Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

Perturbation-Based Method for Estimation of Aircraft Lateral-Directional Parameters

Arleen Kaur
Amity University Uttar Pradesh
Challa Parvathi Rudesh
rudeshchalla@gmail.com

Amity University Uttar Pradesh
Sanjay Singh
Amity University Uttar Pradesh

Research Article

Keywords: System Identification, Double Delta Method, Feed Forward Neural Network, Parameter Estimation, Machine learning, Aerodynamic Derivatives

Posted Date: November 13th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11505258/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

Perturbation-Based Method for Estimation of Aircraft Lateral-Directional Parameters

Arleen Kaur, Challa Parvathi Rudesh, and Sanjay Singh*

Amity Institute of Aerospace Engineering, Amity University Uttar Pradesh, India.

* Corresponding author E-mail: rudeshchalla@gmail.com, ssingh10@amity.edu

Abstract

This paper presents and validates a perturbation-based method in conjunction with feedforward neural network for estimation of aircraft stability and control derivatives. The method, named Double Delta Method, draws its inspiration from Taylor series expansion. Two sets of simulated data were generated using two different sets of flight control inputs of ATTAS aircraft by solving the lateral-directional equations numerically using Runge-Kutta 4th order method in MATLAB®R2020a. First dataset, containing five motion/control variables at input nodes and one of the coefficients at output node at a time, is considered to train neural model. Thereafter, modified input files were created by adding small perturbation values of 0.001 and 0.002 in the motion variables and supplied to trained neural model for predicting the values of corresponding coefficient. For parameter estimation, the method uses modified input files and corresponding predicted values of corresponding coefficient. The robustness of the neural model and the method in estimating the parameters are also evaluated by adding the Gaussian noise of 10% in the simulated data. The estimates obtained from proposed method are also compared with the estimates obtained from existing Delta Method and Maximum Likelihood Method. For 10% noisy data, the $\boldsymbol{C_Y}$ derivatives obtained using Double Delta Method showed an improvement ranging from 4.5% to 41% compared with Delta Method, whereas the improvement compared with Maximum Likelihood Method ranged from 7% to 44%. A significant improvement, ranging from 0.4% to 25%, is also observed in the values of $\boldsymbol{C_l}$ and $\boldsymbol{C_N}$ derivatives obtained from Double Delta Method. The second set of data is used for the validation of all three methods by comparing graphically the simulated and estimated motion variables and coefficients. The results demonstrate significant improvement over those obtained using the Delta method and Maximum Likelihood Method.

Keywords: System Identification**;** Double Delta Method; Feed Forward Neural Network; Parameter Estimation; Machine learning; Aerodynamic Derivatives.

1. Introduction

System identification is a methodology to develop mathematical models of physical systems using measurements or observations. One of the first definitions of system identification was given by Zadeh [1]. There are three elements essential to system identification problem: a system (model), flight tests, and system response [2-4]. The most widely applied subfield of system identification is the field of parameter identification wherein an assumed mathematical model based on the phenomenological considerations is used to estimate the properties of the dynamic system [5]. The model contains a finite number of parameters, the value of which must be deduced from measured data. The assumed model, however, will not be an exact representation of the system, no matter how careful its selection is. Furthermore, the experimental data made with real, and thus imperfect instruments will not be consistent with the assumed model form for identified values of system parameters. To tackle the dual problem of imperfections in the assumed model and the measured data, the problem of system identification is further simplified to evaluation of parameters which would make the mathematical model the best representation of the essential characteristics of the system. This revised task of determining the best estimates, rather than the exact values of the parameters is more properly called parameter estimation.

The development of artificial neural networks (ANNs) based techniques has emerged for aerodynamic modeling and parameter estimation of an aircraft. ANN modeling has been attempted for aircraft dynamics where aircraft motion variables and control inputs are mapped to predict the total aerodynamic coefficients [6-9]. Significant contributions have been made in this direction by Hess [6], Linse and Stengel [8], Youseff and Juang [9], and Raol and Jategaonkar [10]. In all these papers, the emphasis has been on aerodynamic modeling and estimation of aerodynamic coefficients using FFNNs, without any attempt to estimate aircraft parameters as conventionally defined in terms of stability and control derivatives in the equations of motion of an aircraft. Raol and Jategaonkar [10] have used the

recurrent neural network (RNN) to model aircraft aerodynamics in a way that allows aircraft parameters to be estimated from flight data. However, RNNs have only a limited scope for aircraft identification applications and it is the FFNNs which may prove to be more flexible, and thereby to have a higher potential for future applications for aircraft identification and parameter estimation [10].

In the previous research studies, various methods like equation error methods, filter error methods, and output error methods [11] have been used for the aircraft parameter estimation. Applications of these methods require a priori postulations of an aircraft model and numerical integration of aircraft equations of motion. On the other hand, ANNs act as model-free approximators and eliminate the need of solving complex equations of aircraft dynamics. The Delta Method, Zero Method [12-14] and Modified Delta Method [15-17] have been used for explicitly estimating aircraft parameters from the flight data using FFNNs. Recently, extreme machine learning-based methods have been used for aerodynamic modelling and aerodynamic parameter estimation of stable and unstable aircraft [18-20]. Comprehensive investigations to evaluate the performance of various neural networks namely Feedforward Neural Network, Cascade Forward Neural Network, Elman Forward Neural Network along with Levenberg-Marquardt and Bayesian Regularization algorithms have been conducted for the estimation of aircraft aerodynamic parameters [21]. It was demonstrated that Cascade Forward Neural Network together with Bayesian Regularization algorithm estimate parameters with high accuracy. A comprehensive study has been conducted to examine the AI-based methods, including Gated Recurrent Units (GRUs), Long Short-Term Memory (LSTM) networks, Recurrent Neural Networks (RNNs), and various hybrid approaches. The study highlighted key challenges associated with these techniques, particularly issues related to scalability and model interpretability for practical applications [22]. Rudesh et al. [23] research approach the flight system identification by using scaled conjugate gradient algorithm to train a FFNN for longitudinal short period parameter estimations. Thise method used aircraft simulated data to correctly calculate aerodynamic partial coefficients proving convergence and lower mean square error competed to the conventical LMO approach. Validation has been done with actual aircraft data to achieve high accuracy with minimal standard derivation and computational time. The SCG model proved highly efficient for complex aerodynamic parameter estimation in advanced aerospace applications. Further, Rudesh et al. [24] proposed a Bayesian Regularization Neural Network (BRNN)

to estimate short-period longitudinal stability and control derivatives of the HANSA aircraft, using angle of attack, elevator deflection, and pitch rate as inputs to predict lift and pitching moment coefficients. Their results showed BRNN achieved faster convergence and lower mean square error than the Levenberg-Marquardt algorithm, with estimated derivatives closely matching true flight values, demonstrating its robustness for aerodynamic parameter estimation.

