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Multiphase Thermo-Poromechanics on Corner Point Geometry Grids using Finite Volume Method

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Abstract

Geological CO₂ storage is one of the solutions to mitigate anthropological emissions of CO₂ into the atmosphere. The injection of cold CO₂ into hot reservoirs is bringing geomechanical challenges. For industrial numerical packages, the computation of the geomechanics solution is usually based on finite element methods. However, the numerical computation of the moles and heat exchange in porous media is usually based on finite volume methods. The oil & gas industry standard for finite volume grids for flow in porous media is the Corner Point Geometry (CPG). This format allows for faults description and Local Grid Refinements (LGR) that are non-conforming features for the finite element methods. In practice, reservoir simulation users have to modify the CPG grid in order to run the geomechanics solver. This process is not fully automatic and can be long and error prone. Moreover, this adaptation of CPG grid to conforming finite element grid is bringing distorted elements or very elongated elements especially close to the faults. These elements decrease the precision of the finite element solution. To avoid the change of grid for the geomechanics computations and to properly represent the faults, we propose an implementation and a validation of a finite volume method for the geomechanics on CPG grids. We use a variant of the Multi-Point Stress Approximation with weak symmetry (MPSA-W) from Keilegavlen and Nordbotten (2017). We test the method on a set of simple models with a regular grid, with a faulted grid or a grid with pinch-outs with different operating conditions for the pressure and the temperature. Finally we study the geomechanics response of CO₂ injection in a depleted gas field from

the Netherlands. We validate the solutions with reference geomechanics finite element software. This work shows that the MPSA-W method is a good alternative to finite element methods for computing geomechanics on CPG grids.

Keywords: Geomechanics, Thermo-poromechanics, Linear elasticity, Compositional modeling, Finite volume, Corner point geometry, CO₂ storage, Depleted reservoirs

1 Introduction

Geological CO₂ storage is one of the solutions to mitigate anthropological emissions of CO₂ into the atmosphere. It is useful especially for hard-to-abate industries such as waste, cement, biomass, gas power, refineries, and steel. The injection of cold CO₂ into hot reservoirs is presenting geomechanical challenges. The change of pressure and temperature can create fractures, activate faults, and lead to induced seismicity. For industrial numerical packages, the computation of the geomechanics solution is usually based on finite element methods. However, the numerical computation of the moles and heat exchange in porous media is usually based on finite volume methods. The oil & gas industry standard for finite volume grids for flow in porous media is the Corner Point Geometry (CPG) [16]. This format allows for faults description and Local Grid Refinements (LGR) that are non-conforming features for the finite element methods. In practice, reservoir simulation users have to modify the CPG grid in order to run the geomechanics solver. This process is not fully automatic and can be long and error prone.

All industrial packages for geomechanics computation are based on finite element methods with the exception of the attempt of Shaw and Stone [19]. Recent developments in academia show more and more groups developing finite volume methods for the computation of geomechanics with various approaches: Deb and Jenny [4], Keilegavlen and Nordbotten [6], Sokolova et al. [21], Terekhov and Tchelepi [23], Novikov et al. [13], Novikov et al. [14] and Nordbotten and Keilegavlen [12]. We propose a finite volume solution for geomechanics based on the MPSA-W method proposed by Keilegavlen and Nordbotten [6]. This method allows us to simulate the geomechanics equations on CPG grid without modifications of the grid.

We start by the validation of 3 models against analytical solutions. These 3 models have homogeneous properties. One model is a regular grid with all the gridblocks of the same dimension, one model has changes of depth in the gridblocks similar to what can be encountered when modeling faults and one model has a layer disappearing creating pinch-out gridblocks. We then continue our validation using a reference finite element geomechanics simulator [20]. Firstly, we validate an homogeneous model with different injection conditions to vary the pressure and the temperature profiles. Secondly, we continue our validation effort with a heterogeneous model. Thirdly we compare the finite volume solution of the heterogeneous model between the conforming grid and an equivalent non-conforming grid. Fourthly we validate the method on a field model from a depleted gas reservoir from the Netherlands. This model has a conforming grid and can then be simulated with both the reference finite element geomechanics

simulator and our finite volume solution. We also test on the field model the impact of the coupling between the flow equations and the geomechanics equations. Finally, we create a non-conforming model from the depleted reservoir and we validate our MPSA-W solution.

2 Equations

The system of equations modeling mole and energy conservation and thermoporemechanics solution for a system with gas-liquid-water with N components dividing in the gas and liquid phase and one water component in the aqueous phase are the conservation of mole for each component, the conservation of the energy and the mechanics static equilibrium equation (z direction is pointing downward):

$$\frac{\partial}{\partial t} \left[\phi \sum_p x_{p,c} \rho_p S_p \right] + \vec{\nabla} \cdot \left[\sum_p x_{p,c} \rho_p \vec{u}_p \right] = \sum_p x_{p,c} \rho_p q_p \quad (1)$$

$$\frac{\partial}{\partial t} \left[\phi \sum_p (\rho_p H_p - P_p) S_p + (1 - \phi) \rho_r H_r \right] + \vec{\nabla} \cdot \left[\sum_p \rho_p H_p \vec{u}_p - \gamma \vec{\nabla} T \right] = \sum_p \rho_p H_p q_p \quad (2)$$

$$\vec{\nabla} \cdot \vec{\sigma} + \begin{pmatrix} 0 \\ 0 \\ \bar{\rho} g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (3)$$

with P_p the pressure of phase p ($P = P_g$ is the pressure of the gas phase), T the temperature, S_p the saturation of phase p , $x_{p,c}$ the mole fraction of component c in phase p , ϕ the porosity, ρ_p the molar density of phase p , $\vec{u}_p$ the Darcy velocity of phase p , q_p the well source/sink flow for phase p , H_p the molar enthalpy of phase p , $\rho_r H_r$ the rock enthalpy, γ the thermal conductivity of the fluids and the rock, $\vec{\sigma}$ the total stress tensor, and the bulk mass density $\bar{\rho} = \phi(\bar{\rho}_g S_g + \bar{\rho}_l S_l + \bar{\rho}_a S_a) + (1 - \phi) \bar{\rho}_r$, with $\bar{\rho}_p$ for $p = g, l, a$ the mass density of gas, liquid and aqueous phase, respectively and $\bar{\rho}_r$ the mass density of the rock.

The Darcy velocity for each phase is given by

$$\vec{u}_p = -\lambda_p K \vec{\nabla} (P_p - \bar{\rho}_p g D). \quad (4)$$

with λ_p the mobility of phase p (relative permeability divided by viscosity), K the absolute permeability, g the gravity and D the depth.

