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Bottomonium mass shifts in an unquenched quark model

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In this paper, we performed a coupled-channel calculation and evaluated the mass shifts for bottomonium states including ground and excited states, such as $\eta_b(nS)(n=1,2)$, $\Upsilon(nS)(n=1\sim 6)$, $\chi_{bJ}(nP)(J=0,1,2;n=1,2,3)$, $h_b(nP)(n=1,2)$ by incorporating the four-quark components (B,B^*,B_s and B_s^* meson pairs) into the quark model. The transition matrix elements between $b\bar{b}$ mesons and four-quark component is obtained by the transition operator in the $^3P_0$ quark-pair creation model. From our results, the induced mass shifts appear to be large (several hundred MeV) and negative with the original transition operator in the $^3P_0$ model, which raised up challenges for the valence quark model. The transition operator must be modified to justify the valence quark model in describing the low-lying bottomonium spectrum. So herein, we recalculated the mass shifts with the improved $^3P_0$ transition operator introduced in our previous work and the mass shifts are greatly reduced. The new results indicates that the mass shifts of the high $b\bar{b}$ states due to the coupled-channel effects are the order of a few tens MeV; some of the high-lying resonances contain significant non-$b\bar{b}$ components such as $\chi_{bJ}(1P)(J=0,1,2)$, and some states can be classified into the $b\bar{b}$ bottomonium family such as $\Upsilon(10860)$ and so on.

I. INTRODUCTION

Over the last ten years, experimental investigations have comprehensively reported the spectrum of low-lying bottomonium states [1]. The experimentally identified resonances consist of the Υ series: $\Upsilon(1S,2S,3S,4S)$, $\Upsilon(10753)$, $\Upsilon(10860)$, $\Upsilon(11020)$ and $\Upsilon_2(1D)$; the η_b multiplet containing $\eta_b(1S,2S)$; the χ_{bJ} states including $\chi_{b_0}(1P,2P)$, $\chi_{b_1}(1P,2P,3P)$, $\chi_{b_2}(1P,2P,3P)$; as well as the h_b family with $h_b(1P,2P)$. Comprehensive summaries on these states are available in Refs. [2–4]. In the present work, we aim to exploit this abundant set of experimental data to examine how bare quark–antiquark configurations couple with the continuum and the corresponding resulting effects.

For decades, valence quark models have served as a successful framework to investigate hadron spectroscopy [5–10]. Such models are typically categorized as quenched ones, which means hadrons are represented by their minimal Fock-state configurations: mesons correspond to a quark–antiquark pair ($q\bar{q}$), while baryons consist of three valence quarks (qqq). These models may also adopt different forms regarding interaction potentials, numerical calculation schemes, and relativistic correction treatments. Within these quark models, the relativized quark model proposed by Godfrey and Isgur [8] (commonly referred to as the GI model) has evolved into a standard framework, as it achieves great successful in obtaining the mass spectra of mesons and baryons [9–12].

Nevertheless, the discovery of more new particle states has given new challenges to the hadronic spectroscopy in the conventional quark models. To deal with this situation, researchers have attempted to extend the constituent quark model via a variety of approaches, such as introducing gluon degrees of freedom [13, 14] and constructing diquark models [15–17]. There is one more approach, which considers the effect of direct coupling to the hadronic continuum, called "unquenching" [18–25]. The production of quark pairs introduces virtual $q\bar{q}-q\bar{q}$ or $qqq-q\bar{q}$ admixtures into the hadronic wave function, which causes the physical mass to deviate from the bare mass [4, 26]. To account for coupled-channel contributions within a constituent quark model, one must explicitly describe the process of quark pair creation. For the present study, we adopt the so-called $^3P_0$ model, a formalism extensively applied to evaluate strong decays [27–32] as well as coupled-channel effects [18, 33–36].

In our previous work [37, 38], we compute the masses of ground state for the light mesons and charmonium states, incorporating hadron loops in a chiral quark model using the $^3P_0$ model to describe the pair creation, and explored the impact of physically motivated modifications of the associated operator. The results indicated that the virtual quark pair creation in hadronic system leads to a very large mass shift. The large mass shift will challenge the validity of the valence quark model in describing the ground state hadron and convergence of the unquenched quark model. In our previous study [37, 38], we attempted to solve this problem by modifying the transition operator, i.e. introducing energy and separation damping factors. With the improved transition operator, the mass shifts of the low-lying light mesons [37] and charmonium states [38] are significantly reduced. The proportion of the two-quark component increases to approximately 90%. This in turn suppresses the influence of the four-quark components. This ensures the validity of the constituent valence quark model in describing the low-lying hadron states.

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The aim of the present work is to calculate the energy spectra of heavier bottomonium states within the unquenched quark model on the basis of previous investigations, and to analyze and compare the influence of coupled-channel effects on mesons ranging from light ones to heavy ones. The effects of coupled-channels for bottomonium including some valence states are presented. We calculated the mass shifts of these bottomonium states based on the nonrelativistic chiral quark model and solved the quantum mechanics problem using the Gaussian expansion method (GEM) [39] instead of the simple harmonic oscillator (SHO) ones [33, 40, 41]. In Sec. II the chiral quark model, GEM and the $^3P_0$ model are outlined. And Sec. III is devoted to a discussion of the results. In Sec. IV, the paper ends with a short summary.

II. THE CHIRAL QUARK MODEL, GAUSSIAN EXPANSION METHOD AND THE $^3P_0$ MODEL

A. The Chiral Quark Model

We start this section by introducing the chiral quark model, taking a four-quark system as an example. Within this framework, the Hamiltonian constructed for a four-quark system consists of three separate contributions: the rest masses of quarks, kinetic energy terms and potential energy interactions:

$$H = \sum_{i=1}^{4} m_i + \frac{p_{12}^2}{2\mu_{12}} + \frac{p_{34}^2}{2\mu_{34}} + \frac{p_r^2}{2\mu_r} + \sum_{i<j=1}^{4} \left(V_{\rm CON}^{C}(\boldsymbol{r}_{ij}) + V_{\rm OGE}^{C}(\boldsymbol{r}_{ij}) + V_{\rm CON}^{SO}(\boldsymbol{r}_{ij}) + V_{\rm OGE}^{SO}(\boldsymbol{r}_{ij}) + \sum_{\chi=\pi,K,\eta} V_{ij}^{\chi} + V_{ij}^{\sigma} \right). \quad (1)$$

