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Reconstructing End-User Demand under Batch-Level Ambiguity: Flow-Constrained Record Linkage and Audit Allocation in Orthopedic Consignment Networks

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Abstract

This study examines how batch-level records can reveal the commercial provenance of orthopedic implant use and how limited reconciliation resources should be directed toward the most consequential unresolved demand. Methodologically, a flow-constrained probabilistic linkage model connects one month of clinical implantation records to nine months of settlements, adjusting for reporting delays, estimating residual channel states, and comparing audit policies under alternative treatments of repeated records, observation windows, and distributor concentration. Empirical results show that candidate support reaches 42.41% (229/540). Among the 311 unmatched records, the posterior assigns 31.58% to censoring-related reporting lag, 24.57% to batch or capacity competition, and 43.86% to outside-ledger pathways, including wholesale pooling. At a 20% review budget, value-of-information (VoI)-driven auditing captures 67.7% of reducible uncertainty—equivalent to 52.8% of initial uncertainty—outperforming volume-greedy selection by 18.2 percentage points on the initial-uncertainty scale, while conservative duplicate collapse preserves 91.7% of the top-108 audit selection. Ultimately, this research links uncertainty about batch-level provenance to the allocation of audit effort; by combining quantity constraints, reporting delays, and attribution entropy, it demonstrates where digital reconciliation yields the greatest information gain in an asymmetric consignment network.

Keywords: Orthopedic consignment; Probabilistic record linkage; Batch-level traceability; Right censoring; Attribution entropy; Value of information

1 Introduction

An orthopedic consignment network must make suitable implant specifications available before their final use is known. Consignment places these devices close to clinical care while distributing custody, replenishment decisions and financial responsibility across organizations. Once an implant is used, the clinical record locates consumption in a surgical event; an upstream settlement entry recognizes the associated commercial obligation. The two records can differ in timing, location and aggregation. Connecting them is central to healthcare supply-chain integration and to the reliability of inventory decisions based on transaction data [28].

A procedure that uses several screws can leave a fragmented administrative account of its consumption. Several implantation records may enter one settlement, while a single clinical quantity may require allocation across several upstream entries. Transfers between distributors add further distance between physical use and financial recognition. Timing compounds the problem: consumption close to an administrative cutoff has had less opportunity to appear in the settlement ledger. Demand reconstruction must therefore preserve surgical time while identifying the upstream quantities and commercial pathways that can account for the recorded use.

A batch number narrows the possible source of a screw but leaves individual allocations unresolved. Several units can share the same distributor, SKU and production lot, so exact field agreement may produce several eligible upstream entries. Quantity constraints then determine which combinations remain feasible. Repeated clinical rows introduce another ambiguity. Multiple implants of the same specification within a procedure can resemble duplicate entry when the available business fields coincide. The model must carry these competing explanations forward because they affect both attributed demand and the value of further reconciliation.

The analysis follows one spinal implant family through a multi-tier orthopedic supply network. Product family P , a Polyaxial Reduction Pedicle Screw System, contains 30 SKUs. The reference horizon spans M_1 – M_9 : clinical use is observed in M_9 , and the upstream pool contains settlements recorded through Day 29 of that month. During system deployment, coverage expanded from 2 nodes in M_7 to 10 in M_8 and 95 in M_9 . The focal clinical cohort contains 2,951 records from 95 active distributor nodes, representing 6,492 screws. Restricting the analysis to distributors present in both sources yields 540 records and 1,188 screws across 24 SKUs and eight distributors, D_1 – D_8 .

Upstream candidate support varies sharply across records and distributors in the core cohort. Of the 540 records, 229 have effective candidates across six distributors and 19 SKUs; 84 have a unique candidate and 145 have multiple candidates. The remaining 311 span all eight distributors and 22 SKUs. Support is also concentrated: D_1 contributes 142 of the 229 candidate-supported records, and D_1 and D_2 together contribute 181. Variation in attribution entropy adds a further dimension to this uneven coverage. Managers must decide both how to assign consumption and where review effort will most improve those assignments.

Reconstructing demand and deciding what to review require a common account of the uncertainty in each record. The analysis addresses three questions. How can batch-level records support probabilistic demand attribution while satisfying distributor, SKU, batch and quantity constraints? How should reporting delays and repeated

entries affect the distinction between pending recognition and alternative inventory channels? Where should a finite audit budget be spent when volume, uncertainty and attainable information gain differ across records? Audit activity consists of reconciling information already held in the digital record pool, keeping the analysis within the established observation horizon.

The analysis connects record linkage to two operational decisions: how to interpret delayed recognition and where to allocate audit effort. Its flow constraints extend inference from candidate pairs to feasible quantity allocations, which yield unique-candidate, multiple-candidate and residual states. A calibrated delay model then decomposes the residual cohort into 76.4 record equivalents associated with attribution obstruction, 98.2 with latency and 136.4 with alternative recognition pathways. These estimates inform a policy that weights uncertainty by consumption. At a 20% review budget, that policy captures 67.7% of reducible uncertainty, or 52.8% of initial uncertainty, exceeding volume-greedy selection by 18.2 percentage points on the initial-uncertainty scale. Sensitivity analyses assess whether the priorities persist as duplicate handling, follow-up and distributor composition change.

2 Literature Review and Analytical Framework

2.1 Visibility and Information Asymmetry in Multiechelon Medical Consignment

Healthcare supply-chain research places coordination between clinical work and material flows at the center of analysis [28]. Consignment distributes responsibility for custody, monitoring and payment across different organizations. Rosales et al. [22] show how incentives, monitoring and product characteristics shape medical-consignment shrinkage, including differences between general supplies and physician-preference products. For orthopedic implants, this literature directs attention to who observes use, who records it and who bears the consequences of an incomplete account.

A record of implant use answers one operational question but can leave its commercial attribution unresolved. Point-of-use inventory research links inaccurate usage capture to inventory-control performance and the value of reconciliation [20]. Even when a registry captures an implantation accurately, settlement may aggregate it with other uses or recognize it later. Each tier therefore observes a different part of the process. Visibility depends on connecting those observations at the level needed for replenishment or commercial review.