Assuming linear elasticity, the generalized Hooke's law gives the total stress tensor

$$\vec{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \quad (5)$$

as function of the displacement vector

$$\vec{U} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad (6)$$

the strain tensor

$$\vec{\epsilon} = \frac{1}{2} \left(\vec{\nabla} \vec{U} + \vec{\nabla}^T \vec{U} \right) = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{\partial v}{\partial y} & \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) & \frac{\partial w}{\partial z} \end{pmatrix}, \quad (7)$$

the pressure and the temperature (compression is negative and tension is positive) [3]:

$$\vec{\sigma} = \lambda (\vec{\nabla} \cdot \vec{U}) \vec{I} + 2\mu \vec{\epsilon} - \alpha P \vec{I} - (3\lambda + 2\mu) \alpha_T (T - T_0) \vec{I} \quad (8)$$

or

$$\begin{cases} \sigma_{xx} = (\lambda + 2\mu) \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} + \lambda \frac{\partial w}{\partial z} - \alpha P - (3\lambda + 2\mu) \alpha_T (T - T_0) \\ \sigma_{yy} = \lambda \frac{\partial u}{\partial x} + (\lambda + 2\mu) \frac{\partial v}{\partial y} + \lambda \frac{\partial w}{\partial z} - \alpha P - (3\lambda + 2\mu) \alpha_T (T - T_0) \\ \sigma_{zz} = \lambda \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} + (\lambda + 2\mu) \frac{\partial w}{\partial z} - \alpha P - (3\lambda + 2\mu) \alpha_T (T - T_0) \\ \sigma_{xy} = \sigma_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\ \sigma_{xz} = \sigma_{zx} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \sigma_{yz} = \sigma_{zy} = \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \end{cases} \quad (9)$$

with $\vec{I}$ the identity tensor, E the Young's modulus, ν the Poisson's ratio, $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ the first Lamé coefficient, $\mu = \frac{E}{2(1+\nu)}$ the second Lamé coefficient, α the Biot coefficient, α_T the drained thermal expansion coefficient and T_0 the initial temperature. The effective stress tensor is given as:

$$\vec{\sigma}' = \begin{pmatrix} \sigma'_{xx} & \sigma'_{xy} & \sigma'_{xz} \\ \sigma'_{yx} & \sigma'_{yy} & \sigma'_{yz} \\ \sigma'_{zx} & \sigma'_{zy} & \sigma'_{zz} \end{pmatrix} = \begin{pmatrix} \sigma_{xx} + \alpha P & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} + \alpha P & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} + \alpha P \end{pmatrix} = \vec{\sigma} + \alpha P \vec{I}. \quad (10)$$

In order to close the system, we need a set of constraint equations. First the equality of the chemical potentials between the gas and the liquid phases for component c

$$\hat{f}_{c,g}(P, x_{g,1}, \dots, x_{g,N}) = \hat{f}_{c,l}(P, x_{l,1}, \dots, x_{l,N}) \quad (11)$$

with $\hat{f}_{c,p}$ is the chemical potential of component c in phase p . Second the capillary pressure relationships:

$$P_p = P + P_{c_p}, \quad (12)$$

with P_{c_p} the capillary pressure between phase p and the gas phase. Third the mole fraction equality for each phase p :

$$\sum_c x_{p,c} = 1. \quad (13)$$

Finally the saturation constraint:

$$\sum_p S_p = 1. \quad (14)$$

The porosity is function of the volumetric strain change, the pressure change and the temperature change [1, 3] (assuming the solid matrix thermal expansion coefficient equals the drained thermal expansion coefficient):

$$\phi = \phi_0 + \alpha(\epsilon_v - \epsilon_{v_0}) + \frac{(1 - \alpha)(\alpha - \phi_0)}{K_{dr}}(P - P_0) - 3(\alpha - \phi_0)\alpha_T(T - T_0) \quad (15)$$

with ϕ_0 the initial porosity, ϵ_v the volumetric strain, ϵ_{v_0} the initial volumetric strain, $K_{dr} = \frac{3\lambda+2\mu}{3} = \frac{E}{3(1-2\nu)}$ the drained bulk modulus, P_0 the initial pressure. The volumetric strain is the trace of the strain tensor

$$\epsilon_v = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}. \quad (16)$$

We can have a one-way coupling mode that computes the geomechanics solutions at fixed frequencies: at each timestep or at given times for converged flow solutions. We can choose to update the volumetric strain or alternatively to keep it constant in order to have the porosity only function of pressure and temperature. In addition, we can also have a coupled mode where the flow and geomechanics solutions are solved in a fully coupled fully implicit way or with the iterative sequential fixed-stress coupling scheme [7].

3 Discretization

We use the finite volume method with N_b gridblocks to solve the system of equations. We discretize in time with first-order implicit backward Euler. We discretize the Darcy's fluxes with phase-potential upstream weighting [15]. If we integrate the mechanics static equilibrium Eq. 3 over a control volume Ω_I we get the following equation:

$$\int_{\Omega_I} \left[\vec{\nabla} \cdot \vec{\sigma} + \begin{pmatrix} 0 \\ 0 \\ \bar{\rho}g \end{pmatrix} \right] dV = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \quad (17)$$

Using Gauss' theorem, we transform the first part of the integral from a volumetric integral to a surface integral with $\vec{n}$ the outward normal and $\vec{T}^\sigma = \vec{\sigma} \cdot \vec{n}$ the traction vector:

$$\int_{\delta\Omega_I} \vec{T}^\sigma dA + \int_{\Omega_I} \begin{pmatrix} 0 \\ 0 \\ \bar{\rho}g \end{pmatrix} dV = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \quad (18)$$

The traction vectors $\vec{T}^\sigma$ are computed by the MPSA-W method from Keilegavlen and Nordbotten [6]. The default boundary conditions are

- $u = 0$ (roller boundary condition) and $T_y^\sigma = T_z^\sigma = 0$ on faces X- and X+
- $v = 0$ (roller boundary condition) and $T_x^\sigma = T_z^\sigma = 0$ on faces Y- and Y+
- $w = 0$ (roller boundary condition) and $T_x^\sigma = T_y^\sigma = 0$ on face Z+ (face looking downwards)
- $T_x^\sigma = T_y^\sigma = T_z^\sigma = 0$ on Z- (face looking upwards).

We can also fix forces on the external faces as boundary conditions. The discretization of $\int_f T^\sigma dA$ for face f is written as:

$$\begin{cases} \int_f T_x^\sigma dA = \sum_{I \in \Gamma_f} \left(T_{f,I}^{u,xx} u_I + T_{f,I}^{u,xy} v_I + T_{f,I}^{u,xz} w_I - T_{f,I}^x [\alpha_I P_I + (3\lambda_I + 2\mu_I) \alpha_{T_I} (T_I - T_{0_I})] \right) \\ \int_f T_y^\sigma dA = \sum_{I \in \Gamma_f} \left(T_{f,I}^{u,yx} u_I + T_{f,I}^{u,yy} v_I + T_{f,I}^{u,yz} w_I - T_{f,I}^y [\alpha_I P_I + (3\lambda_I + 2\mu_I) \alpha_{T_I} (T_I - T_{0_I})] \right) \\ \int_f T_z^\sigma dA = \sum_{I \in \Gamma_f} \left(T_{f,I}^{u,zx} u_I + T_{f,I}^{u,zy} v_I + T_{f,I}^{u,zz} w_I - T_{f,I}^z [\alpha_I P_I + (3\lambda_I + 2\mu_I) \alpha_{T_I} (T_I - T_{0_I})] \right) \end{cases} \quad (19)$$

with $T_{f,I}^{u,**}$ and $T_{f,I}^*$ the geomechanics transmissibilities for face f and gridblock I . The I subscript in u , v , w , α , P , λ , μ , α_T , T and T_0 specifies properties of gridblock I . Γ_f is the set of gridblocks participating in the traction discretization for face f . For each face between 2 control volumes, the method is based on the continuity of the traction at 4 points across the face. More details on the computation of the geomechanics transmissibilities can be found in Raynaud et al. [17]. The geomechanics transmissibilities MPSA-W are computed in a preprocessing step by the MRST module MPSA [9]. We solve the discretized system of Eqs. 1, 2 and 3 with the Newton-Raphson algorithm. For the one-way coupling mode, the linear solver uses FGMRES with CPR(AMG,BILU0) for the flow equations [2, 8, 18, 24] and FGMRES with AMG [22] for the geomechanics equations. For the coupled mode, we can solve the system in a fully coupled fully implicitly way with the linear solver FGMRES and the preconditioner strategy of White et al. [26] with CPR for the flow subsystem and AMG for the geomechanics subsystem. We can also solve the coupled mode with the iterative sequential fixed-stress coupling scheme of Kim et al. [7] with CPR-FGMRES for the flow system and AMG-FGMRES for the geomechanics system. For the coupled mode algorithms, we need to fix the volumetric mean total stress

$$\sigma_v = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} = K_{dr} \epsilon_v - \alpha P - 3K_{dr} \alpha_T (T - T_0) \quad (20)$$

