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Homotopy surrogate model assisted double-loop approach for efficient structural reliability-based design optimization

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Abstract: The double-loop approach for structural reliability-based design optimization (RBDO) often incurs high computational costs because reliability analysis in the inner loop typically requires numerous finite element analyses (FEAs) or experiments. As a solution, this paper develops a double-loop approach for RBDO by using the stochastic static homotopy surrogate model (SHSM). In the inner loop, the SHSM is combined with Monte Carlo simulation for reliability analysis of candidate designs. The SHSM model is established through the stochastic homotopy method, which is intrusive and does not depend on training samples. In the outer loop, the parallel harmony search (PHS) algorithm is used to optimize the design variables. To further reduce optimization costs by reusing historical optimization information, this study proposes a SHSM reusable strategy. When the reusable criterion established is satisfied, a number of SHSMs are reused for the reliability analysis of subsequent new candidate designs, thereby efficiently utilizing the established SHSMs. The optimization results for a highly nonlinear benchmark problem, two truss structures and a wing box structure show that the new method achieves similar optimization results while substantially reducing the computational cost than the existing methods including reliability index approach (RIA), performance measure approach (PMA), three Kriging-based methods and so on.

Keywords: Reliability-based design optimization, Double-loop approach, Static homotopy surrogate model, Parallel harmony search algorithm, Monte Carlo simulation

1 Introduction

In practical service environments, structural performance is inevitably affected by various uncertain factors, including geometric deviations, material property fluctuations, and external load variations and so on. The deterministic design optimization method treats design parameters as deterministic values and ignores these uncertain factors. When actual parameters deviate from their deterministic optimal designs, due to the influence of the uncertain factors, structural responses may exceed the safety constraints, increase the possibility of failure, and lead to conservative and

uneconomical designs (Meng et al. 2023; Ni et al. 2021; Zhang et al. 2021). To obtain a more reliable design, the reliability-based design optimization (RBDO) was developed. The RBDO method quantifies the uncertainty of design parameters and incorporates reliability constraints or objectives into the optimization framework, thus transforming the design concept from deterministic optimization to reliability optimization. The RBDO has become an important analytical method for structural design and has been widely applied in various fields such as civil, aerospace, and mechanical engineering (Chen et al. 2025; Li and Song 2024; Wu et al. 2024).

The approaches to the RBDO problem can be classified into three categories: single-loop approach (SLA) (Biswas and Sharma 2023), decoupling approach (DA) (Du and Chen 2004), and double-loop approach (DLA) (Zhang et al. 2021). The DLA is the most direct RBDO approach. It uses an outer loop to iteratively optimize the design parameters and a nested inner loop for reliability analysis. Different from the SLA and DA, which establish equivalent probability constraints, the DLA evaluates reliability constraints directly, thereby avoiding the errors introduced by approximation and providing higher accuracy and strict theoretical rigor. Although the DLA requires more computing resources, the superior accuracy makes it applicable to the majority of RBDO problems (An and Shi 2023; Hao et al. 2021; Li et al. 2017; Okoro et al. 2023). Therefore, this study will adopt the DLA to solve the RBDO problems.

In the DLA framework, the reliability analysis performed in the inner loop governs both the optimization cost and the accuracy of the solution. The methods for the reliability analysis are generally classified into three categories: (1) approximation-based methods, such as the first-order reliability method (FORM) (Liu et al. 2021) and the second-order reliability method (SORM) (Hu et al. 2021); (2) simulation-based methods, including MCS (Li et al. 2017) and importance sampling (Song et al. 2024); and (3) surrogate-based methods (Alibrandi 2014; Kleijnen 2017; Yao and Liu 1997). Although approximation-based methods are computationally efficient, they are difficult to deal with high-dimensional problems, and for problems with strong nonlinear performance functions, they usually bring about large approximation errors (Li and Wang 2019). On the other hand, the simulation-based methods have physical intuition

and high statistical accuracy. However, with the increase of problem dimension, its statistical cost is prohibitive (Li et al. 2019). Recently, surrogate-based methods are becoming increasingly popular. Many scholars have carried out research and exploration using a variety of methods. Among these methods, Kriging is widely used because it can provide accurate predictions and quantify the local prediction uncertainty. Ni et al. (2021) proposed a RBDO approach by using Kriging for bridges that accounts for bridge-vehicle interaction and stochastic material uncertainties, enabling the minimum cross-sectional area to be determined under prescribed failure probability constraints. Wang et al. (2025) adopted an adaptive Kriging model to replace the computationally expensive nonlinear response analysis and successfully solved the multi-dimensional RBDO problem of the automobile front subframe. In addition to Kriging, other surrogate techniques, such as artificial neural network (ANN) and response surface model (RSM), have also been introduced into RBDO. Lee (2025) proposed an ANN-ensemble assisted framework to efficiently address the RBDO problem caused by the system uncertainty in high energy laser weapons. Al-Sanad et al. (2021) combined RSM with genetic algorithms for the RBDO design of wind turbine towers. The above studies demonstrate that surrogate models can improve the computational efficiency of RBDO. However, the accuracy of data-driven surrogate models depends on the number and quality of the training datasets. Once there is error or noise, uneven distribution, or insufficient number in the training samples, the surrogate model may produce biased or even misleading results. On the other hand, the performance function evaluations obtained from data-driven surrogate models often lack physical interpretability (Hu et al. 2024).

Besides the inner loop, the outer loop is also an important part in the DLA framework. In the outer loop, the optimization algorithm determines the number of reliability analyses required and the quality of the final optimal design. The existing optimization algorithms used in RBDO can be broadly classified into gradient-based algorithms and metaheuristic algorithms. The gradient-based algorithms tend to converge to local optima when dealing with multimodal problems and are not suitable for discontinuous or non-differentiable performance functions (Zhou et al. 2018).

Compared with gradient-based algorithms, metaheuristic algorithms exhibit superior global exploration and local exploitation capabilities, enabling them to efficiently explore feasible regions with multimodal or discontinuous characteristics. This makes the metaheuristic algorithms more suitable for the RBDO problems (Jafari-Asl et al. 2021; Xu et al. 2025). Among them, the harmony search (HS) algorithm (Geem 2001) and its variants have been successfully applied to various optimization problems because of its simple structure and few control parameters (Ezugwu et al. 2021). Therefore, in this study, the parallel harmony search (PHS) algorithm (Cao et al. 2024) is employed as the outer optimizer to efficiently search for the optimal design. It should be noted that the proposed method is not limited to the PHS algorithm and can also be combined with other suitable optimization algorithms.

Recently, a new homotopy surrogate model was proposed by Huang et al. (2023) for solving stochastic problems. Unlike the data-driven surrogate models, the homotopy surrogate model establishes an explicit approximation relationship between parameters and structural responses based on a homotopy mapping, which can quickly provide structural responses without relying on training samples once it has been determined. This method has been successfully applied in several fields, such as the eigenvalue problem (Zhang et al. 2024), static analysis (Zhang et al. 2023), and stochastic finite element model updating (Chen et al. 2024) and so on. These researches demonstrate that homotopy surrogate model has high accuracy and efficiency in solving stochastic problems. Therefore, it deserves expectation to introduce the homotopy surrogate model into RBDO to provide a new and efficient surrogate model in the inner loop of the DLA framework.

To efficiently realize the double-loop RBDO, this paper develops a new method based on the static homotopy surrogate model (SHSM). In the inner loop, the SHSM is established based on the random homotopy finite element method and used for reliability analysis. To achieve the efficient utilization of SHSM, a SHSM reusable strategy is proposed. The built SHSMs during the optimization process are retained in a database, and when a new candidate design satisfies the reusable criterion, an appropriate SHSM is reused for its reliability analysis and constraint evaluations. In the

outer loop, the PHS algorithm is adopted as an efficient optimizer in the RBDO process. The numerical results of a highly nonlinear benchmark problem, a 10-bar truss, a 72-bar truss and a wing box structure show that, the new method has higher efficiency with similar optimization results than the existing methods including RIA, PMA, three Kriging-based methods and so on.

The remaining part of this paper is organized as follows. Section 2 introduces the mathematical formula of RBDO. Section 3 provides a detailed introduction to the derivation process of SHSM, the SHSM reusable strategy, implementation of PHS algorithm, and steps of the proposed RBDO method. Section 4 compares the proposed method with several existing methods through four numerical examples. Finally, conclusions are drawn.

