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DEVELOPING ROBUST EARTHQUAKE MAGNITUDE CONVERSION RELATIONSHIPS FOR INDIAN REGION

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Abstract

This paper has developed improved magnitude conversion from magnitude types M_S, m_b and M_L to moment magnitude M_W for India and adjoining areas using carefully curated datasets for each magnitude pair compiled from multiple reliable agencies. The study has followed a systematic approach that includes critical evaluation of commonly adopted functional forms and regression methodologies, along with identification of the optimum range of applicability for each conversion relationship. The relationships obtained can be considered more robust compared to the existing relations for the Indian region due to lower standard deviations, broader ranges of applicability, and theoretically consistent functional forms. For applying these relationships to homogenize the earthquake catalogues, the paper also presents a structured step-by-step framework in a hierarchical manner, depending on the type and range of the reported magnitudes. This approach is expected to minimize the biases in magnitude conversion and improve considerably the reliability of seismic hazard assessments in India.

KEYWORDS: Magnitude Conversion Relations, Indian region, Moment magnitude, Earthquake catalog homogenization, Central least squares regression, seismic hazard analysis.

1 Introduction

The accuracy of seismic hazard and risk assessment depends critically on the reliability of the earthquake catalogue, which requires standardized magnitude scales to represent the size of seismic events uniformly. The origin of instrumental magnitude scaling dates back to 1935, when Richter (1935) introduced the local magnitude scale (M_L) based on observations from Southern California. This was followed by the development of the surface-wave magnitude scale (M_S) based on 20 s period surface wave amplitudes by Gutenberg (1945a) and the

broadband body-wave magnitude scale (m_B) by Gutenberg (1945b, c). After commissioning of World-Wide Standard Seismograph Network (WWSSN), the body wave magnitude is being determined using short period ($T \leq 3$ s) P waves amplitudes, which is denoted by m_b and is about 0.3 – 0.6 units lower than m_B. The surface wave magnitude is also being defined using Prague formula based on the ratio of maximum displacement to associated period of the vertical component of Rayleigh surface waves (Vanek et al., 1962).

Although the development of the various magnitude scales significantly advanced the quantitative seismology, they were recognized to saturate at different e levels for large events, resulting in systematic underestimation of earthquake size. To overcome the limitation of saturation, Kanamori (1977) proposed the moment magnitude scale (M_W) for very large earthquakes. Subsequently, Hanks and Kanamori (1979) extended the applicability of the M_W scale to smaller magnitudes using the same conceptual framework. Thus, the moment magnitude has become the standard magnitude scale in seismic hazard analysis. Given the coexistence of multiple magnitude scales in earthquake catalogues, magnitude conversion relations have become essential for transforming different magnitude types into a unified and consistent moment magnitude scale.

Such conversions are particularly important in regions like India, where earthquake catalogues span long historical periods and include data from different agencies, instruments, and magnitude scales. Although, several global conversion relations exist, they often fail to capture regional tectonic characteristics, crustal properties, and attenuation behaviour (Scordilis 2006; Storchak et al., 2013). Consequently, region-specific magnitude conversion relations are required to represent better the seismic characteristics of the Indian subcontinent. The region-specific magnitude conversion relations are expected to significantly improve the reliability of homogeneous catalogues, enhance the performance of ground-motion prediction equations, and lead to more accurate seismic hazard estimates.

Though, simple linear relationship is the most widely used functional form for developing the magnitude conversion models, many other forms have also been proposed by some investigators to match the observed behaviour of the data. Important among these are the quadratic (Tsampas et al., 2016), exponential (Gasperini et al., 2013; Lolli et al., 2014), and circularly connected bilinear (Lolli et al., 2014) forms of equations. Among the methods of regression analysis, the recent trend is to use some form of orthogonal regression scheme that may include the standard orthogonal regression (SOR) (Fuller 1987), general orthogonal

regression (GOR) (Castellaro et al., 2006), modified GOR (Wason et al., 2012), chi-square (CSQ) orthogonal regression (Gasperini et al., 2013), orthogonal distance regression (ODR) (Boggs et al., 1988), and York orthogonal regression (York et al., 2004), for example. A comparative assessment by Gasperini et al. (2015) evaluated OLS, GOR, and modified GOR approaches, concluding that GOR more effectively accounts for uncertainties in both variables, whereas modified GOR does not yield unbiased estimates. The present study has proposed a central least squares (CLS) regression method in which the regression coefficients are taken as the weighted average of the coefficients obtained by ordinary least squares (OLS) regression performed directly for y on x and inversely for x on y, termed as inverse least squares (ILS). The results of CLS are shown to be better than those from the SOR method because it is able to account for the uncertainties in both the variables in a balanced way as per the observed data without any subjective assumption regarding the quantification of the uncertainties as in all the orthogonal regression methods.

Over the past two decades, several studies have developed magnitude conversion relations for different parts of India, reflecting the need to account for the country's diverse tectonic environments and heterogeneous crustal properties. Thingbaijam et al. (2008, 2009) converted M_S and m_b into M_W using the OLS, ILS, SOR, and GOR technique for northeast and northwest India. Among these, GOR was identified as the most suitable method, as it incorporates an optimized variance ratio (η), leading to improved fitting and reduced uncertainty in the regression coefficients. Consequently, GOR-based relations were recommended for magnitude conversion. Yadav et al. (2009, 2012) prepared homogeneous M_W catalogues using OLS, ILS and SOR. They noted that modelling the error variance across different magnitude scales is challenging, although a linear relationship generally exists between magnitude scales; hence, the standard linear least-squares approach is commonly adopted. However, the specific regression method used for the final magnitude conversion was not clearly specified. Das et al. (2012) derived empirical relations among M_W, M_S, and m_b for northeast India and the Himalayan region using GOR, SOR and ISR. They concluded that the SOR and ISR methods yield improved conversion relations for the study area, with reduced uncertainties in the regression parameters. Wason et al. (2012) addressed the magnitude conversion problem using GOR technique and developed empirical conversion relations between different magnitude scales. Baruah et al. (2012) established relations between M_W and M_L using GOR, OLS and ILS approaches, and concluded that the standard least squares

method provides the best-fit line for the regression analysis. Kolathayar and Sitharam (2012) proposed empirical relations among M_W, M_S, m_b, and M_L for the entire Indian region using OLS.

Subsequent studies further expanded both regional coverage and methodological approaches. Chingtham et al. (2014) developed relationships among m_b, M_L, and M_D using GOR, SOR, OLS and ILS for the northwest Himalayas and adopted the GOR approach for magnitude conversion. Sitharam and Sil (2014) established relations between M_W and m_b, M_L and M_S using simple linear regression for northeast India. Bora (2016) developed relations among M_W, M_L, and M_D using linear regression for northeast India. Nath et al. (2017) compiled a unified M_W catalogue for the south east Asia using GOR, while Pandey et al. (2017) converted M_S, m_b, and M_L into M_W using GOR for northeastern India. Anbazhagan and Balakumar (2019) developed relations among M_S, m_b, M_L, and M_W using OLS and SOR and concluded that the SOR approach is more robust, as it yields the lowest standard deviation and comparatively lower mean values. Kumar et al. (2020) established relations among M_W, M_S, m_b, M_L, and M_D using GOR and ODR for the Himalayan seismic belt. The results indicate that there is no significant difference between the GOR and ODR methods; therefore, the GOR technique was adopted for magnitude conversion.

More recent studies have continued to refine magnitude homogenization approaches. Chatterjee et al. (2022) developed linear relations between M_S, m_b and M_W using a bivariate fit-shell approach for the eastern Himalayan region. Shahabuddin and Mohanty (2025) established relations among M_W, M_S, m_b, M_L, and M_D using OLS, ILS, GOR, and ODR, including exponential functional forms. They evaluated the models through residual analysis to identify the best-fitting regression, and adopted OLS for homogenization. Das et al. (2025) employed GOR, modified GOR, OLS and ILS to establish relationships between M_W and other magnitude scales (m_b, M_S, and M_L). They concluded that the modified GOR approach provides improved conversion equations, with lower uncertainties, higher correlation coefficients, and reduced errors in the slope and intercept compared to conventional methods.

The above cited studies for India in general suffer from one or more of the following four problems that no definite recommendation is made in the face of multiple relations developed

using different functional forms and regression techniques, that the magnitude range of applicability is not defined realistically, that the dependence on focal depth has not been analysed in detail, and that no guidelines are proposed on how to apply the developed relations for homogenization of earthquake catalogues. The present study has addressed all these aspects in a comprehensive and scientifically logical manner to develop improved magnitude conversion models for use in catalog homogenization for seismic hazard analysis in India.

2 Forms of Equations and Regression Methods

The reliability of the magnitude conversion relations is expected to depend strongly on the functional form of the equation and the regression methodology adopted for parameter estimation. The possible forms of the equation and the various regression methods used in developing the magnitude conversion relationships are reviewed briefly in the following two subsections.

2.1 Functional Forms of Conversion Relations

The first comprehensive study due to Scordilis (2006) on developing global conversion relations from M_S and m_b to M_W identified a bilinear trend between M_W and M_S, and a single linear trend between m_b and M_W. Most other studies have developed a bilinear relationship for M_S and unilinear relationship for both m_b and M_L. The linear segments in such relations are fitted independently using following form of straight-line equation

$$y = a + bx \tag{1}$$

In this equation, y represents M_W and x represents M_S, m_b or M_L. However. the linear model of Eqn. (1) is not seen to follow closely the distribution of the observed data for m_b and M_L, particularly at low and high magnitude ends. Several different forms of nonlinear relationships have been therefore proposed by many investigators.

Gasperini et al. (2013) showed that the following form of exponential model is able to follow closely the banana shaped distribution of M_W versus m_b data, which was used by Lolli et al. (2014) for M_W versus M_S data also

$$y = \exp(a + bx) + c \tag{2}$$

Another form of nonlinear relationship by Lolli et al. (2014) is the circularly connected bilinear (CBL) model in which the two linear segments are connected by a circular arc to preserve the

158 continuity. The quadratic relationship is yet another form of nonlinear conversion relation
159 recognized first by Gutenberg and Richter (1956) between older broadband m_B and M_L. Some
160 recent studies (Tsampas et al., 2016) have developed following form of quadratic relations by
161 empirical regression analysis

162 $$y = a + bx + cx^2 \quad (3)$$

163 A nonlinear form of the magnitude conversion relation with continuously varying slope is
164 not realistic on physical grounds. From theoretical considerations, the relationship between
165 M_W and other magnitude types has to be linear with slope of 2/3, 1 and 2 for small, intermediate
166 and large magnitude ranges, respectively (Kanamori and Anderson, 1975; Hanks and Boore,
167 1984; Ekstrom and Dziewonsi, 1988). For the cases of m_b and M_L magnitudes, the linear
168 segment with slope of 2 generally occur beyond the saturation limits for these magnitudes,
169 while that for M_S it occurs beyond magnitude of about 8.0. On the other hand, the segments
170 with slope of 2/3 for m_b and M_L occur below the lower limits up to which these magnitudes
171 are commonly estimated, and for M_S it occurs below some magnitude between 6.0 and 6.5.
172 Thus, the conversion relations for m_b and M_L in this study are defined by a single straight-
173 line segment with the range of applicability identified in a physically realistic manner. Whereas,
174 the conversion relation for M_S is developed in the form of two independent straight-line
175 segments with the change point searched iteratively to minimise the offset between the two-
176 line segments without compromising the goodness of fitting of both the segments. The
177 conversion relations thus developed can be considered more suitable than any form of nonlinear
178 relations forced to match the trend of the observed data over the complete magnitude range.
179

180 **2.2 Regression Methods**

181 Regression analysis can be performed to estimate the parameters a, b and c in the functional
182 forms of Eqns. (1) to (3) using several different methods. The ordinary least squares (OLS)
183 regression method is the simplest approach to estimate the parameters in the magnitude
184 conversion relations, which minimizes the sum of squared residuals between observed and
185 predicted y-values. However, it assumes that the independent variable x is free of error. In
186 magnitude conversion, where both magnitude scales are associated with measurement errors,
187 this assumption is considered unrealistic and may lead to biased slope estimates. The regression
188 methods that are able to account for the uncertainties in both the variables are therefore

preferred in developing the magnitude conversion relationships. The uncertainties in majority of these methods are quantified by a parameter $\eta = \sigma_\varepsilon^2 / \sigma_\delta^2$ with σ_ε^2 and σ_δ^2 as the variances of the errors in the observed values of predicted and predictor variables y and x, respectively. The value of η in real applications is commonly assigned somewhat subjectively that cannot be considered to realize the intended improvements. This study has proposed a much simpler central least squares (CLS) method which is able to account for the uncertainties in both the variables in a balanced manner as per the observed database without making any prior assumption about the quantification of the errors. The proposed CLS method and three widely used variations of the orthogonal regression method, viz. standard orthogonal regression (SOR), general orthogonal regression (GOR), and modified GOR are summarized as follows:

(i) CLS Method

In this method, the best fit values $\hat{a}$ and $\hat{b}$ of parameters a and b in the straight-line relationship of Equation (1) are first estimated by OLS method to get the predictive relationship

$$y = \hat{a} + \hat{b}x \text{ with variance } \sigma_y^2 \tag{4}$$

This relationship is able to account for the observed errors in the variable y as per the available observed dataset. Next, the OLS is performed on the inverse linear equation $x = c + dy$ to get the best fit estimates $\hat{c}$ and $\hat{d}$ of parameters c and d that gives the predictive relationship

$$x = \hat{c} + \hat{d}y \text{ with variance } \sigma_x^2 \tag{5}$$

This has been termed as inverse least squares (ILS) method that accounts for the observed errors in the variable x as per the observed database. The ILS based predictive relationship can also be written as follows

$$y = -\hat{c}/\hat{d} + x/\hat{d} \tag{6}$$

The prediction equation by CLS method that is able to account for the uncertainties in both the variables in a balanced way is proposed to be taken as the weighted average of the OLS and ILS equations with weights proportional to the reciprocal of their respective variances as follows

$$y = \alpha + \beta x; \text{ with } \alpha = \frac{1}{\sigma_x^2 + \sigma_y^2}\left(\sigma_x^2 \hat{a} + \frac{\sigma_y^2}{\hat{d}}\right) \quad \& \quad \beta = \frac{1}{\sigma_x^2 + \sigma_y^2}\left(\sigma_x^2 \hat{b} - \frac{\hat{c}\sigma_y^2}{\hat{d}}\right) \tag{7}$$

(ii) SOR and GOR Methods

Both these methods fit the straight-line relation of Eqn. (1) by accounting explicitly for the uncertainties in both predicted (y) and the predictor (x) variables. As against the minimization of the sum of orthogonal distances from the pairs of observed data (x_i, y_i) to the regression line, which assumes equal errors in both the variables in the SOR method, the GOR method is based on minimizing the sum of statistical Euclidean distances defined with different errors in the two variables (Madansky 1959; Kendall and Stuart 1979; Fuller 1987; etc.).

In the GOR method, let the true values x_t and y_t of the observed values x and y be associated with errors δ and ε as follows

$$x = x_t + \delta \quad \text{and} \quad y = y_t + \varepsilon \tag{8}$$

The GOR estimator is obtained by minimizing the following sum of Euclidean distances (weighted orthogonal distance) between the set of observed points (x_i, y_i) and the unknown true values (x_{ti}, y_{ti}) on the GOR line

$$\sum_{i=1}^{n} \left\{ (y_i - a - b x_{ti})^2 / \eta + (x_i - x_{ti})^2 \right\} \tag{9}$$

with $\eta = \sigma_\varepsilon^2 / \sigma_\delta^2$ as the error variance ratio for the dependent and the independent variables as mentioned before. The required estimate $\hat{b}$ of the slope of GOR line can be obtained in terms of the variances S_{xx} and S_{yy} of the observed data (x_i, y_i) and their covariance S_{xy} as follows

$$\hat{b} = \frac{S_{yy}^2 - \eta S_{xx}^2 + \sqrt{(S_{yy}^2 - \eta S_{xx}^2)^2 + 4\eta S_{xy}^2}}{2 S_{xy}} \tag{10}$$

As the errors δ and ε are randomly distributed around the true values, the average values of the observed and the true values will be identical. Thus, the estimator $\hat{a}$ of parameter a can be defined from Eqn. (1) in terms of average values $\bar{x}$ and $\bar{y}$ of the observed values as

$$\hat{a} = \bar{y} - \hat{b}\bar{x} \tag{11}$$

Using the parameters $\hat{a}$ and $\hat{b}$ defined by the expressions of Eqns. (11) and (10), respectively, gives the GOR based conversion relationship. The SOR based conversion relationship with equal errors in both the dependent and independent variables can be obtained simply by using $\eta = 1$ in the foregoing formulation.

(iii) Modified GOR Method

Some researchers (Wason et al., 2012; Das et al., 2014; Nath et al., 2017) have erroneously interpreted that the regression parameters $\hat{a}$ and $\hat{b}$ estimated by GOR method gives a conversion relationship in terms of the true values of both the variables

$$y_t = \hat{a} + \hat{b}x_t \tag{12}$$

They have thus argued that the GOR relationship cannot be used with the observed values of the given type of magnitude with errors, and have proposed a modified GOR method in which the true values x_t of the predictor magnitude are obtained by developing an intermediate prediction relation between the true and the observed values as follows

$$x_t = c + dx \tag{13}$$

For this purpose, the true values, x_{ti}, of the predictor variable are obtained from the available pairs (x_i, y_i) of observed data using the following relationship (Fuller 1987)

$$x_{ti} = \frac{\hat{\beta}(y_i - \hat{a}) + \eta x_i}{\eta + \hat{b}^2} \tag{14}$$

As the true value of the predictor variable in this relationship depends on the observed values of both the predicted and the predictor variables, it cannot be defined accurately by Equation (13) in terms of the predictor variable alone. Also, the interpretation that the GOR method gives a conversion relationship between the error free true values of both the variables is not true. In reality, the method of regression analysis cannot change the physical nature of the variables in a prediction equation. Thus, the modified GOR method is not based on sound technical grounds and needs to be discarded forthwith.

In summary, the practical implementation of the GOR method is constrained by the prior specification of ratio of error variances that may introduce additional uncertainty, and the above-mentioned limitations associated with the modified GOR further reinforce this concern. Considering these aspects, the CLS method provides a simple, balanced, and practical solution, that does not require a priori assignment or explicit definition of variances in either variable and yields stable parameter estimates. The CLS is therefore adopted as the preferred regression method in this study. In addition, the SOR method based on the most unbiased equal variances in both the variables is also applied for comparison purposes.

3 Databases Used

The databases on magnitude pairs of M_S, m_b and M_L with M_W, used to develop the conversion relations in this study are taken from the ISC Bulletin (www.isc.ac.uk/iscbulletin) for the region bound by 5° N - 40° N and 65° E to 100° E. The ISC collects reports from about 130 seismological agencies worldwide operating at global, regional or local scales and gives the recomputed M_S and m_b magnitudes in the reviewed Bulletin along with the various types of magnitude reported originally by the respective agencies. It is thus important to set an order of priority of agencies for each magnitude type (Di Giacomo and Storchak 2016).

Due to teleseismic nature of m_b and M_S magnitude types, the ISC recomputed values using large number of stations is given first priority. The subsequent preference for both these magnitude scales is set as National Earthquake Information Center (NEIC), the Geophysical Survey of Russian Academy of Science (MOS), and the International Data Centre (IDC) (http://www.ctbto.org/) in this order. The NEIC, MOS and IDC report m_b data from 920, 400 and 140 worldwide stations, report M_S data from 840, 250 and 140 stations, respectively. The ISC bulletin does not report the moment magnitude M_W for events below magnitude 4.5 to 5.0, because the global agencies like Global Centroid-Moment-Tensors (GCMT) and NEIC do not estimate the moment tensor solutions for smaller earthquakes. In addition, the National Center for Seismology, New Delhi (NDI) reports the moment magnitudes for many Indian earthquakes of smaller magnitudes also. Thus, the preferred agencies for M_W are set as GCMT, NEIC and NDI in this order.

Finally, the local magnitude M_L by Richter (1935) is normally estimated by agencies running regional/local networks using their own calibration functions. Among the agencies running global networks, the IDC provides M_L estimates, which are generally found to be grossly underestimated and thus not included in the present study. The local/regional networks considered for M_L includes the National Center for Seismology, New Delhi (NDI), Department of Mines and Geology, Kathmandu, Nepal (DMN), China Earthquake Networks Center, Beijing (BJI), Thai Meteorological Department, Bangkok (BKK), and the Badan Meteorologi, Klimatologi dan Geofisika, Djakarta (DJI).

The data on the pairs of different types of magnitudes with M_W compiled as per the above-described hierarchy are listed in Table 1. The epicentral plots in Figs. 1(a) to 1 (c) show the spatial distribution of events overlain on the tectonic features for the magnitude pairs (M_S, M_W), (m_b, M_W), and (M_L, M_W), respectively, in four different depth ranges of < 35 km, 35 – 70 km, 70 – 150 km, and 150 – 300 km for each magnitude pair. These depth intervals are selected to separate out the earthquakes into crustal, sub-crustal, upper intermediate depth, and lower intermediate depth categories. There are no events deeper than 70 km for the (M_S, M_W) pair, while the events deeper than 70 km for both (m_b, M_W) and (M_L, M_W) are mainly concentrated in Hindu Kush, Indo-Burmese, and Andaman subduction zones, with a few stray events in the depth range of 70 – 150 km located in the eastern Nepal and Sikkim Himalaya.

Table 1. Details of the number of events used for three different types of magnitude pairs from various international and regional/ local agencies.

Mag. Pair	No. of Events (Period)	Depth Range (km)	Mag. Type	Mag. Range	Agencies and Number of Events
(M_S, M_W)	2359 (1976 – 2022)	1.2 – 60	M_S	2.7 – 8.0	ISC – 2344, NEIC – 15
			M_W	3.2 – 7.9	GCMT – 2013, NEIC – 37 & NDI - 309
(m_b, M_W)	3668 (1964 – 2022)	0.0 – 268	m_b	3.1 – 7.2	ISC – 3528 & NEIC – 143
			M_W	2.8 – 7.9	GCMT – 2661, NEIC – 58 & NDI - 949
(M_L, M_W)	2273 (1988 – 2022)	0.2 - 261	M_L	1.6 – 7.8	NDI – 1448, DMN – 34, BJI – 561, BKK – 114 & DJA – 116
			M_W	2.0 – 7.9	GCMT – 1078, NEIC – 34 & NDI - 1161

In addition to the databases summarized in Table 1, 566, 56 and 539 number of events are also available from IDC for the magnitude pairs (M_S, M_W), (m_b, M_W) and (M_L, M_W) for the recent periods of 2000 – 2022, 2018 – 2022 and 2000 – 2022, respectively. But these are not utilised in developing the magnitude conversion relationships in the present study due to strong biases and very large dispersion observed in the IDC database compared to the data from other agencies. This may perhaps be due to different measurement procedures adopted by the IDC agency, which generally results in significant underestimation of all three types of magnitude (M_S, m_b and M_L) compared to M_W. This could be seen clearly from a comparison of the

data available from IDC with those from other agencies in Figs. 2(a) to 2(c) for the magnitude pairs (M_S, M_W), (m_b, M_W) and (M_L, M_W), respectively.