Here m_i denotes the constituent mass of the i-th quark (or antiquark). The terms $\frac{\mathbf{p_{ij}^2}}{2\mu_{ij}}$ $(ij = 12; 34)$ and $\frac{\mathbf{p_r^2}}{2\mu_r}$ correspond respectively to internal kinetic energies inside each two-body cluster and the kinetic energy associated with relative motion between the two clusters. Relevant definitions are presented below:

$$\mathbf{p_{12}} = \frac{m_2\mathbf{p}_1 - m_1\mathbf{p}_2}{m_1 + m_2}, \quad (2a)$$

$$\mathbf{p}_{34} = \frac{m_4\mathbf{p}_3 - m_3\mathbf{p}_4}{m_3 + m_4}, \quad (2b)$$

$$\mathbf{p}_r = \frac{(m_3 + m_4)\mathbf{p}_{12} - (m_1 + m_2)\mathbf{p}_{34}}{m_1 + m_2 + m_3 + m_4}, \quad (2c)$$

$$\mu_{ij} = \frac{m_i m_j}{m_i + m_j}, \quad (2d)$$

$$\mu_r = \frac{(m_1 + m_2)(m_3 + m_4)}{m_1 + m_2 + m_3 + m_4}. \quad (2e)$$

$V_{\rm CON}^{C}$ and $V_{\rm OGE}^{C}$ stand for the central component of confinement potential and the central part arising from one-gluon exchange. Meanwhile, $V_{\rm CON}^{SO}$ and $V_{\rm OGE}^{SO}$ represent non-central potential terms. The potentials labelled V_{ij}^{χ} with $\chi = \pi, K, \eta$, as well as the σ-exchange term, describe interactions mediated by Goldstone boson exchange.

The explicit analytical forms for all potential terms can be found in Ref. [9]:

$$V_{\rm CON}^{C}(\boldsymbol{r}_{ij}) = (-a_c r_{ij}^2 - \Delta)\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c, \quad (3a)$$

$$V_{\rm CON}^{\rm SO}(\boldsymbol{r}_{ij}) = \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \cdot \frac{-a_c}{2m_i^2 m_j^2}\Big\{\Big((m_i^2 + m_j^2)(1 - 2a_s) + 4m_i m_j(1 - a_s)\Big)(\boldsymbol{S}_+ \cdot \boldsymbol{L}) + (m_j^2 - m_i^2)(1 - 2a_s)(\boldsymbol{S}_- \cdot \boldsymbol{L})\Big\}, \quad (3b)$$

$$V_{\rm OGE}^{C}(\boldsymbol{r}_{ij}) = \frac{\alpha_s}{4}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left[\frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \delta(\boldsymbol{r}_{ij})\right], \quad (3c)$$

$$V_{\rm OGE}^{\rm SO}(\boldsymbol{r}_{ij}) = -\frac{1}{16} \cdot \frac{\alpha_s}{m_i^2 m_j^2}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \Big\{\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3} \cdot (1 + \frac{r_{ij}}{r_g(\mu)})\Big\} \times \Big\{\Big((m_i + m_j)^2 + 2m_i m_j\Big)(\boldsymbol{S}_+ \cdot \boldsymbol{L}) + (m_j^2 - m_i^2)(\boldsymbol{S}_- \cdot \boldsymbol{L})\Big\}, \quad (3d)$$

$$\delta(\boldsymbol{r}_{ij}) = \frac{e^{-r_{ij}/r_0(\mu_{ij})}}{4\pi r_{ij} r_0^2(\mu_{ij})}, \mathbf{S}_\pm = \mathbf{S}_1 \pm \mathbf{S}_2, \quad (3e)$$

$$V_\pi(\boldsymbol{r}_{ij}) = \frac{g_{ch}^2}{4\pi}\frac{m_\pi^2}{12m_i m_j}\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2} m_\pi v_{ij}^\pi \sum_{a=1}^{3} \lambda_i^a \lambda_j^a, \quad (3f)$$

$$V_K(\boldsymbol{r}_{ij}) = \frac{g_{ch}^2}{4\pi}\frac{m_K^2}{12m_i m_j}\frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2} m_K v_{ij}^K \sum_{a=4}^{7} \lambda_i^a \lambda_j^a, \quad (3g)$$

$$V_\eta(\boldsymbol{r}_{ij}) = \frac{g_{ch}^2}{4\pi}\frac{m_\eta^2}{12m_i m_j}\frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2} m_\eta v_{ij}^\eta \times \left[\lambda_i^8 \lambda_j^8 \cos\theta_P - \lambda_i^0 \lambda_j^0 \sin\theta_P\right], \quad (3h)$$

$$v_{ij}^\chi(\boldsymbol{r}_{ij}) = \left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij})\right]\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \quad (3i)$$

$$V_\sigma(\boldsymbol{r}_{ij}) = -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2} m_\sigma \times \left[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij})\right], \quad (3j)$$

In these expressions, $\mathbf{S}_1$ and $\mathbf{S}_2$ represent spin operators of the two meson clusters. The function $Y(x) = e^{-x}/x$ and $r_0(\mu_{ij}) = s_0/\mu_{ij}$. Matrices $\boldsymbol{\sigma}$ correspond to the $SU(2)$ Pauli matrices, whereas $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}^c$ denote the $SU(3)$ flavour and colour Gell-Mann matrices respectively. The parameters Λ_i $(i = \pi, K, \eta, \sigma)$ serve as cutoff quantities, which effectively act as form factors. Their purpose is

to eliminate short-range contributions originating from Goldstone boson exchanges. Since each Λ_i relies on the mass of the corresponding Goldstone boson, we do not adopt a universal value for all of them. The constant $g_{ch}^2/4\pi$ stands for the chiral coupling strength, whose magnitude is determined from the π-nucleon coupling. Meanwhile, α_s is treated as an effective scale-dependent running coupling as described in Ref. [9]:

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln\left[(\mu_{ij}^2+\mu_0^2)/\Lambda_0^2\right]}. \tag{4}$$

Within our numerical calculations for two-quark systems, both central and non-central potential contributions are incorporated. Nevertheless, computations performed for four-quark systems reveal that non-central potentials exert only negligible influence on mass shifts of the resulting eigenstates.

Model parameters extracted from Ref. [9] are summarised in Table I. It should be highlighted that the confinement term employed in Ref. [9] reads $V_{ij}^C = \left(-a_c(1-e^{-\mu_c r_{ij}}\right)+\Delta)(\boldsymbol{\lambda}_i^c\cdot\boldsymbol{\lambda}_j^c)$. In contrast, the present work utilises the widely adopted quadratic confinement form $V_{ij}^C = (-a_c r_{ij}^2-\Delta)\boldsymbol{\lambda}_i^c\cdot\boldsymbol{\lambda}_j^c$. For meson configurations, spatial separations between quark and antiquark remain relatively small. Consequently, the discrepancy induced by these two confinement forms can be minimised via tuning confinement-related parameters. Both forms are capable of reproducing linear Regge trajectories for $q\bar{q}$ mesons, accompanied by differences in parameters such as quark masses, a_c and Δ.

Making use of the parameter set shown in Table I, we compute masses for various mesons ranging from light to heavy flavours within the chiral quark model, paying particular attention to bottomonium $b\bar{b}$ states relevant to our investigation. Numerical outcomes are listed in Table II. From the tabulated results, one can observe that this quark model achieves considerable success in reproducing hadron spectra, especially for ground-state mesons including most light mesons and several heavy meson species. However, challenges persist when describing excited charmonium and bottomonium states such as $\eta_c(2S), \psi(2S), \chi_{cJ}(1P,2P)$, $\Upsilon(nS)$ and $\chi_{bJ}(1P,2P)$.