The same production lot can pass through intermediaries that handle both consignment and wholesale inventory. That overlap makes timing, record structure and inventory mode relevant to the attribution of a documented clinical use. Restricting the analysis to P retains SKU differences within a coherent product family; selecting M_9 separates the demand comparison from changes in capture during deployment. Visibility is then defined by the ability to connect observed consumption to feasible upstream recognition and to quantify the uncertainty that remains.

2.2 Probabilistic Record Linkage and Reconstruction under Incomplete Information

The statistical record-linkage tradition addresses the integration of records that lack a shared unique identifier. Newcombe et al. [21] established an early basis for automated linkage, and Fellegi and Sunter [8] formalized comparison-based decisions with an explicit unresolved category. Bayesian approaches subsequently made the complete matching configuration an inferential object, allowing uncertainty and dependence among candidate links to enter estimation [24]. Graphical linkage and deduplication further connect observed records to latent entities while propagating uncertainty about identity and duplication [26].

Because recorded quantities frequently span multiple clinical events, attributing orthopedic settlement requires allocating available capacity under strict flow constraints. A clinical row and a settlement entry may aggregate consumption differently, requiring one-to-many or many-to-one allocations. Distributor, SKU and batch agreement establish the eligible links. Settlement quantities determine how much each link can carry. Assigning quantity to one clinical record changes the options available to others in the same block, so the posterior must represent complete allocations. This dependence connects record linkage directly to a constrained flow problem.

Recent surgical consumption has had less time to enter the upstream settlement pool. Survival analysis retains observations whose recognition time falls beyond available follow-up [14], while reporting-delay models distinguish when an event occurs from when it becomes observable [11]. A residual clinical record may therefore represent pending consignment recognition, an obstruction in the linkage structure or another inventory mode. A mixture model assigns probabilities to these explanations and carries them into the demand reconstruction.

2.3 Data-Driven Inventory Reconciliation and Value-of-Information Allocation

Inventory-record inaccuracy varies with operational conditions and audit practices [6]. Inspection models connect the cost of acquiring better information to the value of subsequent inventory decisions [15]. These ideas motivate an explicit allocation problem for digital reconciliation: the network must choose which uncertain records or batch components to examine within a finite resource budget. Random review distributes effort uniformly, whereas volume-greedy selection directs it toward the largest recorded quantities.

Establishing optimal audit priorities requires evaluating both the attribution entropy of a record and its material consumption volume. Following Shannon [25], attribution entropy measures uncertainty across possible allocations in bits. Multiplying entropy by consumption quantity gives greater weight to unresolved records with substantial material exposure. Attainable gain adds a practical consideration: how much of that uncertainty can the available review action resolve? It also accounts for evidence shared by records in the same batch component.

The value of better information depends on the decisions it changes [12]. Here, the operational objective is to reduce quantity-weighted uncertainty within a review

budget, with all policies evaluated under the same resolution assumptions. The three literature streams address different parts of that task. Healthcare visibility locates the organizational problem, probabilistic linkage represents uncertain relationships, and audit optimization allocates effort among them. Combining these elements makes it possible to test whether targeting ambiguous consumption improves reconciliation efficiency.

3 Operational Setting, Event Architecture, and Data Anatomy

3.1 Network Structure and De-identification Protocol

The focal network links a manufacturer, intermediary distributors and hospital points of clinical use through multiple organizational tiers. Consignment separates custody from financial recognition, while wholesale transactions can coexist within the broader distribution system. Industrial consignment research distinguishes ownership, payment timing and replenishment responsibility as separate design dimensions [2, 27]. Vendor-managed inventory adds a further allocation of decision rights [10]. The present analysis follows the recorded relationship between clinical consumption and upstream consignment settlement.

The focal spinal implant product family P is a Polyaxial Reduction Pedicle Screw System containing 30 SKUs. Specification identifiers remain distinct throughout linkage. The complete M_9 clinical cohort contains 2,951 records across 95 active distributor nodes and all 30 SKUs, corresponding to 6,492 screws. The core analytical cohort contains 540 records and 1,188 screws across eight shared distributors, D_1 – D_8 , and 24 SKUs. These masked entities provide the common support for the two-source analysis.

To strictly comply with institutional confidentiality and data protection protocols, all proprietary identifiers—including manufacturer identity, distributor codes, hospital names, SKU catalog numbers, and exact calendar timestamps—have been systematically anonymized and indexed into relative periods (M_1 – M_9) prior to modeling. Entity masking preserves relational consistency, and time-shifting preserves the ordering and elapsed intervals required for temporal linkage. Relative periods and masked entity codes provide the coordinates used throughout the empirical presentation.

The analysis assigns registered consumption to commercial pathways; batch-level compatibility supports probabilistic provenance rather than verified item identity. Its scope covers one network, one product family and one focal terminal month. Channel states remain probabilistic, and the reported gains measure what digital reconciliation can resolve within that observation boundary.