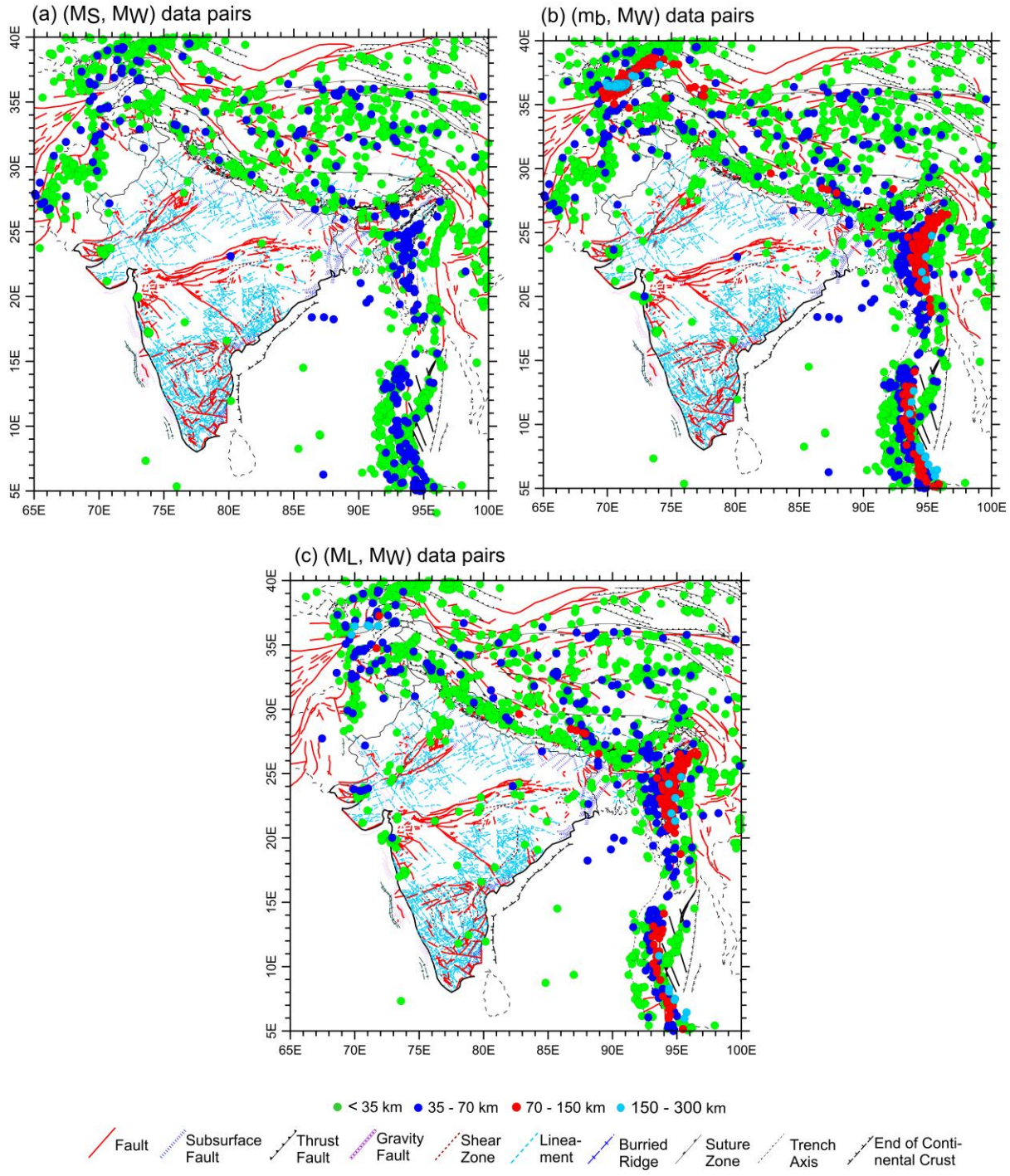


Figure 1. Epicentral plots for the database used in the present study for magnitude pairs (M_S, M_W), (m_b, M_W) and (M_L, M_W) in four different depth ranges.

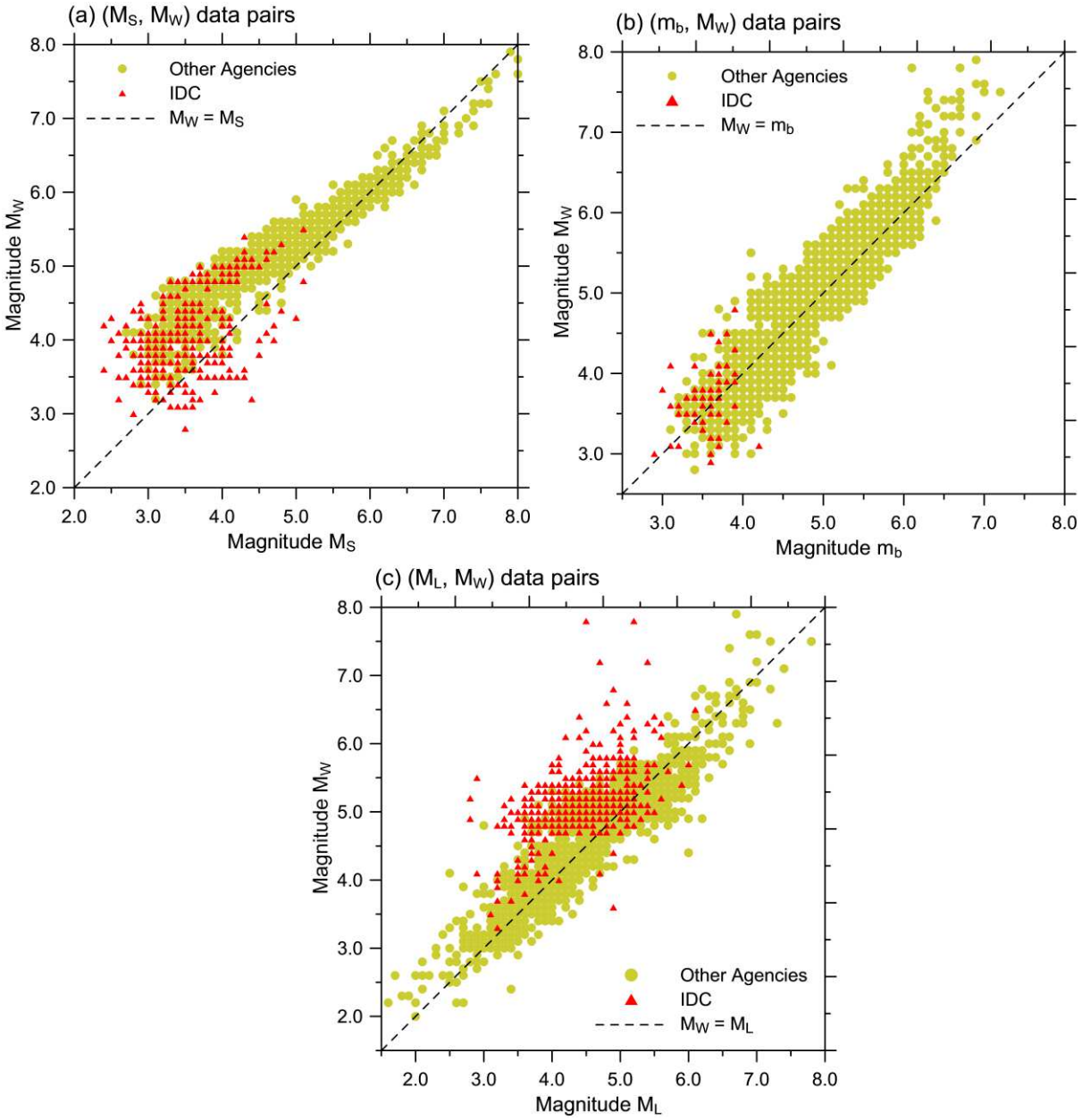


Figure 2. Comparison of the database used in the present study for magnitude pairs (M_S, M_W), (m_b, M_W) and (M_L, M_W) compiled from various other agencies with those available from International Data Centre (IDC).

4 Development of Conversion Relations

It has already been mentioned that various nonlinear forms of the magnitude conversion relations used to match the observed trend of the recorded data are not governed by physical considerations and stem mainly from the incompleteness of smaller magnitudes and saturation of the largest magnitudes, among other factors introducing inaccuracies in magnitude determination due to inaccuracies in the application of corrections for instrument response, noise contamination and the path attenuation characteristics. The nonlinear relations also have

the undesired effect that the linearity of the G-R relationship will not hold for the converted magnitudes. On physical grounds, the conversion relations from other types of magnitude to M_W are expected to follow three separate straight-line segments with slopes of 2/3, 1.0 and 2.0 in small, intermediate and large magnitude ranges, respectively. However, these magnitude ranges will be different for each type of magnitude and may also have significant regional dependence. For developing the magnitude conversion relations for M_S, m_b and M_L to M_W in the present study, these magnitude intervals are identified in a physically realistic manner by detailed analysis of the residuals between M_W and other types of magnitudes for the databases described in Table 1. No previous study in India is known to have performed such a critical analysis for deciding the form and magnitude interval of applicability of the conversion relations developed. The details of developing the three sets of conversion relations for M_S to M_W, m_b to M_W, and M_L to M_W developed in this study are given below.

4.1 Conversion Relations for $M_S \rightarrow M_W$

The database on (M_S, M_W) pairs of magnitudes available for the present study cover M_S range of 2.7 to 8.0, which can be considered to lie within the small and intermediate magnitude ranges only. To identify the boundaries of these ranges, we have first identified three different magnitude ranges by analysis of the residuals $\Delta M_S = M_W - M_S$ such that each magnitude range is characterized by distinctly different mean value of the residuals in different ranges of focal depth as given along with the corresponding standard deviation (SD) values in Table 2. The mean residual for each magnitude interval is found to be slightly higher for deeper earthquakes, but the differences are not very significant to warrant developing separate conversion relations for crustal (≤35 km) and subcrustal (>35 km) events, because the maximum depth is not very large and limited to 70 km only. The detailed distribution of the residuals as a function of focal depth is illustrated in Figure 3 with the data in three identified magnitude ranges plotted in different symbols and colours.

Table 2. Mean and SD values of the residuals $\Delta M_S = M_W - M_S$ for different magnitude and depth ranges.

Sr. No.	M_S Range	Depth Range (km)	No. of Events	Mean Value	SD Value
1.	2.7 – 3.0	10 – 35	10	1.0200	0.0696
		35 – 57.7	3	1.0667	0.0962
		10 – 57.7	13	1.0308	0.0640

2.	3.1 – 6.5	1.3 – 35	2038	0.4636	0.0103
		35 – 60	256	0.6219	0.0201
		1.3 – 60	2294	0.4813	0.0097
3.	6.6 – 8.0	1.2 – 35	51	-0.2078	0.0011
		35 – 58.1	1	0.1000	0.0000
		1.2 – 58.1	52	–0.2019	0.0003

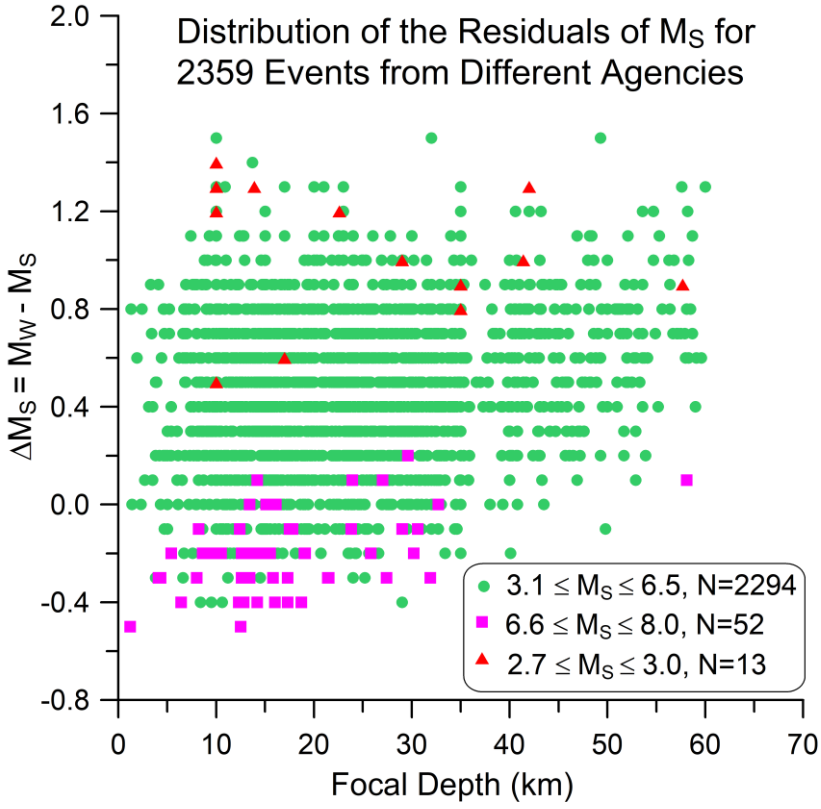


Figure 3. Distribution of the residuals $\Delta M_S = M_W - M_S$ as a function of focal depth for the available data points in three identified M_S ranges.

For a limited number of 13 events in the lowest M_S range of 2.7 – 3.0, the M_W is seen to be on average one magnitude units higher than M_S which may be a combined effect of underestimation of M_S and overestimation of M_W. For such small magnitude earthquakes, the long-period teleseismic surface waves amplitudes are not fully developed, leading to underestimation of M_S. The M_W for all the 13 events, available from NDI, are based on the low-frequency displacement spectrum amplitudes from local recordings (Brune, 1970), which are overestimated due to contamination by the low-frequency noise. Thus, the data in this magnitude range are not included in developing the conversion relationship in this study.

For the highest M_S range of 6.6 – 8.0 considered, 51 of the 52 events have focal depths within 35 km, and all the events are on average characterized by M_W with about 0.2 magnitude units lower than M_S. This appears to be consistent with the fact that the older M_S values against which the moment magnitude scale has been calibrated ($E_s = 1.5M_S + 11.8$ by Richter, 1958) are underestimated by about 0.2 magnitude units (Pacheco and Sykes, 1992). The residuals for the M_S range of 3.1 – 6.5 are seen to be distributed almost symmetrically around mean values of about 0.46 and 0.62 for shallower and deeper events, respectively. The slight increase in the mean residual for deeper earthquakes may be attributed to the underestimation of M_S due to attenuation of the surface wave amplitudes with depth. The foregoing two magnitude ranges have actually been finalized by trial and error to minimize the standard deviation and the offset at the transition magnitude.

