-024-09555-9.

Cao J, Shao Q. Outsourcing and authorized remanufacturing under voluntary emission reduction mechanism. Flexible Services and Manufacturing Journal. 2025;37(3):776–815. https://doi.org/https://doi.org/10.1007/s10696-024-09555-9.

Cao S, Xu H, Bryceson KP. Blockchain traceability for sustainability communication in food supply chains: an architectural framework, design pathway and considerations. Sustainability. 2023;15(18):13486. https://doi.org/https://doi.org/10.3390/su151813486.

Chen X, Wang X, Zhou M. Firms' green R&D cooperation behaviour in a supply chain: Technological spillover, power and coordination. International Journal of Production Economics. 2019;218:118–134.

Table 7 The Optimal Solution for the Model *RO* and *RA*.

	Model *RO*
E^{RO*}	$\frac{p_e\left(\delta\left(c+\lambda t\right)-c+\lambda t+A_3\right)}{2\left(\xi A_1-{p_e}^2\right)}$.
w_r^{RO*}	$\frac{2\left(\rho c_a\delta^2+\left(g\lambda t-c_a\rho\right)\delta+g\left(\lambda t-2c_a\rho\right)\right)\xi-{p_e}^2\left(\lambda t\left(\delta+1\right)-2\rho c_a\right)}{2\left(\xi A_1-{p_e}^2\right)}$ $+\frac{2\left(\left(-c+s\right)\delta^2+\left(cg-s+c\right)\delta+g\left(-s+c\right)\right)\xi-{p_e}^2\left(c\delta-s+c\right)}{2\left(\xi A_1-{p_e}^2\right)}$ $+\frac{\left(\left(\delta\left(1-\delta\right)+A_1\right)c_a-2gp_0\right)\xi+{p_e}^2\left(p_0-c_a\right)}{2\left(\xi A_1-{p_e}^2\right)}$.
p_n^{RO*}	$\frac{c+Q}{2}$.
p_r^{RO*}	$\frac{\left(\delta^2Q+\left(t\left(4g+1\right)\lambda+2gQ\right)\delta-\left(\lambda t+Q\right)\delta^3+4g\lambda t\right)\xi-2\lambda t{p_e}^2-{p_e}^2\left(2\lambda t+Q\right)\delta}{2\xi A_1-2{p_e}^2}$ $+\frac{\left(\left(-c+s-c_a\right)\delta^2+\left(\left(2g+1\right)c-s+c_a\right)\delta-p_0\left(A_1+2g\right)\right)\xi-{p_e}^2\left(-2p_0+c\delta\right)}{2\xi A_1-2{p_e}^2}$.
d_n^{RO*}	$\frac{\left(-\left(\lambda t+Q\right)\delta^2+\left(Q-\left(\lambda t+A_3\right)\right)\delta+2g\left(Q-c\right)\right)\xi-{p_e}^2\left(Q-c\right)}{2A_1\xi-2{p_e}^2}$.
d_r^{RO*}	$\frac{\xi\left(\left(c+\lambda t\right)\delta-c+\lambda t+A_3\right)}{2\left(\xi A_1-{p_e}^2\right)}$.
	Model *RA*
E^{RA*}	$\frac{\left(\left(c+\lambda t\right)\delta-c+\lambda t+A_3\right)p_e}{2\left(\xi\left(\delta+A_1\right)-{p_e}^2\right)}$.
p_n^{RA*}	$\frac{c+Q}{2}$.
p_r^{RA*}	$\frac{-\xi\left(\lambda t+Q\right)\delta^2\left(\delta-2\right)+\left(\left(t\left(4g+3\right)\lambda+2Qg\right)\xi-{p_e}^2\left(2\lambda t+Q\right)\right)\delta+2\lambda t\left(2g\xi-{p_e}^2\right)}{2\left(\left(\delta+A_1\right)\xi-{p_e}^2\right)}$ $+\frac{\left(\left(s-c_a+p_0\right)\delta^2+\left(-3p_0+\left(2g+1\right)c-s+c_a\right)\delta-4p_0g\right)\xi-{p_e}^2\left(\delta c-2p_0\right)}{2\left(\xi\left(\delta+A_1\right)-{p_e}^2\right)}$.
d_n^{RA*}	$\frac{\left(\left(-\lambda t-Q\right)\delta^2+\left(2Q-\left(c+\lambda t+A_3\right)\right)\delta+2g\left(Q-c\right)\right)\xi-{p_e}^2\left(Q-c\right)}{2\left(\left(\delta+A_1\right)\xi-{p_e}^2\right)}$
d_r^{RA*}	$\frac{\left(\left(c+\lambda t\right)\delta-c+\lambda t+A_3\right)\xi}{2\left(\xi\left(\delta+A_1\right)-{p_e}^2\right)}$
f^{RA*}	$\frac{\left(1+\delta\right)\lambda t+2\rho c_a+\delta Q+A_3-c}{2}$