3.2 Asynchronous Operational Events

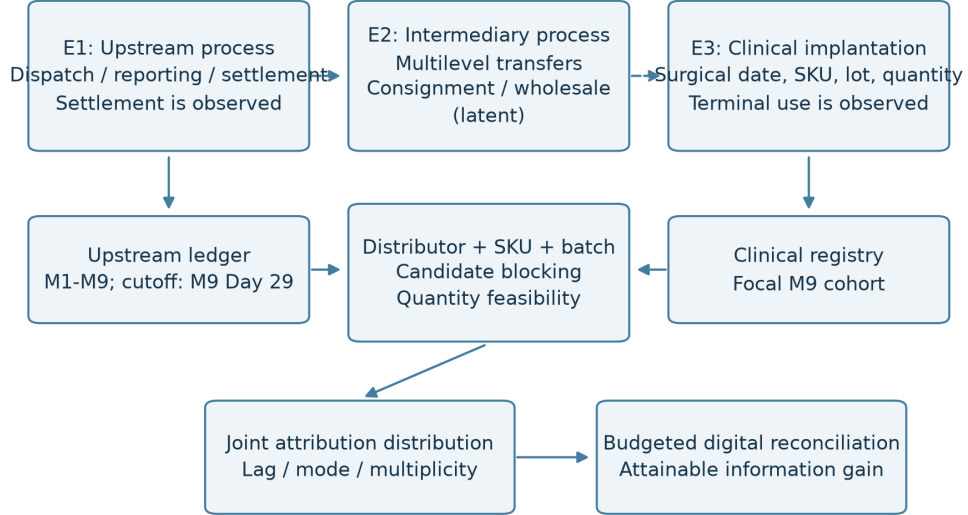
Let the operational event space contain three classes, $E = E_1 \cup E_2 \cup E_3$. Class E_1 comprises the upstream dispatch–reporting–settlement process; E_2 comprises intermediary transfers and potential mixing of consignment and wholesale inventory; and E_3 comprises clinical implantation. The operators $\tau_{ship}(e)$, $\tau_{surg}(e)$ and $\tau_{settle}(e)$ identify the associated event times. The Upstream Consignment Settlement

Ledger, $\mathcal{L}_U$, observes settlement recognition within E_1 , while the Downstream Clinical Implantation Registry, $\mathcal{R}_D$, observes clinical consumption within E_3 .

$$\mathcal{L}_U = \pi_U(E_1), \quad \mathcal{R}_D = \pi_D(E_3), \quad E_2 \text{ is latent.} \quad (1)$$

The mappings π_U and π_D are observation projections. A clinical record has attributes $i = (d_i, s_i, b_i, t_i, q_i)$ and a settlement record has attributes $j = (d_j, s_j, b_j, v_j, Q_j)$, where d , s and b denote distributor, SKU and batch; t_i is surgical time; v_j is settlement time; and quantities share a common unit. Clinical and financial events can aggregate consumption differently. Their temporal relation is captured by reporting latency, $\ell_{ij} = v_j - t_i$, whose calibrated median is 6.4 days and empirical interquartile range is 3.0–11.5 days.

Surgical timestamps and settlement timestamps capture distinct operational dimensions of the same underlying material event. Surgery locates clinical use and starts the follow-up period; settlement places an eligible upstream entry within the reference pool. The model keeps these roles separate while representing intermediary routing and inventory mode through the posterior state. Financial aggregation and reporting delay can then be incorporated without displacing the clinical event from its recorded date.



Dashed: latent physical pathways. Solid: observed inputs or analytical processing.

Fig. 1 Asynchronous Event Architecture and Probabilistic Record Linkage Framework. The upper layer separates upstream dispatch and settlement, intermediary transfers with potential consignment–wholesale mixing, and terminal implantation. Observation arrows connect settlement and clinical endpoints to $\mathcal{L}_U$ and $\mathcal{R}_D$. The lower layer applies distributor–SKU–batch blocking, quantity conservation, reporting-delay adjustment and posterior channel attribution before allocating the digital reconciliation budget. The upstream pool spans M_1 – M_9 through Day 29, and the terminal cohort comprises M_9 clinical consumption.

3.3 Observation Window and Sample Attrition

Digital-system coverage expanded from 2 nodes in M_7 to 10 in M_8 and 95 by M_9 during deployment across adjacent orthopedic product divisions. The focal design selects M_9 as the steady-state terminal window and uses settlements from M_1 – M_9 as the reference pool. This separation keeps deployment changes distinct from the comparison of demand records. The administrative cutoff is c , defined as Day 29 of M_9 ; the relative focal month ends on Day 30.

Table I: Sample Attrition Process and Data Quality Anatomy

Stage or feature	Records / volume	Distributors	SKUs / groups	Empirical interpretation
Eligible M_9 clinical cohort	2,951 records; 6,492 screws	95 active nodes	30 SKUs	Explicit surgical date and positive consumption quantity.
Shared active-distributor cohort	540 records; 1,188 screws	8: D_1 – D_8	24 SKUs	18.30% of initial records; 2,411 records outside the intersection.
Candidate- consistent subset	229 records	6: D_1 – D_6	19 SKUs	42.41% of the core cohort; unique and multiple effective candidates.
Residual core cohort	311 records	8: D_1 – D_8	22 SKUs	57.59% of the core cohort; D_7 and D_8 contribute 18 and 14 records with zero candidates.
Identical business-field configurations	68 records; 37 rows removed under A1	Within the core cohort	31 configura- tion groups	Exact agreement on distributor, surgical date, SKU, batch and quantity.
Head-distributor concentration	D_1 : 248 core records; 142 candidates	D_1 ; top two D_1 – D_2	19 candidate SKUs in the full subset	D_1 : 45.93% of core and 62.01% of candidates; top two: $181/229 = 79.04\%$.
Late-month recognition exposure	94 records on Days 25–30; 80 residual	Within the core cohort	75 records on Days 25–29; 19 on Day 30	High concentration of unresolved records near the Day 29 administrative cutoff.

Notes: SKU = stock-keeping unit. Counts refer to clinical registry records; consumption is measured in screws. The clinical window is M_9 ; the upstream pool spans M_1 – M_9 through Day 29. Retention and concentration percentages use the denominator stated in each row. Exact duplicate configurations share distributor, surgical date, SKU, batch and quantity. The late-month exposure window covers Days 25–30, including 19 post-cutoff records.

The matching funnel retains 18.30% of the initial records in the shared-distributor cohort. Exact-key and quantity screening identifies 229 supported records, representing 42.41% of this core cohort. The residual 311 records span all eight distributors,

providing a broad basis for posterior channel decomposition. Identical configurations involve 68 records in 31 groups; collapsing each group to one retained row removes 37 records. Candidate support is concentrated in D_1 and D_2 , which together contribute 181 records. Temporal exposure is handled separately through the Day 29 cutoff and the cohort-specific survival weights. Days 25–30 contain 94 core records, including 80 residual records; 19 of these clinical observations occur on Day 30 and therefore have zero pre-cutoff recognition opportunity.