For the two magnitude ranges of M_S finalized as above, the conversion relationships are developed by both SOR and CLS regression techniques as listed in Table 3. Combined relationships for the entire depth range of 1.2 – 60 km as well as separate relationships for the shallower (1.2 – 35 km) and deeper (35 – 60 km) events have been developed only for the first magnitude range of 3.1 – 6.5 with sufficient number of data points in both the depth ranges. Due to insufficient data points, only a combined relationship has been developed for the second magnitude range of 6.6 – 8.0. The matching between the developed relationships and the observed data points is shown in Figure 4. For the lower magnitude range, the relationships for the shallower and the entire depth ranges are seen to be almost identical, while those for the deeper events are seen to overestimate M_W by magnitude units of up to about 0.2. This is a manifestation of the underestimation of M_S for deeper earthquakes as mentioned before. In view of this, the relationship based on the data for the complete depth range is recommended to be used in real applications. Further, the relationships for both the magnitude ranges based on the CSL regression method are seen to have smaller standard deviations, σ, and higher coefficients of determination, R^2, and the same are therefore preferred over the relations based on SOR method.

Table 3. Empirical conversion relationships, $M_W = a + bM_S$, developed using CLS and SOR regression methods for different combinations of magnitude and depth ranges.

Sr. No.	Depth Range (km)	Regression Method	Slope, b	Intercept, a	σ	R^2	N	M_S Range
1.	1.2 – 35	CLS	0.6836±0.0056	1.9301±0.0263	0.1821	0.8692	2038	3.1 – 6.5
		SOR	0.7497±0.0042	1.6194±0.0200	0.1939	0.8516		
2.	35.1 – 60	CLS	0.6859±0.0173	2.0479±0.0794	0.1747	0.8452	256	
		SOR	0.7812±0.0058	1.6339±0.0231	0.1935	0.8103		
3.	1.2 – 60	CLS	0.6832±0.0055	1.9466±0.0255	0.1856	0.8608	2294	
		SOR	0.7445±0.0041	1.6588±0.0196	0.1962	0.8445		
4.	1.2 – 60	CLS	1.0030±0.0528	-0.2229±0.3739	0.1617	0.8580	52	6.6 – 8.0
		SOR	0.9950±0.0194	-0.0379±0.0175	0.2071	0.7670	52	

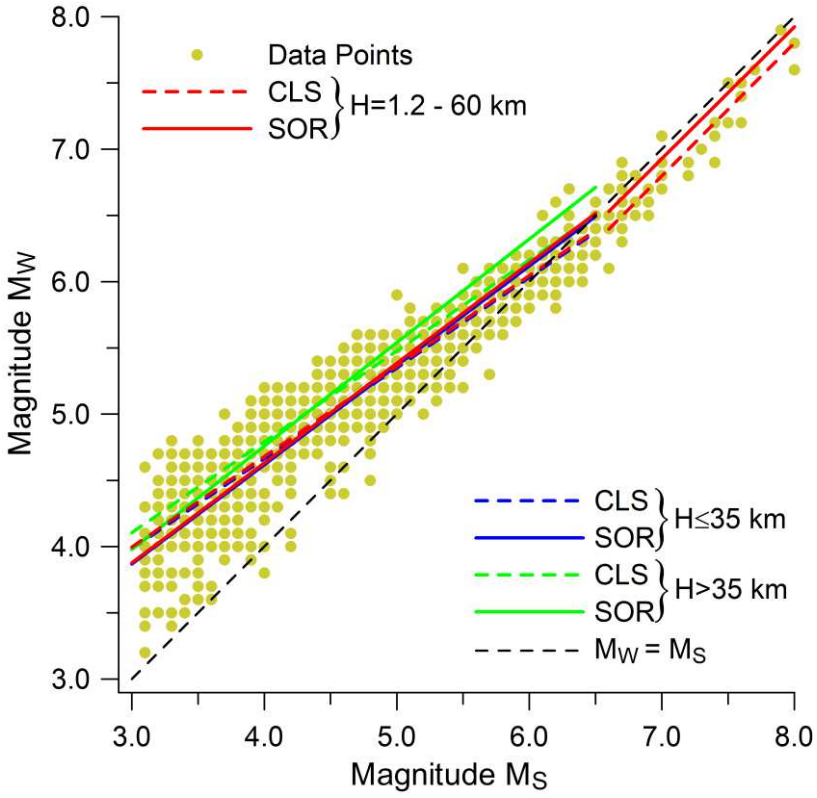
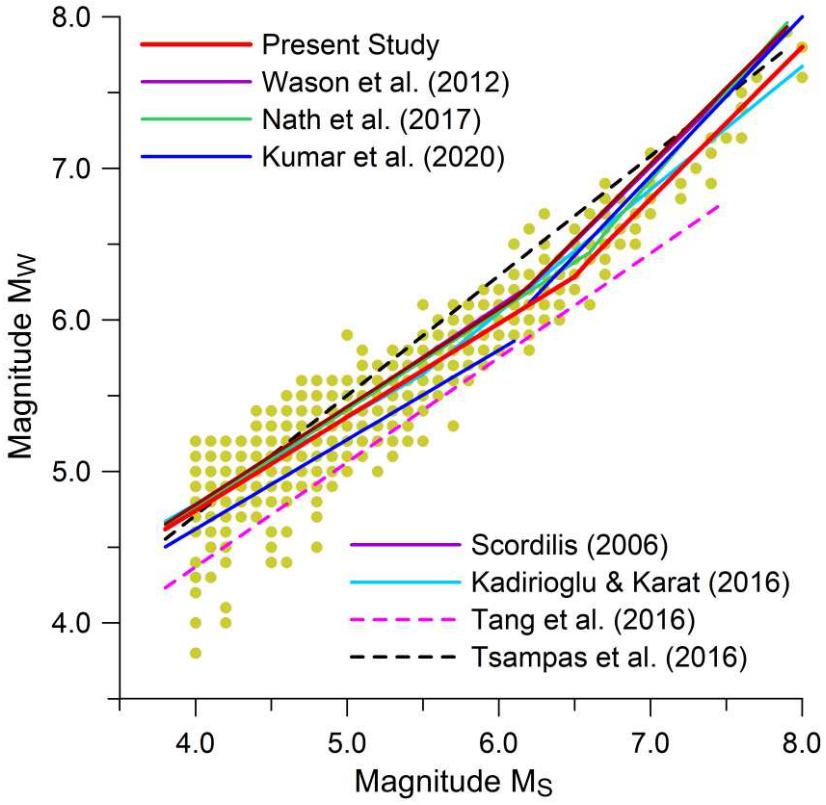


Figure 4. Fitting of the M_S to M_W conversion relationships to observed data points by CLS and SOR regression methods.

Figure 5 shows the comparison of the proposed conversion relationship developed using CLS method in the present study with several typical global and regional relationships developed by other investigators. The details of these relations are given in Table 4, which includes two relations for India, three for Global data, and one each for China and Turky. None

425 of the other relationships closely follow the present relationship over the entire M_S range. Two
426 of these relations due to Tang et al. (2016) and Tsampas et al. (2016) are defined by a single
427 straight-line segment up to maximum M_S values of 7.5 and 7.9, respectively. The remaining
428 five relations are developed in terms of two-line segments and the ranges of their applicability
429 listed in Table 4 vary widely between 3.0 and 8.5, but the plots in Fig. 5 are limited up to M_S
430 of 8.0 only. The transition magnitude for these relations is also seen to vary widely between
431 5.5 and 6.7. Further, the slopes of the lower and higher magnitude intervals in many of the
432 relations deviate significantly from the theoretically expected values of 0.67 and 1.0,
433 respectively.



434

435 **Figure 5**. Comparisons of the M_S to M_W conversion relationships developed in this study with
436 seven existing global and regional relations developed by various other investigators.

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Table 4. Details of the typical global and regional conversion relationships from M_S to M_W compared with the proposed relationship developed for the Indian region in the present study.

Sr. No.	Region	Reference	Regression Method	Slope	Intercept	σ	N	M_S Range
1.	India	Present Study	CLS	0.6832	1.9466	0.1856	2294	3.1 – 6.5
				1.0030	-0.2229		52	6.6 – 8.0
2.	Global	Wason et al. (2012)	M-GOR	0.669	2.08	0.2024	14728	3.1 – 6.1
				1.09	0.571		981	6.2 – 8.4
3.	South Asia	Nath et al. (2017)	M-GOR	0.649	2.163	0.1939	1872	3.5 – 6.6
				1.157	-1.179		41	6.7 – 8.5
4.	Himala-yan Belt	Kumar et al. (2020)	GOR	0.59	2.26	0.2180	314	3.8 – 6.1
				1.05	-0.4		19	6.2 – 7.6
5.	Global	Scordilis (2006)	OLS	0.67	2.07	0.1970	23921	3.0 – 6.1
				0.99	0.08		2382	6.2 – 8.2
6.	Turkey	Kadirioglu & Karat (2016)	OLS	0.5716	2.4980	0.1996	462	3.4 – 5.4
				0.8126	1.1723			≥ 5.5
7.	China	Tang et al. (2016)	SOR	0.69	1.61	0.3590	487	4.3 – 7.5
8.	Global	Tsampas et al. (2016)	GOR	0.790	1.551	0.2338	3243	3.1 – 7.9

To assess the applicability of the other relations to Indian region, the standard deviations for all the relations are estimated with respect to the dataset used in developing the present relationship and reported in antepenultimate column of Table 4. All the seven existing relationships are characterized by higher standard deviations than that for the present relationship, but the differences are not very large for five of the relations developed in terms of two straight-line segments. The standard deviations for the relations due to Tang et al. (2016) and Tsampas et al. (2016), developed as a single straight-line, are unacceptably large. On theoretical grounds also, the wide range of M_S is required to be described by two-line segments with different slopes and intercepts. Thus, the $M_S \rightarrow M_W$ conversion relation proposed in this study can be considered more appropriate for the Indian region compared to typical exiting relations listed in Table 4 and shown in Figure 5.

461 **4.2 Conversion Relations for $m_b \rightarrow M_W$**

462 On theoretical grounds (Kanamori & Anderson 1975), the relationship between M_W and
463 m_b is also expected to be characterized by slopes of 2/3, 1 and 2 in the low, intermediate and
464 large magnitude m_b ranges, but the transition magnitudes shall be much lower than that for
465 M_S because m_b saturates at much lower magnitude. The segment with slope of 2 is expected
466 to start at magnitudes greater than the saturation limit for m_b around 6.0 – 6.5, while the lowest
467 segment with slope of 2/3 may occur even below the minimum of current m_b determinations
468 (~ 3.0 – 3.5). This range has been recognized in the present study by analysis of the residuals
469 in different magnitude ranges of the available data base and by minimizing the standard
470 deviation in the developed conversion relationship.

471 The mean values of the residuals in the low, intermediate, and large m_b ranges identified are
472 given along with the corresponding standard deviation (SD) values in Table 5 for three different
473 ranges of focal depth. In the lowest m_b range of 3.1 – 3.2, the M_W for a highly limited number
474 of 8 events is seen to be biased positively by about 0.4 magnitude units, which may be attributed
475 to the overestimation of the seismic moment by low-frequency displacement spectrum
476 amplitude by NDI as mentioned before in case of M_S. Thus, these data points are not included
477 in developing the magnitude conversion relationship in this study. The M_W is also seen to be
478 higher by about 0.5 magnitude units in the higher m_b range of 6.6 – 7.2. The M_W for 17 out
479 of 18 events in this magnitude range are available from GCMT and one event from NEIC, all
480 of which are thus considered to be quite reliable. However, the m_b estimates are considered to
481 be underestimated due to saturation effects and these are also not included in the present
482 analysis. Therefore, the present conversion relationships for $m_b \rightarrow M_W$ have been developed
483 using 3641 data points available in the intermediate m_b range of 3.3 – 6.5, for which the mean
484 residuals in all the depth ranges are seen to be quite close to zero, indicating 1:1 correspondence
485 between M_W and m_b on average. The detailed distribution of the residuals $\Delta m_b = M_W - m_b$ as
486 a function of focal depth is illustrated in Figure 6 with the data in three identified magnitude
487 ranges plotted in different symbols and colours.

488

Table 5. Mean and SD values of the residuals $\Delta m_b = M_W - m_b$ for different magnitude and depth ranges.