In the present work, a perturbation-based method 'Double Delta method' in conjunction with feedforward neural network is proposed for aircraft parameter estimation. The proposed method is first applied on simulated flight data and then on the 10% noisy simulated data to evaluate its robustness. The results obtained from proposed method are also compared with the estimated value of parameters obtained from the existing delta method and Maximum Likelihood Method. For 10% noisy data, the C_Y derivatives obtained using Double Delta Method showed an improvement ranging from 4.5% to 41% compared with Delta Method, whereas the improvement compared with Maximum Likelihood Method ranged from 7% to 44%. A significant improvement, ranging from 0.4% to 22.5%, is also observed in the values of C_l derivatives obtained from Double Delta Method. Similarly, C_N derivatives showed improvement, ranging from 0.4% to 25% compared to Delta Method and Maximum Likelihood Method. The results show that the proposed method has capability to estimate parameters with high accuracy and lower standard deviation under realistic flight conditions of a flight vehicle.

2. Methodology

The lateral directional parameters $[C_{Y_0}, C_{Y_p}, C_{Y_r}, C_{Y_\beta}, C_{Y_{\delta a}}, C_{Y_{\delta r}}, C_{l_0}, C_{l_p}, C_{l_r}, C_{l_\beta}, C_{l_{\delta a}}, C_{l_{\delta r}}, C_{N_0}, C_{N_p}, C_{N_r}, C_{N_\beta}, C_{N_{\delta a}}, C_{N_{\delta r}}]$ occurring in the aerodynamic model represent partial derivatives of C_Y, C_l and C_N with respect to the motion variables β, p, r and control inputs δ_a, δ_r. The Double Delta Method is based on the Taylor series expansion and is associated with one-sided finite difference expression. To demonstrate the application of Taylor series, $A\ (t)$ is defined as a continuous function of time with all its derivatives known at 't'. According to Taylor Series expansion, the value of A at time $t + \Delta t$ can be estimated using Eqn (1).

$$A(t + \Delta t) = A(t) + (dA/dt)\Delta t + (d^2A/dt^2)(\Delta t^2/2!) + \cdots + (d^nA/dt^n)(\Delta t^n/n!) \quad (1)$$

Similarly, the value of A at time $t + 2\Delta t$ can be estimated using Eqn (2).

$$A(t + 2\Delta t) = A(t) + (dA/dt)(2\Delta t) + (d^2A/dt^2)(2\Delta t^2/2!) + \cdots + (d^nA/dt^n)(2\Delta t^n/n!) \quad (2)$$

To elaborate, when finding the value of $A(t + \Delta t)$, the first term $A(t)$ does not give accurate estimate. To improve accuracy, the second term is taken which considers the slope of the curve dA/dt at a point on the curve. To improve the estimate further, a third term is taken, which considers the curvature between two points on the curve. Thus, considering higher terms gives better accuracy. Eqn (1) is quadrupled and subtracted from Eqn (2) to find the slope of the curve, dA/dt. The first three terms of Eqns (1-2) are considered to obtain Eqn (3).

$$dA/dt = [-3A(t) + 4A(t + \Delta t) - A(t + 2\Delta t)]/2\Delta t + E_t \quad (3)$$

The magnitude of E_t is small and is therefore neglected to find the slope of the curve. The expression given in equation (3) is named as Double Delta method. For the estimation of lateral-directional parameters from Double Delta and Delta methods, the input file containing motion and flight control variables, $(\beta, ps/V, rs/V, \delta_a and\ \delta_r)$ and the output file containing one of the coefficients, $(C_Y, C_l\ or\ C_N)$ are chosen to train the feed forward neural network. The architecture of the feed forward neural network training model is shown in Figure 1. First part of the training process is the forward propagation, where the inputs are passed through the network layers to produce an output. The forward propagation uses randomly assigned weights and biases to compute activations at the node of each layer. The output from forward computation is compared with the target output to estimate the error. In the second part of the training, the network iteratively minimizes the estimated error by updating the weights during the backpropagation phase using Levenberg-Marquard algorithm. The training process is terminated either mean square error is reached the assigned value or number of epochs (iteration) exceeds the target value. Once the network is trained then the network weights are kept fixed for predicting the output corresponding to the modified input files. Thereafter, for estimating any one of the parameters, say C_{l_β}, from the Double Delta method, same is input file containing $(\beta, ps/V, rs/V, \delta_a\ and\ \delta_r)$ is supplied to the trained network for predicting the corresponding rolling moment coefficient, C_l. The C_l predicted by the trained network is represented by $C_l\ (\beta)$. The input file is then modified by adding the small perturbations, $\Delta\beta\ and\ 2\Delta\beta$ to β at each time point, while other variables

($ps/V, rs/V, \delta_a and\ \delta_r$) are kept at their original values. These modified input files containing ($\beta + \Delta\beta, ps/V, rs/V, \delta_a\ and\ \delta_r$) and ($\beta + 2\Delta\beta, ps/V, rs/V, \delta_a and\ \delta_r$) are then supplied to the trained network to predict the value of rolling moment coefficient, C_l. The predicted value of C_l corresponding to modified input files are denoted by $C_l\ (\beta + \Delta\beta)$ and $C_l\ (\beta + 2\Delta\beta)$. The stability derivative C_{l_β} can be estimated using equation (4).

$$C_{l_\beta} = \{-3\ C_l(\beta) + 4\ C_l\ (\beta\ +\ \Delta\beta) - C_l\ (\beta\ +\ 2\Delta\beta)\}/2\Delta\beta \tag{4}$$

Similarly, by perturbing, $ps/V, rs/V, \delta_a and\ \delta_a$, the remaining derivatives of $C_l - C_{l_p}, C_{l_r}, C_{l_\beta}, C_{l_{\delta a}}, C_{l_{\delta r}}$ can be estimated using equations (5-8).

$$C_{l_p} = \{-3\ C_l(ps/V) + 4\ C_l\ [(ps/V)\ +\ \Delta(ps/V)] - C_l[(ps/V)\ +\ 2\Delta(ps/V)]\}/2\Delta(ps/V) \tag{5}$$

$$C_{l_r} = \{-3\ C_l(rs/V) + 4\ C_l\ [(rs/V)\ +\ \Delta(rs/V)] - C_l[(rs/V)\ +\ 2\Delta(rs/V)]\}/2\Delta(rs/V) \tag{6}$$

$$C_{l_{\delta a}} = \{-3\ C_l(\delta a) + 4\ C_l\ (\delta a +\ \Delta\delta a) - C_l\ (\delta a\ +\ 2\Delta\delta a)\}/2\Delta\delta a \tag{7}$$

$$C_{l_{\delta r}} = \{-3\ C_l(\delta r) + 4\ C_l\ (\delta r +\ \Delta\delta r) - C_l\ (\delta r\ +\ 2\Delta\delta r)\}/2\Delta\delta r \tag{8}$$

The lateral-directional parameters of $C_Y - C_{Y_p}, C_{Y_r}, C_{Y_\beta}, C_{Y_{\delta a}}, C_{Y_{\delta r}}$ and $C_N - C_{N_p}, C_{N_r}, C_{N_\beta}, C_{N_{\delta a}}, C_{N_{\delta r}}$ can be estimated using the same approach.