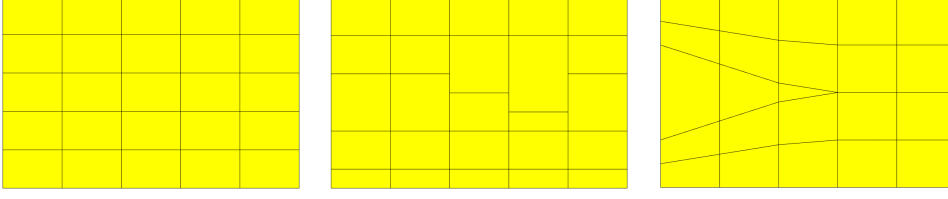


Fig. 1 First model is regular, second model has faults and third model has pinch-out gridblocks.

to get the variation of the volumetric strain as a variation of pressure and temperature:

$$\delta\epsilon_v = \frac{\alpha}{K_{dr}}\delta P + 3\alpha_T\delta T. \quad (21)$$

Both the pressure system from CPR and the geomechanics system are solved with BoomerAMG [5]. The test cases have been done with an in-house research platform: TotalEnergies Prototyping Platform (TPP) [10, 11].

4 Validation

4.1 Validation of Geomechanics against Analytical Solutions

We have 3 models with homogeneous conditions filled with aqueous phase. In case of constant mass densities for the aqueous phase and the rock and constant geomechanics properties, the total stresses have analytical expressions dependent on depth:

$$\begin{cases} \sigma_{xx} = \frac{\nu}{1-\nu} (-\bar{\rho}g(D - D_{Z-}) + \alpha P) - \alpha P \\ \sigma_{yy} = \sigma_{xx} \\ \sigma_{zz} = -\bar{\rho}g(D - D_{Z-}) \end{cases} \quad (22)$$

with D the depth and D_{Z-} the depth of face Z- (top face looking upwards). The pressure is fixed at 10 bars at the top of the reservoir. The density of the aqueous phase is 937.9 kg/m³, the density of the rock is 2640.5 kg/m³, the porosity is 0.2, Young modulus is 25 GPa, Poisson ratio is 0.25 and Biot coefficient is 0.8. Figure 2 shows the analytical solutions for both $\sigma_{xx}(D)$ and $\sigma_{zz}(D)$.

Figure 3 shows the difference between the column that has the worst results for each of the 3 models and the analytical solution for $\sigma_{xx}(D)$. The regular model has perfect agreement with the analytical solution. We observe some errors of the order of 0.15 bars for the fault model and 0.23 bars for the pinch-out model. If we compare to the variations of $\sigma_{xx}(D)$ of 10 bars it corresponds to an error of 1.5% and 2.3% on these highly distorted grids. Figure 4 shows the difference between the column that has the worst results for each of the 3 models and the analytical solution for $\sigma_{yy}(D)$. The regular model has perfect agreement with the analytical solution. We observe some errors of the order of 0.33 bars for the fault model and 0.04 bars for the pinch-out model.

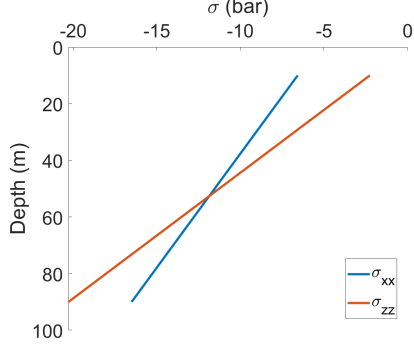


Fig. 2 Analytical solutions for both $\sigma_{xx}(D)$ and $\sigma_{zz}(D)$.

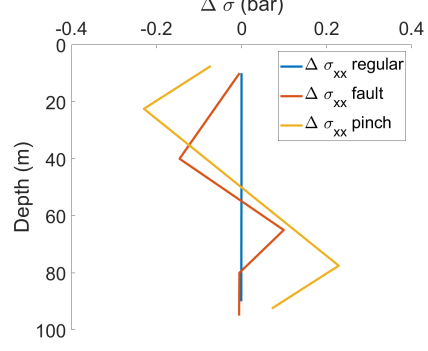


Fig. 3 Difference between numerical σ_{xx} and analytical values.

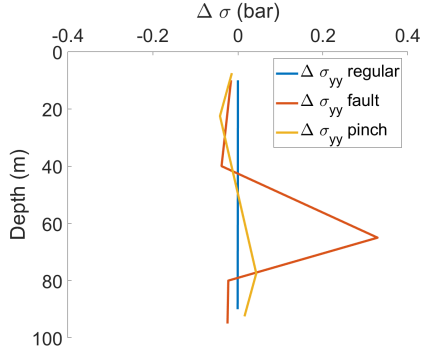


Fig. 4 Difference between numerical σ_{yy} and analytical values.

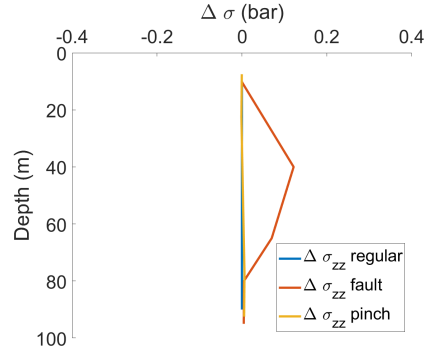


Fig. 5 Difference between numerical σ_{zz} and analytical values.

Figure 5 shows the difference between the column that has the worst results for each of the 3 models and the analytical solution for $\sigma_{zz}(D)$. The regular model has perfect agreement with the analytical solution. We observe some errors of the order of 0.12 bars for the fault model and 0.006 bars for the pinch-out model.

4.2 Validation of Geomechanics on an Homogeneous Model

In this section, we focus on the validation of the geomechanics computations on an homogeneous model for the geomechanics properties. The model is of size $25 \times 25 \times 55$ with $\Delta X = 200$ m, $\Delta Y = 120$ m, $\Delta Z = 9 \times 277.8$ m, 9×55.56 m, 9×22.22 m, 19×5.26 m, 9×188.89 m. Porosity is 0.2 in layers 28 to 46 and 0.1 otherwise. Permeabilities are 1013.25 mD in layers 28 to 46 and $1e-6$ mD otherwise. The rock enthalpy is $\rho_r H_r = 2350.0 \times (T - 120 \text{ degC})$ kJ/m³, the thermal conductivity is 150.0 kJ/m/day/degC,

the rock density is 2650.0 kg/m³, Young's modulus is 50 GPa, Poisson's ratio is 0.25, Biot coefficient is 1.0 and the drained thermal expansion is $1e-5$ 1/degC. The top of the reservoir is at the depth 0 m. We initialize the reservoir full of aqueous phase with $P = 325$ bar at depth 3250 m and $T = 120$ degC everywhere.

The first validation are the total stresses at the initialization. For this homogeneous model, the initial total stresses can be given with Eqs. 22. We compare our initial total stresses against the analytical solution of Eqs. 22 and a reference finite element solution [20]. Figure 6 plots the analytical solutions for σ_{xx} and σ_{zz} as well as the finite volume (FV) and finite element (FE) solutions. The analytical solutions, the FV solutions and the FE solutions are in very good agreement. For clarity, we only show the σ_{xx} and the σ_{zz} , the σ_{yy} are similar to the σ_{xx} .

