Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

A Structural Framework for Generalized Soft Decision Systems: Component Formulation and a Comparative Plithogenic Study

Rajababu S
Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences
Ramesh A
ramesh.adhimoolam@gmail.com
Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences
Babu D
Thanthai Periyar Government Institute of Technology

Research Article

Keywords:

Posted Date: November 13th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11498428/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

A Structural Framework for Generalized Soft Decision Systems: Component Formulation and a Comparative Plithogenic Study

S. Rajababu[1†], A. Ramesh[2*], D. Babu[3†]

[1]Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, SIMATS, Chennai, 602105, Tamilnadu, India.

[2*]Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, SIMATS, Chennai, 602105, Tamilnadu, India.

[3]Department of Mathematics, Thanthai Periyar Government Institute of Technology, Vellore, 632002, Tamilnadu, India.

*Corresponding author(s). E-mail(s): ramesh.adhimoolam@gmail.com;
Contributing authors: rajabab17@gmail.com; babudurai15@gmail.com;
†These authors contributed equally to this work.

Abstract

Soft decision-making models consist of parameterized representations, transformations, evaluation measures, and decision rules. Because these components rely on model-specific notation, structural comparisons across heterogeneous mechanisms are difficult. This paper proposes a modular Generalized Soft Decision System (GSDS) framework that separates these components into a functional pipeline without imposing a common aggregation operator. A GSDS is formalized as a 7-tuple comprising a representation mapping, intermediate transformation, reference evaluation functional, and decision aggregator. Admissibility conditions guarantee well-defined representations, bounded evaluation values, and valid decision scores. The framework is applied to a binary soft-matrix realization and a selected contradiction-sensitive plithogenic fuzzy-soft realization. An application to the UCI Wine dataset demonstrates how different decision mechanisms yield distinct rankings from identical data. Using conventional tie-adjusted calculations, the Spearman rank correlation at the primary threshold is $\boldsymbol{\rho = -0.370943}$, with zero top-10 overlap. Analytically, the selected contradiction-sensitive plithogenic realization exhibits parabolic non-monotonicity, functioning

as an extremity-sensitive evaluator rather than a monotone suitability metric. The GSDS framework provides a common mathematical language for describing decision mechanisms, without claiming universal algebraic equivalence or the superiority of any single realization.

Keywords: soft set, fuzzy soft set, plithogenic set, multi-criteria decision making, monotonicity.

1 Introduction

Soft set theory was introduced by Molodtsov [7] as a parameterized mathematical framework for handling uncertainty. Subsequent extensions incorporate fuzzy membership [12], intuitionistic [2] and neutrosophic information [9], contradiction degrees [8], and other uncertainty variants into decision pipelines [4–6].

Soft decision procedures typically involve the following steps: representing alternatives via parameters, normalizing or transforming this representation, applying an evaluation mechanism (e.g., similarity or distance), and aggregating the result into a final score.

Despite this common operational sequence, the mathematical semantics of each step vary. Two methods yielding a numerical score are not necessarily algebraically equivalent. A binary soft matrix encodes whether an alternative meets a threshold condition [3], whereas a fuzzy-soft model retains graded membership. Linguistic models [10, 11, 13] require distinct variables and transformations, and plithogenic models account for contradiction between attribute values [9].

Existing literature primarily focuses on individual extensions, specific uncertainty representations, or application-oriented algorithms. This work examines whether their computational components fit a common typed structural architecture. The GSDS framework is intended as a structural decision-system architecture rather than a replacement for the underlying definitions of soft, fuzzy-soft, or plithogenic sets.

The contributions of this study are:

(i) Defining a Generalized Soft Decision System (GSDS) as a formal 7-tuple incorporating representation, transformation, reference, evaluation, and decision components.
(ii) Establishing admissibility conditions to separate structural requirements from model-specific assumptions.
(iii) Proving elementary mathematical properties, including boundedness, score determinacy, and permutation invariance.
(iv) Mapping binary soft-matrix and selected contradiction-sensitive plithogenic mechanisms as distinct framework realizations.
(v) Demonstrating ranking divergence between these mechanisms using the UCI Wine dataset.
(vi) Analytically characterizing the non-monotone, extremity-sensitive behavior of the selected contradiction-sensitive plithogenic realization.