4 Methodology: Flow-Constrained Probabilistic Linkage and VoI Audit Prioritization

4.1 Hierarchical Matching and Network Flow Conservation Constraints

When a procedure uses several screw sizes, each SKU must remain distinct during linkage. Units and entity masks are first harmonized; exact distributor, SKU and batch agreement then defines the candidate blocks. The procedure combines probabilistic comparison with inference over joint allocations [8, 24]. Each clinical row provides a unit for linkage classification, while its recorded quantity measures material exposure. Repeated field configurations remain explicit until the analysis selects a rule for treating multiplicity.

$$a_{ij} = \mathbf{1}\{d_i = d_j, s_i = s_j, b_i = b_j\}, \quad x_{ij} = 0 \text{ if } a_{ij} = 0. \quad (2)$$

Let x_{ij} denote clinical quantity assigned to settlement record j , r_i the residual quantity of clinical record i , and u_j the unused quantity of settlement record j . The feasible set $\mathcal{F}$ satisfies row balance and column capacity. Implant counts are integer-valued after unit harmonization.

$$\begin{aligned} \sum_{j \in \mathcal{J}} x_{ij} + r_i &= q_i, \\ \sum_{i \in \mathcal{I}} x_{ij} + u_j &= Q_j, \end{aligned} \quad (3)$$

$$\begin{aligned} x_{ij}, r_i, u_j &\geq 0. \\ \sum_{i: (d_i, s_i, b_i) = (d, s, b)} \sum_j x_{ij} &\leq \sum_{j: (d_j, s_j, b_j) = (d, s, b)} Q_j. \end{aligned} \quad (4)$$

A settlement entry can support several clinical uses only up to its recorded quantity. Equation (4) enforces this limit within each distributor–SKU–batch block using the historical and contemporaneous settlement pool. Clinical quantity left unassigned retains its uncertain provenance; unused upstream capacity can relate to terminal periods outside the focal month. Each recorded settlement quantity enters the graph once, preventing competing clinical rows from drawing on the same capacity twice.

Effective candidate support is determined within the feasible allocation set:

$$\Gamma_i^* = \{j : \exists X \in \mathcal{F}, r_i = 0, x_{ij} > 0\}. \quad (5)$$

Records with $|\Gamma_i^*| = 1$ form the unique-candidate tier $\mathcal{U}$; records with $|\Gamma_i^*| > 1$ form the multiple-candidate tier $\mathcal{M}$; and records with $|\Gamma_i^*| = 0$ form the effectively isolated tier $\mathcal{I}_0$. Effective isolation includes records with raw key-compatible entries whose capacities cannot support full allocation. Joint posterior allocations enforce shared capacity across records, while the support classification summarizes each record’s admissible links.

An allocation to one clinical record changes the options available to other records sharing its settlement pool. The posterior captures this dependence by combining timing, eligible quantities and residual-channel states over $\mathcal{F}$. It also separates uncertainty about an allocation from a known allocation split across several entries. Only the former contributes allocation entropy. The candidate hierarchy in Section 5 therefore reflects both the links available to each record and the quantity restrictions that connect those records.

4.2 Reporting Latency Kernel and Right-Censoring Adjustment

The duration of reporting latency defines the temporal exposure window available for a clinical record to secure upstream settlement. For a compatible pair, the delay is $\ell_{ij} = v_j - t_i$. The post-use recognition process has density $f_\theta(\ell)$, cumulative distribution F_θ and survivor $S_\theta = 1 - F_\theta$ on nonnegative time. The reported log-normal survival calibration ($\hat{\mu} = 1.86, \hat{\sigma} = 0.72$) weights compatible pairs by their joint linkage support and accounts for the recognition window, yielding a median lag of 6.4 days and an empirical interquartile range of 3.0–11.5 days.

An empirical weighted lag distribution summarizes the compatible observations:

$$\hat{F}_{obs}(\ell) = \frac{\sum_{i,j} w_{ij} \mathbf{1}\{0 \leq v_j - t_i \leq \ell, v_j \leq c\}}{\sum_{i,j} w_{ij} \mathbf{1}\{0 \leq v_j - t_i, v_j \leq c\}}. \quad (6)$$

The weights w_{ij} reflect candidate-pair involvement under joint feasible allocation. Parametric calibration then accounts for the greater probability of observing shorter lags within a finite window. For a recognized delay, the likelihood contribution conditional on recognition by c is

$$f_\theta(\ell \mid L \leq c - t_i) = \frac{f_\theta(\ell)}{F_\theta(c - t_i)}, \quad 0 \leq \ell \leq c - t_i. \quad (7)$$

The surgical date fixes the follow-up period available before the settlement pool closes. Survival and reporting-delay methods account for this unequal exposure [11, 14], and covariate-dependent hazards provide a way to represent distributor heterogeneity [5]. Setting $F_i = \mathbf{1}\{t_i \leq c\} F_\theta(c - t_i)$ assigns zero pre-cutoff recognition opportunity to the 19 Day 30 records.

Let π_i denote the probability that consumption belongs to the observed consignment-recognition process. Let κ_i denote the probability of recognizing a timely settlement, conditional on that membership and integrated over competing allocations. For records in $\mathcal{I}_0$, the conditioning probability is $D_i = 1 - \pi_i \kappa_i F_i$. The residual posterior weights are

$$\begin{aligned}
p_{i,L} &= \frac{\pi_i(1 - F_i)}{D_i}, \\
p_{i,A} &= \frac{\pi_i(1 - \kappa_i)F_i}{D_i}, \\
p_{i,O} &= \frac{1 - \pi_i}{D_i}.
\end{aligned} \tag{8}$$

The three posterior states distinguish reasons that a clinical use may lack effective settlement support. They represent later recognition (L), timely recognition obscured by batch or quantity competition (A), and a pathway outside the observed consignment process (O). Each record's weights sum to one. Legitimate wholesale pooling and other outside-ledger pathways enter the O state.