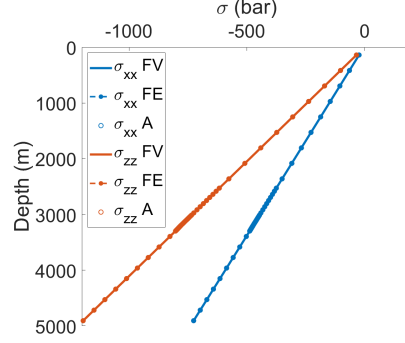


Fig. 6 Analytical, FV and FE solutions for both σ_{xx} and σ_{zz} for homogeneous case.

Now that the initial total stresses have been validated, we look at the impact of the pressure and the temperature on the total stresses. We set an injector at the gridblock (13 13 37) and 4 producers at the gridblocks (1 1 37), (25 1 37), (1 25 37), (25 25 37). We run 3 simulations. One isothermal injection of water without producers at 766368.0 kg/day for 20 years, one injection of water at 766368.0 kg/day at 20 degC without producers resulting in a sharp temperature gradient and finally one injection of water at 766368.0 kg/day at 20 degC with the 4 producers resulting in a smooth temperature gradient. We compare our finite volume stresses computations against the finite element solutions. We show the stresses versus depth along the column of the injection gridblock: (13 13 1-55). For the isothermal injection case, the pressure in the reservoir layers 28 to 46 increases from 380 bars to 700 bars while the pressure in the overburden (layers 1 to 27) and the underburden (layers 47 to 55) remains close to the initial values. Figure 7 shows the total stresses for FV and FE. The solutions are in good agreement. Figure 8 shows the total stresses for FV and FE for the injection at 20 degC in a reservoir at 120 degC. In this solution, the temperature has a very sharp profile in the XY plane. The gridblock of injection (13 13 37) has a temperature of 20 degC and the next surrounding gridblocks have temperature around 59 degC and 82 degC. The temperature profile is not fully resolved. As the numerical methods are different, the results have differences close to the sharp temperature profile. We

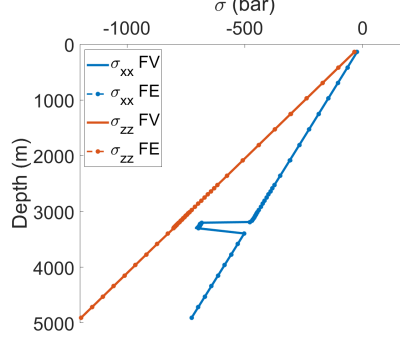


Fig. 7 σ_{xx} and σ_{zz} for FV and FE for isothermal injection of homogeneous case.

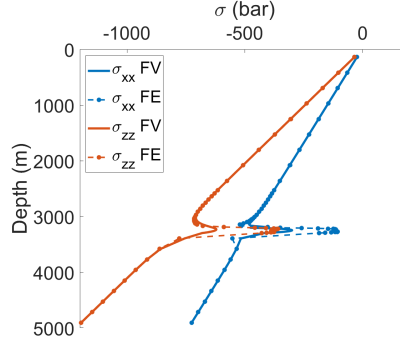


Fig. 8 σ_{xx} and σ_{zz} for FV and FE for first thermal injection of homogeneous case.

now look at the same simulation with a refined grid around the injector. We refine by 5 in X and Y directions for the 3×3 gridblocks in the XY plane around the injector. In this situation, it takes 35 small gridblocks in the XY plane and 5 gridblocks in the vertical direction to change from 20 degC close to the injector to 120 degC in the rest of the reservoir. Figure 9 shows the FV and FE solution for the refined model. The FV and FE solutions are now close. The coarse scale σ_{xx} with FE is closer to the fine scale solution than the coarse scale σ_{xx} with FV. It is the contrary for σ_{zz} : the coarse scale with the FV solution is closer to the fine scale solution than the coarse scale FE solution. In this situation, we cannot say which solution is better at the coarse scale. This highlights the importance of grid refinement studies to stabilize the numerical solution. Figure 10 shows the total stresses for FV and FE for the injection at 20 degC in a reservoir at 120 degC with 4 producers to have a smoother temperature. In this solution, it takes 8 gridblocks in the XY plane to increase from 20 degC of the gridblock of injection (13 13 37) to the 120 degC in the reservoir. The FV solution of σ_{xx} is in good agreement with the FE solution but the peak in the σ_{zz} is not as sharp in the FV method compared to the FE method.

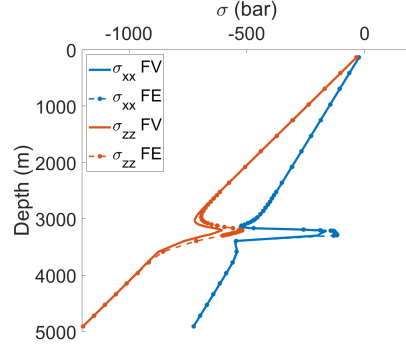


Fig. 9 σ_{xx} and σ_{zz} for FV and FE for first thermal injection of refined homogeneous case.

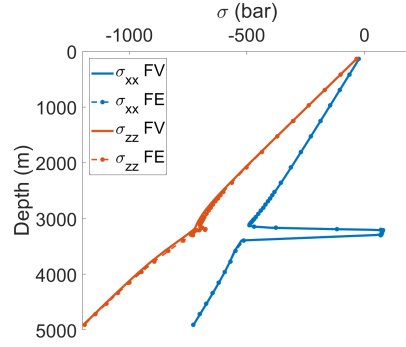


Fig. 10 σ_{xx} and σ_{zz} for FV and FE for second thermal injection of homogeneous case.

4.3 Validation of Geomechanics on an Heterogeneous Model

In this section, we focus on the validation of the geomechanics computations on an heterogeneous model for the geomechanics properties. We use the same settings as for the homogeneous test case except for heterogeneous rock density, Young's modulus and Poisson's ratio. Rock density is 2475 kg/m³ in layers 1 to 9, 2400 kg/m³ in layers 10 to 18, 2700 kg/m³ in layers 19 to 27, 2642 kg/m³ in layers 28 to 46 and 2600 kg/m³ in layers 47 to 55. Young's modulus is 5 GPa in layers 28 to 46 and 10 GPa otherwise. Poisson's ratio is 0.3 in layers 1 to 18, 0.35 in layers 19 to 27, 0.2 in layers 28 to 46 and 0.25 in layers 47 to 55.

The first validation are the total stresses at the initialization. For this heterogeneous model layer by layer, the initial total stresses can be given by Eqs. 23.

$$\begin{cases} \sigma_{xx} = \frac{\nu}{1-\nu} \left(- \int_{D_{Z-}}^D \bar{\rho} g dD + \alpha P \right) - \alpha P \\ \sigma_{yy} = \sigma_{xx} \\ \sigma_{zz} = - \int_{D_{Z-}}^D \bar{\rho} g dD \end{cases} \quad (23)$$

We compare our initial total stresses against the analytical solution of Eqs. 23 and the FE solution. Figure 11 shows the analytical σ_{xx} and the σ_{zz} as functions of depth as well as the FV solution and the FE solution. The initial FV and FE are in very good agreement with the analytical solutions. Now that the initial total stresses have

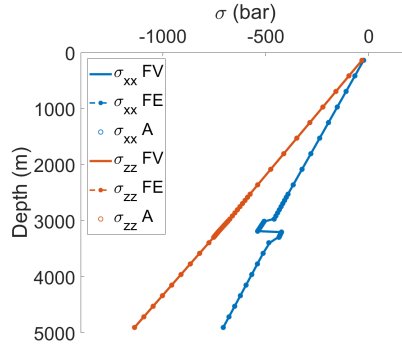


Fig. 11 Analytical, FV and FE solutions for both σ_{xx} and σ_{zz} for heterogeneous case.

been validated, we look at the impact of the pressure and the temperature on the total stresses. The three simulations are the same as for the homogeneous case, only the geomechanics coefficients are modified. We compare our FV stresses computations against the FE solution. Figure 12 shows the total stresses for FV and FE for the isothermal injection of water. The solutions are in very good agreement. Figure 13 shows the total stresses for FV and FE for the cold water injection with sharp temperature front. We still observe some differences between the FV and the FE solutions but they are smaller than for the homogeneous case. Figure 14 shows the total stresses for FV and FE for the cold water injection with smooth temperature front. The FV and the FE solutions are in very good agreement.