2 The mathematical formulation of RBDO problem

The RBDO problem is an uncertain optimization problem, which can be described, by imposing probabilistic constraints on structural performances, as

$$\begin{aligned}
 &\text{Find} && \mathbf{d}, \boldsymbol{\mu}_x \\
 &\text{Min} && f(\mathbf{d}, \boldsymbol{\mu}_x) \\
 &\text{S.t.} && \begin{cases} P_{\text{failure}} \{V_i(\mathbf{d}, \mathbf{X}) \leq 0\} \leq P_{f_i}^{\text{tar}} \text{ or } \Phi(-\beta_i^{\text{tar}}), i = 1, 2, \dots, n \\ \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U, \quad \boldsymbol{\mu}_x^L \leq \boldsymbol{\mu}_x \leq \boldsymbol{\mu}_x^U \end{cases}
 \end{aligned} \tag{1}$$

where $\mathbf{d}$ represents the vector of deterministic design variables, and $\mathbf{d}^U$ and $\mathbf{d}^L$ are their upper and lower bounds, respectively. $\boldsymbol{\mu}_x$ is the mean of the random design variable vector $\mathbf{X}$, and the range of which is determined by the lower limit $\boldsymbol{\mu}_x^L$ and the upper limit $\boldsymbol{\mu}_x^U$. $f(\mathbf{d}, \boldsymbol{\mu}_x)$ is the objective function. $V_i(\mathbf{d}, \mathbf{X})$ denotes the i th performance function. n is the total number of performance functions. The structural failure occurs when $V_i(\mathbf{d}, \mathbf{X}) \leq 0$. $P_{\text{failure}}(\bullet)$ is the probability operator, and $P_{f_i}^{\text{tar}}$ is the failure probability corresponding to the i th constraint. β_i^{tar} is the target reliability index for the i th constraint. $\Phi(\bullet)$ denotes the cumulative distribution function of standard normal distribution.

About Eq. (1), it is worth noting that, for complex engineering structures, the performance functions involving in reliability constraints are generally implicit and must be evaluated through FEA or other high-fidelity numerical simulations.

Consequently, repeated reliability analysis often led to a large amount of structural computation, making the efficient treatment of reliability constraints a great challenge in RBDO.

3 The RBDO method based on static homotopy surrogate model

To reduce the computational cost caused by reliability analysis, this paper uses SHSM which derived through a stochastic homotopy expansion for reliability analysis. For further computational savings, a SHSM reusable strategy is proposed, in which the derived SHSMs are reused for candidate designs that satisfy the reusable criterion, thereby reducing the number of SHSMs established. Meanwhile, the PHS algorithm is employed for design optimization.

3.1 Static homotopy surrogate model

For a structural subjected to static loads, assuming that the structural stiffness parameters such as elastic modulus, and the loads are random, the static equilibrium equation of the structure can be expressed as

$$\mathbf{K}(\boldsymbol{\theta})\mathbf{Y}(\boldsymbol{\theta}) - \mathbf{F}(\boldsymbol{\theta}) = 0 \quad (2)$$

where $\mathbf{K}$ and $\mathbf{Y}$ represent the stiffness matrix and static displacement vector, respectively, and $\mathbf{F}$ is the static load vector. $\boldsymbol{\theta}$ is the random parameter vector involving in the randomness of stiffness and load. Further, the stiffness matrix and load vector can be decomposed into their mean components and stochastic deviations, respectively, as

$$\mathbf{K}(\boldsymbol{\theta}) = \mathbf{K}_0 + \Delta\mathbf{K}, \quad \Delta\mathbf{K} = \sum_{i=1}^r \mathbf{K}_i \theta_i \quad (3)$$

$$\mathbf{F}(\boldsymbol{\theta}) = \mathbf{F}_0 + \Delta\mathbf{F}, \quad \Delta\mathbf{F} = \sum_{i=r+1}^n \mathbf{F}_i \theta_i \quad (n = r + s) \quad (4)$$

where $\mathbf{K}_0$ and $\mathbf{F}_0$ are the stiffness matrix and load vector evaluated at the mean values of the uncertain parameters, respectively. $\mathbf{K}_i$ denotes the expanded matrix of the i th element stiffness matrix. $\mathbf{F}_i$ is the extended vector corresponding to the i th load. r and s are the numbers of random variables contained in the stiffness matrix and the static load, respectively. Specifically, $\mathbf{F}_i = 0$ for $i = 1, \dots, r$, and $\mathbf{K}_i = 0$ for $i = r+1, \dots, n$. n is

the total number of random variables in the structure.

To solve the stochastic equilibrium equation in Eq. (2), the homotopy stochastic finite element method is used to achieve a homotopy path that continuously connects the deterministic response with mean value parameters to the target response with uncertain variables (Zhang and Huang 2019). Let $\mathbf{Y}_0 = \mathbf{K}_0^{-1}\mathbf{F}_0$ denote the deterministic displacement associated with the mean value parameter system. Based on the homotopy stochastic finite element method, the zero-order deformation equation of Eq. (2) can be formulated as

$$(1-p)\mathbf{K}_0[\Psi(\boldsymbol{\theta}, h, p) - \mathbf{Y}_0] = ph[(\mathbf{K}_0 + \sum_{i=1}^n \mathbf{K}_i \theta_i) \Psi(\boldsymbol{\theta}, h, p) - (\mathbf{F}_0 + \sum_{i=1}^n \mathbf{F}_i \theta_i)] \quad (5)$$

where $\Psi(\boldsymbol{\theta}, h, p)$ represents the structural displacement response obtained through the homotopy derivation. h is a parameter that controls the convergence region and convergence rate of the homotopy series, and $p \in [0, 1]$. At the two endpoints of p , according to Eq. (5), the equations associated with the initial and final solutions can be derived, respectively, as

$$p=0: \quad \mathbf{K}_0[\Psi(\boldsymbol{\theta}, h, 0) - \mathbf{Y}_0] = 0 \quad (6)$$

$$p=1: \quad h[(\mathbf{K}_0 + \sum_{i=1}^n \mathbf{K}_i \theta_i) \Psi(\boldsymbol{\theta}, h, 1) - (\mathbf{F}_0 + \sum_{i=1}^n \mathbf{F}_i \theta_i)] = 0 \quad (7)$$

Through solving Eqs. (6) and (7), the initial solution, $\Psi(\boldsymbol{\theta}, h, 0)$, of displacement and the final stochastic response $\Psi(\boldsymbol{\theta}, h, 1)$ can be achieved, respectively. Hence, as p evolves from 0 to 1, the homotopy solution of Eq. (5) continuously moves from the initial displacement solution to the final stochastic response. Then, by using the Maclaurin expansion with respect to the parameter p , $\Psi(\boldsymbol{\theta}, h, p)$ is represented as

$$\Psi(\boldsymbol{\theta}, h, p) = \Psi_0 + \sum_{m=1}^{\infty} \frac{\Psi_m}{m!} p^m \quad (8)$$

where Ψ_0 denotes the zero-order term of the homotopy series and is equal to $\mathbf{Y}_0$. Ψ_m represents the m th-order homotopy coefficient, and $\Psi_m = \left. \frac{\partial^m \Psi(\boldsymbol{\theta}; h, p)}{\partial p^m} \right|_{p=0}$. Here, m denotes the order of the derivative.

As $p=1$, from Eq. (8), the final homotopy solution can be obtained as

$$\mathbf{Y}(\boldsymbol{\theta}, h) = \mathbf{\Psi}(\boldsymbol{\theta}, h, 1) = \sum_{m=0}^{\infty} \frac{\mathbf{\Psi}_m}{m!} \quad (9)$$

where $\mathbf{\Psi}_m$ can be determined for the cases of $m=1$ and $m>1$, respectively, by

$$\mathbf{\Psi}_1 = (-h)\mathbf{K}_0^{-1}(\Delta\mathbf{F} - \Delta\mathbf{K}\mathbf{Y}_0), \quad \Delta\mathbf{F} = \sum_{i=r+1}^n \mathbf{F}_i \theta_i \quad (10)$$

$$\mathbf{\Psi}_m = \left[m(1+h)\mathbf{I} + mh\mathbf{K}_0^{-1}\Delta\mathbf{K} \right] \mathbf{\Psi}_{m-1} \quad (m > 1), \quad \Delta\mathbf{K} = \sum_{i=1}^r \mathbf{K}_i \theta_i \quad (11)$$

where $\mathbf{I}$ is an N -dimensional identity matrix, and N denotes the number of degrees of freedom in the structure.