Let $A_1=\delta+2g-\delta^2$; $A_2=c+p_0+c_a$; $A_3=s-\left(c_a+p_0\right)$; $A_4=\delta+g-\delta^2$; $A_5=1-\delta+2g$; $W_1=\left(1+\delta\right)\lambda t+c\left(\delta-1\right)$; $S=c+2\lambda t+Q$; and $L=\delta S+2\left(\lambda t+A_3-c\right)$.

https://doi.org/https://doi.org/10.1016/j.ijpe.2019.04.033.

Cheong BC. Leveraging blockchain for enhanced transparency and traceability in sustainable supply chains. Discover Analytics. 2025;3(1):6. https://doi.org/https://doi.org/10.1007/s44257-025-00032-7.

Duncan C, Primavera JH, Hill NA, Wodehouse DC, Koldewey HJ. Potential for return on investment in rehabilitation-oriented blue carbon projects: accounting methodologies and project strategies. Frontiers in Forests and Global Change. 2022;4:775341. https://doi.org/https://doi.org/10.3389/ffgc.2021.775341.

El Saadany AM, Jaber MY. A production/remanufacturing inventory model with price and quality dependant return rate. Computers & Industrial Engineering. 2010;58(3):352–362. https://doi.org/https://doi.org/10.1016/j.cie.2009.01.017.

Fang C, Fan S, Chi M, Wang W. The optimal remanufacturing strategy, returned quality choice and independent remanufacturers' advantage for tackling extreme weather. International Journal of Production Economics. 2023;259:108819. https://doi.org/https://doi.org/10.1016/j.ijpe.2023.108819.

Fang W, Guangzhi L, Chunmei S, Feng W, Jianguo M, Yongzhi Y. Carbon emission reduction accounting method for a CCUS-EOR project. Petroleum Exploration and Development. 2023;50(4):989–1000. https://doi.org/https://doi.org/10.1016/S1876-3804(23)60444-6.

Francisco K, Swanson D. The supply chain has no clothes: Technology adoption of blockchain for supply chain transparency. Logistics. 2018;2(1):2. https://doi.org/https://doi.org/10.3390/logistics2010002.

Friedman N, Ormiston J. Blockchain as a sustainability-oriented innovation?: Opportunities for and resistance to Blockchain technology as a driver of sustainability in global food supply chains. Technological Forecasting and Social Change. 2022;175:121403. https://doi.org/https://doi.org/10.1016/j.techfore.2021.121403.

Gao F, Souza GC. Carbon offsetting with eco-conscious consumers. Management Science. 2022;68(11):7879–7897. https://doi.org/https://doi.org/10.1287/mnsc.2021.4293.

Ghosh D, Shah J. A comparative analysis of greening policies across supply chain structures. International Journal of Production Economics. 2012;135(2):568–583. https://doi.org/https://doi.org/10.1016/j.ijpe.2011.05.027.

Guo Y, Wang M, Yang F. Joint emission reduction strategy considering channel inconvenience under different recycling structures. Computers & Industrial Engineering. 2022;169:108159. https://doi.org/https://doi.org/10.1016/j.cie.2022.108159.

Huang S, Wang Z, Wang J, Lu Y, Liang X, Liu L, et al. A Framework for Blockchain Supported Carbon Emission Data Sharing with Flexible Access Control and Enhanced Data Privacy. In: Proceedings of the 2024 10th International Conference on Advances in Energy Resources and Environment Engineering (ICAESEE 2024), vol. 36 Springer Nature; 2025. p. 407.

