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A Nonequilibrium Kinetic Theory of Macroeconomics

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A Nonequilibrium Kinetic Theory of Macroeconomics

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Abstract

Modern macroeconomics struggles to bridge micro-level behavior with aggregate outcomes, while traditional exchange models often treat transactions as zero-sum particle collisions. Real economic life is non-conservative: voluntary trade generates net value, accumulated assets continuously depreciate, and decentralized exchange halts without liquid cash. We present a kinetic framework modeling the macroeconomy as an out-of-equilibrium flow system governed by three primitives: surplus creation, hard liquidity constraints, and continuous wealth decay. Rather than imposing top-down equilibrium or a representative agent, aggregate observables (including total wealth, velocity, and inflation) emerge as statistical moments of an evolving distribution. We show that macroeconomic stability is a dynamic, stationary state where surplus generation balances continuous decay. When exchange rules heavily favor asset-rich agents, circulating cash pools at the top, triggering an endogenous coordination failure: ordinary transactors run out of liquidity, trade freezes, monetary velocity collapses, and the economy stagnates. Finally, we demonstrate how fiscal and structural policies can be modeled as direct modifications of microscopic interaction rules to restore systemic circulation.

Keywords: Agent-based model, kinetic theory, Fokker–Planck, non-equilibrium systems, wealth distribution

1 Introduction

Economics traditionally divides into two largely disconnected domains: microeconomics, which models individual decision-making, and macroeconomics, which describes aggregate outcomes. The bridge between the two remains structurally weak.[1] While agent-based computational economics has demonstrated how aggregate regularities emerge from decentralized interactions (Tesfatsion 2006; Delli Gatti et al.

[1] The theoretical limitations of constructing macro-aggregates from representative-agent microfoundations are well documented; see, e.g., the Sonnenschein–Mantel–Debreu theorem (Sonnenschein 1972; Mantel 1974; Debreu 1974) and Kirman (1992).

2010), an analytically tractable bridge between microscopic transaction rules and macroeconomic distribution dynamics remains an open challenge.

Yet at its origin, economics, following Smith (1776), is fundamentally about trade: *mutual surplus generated through voluntary exchange*. Trade, however, is not symmetric. For a trade to occur, one must sell and another must buy. This asymmetry demands heterogeneity in agent needs and endowments, making analytic modeling intractable under standard equilibrium frameworks.

The introduction of money further complicates this landscape. By decoupling barter into two separate transactions, selling for money and buying with money, it introduces temporal and directional asymmetry. Though mutual surplus remains the driving force, monetary exchange creates liquidity constraints and asymmetric exposures, especially when money itself accumulates, decays, or flows unevenly.

In standard kinetic exchange models in econophysics (Drăgulescu and Yakovenko 2000; Chakraborti and Chakrabarti 2000), monetary transactions are traditionally formalized as conservative energy-exchanging collisions, analogous to elastic particle scattering in closed Hamiltonian systems. While this paradigm successfully recovers power-law and exponential tails, it treats economic interaction as a zero-sum reallocation of a fixed stock. Real economic metabolism, however, departs from strict conservation in two fundamental ways: (i) voluntary exchange is non-conservative, generating net economic surplus ($\Delta S > 0$) shared between transacting parties (Smith 1776; Samuelson 1939), and (ii) accumulated wealth represents retained surplus that is continuously subject to physical obsolescence and entropic depreciation (Hotelling 1925; Georgescu-Roegen 1971). Furthermore, decentralized exchange requires non-zero liquid solvency; agents face operational cash-in-advance constraints (Clower 1967; Kiyotaki and Wright 1989) below which transactions endogenously freeze regardless of aggregate endowments.

This paper presents a non-conservative kinetic framework for macroeconomics that captures these asymmetric micro-level interactions and derives their macroeconomic consequences. This extends the foundational tradition of kinetic wealth and income modeling (Bouchaud and Mézard 2000; Yakovenko and Rosser 2009) by coupling monetary liquidity constraints directly to production and value metabolism. Using a modified Boltzmann collision equation, we model the evolution of agent distributions over wealth and money. We then derive aggregate quantities such as total wealth, money supply, surplus production, decay, and inflation dynamics—grounding macroeconomic variables in microstructural interaction flows.

This paper bridges micro-level transactions and macro-level dynamics from the bottom up, avoiding representative-agent assumptions (Delli Gatti et al. 2010). Instead of imposing top-down equilibrium, we derive macroeconomic indicators—such as total wealth, surplus flows, and inflation—as statistical moments of an evolving agent distribution.

By endogenizing surplus creation, liquidity constraints, and wealth decay, we show that macro phenomena like laminar attractors and structural collapse emerge endogenously. Furthermore, the framework functions as a policy sandbox, allowing institutional interventions to be modeled directly as structural modifications to the underlying interaction rules rather than arbitrary shifts in static equilibrium.

This approach provides a generative, nonequilibrium foundation for macroeconomics rooted in the asymmetric metabolism of trade.

2 Agent State Space

To model economic interactions analogously to physical collisions, we begin by thinking of each agent's state as a point in a minimal two-dimensional space:

$$\boldsymbol{x}_i(t) = \begin{pmatrix} w_i(t) \\ m_i(t) \end{pmatrix},$$

where $w_i(t)$ denotes the agent's real wealth—interpreted as the utility-equivalent value of goods, services, and productive assets—and $m_i(t)$ is the amount of liquid money held. These two quantities together capture an agent's ability to engage in trade (via money) and to benefit from trade (via changes in real wealth).

This construction intentionally abstracts away from preferences, utility functions, and production functions, in order to formulate a physically tractable kinetic model. Real wealth increases through surplus-generating trades and decreases via decay, while money serves as a medium of exchange subject to liquidity constraints. The state space $\mathbb{R}^2_{\geq 0}$ thus defines a minimal but sufficient foundation for agent-level dynamics in a monetary exchange economy.

Formally, we define the state of an individual agent i at time t as:

$$\boldsymbol{x}_i(t) = \begin{pmatrix} w_i(t) \\ m_i(t) \end{pmatrix} \in \mathbb{R}^2_{\geq 0},$$

where:

- $w_i(t) \geq 0$: real wealth (utility-equivalent value of assets, goods, and productive capacity),
- $m_i(t) \geq 0$: money held (liquid funds available for trade).

The macrostate of the economy is described by the agent distribution function $f(w, m, t)$, where $f(w, m, t)\, dw\, dm$ represents the number of agents with real wealth in the range $[w, w + dw]$ and money holdings in $[m, m + dm]$ at time t. This distribution evolves over time as agents interact, trade, and experience gains or losses in both wealth and money.

3 Microscopic Interaction (Collision) Kernel

3.1 Model Assumptions

This interaction model rests on several key assumptions:

- **Heterogeneous Agents:** Agents differ in their wealth w and money m, allowing for trade to generate surplus. Without heterogeneity, no gain from trade would be possible.

- **Surplus-Generating Exchange:** Trade produces real surplus $\phi > 0$, which is split between the two parties. This reflects the classical economic view that voluntary trade increases overall value (Smith 1776; Samuelson 1939).
- **Price is Emergent:** The function $p(\boldsymbol{x}_i, \boldsymbol{x}_j)$ determines the price of the transaction, possibly based on bargaining power, liquidity, or matching quality. Prices are not fixed externally, but arise from the agent states.
- **Money is Conserved:** During trade, money flows from buyer to seller. No new money is created or destroyed in the transaction itself. This is a deliberate modeling assumption: while real-world money supply evolves through institutional mechanisms (e.g., central bank operations or credit creation), the model isolates agent-level trade to later explore how such systemic changes emerge from or interact with micro-level dynamics.
- **Money is the Structural Enabler:** In this framework, money is not just a record or token—it is the conserved flow variable that gates trade, enables surplus, and regulates systemic throughput (Kiyotaki and Wright 1989). By treating money as the critical state variable, the model allows micro-level liquidity constraints to translate into macro-level economic dynamics, such as inflation, decay, and efficiency.
- **Wealth Decays Over Time:** Wealth represents accumulated surplus. In the absence of trade, this value erodes through two mechanisms (Hotelling 1925; Georgescu-Roegen 1971):
  - *Frictional decay* — a continuous, time-dependent loss reflecting depreciation, maintenance, or entropy,
  - *Unmet-needs decay* — an accelerated loss triggered by lack of trade, representing unfulfilled consumption, obsolescence, or system detachment.