Substituting Eq. (10) into Eq. (9), the homotopy solution of the stochastic static equilibrium equation can be written as:

$$\begin{aligned} \mathbf{Y}(\boldsymbol{\theta}; h) = & \mathbf{Y}_0 + \beta_{m,1}(h) \sum_{i_1=1}^n \mathbf{Y}_{i_1} \theta_{i_1} + \beta_{m,2}(h) \sum_{i_1=1}^n \sum_{i_2=1}^{i_1} \mathbf{Y}_{i_1 i_2} \theta_{i_1} \theta_{i_2} + \\ & \cdots + \beta_{m,m}(h) \sum_{i_1=1}^n \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} \cdots \sum_{i_m=1}^{i_{m-1}} \mathbf{Y}_{i_1 i_2 \cdots i_m} \theta_{i_1} \theta_{i_2} \cdots \theta_{i_m} \\ & \sum_{i_1=1}^n \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} \cdots \sum_{i_m=1}^{i_{m-1}} \mathbf{Y}_{i_1 i_2 \cdots i_m} \theta_{i_1} \theta_{i_2} \cdots \theta_{i_m} = \frac{\mathbf{\Psi}_m}{m!} \Big|_{h=-1} \end{aligned} \quad (12)$$

The approach function term $\beta_{m,k}(h)$ in Eq. (12) can be expressed as

$$\beta_{m,k}(h) = \begin{cases} 0 & (k > m) \\ (-h)^k \sum_{l=0}^{m-k} \binom{k+l-1}{l} (1+h)^l & (1 \leq k \leq m) \\ 1 & (k \leq 0) \end{cases} \quad (13)$$

The optimal value of h can be determined by the stochastic residual minimization method (Zhang et al. 2025). As long as the value of h is determined, the static response of the structure can be calculated through Eq. (12).

3.2 The SHSM reusable strategy

In the proposed method, numerous SHSMs are established for reliability analysis of candidate designs. Reusing the established SHSM can reduce computational costs. Similar historical information reuse ideas have been applied to deterministic structural optimization. For example, Cao et al. (2022) selected historical reference designs based

on design similarity and used the FEA information of reference designs for structural reanalysis, thus decreasing the count of FEAs throughout the optimization process. However, structural reanalysis mainly focuses on the evaluation of deterministic structural responses, while reliability analysis requires the calculation of structural responses corresponding to a large number of random samples. The SHSM establishes an explicit relationship between random variables and structural response, which can be used for predicting the response of random samples. Based on this characteristic, this article proposes a SHSM reusable strategy. This strategy employs a database H_{DB} to store all the built SHSMs together with their corresponding expansion centers. Here, each expansion center denotes the mean value vector corresponding to the random variables used to construct the corresponding SHSM. When a new candidate design requires reliability constraint evaluation, its normalized distance from each historical expansion center is first calculated. The distance to the expansion center of the j th historical SHSM is defined as the maximum absolute relative difference across all design variables, as:

$$D_j = \max_{i=1,2,\dots,nd} \frac{|d_i - d_i^j|}{|d_i^j|}, \quad j = 1, 2, \dots, N_h \quad (14)$$

where d_i ($i = 1, \dots, nd$) is the i th variable of the new candidate design, and nd is the number of design variables. d_i represents the i th variable of the new candidate design. d_i^j denotes the i th variable of the expansion center of the j th SHSM in H_{DB} . N_h is the number of SHSMs stored in the H_{DB} .

Then, the index J of the SHSM whose expansion center is closest to the new candidate design is defined as

$$J = \arg \min_j D_j, \quad j = 1, 2, \dots, N_h \quad (15)$$

If D_J is the minimum distance, when $D_J < D_{lim}$ and D_{lim} is the prescribed reuse threshold, the J th SHSM is reused to evaluate the reliability constraints of the new candidate design. Otherwise, a new SHSM is derived with the new candidate design as its expansion center and used for reliability constraint evaluation. The new SHSM and its expansion center are then added to H_{DB} for subsequent reuse.

3.3 Parallel harmony search algorithm

The HS algorithm is a metaheuristic optimization with a simple structure and few control parameters (Geem 2001; Ezugwu et al. 2021), which is often used in optimization problems. In the HS algorithm, an initial population consisting of N_{HMS} candidate designs is maintained and updated during the optimization process. A new design $\mathbf{Z}^{new} = [z_1^{new}, z_2^{new}, \dots, z_D^{new}]$ is generated through memory consideration and pitch adjustment. For the j th design variable, there is

$$z_j^{new} = \begin{cases} z_j^k, & k_1 \in (1, 2, \dots, N_{HMS}), \text{ Probability} = P_{HMCR} \\ lb_j + rand(0,1) \times (up_j - lb_j), & \text{ Probability} = 1 - P_{HMCR} \end{cases} \quad (16)$$

where k_1 is a random integer ranging from 1 to N_{HMS} , and lb_j and up_j are the lower and upper bounds of the j th design variable, respectively. P_{HMCR} denotes the harmony memory considering rate. The selected variable z_j^{new} can be further adjusted with a probability of P_{PAR} , which is written as

$$z_j^{new} = \begin{cases} z_j^{new} \pm rand(0,1) \times BW, & \text{ Probability} = P_{PAR} \\ z_j^{new}, & \text{ Probability} = 1 - P_{PAR} \end{cases} \quad (17)$$

where BW denotes the bandwidth and P_{PAR} is the pitch adjusting rate. These two operations are referred to as the HS operator in this study.

Different from the standard HS algorithm, the PHS algorithm divides the initial population into N_{HM} subpopulations (Cao et al. 2024). The subpopulations generate and update candidate designs in parallel. To exchange local and global search information, the PHS algorithm employed a swarm information sharing (SIS) operator, which is represented as:

$$\mathbf{Z}_g^{new} = \mathbf{Z}_g^R + c_1 r_1 (\mathbf{P}_g^{bh} - \mathbf{Z}_g^R) + c_2 r_2 (\mathbf{G}_{bh} - \mathbf{Z}_g^R) \quad (18)$$

where $\mathbf{Z}_g^R$ is a randomly selected design from the g th subpopulation, $\mathbf{P}_g^{bh}$ is the best design in this subpopulation, and $\mathbf{G}_{bh}$ is the best design among all subpopulations. c_1 and c_2 are acceleration coefficients, while r_1 and r_2 are random numbers in the range of $[0, 1]$. The local best design guides each subpopulation to exploit promising regions, whereas the global best design enables information sharing among different subpopulations.

During the optimization process, the HS and SIS operators are adaptively selected. The probability of using the HS operator is defined as

$$G = G_{\min} + \frac{(G_{\max} - G_{\min}) \times (T_{\max} - Iter)}{T_{\max}} \quad (19)$$

where $G_{\min}$ and $G_{\max}$ are the lower and upper bounds of G , respectively. In this study, $G_{\min} = 0.05$ and $G_{\max} = 0.95$. $T_{\max}$ and $Iter$ denote the maximum and current numbers of iterations, respectively. Thus, the HS operator is used more frequently in the early stage, whereas the SIS operator gradually plays a greater role as the search proceeds.

3.4 Procedure of the new RBDO method

To demonstrate the details on using SHSM, the SHSM reusable strategy and the PHS algorithm to realize the double-loop RBDO, the flowchart of the proposed new method is shown in **Fig. 1**. Correspondingly, the implementation steps of the procedure are summarized as follows:

Step 1: Initialize the parameters of the PHS algorithm, the reusable threshold $D_{\lim}$, and an empty historical database H_{DB} . Generate the initial optimization population and evaluate the objective function and reliability constraints of all initial designs using SHSMs. Store the derived SHSMs and their expansion centers in the H_{DB} .

Step 2: Divide the initial population into multiple subpopulations.

Step 3: At each musical improvisation process, each subpopulation generates a new candidate design using either the HS operator defined in Eqs. (16) and (17) or the SIS operator described by Eq. (18).

Step 4: If a newly generated candidate design exceeds the lower or upper bounds, it is projected onto the nearest feasible boundary.

Step 5: Calculate the objective function in Eq. (1) for each new candidate design. If a new candidate design has a better objective function value F_{new} than the objective function value F_{worst} of the worst design in its corresponding subpopulation, it proceeds to Step 6.

Step 6: For each new candidate design, reuse or build a SHSM according to the reusable strategy in Section 3.2. If H_{DB} is nonempty and $D_J < D_{\lim}$, reuse the SHSM associated with the nearest historical expansion center. Otherwise, build a new SHSM at the candidate design using Eq. (12), and add the new SHSM and its expansion center to H_{DB} . Monte Carlo simulation samples are generated according to the random

parameters, and the reused or new SHSM is used to efficiently calculate the structural responses of the Monte Carlo simulation samples. Then, based on these response samples, the structural failure probability can be calculated.

Step 7: For all the new candidate designs, if the g th new candidate design $\mathbf{Z}_g^{\text{new}}$ satisfies the reliability constraints and there is the worst design $\mathbf{Z}_g^{\text{worst}}$ in the g th subpopulation, the $\mathbf{Z}_g^{\text{worst}}$ will be replaced by $\mathbf{Z}_g^{\text{new}}$. Otherwise, the g th subpopulation is retained.

Step 8: The iterations covering Steps 3–7 are performed continuously until the predefined upper limit of iteration count is reached, after which the optimal design result is exported.