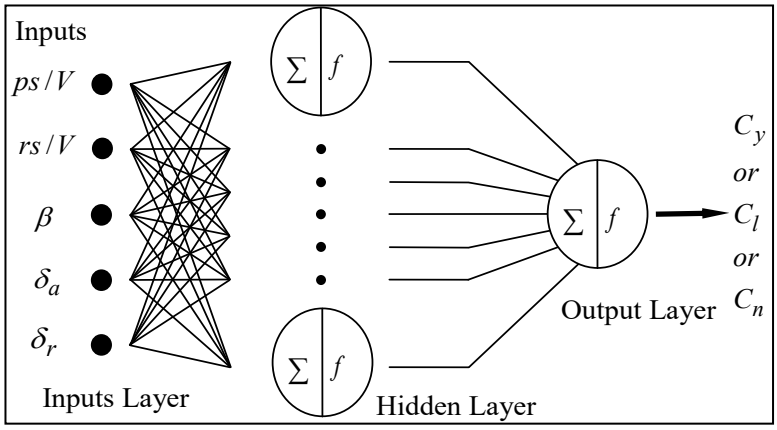


Figure 1. Schematic of Feed Forward Neural Network Training Model.

For the estimation of C_{l_β} using Delta Method, the input file is again modified by adding and subtracting the small perturbations, $\Delta\beta$ to β at each time point with no change, while other variables ($ps/V, rs/V, \delta_a and\ \delta_r$) are kept at their original values. These modified input files containing ($\beta +$

$\Delta\beta, ps/V, rs/V, \delta_a \; and \; \delta_r$) and ($\beta - \Delta\beta$, $ps/V, rs/V, \delta_a and \; \delta_r$) are then supplied to the same trained network to predict the value of rolling moment coefficient, C_l. The predicted value of C_l corresponding to modified input files are denoted by $C_l \; (\beta + \Delta\beta)$ and $C_l \; (\beta - \Delta\beta)$. The stability derivative C_{l_β} is now estimated using the equation (9) which is named as Delta method.

$$C_{l_\beta} = \{C_l(\beta + \Delta\beta) - C_l(\beta - \Delta\beta)\}/\{(\beta + \Delta\beta) - (\beta - \Delta\beta)\} = \{C_l\,(\beta\; +\; \Delta\beta) - C_l\,(\beta -\; \Delta\beta)\}/ \; 2\Delta\beta \tag{9}$$

3. Numerical Integration of Aircraft Equations

Standard lateral-directional equations (10-13) have been solved numerically using 4th order Runge-Kutta method [25] in MATLAB®R2020a for the generation of simulated flight data using the real flight control inputs of research aircraft ATTAS [26]. The flight aileron and rudder inputs used during the MATLAB simulation are depicted in Figure (2). The simulation is performed for 30 seconds with time step of 0.05 second.

$$\dot{\beta} = \; (\bar{q}S/mV)C_Y - r + (g/V)\sin\phi \tag{10}$$

$$\dot{p} = [(1/(I_x I_z - I_{xz}^2)][\bar{q}Ss(I_z C_l + I_{xz} C_N)] \tag{11}$$

$$\dot{r} = \; [(1/(I_x I_z - I_{xz}^2)][\bar{q}Ss(I_x C_N + I_{xz} C_l)] \tag{12}$$

$$\dot{\phi} \; = p \tag{13}$$

The linear aerodynamic model considered for lateral-directional dynamics for the ATTAS aircraft [26] is considered and represented by equations (14-16). The acceleration along y-axis is calculated using the equation (17).

$$C_Y = \; C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}(ps/V) + C_{Y_r}(rs/V) + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r \tag{14}$$

$$C_l = \; C_{l_0} + C_{l_\beta}\beta + C_{l_p}(ps/V) + C_{l_r}(rs/V) + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r \tag{15}$$

$$C_N = \; C_{N_0} + C_{N_\beta}\beta + C_{N_p}(ps/V) + C_{N_r}(rs/V) + C_{N_{\delta_a}}\delta_a + C_{N_{\delta_r}}\delta_r \tag{16}$$

$$a_y = (\bar{q}S/m)C_Y \tag{17}$$

Total 600 data points of variables $\beta, \; ps/V \, , ps/V \, , \delta_a, \delta_r, C_Y, C_l, C_N$ were generated for parameter estimation and validation of the results. Table 1 represents the values of flight velocity, density, mass and moment of inertia and Table 2 shows the true values of lateral-directional parameters. The values given in Table 1 and Table 2 were used as initial inputs (one time) to generate the simulated data during

numerical integration of lateral-directional equations. The variation in motion variables, force and moment coefficients corresponding to real flight control inputs during the simulation are shown in Figure 3 and Figure 4.

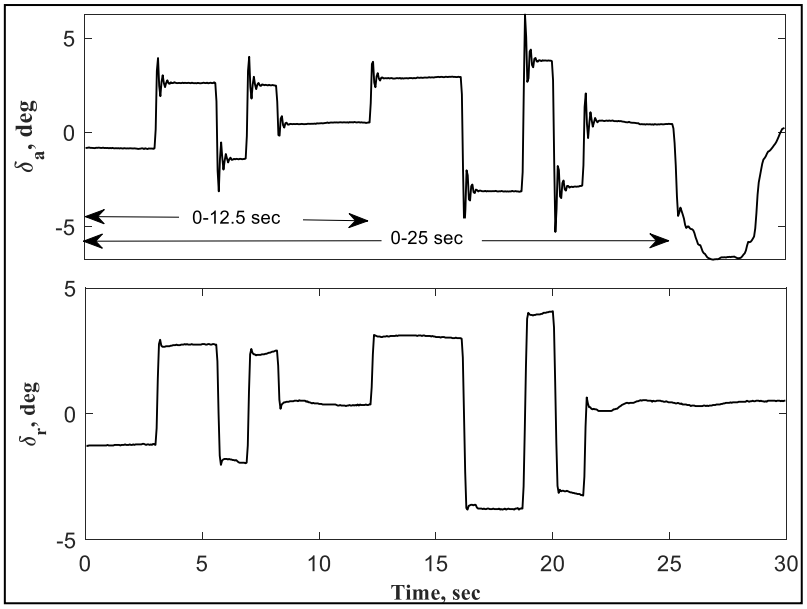


Figure 2. Control inputs used for generating simulated flight data.

Table 1. Flight conditions, geometric, mass and inertia characteristics.

True Air Speed	V = 106.70 m/s
Air Density	ρ = 0.731 kg-m^3
Mean Aerodynamic Chord	c = 3.159 m
Half Wing Span	s = 10.75 m
Wing Area	S = 64.00 m^2
Mass	m = 19633.23 kg
Moment of Inertia	I_X = 162392.80 kg-m^2,
	I_Y = 252687.00 kg-m^2
	I_Z = 386737.83kg-m^2,
	I_{XZ} = 11442.00 kg-m^2

Table 2. True values of lateral-directional derivatives.