  This erosion is modeled via a nonlinear decay function $\delta(w) = cw^\gamma$, where c and γ govern the rate and curvature of decay.
- **Liquidity is Required:** Trade cannot occur unless the buyer has enough money to pay the price. This introduces a hard liquidity constraint (Clower 1967) and prevents surplus from being generated without monetary exchange.
- **Trade is Pairwise and Instantaneous:** All interactions are modeled as binary collisions between agents, occurring over infinitesimal time intervals.

Together, these assumptions create an asymmetric but generative system: trade redistributes money, increases value, and sustains wealth in the face of decay. Without money flow, surplus cannot be realized. Without surplus, decay dominates. This captures the metabolic structure of a living economy.

3.2 State Update Rule

Let agents i and j have pre-trade states $\boldsymbol{x}_i = (w_i, m_i)$ and $\boldsymbol{x}_j = (w_j, m_j)$. Agent i is the buyer and agent j the seller.

The post-trade state is given by:

$$w_i' = w_i + \phi_i^{\text{real}} - \delta(w_i),$$
$$w_j' = w_j + \phi_j^{\text{real}} - \delta(w_j),$$

$$m_i' = m_i - p(\boldsymbol{x}_i, \boldsymbol{x}_j) \cdot \phi(\boldsymbol{x}_i, \boldsymbol{x}_j),$$
$$m_j' = m_j + p(\boldsymbol{x}_i, \boldsymbol{x}_j) \cdot \phi(\boldsymbol{x}_i, \boldsymbol{x}_j),$$

with

$$\phi_i^{\text{real}} + \phi_j^{\text{real}} = \phi(\boldsymbol{x}_i, \boldsymbol{x}_j).$$

3.2.1 Collision Admissibility

A trade occurs only if two conditions are satisfied:

- **Positive surplus condition:** both agents must realize positive surplus,

$$\phi(\boldsymbol{x}_i, \boldsymbol{x}_j) > 0,$$

- **Liquidity constraint:** the buyer must be able to afford the transaction,

$$m_i \geq p(\boldsymbol{x}_i, \boldsymbol{x}_j)\, \phi(\boldsymbol{x}_i, \boldsymbol{x}_j).$$

These conditions define the admissible set of interactions and ensure consistency with voluntary exchange under liquidity constraints.

3.3 Primitive Functions

Every trade in the system is governed by three primitive functions:

- **Surplus function** $\phi(\boldsymbol{x}_i, \boldsymbol{x}_j)$,
- **Price function** $p(\boldsymbol{x}_i, \boldsymbol{x}_j)$,
- **Decay function** $\delta(w) = cw^{\gamma}$.

These functions do not represent preferences or utilities. Instead, they encode the structural logic of interaction: how value is created, how money flows, and how stored value erodes. They are primitives of the kinetic process, not derived from optimization or equilibrium.

3.3.1 Surplus Function

The surplus function quantifies the real (non-monetary) value created by a trade between two agents with states $\boldsymbol{x}_i = (w_i, m_i)$ and $\boldsymbol{x}_j = (w_j, m_j)$. It formalizes the classical economic idea that voluntary exchange generates mutual gain due to heterogeneity in needs, timing, liquidity, or resources.

Properties of the surplus function.

- $\phi(\boldsymbol{x}_i, \boldsymbol{x}_j) > 0$ whenever trade occurs,
- Surplus is created in the real domain and accumulates in w,
- Surplus is generally asymmetric: $\phi_i^{\text{real}} \neq \phi_j^{\text{real}}$,
- Surplus depends on agent states rather than explicit utility functions.

Surplus captures the generative asymmetry of the system: trade increases total real wealth.

3.3.2 Price Function

The price function determines the exchange rate between money and surplus during trade. It sets how much money the buyer must pay per unit of surplus gained.

Price is an emergent property of interaction and may reflect liquidity constraints, urgency, bargaining power, or structural asymmetries between agents. It gates the flow of money and constrains the realization of surplus.

Role of price.

The price function does not determine the amount of surplus created. It governs only the monetary transfer required to realize that surplus. Real value is generated by $\phi(\boldsymbol{x}_i, \boldsymbol{x}_j)$, while $p(\boldsymbol{x}_i, \boldsymbol{x}_j)$ regulates the liquidity required for its realization.

3.3.3 Decay Function

The decay function captures the erosion of wealth over time:

$$\delta(w) = cw^{\gamma}.$$

It represents depreciation, consumption, and maintenance costs. Decay introduces dissipation into the system, ensuring that wealth must be continually regenerated through trade. Without decay, agents could accumulate wealth passively; with decay, ongoing interaction becomes necessary.

Together, these functions define the microstructure of value flow:

- Surplus creates value,
- Price channels liquidity,
- Decay dissipates stored value.

3.4 Liquidity Constraint

A trade occurs only if the buyer has sufficient liquidity:

$$m_i \geq p(\boldsymbol{x}_i, \boldsymbol{x}_j) \cdot \phi(\boldsymbol{x}_i, \boldsymbol{x}_j).$$

This condition enforces a hard constraint on interaction: surplus cannot be realized without monetary flow. Money therefore functions as the structural enabler of economic activity.

4 From Boltzmann Kinetics to Economic Exchange

The Boltzmann equation (Boltzmann 1872) describes how the distribution of particles evolves over time through pairwise collisions. Each collision conserves mass, momentum, and energy. In the present setting, we borrow this structure, but modify the dynamics to match economic behavior.

Here, agents collide not in space, but in trade. A "collision" is a transaction between a buyer and a seller. Money moves, and real wealth changes. The kernel $K(\boldsymbol{x}, \boldsymbol{x}^*)$

plays the role of a matching rate: it determines how likely two agents are to interact, given their states.

The core difference from physical systems is that conservation laws are altered. In physics, energy is conserved. In this framework, real value can be created through trade, while wealth simultaneously decays over time. The system is therefore intrinsically non-conservative and nonequilibrium.

4.1 Matching Kernel

The function $K(\boldsymbol{x}, \boldsymbol{x}^*)$ represents the *matching rate kernel*, defining the frequency at which agents encounter potential trading partners in a decentralized market.

Unlike Walrasian frameworks that assume a centralized clearinghouse, decentralized exchange requires physical or institutional contact. In alignment with standard monetary search theory, we treat the baseline contact rate as a function of active market participation, where trade opportunities arise as agents circulate through the economic environment. To maintain analytical parsimony and avoid unobservable micro-behavioral assumptions regarding hidden complementarities or network frictions, the foundational baseline adopts a uniform contact frequency:

$$K(\boldsymbol{x}, \boldsymbol{x}^*) = K_0 > 0,$$

where K_0 represents the baseline aggregate meeting rate. This ensures that any emergent macro-level structure or phase transition is driven strictly by the observable mechanics of surplus generation, monetary liquidity gating, and wealth decay, rather than by arbitrary or unmeasurable structural assumptions.