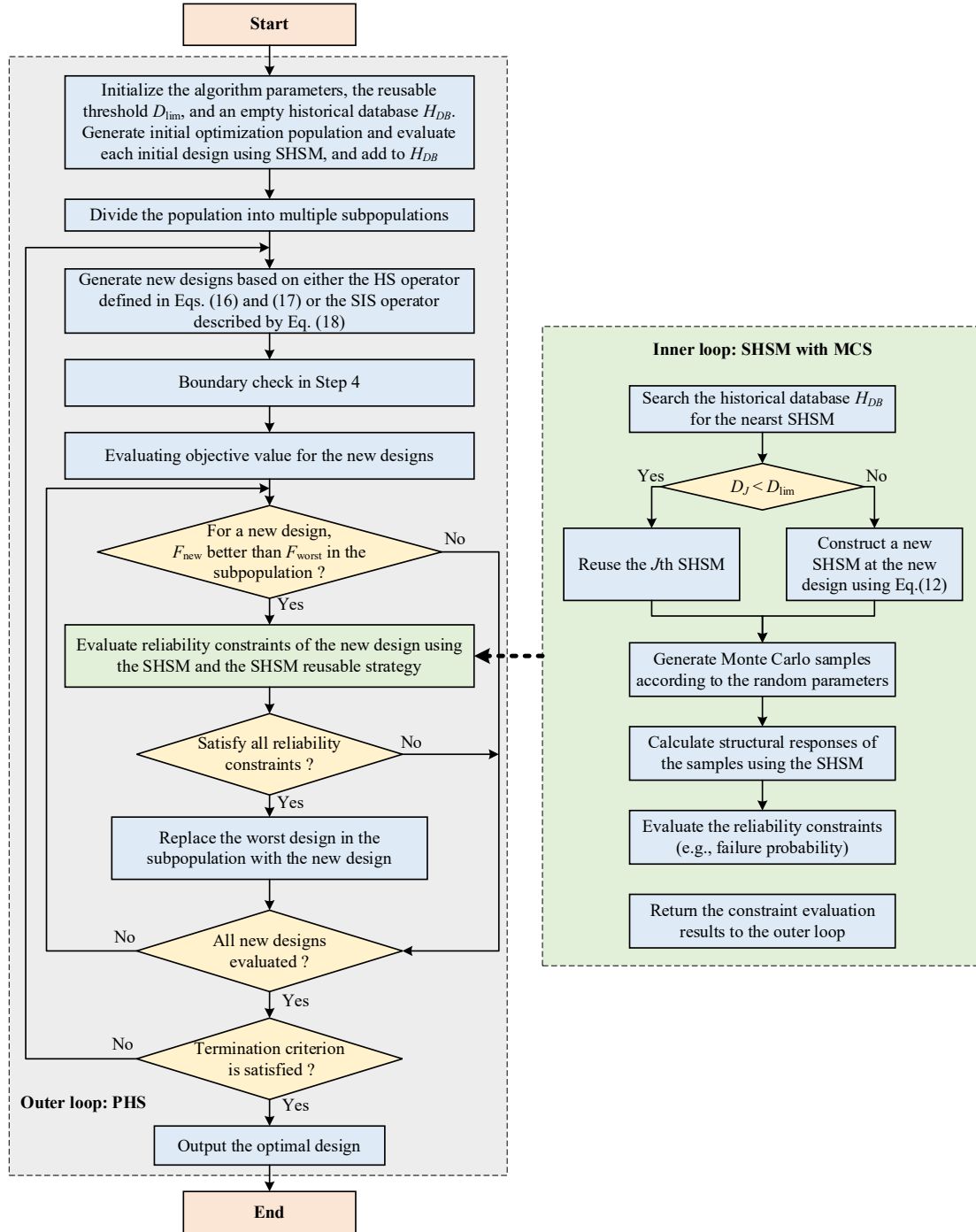


Fig. 1 Flowchart of the proposed method

To visually illustrate the optimization mechanism of the proposed method, **Fig. 2** presents a schematic diagram of the proposed method assisted by SHSM. As can be seen from **Fig. 2**, during the outer loop optimization process, the search path moves from $\mathbf{D}_{(1)}$ to $\mathbf{D}_{(2)}$, gradually approaching the global optimal design $\mathbf{D}_{best}$. For any candidate design $\mathbf{D}_{(k)}$, first, a SHSM is established based on Eq. (12), thereby obtaining an explicit approximate relationship between the parameters and the structural responses. Subsequently, this explicit response expression is directly used in the inner

loop reliability analysis to quickly calculate the failure probability of $\mathbf{D}_{(k)}$ and evaluate its reliability constraints. Based on the evaluation results of the reliability constraints and the objective function value, it is determined whether $\mathbf{D}_{(k)}$ is accepted. Through the continuous iteration of the above process, the design points gradually approach the global optimal design $\mathbf{D}_{\text{best}}$ that satisfies the reliability constraints.

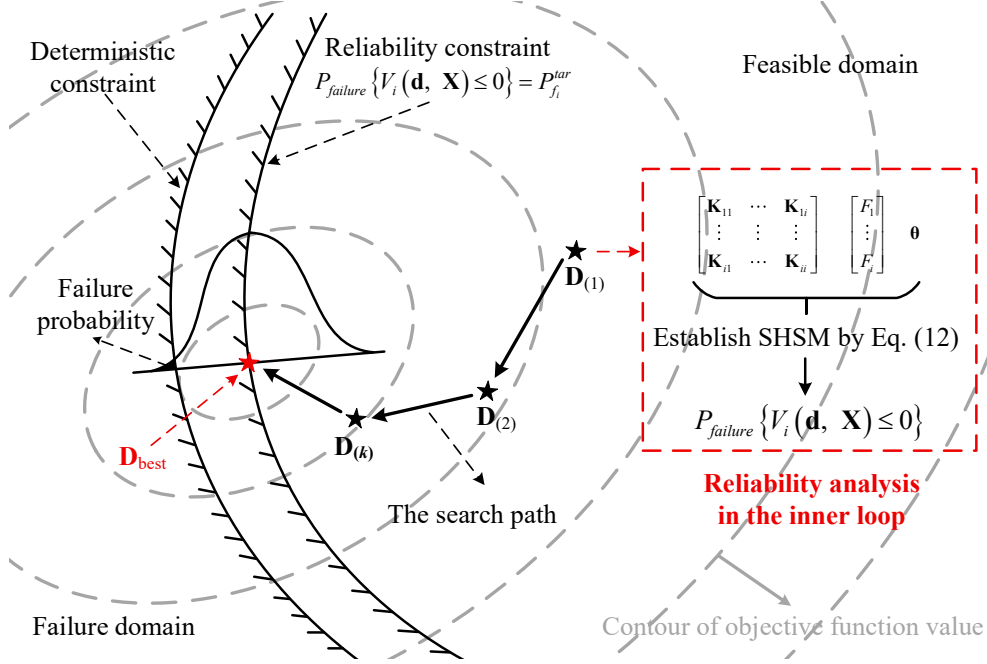


Fig. 2 Schematic of the proposed method based on the SHSM

4 Examples

This section uses four examples, including a highly nonlinear benchmark example, a 10-bar planar truss, a 72-bar spatial truss, and a wing-box structure, to evaluate the performance of the proposed method. The influence of the reusable threshold D_{lim} on the optimization results is also investigated. All computations are performed on a workstation equipped with an Intel (R) Core (TM) i5-14600KF CPU running at 3.50 GHz and 64 GB of RAM. The program implementation is carried out in MATLAB (R2020a).

4.1 A two-dimensional highly nonlinear example

This example considers an optimization problem with a highly nonlinear constraint function $g_1(\mathbf{X})$ and a linear constraint function $g_2(\mathbf{X})$ (Wu et al. 2021), which is described as the formula in Eq. (20). In the formula, the design variables d_1 and d_2

are the mean values of two independent normally distributed random variables X_1 and X_2 , respectively. For each constraint, the target reliability index is set to $\beta_t = 2$, corresponding to a target failure probability of 2.28×10^{-2} . Both random variables have a standard deviation of 0.1. To evaluate the failure probability in the constraint, the number of Monte Carlo samples is set to 5×10^5 . The maximum number of generations is set to 200. The parameters in the PHS algorithm are set as follows: $N_{HMS} = 10$, $N_{HM} = 3$, $P_{HMCr} = 0.9$, and $P_{PAR} = 0.3$.

$$\begin{aligned}
&\text{find: } \mathbf{d} = [d_1, d_2]^T \\
&\text{min: } f(\mathbf{d}) = (d_1 - 3.7)^2 + (d_2 - 4)^2 \\
&\text{s.t.: } \text{prob}[g_i(\mathbf{X}) \leq 0] \leq \Phi(-2), \quad i = 1, 2 \\
&\quad g_1(\mathbf{X}) = -X_1 \sin(4X_1) - 1.1X_2 \sin(2X_2) \\
&\quad g_2(\mathbf{X}) = X_1 + X_2 - 3 \\
&\quad 0 \leq d_1 \leq 3.7, \quad 0 \leq d_2 \leq 4 \\
&\quad x_j \sim N(d_j, 0.1^2), \quad j = 1, 2
\end{aligned} \tag{20}$$