Sr. No.	m_b Range	Depth Range (km)	No. of Events	Mean Value	SD Value
1.	3.1 – 3.2	17 – 35	2	0.3000	0.0999
		35 – 107.9	6	0.4167	0.0074
		17 – 107.9	8	0.3875	0.0047
2.	3.3 – 6.5	0 – 35	2657	0.0446	0.0009
		35 – 268	984	0.0033	0.0065
		0 – 268	3641	0.0334	0.0006
3.	6.6 – 7.2	8.2 – 35	10	0.6100	0.03667
		35 – 228.5	8	0.3875	0.1465
		8.2 – 228.5	18	0.5111	0.1240

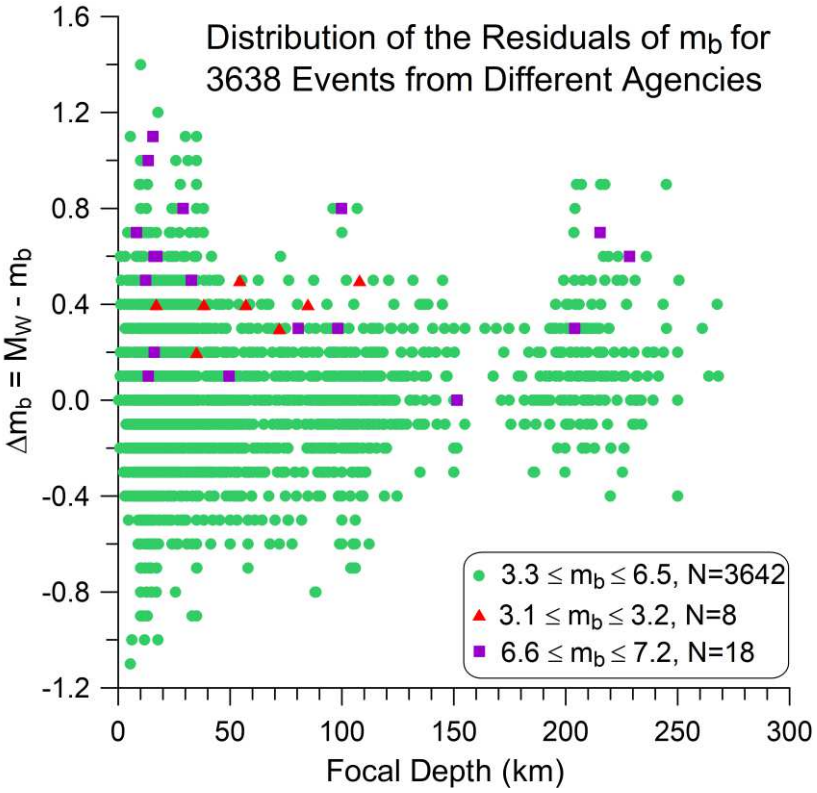


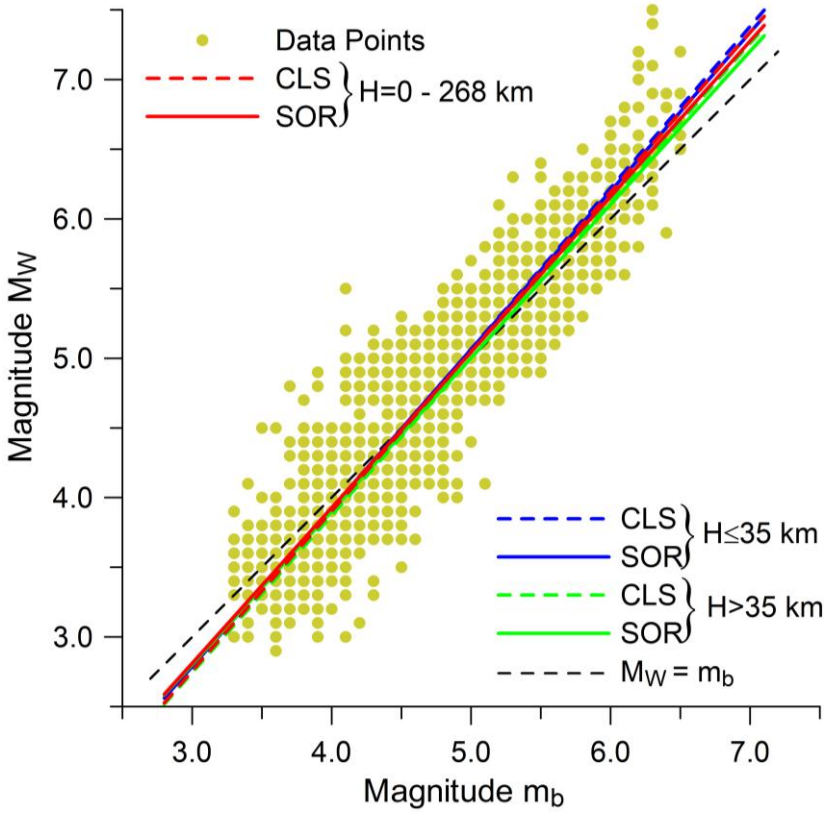
Figure 6. Distribution of the residuals $\Delta m_b = M_W - m_b$ as a function of focal depth for the available data points in three identified m_b ranges.

The limited number of 8 events in the lowest m_b range of 3.1 – 3.2 and 18 events in the highest m_b range of 6.6 – 7.2 are all seen to be skewed towards higher residuals compared to

almost symmetric distribution of the residuals around zero value for the intermediate m_b range of 3.3 – 6.5. Further, the mean trend of the residuals for the 3642 data points in m_b range of 3.3 – 6.5 is not seen to have any apparent dependence on the focal depth, and hence a single conversion relationship for the complete depth range is considered appropriate. However, just for the purpose of comparison, the relationships have also been developed for the shallower and deeper events separately by both SOR and CLS regression methods as summarized in Table 6. The matching of these relations with the database used is shown in Figure 7, which does not reveal any significant difference among the relations for different depth ranges. Thus, the relationship based on the data for the complete depth range is recommended to be used. Here, the relationships based on the SOR regression method are seen to be characterized by slightly smaller standard deviation, σ, and higher coefficient of determination, R^2, but the differences being only at the third decimal place can be considered negligible. In Figure 7, the relationships based on both SOR and CLS regression method are seen to be practically identical, and hence the relationship based on CLS method is proposed to be adopted for reasons of uniformity. Though, the lowest m_b considered in developing the present relationship is 3.3, it is expected to be valid for lower magnitudes up to 3.1 if the overestimation of the corresponding M_W in the observed database is assumed to have been rectified by filtering out the low-frequency noise and application of the baseline correction (Gupta, 2018).

Table 6. Empirical conversion relationships, $M_W = a + bm_b$, developed using CLS and SOR regression methods for three different depth ranges and m_b range of 3.3 to 6.5.

Sr. No.	Depth Range (km)	Regression Method	Slope, b	Intercept, a	σ	R^2	N
1.	0 – 35	CLS	1.1577±0.0093	-0.7228±0.0458	0.2942	0.8219	2657
		SOR	1.1371±0.0047	-0.6227±0.0211	0.2913	0.8253	
2.	35 – 268	CLS	1.1289±0.0129	-0.6431±0.0654	0.2573	0.8689	984
		SOR	1.0998±0.0065	-0.4966±0.0327	0.2542	0.8721	
3.	0 – 268	CLS	1.1452±0.0076	-0.6791±0.0376	0.2852	0.8350	3641
		SOR	1.1160±0.0031	-0.5359±0.0135	0.2815	0.8393	



519

520 **Figure 7**. Fitting of the m_b to M_W conversion relationships to observed data points by CLS
521 and SOR regression methods.

522

523 To examine the validity of other typical global and regional relationships for the Indian
524 region, Figure 8 compares seven existing conversion relationships from m_b to M_W with that
525 developed in this study using CLS method. The details of these relations are given in Table 7,
526 which includes two relations for India, two for Global data, and one each for New Zealand,
527 Turky, and China. The minimum and maximum magnitudes of applicability for the relations
528 listed in Table 7 vary over wide ranges of 3.5 to 4.4 and 6.2 to 7.0, respectively, but all the
529 plots in Fig. 5 are limited between m_b values of 3.1 and 6.5. None of the existing relationships
530 is seen to closely follow the proposed relationship developed in the present study over the
531 complete m_b range. All existing relationships by other investigators are in general seen to
532 overestimate the proposed relationship to varying extent for m_b up to 5.5, with the largest
533 overestimation by the global relationship due to Scordilis (2006) exceeding 0.5 magnitude
534 units.

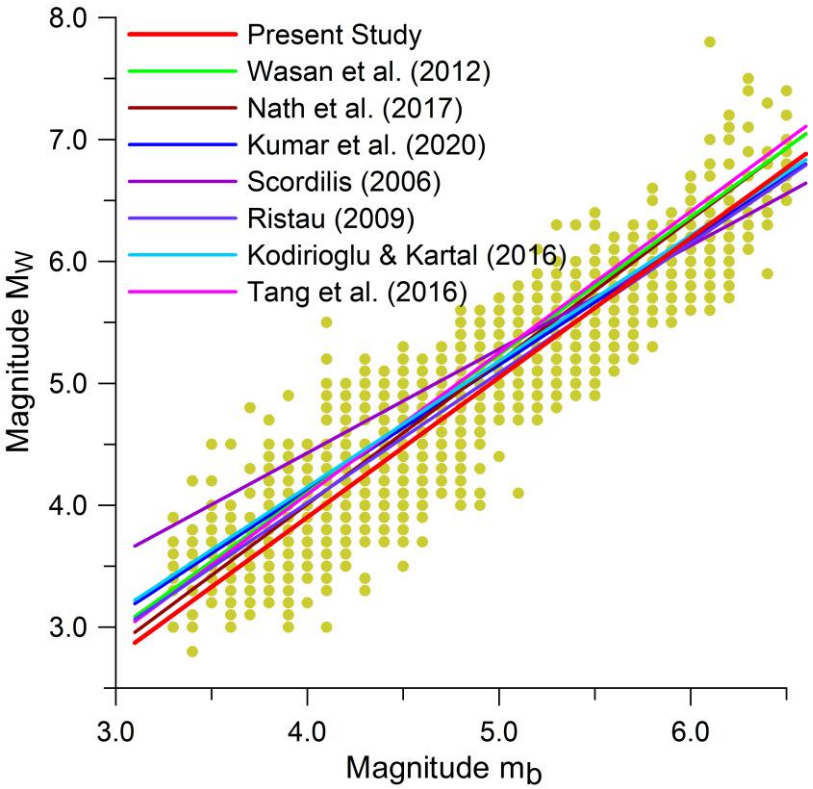


Figure 8. Comparisons of the m_b to M_W conversion relationships developed in this study with seven existing global and regional relations developed by various other investigators.

Table 7. Details of the typical global and regional conversion relationships from m_b to M_W compared with the relationship developed for the Indian region in the present study.

Sr. No.	Region	Reference	Regression Method	Slope	Intercept	σ	N	m_b Range
1.	India	Present Study	CLS	1.1452	-0.6791	0.2852	3641	3.3 – 6.5
2.	Global	Wason et al. (2012)	M-GOR	1.13	-0.416	0.3407	19466	4.4 – 6.2
3.	South Asia	Nath et al. (2017)	M-GOR	1.168	-0.663	0.3166		3.8 – 7.0
4.	Himala-yan Belt	Kumar et al. (2020)	GOR	1.03	-0.00	0.3008	348	4.4 – 6.7
5.	Global	Scordilis (2006)	OLS	0.85	1.03	0.3989	39784	3.5 – 6.2
6.	New Zealand	Ristau (2009)	ODR	1.087	-0.380	0.2828	314	3.1 – 6.3
7.	Turkey	Kadirioglu & Karat (2016)	OLS	1.0319	0.0223	0.3140	488	3.9 – 6.8
8.	China	Tang et al. (2016)	SOR	1.16	-0.55	0.3517	487	4.3 – 6.3

Similar to that for the $M_S \rightarrow M_W$ relationships, Table 7 also gives the standard deviations, σ, for all the relations estimated with respect to the dataset used in developing the present relationship. Except for the relationship due to Ristau (2009) for New Zealand, all other existing relationships are characterized by much higher standard deviations compared to that for the present relationship. It may thus be concluded that the existing relationships for $m_b \rightarrow M_W$ conversion are in general not suitable for the Indian region. The smaller standard deviation for Ristau (2009) may just be a coincidence.