4.2 Collision Operator

Economic interaction is modeled as a pairwise transaction between agents. The collision operator $C[f](\boldsymbol{x}, t)$ captures the net effect of these interactions on the agent distribution $f(w, m, t)$. It consists of a loss term and a gain term.

Crucially, while the joint distribution $f(w, m, t)$ tracks agents across both coordinates simultaneously, money and wealth do not operate under the same conservation rules. Money acts as a conserved medium of exchange locally transferred during trade, whereas wealth is non-conservative, driven by active surplus generation and independent decay.

4.2.1 Loss Term

$$C_{\text{loss}}[f](\boldsymbol{x}, t) = f(\boldsymbol{x}, t) \int K(\boldsymbol{x}, \boldsymbol{x}^*)\, f(\boldsymbol{x}^*, t)\, d\boldsymbol{x}^*$$

This term measures the rate at which agents leave state $\boldsymbol{x} = (w, m)$ to engage in trade. It depends on the local density of agents, the distribution of potential trading partners, and the decentralized matching kernel.

4.2.2 Gain Term

$$C_{\text{gain}}[f](\boldsymbol{x},t) = \iint \left[\delta(\boldsymbol{x}-\boldsymbol{x}'_{\text{post,B}}) + \delta(\boldsymbol{x}-\boldsymbol{x}'_{\text{post,S}})\right] K(\boldsymbol{x}',\boldsymbol{x}'^{*}) f(\boldsymbol{x}',t) f(\boldsymbol{x}'^{*},t)\, d\boldsymbol{x}' d\boldsymbol{x}'^{*}$$

This term accounts for agents entering state $\boldsymbol{x}$ as a result of trade. The integration spans all pre-trade states $(\boldsymbol{x}', \boldsymbol{x}'^{*})$ generating post-trade states equal to $\boldsymbol{x}$, where $\boldsymbol{x}'$ and $\boldsymbol{x}'^{*}$ represent the buyer and seller pre-trade states, and $\boldsymbol{x}'_{\text{post,B}}$ and $\boldsymbol{x}'_{\text{post,S}}$ denote their respective post-trade configurations under the update rules. The Dirac delta functions ensure that only transitions resulting precisely in state $\boldsymbol{x}$ contribute to the gain current.

4.2.3 Full Collision Operator

$$C[f](\boldsymbol{x},t) = C_{\text{gain}}[f](\boldsymbol{x},t) - C_{\text{loss}}[f](\boldsymbol{x},t)$$

This operator describes how decentralized transactions redistribute the population across the state space. Because the update rules enforce local money conservation alongside non-conservative real wealth generation, the collision operator serves as the microscopic mechanism translating asymmetric trade into macroeconomic structure, avoiding the false equivalence of treating wealth and money as interchangeable conserved quantities.

5 Generalized Kinetic Equation

The evolution of the agent distribution $f(\boldsymbol{x},t)$ over time is governed by a generalized kinetic equation of the form:

$$\frac{\partial f(\boldsymbol{x},t)}{\partial t} = C[f](\boldsymbol{x},t) - D[f](\boldsymbol{x},t).$$

This formulation follows the tradition of continuous master-equation dynamics for heterogeneous economic distributions (Aoyama et al. 2010), modeling how agents move through the state space $\boldsymbol{x} = (w,m)$ through decentralized interactions rather than physical motion. The right-hand side consists of two operators: the collision operator $C[f]$, representing surplus-generating pairwise trade, and the decay operator $D[f]$, representing the continuous erosion of wealth.

5.1 Interpretation

This equation defines a non-equilibrium flow system. The distribution $f(\boldsymbol{x},t)$ is not assumed to converge to a fixed point unless structural balance conditions are satisfied. Instead, the system evolves through the interaction of value creation and value dissipation.

Unlike classical kinetic theory, real value or wealth is created through trade, value is destroyed through decay, and money is conserved locally while being redistributed globally. Each point $\boldsymbol{x} = (w,m)$ represents a class of agents, and the distribution $f(\boldsymbol{x},t)$ describes how population mass flows through this space. Specifically, trade shifts agents diagonally through changes in both wealth and money, decay shifts agents

toward lower wealth levels, and the resulting distribution encodes macroeconomic outcomes such as inequality and structural stagnation.

5.2 Decay Operator

The decay operator captures the continuous erosion of wealth:

$$D[f](\boldsymbol{x}, t) = \frac{\partial}{\partial w} \left[\delta(w) f(w, m, t)\right].$$

This term represents a flux along the wealth dimension, where the function $\delta(w) = cw^{\gamma}$ defines the decay rate. Multiplying by the density $f(w, m, t)$ yields the total decay flux at each state, and the derivative with respect to wealth measures how this flux varies across different asset levels.

Decay is a continuous and deterministic force acting on all agents, representing asset depreciation, consumption, and maintenance costs where larger wealth stocks incur higher upkeep expenses. In the absence of trade, the distribution would drift toward lower wealth levels, meaning decay acts as the entropic counterforce to surplus generation.

At the lower boundary where $w = 0$, agents reach zero wealth. Depending on the model specification, this boundary may be absorbing, where agents remain at zero wealth, reflecting or redistributive, where agents re-enter the system, or mass-removing, where agents exit entirely. These choices influence long-run distributional outcomes and system stability.

5.3 Continuous and Transactional Decay

The model incorporates two distinct decay mechanisms operating simultaneously: continuous decay, represented by the operator $D[f]$, and transactional decay, applied at the moment of interaction within the state update rule such that $w_i' = w_i + \phi_i^{\text{real}} - \delta(w_i)$. Continuous decay governs baseline dissipation, while transactional decay captures costs associated with economic transformation and interaction, defining a system where both inactivity and activity carry costs.

5.4 Modeling Limitations

The framework is intentionally minimalist, designed not to reproduce granular institutional details, but to isolate the structural interaction between surplus generation, liquidity constraints, and decay.

The surplus function $\phi(x_i, x_j)$ is inspired by classical notions of mutual gain from trade, but it represents state-dependent realized gains rather than an objectively measurable value. Surplus arises from heterogeneous valuations implicitly encoded in agent states, and trade occurs strictly when surplus is positive. The model abstracts from explicit preference formation, bargaining processes, and informational asymmetries, meaning that wealth w represents accumulated realized surplus rather than a universal value metric.

Similarly, trades occur only when both agents realize positive surplus under a memoryless, state-dependent matching kernel $K(x, x^*)$. The model abstracts from complex network structures, repeated interactions, or institutional persistence, aligning the framework with mean-field kinetic theory. The decay function $\delta(w) = cw^\gamma$ is a stylized representation of depreciation and maintenance, chosen because it captures dissipative dynamics while preserving analytical tractability.

By design, the framework abstracts away from secondary asset markets and speculation, focusing strictly on primary surplus-generating production where surplus is greater than zero. Purely zero-sum asset reallocations and speculative price dynamics are excluded to isolate baseline metabolic flow. Furthermore, the model omits strategic optimization, endogenous institutional price formation, and credit creation within the collision operator.

These omissions are deliberate. The objective is to define a minimal generative structure capable of producing macroeconomic flow phenomena within a non-conservative kinetic framework. By explicitly bounding the scope of this approximation, we insulate the core framework from the fundamental attribution problem that routinely penalizes highly dense, hyper-parameterized macroeconomic forecasting engines. In highly non-linear economic networks, attempting to eliminate structural discrepancies by exogenously appending high-dimensional parameters renders systematic model errors fundamentally intractable to isolate. Because predictive errors in multi-scale open systems expand exponentially over time, establishing an uncompromised, minimal causal anatomy must structurally precede data calibration. Consequently, this work operates as a disciplined, falsifiable foundational proof-of-concept, preserving analytical parsimony so that subsequent empirical specifications can map causal feedback loops with mathematical clarity.