Huang Y, Wang Z. Information sharing in a closed-loop supply chain with technology licensing. International Journal of Production Economics. 2017;191:113–127. https://doi.org/https://doi.org/10.1016/j.ijpe.2017.06.008.

Jaber MY, Glock CH, El Saadany AM. Supply chain coordination with emissions reduction incentives. International Journal of Production Research. 2013;51(1):69–82. https://doi.org/https://doi.org/10.1080/00207543.2011.651656.

Jalapathy P, Unnissa MM. Revolutionizing sustainability and adoption of blockchain technology: A dual-channel approach to (Re) manufacturing strategy with an extended warranty. Computers & Industrial Engineering. 2025;p. 111566. https://doi.org/https://doi.org/10.1016/j.cie.2025.111566.

Jiang X, Sun L, Wang Y. Effects of interactions between technology spillovers and carbon transfers on carbon emission reduction decisions and coordination within supply chains. Journal of Business & Industrial Marketing. 2024;39(7):1497–1512. https://doi.org/https://doi.org/10.1108/JBIM-04-2023-0215.

Jin M, Li G, Reimann M. Team of rivals: How should original equipment manufacturers cooperate with independent remanufacturers via authorisation? European Journal of Operational Research. 2022;296(3):837–845. https://doi.org/https://doi.org/10.1016/j.ejor.2021.04.014.

Li W, Sun H, Dong H, Gan Y, Koh L. Outsourcing decision-making in global remanufacturing supply chains: The impact of tax and tariff regulations. European Journal of Operational Research. 2023;304(3):997–1010. https://doi.org/https://doi.org/10.1016/j.ejor.2022.05.016.

Li W, Wang P, Cheng W, Nie K. Transnational remanufacturing decisions under carbon taxes and tariffs. European Journal of Operational Research. 2024;312(1):150–163. https://doi.org/https://doi.org/10.1016/j.ejor.2023.06.019.

Li Y, Huang L, Tong Y. Cooperation with competitor or not? The strategic choice of a focal firm's green innovation strategy. Computers & Industrial Engineering. 2021;157:107301. https://doi.org/https://doi.org/10.1016/j.cie.2021.107301.

Liu B, Holmbom M, Segerstedt A, Chen W. Effects of carbon emission regulations on remanufacturing decisions with limited information of demand distribution. International Journal of Production Research. 2015;53(2):532–548. https://doi.org/https://doi.org/10.1080/00207543.2014.957875.

Liu H, Lei M, Huang T, Leong GK. Refurbishing authorization strategy in the secondary market for electrical and electronic products. International Journal of Production Economics. 2018;195:198–209. https://doi.org/https://doi.org/10.1016/j.ijpe.2017.10.012.

Liu Q, Zhu X. Carbon reduction decisions in green technology collaborative R&D and spillover time lag effects. Journal of Cleaner Production. 2023;429:139595. https://doi.org/https://doi.org/10.1016/j.jclepro.2023.139595.

Luo R, Zhou L, Song Y, Fan T. Evaluating the impact of carbon tax policy on manufacturing and remanufacturing decisions in a closed-loop supply chain. International Journal of Production Economics. 2022;245:108408. https://doi.org/https://doi.org/10.1016/j.ijpe.2022.108408.

Malik S, Dedeoglu V, Kanhere SS, Jurdak R. PrivChain: Provenance and privacy preservation in blockchain enabled supply chains. In: 2022 IEEE International Conference on Blockchain (Blockchain) IEEE; 2022. p. 157–166.

Manzoor R, Sahay B, Singh SK. Blockchain technology in supply chain management: an organizational theoretic overview and research agenda. Annals of Operations Research. 2025;348(3):1307–1354. https://doi.org/https://doi.org/10.1007/s10479-022-05069-5.

Merlo AL, Mendonça DS, Santos J, Carvalho ST, Guerra R, Brandão D. Blockchain for the carbon market: a literature review. Discover Environment. 2025;3(1):68. https://doi.org/https://doi.org/10.1007/s44274-025-00260-4.