4.3 Conversion Relations for $M_L \rightarrow M_W$

Hanks and Boore (1984) have shown that the $\log M_0 - M_L$ plot is characterized by a continuously increasing slope with increase in magnitude, which reinforces the theoretical basis that M_W versus M_L can also be characterized by slopes of 2/3, 1.0 and 2.0 in the low, intermediate and higher M_L ranges, respectively. Similar to that for m_b, the highest magnitude range with slope of 2.0 lies beyond the saturation limit for M_L and the lowest magnitude range with slope of 2/3 lies below the practical lower limit for estimation of M_L. The optimum intermediate range of M_L for developing a conversion relationship for $M_L \rightarrow M_W$ in the present study has been identified by detailed analysis the residuals between M_W and M_L in different M_L and depth ranges.

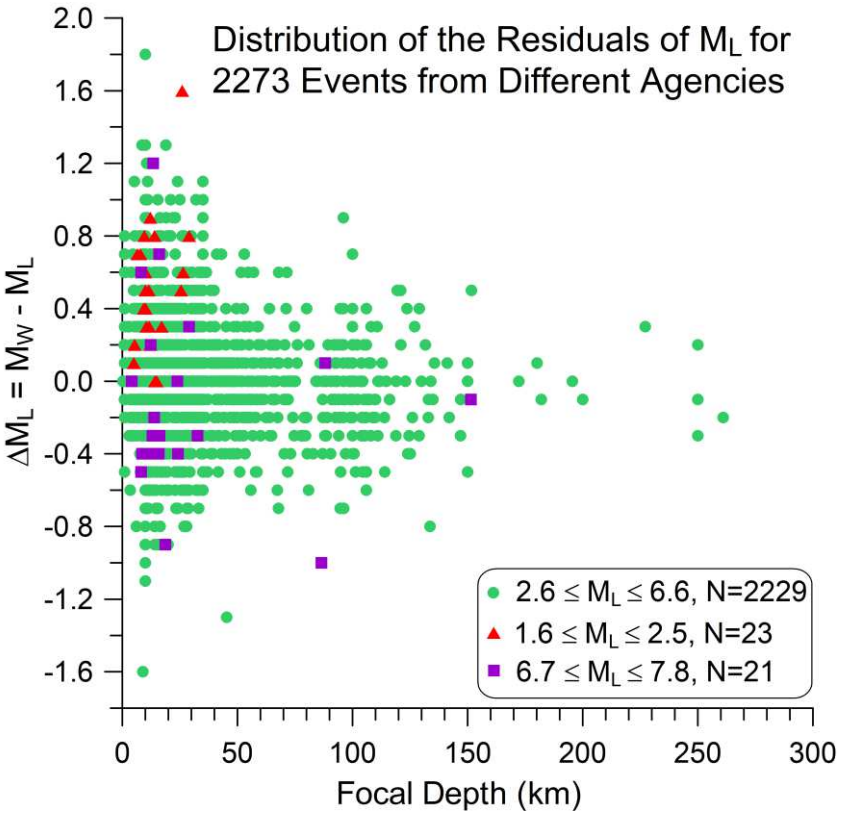
The mean values of the residuals in the identified low, intermediate, and large M_L ranges are given in Table 8 for three different ranges of focal depth. In the lowest M_L range of 1.6 – 2.5, the M_W for all the 23 events is seen to be biased positively by about 0.5 magnitude units, which may be attributed to the overestimation of the seismic moment for smaller earthquakes by low-frequency displacement spectrum amplitude as mentioned before. Thus, these data points are not included in developing the magnitude conversion relationship in the present study. For 18 out of 21 events in the highest M_L range of 6.7 – 7.8 with shallower focal depth of less than 35 km, the mean value of the residuals is quite close to zero. The residuals for two of the remaining three deeper events in the depth range of 35 – 151.4 km are also seen to be close to zero. However, one event on 04-02-2011 in northeast India has been assigned M_L of 7.3 by BKK with much smaller M_W of 6.3 by GCMT. The ISC has assigned m_b of 6.4 to this

572 event and hence the available value of M_L is not considered reliable for this event. Barring
573 this event, the other 20 events with M_L greater than 6.6 are seen to follow a 1:1 mean trend.
574 But the M_L in this range of magnitude is expected to suffer from the saturation effects, and
575 thus not used in developing the magnitude conversion relation in the present study. The data
576 base of 2229 events in the intermediate M_L range of 2.6 – 6.6 is seen to be characterized by
577 average value of residuals close to zero in all the distance ranges. This is also supported by the
578 detailed distribution of the residuals $\Delta M_L = M_W - M_L$ as a function of focal depth shown in
579 Figure 9 with the data in three identified magnitude ranges plotted in different symbols and
580 colours.

581 **Table 8.** Mean and SD values of the residuals $\Delta M_L = M_W - M_L$ for different magnitude
582 and depth ranges.

Sr. No.	M_L Range	Depth Range (km)	No. of Events	Mean Value	SD Value
1.	1.6 – 2.5	5 – 29	23	0.5130	0.0881
2.	2.6 – 6.6	0.2 – 35	1809	0.0381	0.0197
		35 – 261	420	-0.0376	0.0616
		0.2 – 261	2229	0.0238	0.0174
3.	6.7 – 7.8	4.1 – 35	18	-0.0833	0.0283
		35 – 151.4	3	-0.3333	0.3064
		4.1 – 151.4	21	-0.1190	0.0181

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585

586 **Figure 9.** Distribution of the residuals $\Delta M_L = M_W - M_L$ as a function of focal depth for the
587 available data points in three identified M_L ranges.
588
589 The $M_L \rightarrow M_W$ conversion relationships developed for the M_L range of 2.6 to 6.6 by both
590 CLS and SOR regression methods for the entire depth range of 0 – 261 km as well as separately
591 for the shallower and deeper events are summarized in Table 9. The matching of these relations
592 with the database used is shown in Figure 10, which indicates slight underestimation of M_W
593 for deeper events with M_L greater than about 4.0. However, this underestimation is very small
594 to be of any practical significance. Also, the number of data points available for deeper events
595 is much smaller to obtain a separate relationship with good reliability. The relationship for the
596 complete depth range is therefore recommended to be used in the present study. Here again,
597 the relationships based on the SOR regression method are seen to be characterized by slightly
598 smaller standard deviation, σ, and higher coefficient of determination, R^2, but the differences
599 are at the third or fourth decimal places only. Also in Figure 10, the relationships based on the
600 SOR and CLS regression method are seen to be indistinguishable. Thus, similar to that for the
601 case of m_b, the relationship based on CLS method is proposed to be adopted. Though, the
602 lowest M_L considered in developing the present relationship is 2.6, it is expected to be

applicable for lower M_L values up to 2.0 if the overestimation of the M_W for these magnitudes is assumed to have been rectified for low-frequency noise as mentioned before in case of m_b.

Table 9. Empirical conversion relationships, $M_W = a + bM_L$, developed using CLS and SOR regression methods for three different depth ranges and M_L range of 2.6 to 6.6.

Sr. No.	Depth (km)	Regression Method	Slope, b	Intercept, a	σ	R^2	N
1.	0 – 35	CLS	0.9515±0.0094	0.2562±0.0427	0.3175	0.8253	1809
		SOR	0.9493±0.0050	0.2673±0.0257	0.3171	0.8257	
2.	35 – 261	CLS	0.9182±0.0153	0.3256±0.0690	0.2456	0.8845	420
		SOR	0.9023±0.0161	0.3953±0.0678	0.2439	0.8861	
3.	0 – 261	CLS	0.9472±0.0082	0.2605±0.0372	0.3069	0.8345	2229
		SOR	0.9415±0.0057	0.2841±0.0231	0.3060	0.8354	

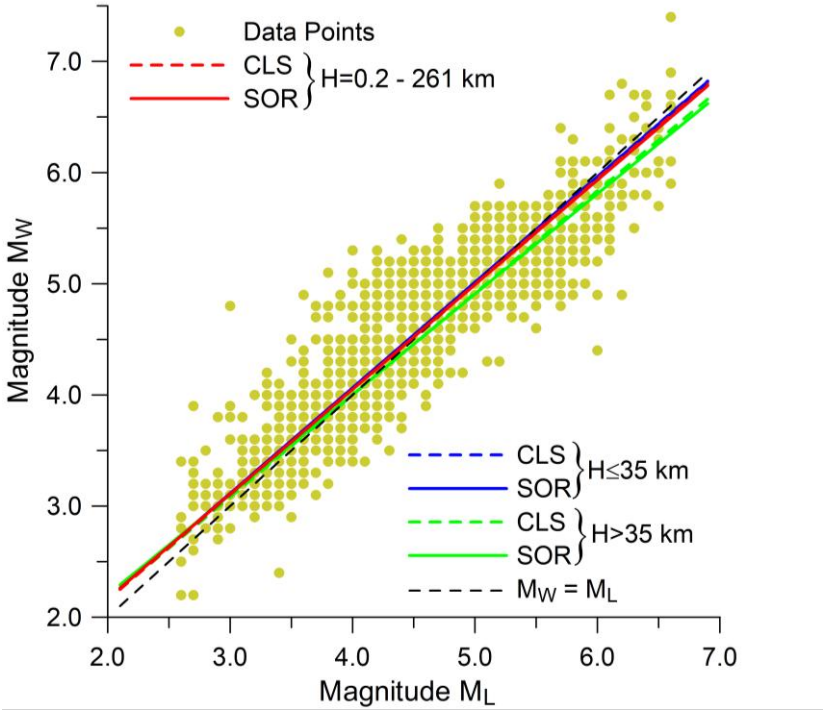


Figure 10. Fitting of the M_L to M_W conversion relationships to observed data points by CLS and SOR regression methods.

Fig. 11 shows the comparison of the conversion relationship developed and recommended in this study with several typical regional relationships developed by other investigators for India and other countries. The details of these relations are given in Table 10, which includes three relations for India and one each for Croatia, China, Turky, and California (USA). Despite

the fact that M_L range of applicability varies widely among the various relationships, all the plots in Figure 10 are given for the M_L range of 2.0 and 7.0. The relationship due to Chung and Bernreuter (1981), which is obtained by combining the $\log_{10} M_0 - M_L$ relationship for California with $\log_{10} M_0 = 1.5M_W + 16.1$ (Hanks and Kanamori 1979), is seen to follow most closely the relationship proposed in this study. All other relationships are seen to overestimate or underestimate the present relationship significantly in different M_L ranges, which is also manifested by the standard deviations for the various relationships estimated with respect to the dataset used in developing the present relationship and given in Table 10. Thus, proposed conversion relationship developed in this study can be considered the most suitable for the Indian region.

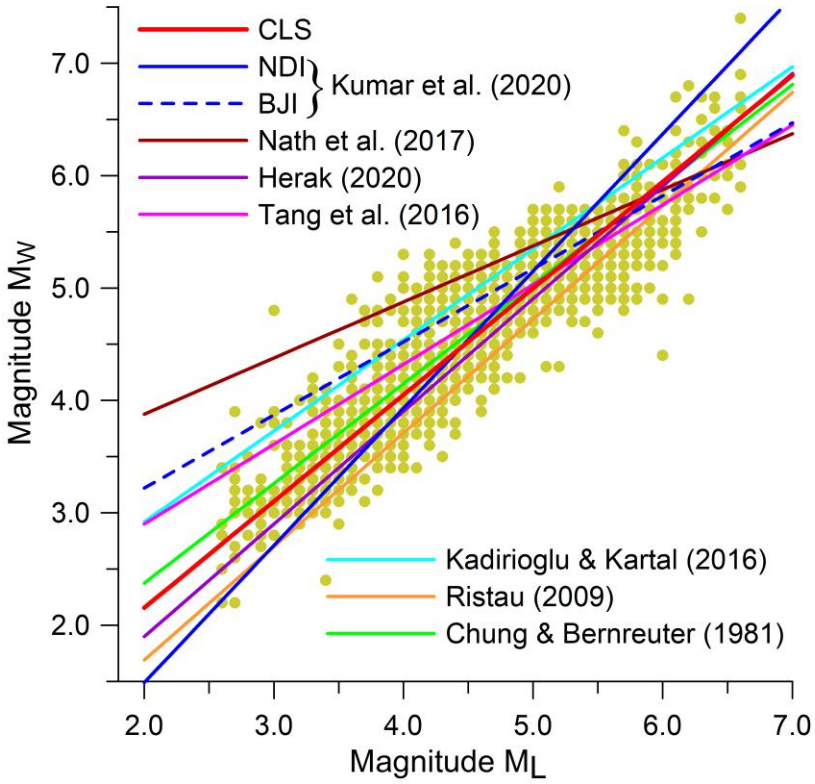


Figure 11. Comparisons of the M_L to M_W conversion relationships developed in this study with eight existing relations developed by various other investigators.