Parameters	True Value	Parameters	True Value	Parameters	True Value
C_{Y_p}	0.303	C_{l_p}	-0.978	C_{N_p}	-0.115
C_{Y_r}	0.727	C_{l_r}	0.418	C_{N_r}	-0.495
C_{Y_β}	-1.333	C_{l_β}	-0.126	C_{N_β}	0.281
$C_{Y_{\delta_a}}$	0.029	$C_{l_{\delta_a}}$	-0.247	$C_{N_{\delta_a}}$	0.0
$C_{Y_{\delta_r}}$	0.191	$C_{l_{\delta_r}}$	0.046	$C_{N_{\delta_r}}$	-0.166

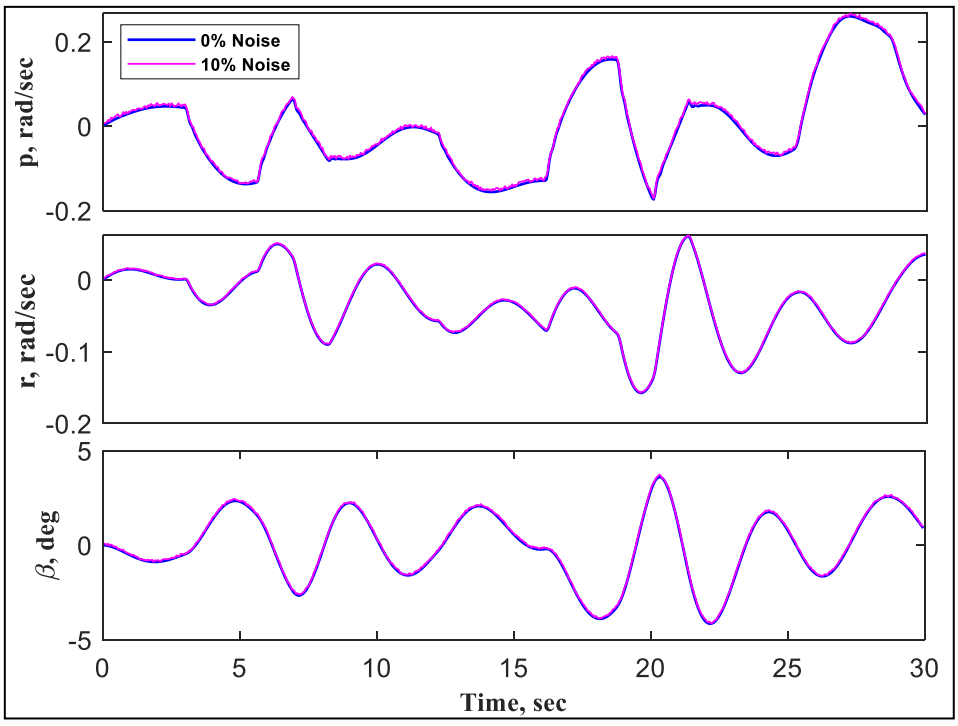


Figure 3. Variation in motion variables with respect to flight control inputs.

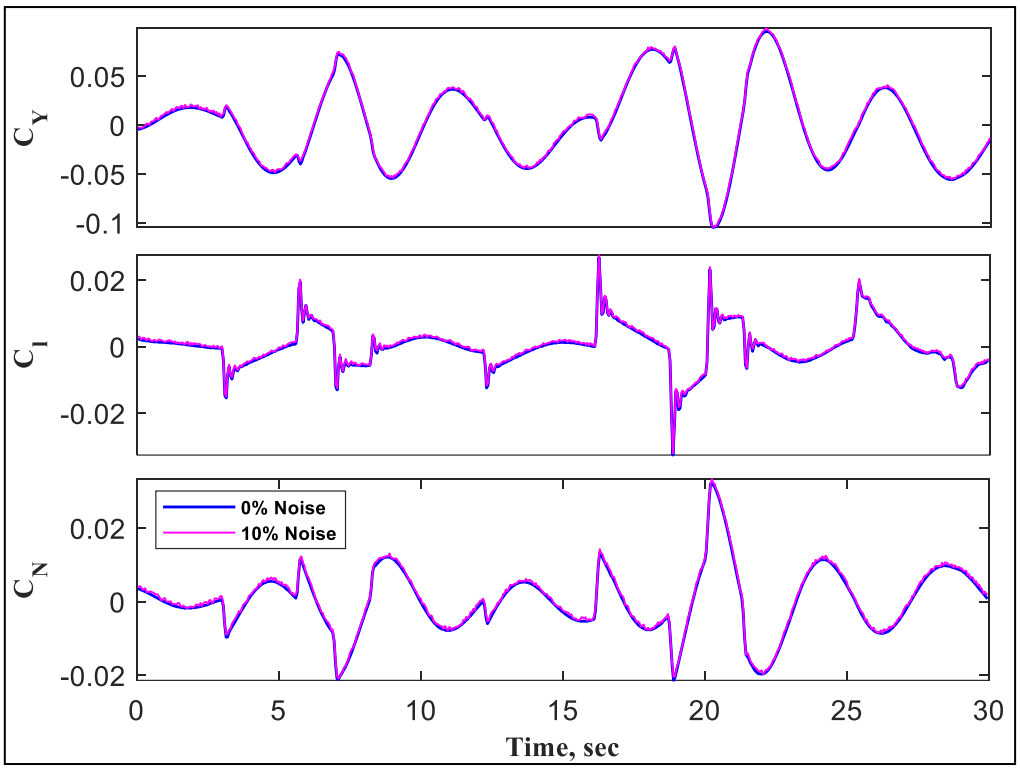


Figure 4. Variation in force and moment coefficients with respect to flight control inputs.

4. Results and Discussion

The simulated data were generated using two different sets of flight control inputs shown in Figure 1. The simulated data from 0-12.5 second (250 data points) generated using control inputs 1 is used for parameter estimation. As mentioned in the methodology section, the feedforward neural network is first trained by supplying the variables $\beta, ps/V, rs/V, \delta_a, \delta_r$ as inputs and one of the corresponding coefficients (C_Y *or* C_l *or* C_N) as output. The mean square error (MSE) is calculated by equation (18).

$$Mean\ Square\ Error = \frac{1}{n}\sum_{i=1}^{n}[¥(i) - €(i)]^2 \tag{18}$$

Where n is the number of data points, ¥ is the desired output (i.e. known C_Y or C_l or C_N) and € is the predicted output (i.e. C_Y or C_l or C_N). The target mean square error of 10^{-19} and 2000 number of iterations were set to stop the training of the network. Once the network is trained, the modified input files of input variables are presented to the network for predicting the corresponding values of lateral-directional force and moment coefficients. The Double Delta and Delta methods now use the modified inputs and predicted outputs for extracting the values of 15 parameters using the equations (4-9). The

parameters are also estimated using the most developed and implemented Maximum likelihood method. The Maximum Likelihood Method is documented by Maine and Illiff [11]. The method provides a measure of accuracy in terms of Cramer-Rao bound. The random disturbances caused by the surrounding atmosphere are always present in measured real flight data. In the absence of real flight data, it is always recommended in literature to estimate parameters in real flight like conditions by adding the Gaussian noise of various intensity. Therefore, Gaussian noise of 10% is introduced in the simulated data containing motion variables, $\beta, p, r, a_{y,}\, \dot{p}$ and $\dot{r}$. The values of $a_{y,}\, \dot{p}$ and $\dot{r}$ are obtained from equations (11-12) and equation (17) respectively. The noise was added to the simulated data to test the robustness of the Double Delta method along with Delta Method and Maximum likelihood method in estimating the parameters in real flight like conditions. The noisy data for lateral-directional force and moment coefficients are obtained from equations (19-21) by substituting the noisy values of $a_{y,}\, \dot{p}$ and $\dot{r}$.