Table 1 summarizes the optimization results obtained by RIA (Chen et al. 2013), PMA (Chen et al. 2013), LHS-Kriging (Liu et al. 2024), U-ADK (Shen et al. 2026), EFF-ADK (Shen et al. 2026), and the proposed method with $D_{lim} = 0, 0.15$, and 0.2 , together with the analytical method (Li et al. 2016). As shown in **Table 1**, RIA, PMA, LHS-Kriging, and U-ADK yield lower objective function values than those of the proposed method under the three reusable thresholds. However, the reliability indicators of the results obtained by these four methods do not satisfy the first reliability constraint. The objective function value obtained by EFF-ADK satisfies the reliability constraint, but produces a more conservative design compared with all other comparison methods. In contrast, the proposed method obtains the objective function value of 1.3262 for $D_{lim} = 0$, and 0.15 , which is very close to the analytical value and satisfy the reliability constraints. Meanwhile, the proposed method evaluates the constraint function much less than the RIA, PMA, and LHS Kriging methods. Specifically, the proposed method requires 24 and 32 constraint function evaluations when $D_{lim} = 0.15$, and 0.20 , respectively. Compared to the 40 and 45 sample points reported by U-ADK and EFF-ADK, the number of evaluating constraint functions required by the proposed method needs to be halved, which means that the proposed

method requires the fewer sample points. Moreover, the number of reusing SHSM, N_{re} , is approximately 490 for the proposed method when $D_{lim} = 0.15$ and 0.20, indicating that about 490 SHSMs are reduced compared to that generated by the method without the SHSM reusable strategy where $D_{lim} = 0$. This demonstrates that the effectiveness of the strategy in reducing computational cost is very well.

Fig. 3 shows the convergence histories of objective function value by the proposed method for the three reusable thresholds where D_{lim} are 0, 0.15 and 0.2, respectively. It can be observed from **Fig. 3** that the convergence rates of objective function values are similar under the three thresholds. However, when $D_{lim} = 0.2$, the convergent value of the objective function value is larger than those under other two thresholds, which leads to the decrease of the prediction accuracy of the constraint functions g_1 and g_2 in Eq. (20), thereby resulting in worse optimization results.

Table 1 Results of the two-dimensional highly non-linear example

Methods	f_{min}	Optimal design	β_1	β_2	N_c	N_{re}
Anal.	1.3259	[2.8421, 3.2320]	2.0000	Inf.	/	/
RIA	1.3038	[2.8163, 3.2769]	1.8570	Inf.	1382	/
PMA	1.3038	[2.8163, 3.2769]	1.8570	Inf.	574	/
LHS-Kriging	1.3049	[2.8173, 3.2749]	1.8663	Inf.	130	/
EFF-ADK	1.3434	[2.8798, 3.1810]	2.0139	Inf.	45 [†]	/
U-ADK	1.3103	[2.8159, 3.2729]	1.8898	Inf.	40 [†]	/
$D_{lim} = 0$	1.3262	[2.8412, 3.2327]	2.0018	Inf.	521	0
Proposed $D_{lim} = 0.15$	1.3262	[2.8363, 3.2383]	2.0012	Inf.	32	495
$D_{lim} = 0.2$	1.3379	[2.8788, 3.1854]	2.0068	Inf.	24	494

Note: N_c denotes the number of evaluations of the two constraint functions. For EFF-ADK and U-ADK, the values marked with [†] represent the number of evaluated samples points.

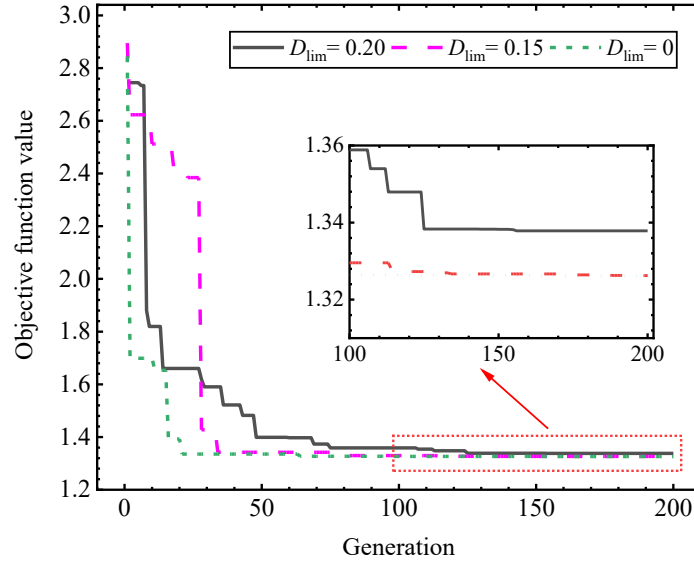


Fig. 3 Convergence history of objective function value by the proposed method for the three reusable thresholds in the highly nonlinear example

In summary, these results indicate that for this highly nonlinear problem, the proposed method can achieve optimization results that are closer to those of the analytical method with less optimization cost than RIA, PMA, LHS-Kriging and the active Kriging methods with the U or EFF learning function. The SHSM reusable strategy can effectively reduce the number of evaluations of the constraint functions during the optimization process, but the reusable threshold should not be too large so as to remain the accuracy of optimization results.

4.2 10-bar planar truss

One 10-bar planar truss is considered, as shown in **Fig. 4**. This truss consists of 6 nodes and 10 bar elements. The elastic modulus of each element is modeled as a normally distributed random variable with a mean value of 1.0×10^4 ksi. The material density is 0.1 lb/in^3 . Vertical concentrated loads are applied at Nodes 2 and 4, respectively, and each load is assumed to follow a normal distribution with a mean value of 5 lb. The design variables are the mean cross-sectional areas of the ten members, which are bounded between 0.1 and 35 in^2 . The objective of the optimization problem is to minimize the mean structural weight W while satisfying a reliability constraint. The reliability constraint stipulates that when the lateral displacement of Node 2 exceeds 2 in, a structural failure will occur. The target reliability index is set to 3, and the corresponding failure probability is 1.35×10^{-3} . For all 12 random quantities,

including 10 cross-sectional areas, elastic modulus, and load, their coefficients of variation (COV) are 0.05. To evaluate the failure probability in the constraint, the number of Monte Carlo simulation samples is set to 1×10^6 . The maximum number of generations is set to 1600. The parameters in the PHS algorithm are set as follows: $N_{HMS} = 50$, $N_{HM} = 5$, $P_{HMCr} = 0.9$, and $P_{PAR} = 0.3$.

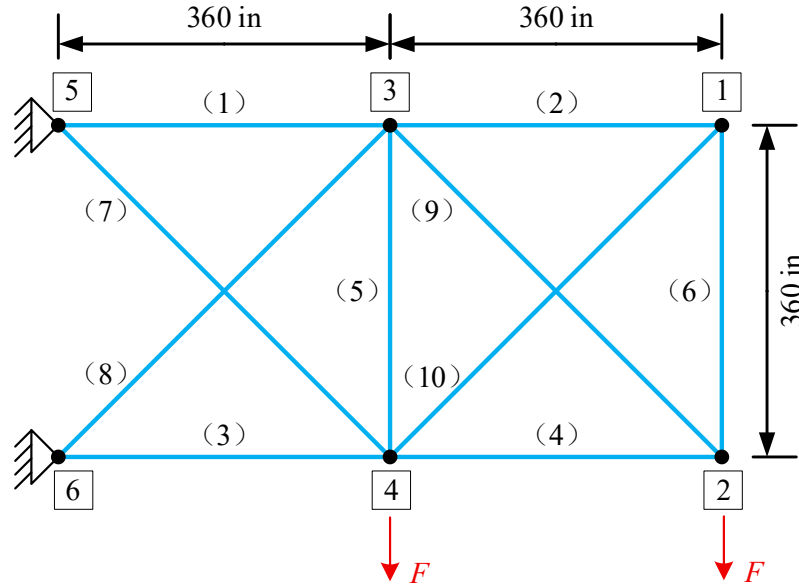


Fig. 4 10-bar truss structure

Table 2 summarizes the optimization results obtained by NDL-IDE (Khodam et al. 2021), ESORA-IDE (Khodam et al. 2021), SORA-DESSOA (Kaveh and Zaeerreza 2022) and the proposed method with the threshold $D_{lim} = 0, 0.03$ and 0.05 , respectively. From **Table 2**, it is found that the objective function values obtained by NDL-IDE and ESORA-IDE are very close, and both have the reliability index of 3.146. SORA-DESSOA achieves the minimum objective function value (6072.830 lb) in the four comparison methods, with a reliability index of 3.02. Under the three thresholds, the objective function values obtained by the proposed method are similar to those of the three comparison methods. When $D_{lim} = 0.03$, the proposed method reduces the number of FEAs (N_{FEA}) by approximately 99.87%, 95.21%, and 96.57% compared to NDL-IDE, ESORA-IDE and SORA-DESSOA, respectively. One of the reasons why the proposed method is so efficient is that due to the SHSM reusable strategy, the number of SHSMs reused reaches 2693, indicating that about 40% SHSMs are reduced compared to that where $D_{lim} = 0$. **Fig. 5** shows the convergence histories of objective

function value by the proposed method for the three reusable thresholds where D_{lim} are 0, 0.03 and 0.05, respectively. As shown in **Fig. 5**, the convergence rates of objective function values are similar under the three thresholds. The optimal values of weight for $D_{lim} = 0$ and 0.03 are close, whereas $D_{lim} = 0.05$ yields a slightly higher objective function value. These results further suggest that a large value of D_{lim} should be avoided to maintain the quality of the optimization results. Overall, the proposed method achieves optimization results comparable to those of the reference methods with very low calculation cost when solving the 10-bar truss problem.