638 **Table 10**. Details of the typical conversion relationships from M_L to M_W developed by other
639 investigators with that developed for the Indian region in the present study.

Sr. No.	Region	Reference	Regression Method	Slope	Intercept	σ	N	M_L Range
1.	India	Present Study	CLS	0.9472	0.2605	0.3069	2229	2.6 – 6.6
2.	Himala-yan Belt	Kumar et al. (2020)	GOR	1.22	-0.95	0.4102	78 (NDI)	4.0 – 7.0
				0.65	1.92	0.4771	42 (BJI)	4.0 – 6.7
3.	South Asia	Nath et al. (2017)	M-GOR	0.499	2.88	0.7409		3.5 – 7.3
4.	Croatia	Herak (2020)	Y-GOR	1.002	-0.106	0.3407	153	3.5 – 6.5
5.	China	Tang et al. (2016)	SOR	0.71	1.48	0.3615	487	4.0 – 7.1
6.	Turkey	Kadirioglu & Karat (2016)	OLS	0.8095	1.3003	0.5205	404	3.3 – 6.6
7.	New Zealand	Ristau (2009)	ODR	1.01	-0.33	0.4454	479	4.0 – 7.0
8.	California	Chung & Bernreuter (1981)	OLS	0.887	0.60	0.3088		

640

641 **5. Conversion Relationship for $m_B \rightarrow M_S$**

642 For many older earthquakes prior to 1963, the broad-band body wave magnitude m_B is the
643 only type of magnitude available. Thus, a conversion relationship for $m_B \rightarrow M_W$ is also required
644 for the Indian region, but no database on the pairs of m_B and M_W magnitudes is available to
645 develop such a conversion relationship. A conversion relationship has been therefore developed
646 for $m_B \rightarrow M_S$ so that the already developed $M_S \rightarrow M_W$ relationship can be used to get the
647 M_W. A dataset of 73 earthquakes could be compiled for the period 1902 to 1963 for which
648 both m_B and M_S are available. The m_B for majority of this dataset is from Abe (1981) and
649 M_S from PAS (California Institute of Technology, Pasadena). This database covers the m_B
650 range of 5.7 to 8.2 and M_S range of 5.7 to 8.6 with focal depths up to 260 km, which has been
651 used to develop the relationships of Equations (12) and (13) by CLS and SOR methods,
652 respectively.

653 $$M_S = (1.1420 \pm 0.0748) m_B - (0.9332 \pm 0.5322); \sigma = 0.3229 \qquad (12)$$

$$M_S = (1.0686 \pm 0.0185) m_B - (0.3300 \pm 0.0854); \sigma = 0.3232 \quad (13)$$

Comparison of these relationships with the observed database used is shown in Fig. 12. The only known relationship between m_B and M_S by Gutenberg and Richter (1956) based on ILS method is also plotted in this figure.

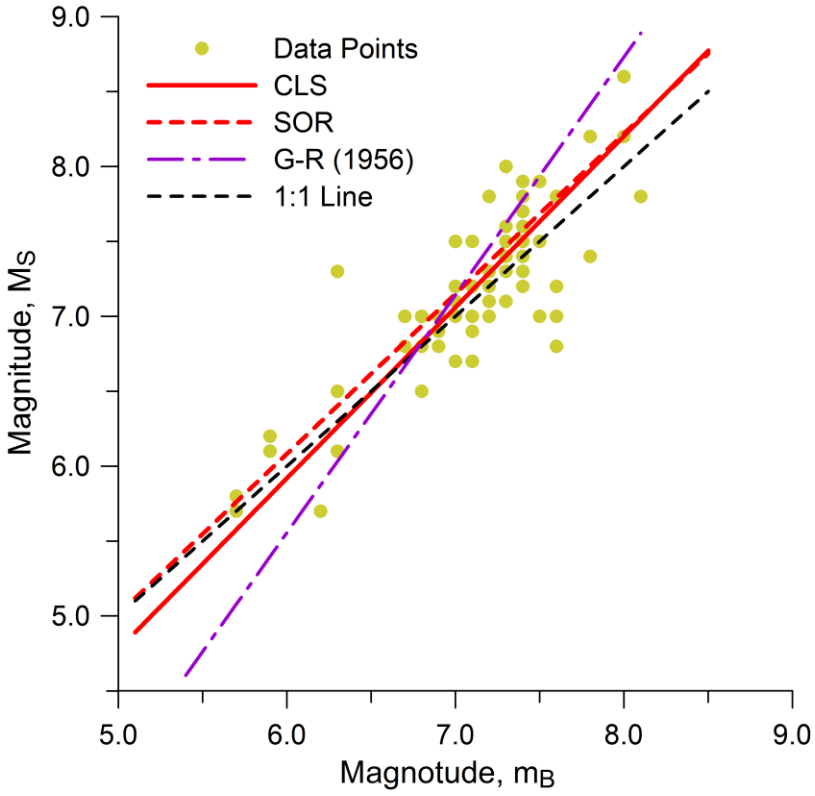


Figure 12. The m_B to M_S conversion relationships developed by CLS and SOR regression methods in this study along with the corresponding relationship by Gutenberg and Richter (1956) compared with the available observed data points.

The relationship due to Gutenberg and Richter (1956) is not seen to be suitable for the Indian region. The relationship developed by CLS method in the present study is characterized by slightly lower standard deviation than that developed by SOR method. Similar to the other conversion relations developed in this study, the CLS based relation of Equation (12) is proposed to be used for $m_B \rightarrow M_S$ conversion in the present study.

5 Recommended Procedure for Homogenization of Earthquake Catalog

The magnitude conversion relations developed and proposed to be used for converting the magnitude types M_S, m_b, and M_L to moment magnitude M_W for the Indian region are given below in Equations (14), (15) and (16), respectively.

673 $$M_W = \begin{cases} 0.6832M_S + 1.9466; \sigma = 0.1856, 3.1 \le M_S \le 6.5 \\ 1.0030M_S - 0.2292; \sigma = 0.1617, 6.6 \le M_S \le 8.0 \end{cases} \quad (14)$$

674 $$M_W = 1.1452m_b - 0.6791; \sigma = 0.2852, 3.3 \le m_b \le 6.5 \quad (15)$$

675 $$M_W = 0.9472M_L + 0.2606; \sigma = 0.3069, 2.6 \le M_L \le 6.6 \quad (16)$$

676 This section outlines a step-by-step procedure to use these relationships in a hierarchical way
677 to homogenize the earthquake catalog for India compiled from multiple sources. It is assumed
678 that the catalog has been compiled by following the same hierarchy for the preferred agencies
679 for various magnitude types as used in compiling the databases for developing the present
680 conversion relationships. It is also assumed that the moment magnitudes for great earthquakes
681 with 8.0(+) magnitudes have been already finalized by critical review of the available literature
682 in each case. It is further assumed that the longer period body wave magnitudes m_B have
683 already been converted to the surface wave magnitude M_S using the conversion relationship
684 proposed in the present study. Finally, values below 2.0 for all the magnitude types are not
685 recommended to be reported in the catalog as these are not considered reliable and of much
686 significance in the seismic hazard estimation.

687 The proposed methodology for homogenization of the earthquake catalog to moment
688 magnitude, when all or limited types of other magnitudes are reported for each event in the
689 compiled catalog, can be described in a sequential manner as follows:

690 (i) If M_W is already available and $M_W \ge 4.5$, take it directly without using any conversion
691 relationship.

692 (ii) Else if M_S is available and $M_S \ge 6.6$, use the proposed conversion relationship for
693 $M_S \rightarrow M_W$.

694 (iii) Else if m_b is available and $3.3 \le m_b \le 6.5$, use the proposed conversion relationship for
695 $m_b \rightarrow M_W$.

696 (iv) Else if M_L is available and $2.6 \le M_L \le 6.6$, use the proposed conversion relationship for
697 $M_L \rightarrow M_W$.

(v) Else if M_S is available and $3.1 \le M_S \le 6.5$, use the proposed conversion relationship for $M_S \to M_W$.

(vi) Else if m_b is available and $3.1 \le m_b \le 3.2$, use the proposed conversion relationship for $m_b \to M_W$.

(vii) Else if M_L is available and $2.0 \le M_L \le 2.5$, use the proposed conversion relationship for $M_L \to M_W$.

(viii) Else if m_b is available and $2.0 \le m_b \le 3.0$, take $M_W = m_b - 0.2$, which is proposed subjectively to constrain further decrease in M_W with decrease in m_b below 3.0.

(ix) Finally, if an event in the catalog is not covered by any of the above conditions (i) to (vii), it may be discarded because it is expected to of very small size for which no reliable estimation of M_W is possible.

The above hierarchical procedure stems from the relative reliability of different types of magnitudes in different ranges. The recommendation on using directly the reported M_W values for magnitudes greater than or equal to 4.5 is based on the fact that the GCMT catalog does not reports values for smaller magnitudes and the estimates for smaller magnitudes based on regional/local networks are generally overestimated and not reliable as discussed before. However, the overestimations are largely normalized in the conversion relations. Conversion from larger M_S greater than or equal to 6.6 is recommended because the other magnitude scales are saturated in this range of magnitude. Conversion from m_b, M_L and M_S is suggested in the sequence within the respective intermediate ranges of the periods of applicability. The M_S is given the lowest priority because it relates the most poorly to M_W because the M_S estimates are increasingly underestimated for magnitudes decreasing below 6.0. Though, both m_b and M_L correlate equally well to M_W, the m_b is given higher priority because reliable estimates of m_b are available for much larger number of events due to its teleseismic nature and recording by global networks. Steps (vi) and (vii) in the recommended sequence are based on ranges for which the m_b and M_L conversion relations could be extrapolated reliably as already discussed in the paper. The proposed $m_b \to M_W$ relationship can be extrapolated

reliably up to m_b value of 3.0. However, to cover more of the events in the catalog, further extrapolation has been proposed in step (viii) by constraining M_W to 0.2 magnitude units lower than the reported m_b value, which is the decrease at m_b value of 3.0. Very limited number of small sizes of events in the catalog not covered under any of the proposed sequence of steps are recommended to be discarded. Lacking such definite recommendations on catalogue homogenization in the previous studies on magnitude conversion relations may lead to erroneous applications of the relationships proposed by them.

6. Discussion

The present study has addressed several crucial issues often disregarded in the development and application of earthquake magnitude conversion relationships. A key improvement lies in the adoption of physically consistent linear and bilinear functional forms instead of purely empirical nonlinear models. Another important contribution is the introduction of the Central Least Squares (CLS) regression method, which gives better or equally good results to the SOR method. Unlike conventional approaches such as OLS, which assume error-free independent variables, or orthogonal regression techniques that require subjective estimation of error variance ratios, CLS provides a balanced and unbiased alternative. The CLS method effectively captures the inherent uncertainties in both predictor and the predicted magnitude scales without relying on unverifiable prior assumptions regarding their error structure. The so-called modified general orthogonal regression (M-GOR) method, advocated in a number of recent studies, has been indicated to suffer from serious theoretical and physical drawbacks and discrepancies.

The study also emphasizes the importance of careful data selection and preprocessing. The exclusion of biased datasets, particularly those from IDC and low-magnitude ranges affected by noise contamination, is shown to significantly improve the robustness of the derived relationships. The analysis of residuals across different magnitude and depth ranges provides a systematic basis for identifying reliable ranges of applicability, an aspect often overlooked in existing studies. The findings confirm that apparent nonlinear trends in magnitude relationships are largely attributable to measurement biases, magnitude saturation, and data incompleteness rather than being the physical reality.

Another factor rarely examined in earlier works, is the explicit analysis of the depth dependence. Although minor variations are observed, particularly for surface-wave magnitudes

of deeper events, the differences are not substantial enough to warrant development of separate conversion relations for different depth ranges. A unified relationships has been thus developed for the full depth range, simplifying practical implementation without significant loss of accuracy.

The comparative analysis with several typical existing regional and global relationships further highlights the superiority of the proposed models. Most previously published relations exhibit larger standard deviations when tested against the present dataset, and exhibit wide variations in the slopes as well as intercepts.

Finally, the present study provides a hierarchical framework for applying the magnitude conversion relations in catalog homogenization. The absence of such guidelines in earlier works has often led to inconsistent practices and potential biases in seismic hazard assessments. The proposed step-by-step approach, based on the relative reliability of different magnitude scales in different ranges, ensures a logical and reproducible procedure for converting heterogeneous catalogue into a unified moment magnitude scale.