TABLE I. Model parameters, determined by fitting the meson spectrum, taking from Ref. [37].

Quark masses	$m_u = m_d$	313
(MeV)	m_s	536
	m_c	1728
	m_b	5112
Goldstone bosons	m_π	0.70
($\mathrm{fm}^{-1} \sim 200\,\mathrm{MeV}$)	m_σ	3.42
	m_η	2.77
	m_K	2.51
	$\Lambda_\pi = \Lambda_\sigma$	4.2
	$\Lambda_\eta = \Lambda_K$	5.2
	$g_{ch}^2/(4\pi)$	0.54
	$\theta_p(^\circ)$	-15
Confinement	a_c (MeV fm^{-2})	101
	Δ (MeV)	-78.3
OGE	α_0	3.67
	$\Lambda_0(\mathrm{fm}^{-1})$	0.033
	μ_0(MeV)	36.98
	s_0(MeV)	28.17

B. Gaussian Expansion Method

Within the framework of nonrelativistic quark models, meson energy spectra can be acquired through solving the Schrödinger equation:

$$H\Psi_{M_I M_J}^{IJ}(1,2) = E^{IJ}\Psi_{M_I M_J}^{IJ}(1,2)\,, \tag{5}$$

where labels 1 and 2 correspond to quark and antiquark respectively. The wave function $\Psi_{M_I M_J}^{IJ}(1,2)$ describes a meson formed by one quark and one antiquark carrying quantum numbers IJ^{PC}, and takes the following form:

$$\Psi_{M_I M_J}^{IJ}(1,2) = \sum_\alpha C_\alpha \left[\psi_l(\mathbf{r})\chi_s(1,2)\right]_{M_J}^J \omega^c(1,2)\phi_{M_I}^I(1,2), \tag{6}$$

In this expression, $\psi_l(\mathbf{r})$, $\chi_s(1,2)$, $\omega^c(1,2)$ and $\phi^I(1,2)$ are orbital, spin, colour and flavour wave functions separately. The symbol α collectively stands for intermediate quantum numbers, including l, s and relevant flavour indices. In our numerical implementation, orbital wave functions are expanded by a basis composed of Gaussian functions:

$$\psi_{lm}(\mathbf{r}) = \sum_{n=1}^{n_{\max}} c_n \psi_{nlm}^G(\mathbf{r}), \tag{7a}$$

$$\psi_{nlm}^G(\mathbf{r}) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}), \tag{7b}$$

Gaussian width parameters are selected following a geometric progression rule:

$$\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 a^{n-1}, \quad a = \left(\frac{r_{n_{\max}}}{r_1}\right)^{\frac{1}{n_{\max}-1}}. \tag{8}$$

This treatment enables adequate coverage over different spatial ranges using only a limited number of Gaussian basis functions.

After basis expansion, the wave function shown in Eq. (6) can be rewritten as

$$\Psi_{M_I M_J}^{IJ}(1,2) = \sum_{n\alpha} C_\alpha c_n \left[\psi_{nl}^G(\mathbf{r})\chi_s(1,2)\right]_{M_J}^J \omega^c(1,2)\phi_{M_I}^I(1,2). \tag{9}$$

TABLE II. The mass spectrum in the chiral quark model, in comparison with the experimental data [1] (in unit of MeV).

Name	$J^{P(C)}$	Mass	PDG [1]
π	0^-	134.9	135.0
K	0^-	489.4	493.7
ρ	1^{--}	772.3	775.3
K^*	1^-	913.6	892.0
ω	1^{--}	701.6	782.7
η	0^{-+}	669.2	547.9
$\phi(1020)$	1^{--}	1015.9	1019.5
D^0	0^-	1861.9	1864.8
D^{*0}	1^-	1980.6	2006.9
D_s^+	0^-	1950.1	1968.4
D_s^{*+}	1^-	2079.9	2112.2
B^-	0^-	5280.7	5279.3
B^*	1^-	5319.5	5324.7
B_s^0	0^-	5367.7	5366.9
B_s^*	1^-	5410.2	5415.4
$\eta_c(1S)$	0^{-+}	2964.4	2983.9
$\eta_c(2S)$	0^{-+}	3507.8	3637.5
J/ψ	1^{--}	3096.4	3096.0
$\psi(2S)$	1^{--}	3605.0	3686.1
$\chi_{c0}(1P)$	0^{++}	3362.8	3414.7
$\chi_{c0}(2P)$	0^{++}	3814.7	$\chi_{c0}(3915)$?
$\chi_{c1}(1P)$	1^{++}	3393.9	3510.7
$\chi_{c1}(2P)$	1^{++}	3851.9	$\chi_{c1}(3872)$?
$\chi_{c2}(1P)$	2^{++}	3435.8	3556.2
$\chi_{c2}(2P)$	2^{++}	3901.1	$\chi_{c2}(3930)$?
$h_c(1P)$	1^{+-}	3416.1	3525.4
$h_c(2P)$	1^{+-}	3877.4	$Z_c(3900)$?
$\eta_b(1S)$	0^{-+}	9561.7	9398.7
$\Upsilon(1S)$	1^{--}	9647.9	9460.3
$\Upsilon(2S)$	1^{--}	10016.6	10023.3
$\chi_{b0}(1P)$	0^{++}	9917.0	9859.4
$\chi_{b0}(2P)$	0^{++}	10198.4	10232.5
$\chi_{b1}(1P)$	1^{++}	9925.6	9892.8
$\chi_{b1}(2P)$	1^{++}	10208.4	10255.5
$\chi_{b2}(1P)$	2^{++}	9939.3	9912.2
$\chi_{b2}(2P)$	2^{++}	10222.9	10268.7
$h_b(1P)$	1^{+-}	9932.8	9899.3
$h_b(2P)$	1^{+-}	10216.4	10259.8
$\eta_{c2}(1D)$	2^{-+}	3675.1	?
$\psi(1D)$	1^{--}	3653.3	$\psi(3770)$?
$\psi_2(1D)$	2^{--}	3668.3	$\psi_2(3823)$?
$\psi_3(1D)$	3^{--}	3688.1	$\psi_3(3842)$?

Given the nonorthogonality among Gaussian basis states, we adopt the Rayleigh-Ritz variational principle to tackle the Schrödinger equation, which transforms the original differential equation into a generalised matrix eigenvalue problem:

$$\sum_{n',\alpha'}(H^{IJ}_{n\alpha,n'\alpha'} - E^{IJ}N^{IJ}_{n\alpha,n'\alpha'})C^{IJ}_{n'\alpha'} = 0, \tag{10a}$$

$$H^{IJ}_{n\alpha,n'\alpha'} = \langle\Phi^{IJ}_{M_IM_J,n\alpha}|H|\Phi^{IJ}_{M_IM_J,n'\alpha'}\rangle, \tag{10b}$$

$$N^{IJ}_{n\alpha,n'\alpha'} = \langle\Phi^{IJ}_{M_IM_J,n\alpha}|1|\Phi^{IJ}_{M_IM_J,n'\alpha'}\rangle, \tag{10c}$$

with $\Phi^{IJ}_{M_IM_J,n\alpha} = [\psi^G_{nl}(\mathbf{r})\chi_s(1,2)]^J_{M_J}\omega^c(1,2)\phi^I_{M_I}(1,2)$, $C^{IJ}_{n\alpha} = C_\alpha c_n$.