6 Aggregate Macroeconomic Variables

Macroeconomic quantities are defined as moments of the agent distribution $f(w, m, t)$. These variables aggregate micro-level interactions into system-level observables.

6.1 Definitions

Total Wealth.

Total wealth $W(t)$ measures the accumulated surplus stored across all agents:

$$W(t) = \iint w\, f(w, m, t)\, dw\, dm.$$

Wealth represents real value stored in non-liquid form, including assets, capabilities, and productive capital. It is not tied to market prices, but to accumulated realized surplus net of decay.

Money Supply.
The total money supply $M(t)$ is defined as:

$$M(t) = \iint m\, f(w, m, t)\, dw\, dm.$$

Money represents liquid purchasing power available for trade. It is conserved at the level of individual transactions, but redistributed across agents.

Aggregate Surplus.
The total surplus generated through trade is:

$$\Phi(t) = \frac{1}{2} \iint \phi(\boldsymbol{x}, \boldsymbol{x}^*) K(\boldsymbol{x}, \boldsymbol{x}^*) f(\boldsymbol{x}, t) f(\boldsymbol{x}^*, t)\, d\boldsymbol{x}\, d\boldsymbol{x}^*.$$

The factor $\frac{1}{2}$ avoids double counting symmetric interactions. This quantity represents the rate of real value creation, analogous to real output.

Aggregate Decay.
Total decay is given by:

$$D(t) = \int \delta(w) \left(\int f(w, m, t)\, dm \right) dw.$$

This measures the total loss of stored value due to dissipation.

6.2 Macroscopic Conservation Laws

The time evolution of total wealth follows directly from the kinetic equation:

$$\frac{\partial f}{\partial t} = C[f] - D[f].$$

Differentiating $W(t)$:

$$\frac{dW}{dt} = \iint w \frac{\partial f}{\partial t}\, dw\, dm = \iint w \left(C[f] - D[f] \right)\, dw\, dm.$$

6.2.1 Decay Contribution

Using the decay operator

$$D[f](w, m, t) = \frac{\partial}{\partial w} \left[\delta(w) f(w, m, t) \right],$$

we obtain

$$\iint w D[f]\, dw\, dm = \iint w \frac{\partial}{\partial w} \left[\delta(w) f(w, m, t) \right]\, dw\, dm.$$

Applying integration by parts in w gives:

$$= -\iint \delta(w) f(w, m, t)\, dw\, dm = -D(t).$$

6.2.2 Collision Contribution

The contribution from the collision operator equals total surplus generation:

$$\iint wC[f]\, dw\, dm = \Phi(t).$$

6.2.3 Wealth Evolution

Combining terms:

$$\frac{dW}{dt} = \Phi(t) - D(t).$$

Thus, total wealth evolves as the difference between surplus creation and decay.

6.2.4 Money Conservation

For total money:

$$M(t) = \iint mf(w, m, t)\, dw\, dm,$$

we obtain:

$$\frac{dM}{dt} = \iint mC[f]\, dw\, dm = 0.$$

Money is conserved under the collision operator, as it is only redistributed between agents.

6.3 Additional Observables: Participation as Net Surplus Flow

Traditional macroeconomic analysis classifies agents by employment status. In this framework, economic viability is instead determined by whether agents generate sufficient surplus to offset decay.

6.3.1 Net Surplus Participation Rate

Define the expected surplus generation rate $\mathbb{E}[\phi(x, t)]$. The Net Surplus Participation Rate (NSPR) is:

$$\text{NSPR}(t) = \frac{1}{N} \int_{\mathbb{R}^2_{\geq 0}} \mathbf{1}\left[\mathbb{E}[\phi(x, t)] - \delta(w) > 0\right] f(w, m, t)\, dw\, dm,$$

where

$$N = \int f(w, m, t)\, dw\, dm.$$

This measures the fraction of agents whose expected surplus exceeds decay.

6.3.2 Surplus Flow Index

A continuous analogue is the Surplus Flow Index:

$$\mathrm{SFI}(t) = \frac{1}{N} \int_{\mathbb{R}^2_{\geq 0}} \left(\mathbb{E}[\phi(x,t)] - \delta(w)\right) f(w,m,t)\, dw\, dm.$$

Positive values indicate surplus-dominated regimes, while negative values indicate decay-dominated states.

6.3.3 Interpretation

These observables capture participation in economic flow rather than categorical employment. High NSPR corresponds to sustained economic activity, while low values indicate widespread decay dominance and systemic contraction.

7 Monetization, Velocity, and Inflation Dynamics

The previous section established that, under the collision operator alone, total money is conserved:

$$\frac{dM}{dt} = 0.$$

In real economies, however, money supply evolves through institutional mechanisms such as credit creation, central bank operations, and financial intermediation. To incorporate this, we extend the framework to an open system in which money is created in relation to real economic activity.

7.1 Monetization of Surplus

We define the rate of money creation as proportional to real surplus generation:

$$\frac{dM(t)}{dt} = \theta(t)\, \Phi(t),$$

where $\theta(t)$ is the *monetization rate*.

This parameter governs how much new money enters the system per unit of real value created. It may reflect institutional credit creation, monetary policy, or financial innovation.

This formulation decouples inflation from monetary growth alone: money can expand without inflation if it grows in proportion to real value creation.

To clarify the structural interface between our scales, we emphasize that the aggregate monetization parameter $\theta(t)$ functions strictly as a mean-field scaling operator on the systemic distribution, rather than an arbitrary micro-injection mechanism. At the microscopic level, money functions exclusively as a localized trade enabler and an immediate velocity-gating constraint, meaning that individual agent interactions strictly conserve currency localized within the collision operator. At the macroscopic scale, however, money operates fundamentally as a decentralized ledger tracking

aggregate systemic obligations. In a real-world economy, this ledger is continuously modified by institutional mechanisms such as banking-sector credit expansion and fractional-reserve deposit multiplication, dynamics that require a distinct state space of non-trading agents with custom balance-sheet drift operators to be fully modeled from first principles. By restricting the scope of this baseline architecture to a clean, unperturbed sandbox model of pure exchange, we model $\theta(t)$ as a uniform spatial transformation that preserves local exchange rules while continuously shifting the mean-field monetization boundaries. This clear scale separation provides an honest, baseline sandbox model of trade mechanics, leaving the microfoundations of institutional credit cycles to be grafted endogenously within a dedicated follow-up framework.

7.2 Inflation as Misalignment

Inflation is defined as the time derivative of the logarithmic ratio between money supply and real surplus:

$$\pi(t) = \frac{\mathrm{d}}{\mathrm{d}t} \ln\left(\frac{M(t)}{\Phi(t)}\right) = \frac{\dot{M}(t)}{M(t)} - \frac{\dot{\Phi}(t)}{\Phi(t)}.$$

Price stability requires:

$$\pi(t) = 0 \quad \Longleftrightarrow \quad \frac{\dot{M}}{M} = \frac{\dot{\Phi}}{\Phi}.$$

Thus, inflation arises when money grows faster than real value, while deflation arises when real value grows faster than money.

7.3 Velocity of Money

Define the velocity of money as:

$$V(t) = \frac{\Phi(t)}{M(t)}.$$

Substituting into the inflation expression yields:

$$\pi(t) = -\frac{\dot{V}(t)}{V(t)}.$$

Inflation therefore corresponds to declining velocity, while increasing velocity corresponds to deflation.

In this framework, velocity is not an exogenous constant. It emerges from the interaction between liquidity constraints, trade conditions, and the structure of the agent distribution.