4.3 Attribution Entropy and Budget-Constrained Audit Optimization

Let Z_i denote the complete quantity allocation and residual-channel state of record i , with posterior support Ω_i . Entropy is measured in bits using a base-two logarithm, and material exposure is incorporated through recorded consumption q_i :

$$\begin{aligned}
H_i &= - \sum_{z \in \Omega_i} p_i(z) \log_2 p_i(z), \\
p_i(z) &= P(Z_i = z | \mathcal{E}_0), \\
U_0 &= \sum_i q_i H_i.
\end{aligned} \tag{9}$$

A unique candidate can still leave uncertainty about the inventory mode of the recorded consumption. Entropy measures that uncertainty alongside uncertainty about allocation, following Shannon [25]. A fully resolved allocation vector contributes zero allocation entropy.

The evidence state $\mathcal{E}_0$ contains distinctions already processed by the linkage model. A digital reconciliation action exposes usable information remaining in the frozen record pool. For selected actions A with outputs Y_A , expected gain is

$$G(A) = U_0 - \mathbb{E}_{Y_A | \mathcal{E}_0} \left[\sum_i q_i H(Z_i | \mathcal{E}_0, Y_A) \right]. \tag{10}$$

$$\max_{z_g \in \{0,1\}} G(\{g : z_g = 1\}) \quad \text{subject to} \quad \sum_g c_g z_g \leq B. \tag{11}$$

Audit effort is valuable when it changes how records can be allocated or interpreted. The cost c_g measures the review units or reconciliation time required for action g . When an action resolves information shared by several batch-linked records, the remaining priorities are updated to reflect that gain. This batch-set adjustment credits each subsequent action for the additional information it contributes. With separable gains, the problem reduces to an integer knapsack with values v_g . Equal costs and a common resolvable fraction yield the $q_i H_i$ ranking; the set-based implementation also accounts for overlapping evidence.

All three policies are evaluated with the same resolvable fraction, $\bar{\eta} = 0.78$. This gives a maximum reducible total of 1,141.0 screw-bits. Random sampling selects records uniformly, volume-greedy selection ranks q_i , and VoI-driven selection uses uncertainty-weighted exposure with batch-set adjustment.

Howard [12] provides the decision-theoretic foundation for valuing information. The operational objective here is quantity-weighted uncertainty reduction, with exact knapsack solutions serving as references for the reported regret comparisons. Classical and adaptive-submodular theory explains when diminishing-return structure supports greedy approximation [9, 19]. Empirical evaluation across the distributor-deletion scenarios complements that structural analysis by measuring the attained gap to the stated optimization benchmark.

5 Empirical Analysis, Strategy Evaluation, and Sensitivity Tests

5.1 Baseline Linkage Stratification and Ambiguity Decomposition

The core cohort comprises 540 records and 1,188 screws across eight distributors and 24 SKUs. Effective candidate support is obtained for 229 records, or 42.41%, spanning D_1 – D_6 and 19 SKUs. The support hierarchy evaluates Γ_i^* directly: 84 records have one effective candidate and 145 have multiple effective candidates. Their shares of the core cohort are 15.56% and 26.85%. The residual 311 records represent 57.59% of the cohort and span 22 SKUs across all eight distributors.

Records with multiple effective candidates carry substantially more uncertainty than those with a single candidate. Table II reports mean entropy of 0.18 bits for the unique-candidate tier, which covers 14 SKUs and five distributors, and 1.42 bits for the multiple-candidate tier, which covers 18 SKUs and six distributors. The latter mean is approximately 7.9 times the former. Across all 229 candidate-supported records, mean entropy is 0.97 bits. These differences give uncertainty-weighted selection a basis for prioritizing review even among records with upstream support.

Table II: Hierarchical Linkage Tiers, Flow Constraint Outcomes, and Unmatched Decomposition (N = 540)

Linkage tier	Records / equiva- lents	Share of core	SKUs	Distributors	Mean entropy (bits)
U: unique effective candidate	84	15.56%	14	5	0.18
M: multiple effective candidates	145	26.85%	18	6	1.42
Candidate subtotal U $\cup$ M	229	42.41%	19	6	0.97

Continued on next page

Table II (continued)

Linkage tier	Records / equiva- lents	Share of core	SKUs	Distributors	Mean entropy (bits)
I_0 -A: attribution / capacity competition	76.4	14.15%	16	6	1.85
I_0 -L: latency / post-cutoff recognition	98.2	18.19%	19	7	1.16
I_0 -O: outside observed consignment process	136.4	25.26%	21	8	0.94
Residual subtotal I_0	311.0	57.59%	22	8	1.23
Core total	540	100.00%	24	8	1.12

Notes: $\mathcal{U}$ and $\mathcal{M}$ denote unique and multiple effective candidates; $\mathcal{I}_0$ denotes residual records. A , L and O denote batch/capacity competition, reporting latency and outside-ledger pathways. Residual counts are posterior expected record equivalents. Percentages use $N = 540$ and are rounded independently. Entropy is in bits; residual-tier means use posterior membership weights. SKU and distributor columns count distinct entities. $U_0 = 1,462.8$ screw-bits across 1,188 screws.

Summing the individual residual-state probabilities yields the estimated decomposition:

$$N_A = \sum_{i \in \mathcal{I}_0} p_{i,A} = 76.4 \quad (12a)$$

$$N_L = \sum_{i \in \mathcal{I}_0} p_{i,L} = 98.2 \quad (12b)$$

$$N_O = \sum_{i \in \mathcal{I}_0} p_{i,O} = 136.4. \quad (12c)$$

$$\begin{aligned} 76.4 + 98.2 + 136.4 &= 311, \\ 84 + 145 + 76.4 + 98.2 + 136.4 &= 540. \end{aligned} \quad (13)$$

Attribution and capacity competition account for 24.57% of residual record equivalents, reporting latency for 31.58%, and outside-ledger pathways for 43.86%. Their mean entropies are 1.85, 1.16 and 0.94 bits, respectively. Notably, the outside-ledger state exhibits a lower mean entropy (0.94 bits) than the multiple-candidate tier (1.42 bits) because mature records lacking any eligible consignment batches concentrate their posterior mass onto a single residual channel state rather than splitting probability across multiple combinatorial batch allocations. The obstruction state is the most uncertain on a per-record basis, whereas the outside-ledger state is the largest in expected membership. The residual subtotal has mean entropy of 1.23 bits. Across all records, mean entropy is 1.12 bits, and quantity weighting yields $U_0 = 1,462.8$ screw-bits.