Mass eigenvalues for four-quark systems are also obtained by solving the corresponding Schrödinger equation:

$$H\,\Psi^{4\,IJ}_{M_IM_J} = E^{IJ}\Psi^{4\,IJ}_{M_IM_J}, \tag{11}$$

where $\Psi^{4\,IJ}_{M_IM_J}$ represents the wave function for the four-quark configuration, whose construction procedure is outlined below. Within coupled-channel calculations carried out in this paper, we only consider meson-meson configurations with colour-singlet clusters for four-quark systems. We first write wave functions for the two separate meson clusters:

$$\Psi^{I_1J_1}_{M_{I_1}M_{J_1}}(1,2) = \sum_{\alpha_1 n_1}\mathcal{C}^{\alpha_1}_{n_1} \times\left[\psi^G_{n_1l_1}(\mathbf{r}_{12})\chi_{s_1}(1,2)\right]^{J_1}_{M_{J_1}}\omega^{c_1}(1,2)\phi^{I_1}_{M_{I_1}}(1,2), \tag{12a}$$

$$\Psi^{I_2J_2}_{M_{I_2}M_{J_2}}(3,4) = \sum_{\alpha_2 n_2}\mathcal{C}^{\alpha_2}_{n_2} \times\left[\psi^G_{n_2l_2}(\mathbf{r}_{34})\chi_{s_2}(3,4)\right]^{J_2}_{M_{J_2}}\omega^{c_2}(3,4)\phi^{I_2}_{M_{I_2}}(3,4), \tag{12b}$$

The total wave function for the four-quark state is then expressed as

$$\begin{aligned}\Psi^{4\,IJ}_{M_IM_J} &= \mathcal{A}\sum_{L_r}\left[\Psi^{I_1J_1}(1,2)\Psi^{I_2J_2}(3,4)\psi_{L_r}(\mathbf{r}_{1234})\right]^{IJ}_{M_IM_J}\\ &= \sum_{\alpha_1\,\alpha_2\,n_1\,n_2\,L_r}\mathcal{C}^{\alpha_1}_{n_1}\mathcal{C}^{\alpha_2}_{n_2}\Bigg[\left[\psi^G_{n_1l_1}(\mathbf{r}_{12})\chi_{s_1}(1,2)\right]^{J_1}\\ &\quad\times\left[\psi^G_{n_2l_2}(\mathbf{r}_{34})\chi_{s_2}(3,4)\right]^{J_2}\psi_{L_r}(\mathbf{r}_{1234})\Bigg]^J_{M_J}\\ &\quad\times[\omega^{c_1}(1,2)\omega^{c_2}(3,4)]^{[1]}\left[\phi^{I_1}(1,2)\phi^{I_2}(3,4)\right]^I_{M_I},\end{aligned} \tag{13}$$

Here $\mathcal{A}$ stands for the antisymmetrization operator. Treating all quarks (antiquarks) as identical particles, we define

$$\mathcal{A} = \frac{1}{2}(1 - P_{13} - P_{24} + P_{13}P_{24}). \tag{14}$$

The function $\psi_{L_r}(\mathbf{r}_{1234})$ describes relative motion between the two clusters and is also expanded on Gaussian basis functions. L_r denotes orbital angular momentum associated with inter-cluster relative motion. It is

worth noting that internal orbital angular momenta l_1 and l_2 of both mesons are fixed to zero in our computations. As a consequence, for bottomonium nS states, the inter-cluster angular momentum L_r in four-quark configurations equals 1, corresponding to P-wave relative motion. For nP bottomonium states, we include both S-wave ($L_r = 0$) and D-wave ($L_r = 2$) components for cluster relative motion.

C. The $^3P_0$ Model

The $^3P_0$ quark-pair creation model [42–44] constitutes a widely adopted framework for describing two-body strong decays of hadrons allowed by the OZI rule [41, 45–49]. We label the quark and antiquark contained within the initial meson as particle 1 and particle 2 respectively. Meanwhile, the quark–antiquark pair $(u\bar{u}, d\bar{d}, s\bar{s})$ generated from the vacuum is assigned labels 3 and 4. Within this notation scheme, the transition operator corresponding to the $^3P_0$ model reads

$$T_0 = -3\,\gamma \sum_m \langle 1m1(-m)|00\rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3+\mathbf{p}_4) \times \mathcal{Y}_1^m(\frac{\mathbf{p}_3-\mathbf{p}_4}{2})\chi^{34}_{1-m}\phi_0^{34}\omega_0^{34}b_2^\dagger(\mathbf{p}_3)d_3^\dagger(\mathbf{p}_4), \quad (15)$$

The parameter γ characterises the probability of producing a quark–antiquark pair with momenta $\mathbf{p}_3$ and $\mathbf{p}_4$ from the 0^{++} vacuum. Generally, its numerical value is determined via fitting experimental measurements of various hadronic strong decay channels. The fitted value $\gamma = 6.95$ is used for generating $u\bar{u}$ and $d\bar{d}$ pairs, whereas $\gamma = 6.95/\sqrt{3}$ is utilised for the creation of $s\bar{s}$ pairs [50]. Moreover, ω_0^{34} and ϕ_0^{34} correspond to colour and flavour wave function components of the quark pair labelled 3 and 4.

III. NUMERICAL RESULTS

By introducing four-quark configurations of the form $b\bar{b}q\bar{q}$ ($q = u, d, s$) into conventional $b\bar{b}$ bottomonium states, we may solve the corresponding Schrödinger equation to extract the eigenvalues of the mixed $b\bar{b} + b\bar{b}q\bar{q}$ system:

$$H\Psi = E\Psi, \quad (16)$$

in this expression, Ψ stands for the total wave function of the whole system and H represents its Hamiltonian operator, which is demonstrated as follows:

$$\Psi = c_1\Psi_{2q} + c_2\Psi_{4q}\,, \quad (17)$$

$$H = H_{2q} + H_{4q} + T_0\,. \quad (18)$$

Within the framework of the nonrelativistic quark model, particle number is conserved. For this reason, a prescription is given below. The Hamiltonian component H_{2q} only acts upon the wave function Ψ_{2q} belonging to the two-body $b\bar{b}$ system, while H_{4q} exclusively operates on the four-quark wave function Ψ_{4q}. The transition operator T_0 defined in Eq. (15) from the $^3P_0$ model serves to couple the two-quark sector and four-quark sector with each other. Following this setup, we are able to compute the matrix elements of the Hamiltonian.