Stock–flow relation.

While the flow ratio $M(t)/\Phi(t) = 1/V(t)$ links money to real economic throughput, the ratio $W(t)/M(t)$ provides a complementary stock-based measure relating accumulated real value to circulating liquidity. This distinction provides an analytical, kinetic counterpart to computational stock-flow consistent agent-based frameworks (Caiani et al. 2016), separating the flow dynamics that gate transactions from the valuation structure of accumulated capital.

7.4 Interpretation

This formulation reframes inflation as a relational imbalance between money and real value:

- Inflation is not driven by money creation alone, but by its growth relative to surplus,
- Velocity reflects how effectively liquidity is matched to surplus-generating opportunities,
- Declining velocity can produce inflation even without expansion of the money supply,
- Increasing velocity can stabilize or reduce prices even under monetary growth.

Macroeconomic outcomes thus depend on the alignment between liquidity and value creation, rather than on monetary aggregates in isolation.

8 Stability, Cycles, and Laminar Flow Attractors

A laminar flow attractor is defined as a stationary solution of the kinetic equation:

$$\frac{\partial f_{st}(\boldsymbol{x})}{\partial t} = 0 \quad \Longrightarrow \quad C[f_{st}] = D[f_{st}].$$

In this regime, surplus generation through trade exactly offsets wealth decay. The system does not freeze, but maintains a steady throughput of value. The distribution remains stable in shape while agents continue to move through state space.

8.1 Laminar Flow Attractor

A laminar attractor is characterized by three conditions:

Metabolic Balance.

$$\Phi(t) = D(t).$$

Total surplus generation equals total decay.

Price Stability.

$$\pi(t) = \frac{d}{dt} \ln \left(\frac{M(t)}{\Phi(t)} \right) = 0.$$

Money and surplus grow at equal rates.

Flow Efficiency.

The distribution $f_{st}(w, m)$ supports continuous trade, with liquidity circulating toward surplus-generating states and away from decay-dominated regions.

This regime represents a balanced economic metabolism, with no accumulation imbalance, no inflationary divergence, and no systemic decay collapse.

8.2 Attractor Typology

The system does not necessarily converge to a laminar attractor. Depending on the interaction between surplus generation, liquidity distribution, and decay, several dynamic regimes may arise.

8.2.1 Cyclic Attractors (Boom–Bust Dynamics)

If monetization and surplus generation become out of phase, oscillatory dynamics emerge where periods of excess liquidity drive expansion, followed by phases where decay dominates and contraction occurs. These dynamics correspond to macroeconomic cycles, with agents moving through alternating accumulation and depletion phases in state space.

8.2.2 Degenerate Attractors (Recession or Collapse)

When surplus persistently falls below decay,

$$\Phi(t) < D(t),$$

the system drifts toward low-wealth states. The distribution concentrates near $w = 0$, with reduced trade activity and low liquidity utilization, corresponding to deep economic stagnation or systemic collapse.

8.2.3 Inflationary Divergence

If the money supply grows faster than surplus,

$$M(t) \gg \Phi(t),$$

liquidity accumulates without corresponding value creation. This produces horizontal flow in the state space characterized by increases in money without increases in real wealth, driving the system into an inflationary regime lacking an intrinsic restoring force.

8.2.4 Inequality-Driven Bifurcations

If the matching kernel or feedback mechanisms favor high-wealth agents, the distribution may split into distinct regions:

$$f(w, m, t) \to f_{\text{low}} + f_{\text{high}}.$$

This generates persistent structural inequality, often manifesting as heavy-tailed or bimodal distributions.

8.3 Geometry of Economic Evolution

In this framework, macroeconomic states are not static equilibrium points, but dynamic regions of flow within the distribution space. Growth, stability, recession, and inflation correspond to distinct flow geometries determined by surplus generation, monetary flow, decay, and the underlying matching structure. The attractor is therefore not a single point, but an evolving shape toward which the distribution converges. Future analysis may characterize attractor basins, bifurcation thresholds, hysteresis effects, and policy-induced transitions, reframing macroeconomics as the study of nonequilibrium flows in a dissipative system.

9 Simulation and Results

To evaluate the structural properties and internal consistency of the proposed non-conservative kinetic framework, this section presents agent-based Monte Carlo simulations of the microscopic interaction and decay operators. By transitioning from generalized operators to a concrete, parameterized environment, we demonstrate how macro-level stability and non-equilibrium phase transitions emerge directly from the underlying microstructural rules. These numerical experiments function as illustrative realizations of the theory designed to verify the model's qualitative behavioral capacity.

Code Availability: The underlying simulation engine, parameter files, and visualization scripts developed for this framework are hosted openly on GitHub to ensure full empirical reproducibility and will be made publicly accessible upon publication (anonymized repository available to referees upon request).

9.1 Initialization and Core Engine Dynamics

To preserve analytical focus on the generative sieve of the system, the simulation is initialized under strict equality constraints: every agent begins with identical real wealth and liquid money endowments. This singular initial condition means the distribution starts as a highly spiked, near-singular delta mass:

$$f(w, m, 0) = \delta(w - w_0)\delta(m - m_0). \tag{1}$$

Consequently, any subsequent variance, tail formation, or clustering observed over time is entirely endogenous, driven exclusively by the recursive action of the kinetic operators rather than initial heterogeneity.

At each time step, pairs of agents are selected via a uniform matching kernel to engage in trade. Their states are updated according to the transactional rules under a hard liquidity constraint, while real wealth is simultaneously subjected to continuous dissipation via the decay operator. To illustrate how systemic macro-dynamics depend

on microstructural design, we contrast two distinct simulation scenarios using this core engine.

9.2 Baseline Scenario: Symmetric Surplus and the Laminar Flow Attractor

The baseline scenario implements highly symmetric parameters to establish a macroeconomic benchmark for an unperturbed economy. Specifically, real surplus creation scales cooperatively with the aggregate accumulated wealth of both interacting parties, reflecting an economy where asset accumulation naturally yields production efficiencies. This generated surplus is split completely equally between the buyer and the seller, and all agents are subjected to an identical, linear wealth decay representing standard asset depreciation.

As visualized in Figures 1 and 2, the system exhibits a qualitative convergence to a stable Laminar Flow Attractor. Because initialization begins at a single uniform point, the distribution at early timesteps manifests as a highly narrow density spike. As repeated mixing occurs, the transaction operator spreads the probability mass into a smooth, stable cloud, while the balancing force of uniform decay confines the upper tail.

As the simulation progresses, Total Wealth $W(t)$ flattens into a stable asymptote. This stationary state illustrates a balanced economic metabolism where aggregate surplus generation perfectly offsets continuous decay ($\Phi(t) = D(t)$). The Net Surplus Participation Rate (NSPR) stabilizes at a sustainable plateau, confirming that the uniform circulation of liquid money successfully sustains widespread agent participation in trade.

9.3 Contrasting Scenario: Asymmetric Surplus Capture and Stagnation

To explore the emergence of structural vulnerabilities, the contrasting scenario introduces micro-level behavioral asymmetries that alter the transaction rules. Here, bargaining power is biased toward the wealthier node, allowing the agent holding higher asset levels to capture a disproportionately larger share of the transaction surplus. Furthermore, wealthier agents experience a slower rate of nonlinear decay, capturing the economic reality where low-asset agents must spend a higher fraction of their discretionary endowments on basic structural upkeep. Finally, if an agent encounters a transaction opportunity but faces an active liquidity constraint, a wealth-for-money swap pre-step hook is triggered, forcing the illiquid agent to liquidate real assets at a steep penalty to acquire circulating cash.