Nan J, Wu X, Zhang M. Production competition and emission reduction technology cooperation between manufacturers under the low-carbon market based on blockchain technology. Chinese Journal of Management Science. 2025;33(6):360–368. https://doi.org/https://doi.org/10.16381/j.cnki.issn1003-207x.2023.1421.

Niu B, Dong J, Liu Y. Incentive alignment for blockchain adoption in medicine supply chains. Transportation Research Part E: Logistics and Transportation Review. 2021;152:102276. https://doi.org/https://doi.org/10.1016/j.tre.2021.102276.

Pun H, Ghamat S. The value of partnership under competition: When competitors may be R&D joint-venture and supply-chain partners for a critical component. International Journal of Production Economics. 2016;177:1–11. https://doi.org/https://doi.org/10.1016/j.ijpe.2016.03.018.

Pun H, Swaminathan JM, Hou P. Blockchain adoption for combating deceptive counterfeits. Production and Operations Management. 2021;30(4):864–882. https://doi.org/https://doi.org/10.1111/poms.13348.

Saberi S, Kouhizadeh M, Sarkis J, Shen L. Blockchain technology and its relationships to sustainable supply chain management. International journal of production research. 2019;57(7):2117–2135. https://doi.org/https://doi.org/10.1080/00207543.2018.1533261.

Sabzevar N, Enns S, Bergerson J, Kettunen J. Modeling competitive firms' performance under price-sensitive demand and cap-and-trade emissions constraints. International Journal of Production Economics. 2017;184:193–209. https://doi.org/https://doi.org/10.1016/j.ijpe.2016.10.024.

Schweitzer V, Bach V, Holzapfel PK, Finkbeiner M. Differences in science-based target approaches and implications for carbon emission reductions at a sectoral level in Germany. Sustainable Production and Consumption. 2023;38:199–209. https://doi.org/https://doi.org/10.1016/j.spc.2023.04.009.

Sharabati AAA, Jreisat ER. Blockchain technology implementation in supply chain management: A literature review. Sustainability. 2024;16(7):2823. https://doi.org/https://doi.org/10.3390/su16072823.

Sharma I, Kaur G, Dey BK, Majumder A. Leveraging blockchain and consignment contracts to optimize food supply chains under uncertainty. Applied Sciences. 2024;14(24):11735. https://doi.org/https://doi.org/10.3390/app142411735.

Subramanian R, Subramanyam R. Key factors in the market for remanufactured products. Manufacturing & Service Operations Management. 2012;14(2):315–326. https://doi.org/https://doi.org/10.1287/msom.1110.0368.

Sun Y, Shen W, Li J, Liao Y. Competition in remanufacturing with asymmetric demand information. Sustainability. 2024;16(2):471. https://doi.org/https://doi.org/10.3390/su16020471.

Swami S, Shah J. Channel coordination in green supply chain management. Journal of the operational research society. 2013;64(3):336–351. https://doi.org/https://doi.org/10.1057/jors.2012.44.

Times F. EU carbon price tops€ 100 a tonne for first time. Abgerufen am. 2023;15:2024.

Wang X, Zhu Y, Sun H, Jia F. Production decisions of new and remanufactured products: Implications for low carbon emission economy. Journal of Cleaner Production. 2018;171:1225–1243. https://doi.org/https://doi.org/10.1016/j.jclepro.2017.10.053.

Wang Y, Xu X, Zhu Q. Carbon emission reduction decisions of supply chain members under cap-and-trade regulations: A differential game analysis. Computers & Industrial Engineering. 2021;162:107711. https://doi.org/https://doi.org/10.1016/j.cie.2021.107711.

Wei Q, Xiao S. Assessing barriers to the internationalization of China's certified emission reductions (CCERs): A Delphi survey. Climate Policy. 2022;22(7):906–917. https://doi.org/https://doi.org/10.1080/14693062.2022.2090892.

Wu X, Miao W, Zhang X, Zuo W, Shen B. Recycling and remanufacturing or technology upgrading? emission reduction decisions of supply chain under carbon cap-and-trade mechanism. PloS one. 2025;20(3):e0318952. https://doi.org/https://doi.org/10.1371/journal.pone.0318952.