$$\begin{aligned}\langle\Psi|H|\Psi\rangle &= \langle c_1\Psi_{2q} + c_2\Psi_{4q}|H|c_1\Psi_{2q} + c_2\Psi_{4q}\rangle \\ &= c_1^2\langle\Psi_{2q}|H_{2q}|\Psi_{2q}\rangle + c_2^2\langle\Psi_{4q}|H_{4q}|\Psi_{4q}\rangle \\ &\quad + c_1c_2^*\langle\Psi_{4q}|T_0|\Psi_{2q}\rangle + c_1^*c_2\langle\Psi_{2q}|T_0^\dagger|\Psi_{4q}\rangle, \end{aligned} \quad (19)$$

and the block-matrix structure for the Hamiltonian and overlap takes,

$$(H) = \begin{bmatrix} (H_{2q}) & (H_{24}) \\ (H_{42}) & (H_{4q}) \end{bmatrix}, (N) = \begin{bmatrix} (N_{2q}) & (0) \\ (0) & (N_{4q}) \end{bmatrix}, \quad (20)$$

with

$$(H_{2q}) = \langle\Psi_{2q}|H_{2q}|\Psi_{2q}\rangle, \quad (21a)$$
$$(H_{24}) = \langle\Psi_{4q}|T_0|\Psi_{2q}\rangle, \quad (21b)$$
$$(H_{4q}) = \langle\Psi_{4q}|H_{4q}|\Psi_{4q}\rangle, \quad (21c)$$
$$(N_{2q}) = \langle\Psi_{2q}|1|\Psi_{2q}\rangle, \quad (21d)$$
$$(N_{4q}) = \langle\Psi_{4q}|1|\Psi_{4q}\rangle. \quad (21e)$$

Where (H_{2q}) and (H_{4q}) is the matrix for the pure two-quark $b\bar{b}$ system and pure four-quark system, respectively. (H_{24}) is the coupling matrix of two-quark system and four-quark system.

Finally the eigenvalues (E_n) and eigenvectors (C_n) of the system are obtained by solving the diagonalization problem,

$$\Big[\,(H) - E_n(N)\,\Big]\Big[\,C_n\,\Big] = 0. \quad (22)$$

For the purpose of carrying out Fourier transformation in our numerical computations, we introduce a convergence factor $e^{-f^2\mathbf{p}^2}$ into the transition operator T_0 given in Eq. (15). This treatment is necessary since our solution for both the two-quark and four-quark systems is performed within coordinate space. The explicit form of this Fourier-transformed factor is expressed as:

$$T_1 = -i3\gamma \sum_m \langle 1m1(-m)|00\rangle \int d\mathbf{r_3} d\mathbf{r_4} (\frac{1}{2\pi})^{\frac{3}{2}} 2^{-\frac{5}{2}} f^{-5} r Y_{1m}(\hat{\mathbf{r}}) \mathrm{e}^{-\frac{\mathbf{r}^2}{4f^2}} \chi^{34}_{1-m}\omega_0^{34}\phi_0^{34}b_3^\dagger(\mathbf{r_3})d_4^\dagger(\mathbf{r_4}). \quad (23)$$

An additional parameter f exists within the transition operator T_1. Taking the limit of $f \to 0$ will reproduce the original formulation of the $^3P_0$ quark model. Furthermore, $\mathbf{r}$ appearing in Eq. (23) denotes the relative separation of the quark–antiquark pair generated from the vacuum, which satisfies the relation $\mathbf{r} = \mathbf{r}_3 - \mathbf{r}_4$.

By solving Eq. (22) with the transition operator in Eq. (23) and in the limit $f \to 0$, $\gamma = 6.95$, we obtained the mass shifts for $\eta_b(1S, 2S)$, $\Upsilon(nS)(n = 1 \sim 6)$,

$\chi_{b_J}(nP)(J=0,1,2;n=1,2,3)$ and $h_b(1P,2P)$ bottomonium valence states, by incorporating the four-quark components (B, B^*, B_s and B_s^* meson pairs) into the two-quark $b\bar{b}$ system in the unquenched quark model. The results are shown in Table III.

In Table III, the angular momentum of the mesons within each cluster is set to 0 for all states. For the $\eta_b(nS)(n=1,2)$ and $\Upsilon(nS)(n=1\sim 6)$ states, the possible angular momentum between two clusters L_r are 1 and 3. In the present work, owing to the considerable computational complexity for the higher partial wave $L_r=3$ (F-wave), only the P-wave case ($L_r=1$) is considered. For the highly excited $\chi_{b_J}(nP)(J=0,1,2)(n=1,2,3)$ and $h_b(nP)(n=1,2)$ states, the angular momentum L_r can take values of 0 or 2. In our calculations, we take into consideration not only the S-wave case but also the D-wave case. The bare mass of the states are obtained in the conventional quenched quark model, viz. solved with only the $b\bar{b}$ component. In the table, the experimental values (Exp) are taken from the latest numerical results of the PDG [1].

Considering the isospin symmetry and the total angular momentum conservation, the four-quark channels that can couple to the two-quark system are listed in Table III. It should be noted that possible structures for four-quark systems include meson-meson structure, diquark-antidiquark structure, K-configuration, etc. Within the unquenched quark model, we preliminarily take into account the coupling effects between the meson-meson structure and the two-quark system. In our calculations, open channels exist when the theoretical bare mass is larger than the theoretical threshold mass, the sum of masses of two mesons. For example, the $B\bar{B}$ is the open channel for the $\Upsilon(4S)$ state; For higher-lying states $\Upsilon(5S)$ and $\Upsilon(6S)$, the total eight channels list in the table are all open channels. For the coupled-channel calculations between $b\bar{b}$ configuration and open channels, the mass shift of the state will change with the Gaussian distribution of the relative motion between two mesons. Specifically, we determine that the mass shift will periodically change with an increase in spatial volume. In our calculations, we selected the biggest mass shift as the contribution of this open channel. For $B\bar{B}$ state, it is a scattering one with discrete energy levels that vary with the Gaussian distribution in the theoretical calculations due to the limitation of finite volume. When considering the coupling of $B\bar{B}$ and $\Upsilon(4S)$, the strength of coupling will be increased as one of the energy of $B\bar{B}$ state is close to that of $\Upsilon(4S)$, and the induced mass shift will increase. We consider the biggest one as the mass shift of state $\Upsilon(4S)$ to $B\bar{B}$ state. Furthermore, if we expand the space further with higher values, the same biggest mass shift will be repeated. In the last column, the mass with unquenched effects (Unquenched mass) is equal to the bare mass plus the mass shifts from all channel couplings.