4.4 Validation of Geomechanics on an Heterogeneous Non-Conforming Model

We use the same model as in the previous section, but in addition we add 10 m to the depths of all the gridblocks with J equals to 1 to 13. This represents the situation of a

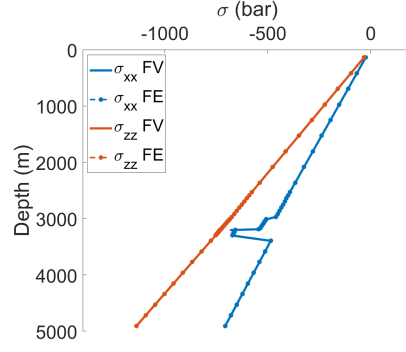


Fig. 12 σ_{xx} and σ_{zz} for FV and FE for isothermal injection of heterogeneous case.

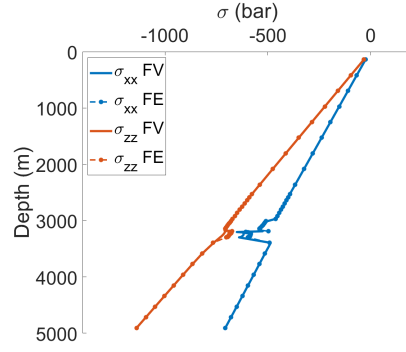


Fig. 13 σ_{xx} and σ_{zz} for FV and FE for first thermal injection of heterogeneous case.

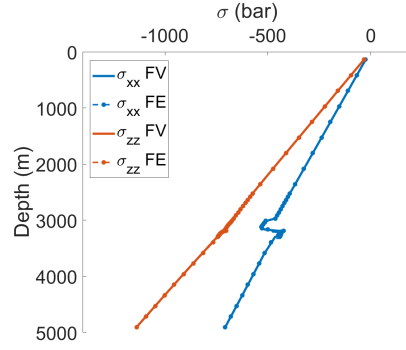


Fig. 14 σ_{xx} and σ_{zz} for FV and FE for second thermal injection of heterogeneous case.

fault. This model has a non-conforming grid and cannot be solved with a FE method. Figure 15 shows the Poisson's ratio for this new model. Figure 16 shows the σ_{xx} and

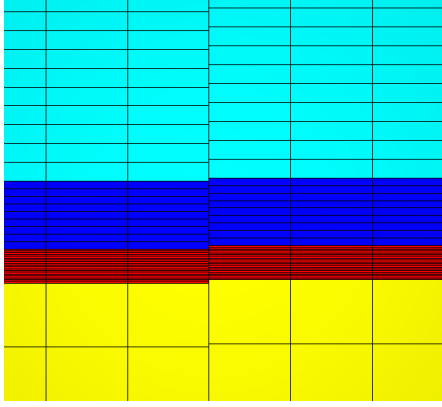


Fig. 15 Poisson's ratio for the heterogeneous model with a fault, creating a non-conforming grid.

the σ_{zz} as functions of depth for the column [1 21] for the new model named "FLT" and the previous reference model named "REF". The column [1 21] is in the region of the 2 models where the depths for the gridblocks are identical, so the solutions should be very close. The solutions are in exact agreement.

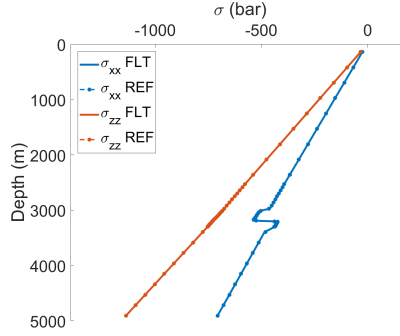


Fig. 16 σ_{xx} and σ_{zz} for FLT and REF cases at initialization.

4.5 Validation of Geomechanics on a CO₂ Injection Project into a Depleted Gas Reservoir

We now validate our solution with a CO₂ injection project into a depleted gas reservoir from the Netherlands. We call the model case A. The gas reservoir is surrounded by over, under and side burdens. The model has dimensions of 12,700 m by 17,200 m by 15,000 m. The total number of gridblocks in the model is 1,769,364. The model has six gas producers and one water injector. The field has been produced for 46 years. After this period of production, the wells are closed and the water injector well is converted to a CO₂ injection well. Pure CO₂ at 23.5 degC is injected into the reservoir at 130 degC

for 26 years. The Young modulus has values ranging from 0.64 GPa to 49 GPa. The Poisson's ratio has values ranging from 0.17 to 0.48. The values of Poisson's ratio at 0.48 correspond to a salt dome inside the model. These heterogeneities give large Lamé coefficients of around 43 GPa next to values at 5 GPa. The initial model had a value of 0.499 for the Poisson's ratio in the salt dome area. This gave even larger heterogeneous Lamé coefficients. Converging the geomechanics equations at a relative tolerance of $1.0e-6$ required 242 iterations with AMG-FGMRES. In a second time, we tested a Poisson's ratio of 0.48 in the salt dome area. The physical results are very close, and the AMG-FGMRES solver is now converging in 42 iterations. First, we validate the total stresses versus depth at the initialization time, when the gas field has not started production. We validate the solution in several vertical columns in the model. For simplicity, we present the solution only in one column along the injection well. Figure 17 shows the total stresses σ_{xx} , σ_{yy} and σ_{zz} for the FV and FE solutions at initialization before production of the gas field for the full model and Figure 18 shows for the first 5000 m of the model. We see that the FV and FE solutions are in very good agreement. We observe an unstable point around 3500 m for the σ_{zz} of the FE solution that does not exist in the FV solution where the σ_{zz} is smoother. The

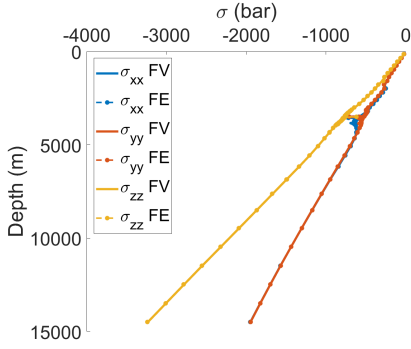


Fig. 17 σ_{xx} , σ_{yy} and σ_{zz} for FV and FE at initialization before production of the field. View for the full model depth.