The macroeconomic consequences of these asymmetric modifications are observed in Figures 3 and 4. The early distribution at Step 500 again shows a sharp peak due to identical initial endowments, but the systemic trajectory quickly diverges from the baseline. Instead of relaxing into a smooth, balanced cloud, the distribution undergoes a distinct spatial bifurcation.

Wealthier agents, insulated from rapid decay and capturing the bulk of trade surplus, rapidly drift toward higher wealth coordinates. Conversely, the bulk of the

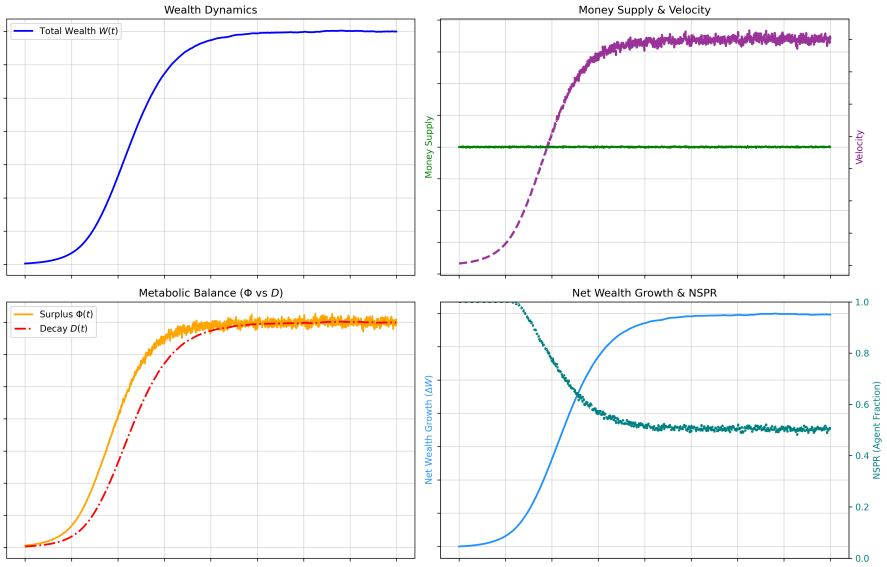


Fig. 1 Macroscopic variables for the baseline scenario. Total wealth $W(t)$ and monetary velocity $V(t)$ stabilize as aggregate surplus $\Phi(t)$ reaches metabolic balance with aggregate decay $D(t)$ (Delli Gatti et al. 2010).

population is repeatedly forced into asset liquidation via the pre-step hook, causing agent mass to pool heavily against the lower wealth boundary.

This structural bifurcation triggers a critical coordination failure (Leijonhufvud 1973). As wealth concentrates in the tail, the broader population loses the liquid money required to gate voluntary transactions under the cash-in-advance constraint. Consequently, monetary velocity $V(t)$ suffers a catastrophic decay, and the Net Surplus Participation Rate (NSPR) collapses toward zero. Even though total wealth superficially appears to expand, the economy drops into a decay-dominated, stagnant regime where the macroscopic throughput of the system is effectively paralyzed.

10 Policy Implications and Institutional Transformations

A key strength of the non-conservative kinetic framework is its ability to represent institutional and policy interventions as structural transformations of the underlying interaction rules, rather than as exogenous shifts in static equilibrium. Since all macroeconomic dynamics arise from admissible interactions, matching rates, and state transitions, any effective policy must act through one or more of the primitive components of the system: the surplus function $\phi(x_i, x_j)$, the pricing rule $p(x_i, x_j)$, the liquidity constraint $m_i \geq p(x_i, x_j)\phi(x_i, x_j)$, the matching kernel $K(x, x^*)$, or the decay operator $\delta(w)$.

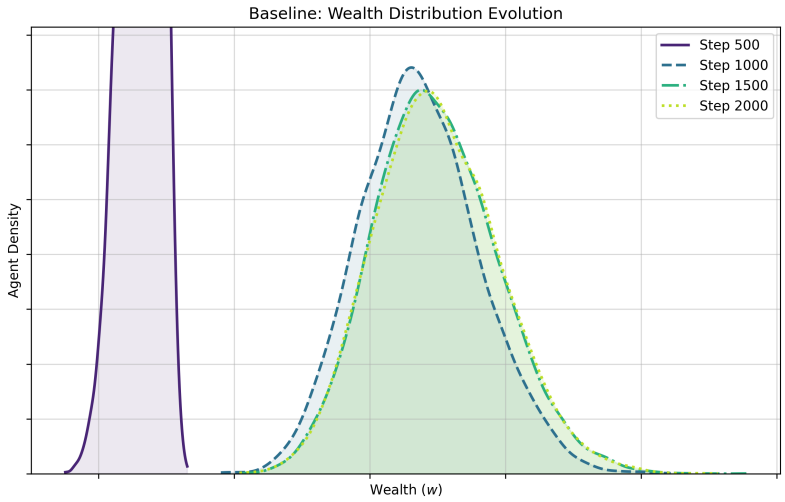


Fig. 2 Temporal evolution of the wealth distribution $f(w, m, t)$ under symmetric exchange. The highly spiked initial delta distribution relaxes over time into a stable, smooth gamma-type profile.

Under this formulation, policy interventions operate by altering the geometry of admissible interactions and the rate of realized flow $\Phi(t)$, which governs the macroscopic evolution of the system. This provides a unified mechanism to analyze diverse institutional structures within a common dynamical framework without requiring closed-form solutions for the resulting distribution. Specific policy applications include:

- **Surplus Floors and Labor Standards:** Constraints that impose minimum feasible compensation can be modeled as lower bounds on admissible surplus,

$$\phi(x_i, x_j) \geq \phi_{\min}, \tag{2}$$

  which restrict the set of realizable interactions and thereby reduce effective collision rates among disadvantaged segments.
- **Transaction Taxes and Frictions:** Transaction costs or levies can be introduced either as wedges in the transfer term of the state update rule,

$$m_i' = m_i - (1+\tau)p(x_i, x_j)\phi(x_i, x_j), \qquad m_j' = m_j + p(x_i, x_j)\phi(x_i, x_j), \tag{3}$$

  or as reductions in the effective matching kernel, $K_\tau(x, x^*) = (1 - \kappa(\tau))K(x, x^*)$, both of which act to lower realized throughput.
- **Wealth and Asset Taxation:** Stock-based taxation can be incorporated directly through the decay operator as an additional dissipation term,

$$\delta(w) = cw^\gamma + \tau_w w, \tag{4}$$

  modifying the long-run dynamics of wealth accumulation and mitigating extreme tail divergence.

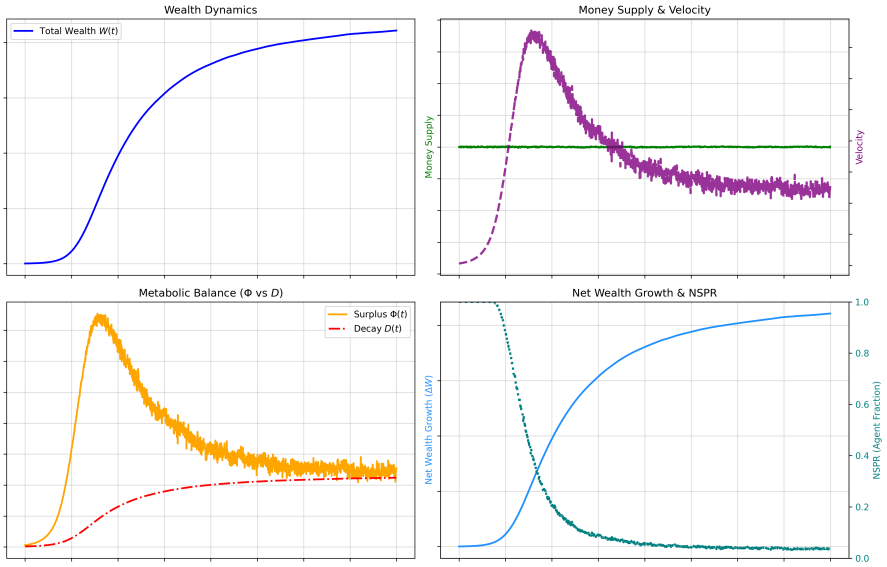


Fig. 3 Macroscopic metrics under asymmetric trading parameters. A structural drop in monetary velocity $V(t)$ and the near-zero collapse of the Net Surplus Participation Rate (NSPR) signal systemic stagnation.