7. Conclusions

This study presents a comprehensive and physically consistent framework for developing earthquake magnitude conversion relationships with illustrative application to India and adjoining areas. Based on the analysis performed, the following important conclusions are drawn:

- Linear functional forms are found to be the most appropriate for $m_b \rightarrow M_W$ and $M_L \rightarrow M_W$ conversion relations, while a bilinear model is required for $M_S \rightarrow M_W$ with the interface magnitude identified in an iterative manner such that the offset is the minimum and the slopes of the lower and higher magnitude segments deviate the minimum from the theoretically expected values of 2/3 and 1.0, respectively. Nonlinear empirical forms, though statistically appealing, are not supported by physical considerations, and are also not able to preserve the linearity of the G-R relationship.
- The proposed Central Least Squares (CLS) method provides a simple, robust, and unbiased alternative to existing regression techniques. It avoids the need for subjective assumptions regarding error variances while delivering improved statistical performance. This paper has also elaborated the fallacies in the much-hyped modified general

orthogonal regression (M-GOR) method in several recent papers, which have not been unidentified in any of the previous papers.

- The conversion relationships are developed for carefully identified magnitude ranges based on residual analysis, ensuring realistic and reliable application. Data outside these ranges, particularly at very low and very high magnitudes, are shown to be affected by systematic biases and are excluded. The variations with focal depth are not found to be significant enough, and unified relations for the full depth range are therefore recommended for convenience in practical applications.

- Compared to a large number of existing regional and global models, the proposed relationships demonstrate lower standard deviations and better agreement with observed data, confirming their suitability for the Indian tectonic setting. Majority of the existing relationships for $M_S \rightarrow M_W$ and $M_L \rightarrow M_W$ conversion is seen to be grossly inconsistent with the present datasets.

- A new relationship for converting the older long-period body wave magnitude m_B to M_S has also been developed, enabling the inclusion of older earthquakes in the homogenized catalogues. The m_B is the only magnitude type available for the Indian region for several important earthquakes prior to 1963.

- The paper has also proposed a structured, hierarchical procedure for converting different magnitude types to moment magnitude. This framework minimizes conversion bias and ensures consistency in seismic hazard applications.

Overall, this study provides a scientifically rigorous and practically implementable solution to the problem of magnitude homogenization in India. The proposed methodology is expected to significantly enhance the reliability of earthquake catalogues and, consequently, improve the accuracy of seismic hazard and risk assessments in the region.

REFERENCES

Abe, K. (1981). Magnitudes of large shallow earthquakes from 1904 to 1980. *Physis of the Earth and Planetary Interior,* 27, 72-92.

Anbazhagan, P. and Balakumar, B. (2019). Magnitude conversion for peninsular India using orthogonal regression analysis. *Journal of Seismology*, 23(3), 567-585.

Baruah, S., Baruah, S., Bora, P. K., Duarah, R., Kalita, A., Biswas, R., Gogoi, N., and Kayal, J. R. (2012). Moment magnitude (M_W) and local magnitude (M_L) relationship for earthquakes in Northeast India. *Pure and Applied Geophysics*, *169*(11), 1977-1988.

Boggs, P. T., Spiegelman, C. H., Donaldson, J. R., and Schnabel, R. B. (1988). A computational examination of orthogonal distance regression. *Journal of Econometrics*, *38*(1-2), 169-201.

Bora, D. K. (2016). Scaling relations of moment magnitude, local magnitude, and duration magnitude for earthquakes originated in northeast India. *Earthquake Science*, *29*(3), 153-164.

Brune, J.N. (1970). Tectonic stress and spectra of seismic shear waves from earthquakes. Jour. Geophys. Res., 75, 4997-5009.

Castellaro, S., Mulargia, F., and Kagan, Y.Y. (2006). Regression problems for magnitudes, Geophys. Jour. Int., 165, 913–930.

Chatterjee, A., Ghosh, S., and Kayal, J. R. (2022). Homogenization of earthquake magnitudes using bivariate fit shell approach: Eastern Himalaya region. *Physics and Chemistry of the Earth, Parts A/B/C*, *126*, 103144.

Chingtham, P., Chopra, S., Baskoutas, I., and Bansal, B. K. (2014). An assessment of seismicity parameters in northwest Himalaya and adjoining regions. *Natural hazards*, *71*(3), 1599-1616.

Chung, D. H. and Bernreuter, D. L. (1981). Regional relationships among earthquake magnitude scales. *Reviews of Geophysics*, *19*(4), 649-663.

Das, A., Das, R., and Choudhury, D. (2025). Improved regional earthquake magnitude scaling relations for the Indian Subcontinent. *Natural Hazards Review*, *26*(4), 04025048.

Das, R., Wason, H. R., and Sharma, M. L. (2012). Magnitude conversion to unified moment magnitude using orthogonal regression relation. *Journal of Asian Earth Sciences*, *50*, 44-51.

Das, R., Wason, H. R., and Sharma, M. L. (2014). Unbiased estimation of moment magnitude from body-and surface-wave magnitudes. *Bulletin of the Seismological Society of America*, *104*(4), 1802-1811.

Di Giacomo, D. and Storchak, D. A. (2016). A scheme to set preferred magnitudes in the ISC Bulletin. *Journal of Seismology*, *20*(2), 555-567.

Ekström, G. and Dziewonski, A. M. (1988). Evidence of bias in estimations of earthquake size. *Nature*, *332*(6162), 319-323.

Fuller W. A. (1987). Measurement Error Models, Wiley, New York.

Gasperini, P., Lolli, B., and Castellaro, S. (2015). Comparative analysis of regression methods used for seismic magnitude conversions. *Bulletin of the Seismological Society of America*, *105*(3), 1787-1791.

Gasperini, P., Lolli, B., and Vannucci, G. (2013). Empirical calibration of local magnitude data sets versus moment magnitude in Italy. *Bulletin of the Seismological Society of America*, *103*(4), 2227-2246.

Gupta, I. D. (2018). Uniformly processed strong motion database for Himalaya and northeast region of India. *Pure and Applied Geophysics*, *175*(3), 829-863.

Gutenberg, B. (1945a). Amplitudes of surface waves and magnitudes of shallow earthquakes. *Bulletin of the Seismological Society of America*, 35(1), 3-12.

Gutenberg, B. (1945b). Amplitude of P, PP, and S waves and magnitudes of shallow earthquakes. *Bulletin of the Seismological Society of America*, 35, 57-69.

Gutenberg, B. (1945c). Magnitude determination for deep-focus earthquakes. *Bulletin of the Seismological Society of America,* 35, 117-130.

Gutenberg, B., and Richter, C. F. (1956). Earthquake magnitude, intensity, energy, and acceleration: (Second paper). *Bulletin of the seismological society of America*, *46*(2), 105-145.

Hanks, T. C., and Boore, D. M. (1984). Moment-magnitude relations in theory and practice. *Journal of Geophysical Research: Solid Earth*, *89*(B7), 6229-6235.

Hanks, T. C., and Kanamori, H. (1979). A moment magnitude scale. *Journal of Geophysical Research*, 84(B5), 2348-2350.

Herak, M. (2020). Conversion between the local magnitude (ML) and the moment magnitude (Mw) for earthquakes in the Croatian Earthquake Catalogue. *Geofizika*, *37*(2), 197-211.

Kadirioğlu, F. T., and Kartal, R. F. (2016). The new empirical magnitude conversion relations using an improved earthquake catalogue for Turkey and its near vicinity (1900-2012). *Turkish Journal of Earth Sciences*, *25*(4), 300-310.

Kanamori, H. (1977). The energy release in great earthquakes. *Journal of Geophysical Research*, 82(20), 2981-2987.

Kanamori, H., and Anderson, D. L. (1975). Amplitude of the Earth's free oscillations and long-period characteristics of the earthquake source. *Journal of Geophysical Research*, *80*(8), 1075-1078.

Kendall, M. G., and Stuart, A. (1979). Functional and structural relationship. *The advanced theory of statistics*, *2*, 399-343.

Kolathayar, S., Sitharam, T. G., and Vipin, K. S. (2012). Spatial variation of seismicity parameters across India and adjoining areas. *Natural hazards*, *60*(3), 1365-1379.

Kumar, R., Yadav, R. B. S. and Castellaro, S. (2020). Regional earthquake magnitude conversion relations for the Himalayan seismic belt. *Seismological Society of America*, *91*(6), 3195-3207.

Lolli, B., Gasperini, P., and Vannucci, G. (2014). Empirical conversion between teleseismic magnitudes (mb and Ms) and moment magnitude (M w) at the Global, Euro-Mediterranean and Italian scale. *Geophysical Journal International*, *199*(2), 805-828.

Madansky, A. (1959). The fitting of straight lines when both variables are subject to error. *Journal of the American Statistical Association*, *54*(285), 173-205.

Nath, S. K., Mandal, S., Das Adhikari, M., and Maiti, S. K. (2017). A unified earthquake catalogue for South Asia covering the period 1900–2014. *Natural Hazards*, *85*(3), 1787-1810.

Pandey, A. K., Chingtham, P., and Roy, P. N. S. (2017). Homogeneous earthquake catalogue for Northeast region of India using robust statistical approaches. *Geomatics, Natural Hazards and Risk*, *8*(2), 1477-1491.

Pacheco, J.F. and Sykes, L.R. (1992). Seismic moment catalog of large shallow earthquakes, 1900 to 1989, *Bull. Seism. Soc. Am.*, **82**, 1306–1349.

Richter, C. F. (1935). An instrumental earthquake magnitude scale. *Bulletin of the Seismological Society of America*, 25(1), 1-32.

Ristau, J. (2009). Comparison of magnitude estimates for New Zealand earthquakes: moment magnitude, local magnitude, and teleseismic body-wave magnitude. *Bulletin of the Seismological Society of America*, *99*(3), 1841-1852.

Scordilis, E. M. (2006). Empirical global relations converting MS and mb to moment magnitude. *Journal of Seismology,* 10, 225-236.

Shahabuddin, M. and Mohanty, W.K. (2025). An updated and unified complete earthquake catalog of the Shillong Plateau and adjoining region. *Seismological Research Letters*, *96*(4), 2603-2624.

Sitharam, T. G. and Sil, A. (2014). Comprehensive seismic hazard assessment of Tripura and Mizoram states. *Journal of Earth System Science*, *123*(4), 837-857.

Storchak, D. A., Di Giacomo, D., Bondár, I., Engdahl, E. R., Harris, J., Lee, W. H., Villaseñor, A., and Bormann, P. (2013). Public release of the ISC–GEM global instrumental earthquake catalogue (1900–2009). *Seismological Research Letters*, *84*(5), 810-815.

Tang, C. C., Zhu, L., and Huang, R. (2016). Empirical Mw–ML, mb, and Ms conversions in Western China. *Bulletin of the Seismological Society of America*, *106*(6), 2614-2623.

Thingbaijam, K. K. S., Chingtham, P., and Nath, S. K. (2009). Seismicity in the North-West Frontier province at the Indian-Eurasian plate convergence. *Seismological Research Letters*, *80*(4), 599-608.

Thingbaijam, K. K. S., Nath, S. K., Yadav, A., Raj, A., Walling, M. Y., and Mohanty, W. K. (2008). Recent seismicity in Northeast India and its adjoining region. *Journal of Seismology*, *12*(1), 107-123.

Tsampas, A. D., Scordilis, E. M., Papazachos, C. B., and Karakaisis, G. F. (2016). Global-magnitude scaling relations for intermediate-depth and deep-focus earthquakes. *Bulletin of the Seismological Society of America*, *106*(2), 418-434.

Vanek, J., Zatopek, A., Karnik, V., Kondorskaya, N.V., Riznichenko, Y.N., Savarensky, E.F., Soloview, S.L., and Shebalin, N.V. (1962). Standardization of magnitude scales, *Bull. Acad. Sci. USSR Geophys. Ser.* 108-111.

Wason, H. R., Das, R., and Sharma, M. L. (2012). Magnitude conversion problem using general orthogonal regression. *Geophysical Journal International*, *190*(2), 1091-1096.

Yadav, R. B. S., Bayrak, Y., Tripathi, J. N., Chopra, S., Singh, A. P., and Bayrak, E. (2012). A probabilistic assessment of earthquake hazard parameters in NW Himalaya and the adjoining regions. *Pure and applied geophysics*, *169*(9), 1619-1639.

Yadav, R. B. S., Bormann, P., Rastogi, B. K., Das, M. C., and Chopra, S. (2009). A homogeneous and complete earthquake catalog for northeast India and the adjoining region. *Seismological Research Letters*, *80*(4), 609-627.

York, D., Evensen, N. M., Martínez, M. L., and De Basabe Delgado, J. (2004). Unified equations for the slope, intercept, and standard errors of the best straight line. *American journal of physics*, *72*(3), 367-375.