The latency scale links this decomposition to the observation window. Compatible pair differences ℓ_{ij} have a calibrated median of 6.4 days and an empirical interquartile

range of 3.0–11.5 days. Days 25–30 contain 94 core records, of which 80 are residual; the 19 Day 30 observations have zero pre-cutoff recognition opportunity. These temporal features concentrate uncertainty near the observation boundary and motivate the cutoff-aligned comparisons in Section 5.3.

The network also exhibits pronounced concentration. Distributor D_1 contributes 248 core records, or 45.93%, and 142 candidate-supported records, or 62.01%. The first two distributors contribute 181 candidate records, or 79.04%. At the opposite end of the support distribution, D_7 and D_8 contribute 18 and 14 clinical records with zero effective candidates. The empirical framework therefore combines a concentrated supported core with residual demand distributed across the entire eight-distributor network.

5.2 Comparative Evaluation of Audit Prioritization Strategies

The evaluation considers budget fractions of 10%, 20%, 30% and 50%, corresponding to 54, 108, 162 and 270 reviewed records. The baseline resolution environment has $\bar{\eta} = 0.78$, so the maximum reducible uncertainty is 1,141.0 screw-bits. Two normalizations support interpretation: resolved initial uncertainty is $100G(A)/U_0$, and captured reducible uncertainty is $100G(A)/(\bar{\eta}U_0)$. Figure 2 displays both scales.

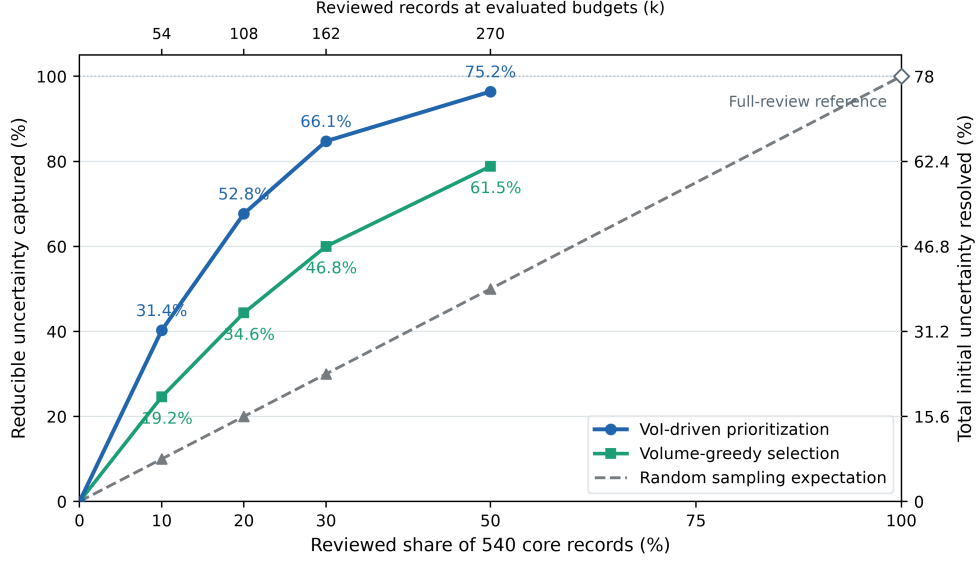
$$RDU(A) = 100 \frac{G(A)}{1462.8}, \quad RC(A) = 100 \frac{G(A)}{1141.0}. \quad (14)$$

At the four budgets, VoI-driven prioritization resolves 31.4%, 52.8%, 66.1% and 75.2% of initial uncertainty. These gains correspond to 40.3%, 67.7%, 84.7% and 96.4% of the reducible total. Volume-greedy selection resolves 19.2%, 34.6%, 46.8% and 61.5% of initial uncertainty. The random benchmark follows the linear expectation of 7.8%, 15.6%, 23.4% and 39.0%, capturing the same budget fractions of the reducible total.

The VoI advantage over volume-greedy selection is 12.2, 18.2, 19.3 and 13.7 percentage points of initial uncertainty at the respective budgets. At 108 reviews, the proposed policy recovers 772.4 screw-bits, compared with 506.1 for volume-greedy selection and 228.2 for random sampling. The largest absolute advantage among the evaluated budgets occurs at 30%, where VoI captures 66.1% of initial uncertainty. This exceeds the 61.5% attained by volume-greedy selection with 270 reviews, yielding higher information recovery with 40% fewer reviewed records.

Large quantities alone provide a weak guide when high-volume accounts already have relatively clear attribution. Within the reported high-volume stratum at D_1 , 64% of rows belong to the low-entropy unique-candidate tier. Quantity ranking consequently spends substantial review effort on comparatively stable records. VoI prioritization directs effort toward material consumption whose batch or channel allocation remains uncertain, then updates priorities as shared evidence is resolved. The first 54 reviews recover 31.4% of initial uncertainty. The next two increments of 54 reviews add 21.4 and 13.3 percentage points, respectively. A further 108 reviews add 9.1 points, showing how marginal information yield declines as the most informative cases are resolved.

Across the four evaluated budgets, VoI-driven prioritization occupies the budget–information efficiency frontier among the three policies. Its empirical advantage



Point labels report the share of initial uncertainty resolved; baseline resolvability = 0.78.

Fig. 2 Cumulative Information Gain under Competing Audit Prioritization Strategies. The horizontal axis shows reviewed records as a percentage of the 540-record core cohort. Markers identify the evaluated budgets $k = 54, 108, 162$ and 270 . The left axis measures the share of reducible uncertainty captured; the right axis measures the share of initial uncertainty U_0 resolved, with conversion factor $\bar{\eta} = 0.78$. VoI and volume-greedy segments join reported budget points, while the random expectation follows the linear budget benchmark. The full-review reference corresponds to 100% of reducible uncertainty, or 78% of initial uncertainty.

is measured in recovered attribution information. Channel-state probabilities guide reconciliation toward consequential uncertainty, and the information metric provides a common objective for comparing policy performance and exact optimization benchmarks.