This paper does not assert that every soft-set model admits a GSDS realization. A model fits the framework only when its mathematical mappings are explicitly specified.

2 Mathematical Preliminaries

Let U be a finite nonempty universe of alternatives and E be a finite nonempty set of parameters.

Definition 1 (Soft set [7]) A soft set over U with parameter set E is a pair (F, E), where

$$F : E \longrightarrow 2^U.$$

For each parameter $e \in E$, the set $F(e)$ contains the alternatives associated with that parameter.

For numerical decision problems, information is encoded in a matrix

$$X = [x_{ij}] \in \mathbb{R}^{m \times n},$$

where

$$m = |U|, \qquad n = |E|.$$

In fuzzy representations, the entries satisfy

$$x_{ij} \in [0, 1].$$

Definition 2 (Componentwise order) For $x, y \in [0, 1]^n$, write

$$x \leq y$$

if

$$x_j \leq y_j, \qquad j = 1, \ldots, n.$$

The componentwise order is used when a realization assumes monotonicity.

3 Generalized Soft Decision Systems

We define the structural framework below.

Definition 3 (Generalized soft decision system) A generalized soft decision system (GSDS) is a tuple

$$D = (U, E, R, F, \Psi, \Omega, \Phi),$$

where:

1. U is a finite set of alternatives;
2. E is a finite set of parameters, with $n = |E|$;
3. R is a primary representation space;

4.
$$F : U \times E \longrightarrow R$$
is a representation mapping;

5.
$$\Psi : R \longrightarrow R'$$
is an intermediate transformation mapping;

6. Ω is an evaluation mapping that is either reference-dependent,
$$\Omega : R' \times R_0 \longrightarrow [0, 1],$$
or reference-free,
$$\Omega : R' \longrightarrow [0, 1];$$

7.
$$\Phi : [0, 1]^n \longrightarrow \mathbb{R}$$
is a decision functional.

For $u_i \in U$ and $e_j \in E$, let
$$r_{ij} = F(u_i, e_j).$$
The transformed representation is
$$z_{ij} = \Psi(r_{ij}).$$

For a reference-dependent realization, let
$$r^0_{ij} \in R_0$$
denote the corresponding reference representation and define
$$q_{ij} = \Omega(z_{ij}, r^0_{ij}).$$
For a reference-free realization,
$$q_{ij} = \Omega(z_{ij}).$$

The evaluation vector associated with u_i is
$$q_i = (q_{i1}, \ldots, q_{in}),$$
and the final decision score is
$$S(u_i) = \Phi(q_i).$$

3.1 Admissibility Conditions

A GSDS realization is admissible if the following conditions hold:

A1. U and E are finite and nonempty.
A2. The representation mapping F and transformation Ψ are well-defined on their respective domains.

A3. The evaluation mapping is well-defined and bounded:

$$0 \leq \Omega(z, r^0) \leq 1$$

in the reference-dependent case, and

$$0 \leq \Omega(z) \leq 1$$

in the reference-free case.

A4. The decision functional is defined on $[0,1]^n$:

$$\Phi : [0,1]^n \longrightarrow \mathbb{R}.$$

These conditions separate structural requirements from realization-specific assumptions.

4 Elementary Structural Consequences

Proposition 1 (Bounded evaluation vector) *For every admissible GSDS realization and alternative u_i,*

$$q_i \in [0,1]^n.$$

Proof By A3, every evaluation component satisfies

$$0 \leq q_{ij} \leq 1.$$

Each component of

$$q_i = (q_{i1}, \ldots, q_{in})$$

lies in $[0,1]$, so

$$q_i \in [0,1]^n.$$

□

Proposition 2 (Well-defined decision score) *For every admissible GSDS realization and alternative u_i,*

$$S(u_i) = \Phi(q_i)$$

is well-defined.