$$C_{Y_{noise}} = 2ma_y/\rho V^2 S \tag{19}$$

$$C_{l_{noise}} = 2(I_x\dot{p} - I_{xz}\dot{r})/\,\rho V^2 Ss \tag{20}$$

$$C_{N_{noise}} = 2(I_z\dot{r} - I_{xz}\dot{p})/\,\rho V^2 Ss \tag{21}$$

The input and output files containing noisy data of motion variables (β_{noise}, p_{noise}, r_{noise}, δ_a and δ_r) and coefficients ($C_{Y_{noise}}$, $C_{l_{noise}}$ and $C_{N_{noise}}$) are prepared for training the neural model and parameter estimation using Double Delta and Delta methods as per the process stated in the methodology section. The comparison of simulated data with and without noise is shown in Figure 2 and Figure 3. The parameters are also estimated using 10% noisy data from Maximum Likelihood method. For neural network model, various tuning parameters such as number of hidden layers (1-2), number of nodes in hidden layers (4-6), learning rate of the model (0.2-0.3), momentum rate (0.3-0.5), transfer function (Tan-sigmoid, Log-sigmoid, Linear), initial random weights (0-1) were changed within a specified range for training of the neural model. Levenberg - Marquardt back propagation

optimization is used such that minimum mean squared error is obtained. After completing the tunning process, the selected parameters were one hidden layer with five nodes, learning rate as 0.3 and

momentum rate as 0.5. Logistic-sigmoid transfer function is used for input and linear (purelin) transfer function for the output.

Table 3 shows the values of 15 parameters along with standard deviation and Cramer-Rao bounds estimated using Delta method, Double Delta method and Maximum likelihood method from the simulated flight data having 0% noise and 10% noise.

The estimated parameter values are almost identical as the true target values of parameters for the 0% noisy data whereas less standard deviation is obtained for Double Delta method compared to Delta method. The simulated and estimated motion variables (β, p, r) and coefficients (C_Y, C_l, C_N) are compared in the Figure 5 and Figure 6. All three methods estimate the parameters with high accuracy resulting in substantial overlap between the simulated and estimated responses of lateral-directional motion.

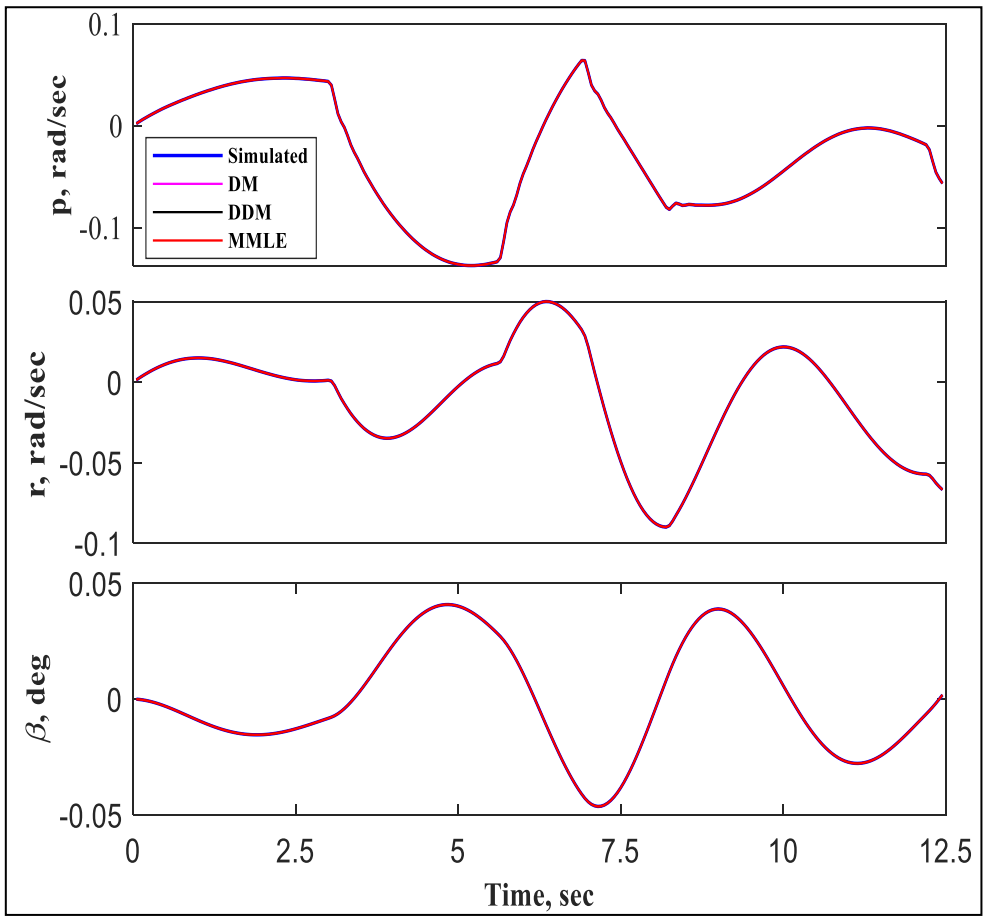


Figure 5. Comparison of simulated and estimated motion variables for 0% noise.