Xia X, Li M, Li B, Wang H. The impact of carbon trade on outsourcing remanufacturing. International Journal of Environmental Research and Public Health. 2021;18(20):10804. https://doi.org/https://doi.org/10.3390/ijerph182010804.

Xia X, Li M, Wang W. Impact of three emission reduction decisions on authorized remanufacturing under carbon trading. Expert Systems with Applications. 2023;216:119476. https://doi.org/https://doi.org/10.1016/j.eswa.2022.119476.

Xia X, Lu M, Wang W. Emission reduction and outsourcing remanufacturing: A comparative study under carbon trading. Expert Systems with Applications. 2023a;227:120317. https://doi.org/https://doi.org/10.1016/j.eswa.2023.120317.

Xu L, Wang C. Contracting pricing and emission reduction for supply chain considering vertical technological spillovers. The International Journal of Advanced Manufacturing Technology. 2017;93(1):481–492. https://doi.org/https://doi.org/10.1007/s00170-017-0116-5.

Xu X, Yan L, Choi TM, Cheng T. When is it wise to use blockchain for platform operations with remanufacturing? European Journal of Operational Research. 2023;309(3):1073–1090. https://doi.org/https://doi.org/10.1016/j.ejor.2023.01.063.

Yadav N, Luthra S, Garg D. Blockchain technology for sustainable supply chains: a network cluster analysis and future research propositions. Environmental Science and Pollution Research. 2023;30(24):64779–64799. https://doi.org/https://doi.org/10.1007/s11356-023-27049-3.

Yang L, Hu Y, Huang L. Collecting mode selection in a remanufacturing supply chain under cap-and-trade regulation. European Journal of Operational Research. 2020;287(2):480–496. https://doi.org/https://doi.org/10.1016/j.ejor.2020.04.006.

Yenipazarli A. Managing new and remanufactured products to mitigate environmental damage under emissions regulation. European Journal of Operational Research. 2016;249(1):117–130. https://doi.

org/https://doi.org/10.1016/j.ejor.2015.08.020.

Zheng Y, Zhang B. The impact of carbon market on city greening: Quasi-experimental evidence from China. Resources, Conservation and Recycling. 2023;193:106960. https://doi.org/https://doi.org/10.1016/j.resconrec.2023.106960.

Zhou Q, Meng C, Sheu JB, Yuen KF. Remanufacturing mode and strategic decision: A game-theoretic approach. International Journal of Production Economics. 2023;260:108841. https://doi.org/https://doi.org/10.1016/j.ijpe.2023.108841.

Zhou Q, Meng C, Yuen KF. Remanufacturing authorization strategy for competition among OEM, authorized remanufacturer, and unauthorized remanufacturer. International Journal of Production Economics. 2021;242:108295. https://doi.org/https://doi.org/10.1016/j.ijpe.2021.108295.

Zhou Q, Meng C, Yuen KF, Sheu JB. Remanufacturing authorization strategy for an original equipment manufacturer-contract manufacturer supply chain: Cooperation or competition? International Journal of Production Economics. 2021a;240:108238. https://doi.org/https://doi.org/10.1016/j.ijpe.2021.108238.

Zhou Y, Hou M, Wong KH. The optimal remanufacturing strategy of the closed-loop supply chain network under government regulation and the manufacturer's design for the environment. Sustainability. 2023;15(9):7342. https://doi.org/https://doi.org/10.3390/su15097342.

Zhu C, Ma J, Li J. Dynamic production and carbon emission reduction adjustment strategies of the brand-new and the remanufactured product under hybrid carbon regulations. Computers & Industrial Engineering. 2022;172:108517. https://doi.org/https://doi.org/10.1016/j.cie.2022.108517.

Zou ZB, Wang JJ, Deng GS, Chen H. Third-party remanufacturing mode selection: Outsourcing or authorization? Transportation Research Part E: Logistics and Transportation Review. 2016;87:1–19. https://doi.org/https://doi.org/10.1016/j.tre.2015.12.008.