In Table III, we get the large negative mass shifts. Such large shifts invalidate the traditional quenched quark model in describing the low-lying quarkonium spectrum. In our previous work [37], when we investigate the hadron loop effects of the $n\bar{n}$ ($n=u,d$) states in the unquenched quark model, similarly, large mass shifts are obtained with the original operator T_1 in Eq. (23) in the limit $f\to 0$. In order to develop a more realistic unquenching procedure, in the work [37], we gave some modifications of the operator T_1. It reads,

$$T_2=-3\gamma\sum_m\langle 1m1(-m)|00\rangle\int d\mathbf{r_3}d\mathbf{r_4}(\frac{1}{2\pi})^{\frac{3}{2}}ir2^{-\frac{5}{2}}f^{-5}$$
$$Y_{1m}(\hat{\mathbf{r}})\mathrm{e}^{-\frac{\mathbf{r}^2}{4f^2}}\mathrm{e}^{-\frac{R_{AV}^2}{R_0^2}}\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34}b_3^\dagger(\mathbf{r_3})d_4^\dagger(\mathbf{r_4}),\qquad(24)$$

compared with Eq. (23), the factor $e^{-\frac{R_{AV}^2}{R_0^2}}$ is introduced because the creation of quark-antiquark pairs should become less likely as the distance from the bare-hadron source is increased. R_{AV} is the relative distance between the source particle and quark-antiquark pair in the vacuum. In Eq. (24), there are three parameters need to identify, γ, f and R_0. According to our previous work [37], we find

$$\gamma=32.2,\quad f=0.5\,\mathrm{fm},\quad R_0=1\,\mathrm{fm}.\qquad(25)$$

In the present work, we also apply the transition operator in Eq. (24) and the values of parameters γ, f and R_0 in Eq. (25). The newly mass shifts of the bottomonium valence states are demonstrated in Table IV.

Compared with the results in Table III, the most mass shifts of all channels are significantly reduced greatly and remaining negative. And we find that the modified $^3P_0$ pair-creation model yields moderate unquenching corrections, where the mass renormalizations are reasonable compared with the bare mass for each considered meson. So the effects of including the new term $e^{-\frac{R_{AV}^2}{R_0^2}}$ on the mass shifts are relatively stable. On the other hand, we calculated the decay width of mesons using the transition amplitude T_2 in Eq. (24), and we find that the results are similar with the ones using the original transition amplitude T_1 with $f\to 0$ in Eq. (23).

Combining the results of Table III and Table IV, we observe that, for channel couplings with $L_r=S$-wave and $L_r=D$-wave, the coupling strength between the two-quark and four-quark system is larger for $L_r=D$-wave than for $L_r=S$-wave. This conclusion is consistent with Ref. [51]. By modifying the transition operator of the $^3P_0$ model, the mass shifts are reduced. Correspondingly, the unquenched masses also become smaller in comparison with Table III. From the Table IV, one can observe that the mass shift increases from the $\eta_b(1S)$ to $\eta_b(2S)$ states, as well as the $\Upsilon(1S)$ to $\Upsilon(4S)$ states. This arises because the energy gap between the four-quark system and two-quark system states decreases; when the two states couple, the induced mass shift is inversely proportional to their energy difference. At the $\Upsilon(5S)$ state, the energy of the two-quark state exceeds that of the four-quark state, and several open channels become available.

TABLE III. Mass shifts computed for $b\bar{b}$ bottomonium states using the transition matrix constructed from T_1 in Eq. (23) with $f \to 0$, $\gamma = 6.95$. (Units of MeV)

Bare $b\bar{b}$ state			Mass shifts by channels									$b\bar{b} + qq\bar{q}\bar{q}$
State	Bare mass	Exp[1]	$B\bar{B}$	$B\bar{B}^*$	$B^*\bar{B}$	$B^*\bar{B}^*$	$B_s\bar{B}_s$	$B_s\bar{B}_s^*$	$B_s^*\bar{B}_s$	$B_s^*\bar{B}_s^*$	Total	Unquenched mass
$\eta_b(1S)$	9561.7	9398.7	...	-112.6	-112.6	-223.5	...	-39.7	-39.7	-72.1	-600.2	8961.5
$\eta_b(2S)$	9976.1	...	...	-79.9	-79.9	-151.5	...	-22.0	-22.0	-42.2	-397.5	9577.6
$\Upsilon(1S)$	9647.9	9460.4	-40.8	-83.5	-83.5	-261.6	-12.6	-24.5	-24.5	-78.7	-609.7	9038.2
$\Upsilon(2S)$	10016.6	10023.4	-28.3	-54.7	-54.7	-179.7	-7.3	-14.2	-14.2	-48.4	-401.6	9615.0
$\Upsilon(3S)$	10309.0	10355.1	-22.7	-70.9	-70.9	-138.6	-5.4	-10.6	-10.6	-35.6	-365.3	9943.7
$\Upsilon(4S)$	10587.6	10579.4	-23.4	-37.7	-37.7	-115.6	-4.5	-8.6	-8.6	-26.9	-263.0	10324.6
$\Upsilon(5S)$	10881.7	$\Upsilon(10860)$?	-1.6	-3.9	-3.9	-8.6	-2.4	-4.0	-4.0	-1.5	-29.9	10851.8
$\Upsilon(6S)$	11197.3	$\Upsilon(11020)$?	-15.4	-19.3	-19.3	-41.9	-5.9	-7.0	-7.0	-10.4	-126.2	11071.1
$\chi_{b_0}(1P)(L_r = S)$	9917.0	9859.4	-82.6	...	...	-24.9	-21.1	...	...	-6.5		
$\chi_{b_0}(1P)(L_r = D)$	9917.0	9859.4	...	...	...	-261.5	...	...	...	-73.7	-470.3	9446.7
$\chi_{b_0}(2P)(L_r = S)$	10198.4	10232.5	-66.5	...	...	-20.1	-15.9	...	...	-4.9		
$\chi_{b_0}(2P)(L_r = D)$	10198.4	10232.5	...	...	...	-195.4	...	...	...	-53.2	-356.0	9842.4
$\chi_{b_0}(3P)(L_r = S)$	10470.1	...	-59.7	...	...	-17.6	-12.1	...	...	-3.7		
$\chi_{b_0}(3P)(L_r = D)$	10470.1	...	...	...	...	-154.8	...	...	...	-41.1	-289.0	10181.1
$\chi_{b_1}(1P)(L_r = S)$	9925.6	9892.7	...	-53.0	-53.0	...	...	-13.4	-13.4	...		
$\chi_{b_1}(1P)(L_r = D)$	9925.6	9892.7	...	-33.5	-33.5	-194.4	...	-9.2	-9.2	-54.1	-466.7	9458.9
$\chi_{b_1}(2P)(L_r = S)$	10208.4	10255.4	...	-42.6	-42.6	...	...	-10.1	-10.1	...		
$\chi_{b_1}(2P)(L_r = D)$	10208.4	10255.4	...	-26.7	-26.7	-147.6	...	-6.8	-6.8	-39.4	-359.4	9849
$\chi_{b_1}(3P)(L_r = S)$	10483.8	10513.4	...	-38.6	-38.6	...	...	-7.3	-7.3	...		
$\chi_{b_1}(3P)(L_r = D)$	10483.8	10513.4	...	-19.2	-19.2	-110.0	...	-4.5	-4.5	-27.3	-276.5	10207.3
$\chi_{b_2}(1P)(L_r = S)$	9939.3	9912.2	...	...	...	-101.9	...	...	...	-25.2		
$\chi_{b_2}(1P)(L_r = D)$	9939.3	9912.2	-40.5	-58.6	-58.6	-102.3	-10.9	-16.0	-16.0	-27.7	-457.7	9481.6
$\chi_{b_2}(2P)(L_r = S)$	10222.9	10268.6	...	...	...	-78.7	...	...	...	-18.2		
$\chi_{b_2}(2P)(L_r = D)$	10222.9	10268.6	-32.4	-46.2	-46.2	-79.4	-7.9	-11.6	-11.6	-20.1	-352.3	9870.6
$\chi_{b_2}(3P)(L_r = S)$	10498.9	10524.0	...	...	...	-68.9	...	...	...	-13.8		
$\chi_{b_2}(3P)(L_r = D)$	10498.9	10524.0	-24.2	-32.8	-32.8	-55.2	-5.2	-7.6	-7.6	-13.2	-261.3	10237.6
$h_b(1P)(L_r = S)$	9932.8	9899.3	...	-26.5	-26.5	-49.7	...	-6.6	-6.6	-12.7		
$h_b(1P)(L_r = D)$	9932.8	9899.3	...	-65.7	-65.7	-123.8	...	-18.1	-18.1	-35.1	-455.1	9477.7
$h_b(2P)(L_r = S)$	10216.4	10259.8	...	-21.4	-21.4	-39.9	...	-5.0	-5.0	-9.7		
$h_b(2P)(L_r = D)$	10216.4	10259.8	...	-52.3	-52.3	-96.7	...	-13.5	-13.5	-26.0	-356.7	9859.7