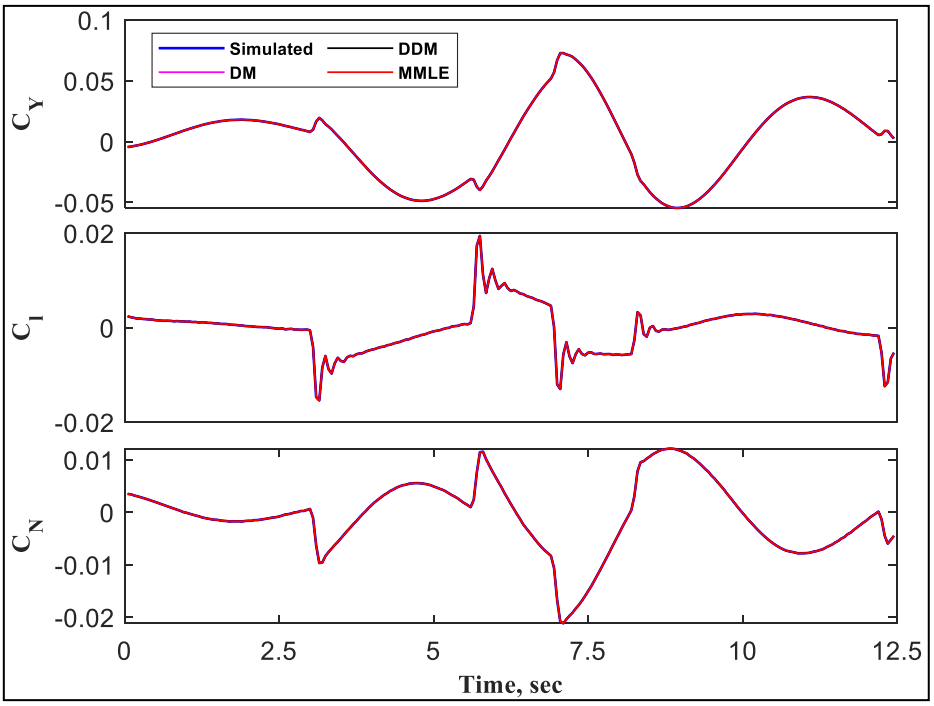


Figure 6. Comparison of simulated and estimated coefficients for 0% and 10% noise.

Table 3. Comparison of estimated parameters obtained using DM, DDM and MMLE.

Para-meters	Target Values	DM (0% noise)	DDM (0% noise)	MMLE (0% noise)	DM (10% noise)	DDM (10% noise)	MMLE (10% noise)
C_{Y_β}	-1.333	-1.3332 (0.0008)	-1.3330 (1.52×10^{-4})	-1.3330 (0.0134)	-1.2666 (0.2226)	-1.3379 (0.0727)	-1.1723 (0.5141)
C_{Y_p}	0.303	0.2991 (0.0090)	0.3032 (2.42×10^{-4})	0.3030 (0.0291)	0.4610 (0.3872)	0.3374 (0.1390)	0.2062 (0.1924)
C_{Y_r}	0.727	0.7267 (0.003)	0.7265 (8.62×10^{-4})	0.7270 (0.0012)	0.8700 (0.3057)	0.7962 (0.1567)	0.6047 (0.2461)
$C_{Y_{\delta a}}$	0.029	0.0291 (0.0003)	0.0290 (0.615×10^{-4})	0.0290 (0.0009)	0.0053 (0.0993)	0.0151 (0.0364)	0.0120 (0.031)
$C_{Y_{\delta r}}$	0.191	0.1908 (0.0007)	0.1910 (0.665×10^{-4})	0.1910 (0.0041)	0.2694 (0.2125)	0.2264 (0.1391)	0.0716 (0.1772)
C_{l_β}	-0.126	-0.1260 (3.17×10^{-5})	-0.1260 (1.201×10^{-5})	-0.1260 (0.0118)	-0.0935 (0.0216)	-0.0941 (0.0465)	-0.1052 (0.0773)
C_{l_p}	-0.978	-0.9780 (1.623×10^{-4})	-0.9780 (8.889×10^{-5})	-0.9780 (0.0078)	-0.6616 (0.1862)	-0.7982 (0.2554)	-0.7143 (0.2190)
C_{l_r}	0.418	0.4180 (1.54×10^{-4})	0.4180 (5.403×10^{-5})	0.4180 (0.0357)	0.4464 (0.2263)	0.4437 (0.1164)	0.5288 (0.194)

$C_{l_{\delta_a}}$	-0.247	-0.2470 (5.2×10^{-5})	-0.2470 (1.888×10^{-5})	-0.2470 (0.0005)	-0.2724 (0.0266)	-0.2654 (0.0461)	-0.3212 (0.0945)
$C_{l_{\delta_r}}$	0.046	0.0460 (3.22×10^{-5})	0.0460 (0.79×10^{-5})	0.0460 (0.0001)	0.0825 (0.0832)	0.0508 (0.0232)	0.1470 (0.0638)
C_{N_β}	0.281	0.2810 (1.91×10^{-5})	0.2810 (1.474×10^{-5})	0.2810 (0.0832)	0.2687 (0.0224)	0.2762 (0.0157)	0.2413 (0.0441)
C_{N_p}	-0.115	-0.1150 (5.97×10^{-5})	-0.1151 (5.106×10^{-5})	-0.1150 (0.00328)	-0.1630 (0.1158)	-0.1338 (0.0579)	-0.1421 (0.0866)
C_{N_r}	-0.495	-0.4949 (4.67×10^{-5})	-0.4949 (9.76×10^{-5})	-0.4950 (0.0099)	-0.5202 (0.1362)	-0.4872 (0.0511)	-0.4784 (0.1141)
$C_{N_{\delta_a}}$	0.0	-0.0000 (0.13×10^{-5})	-0.0000 (0.072×10^{-5})	-0.0000 (0.00001)	0.0017 (0.0145)	0.0023 (0.0162)	-0.0018 (0.0293)
$C_{N_{\delta_r}}$	-0.166	-0.1660 (1.36×10^{-5})	-0.1660 (0.657×10^{-5})	-0.1660 (0.0044)	-0.1794 (0.0232)	-0.1630 (0.0160)	-0.1636 (0.0382)

For investigating the effects of 10% noise on the values of estimated parameters, the accuracy of each parameter obtained from all three methods is calculated and presented in Table 4. The accuracy of C_{Y_β}, estimated using DDM is 99.634 with improvement of 4.615% compared to Delta method and 11.99% compared to MMLE. A high improvement of 40.79% and 20.56% for C_{Y_p} parameter is observed compared with the Delta method and MMLE respectively.

A significant improvement in the estimated values of parameters namely C_{Y_r}, $C_{Y_{\delta_a}} and\ C_{Y_{\delta_r}}$ is obtained for Double delta method as compared to

the parameters estimated using Delta method instead of MMLE. MMLE estimates C_{l_β} more accurately compared to Delta method and Double delta method. MMLE couldn't estimate $C_{l_{\delta_r}}$ while Double Delta method estimated its value with the accuracy of 89.565%. The equation (22) is not applicable for finding the accuracy of $C_{N_{\delta_a}}$ as its target value is zero. The estimated values for the parameter $C_{N_{\delta_a}}$ are expressed as an absolute error. Based on overall analysis of Table 3 and Table 4, It is evident that the Double delta method estimates most parameters with high accuracy as compared to Delta method and MMLE.

Table 4. Accuracy of estimated parameters with respect to target values.