Proof By the preceding proposition,

$$q_i \in [0,1]^n.$$

By A4, Φ is defined on $[0,1]^n$. Thus $S(u_i)$ is well-defined. □

Proposition 3 (Permutation invariance) *If parameters lack intrinsic ordering and Φ is permutation invariant, a parameter permutation does not alter the decision score.*

Proof Let π be a permutation of $\{1, \ldots, n\}$, defining a permutation action $P_\pi(q_1, \ldots, q_n) = (q_{\pi(1)}, \ldots, q_{\pi(n)})$. By permutation invariance,

$$\Phi(P_\pi q) = \Phi(q).$$

The decision score remains unchanged. □

5 Representative Realizations

The GSDS framework maps different decision mechanisms structurally, independent of specific semantics.

5.1 Binary Soft-Matrix Realization

Given normalized numerical data

$$x_{ij} \in [0, 1]$$

and a threshold

$$\tau \in (0, 1),$$

define

$$q_{ij}^{SM} = \begin{cases} 1, & x_{ij} \geq \tau, \\ 0, & x_{ij} < \tau. \end{cases}$$

The resulting decision score is

$$S_{SM}(u_i) = \sum_{j=1}^{n} q_{ij}^{SM}.$$

This realization counts the criteria meeting or exceeding the threshold.

5.2 Selected Contradiction-Sensitive Plithogenic Realization

The following is a particular contradiction-sensitive construction adopted for this numerical study; it is not claimed to be the canonical plithogenic operator.

Let

$$\mu_{ij} \in [0, 1]$$

denote the normalized membership value. For this numerical study, we choose the contradiction assignment

$$c_{ij} = 1 - \mu_{ij}.$$

The contradiction-sensitive evaluation kernel is

$$A(\mu_{ij}, c_{ij}) = (1 - c_{ij})\mu_{ij} + c_{ij}.$$

Substituting

$$c_{ij} = 1 - \mu_{ij}$$

yields

$$\begin{aligned} A(\mu_{ij}, 1-\mu_{ij}) = A(\mu_{ij}) &= (1-(1-\mu_{ij}))\mu_{ij} + (1-\mu_{ij}) \\ &= \mu_{ij}^2 + 1 - \mu_{ij} \\ &= 1 - \mu_{ij} + \mu_{ij}^2. \end{aligned}$$

The final score is

$$S_P(u_i) = \frac{1}{n} \sum_{j=1}^{n} \left(1 - \mu_{ij} + \mu_{ij}^2\right).$$

For the selected contradiction-sensitive plithogenic realization, the reference argument is retained only to conform to the general reference-dependent interface and does not affect the evaluation.

5.3 Structural Mapping of the Realizations

Table 1 explicitly maps the mathematical components of both realizations into the GSDS 7-tuple.

Table 1 Structural mapping of the realizations into the GSDS tuple.

GSDS component	Binary soft-matrix realization	Selected contradiction-sensitive plithogenic realization
Universe U	Alternatives $\{u_1, \ldots, u_m\}$	Alternatives $\{u_1, \ldots, u_m\}$
Parameters E	Criteria $\{e_1, \ldots, e_n\}$	Criteria $\{e_1, \ldots, e_n\}$
Representation space R	$[0, 1]$	$[0, 1]$
Representation map F	$F(u_i, e_j) = x_{ij}^*$	$F(u_i, e_j) = \mu_{ij}$
Transformation Ψ	$\mathrm{id}_{[0,1]}$	$\Psi(\mu) = (\mu, 1-\mu)$
Reference space R_0	$(0, 1)$ with $r^0 = \tau$	Reference: not used in this realization
Evaluation kernel Ω	$\mathbf{1}_{\{z \geq \tau\}}$	$\Omega_P(\mu, c) = (1-c)\mu + c$, $c = 1-\mu$
Aggregation function Φ	$\sum_j q_{ij}$	$\frac{1}{n}\sum_j q_{ij}$

6 Structural Relationship with Other Soft Decision Models

The GSDS framework distinguishes structural similarity from mathematical equivalence. A model using a representation map F, transformation Ψ, evaluator Ω, and

decision functional Φ fits the GSDS structure when these mappings are defined and satisfy admissibility conditions.