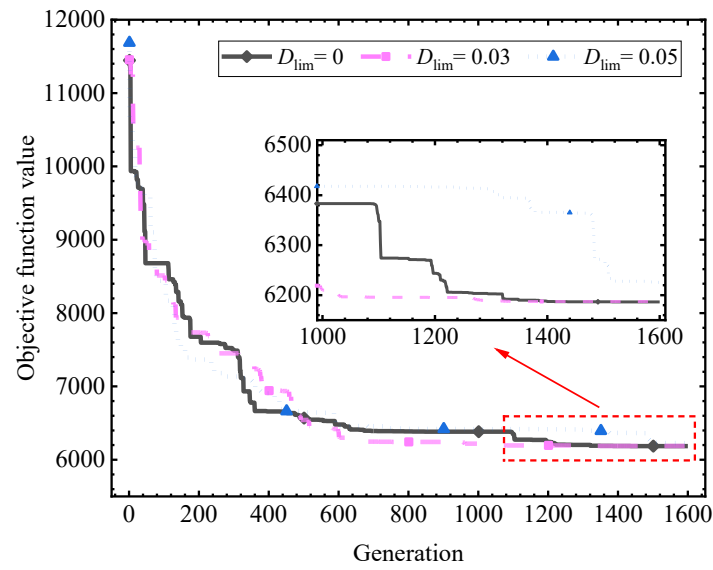


Fig. 5 Convergence history of objective function value by the proposed method for the three reusable thresholds in 10-bar truss example

Table 2 Comparison of optimization results of the 10-bar planar truss

Results	NDL-IDE	ESORA-IDE	SORA-DESSOA	Proposed		
				$D_{lim} = 0$	$D_{lim} = 0.03$	$D_{lim} = 0.05$
A_1 (in ²)	34.999	34.999	34.991	34.716	34.808	34.994
A_2 (in ²)	0.100	0.100	0.105	0.133	0.100	0.130
A_3 (in ²)	27.809	27.948	27.291	28.798	28.585	34.235
A_4 (in ²)	19.895	19.828	19.579	19.429	21.138	20.450
A_5 (in ²)	0.100	0.100	0.100	0.100	0.100	0.111
A_6 (in ²)	0.100	0.100	0.101	0.100	0.100	0.663
A_7 (in ²)	3.435	3.393	3.358	3.365	4.308	4.236
A_8 (in ²)	29.208	29.190	29.375	31.102	26.341	26.275
A_9 (in ²)	28.421	28.429	28.349	27.025	30.001	27.621
A_{10} (in ²)	0.100	0.100	0.100	0.100	0.102	0.100
W (lb)	6102.040	6102.020	6072.830	6133.678	6146.941	6225.674

β	3.146	3.146	3.020	3.000	3.000	3.011
N_{FEA}	2943447	81944	114420	6554	3926	2851
N_{re}	/	/	/	0	2693	3698

4.3 72-bar spatial truss

This example considers a 72-bar spatial truss, which is shown in **Fig. 6**. The members in the truss are divided into 16 groups. The number of design variables is selected as 16, as listed in **Table 3**, where A_1 to A_{16} represent the mean cross-sectional areas of the 1st to 16th member groups, respectively. The allowable range for each design variable is 0.1 to 4.5 in². The material density is 0.1 lb/in³. A three-dimensional random concentrated load $[P_x, P_y, P_z]$ is applied at Node 1, where the mean value vector of the load is $[5, 5, -5]$ kip. Totally, this problem assumed 20 random parameters, including the 16 group areas, the elastic modulus E , and the three load components, which are summarized in **Table 4**. Each design variable follows a normal distribution with a COV of 0.05. The elastic modulus follows a lognormal distribution with mean value of 10^4 ksi and COV of 0.05, while the three load components follow lognormal distributions with COV of 0.1. The optimization objective is to minimize the value of W subject to two reliability constraints on the displacements at Node 1 in the X and Y directions. For the X and Y directions, it is required that the probability of the displacement exceeding 0.3 in is less than 1.35×10^{-3} , corresponding approximately to a target reliability index of 3. The number of samples for Monte Carlo simulation in evaluating the failure probability and the parameter settings of the PHS algorithm are the same as those for the 10-bar truss. The maximum number of generations is set to 2000.

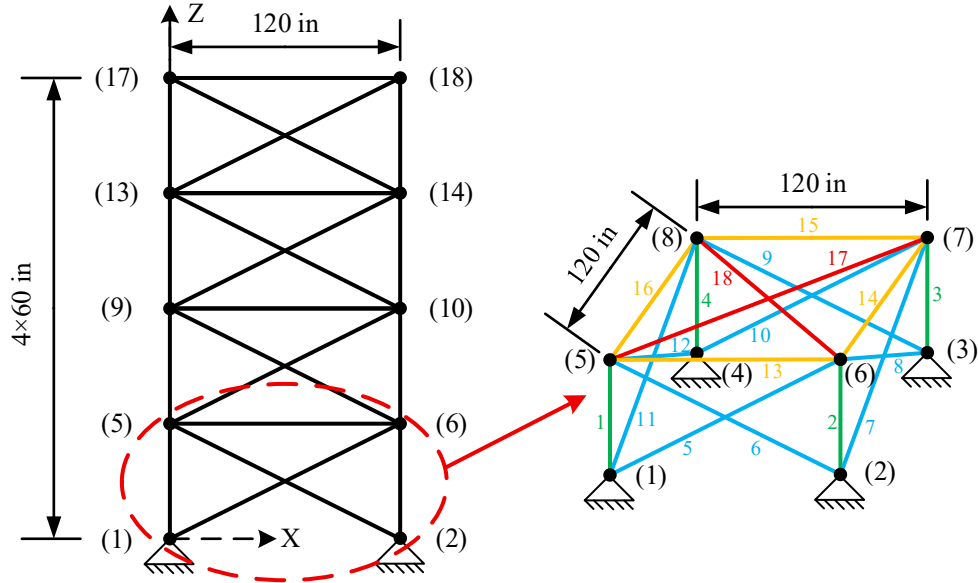


Fig. 6 72-bar spatial truss structure

Table 5 summarizes the optimization results obtained by NDL-GA (Shayanfar et al. 2014), NDL-IDE (Khodam et al. 2021), ESORA-IDE (Khodam et al. 2021) and the proposed method with $D_{lim} = 0, 0.1$ and 0.2 . From **Table 5**, it can be seen that under the three thresholds, the values of the weight W obtained by the proposed method are 499.801, 497.802 and 517.036 lb, respectively, which are lower than the value of weight reported by NDL-GA, and slightly higher than the weight values obtained by NDL-IDE and ESORA-IDE. The optimized designs obtained by the proposed method under the three reusable thresholds satisfy the two reliability constraints.

Table 3 Group assignments for members of 72-bar spatial truss

Member group	Design variable	Number of truss member
1	A_1	Members 1 to 4
2	A_2	Members 5 to 12
3	A_3	Members 13 to 16
4	A_4	Members 17 to 18
5	A_5	Members 19 to 22
6	A_6	Members 23 to 30
7	A_7	Members 31 to 34
8	A_8	Members 35 to 36
9	A_9	Members 37 to 40
10	A_{10}	Members 41 to 48
11	A_{11}	Members 49 to 52
12	A_{12}	Members 53 to 54

13	A_{13}	Members 55 to 58
14	A_{14}	Members 59 to 66
15	A_{15}	Members 67 to 70
16	A_{16}	Members 71 to 72

Table 5 also shows that when using the SHSM reusable strategy with a threshold of 0.1, the proposed method requires 99.98% and 98.56% less FEAs than NDL-IDE and ESORA-IDE, respectively, and the number of FEAs is reduced about 60% compared to the proposed method without the reusable strategy where $D_{\text{lim}} = 0$. **Fig. 7** shows the convergence histories of objective function value by the proposed method for the three reusable thresholds where D_{lim} are 0, 0.1 and 0.2, respectively. According to **Fig. 7** and **Table 5**, when the reusable threshold is 0.2, although the proposed method can obtain a design that satisfies reliability constraints with minimal FEAs, the obtained objective function value is about 4% higher than the results under the other two thresholds. These results further indicate that the reusable threshold should not be too large to balance computational costs and the quality of optimization results. In summary, the results of 72-bar spatial truss indicate that even for structural RBDO problems involving a relatively large number of random parameters including parameters with nonnormal distributions, the proposed method also only needs few FEAs to obtain optimal designs.