Parameter	DM	DDM	MMLE
C_{Y_β}	95.019%	99.634%	87.944%
C_{Y_p}	47.854%	88.646%	68.05%
C_{Y_r}	80.330%	90.481%	83.177%
$C_{Y_{\delta_a}}$	18.27%	52.068%	41.379%
$C_{Y_{\delta_r}}$	58.952%	81.466%	37.487%
C_{l_β}	74.206%	74.682%	83.492%
C_{l_p}	67.648%	81.615%	73.036%
C_{l_r}	93.205%	93.851%	73.492%
$C_{l_{\delta_a}}$	89.716%	92.550%	69.959%
$C_{l_{\delta_r}}$	20.652%	89.565%	-119.56%
C_{N_β}	95.622%	98.291%	85.871%
C_{N_p}	58.260%	83.652%	76.434%
C_{N_r}	94.909%	98.424%	96.646%
$C_{N_{\delta_a}}$	0.0017	0.0023	0.0018
$C_{N_{\delta_r}}$	91.927%	98.192%	98.554%

*Standard Deviation; *Cramer-Rao Bounds

For 10% noisy data, the values of parameters estimated using Double Delta method are closer to the target values of parameters with less standard deviations. A significant improvement in accuracy of estimated parameters is seen for Double Delta method compared to other two methods. The relative accuracy of estimated parameters is calculated using the equation (22).

$$\text{Accuracy} = \left(1 - \frac{[\text{Target values of parameter} - \text{estimated value of parameter}]}{[\text{Target values of parameter}]}\right) \times 100\% \quad (22)$$

The validation of parameters obtained from the simulated data with 10% added noise using Delta method, Double delta method and MMLE method is done. For the validation, the equations (10-13)

were once again solved numerically for the control inputs from 0-25 second (500 data points) as shown in Figure 2. The values of estimated parameters obtained using 10% noisy data were substituted in the aerodynamic model represented by equations (14-16). The responses generated using estimated parameters obtained from Delta method, Double Delta method and MMLE method are contrasted with the simulated responses in Figure 7 and Figure 8. It can be seen for Double Delta method that simulated responses show close agreement with the estimated responses compared to Delta method and MMLE method. The results demonstrate the robustness of the Double Delta method in estimating the parameters with high accuracy, even when applied to simulated data with 0% and 10% noise. The Double Delta method can be considered as an alternative method for estimating the aircraft parameters from real flight data.

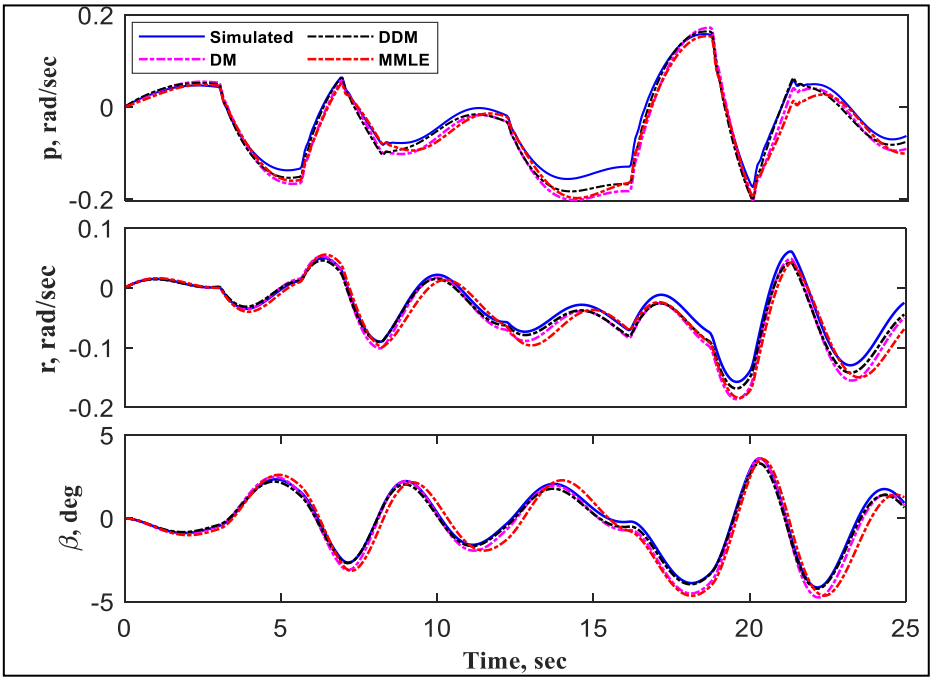


Figure 7. Comparison of simulated and estimated motion variables for 10% noise.

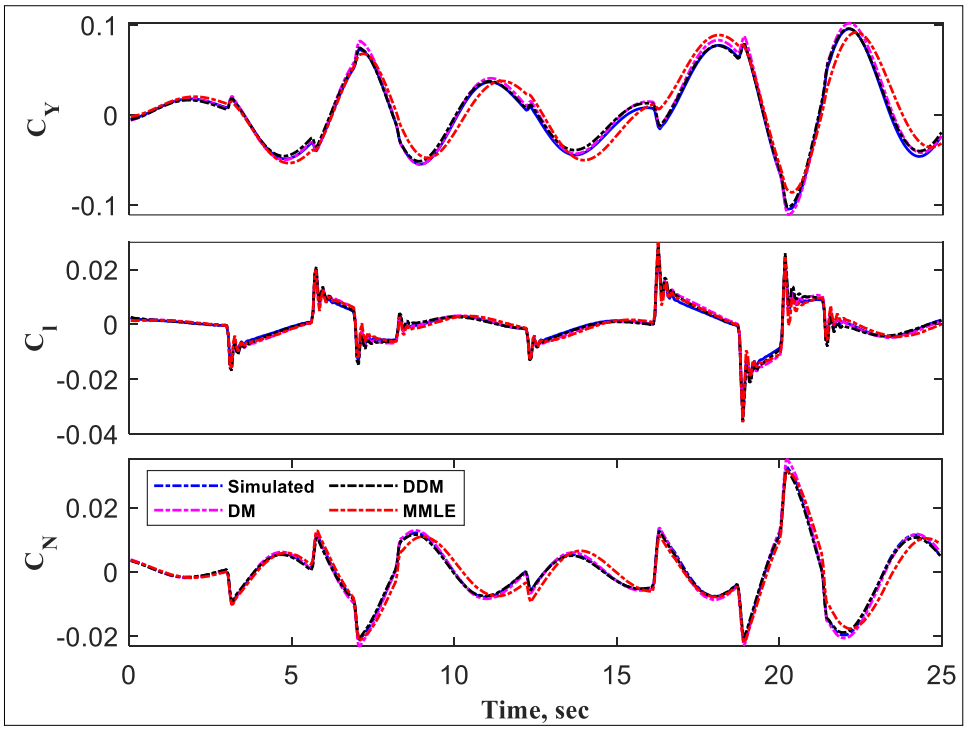


Figure 8. Comparison of simulated and estimated coefficients for 10% noise.