In this case, owing to the oscillatory behavior of the wave function, the transtion matrix elements assume both positive and negative values and cancel one another, leading to an abrupt reduction in the mass shift. For the $\Upsilon(6S)$ state, such cancellation is weakened.

For several highly excited states, their unquenched masses are in very good agreement with the experimental values, such as $\chi_{b_0}(1P)$, $\chi_{b_1}(1P)$, $\chi_{b_2}(1P)$ and $h_b(1P)$ states, with theoretical unquenched masses 9867.5 MeV, 9875.2 MeV, 9886.5 MeV and 9882.0 MeV, respectively. So the four-quark component cannot be neglected for these states. And our results are well agreement with the Ref. [52], where in the Godfrey–Isgur model, the authors get the unquenched mass with 9857.6MeV, 9879.6MeV, 9901.69 MeV, and 9886.7 MeV, respectively.

For $\Upsilon(5S)$ state, the stable theoretical bare mass in the quark model is 10881.7 MeV, and the unquenched mass is 10845.6 MeV. Both of them are very close to the experiment mass of the state $\Upsilon(10860)$. In the references [52, 53], the authors also obtained a very similar unquenched mass of the $\Upsilon(5S)$ state to ours. What's more, in our work, the mass shifts arising from the coupling between the $\Upsilon(5S)$ state and the $B\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}$ and $B^*\bar{B}^*$ channels are very small, which indicates that the coupling of the $\Upsilon(5S)$ state to the four-quark states is weak. Therefore, in our calculations, the $\Upsilon(10860)$ state is regarded as a state dominated by the $b\bar{b}$ two-quark component and can be classified as a member of

TABLE IV. Mass shifts computed for $b\bar{b}$ bottomonium mesons using the transition matrix constructed from T_2 in Eq. (24) with $f = 0.5$ fm, $\gamma = 32.2$, $R_0 = 1$ fm. (Units of MeV)

Bare $b\bar{b}$ state			Mass shifts by channels									$b\bar{b} + qq\bar{q}\bar{q}$
State	Bare mass	Exp[1]	$B\bar{B}$	$B\bar{B}^*$	$B^*\bar{B}$	$B^*\bar{B}^*$	$B_s\bar{B}_s$	$B_s\bar{B}_s^*$	$B_s^*\bar{B}_s$	$B_s^*\bar{B}_s^*$	Total	Unquenched mass
$\eta_b(1S)$	9561.7	9398.7	...	-5.1	-5.1	-9.8	...	-1.3	-1.3	-2.5	-25.1	9536.6
$\eta_b(2S)$	9976.1	...	...	-13.6	-13.6	-25.7	...	-2.3	-2.3	-4.5	-62.0	9914.1
$\Upsilon(1S)$	9647.9	9460.4	-1.9	-3.8	-3.8	-13.3	-0.5	-0.9	-0.9	-3.0	-28.1	9619.8
$\Upsilon(2S)$	10016.6	10023.4	-5.2	-9.9	-9.9	-34.3	-0.9	-1.7	-1.7	-5.7	-69.3	9947.3
$\Upsilon(3S)$	10309.0	10355.1	-7.0	-12.9	-12.9	-51.6	-0.9	-1.6	-1.6	-5.7	-94.2	10214.8
$\Upsilon(4S)$	10587.6	10579.4	-27.9	-24.8	-24.8	-65.7	-1.5	-2.7	-2.7	-8.7	-158.8	10428.8
$\Upsilon(5S)$	10881.7	$\Upsilon(10860)$?	-1.5	-2.0	-2.0	-7.7	-5.6	-8.2	-8.2	-0.9	-36.1	10845.6
$\Upsilon(6S)$	11197.3	$\Upsilon(11020)$?	-6.7	-9.6	-9.6	-23.0	-3.1	-4.6	-4.6	-7.5	-68.7	11128.6
$\chi_{b_0}(1P)(L_r = S)$	9917.0	9859.4	-8.3	...	...	-2.4	-1.4	...	...	-0.5		
$\chi_{b_0}(1P)(L_r = D)$	9917.0	9859.4	...	...	...	-30.8	...	...	...	-6.1	-49.5	9867.5
$\chi_{b_0}(2P)(L_r = S)$	10198.4	10232.5	-15.7	...	...	-4.5	-2.2	...	...	-0.7		
$\chi_{b_0}(2P)(L_r = D)$	10198.4	10232.5	...	...	...	-51.1	...	...	...	-8.8	-83.0	10115.4
$\chi_{b_0}(3P)(L_r = S)$	10470.1	...	-33.4	...	...	-8.8	-3.3	...	...	-1.0		
$\chi_{b_0}(3P)(L_r = D)$	10470.1	...	...	...	...	-47.7	...	...	...	-6.0	-100.2	10369.9
$\chi_{b_1}(1P)(L_r = S)$	9925.6	9892.7	...	-5.4	-5.4	...	...	-0.9	-0.9	...		
$\chi_{b_1}(1P)(L_r = D)$	9925.6	9892.7	...	-4.0	-4.0	-23.6	...	-0.8	-0.8	-4.6	-50.4	9875.2
$\chi_{b_1}(2P)(L_r = S)$	10208.4	10255.4	...	-10.0	-10.0	...	...	-1.4	-1.4	...		
$\chi_{b_1}(2P)(L_r = D)$	10208.4	10255.4	...	-7.1	-7.1	-39.6	...	-1.1	-1.1	-6.7	-85.5	10122.9
$\chi_{b_1}(3P)(L_r = S)$	10483.8	10513.4	...	-23.2	-23.2	...	...	-2.3	-2.3	...		
$\chi_{b_1}(3P)(L_r = D)$	10483.8	10513.4	...	-9.0	-9.0	-41.8	...	-0.9	-0.9	-4.8	-117.4	10366.4
$\chi_{b_2}(1P)(L_r = S)$	9939.3	9912.2	...	...	...	-11.3	...	...	...	-1.9		
$\chi_{b_2}(1P)(L_r = D)$	9939.3	9912.2	-5.2	-7.5	-7.5	-13.1	-1.0	-1.4	-1.4	-2.5	-52.8	9886.5
$\chi_{b_2}(2P)(L_r = S)$	10222.9	10268.6	...	...	...	-20.2	...	...	...	-2.8		
$\chi_{b_2}(2P)(L_r = D)$	10222.9	10268.6	-9.5	-13.5	-13.5	-22.9	-1.5	-2.1	-2.1	-3.7	-91.8	10131.1
$\chi_{b_2}(3P)(L_r = S)$	10498.9	10524.0	...	...	...	-39.0	...	...	...	-4.3		
$\chi_{b_2}(3P)(L_r = D)$	10498.9	10524.0	-14.2	-16.8	-16.8	-25.1	-1.1	-1.5	-1.5	-2.6	-122.9	10376.0
$h_b(1P)(L_r = S)$	9932.8	9899.3	...	-2.8	-2.8	-5.2	...	-0.5	-0.5	-0.9		
$h_b(1P)(L_r = D)$	9932.8	9899.3	...	-8.2	-8.2	-15.4	...	-1.6	-1.6	-3.1	-50.8	9882.0
$h_b(2P)(L_r = S)$	10216.4	10259.8	...	-5.1	-5.1	-9.3	...	-0.7	-0.7	-1.4		
$h_b(2P)(L_r = D)$	10216.4	10259.8	...	-14.3	-14.3	-26.2	...	-2.3	-2.3	-4.5	-86.2	10130.2