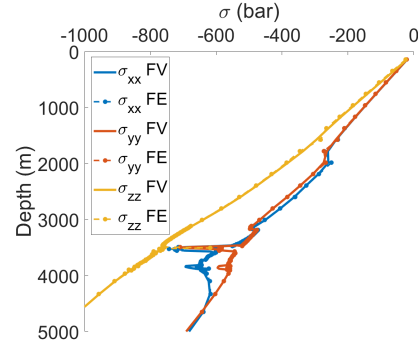


Fig. 18 σ_{xx} , σ_{yy} and σ_{zz} for FV and FE at initialization before production of the field. View for the first 5000 m of the model.

second test is at the end of the production period of 46 years. We plot in Figure 19 the variation of the total stresses between the end of the production and the beginning of the production for the full depth of the model. Figure 20 shows for the first 5000 m of the model. The FV and FE solutions are in good agreement. The total stresses have large peaks at the level of the reservoir at a depth around 3800 m. $\Delta\sigma_{xx}$, $\Delta\sigma_{yy}$, $\Delta\sigma_{zz}$ have maximums of 128 bars, 142 bars, 13 bars, respectively for the FV and 120 bars, 127 bars, 29 bars, respectively for the FE. The last test is at the end of the CO₂ injection period of 26 years. We plot in Figure 21 the variation of the total stresses between the end of the injection period and the end of the production period for the full depth of the model. Figure 22 shows for the first 5000 m of the model. We observe

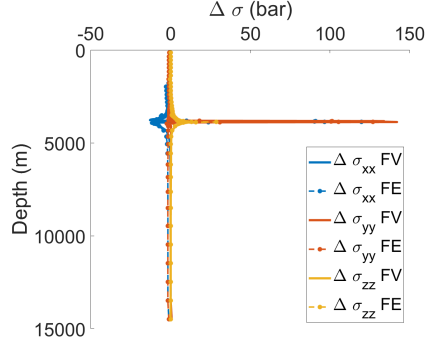


Fig. 19 $\Delta\sigma_{xx}$, $\Delta\sigma_{yy}$ and $\Delta\sigma_{zz}$ for FV and FE between the end of production period of the field and the initial state. View for the full model depth.

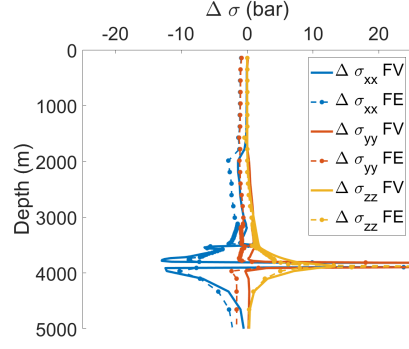


Fig. 20 $\Delta\sigma_{xx}$, $\Delta\sigma_{yy}$ and $\Delta\sigma_{zz}$ for FV and FE between the end of production period of the field and the initial state. View for the first 5000 m of the model.

that the stresses are in very good agreement between the FV and the FE solutions. $\Delta\sigma_{xx}$, $\Delta\sigma_{yy}$, $\Delta\sigma_{zz}$ have maximums of 117 bars, 120 bars, 34 bars, respectively for the FV and 127 bars, 128 bars, 45 bars, respectively for the FE.

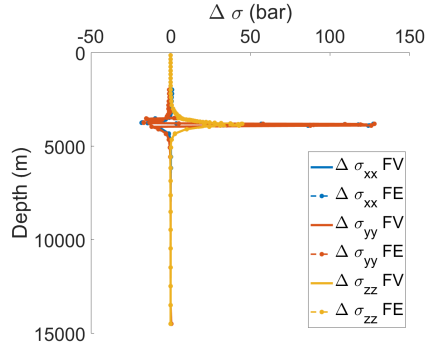


Fig. 21 $\Delta\sigma_{xx}$, $\Delta\sigma_{yy}$ and $\Delta\sigma_{zz}$ for FV and FE between the end of the CO₂ injection period and the end of production period of the field. View for the full model depth.

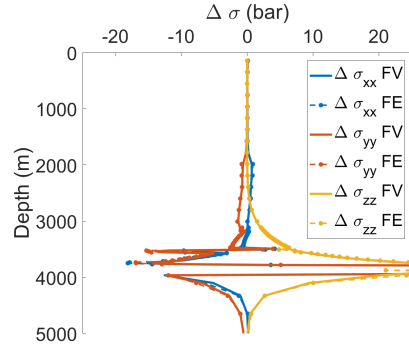


Fig. 22 $\Delta\sigma_{xx}$, $\Delta\sigma_{yy}$ and $\Delta\sigma_{zz}$ for FV and FE between the end of the CO₂ injection period and the end of production period of the field. View for the first 5000 m of the model.

4.6 Impact of Coupling Flow/Geomechanics

We use in this section the model case A. The Coupling Strength Ratio (CSR) defined by Waziri et al. [25] for the model case A is between 0.045 and 2.6. The maximum values of 2.6 are located in the overburden. In the region of the reservoir, the value

of CSR is around 0.2. It is then possible to efficiently run the simulation with the iterative sequential fixed-stress coupling scheme. We compare 3 simulations: 1) one-way coupling scheme (OW), 2) fully coupled fully implicit scheme (FC), 3) iterative sequential fixed-stress coupling scheme (SC). For the one-way coupling scheme, every simulated year, we converge the geomechanics equations with a linear solver tolerance of $1.0e - 6$ and we update the volumetric strain change in the porosity correlation of Eq.15. For the fully coupled fully implicit scheme, we converge the full system of mole, energy and geomechanics equations with a tolerance of $1.0e - 4$ for the linear solver. The tolerance is looser because we solve several linear systems before convergence compared to the one-way coupling scheme that only solves one geomechanics linear system before accepting the solution. For the iterative sequential fixed-stress coupling scheme, we also use the tolerance of $1.0e - 4$ for the flow and the geomechanics linear solvers. The 3 simulations give the same physical results. OW is less stable than the FC and SC scheme and has to take smaller timesteps when the porosity is updated every year. The OW flow system converges in an average of 2 Newton iterations per timestep. FC converges in an average of 4.3 Newton iterations per timestep. The SC scheme has an average of 3.2 outer iterations per timestep. The geomechanics systems of OW converge with an average of 42.2 linear iterations. The linear systems of FC converge with an average of 32.9 linear iterations. The geomechanics systems of SC converge with an average of 16.7 linear iterations. In this situation with small CSR between the flow and the geomechanics equations, the OW scheme is enough to give the good answer.

4.7 Simulation on a Non-Conforming Grid Created from Case A

In this model, we create a non-conforming finite volume grid from the conforming grid of the case A field. For all the gridblocks with J equals to 1 to 83 we add 50 m to their depths. This represents a fault. Gridblocks can now have several neighbors through a face as shown in Figure 23. This mesh cannot be simulated with finite element

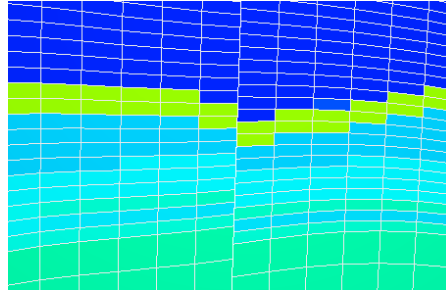


Fig. 23 Young modulus for the non-conforming finite volume grid of case A.

methods. In order to run this model, we have developed a variant of the MPSA-W method where we have modified the influence zones for the gradient computations. Figures 24, 25 and 26 show how we construct gradients on larger domains than in the

conventional MPSA-W method. Using an approximation of the gradient on the region $\Omega_{2,a} \cup \Omega_{2,b}$ instead of different gradients on regions $\Omega_{2,a}$ and $\Omega_{2,b}$ is continuous with the approach when there is no fault, i.e. $s_a = s_b$. More details are given in Raynaud et al. [17].

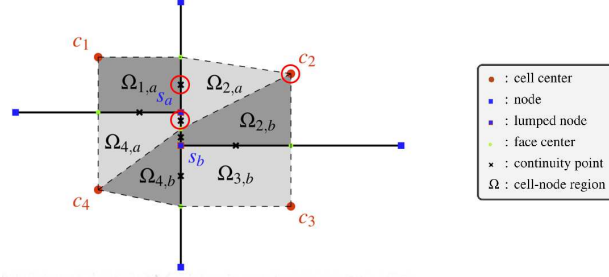


Fig. 24 For this 2D example, the conventional method builds the gradient on $\Omega_{2,a}$ with 3 points circled in red. The gradient is constant on $\Omega_{2,a}$.