5. Conclusion and Future Scope

In this paper, Double Delta method has been proposed for aircraft lateral-directional parameter estimation in conjunction with Feedforward Neural Network. A comparative analysis was conducted to evaluate robustness of two existing methods and one proposed method in estimation of parameters using simulated data having 0% noise and 10% noise. For 10% noisy data, the C_Y derivatives obtained using Double Delta Method showed an improvement ranging from 4.5% to 41% compared with Delta Method, whereas the improvement compared with Maximum Likelihood Method ranged from 7% to 44%. A significant improvement, ranging from 0.4% to 22.5%, is also observed in the values of C_l derivatives obtained from Double Delta Method. Similarly, C_N derivatives showed improvement, ranging from 0.4% to 25% compared to Delta Method and Maximum Likelihood Method. The results obtained using all three methods are shown in graphical and tabular form. When the comparison is drawn among Delta method, MMLE method and proposed Double Delta method, the latter provides more accurate parameter estimates with lesser standard deviation. Based on the result, The Double Delta

Method can be used for the estimation of parameters from the flight data of stable and unstable flight vehicles by considering linear and non-linear aerodynamic models.

List of symbols

C_l, C_N	rolling and yawing moment coefficients
C_y	lateral force coefficient
Ix, Iy, Iz	moments of inertia about x, y, and z axis
p, r	roll, and yaw rates
$\bar{q}$	dynamic pressure
β, φ, θ	side-slip, bank and pitch angles
δa, δr	aileron and rudder deflections

Acknowledgments

One of the authors sincerely acknowledges the Amity University Uttar Pradesh India.

Author Contributions

S.S.: Conceptualization, methodology, software, supervision; C.P.R: Writing original draft preparation; A.K.: Data curation, writing, reviewing and editing. All authors have studied and allowed to the published edition of the manuscript.

Conflicts of Interest

The authors state no conflict of interest.

Data availability

All data generated or analysed during this study are included in manuscript, supplementary information too.

References

[1] Zadeh, L.A., "From Circuit Theory to System Theory, Proceedings of the IRE," Vol. 50, May 1962, pp. 856-965.

[2] Klein, V., and Morelli, E.A., "Aircraft System Identification – Theory and Practice," AIAA Educaton Series, Inc., Reston, Virginia, 2006.

[3] Hamel, P.G., and Jategaonkar, R.V., "The Evolution of Flight Vehicle System Identification," AGARD, DLR Germany, 8-10, May 1995.

[4] Iliff, K. W., "Parameter Estimation for Flight Vehicle," Journal of Guidance, Control and Dynamics, Vol. 12, No. 5, 1989, pp. 609-622.

[5] Hamel, P. G., "Aircraft Parameter Identification Methods and their Applications Survey and Future Aspects," AGARD, 15-104, Nov. 1979.

[6] Hess, R.A. On the use of back propagation with feed forward neural networks for the aerodynamic estimation problem, AIAA Paper 93-3639, August 1993.

[7] Basappa and Jategaonkar, R.V. Aspects of feed forward neural network modeling and its application to lateral-directional flight-data, DLR IB-111-95/30, September 1995.

[8] Linse, D.J. and Stengel, R.F. Identification of aerodynamic coefficients using computational neural networks, *J Guid Cont and Dynam*, November – December 1993, 16, (6), pp. 1018–1025.

[9] Youseff, H.M. and Juang, J.C. Estimation of aerodynamic coefficients using neural networks, AIAA Paper 93-3639, August 1993.

[10] Raol, J.R. and Jategaonkar, R.V. Aircraft parameter estimation using recurrent neural networks -- a critical appraisal, AIAA Paper 95-3004, August 1995.

[11] Maine, K. E., and Iliff, K. W., "Application of Parameter Estimation to Aircraft Stability and Control: The Output Error Approach," NASA RP 1168, Jan. 1986.

[12] Raisinghani, S.C., Ghosh, A.K. and Kalra, P.K. Two new techniques for aircraft parameter estimation using neural networks, *Aeronaut J*, January 1998, 102, (1011), pp. 25-29.

[13] Ghosh, A.K., Raisinghani, S.C. and Khubchandani, S. Estimation of aircraft lateral-directional parameters using neural networks, *J Aircr*, November-December 1998, 35, (6), pp. 876-881.

[14] Akanksha Talwa, Gaurav Lokhande, Ritesh Jain and Singh S., "Estimation of Aerodynamic Parameter using Cascade Forward Back Propagation," IEEE 2nd International Conference on Telecommunication and Networks, India, Aug 2017.

[15] Singh, S., and Ghosh, A. K., "*Estimation of Lateral-Directional Parameters using Neural Networks based Modified Delta Method*," The Aeronautical Journal, Vol. 111, No. 1124, Oct. 2007, pp. 659-667.

[16] Singh, S., and Ghosh, A. K., "Modified Delta Method for Estimation of Parameters from Flight Data of Stable and Unstable Aircraft," 3rd IEEE International Advance Computing Conference, AKG, Ghaziabad, India, Feb 2013, pp. 768-774.

[17] R. K. Chauhan., and Singh, S., “Application of Neural Networks Based Method for Estimation of Aerodynamics Derivatives,” 7th IEEE International Conference on Cloud Computing, Data Science & Engineering, India, Jan 2017.

[18] H. O. Verma and N. K. Peyada, “Parameter estimation of unstable aircraft using extreme learning machine”, Defense Science Journal, Vol. 67, No. 6, pp. 603-611, 2017.

[19] H. O. Verma and N. K. Peyada, “Estimation of aerodynamic parameters near stall using maximum likelihood and extreme learning machine-based methods”, The Aeronautical Journal, 2020.

[20] Priya, D. Gupta, K. Saxena and H. J. Rao, "Estimation of Longitudinal Aerodynamic Parameters Using Extreme Learning Machine and Delta Method," 2025 12th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions) (ICRITO), Noida NCR, India, 2025, pp. 1-6. DOI: 10.1109/ICRITO66076.2025.11241824

[21] Priya, Deepa Gupta and Kumud Saxena, Machine Learning Based Models for Estimation of Stability and Control Derivatives, Journal of Aeronautics, Astronautics and Aviation, 58(2), 159-169, 2026,https://doi.org/10.6125/JoAAA.202602_58(2S).01

[22] N. S. Kashyap and R. Kumar, A Review of AI-Based Aircraft System Identification: Methods, Applications, and Chalenges, Journal of Aeronautics, Astronautics and Aviation, Vol.58 No.2S, Pp. 217-228, 2026, https://doi.org/10.6125/JoAAA.202602_58(2S).06

[23] C. P. Rudesh, S. Singh, P. S. Rao, and H. J. Rao, “Incorporating scaled conjugate gradient (SCG) in system identification for aircraft stability and control derivatives,” *Discover Mechanical Engineering*, vol. 4, no. 1, Dec. 2025, DOI: 10.1007/s44245-025-00117-3

[24] C. P. Rudesh, S. Singh, and P. Srinivasa Rao, “A Bayesian regulation framework for robust estimation of aircraft stability and control derivatives,” *Aerospace Systems*, 2026, DOI: 10.1007/s42401-026-00464-3.

[25] J.C. Butcher, G. Wanner, Runge-Kutta methods: some historical notes, Applied Numerical Mathematics, Volume 22, Issues 1–3,1996, Pages 113-151, ISSN 0168-9274.

[26] Jategaonkar, R.V. Identification of the aerodynamic model of the DLR research aircraft ATTAS from flight test data DLR, DLR-FB 90-40, Braunschweig, Germany, September1990.