5.3 Robustness Checks and Sensitivity Analysis

Table III evaluates three sources of analytical variation: record multiplicity, maturity of observation and distributor concentration. The repeated-configuration analysis is grounded in 68 records belonging to 31 exact groups. Scenario A1 collapses each group to one representative, removing 37 rows; Scenario A2 retains the original clinical rows. This interpretation of repeated observations is compatible with treating duplication as a record-partition problem [23], while retaining the possibility of multiple implants of the same specification within a procedure.

Table III: Sensitivity Analysis across De-duplication Rules, Right-Censoring Windows, and Distributor Concentration

Scenario	Cohort and transformation	Candidate support	Audit outcome / comparison	Empirical interpretation
A1: conservative duplicate collapse	503 records after collapsing 68 rows in 31 exact groups; 37 rows removed.	208/503 = 41.35%	Top-108 VoI overlap: 91.7%; Jaccard similarity: 0.85.	Support changes by -1.06 percentage points; 99 of 108 selected units are shared.
A2: clinical retention baseline	540 records; repeated same-specification clinical entries retained.	229/540 = 42.41%	20% budget: VoI 52.8%; volume 34.6%; random 15.6% of U_0 .	Baseline multiplicity and resolution environment.
B1: literal month-end exclusion	$w = 3, 5, 7$; retain $t_i \leq T_9 - w$, with $T_9 = \text{Day } 30$. $N = 498, 465, 428$.	221/498 = 44.38%; 216/465 = 46.45%; 206/428 = 48.13%.	Monotone support increase across the three retained cohorts.	Support improves as late-month records are excluded.
B2: cutoff-aligned follow-up	$w = 3, 5, 7$; retain $t_i \leq c - w$, with $c = \text{Day } 29$. $N = 479, 446, 410$.	221/479 = 46.14%; 215/446 = 48.21%; 203/410 = 49.51%.	Support interval: 46.14%–49.51%; width: 3.37 percentage points.	Longer minimum follow-up produces higher and more closely grouped support rates.
C1: omit D_1	292 records across D_2 – D_8 ; five distributors retain candidate support.	87/292 = 29.79%	At 20% budget, VoI exceeds volume-greedy by 16.4 percentage points.	The information-allocation advantage persists beyond the head distributor.
C2: eight distributor deletions	Delete each D_k , $k = 1, \dots, 8$, from both sources under the same linkage rules.	Range: 29.79%–46.82%; mean: 41.88%.	Relative objective regret against exact integer knapsack remains below 1.8% in all eight scenarios.	Small optimization gaps accompany substantial variation in linkage coverage.

Notes: A2 is the baseline; A1 collapses exact duplicate configurations. Support rates equal candidate records divided by retained records. For the two top-108 selections, overlap equals intersection/108 and Jaccard similarity equals intersection/union. Temporal exclusions use $w \in \{3, 5, 7\}$, $T_9 = \text{Day } 30$ and $c = \text{Day } 29$. Policy comparisons use $\bar{\eta} = 0.78$; audit gains are percentages of initial uncertainty and differences

are percentage points. Regret is $100(G^* - G_{VoI})/G^*$, relative to the exact knapsack solution under the same budget and objective.

Duplicate collapse reduces the cohort from 540 to 503 records and the candidate count from 229 to 208. The support rate changes from 42.41% to 41.35%, a decrease of 1.06 percentage points. Comparing the top 108 audit selections on the common record-identity basis yields 99 shared selections: 91.7% overlap and a Jaccard similarity of 0.85. These results indicate that the main reconciliation priorities persist under the alternative multiplicity interpretation.

Literal month-end exclusions produce support rates of 44.38%, 46.45% and 48.13%. Cutoff-aligned exclusions produce 46.14%, 48.21% and 49.51%, an interval of 3.37 percentage points. The latter comparison directly increases the minimum recognition opportunity before Day 29. The upward pattern is consistent with the calibrated delay mechanism and with the posterior estimate that 31.58% of residual record equivalents belong to the latency state.

Removing D_1 leaves 292 records and 87 candidates, giving 29.79% support across seven core distributors and five candidate-supporting distributors. Within this reduced sample, the VoI advantage over volume-greedy selection remains 16.4 percentage points at the 20% budget. Across all eight distributor deletions, support ranges from 29.79% to 46.82%, with a mean of 41.88%. Relative objective regret, defined as $100(G^* - G_{VoI})/G^*$ under the common knapsack benchmark, remains below 1.8%. The deletion analysis separates variation in network linkage coverage from the consistency of the audit-selection advantage.

6 Discussion and Managerial Implications

6.1 Bridging the Visibility Gap in Non-Serialized Consignment Networks

Leveraging batch-level records effectively requires evaluating shared quantities and reporting intervals jointly. In the 540-record cohort, 84 records have a unique effective candidate and 145 have multiple candidates. Their mean entropies, 0.18 and 1.42 bits, distinguish relatively stable assignments from unresolved allocation choices. The model thus answers the first research question with a distribution over feasible commercial attributions, accompanied by a measure of uncertainty for each record.

Visibility improves when a network can relate its records to the decisions they support. Barratt and Oke [1] emphasize the organizational capabilities behind visibility, while Caridi et al. [4] examine its operational value. The present analysis connects these perspectives through explicit quantity allocation. Shared identifiers determine eligible links, flow constraints control how settlement quantities are used, and entropy measures uncertainty across the remaining possibilities. Managers receive a clinical demand signal together with an assessment of confidence in its upstream attribution.