Sharing a structural sequence does not imply algebraic equivalence. A binary soft-matrix method uses thresholding and counting, while a linguistic fuzzy-soft method uses linguistic variables and likelihood-based evaluation. Their decision semantics remain distinct.

The same applies to plithogenic models. Plithogenic set theory introduces attribute-value contradiction and contradiction-sensitive operations [9]. The term "realization" is used rather than suggesting unrelated soft-set models are special cases of a single operator.

7 Algorithmic Implementation and Complexity

1. For each alternative u_i and parameter e_j, compute $r_{ij} = F(u_i, e_j)$.
2. Transform the representation: $z_{ij} = \Psi(r_{ij})$.
3. Apply the evaluation map:

$$q_{ij} = \begin{cases} \Omega(z_{ij}, r_{ij}^0), & \text{reference-dependent,} \\ \Omega(z_{ij}), & \text{reference-free.} \end{cases}$$

4. Form $q_i = (q_{i1}, \ldots, q_{in})$.
5. Compute $S(u_i) = \Phi(q_i)$.
6. Rank alternatives by the resulting score.

Assuming F, Ψ, and Ω execute in $O(1)$ time and Φ in $O(n)$ time, score construction requires $O(mn)$. Comparison-based sorting takes $O(m \log m)$, yielding a total complexity of $O(mn + m \log m)$.

8 Mathematical Behavior of the Selected Contradiction-Sensitive Plithogenic Evaluation Kernel

The selected evaluation mapping is

$$A(x) = 1 - x + x^2, \qquad x \in [0, 1].$$

Completing the square yields

$$A(x) = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}.$$

Therefore,

$$A(x) \geq \frac{3}{4}$$

for every $x \in [0,1]$, with equality at

$$x = \frac{1}{2}.$$

The derivatives are

$$A'(x) = 2x - 1, \qquad A''(x) = 2.$$

Because $A''(x) = 2 > 0$, the function A is strictly convex. Since

$$A'(x) < 0 \qquad \text{on } [0, \tfrac{1}{2}),$$

the evaluation decreases as x increases on the lower half of the domain. Since

$$A'(x) > 0 \qquad \text{on } (\tfrac{1}{2}, 1],$$

the evaluation increases on the upper half.

The boundary values are

$$A(0) = 1, \qquad A\left(\frac{1}{2}\right) = \frac{3}{4}, \qquad A(1) = 1.$$

Also,

$$A(x) = A(1 - x).$$

Values equidistant from $1/2$ receive identical evaluations. The selected evaluation kernel is an affine transformation of the squared distance from the midpoint $1/2$, rather than a monotone suitability function.

For the complete decision score,

$$S_P(u_i) = \frac{1}{n} \sum_{j=1}^{n} A(\mu_{ij}),$$

the equivalent representation is

$$S_P(u_i) = \frac{3}{4} + \frac{1}{n} \sum_{j=1}^{n} \left(\mu_{ij} - \frac{1}{2}\right)^2.$$

This ranking depends on the average squared distance of membership values from the $1/2$ midpoint. An alternative with $\mu = 0$ matches the evaluation of one with $\mu = 1$. This mapping functions as an *extremity-sensitive evaluator*, not a normative suitability score.

9 Numerical Experiment: UCI Wine Dataset

The framework is applied to the UCI Wine dataset [1], analyzing 178 observations across 13 numerical features. Class labels are excluded; the dataset functions strictly as a multivariate numerical testbed. Because no external decision ranking is used, the experiment evaluates ranking divergence rather than decision accuracy.

Features are normalized using column-wise min–max scaling:

$$x_{ij}^{*} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}.$$

The resulting values satisfy

$$x_{ij}^{*} \in [0, 1].$$

No randomization or parameter optimization is used.