the bottomonium family, $\Upsilon(5S)$.

For the $\chi_{b_0}(2P)$, $\chi_{b_1}(2P, 3P)$, $\chi_{b_2}(2P, 3P)$ and $h_b(2P)$ states, their theoretical bare masses are close to the experimental values, lying roughly 20∼40 below experiment values. When coupled-channel effects are included, negative mass shifts are obtained. As a result, the unquenched masses are even lower than the experimental data. The main reason is that the mass splitting induced by spin-orbit coupling is insufficient, which can be improved by increasing α_s. Accordingly, these states can be well described within the conventional quark model and classified as members of the bottomonium family. For lower energy states such as $\Upsilon(2S)$, $\Upsilon(3S)$ and $\Upsilon(4S)$ states, their theoretical bare masses also show good agreement with experimental values in the conventional quark model, so they can still be assigned to the $b\bar{b}$ bottomonium family.

For $\eta_b(1S)$, $\Upsilon(1S)$ and $\Upsilon(6S)$ three states, the bare masses are all higher than the experimental values, about 160∼190 MeV. After incorporating coupled-channel effects, the unquenched masses are still approximately 100∼160 MeV above the experimental values, closer to the experiment values. In future work, we will attempt to adjust the model parameters, with the expectation that a single set of parameters can reproduce the spectra of light-meson, charmonium, and bottomonium systems simultaneously.

IV. SUMMARY

We perform coupled-channel calculations for a series of bottomonium valence states (including $\eta_b(nS)(n=1,2)$, $\Upsilon(nS)(n=1\sim 6)$, $\chi_{bJ}(nP)(J=0,1,2; n=1,2,3)$, $h_b(nP)(n=1,2)$) by introducing four-quark components into the quark model. The matrix elements between $b\bar{b}$ mesons and four-quark components are computed via the $^3P_0$ quark-pair creation model. The original $^3P_0$ transition operator yields large negative mass shifts of several hundred MeV, which poses difficulties for the valence quark model. We therefore adopt an improved transition operator from our earlier work, which introduces a suppression factor $e^{-\frac{R_{AV}^2}{R_0^2}}$ to account for the reduced probability of quark-pair creation at large distances from the source hadron, and keep the previously determined parameter set unchanged.

In our calculations, we respect and follow isospin symmetry and total angular-momentum conservation to select allowed two-quark and four-quark coupled channels. We consider both S-wave and D-wave angular momentum between two clusters for P-wave bottomonium states(χ_b and h_b), while only the P-wave ($L_r=1$) is included for S-wave bottomonia (η and Υ) due to the heavy computational cost of the F-wave. Only meson-meson configurations are preliminarily considered in this unquenched quark model study. Bare masses are obtained within the quenched $b\bar{b}$-only quark model, and unquenched masses are the bare masses plus the total mass shifts from all coupled channels. With the modified $^3P_0$ operator, the large mass shifts are reduced largely; the resulting unquenching corrections are moderate, since the strong decay remain consistent with those from the original operator.

Many of the investigated states can be interpreted as conventional bottomonium. The $\Upsilon(10860)$ state (connected to our $\Upsilon(5S)$) shows weak coupling to tetraquark components, supporting its assignment as a predominantly $b\bar{b}$ bottomonium state. For $\chi_{bJ}(2P,3P)$ and $h_b(2P)$, bare masses already lie 20$\sim$40 MeV below experiment; negative coupled-channel mass shifts push the unquenched masses even lower, yet these states are still well described by the conventional quark model. Low-lying states such as $\Upsilon(2S,3S,4S)$, their quenched masses also match experiment values very well in the quenched quark model. So these states can be classified into the $b\bar{b}$ bottomonium family. Otherwise, for $\chi_{bJ}(1P)$ states, the unquenched masses are well consistent with the experimental values, which indicates that these states are mixtures of two-quark and four-quark components, where the four-quark contribution cannot be neglected.

For some other states including $\eta_b(1S)$, $\Upsilon(1S)$, and $\Upsilon(6S)$, the unquenched masses remain 100$\sim$160 MeV above experiment values even after including coupled-channel corrections. They cannot be described very well neither in the quenched nor unquenched quark model. Therefore, it is very necessary to adjust the model parameters. In our previous work, we calculated the coupled-channel effects for light meson systems ($n\bar{n}$) and charmonium systems ($c\bar{c}$) within the unquenched quark model. Following the present work, we have completed calculations for all light and heavy mesons using the unquenched quark model. In the future studies, we will take the mass spectra of all well-established light and heavy mesons as inputs, and adjust the model parameters within the unquenched quark model. Our goal is to obtain a set of model parameters that can describe and explain both light-heavy mesons and some newly observed poorly understood hadrons in experiments.

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V. DATA AVAILABILITY STATEMENT

We do not analyse or generate any datasets, because our work proceeds within a theoretical and mathematical approach.

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