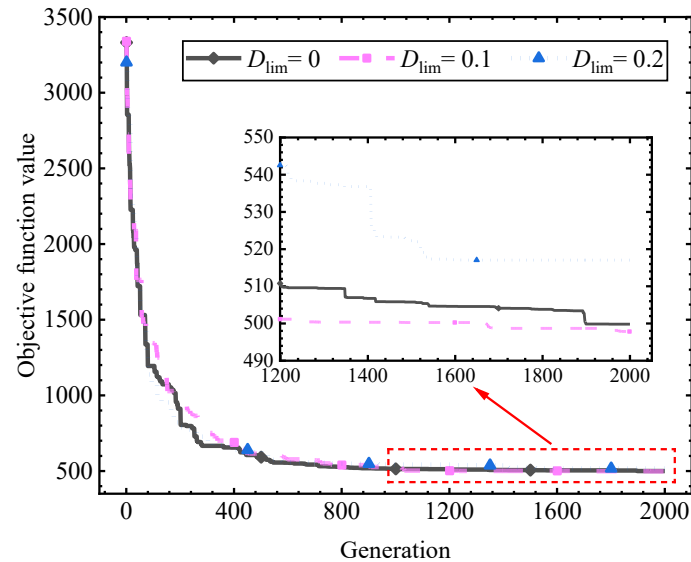


Fig. 7 Convergence history of objective function value by the proposed method for the three reusable thresholds in 72-bar truss example

Table 4 Statistical data of random parameters of 72-bar spatial truss

Random variable	Description	Distribution	Distribution	Mean value
A_{1-16}	Cross-sectional area	Normal	0.05	Design variable
E	Elastic modulus	Lognormal	0.05	10^4 ksi
P_x	External force	Lognormal	0.1	5 kip
P_y	External force	Lognormal	0.1	5 kip
P_z	External force	Lognormal	0.1	-5 kip

Table 5 Comparison of optimization results of the 72-bar spatial truss

Results	NDL-GA	NDL-IDE	ESORA-IDE	Proposed		
				$D_{lim} = 0$	$D_{lim} = 0.1$	$D_{lim} = 0.2$
A_1 (in ²)	0.100	0.100	0.100	0.100	0.107	0.100
A_2 (in ²)	0.900	0.701	0.701	0.622	0.806	0.549
A_3 (in ²)	0.300	0.499	0.499	0.695	0.556	0.661
A_4 (in ²)	0.500	0.701	0.701	0.647	0.523	1.173
A_5 (in ²)	1.100	0.901	0.901	0.813	1.016	0.670
A_6 (in ²)	0.900	0.699	0.699	0.629	0.635	0.655
A_7 (in ²)	0.100	0.100	0.100	0.110	0.108	0.100
A_8 (in ²)	0.100	0.100	0.100	0.145	0.173	0.349
A_9 (in ²)	1.500	1.700	1.500	1.848	1.517	1.841
A_{10} (in ²)	0.500	0.699	0.699	0.700	0.644	0.788
A_{11} (in ²)	0.300	0.100	0.100	0.119	0.124	0.133
A_{12} (in ²)	0.300	0.100	0.100	0.129	0.110	0.100
A_{13} (in ²)	2.300	2.300	2.499	3.394	2.440	2.239
A_{14} (in ²)	0.900	0.701	0.701	0.592	0.758	0.745
A_{15} (in ²)	0.100	0.100	0.100	0.100	0.100	0.100
A_{16} (in ²)	0.100	0.100	0.100	0.111	0.119	0.122
W (lb)	535.79	492.870	492.870	499.801	497.802	517.036
β_1	2.950	3.040	3.037	3.000	3.000	3.000
β_2	2.960	3.040	3.037	3.005	3.003	3.000
N_{FEA}	/	11462892	174266	6384	2506	2157
N_{re}	/	/	/	0	3853	4362

4.4 A wing box structure

A simplified fighter wing box structure consisting of three spars, six ribs, and upper and lower skins is considered, as plotted in **Fig. 8**. The spars and ribs are made of aluminum alloy with the elastic modulus of 72.42 GPa and mass density of 2.768×10^3 kg/m³, while the skins are made of boron/aluminum alloy with the elastic

modulus of 68.98 GPa and mass density of $2.007 \times 10^3 \text{ kg/m}^3$. The aerodynamic lift is equivalently represented by the three static loads, P_1 , P_2 , and P_3 , applied to the three spars, with nominal values of 100, 80, and 20 kN, respectively. The finite element model of the wing box structure consists of 6192 skin elements, 2880 rib elements, and 2160 spar elements. The wing root is fixed. Material properties, geometric dimensions, and load magnitudes are random variables with a COV of 0.1, resulting in eight uncertain quantities. A vertical displacement constraint g_1 is imposed at the point A on the wing tip, requiring the vertical displacement V_A at the point A less than 40 mm. The failure probability for the constraint is 1.350×10^{-3} , corresponding to a target reliability index of 3. The thicknesses of the spars, ribs, and skins are selected as design variables within the range of 3 to 10 mm. The optimization objective of this example is to minimize the mean weight value of structural. The number of Monte Carlo simulation samples is set to 2×10^5 . The maximum number of generations is set to 300. The parameters in the PHS algorithm are set as follows: $N_{HMS} = 10$, $N_{HM} = 3$, $P_{HMCr} = 0.9$, and $P_{PAR} = 0.3$.

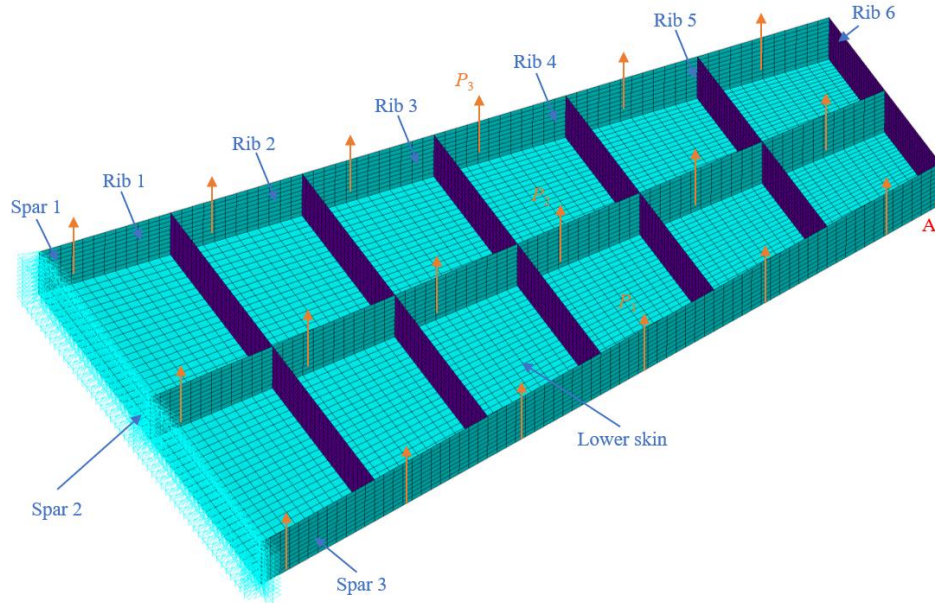


Fig. 8 Finite element model of the wing box structure

Table 6 summarizes the initial design and the optimization results of the proposed methods with the SHSM reusable strategy where $D_{lim} = 0.15, 0.2$ and without the

strategy where $D_{\text{lim}} = 0$, respectively. From **Table 6**, when $D_{\text{lim}} = 0, 0.15$ and 0.2 , the proposed method obtains the objective function values of 82.941, 82.965 and 84.675 kg, respectively, all of which satisfy reliability constraints. Compared with the initial design, the proposed method achieved a reduction of about 13% in the objective function values obtained under the three thresholds, and the corresponding reliability index increased by 13%. **Fig. 9** shows the convergence histories of objective function value by the proposed method for the three reusable thresholds where D_{lim} are 0, 0.15 and 0.2, respectively. It can be seen from **Fig. 9** that when $D_{\text{lim}} = 0$ and 0.15 yield similar final objective values. In comparison, the curve for $D_{\text{lim}} = 0$ stabilizes at a higher objective value. The results obtained by the proposed method when $D_{\text{lim}} = 0.15$ indicate that using the SHSM reusable strategy can reduce the number of FEAs by about 93% compared to not using the strategy, while maintaining good design quality.