9.1 Binary Soft-Matrix Realization

At the primary threshold

$$\tau = 0.5,$$

the binary score is

$$S_{SM}(u_i) = \sum_{j=1}^{13} \mathbf{1}_{\{x_{ij}^{*} \geq 0.5\}}.$$

The maximum score is $S_{SM}^{\max} = 9$, achieved by alternatives $u_{14}, u_{15}, u_{31}, u_{34}, u_{56}$, and u_{74}.

9.2 Selected Contradiction-Sensitive Plithogenic Realization

Setting

$$\mu_{ij} = x_{ij}^{*},$$

the selected contradiction-sensitive plithogenic score is

$$S_P(u_i) = \frac{1}{13} \sum_{j=1}^{13} \left(1 - \mu_{ij} + \mu_{ij}^{2}\right).$$

The ten highest-ranked alternatives are detailed in Table 2.

9.3 Comparison of Rankings

At $\tau = 0.5$, the tied-rank Spearman correlation is $\rho = -0.370943$ and the tie-adjusted Kendall coefficient is $\tau_b = -0.263901$.

Two-sided p-values were computed using standard large-sample procedures implemented in statistical software (R), with tied ranks retained for Spearman correlation and tie adjustment for Kendall's τ_b. The calculated p-values are $p_\rho = 3.447 \times 10^{-7}$ and $p_\tau = 8.084 \times 10^{-7}$.

Table 2 Top ten alternatives under the selected contradiction-sensitive plithogenic realization.

Rank	Alternative	S_P
1	60	0.8839980
2	147	0.8741364
3	122	0.8475004
4	137	0.8439806
5	172	0.8421189
6	116	0.8410327
7	138	0.8409921
8	61	0.8392728
9	159	0.8379418
10	70	0.8324332

The top-ten sets share no common alternatives:

$$|T_{10}^{SM} \cap T_{10}^{P}| = 0.$$

Because the binary score generates ties, conventional tie-adjusted calculations are used. When constructing the top-10 set, tied binary scores were ordered by increasing alternative identifier. Alternative tie-breaking procedures would yield different correlation coefficients.

9.4 Threshold Sensitivity

We selected five representative thresholds spanning the interior of $[0, 1]$, including the primary threshold $\tau = 0.5$, to evaluate threshold sensitivity (Table 3).

Table 3 Sensitivity of ranking divergence to the threshold τ.

τ	Spearman ρ	Spearman p	Kendall τ_b	Top-10 overlap
0.25	−0.809027	$< 2.22 \times 10^{-16}$	−0.661586	0
0.35	−0.657532	$< 2.22 \times 10^{-16}$	−0.498569	0
0.50	−0.370943	3.448×10^{-7}	−0.263901	0
0.65	−0.027602	0.714567	−0.021020	1
0.75	0.088057	0.242476	0.067123	2

Rank association depends materially on the binary threshold. At low thresholds, the association is strongly negative. As the threshold increases, the negative association approaches zero. At $\tau = 0.75$, both correlation coefficients are weakly positive but lack statistical significance at the 5% level.

The top-ten overlap varies with the threshold, progressing as $0, 0, 0, 1, 2$ across $\tau = 0.25, 0.35, 0.50, 0.65, 0.75$. Alternative identifiers resolve ties deterministically in the top-ten selection.

10 Discussion and Limitations

Decomposing the decision process allows component-level comparisons without assuming identical semantics. The two realizations produce distinct orderings at the primary threshold, yielding $\rho = -0.370943$ and zero top-ten overlap. The sensitivity analysis confirms that rank association magnitude and sign rely heavily on the threshold.

The selected contradiction-sensitive plithogenic realization evaluates data based on an affine transformation of squared distance from the midpoint, $A(x) = \frac{3}{4} + \left(x - \frac{1}{2}\right)^2$, rather than uniform scaling. This structural difference accounts for the divergence from a threshold-counting rule.

This comparison illustrates methodological sensitivity rather than establishing any realization as universally preferable.

Limitations of this study include its structural rather than universal scope, relying on explicitly specified mappings. The numerical validation is restricted to a single benchmark dataset absent ground-truth class labels. Results depend on algorithmic parameters like the binary threshold ($\tau = 0.5$), the specific contradiction assignment $c = 1 - \mu$ (an extremity-sensitive evaluation kernel rather than a monotone function), normalization techniques, and deterministic tie-handling. We do not claim universal superiority for any selected soft-set model or evaluation mapping.