By reframing policy as a structural modification of microscopic interaction rules rather than a top-down allocation shock, this framework offers policymakers a diagnostic tool for anticipating how micro-level regulations propagate into macro-level velocity changes, systemic stability, and phase transitions.

11 Conclusion

This paper presents a non-conservative kinetic framework for macroeconomics, rooted in the asymmetric microstructure of trade. Unlike traditional economic models, which impose equilibrium conditions or rely on representative-agent assumptions for analytical tractability, the present framework is constructed from three minimal primitives: surplus generation, monetary constraint, and nonlinear decay. From these, macroeconomic behaviors—such as inequality, cyclical fluctuations, and structural stagnation—emerge as properties of a nonequilibrium flow system.

The objective of this framework is not to produce point predictions, but to provide a structural description of economic dynamics. Its primary role is interpretative and diagnostic. By focusing on the interaction between generative and dissipative processes, the model characterizes how aggregate outcomes arise from the evolution of the distribution $f(w, m, t)$, rather than from equilibrium conditions.

Given a specification of the interaction functions, such as the matching kernel, surplus function, and price formation rule, the framework enables analysis of how local interactions propagate into macro-level dynamics. In this sense, the formulation is fully specified at the level of its kinetic operators, while remaining parametrically general in its functional components.

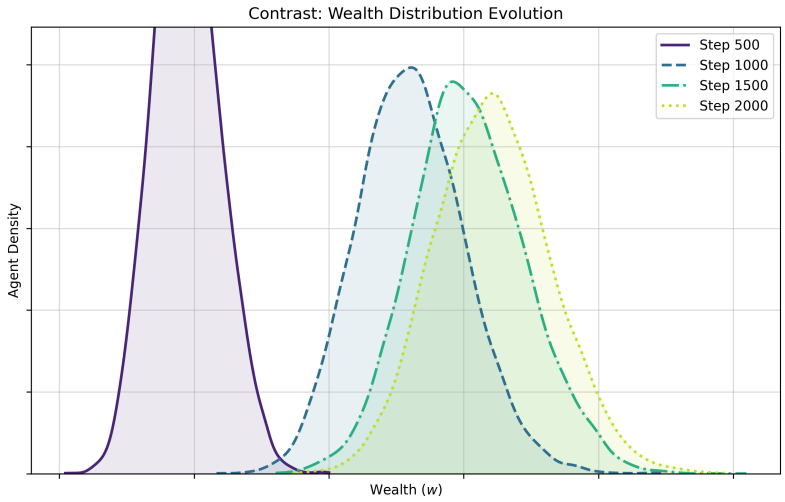


Fig. 4 Temporal evolution of wealth distribution under asymmetric exchange. The system undergoes an asymmetric phase transition, fracturing into an isolated, lower-boundary asset cluster and a high-wealth tail.

In practice, the framework supports four forms of analysis. First, it provides a basis for identifying macro-level constraints. Aggregate quantities such as total wealth, money supply, surplus production, and decay emerge directly from the distribution, enabling consistency conditions and conservation relationships to be examined at the system level. Second, it allows characterization of dynamic regimes. Different configurations of surplus generation, liquidity, and decay give rise to qualitatively distinct regimes, including stable laminar flow, inflationary divergence, and systemic collapse, defined by the balance between generative and dissipative forces. Third, it serves as a foundation for model construction. By introducing specific functional forms or agent-based rules for the interaction terms, the framework can be extended to numerical simulations or empirical calibration while preserving structural consistency. Fourth, it provides a structural sandbox for policy evaluation, enabling institutional interventions—such as surplus floors, transaction frictions, or asset taxation—to be modeled as direct modifications of microscopic interaction rules rather than top-down allocation shocks.

The framework is motivated by the hypothesis that systemic economic behavior arises from the interaction between value creation, liquidity flow, and dissipation. Within this formulation, macroeconomic outcomes such as inflationary divergence, stagnation, and inequality bifurcation emerge endogenously from the recursive dynamics of these processes.

A key implication of the framework is that institutional and policy interventions can be represented as structural transformations of the underlying interaction rules. Rather than shifting a static equilibrium, such interventions modify the admissible set of interactions and the rate of realized flow, thereby altering the macroscopic evolution of the system.

This perspective provides a unified way to analyze diverse economic mechanisms within a common dynamical structure, without requiring closed-form solutions for the resulting distributions.

Mechanistic formulations provide structural understanding that enables analysis and intervention, but do not by themselves determine precise predictions without further specification of functional forms. Accordingly, the contribution of this work is not a closed-form solution or a fully specified empirical model, but a generative platform that defines the admissible dynamics of a non-conservative economic system.

The economy is thus interpreted as a dissipative system in which value must be continuously regenerated to offset decay. Macroeconomic outcomes are not equilibrium allocations, but flow regimes arising from the interaction between surplus generation, liquidity constraints, and dissipation. This perspective provides a basis for analyzing stability, systemic risk, and structural design in economic systems beyond equilibrium-based approaches.

This model is deliberately incomplete; to evolve into a functionally minimal macroeconomic model, it requires the subsequent integration of explicit endogenous credit creation and detailed price-level dynamics. What this work establishes is a clean, parsimonious baseline: a foundational framework grounded in surplus and money that builds a rigorous micro-to-macro bridge, deriving macroeconomic behavior directly from aggregate interactions. Similarly, the simulation serves to demonstrate the mechanism rather than offer an exhaustive proof. To that end, we have made the codebase open-source and object-oriented, inviting the broader community to use this baseline as a foundation for developing increasingly realistic macroeconomic systems.

Appendix A Fokker–Planck Approximation

While aggregate quantities such as wealth, money supply, surplus, and decay can be computed directly from the distribution $f(\boldsymbol{x}, t)$, they do not describe how the distribution evolves in continuous time. For this, we employ a Fokker–Planck approximation (Fokker 1914; Planck 1917; Kolmogorov 1931).

When trades are small but frequent, the Boltzmann dynamics can be approximated by a continuous partial differential equation governing the evolution of $f(\boldsymbol{x}, t)$. This approximation captures the average motion and dispersion of agents in state space.

The Fokker–Planck equation takes the form:

$$\frac{\partial f(\boldsymbol{x}, t)}{\partial t} = -\nabla \cdot (\boldsymbol{A}(\boldsymbol{x}) f(\boldsymbol{x}, t)) + \frac{1}{2} \nabla \cdot (\boldsymbol{B}(\boldsymbol{x}) \nabla f(\boldsymbol{x}, t)) \,.$$

Here, $\boldsymbol{x} = (w, m) \in \mathbb{R}^2_{\geq 0}$ represents the agent state, $f(\boldsymbol{x}, t)$ is the agent density, $\boldsymbol{A}(\boldsymbol{x})$ is the drift vector, and $\boldsymbol{B}(\boldsymbol{x})$ denotes the diffusion tensor.