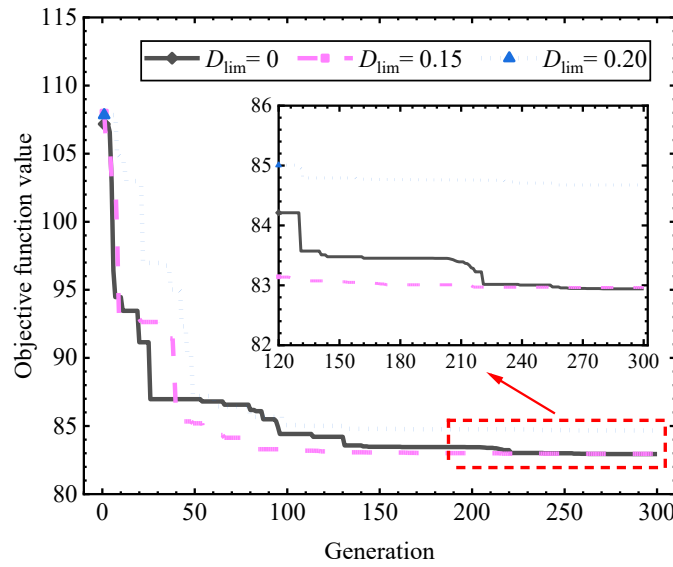


Fig. 9 Convergence history of objective function value by the proposed method for the three reusable thresholds in the wing box example

Fig. 10 and **Fig. 11** present the displacement response contours and the element volume contours of the proposed method at generations 1, 100, and 300 when $D_{\text{lim}} = 0.15$, respectively. It can be watched from **Fig. 10** and **Fig. 11** that as the optimization proceeds, the deformation of wing increases due to the reduction of all structural elements thickness, especially near the wing tip. Nevertheless, the final displacement response at the point A remains within the constraint requirement. In **Fig. 11**, the

proposed method optimized the element volumes (or thickness values) of all structural elements according to the reliability constraints. These results demonstrate the effectiveness of the proposed method in optimizing the weight of structures involving a large number of finite elements.

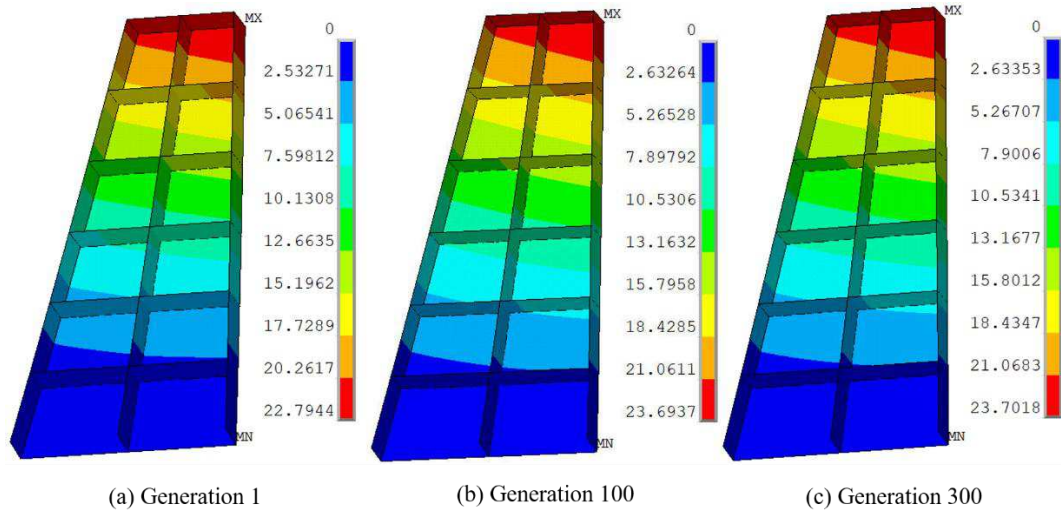


Fig. 10 Displacement response contours by the proposed method when $D_{lim}=0.15$

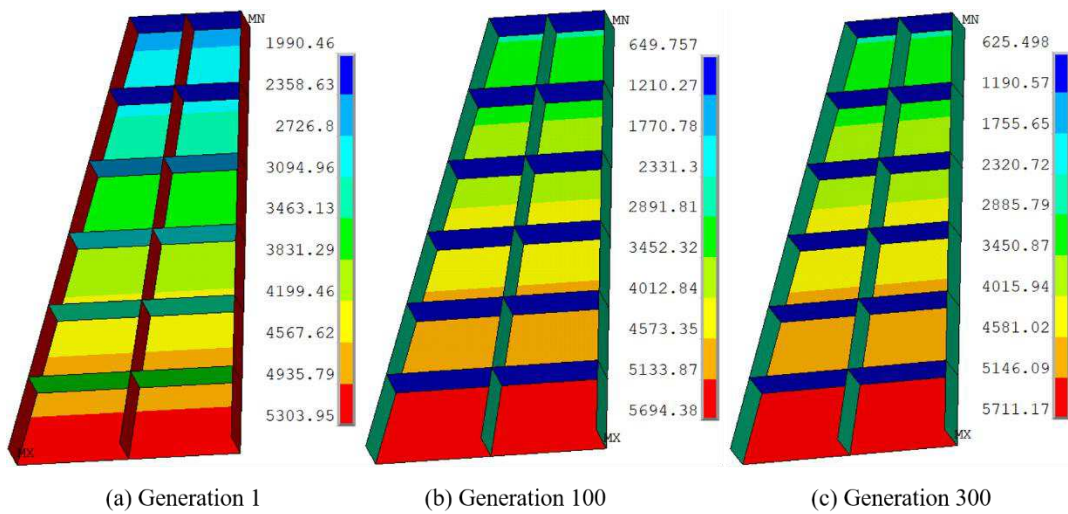


Fig. 11 Contours of element volume by the proposed method when $D_{lim}=0.15$

Table 6 The optimization results of the wing box structure

Results	Initial design	The proposed method		
		$D_{lim} = 0$	$D_{lim} = 0.15$	$D_{lim} = 0.2$
Spar (mm)	8	4.795	4.824	6.293
Rib (mm)	8	3.000	3.002	3.010
Skin (mm)	8	9.917	9.903	9.361
W (kg)	93.975	82.941	82.965	84.675

β	2.539	3.001	3.001	3.000
N_{FEA}	/	776	48	35
N_{re}	/	0	721	748

5 Conclusion

A new efficient RBDO method integrating the SHSM with the SHSM reusable strategy is developed in this paper. The SHSM is derived using the homotopy random finite element method and provides an explicit, accurate and physically interpretable representation of structural static responses. Moreover, unlike conventional data-driven surrogate models that require large number of training samples for model construction, the SHSM does not depend on training samples. This characteristic reduces the cost of using surrogate model during the RBDO process. By combining the SHSM with MCS, the structural static response samples can be evaluated using the SHSM instead of repeated FEAs, enabling accurate and efficient reliability analysis. Meanwhile, the SHSM reusable strategy is employed to reduce the unnecessary derivation of SHSM during the optimization process. Consequently, the proposed method reduces the computational cost of structural RBDO while maintaining high-quality optimization results.

In the highly nonlinear example, compared with the two traditional double-loop methods RIA and PMA, as well as the three Kriging based methods, the proposed method has fewer constraint function evaluations and its optimization results are closer to the results of the analytical method. In the example of the 10-bar truss, when using the SHSM reusable strategy, the proposed method reduces the number of FEAs by more than 95% compared to the NDL-IDE, ESORA-IDE and SORA-DESSOA methods while obtaining similar objective function value. For the 72-bar truss example, in the case that the number of random parameters reaches 20, the optimization results of the proposed method further illustrate its computational advantage, requiring 99.97% and 97.79% fewer FEAs than NDL-IDE and ESORA-IDE, respectively. The application to the wing box structure demonstrates that the proposed method performs well in solving structural RBDO problems that involve a large number of finite elements.

In the future, it is expected to develop an adaptive SHSM to further reduce the number of FEAs and computational costs during the optimization process. The extension of the SHSM to single-loop and decoupled RBDO methods is also worthy of further investigation.

Author Contributions Guanyu Zhang: conceptualization, methodology, software, visualization, validation, writing—original draft. Bin Huang: conceptualization, supervision, writing—review & editing. Hongyou Cao: conceptualization, supervision, writing—review & editing.

Data availability The data and the source codes of the method will be available upon reasonable request.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Replication of results This study introduces the double-loop method of using the SHSM for solving RBDO problems. Readers interested in replicating the results may obtain the code and relevant implementation details from the corresponding author upon reasonable request.

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