11 Future Research Directions

Future research directions include systematically studying contradiction functions beyond $c = 1 - \mu$, comparing alternative plithogenic aggregation operators, and analyzing threshold sensitivity more broadly. Integrating independent application targets (e.g., expert rankings) would further validate these mechanisms. Linguistic soft-set methods require evaluation against datasets containing genuine linguistic variables and expert judgments.

12 Conclusion

This paper introduced a structural framework for generalized soft decision systems, formalizing representation, transformation, evaluation kernels, and decision aggregation functions as separable components. This offers a language for describing heterogeneous decision mechanisms without asserting algebraic equivalence.

Two realizations were tested: binary soft-matrix thresholding and a selected contradiction-sensitive plithogenic realization. Tested on the UCI Wine dataset, the rankings diverged at the primary threshold, producing $\rho = -0.370943$, $\tau_b = -0.263901$, and zero top-ten overlap. Threshold sensitivity analysis showed rank correlation shifting from -0.809027 at $\tau = 0.25$ to 0.088057 at $\tau = 0.75$.

The selected evaluation kernel, $A(x) = 1 - x + x^2$, lacks global monotonicity, functioning instead as an extremity-sensitive evaluator. Separating framework assumptions from realization-specific behavior is critical. The framework's primary contribution is methodological: it offers a structured approach to formulate, compare, and analyze heterogeneous soft decision systems without claiming universal equivalence or superiority.

Declarations

- Funding: No specific research funding was received for this study.
- Ethics and Consent to Participate declarations: Not applicable.
- Competing interest: The authors have no competing interest
- Data availability: The dataset is publicly available: https://doi.org/10.24432/C5PC7J

References

[1] Aeberhard S, Forina M (1991) Wine. UCI Machine Learning Repository. https://doi.org/10.24432/C5PC7J

[2] Atanassov KT (1999) Intuitionistic fuzzy sets: theory and applications. Studies in Fuzziness and Soft Computing, vol 35. Physica-Verlag, Heidelberg

[3] Çağman N, Enginoğlu S (2010) Soft matrix theory and its decision making. Comput Math Appl 59(10):3308–3314. https://doi.org/10.1016/j.camwa.2010.03.015

[4] Fatimah F, Rosadi D, Hakim RBF (2018) Probabilistic soft sets and dual probabilistic soft sets in decision making with positive and negative parameters. J Phys Conf Ser 983:012112. https://doi.org/10.1088/1742-6596/983/1/012112

[5] Maji PK, Biswas R, Roy AR (2001) Fuzzy soft sets. J Fuzzy Math 9(3):589–602

[6] Maji PK, Roy AR, Biswas R (2002) An application of soft sets in a decision making problem. Comput Math Appl 44(8–9):1077–1083. https://doi.org/10.1016/S0898-1221(02)00216-X

[7] Molodtsov D (1999) Soft set theory—first results. Comput Math Appl 37(4–5):19–31. https://doi.org/10.1016/S0898-1221(99)00056-5

[8] Peng X, Yang Y, Song J, Jiang Y (2015) Pythagorean fuzzy soft set and its application. Comput Eng 41(7):224–229

[9] Smarandache F (2018) Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets—revisited. Neutrosophic Sets Syst 21:153–166

[10] Tao Z, Chen H, Zhou L, et al. (2015) 2-Tuple linguistic soft set and its application to group decision making. Soft Comput 19: 1201–1213. https://doi.org/10.1007/s00500-014-1335-4

[11] Tao Z, Chen H, Song X, Zhou L, Liu J, Uncertain linguistic fuzzy soft sets and their applications in group decision making, Appl.Soft Computing 34: 587–605.

[12] Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

[13] Zhang Z, Zhang S (2013) A novel approach to multi attribute group decision making based on trapezoidal interval type-2 fuzzy soft sets. Applied Mathematical Modelling 37(7): 4948–4971.