A.1 Drift Vector

The drift term represents the expected change in state:

$$\boldsymbol{A}(\boldsymbol{x}) = \mathbb{E}[\Delta \boldsymbol{x}] = \begin{pmatrix} \mathbb{E}[\phi^{\text{real}}(\boldsymbol{x})] - \delta(w) \\ \mathbb{E}[\Delta m(\boldsymbol{x})] \end{pmatrix}.$$

The first component is the expected net change in wealth, given by surplus generation minus decay. The second component is the expected change in money due to spending and earning.

A.2 Diffusion Tensor

The diffusion tensor captures variability in state transitions:

$$B_{ij}(\boldsymbol{x}) = \mathbb{E}[\Delta x_i \Delta x_j].$$

Explicitly,

$$\boldsymbol{B}(\boldsymbol{x}) = \begin{pmatrix} \mathbb{E}[(\Delta w)^2] & \mathbb{E}[\Delta w \Delta m] \\ \mathbb{E}[\Delta m \Delta w] & \mathbb{E}[(\Delta m)^2] \end{pmatrix}.$$

The diagonal terms represent variance in wealth and money changes, while off-diagonal terms capture covariance between them.

A.3 Interpretation

The Fokker–Planck equation represents the evolution of the agent distribution as a continuous flow, where the drift term describes the average direction of motion induced by surplus and decay, and the diffusion term describes dispersion arising from variability in interactions. Together, they define a probabilistic flow field over the state space.

This approximation enables the analysis of long-run distributional shapes, stationary solutions, inequality formation, liquidity clustering, and macroscopic responses to changes in drift or diffusion. The system can thus be studied as a continuous, non-equilibrium flow over wealth and money space, rather than as discrete agent interactions.

Appendix B Grazing Asymptotics and Boundary Closures

Following standard grazing collision scaling (Cordier et al. 2005; Pareschi and Toscani 2013), we scale microscopic transfers by $\epsilon \ll 1$ such that $\phi = \epsilon\hat{\phi}$ and $K = \epsilon^{-1}\hat{K}$. Let $\psi(\boldsymbol{x}) \in C_c^2(\mathbb{R}^2_{\geq 0})$ be a test function. Expanding $\psi(\boldsymbol{x}')$ to second order in $\Delta\boldsymbol{x} = \boldsymbol{x}' - \boldsymbol{x}$ yields:

$$\psi(\boldsymbol{x}') - \psi(\boldsymbol{x}) = \sum_k \frac{\partial \psi}{\partial x_k}\Delta x_k + \frac{1}{2}\sum_{k,l}\frac{\partial^2\psi}{\partial x_k \partial x_l}\Delta x_k \Delta x_l + \mathcal{O}(\epsilon^3). \tag{B1}$$

Substituting Eq (B1) into the weak master equation, conditioning on equal buyer/seller encounter probabilities, and taking $\epsilon \to 0$ recovers the continuous bivariate Fokker–Planck equation, where the jump moments are given explicitly by:

$$A_w(\boldsymbol{x}) = \frac{1}{2}\int \hat{K}(\boldsymbol{x}, \boldsymbol{x}^*)\left[\hat{\phi}^{\text{real}}_{\text{buyer}}(\boldsymbol{x}, \boldsymbol{x}^*) + \hat{\phi}^{\text{real}}_{\text{seller}}(\boldsymbol{x}^*, \boldsymbol{x})\right] f(\boldsymbol{x}^*)\,\mathrm{d}\boldsymbol{x}^* - \delta(w), \tag{B2}$$

$$A_m(\boldsymbol{x}) = \frac{1}{2}\int \hat{K}(\boldsymbol{x}, \boldsymbol{x}^*)\left[\sigma(\boldsymbol{x}^*, \boldsymbol{x}) - \sigma(\boldsymbol{x}, \boldsymbol{x}^*)\right] f(\boldsymbol{x}^*)\,\mathrm{d}\boldsymbol{x} j, \tag{B3}$$

$$B_{ww}(\boldsymbol{x}) = \frac{1}{2}\int \hat{K}(\boldsymbol{x}, \boldsymbol{x}^*)\left\{\left[\hat{\phi}^{\text{real}}_{\text{buyer}}(\boldsymbol{x}, \boldsymbol{x}^*)\right]^2 + \left[\hat{\phi}^{\text{real}}_{\text{seller}}(\boldsymbol{x}^*, \boldsymbol{x})\right]^2\right\} f(\boldsymbol{x}^*)\,\mathrm{d}\boldsymbol{x} j, \tag{B4}$$

$$B_{mm}(\boldsymbol{x}) = \int \hat{K}(\boldsymbol{x}, \boldsymbol{x}^*)\left[p(\boldsymbol{x}, \boldsymbol{x}^*)\hat{\phi}(\boldsymbol{x}, \boldsymbol{x}^*)\right]^2 f(\boldsymbol{x}^*)\,\mathrm{d}\boldsymbol{x} j, \tag{B5}$$

$$B_{wm}(\boldsymbol{x}) = -\frac{1}{2}\int \hat{K}(\boldsymbol{x}, \boldsymbol{x}^*) p(\boldsymbol{x}, \boldsymbol{x}^*)\hat{\phi}(\boldsymbol{x}, \boldsymbol{x}^*)\hat{\phi}^{\text{real}}_{\text{buyer}}(\boldsymbol{x}, \boldsymbol{x}^*) f(\boldsymbol{x}^*)\,\mathrm{d}\boldsymbol{x} j, \tag{B6}$$

with $\sigma(\boldsymbol{x}, \boldsymbol{x}^*) \equiv p(\boldsymbol{x}, \boldsymbol{x}^*)\hat{\phi}(\boldsymbol{x}, \boldsymbol{x}^*)$.

Verification of boundary currents and asymptotic regimes

The continuous probability current vector $\boldsymbol{J} = (J_w, J_m)^\top$ has components:

$$J_w = A_w f - \frac{1}{2}\frac{\partial}{\partial w}[B_{ww} f] - \frac{1}{2}\frac{\partial}{\partial m}[B_{wm} f], \tag{B7}$$

$$J_m = A_m f - \frac{1}{2}\frac{\partial}{\partial m}[B_{mm} f] - \frac{1}{2}\frac{\partial}{\partial w}[B_{wm} f]. \tag{B8}$$

Near $m \to 0^+$, the solvency condition mandates that the transaction rate vanishes: $\hat{K}(\boldsymbol{x}, \boldsymbol{x}^*) \to 0$ for buyer states. Consequently, $A_m \to 0$, $B_{mm} \to 0$, and $B_{wm} \to 0$, which ensures $J_m(w, 0, t) = 0$ natively.

At the lower wealth boundary ($w \to 0^+$), boundary behavior is dictated by the dynamic competition between collision-induced surplus generation and continuous dissipation. In the active laminar regime, interaction with positive-wealth counterparties yields a strictly positive expected surplus drift $\lim_{w\to 0^+} \mathbb{E}[\phi^{\text{real}}] > 0$ that overpowers local decay ($\delta(0) = 0$). This emergent outward drift repels probability density from the origin, establishing $J_w(0, m, t) = 0$ natively and preserving smooth interior closure.

Conversely, when systemic liquidity starvation suppresses transactions ($\mathbb{E}[\phi^{\text{real}}] \to 0$), the local drift reduces strictly to dissipation: $A_w \to -c\, w^\gamma$. For linear decay ($\gamma \geq 1$), unreplenished trajectories approach the origin asymptotically as $t \to \infty$. For sub-linear dissipation ($\gamma < 1$), uncompensated decay terminates at the origin in finite time $t^* = w_0^{1-\gamma}/[c(1-\gamma)] < \infty$, causing probability flux to drain directly into the singular zero-wealth absorbing sink.

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