Settlement delay changes how an apparently unmatched implantation record should be handled in practice. The posterior assigns 76.4 residual record equivalents to linkage or capacity obstruction, 98.2 to delayed recognition and 136.4 to pathways outside the observed consignment process. These categories call for different administrative responses even though each begins as an unsupported clinical record. Keeping

clinical occurrence separate from financial recognition also clarifies the signal passed upstream, complementing research on information distortion and inventory-record inaccuracy [13, 16].

A limited review budget makes the operational value of these distinctions directly observable. With 20% of records reviewed, VoI-driven prioritization resolves 52.8% of initial uncertainty, compared with 34.6% under volume-greedy selection and 15.6% under random sampling. Its 18.2-percentage-point advantage over volume ranking comes from considering exposure, unresolved attribution and shared batch evidence together. Across the evaluated budgets, it offers the strongest budget–information performance of the three policies.

6.2 Actionable Decision Rules for Inventory Replenishment and Channel Auditing

Replenishment and reconciliation require different forms of confidence in the same clinical consumption record. Stable quantities with credible channel attribution can supply demand inputs to an existing automated consignment replenishment process. A distributor–SKU aggregate can serve the same purpose when it remains stable across feasible allocations of the underlying records. The replenishment system then combines that demand signal with inventory position and lead-time information. More uncertain commercial attributions enter targeted reconciliation. This separation ties the use of information to the decision it supports, consistent with research on information sharing and belief-based inventory control [3, 7].

Table IV: Decision Matrix for Replenishment Inputs and Targeted Digital Reconciliation

Evidence and empirical anchor	Demand-management use	Digital reconciliation action	Operational decision rule
Stable quantity and channel attribution; unique-candidate tier: 84 records, mean entropy 0.18 bits.	Feed stable distributor–SKU consumption into the existing automated replenishment process.	Routine consistency review of quantity, mode and temporal alignment.	Apply record-specific confidence and stability criteria alongside inventory position and lead time.
Multiple candidates or capacity obstruction; mean entropies 1.42 and 1.85 bits.	Use posterior-attributed quantities and scenario ranges for the relevant consignment pool.	Prioritize high $q_i H_i$ components with material attainable information gain.	Update marginal priorities after shared batch evidence is resolved.

Continued on next page

Table IV (continued)

Evidence and empirical anchor	Demand-management use	Digital reconciliation action	Operational decision rule
Latency state: 98.2 expected record equivalents; median lag 6.4 days.	Retain clinical demand with a distinct settlement-recognition status.	Align existing surgical and settlement timing with the calibrated delay model.	Treat exposure before Day 29 explicitly; preserve the zero-opportunity status of Day 30.
Outside-ledger state: 136.4 expected record equivalents, mean entropy 0.94 bits.	Route mode-sensitive consumption to commercial attribution review.	Compare recorded channel, batch and quantity evidence within selected components.	Separate wholesale-compatible demand from consignment charging and document the resulting attribution.

High-entropy, high-volume records warrant targeted digital reconciliation when existing evidence offers material resolution potential. Supply chain analysts or channel governance teams can examine selected distributor–SKU–batch components, align clinical and settlement timing, and resolve quantity or duplicate-configuration inconsistencies. Reviewing connected components allows one informative distinction to improve several related allocations. After each action, priorities can be updated to reflect information already recovered. The operational score therefore combines exposure, uncertainty, expected resolvability and the cost of obtaining usable evidence.

Managers can recover more information with fewer reviews by targeting uncertainty alongside recorded consumption. Reviewing 162 records under the VoI policy resolves 66.1% of initial uncertainty; reviewing 270 records by volume resolves 61.5%. The first policy therefore recovers more information with 40% fewer reviews. This comparison helps frame the allocation of reconciliation effort between large established accounts and smaller accounts whose demand attribution remains less settled.

The sensitivity results support continuity of these priorities under alternative representations of the evidence. Conservative duplicate collapse changes candidate support from 42.41% to 41.35%, while preserving 91.7% overlap in the top-108 selections and a Jaccard similarity of 0.85. Removing the leading distributor leaves a 16.4-percentage-point VoI advantage at the 20% budget. Relative objective regret below 1.8% across the eight distributor-deletion scenarios further supports the operational usefulness of the selection rule.

The practical sequence is to replenish from stable demand and review consequential uncertainty. Recent consumption retains a separate recognition status, while records with high attainable information gain receive targeted reconciliation. This approach ties inventory investment to demonstrated requirements and directs commercial review toward evidence that can change an attribution. It gives managers a way to act on uncertain records while preserving the information needed to revisit those decisions.

7 Conclusions, Limitations, and Future Research

This study develops and evaluates a flow-constrained probabilistic linkage framework for a non-serialized orthopedic consignment network. The core cohort contains 540 clinical records, comprising 229 with effective upstream candidates and 311 requiring residual-state attribution. Hierarchical linkage, reporting-latency adjustment and quantity-weighted entropy connect these records to a budgeted reconciliation policy. At a 20% review budget, VoI-driven prioritization resolves 52.8% of initial uncertainty, exceeding volume-greedy selection by 18.2 percentage points. Duplicate-treatment and distributor-deletion analyses support the stability of the resulting priorities within the observed cohort.

The study covers one network, one spinal implant family and one steady-state terminal month. Without an independent wholesale-channel comparison group, the outside-ledger state combines legitimate inventory-mode mixing with other recognition pathways. Batch-level support establishes compatible commercial provenance rather than serialized implant identity. Model calibration and policy evaluation are conducted on the de-identified record-level extract under the stated baseline resolvability ($\eta = 0.78$). Policy gains measure information recovery rather than verified discrepancy detection, financial savings or clinical effects. Separating observed quantities from model-dependent attribution follows the logic of partial identification [18].

Longitudinal panels could show whether attribution and audit priorities persist as demand and reporting practices change. Comparisons across product families could examine how specification structure affects the feasible allocation set, while explicit decision-loss functions could connect attribution to replenishment and commercial review. The dependence of information-sharing value on demand dynamics makes these extensions especially relevant [17]. Within the present observation window, the study demonstrates how feasible attribution and quantified uncertainty can guide a limited reconciliation budget.

Statements and Declarations

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