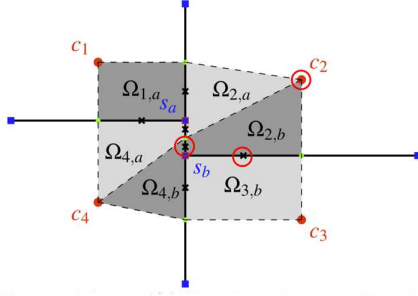


Fig. 25 For this 2D example, the conventional method builds the gradient on $\Omega_{2,b}$ with 3 points circled in red. The gradient is constant on $\Omega_{2,b}$.

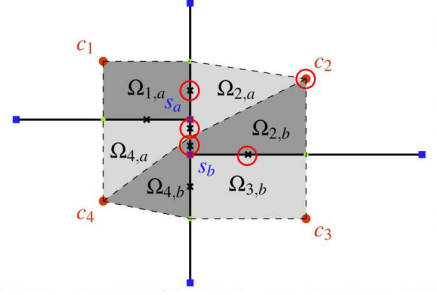


Fig. 26 For this 2D example, the variant method builds the gradient on $\Omega_{2,a} \cup \Omega_{2,b}$ with 5 points circled in red. We use least-square fitting to have a constant gradient on $\Omega_{2,a} \cup \Omega_{2,b}$.

The geomechanics system of this grid converges in 50 iterations with a linear solver tolerance of $1.0e - 6$ with AMG-FGMRES. Figure 27 shows the σ_{xx} , σ_{yy} and the σ_{zz} as functions of depth for the column [75 99] for the new model named "FLT" and the previous reference model named "REF". The column [75 99] is in the region of the 2 models where the depths for the gridblocks are identical, so the solutions should be close. We observe that the solutions are in perfect agreement.

Figure 28 shows the σ_{xx} , σ_{yy} and the σ_{zz} as functions of depth for the column [63 84] for the new model named "FLT" and the previous reference model named "REF". The column [63 84] is a column adjacent to the fault. It is still in the region of the 2 models where the depths for the gridblocks are identical, so the solutions should be

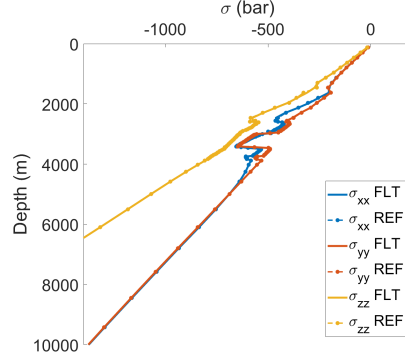


Fig. 27 σ_{xx} , σ_{yy} and σ_{zz} for FLT and REF of case A at initialization for column [75 99].

close. We observe that the solutions in the vicinity of the fault are in perfect agreement.

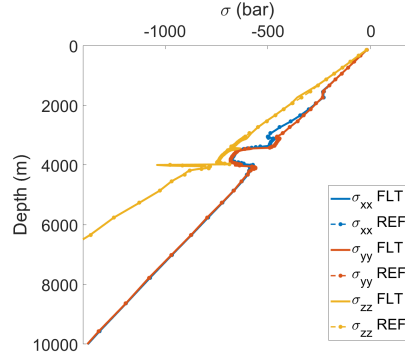


Fig. 28 σ_{xx} , σ_{yy} and σ_{zz} for FLT and REF of case A at initialization for column [63 84].

5 Conclusion

The MPSA-W method has been implemented in a coupled flow/geomechanics solution. We can directly compute the geomechanics on Corner Point Geometry grids used for the flow simulation with non-conforming features such as faults. It is then not necessary to modify the grids used for the flow simulation to compute the geomechanics response, as is the case for finite element methods. The solution has been validated on synthetic models as well as a field model giving the same solution as established software. We have tested the initialization of the stresses computation in addition to various pressure and temperature conditions. This shows that the MPSA-W method is a good alternative to finite element methods on CPG grids. In one situation, with Poisson's ratio of 0.499 and large discontinuities of the geomechanics coefficients, we

have observed a small difference of our FV solution that accumulates at one corner of the model compared to the FE solution. This situation has a singular solution and we have decided that both FV and FE solutions are acceptable. This situation does not impact the solutions in the middle of the model. The MPSA-W solution has been tested on some models with non-conforming features that cannot be solved with finite element methods with very good results. Future work will concentrate on models with Local Grid Refinements (LGR).

Nomenclature

- D = depth
- E = Young's modulus
- $\hat{f}_{c,p}$ = chemical potential of component c in phase p
- g = gravity
- H_p = molar enthalpy of phase p
- $\vec{I}$ = identity tensor
- K = absolute permeability
- $K_{dr} = \frac{3\lambda+2\mu}{3} = \frac{E}{3(1-2\nu)}$ = drained bulk modulus
- $\vec{n}$ = outward normal of surface $\delta\Omega_I$
- P_p = pressure of phase p
- $P = P_g$ = pressure of the gas phase
- P_0 = initial pressure of the gas phase
- P_{c_p} = capillary pressure between phase p and the gas phase
- q_p = well source/sink flow for phase p
- S_p = saturation of phase p
- T = temperature
- T_0 = initial temperature
- $\vec{T}^\sigma$ = traction vector
- T_*^σ with $*$ as x, y, z = components of traction vector
- $T_{f,I}^{u,**}$ and $T_{f,I}^*$ with $*$ as x, y, z = geomechanics transmissibilities for face f and gridblock I
- $\vec{U}$ = displacement vector
- $\vec{u}_p$ = Darcy velocity of phase p
- u = displacement in x direction
- v = displacement in y direction
- w = displacement in z direction
- $x_{p,c}$ = mole fraction of component c in phase p
- α = Biot coefficient
- α_T = drained thermal expansion coefficient
- γ = thermal conductivity of the fluids and the rock
- $\delta\Omega_I$ = surface of the control volume
- $\vec{\epsilon}$ = strain tensor
- $\epsilon_v = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ = volumetric strain
- ϵ_{v_0} = initial volumetric strain

- $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ = first Lamé coefficient
- λ_p = mobility of phase p
- $\mu = \frac{E}{2(1+\nu)}$ = second Lamé coefficient
- ν = Poisson's ratio
- ρ_p = molar density of phase p
- $\rho_r H_r$ = rock enthalpy
- $\bar{\rho}$ = bulk mass density
- $\bar{\rho}_p$ = mass density of phase p
- $\bar{\rho}_r$ = mass density of the rock
- $\vec{\sigma}$ = total stress tensor
- $\vec{\sigma}'$ = effective stress tensor
- $\sigma_v = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$ = volumetric mean total stress
- ϕ = porosity
- ϕ_0 = initial porosity
- Ω_I = control volume

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Declarations

The authors declare no conflicts of interest or competing interests. Data for synthetic cases can be made available from the corresponding author on reasonable request.

References

- [1] M. Boutéca and J.P. Sarda. *Éléments de Thermoporoélasticité. Chapter in book "Ecole de Mécanique des Milieux Poreux", Aussois*, pages 23–32, 1994.
- [2] H. Cao, H.A. Tchelepi, J. Wallis, and H. Yardumian. Parallel Scalable Unstructured CPR-Type Linear Solver for Reservoir Simulation. In *SPE Annual Technical Conference and Exhibition*. Society of Petroleum Engineers (SPE), 2005. doi: 10.2118/96809-ms. URL <http://dx.doi.org/10.2118/96809-MS>.
- [3] O. Coussy. *Poromechanics*. Wiley, 2004. ISBN 9780470849200. doi: 10.1002/0470092718. URL <https://doi.org/10.1002/0470092718>.
- [4] R. Deb and P. Jenny. Finite Volume-Based Modeling of Flow-Induced Shear Failure along Fracture Manifolds. *International Journal for Numerical and Analytical Methods in Geomechanics*, 41(18):1922–1942, 2017. doi: 10.1002/nag.2707. URL <https://doi.org/10.1002/nag.2707>.
- [5] V.E. Henson and U.M. Yang. BoomerAMG: A Parallel Algebraic Multigrid Solver and Preconditioner. *Applied Numerical Mathematics*, 41(1):155 – 177, 2002. ISSN 0168-9274. doi: 10.1016/S0168-9274(01)00115-5. URL <https://doi.org/10.1016/>

- S0168-9274(01)00115-5. Developments and Trends in Iterative Methods for Large Systems of Equations - in memorium Rudiger Weiss.
- [6] E. Keilegavlen and J.M. Nordbotten. Finite Volume Methods for Elasticity with Weak Symmetry. *International Journal for Numerical Methods in Engineering*, 112(8):939–962, 2017. doi: 10.1002/nme.5538. URL <https://doi.org/10.1002/nme.5538>.
 - [7] J. Kim, H.A. Tchelepi, and R. Juanes. Stability, Accuracy, and Efficiency of Sequential Methods for Coupled Flow and Geomechanics. *SPE Journal*, 16(02): 249–262, 01 2011. ISSN 1086-055X. doi: 10.2118/119084-PA. URL <https://doi.org/10.2118/119084-PA>.
 - [8] S. Lacroix, Y. Vassilevski, J. Wheeler, and M. Wheeler. Iterative Solution Methods for Modeling Multiphase Flow in Porous Media Fully Implicitly. *SIAM J. Sci. Comput.*, 25(3):905–926, 2003. doi: 10.1137/S106482750240443X. URL <https://doi.org/10.1137/S106482750240443X>.
 - [9] K.-A. Lie and O. Moyner. *Advanced Modeling with the MATLAB Reservoir Simulation Toolbox*. Cambridge University Press, Nov 2021. ISBN 9781316519967. URL <https://doi.org/10.1017/9781009019781>.
 - [10] A. Moncorgé. Natural Variable Formulation for Simulation of Geological CO₂ Storage. *Geoenergy Science and Engineering*, 253:213977, 2025. ISSN 2949-8910. doi: 10.1016/j.geoen.2025.213977. URL <https://doi.org/10.1016/j.geoen.2025.213977>.
 - [11] A. Moncorgé, L. Patacchini, and R. de Loubens. Multi-Phase, Multi-Component Simulation Framework for Advanced Recovery Mechanisms. In *Abu Dhabi International Petroleum Conference and Exhibition*, Nov 2012. doi: 10.2118/161615-ms. URL <https://doi.org/10.2118/161615-ms>.
 - [12] J.M. Nordbotten and E. Keilegavlen. Two-Point Stress Approximation: A Simple and Robust Finite Volume Method for Linearized (Poro-)elasticity and Stokes Flow. *Comput. Math. Appl.*, 197(C):259–294, November 2025. ISSN 0898-1221. doi: 10.1016/j.camwa.2025.07.035. URL <https://doi.org/10.1016/j.camwa.2025.07.035>.
 - [13] A. Novikov, D. Voskov, M. Khait, H. Hajibeygi, and J.D. Jansen. A Scalable Collocated Finite Volume Scheme for Simulation of Induced Fault Slip. *Journal of Computational Physics*, 469:111598, 2022. ISSN 0021-9991. doi: 10.1016/j.jcp.2022.111598. URL <https://doi.org/10.1016/j.jcp.2022.111598>.
 - [14] A. Novikov, I. Saifullin, H. Hajibeygi, and D. Voskov. A Finite Volume Framework for the Fully Implicit Thermal-Hydro-Mechanical-Compositional Modeling in Subsurface Applications. *Journal of Computational Physics*, 538:114152, 2025. ISSN 0021-9991. doi: 10.1016/j.jcp.2025.114152. URL <https://doi.org/10.1016/j.jcp.2025.114152>.
 - [15] D.W. Peaceman. *Fundamentals of Numerical Reservoir Simulation*. Elsevier Scientific Publishing, Amsterdam, The Netherlands, Jan 1977. ISBN 9780080868608.
 - [16] D.K. Ponting. Corner Point Geometry in Reservoir Simulation. In *ECMOR 1 - European Conference on the Mathematics of Oil Recovery*, 1989. doi: 10.3997/2214-4609.201411305. URL <https://doi.org/10.3997/2214-4609.201411305>.

- [17] X. Raynaud, A. Moncorgé, and H.M. Nilsen. Challenges of Multiple Point Stress Approximation Implementation for Corner Point Geometry Grid. In *Under Review at Computational Geosciences Special Issue for ECMOR 2026 - European Conference on the Mathematics of Geological Reservoirs*, 2027.
- [18] R. Scheichl, R. Masson, and J. Wendebourg. Decoupling and Block Preconditioning for Sedimentary Basin Simulations. *Computational Geosciences*, 7(4): 295–318, Dec 2003. ISSN 1573-1499. doi: 10.1023/B:COMG.0000005244.61636.4e. URL <https://doi.org/10.1023/B:COMG.0000005244.61636.4e>.
- [19] G. Shaw and T. Stone. Finite Volume Methods for Coupled Stress/Fluid Flow in a Commercial Reservoir Simulator. In *SPE Reservoir Simulation Symposium*. Society of Petroleum Engineers (SPE), 2005. doi: 10.2118/93430-MS. URL <https://doi.org/10.2118/93430-MS>.
- [20] SLB. *Visage Finite-Element Geomechanics Simulator*. SLB, 2026. URL <https://www.slb.com/products-and-services/delivering-digital-at-scale/software/visage>.
- [21] I. Sokolova, M. Gusti Bastisya, and H. Hajibeygi. Multiscale Finite Volume Method for Finite-Volume-Based Simulation of Poroelasticity. *Journal of Computational Physics*, 379:309–324, 2019. ISSN 0021-9991. doi: 10.1016/j.jcp.2018.11.039. URL <https://doi.org/10.1016/j.jcp.2018.11.039>.
- [22] K. Stueben. An Introduction to Algebraic Multigrid. *Appendix in book "Multigrid" by U. Trottenberg, C.W. Oosterlee and A. Schueller, Academic Press*, pages 413–532, 2001.
- [23] K.M. Terekhov and H.A. Tchelepi. Cell-Centered Finite-Volume Method for Elastic Deformation of Heterogeneous Media with Full-Tensor Properties. *Journal of Computational and Applied Mathematics*, 364:112331, 2020. ISSN 0377-0427. doi: 10.1016/j.cam.2019.06.047. URL <https://doi.org/10.1016/j.cam.2019.06.047>.
- [24] J.R. Wallis, R.P. Kendall, and T.E. Little. Constrained Residual Acceleration of Conjugate Residual Methods. In *SPE Reservoir Simulation Symposium*. Society of Petroleum Engineers (SPE), 1985. doi: 10.2118/13536-MS. URL <https://doi.org/10.2118/13536-MS>.
- [25] S. Waziri, G. Ren, P. Tomin, and B. Guyaguler. Nonlinear Acceleration of the Iterative-Sequentially Coupled Flow and Geomechanics Fixed Stress Scheme. *Rock Mechanics and Rock Engineering*, 2025. doi: 10.1007/s00603-025-04633-2. URL <https://doi.org/10.1007/s00603-025-04633-2>.
- [26] J.A. White, N. Castelletto, S. Klevtsov, Q.M. Bui, D. Osei-Kuffuor, and H.A. Tchelepi. A Two-Stage Preconditioner for Multiphase Poromechanics in Reservoir Simulation. *Computer Methods in Applied Mechanics and Engineering*, 357:112575, 2019. ISSN 0045-7825. doi: 10.1016/j.cma.2019.112575. URL <https://doi.org/10.1016/j.cma.2019.112575>.