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Chemical Periodicity Was Not Indeterministic: Fourteen Independent Convergences Toward a Deterministic Algebraic Law of Chemical Periodicity

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Systematic Review

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Abstract

Chemical periodicity has been interpreted for over a century as an empirical regularity without derivation from first principles — consistent with the Copenhagen Interpretation, which treats period distribution as contingent. This paper proposes a contrary thesis: chemical periodicity is a necessary algebraic law, and its apparent contingency reflects an incomplete conceptual framework.

The evidence is the independent convergence of fourteen research lines, from Rydberg (1913) to Varlamov (2025), on the same observation: all periods organise in symmetric pairs sharing the same orbital block structure. Only the **Binodic Periodic System (BPS)** of Gutiérrez-Samanez (2004, 2020) provides the complete deductive chain: the 2004 discovery that the topology of period pairs coincides exactly with the parabolic function $Y = 4N^2$ yielded the closure function $Z(N) = 2N(N + 1)(2N + 1)/3$, the binode quantum number N, and period spin $\tau = \pm\frac{1}{2}$. Their formalisation as eigenvalues of **so(4,4)** followed between 2020 and 2026.

Key convergences analysed include Muradjan (2019), Ceulemans & Thyssen (2018) — whose conclusion pointing to Lie algebras led the author to so(4,4) — Schwarz et al. (2021) as limit-convergence, and Varlamov (2025), who reached so(4,4) independently from group theory.

1. INTRODUCTION: THE UNRESOLVED PROBLEM OF PERIOD DOUBLING

The standard Mendeleev-Bohr periodic table organises the 118 known chemical elements in seven periods of unequal length — 2, 8, 8, 18, 18, 32, 32 elements respectively — governed by the Madelung rule and the Aufbau principle. This empirical organisation has proven extraordinarily useful for predicting chemical properties and reactivity. Yet it conceals a deeper structural regularity that has been independently identified, named, and partially formalised by at least fourteen groups of researchers across more than a century: **all periods organise in pairs** sharing the same orbital block structure (s, p, d, f).

This regularity has been called by different names in different traditions: "double periods" (Rydberg, 1913), "left-step periods / Dyades" (Janet, 1928/1929), "bínodos" (Baca Mendoza, 1953), "binodes" (Gutiérrez-Samanez, 2004), "eight-period table" (Katz, 2001), "square cross" (Stowe, 1989; Scerri & Rihani, 2018), "mirror-image two-period cycles" (Vyatkin, 2019), "Dyads" (Muradjan, c. 2019), "binodic helical system" (Kudan, 2022), "bilateral symmetry" (Vernon, 2024), "chemical double orbit" (Ceulemans & Thyssen, 2018), "limit-convergence" (Schwarz et al., 2021), "SO(4,4) projections" (Varlamov, 2025). The sheer multiplicity of independent rediscoveries suggests that period doubling is not a matter of representation or convention, but an objective feature of the algebraic structure underlying chemical periodicity.

The central question this paper addresses is: if fourteen independent researchers from such different traditions have all arrived at the same structural observation, what is the mathematical foundation that

makes it necessary? The answer we propose is that period doubling is the direct consequence of the Lie algebra **so(4,4)** with a quadratic Casimir whose spectrum generates multiplicities of exactly $4N^2$ per binode — a result first identified analytically by the author in 2004 and published in peer-reviewed form in 2020 (Gutiérrez-Samanez, *Foundations of Chemistry*, DOI: 10.1007/s10698-020-09359-3). The formalisation via so(4,4) was developed between 2020 and 2026.

2. THE HISTORICAL RECORD: FOURTEEN INDEPENDENT CONVERGENCES

2.1 Rydberg (1913): the empirical formula $4N^2$

Johannes Rydberg was the first to note in 1913 that the lengths of the double periods in the periodic table follow the sequence 4, 16, 36, 64 — exactly the values of $4N^2$ for N = 1, 2, 3, 4. Rydberg arrived at this observation purely empirically, without any group-theoretic or quantum-mechanical framework. This is the earliest documented recognition of the formula that the BPS subsequently derives from first principles.

2.2 Janet (1928/1929): the left-step table or LSPT, the term Dyades, and implicit period doubling

Charles Janet proposed in 1928 the "left-step" periodic table or LSPT, which organises the periods in a form that makes the pairing of periods visually explicit. In 1929, Janet also introduced the term **"Dyades"** to designate the paired periods sharing the same orbital block structure — the **earliest documented use of this nomenclature**, ninety years prior to Muradjan's 2019 proposal. The credit for the designation of paired periods as Dyades belongs to Janet (1929). From the algebraic perspective of the BPS, the first binode reads horizontally: H→He (first semi-binode α) followed by Li→Be (first semi-binode β) — as the binodic function $4N^2$ requires for N = 1. The subsequent binodes read: B-...-Mg and Al-...-Ca; Sc-...-Sr and Y-...-Ba as pairs within each binode.

2.3 Baca Mendoza (1953, 1965): binodes and the 45° oblique line

Dr. Oswaldo Baca Mendoza (1908–1962), first Dean of the Faculty of Chemical Sciences at UNSAAC, Cusco, Peru, introduced in his 1953 work *"Leyes genéticas de los elementos químicos: Nuevo sistema periódico"* the formal concept of **"bínodos"** and demonstrated empirically that each binode has capacity $2N^2 + 2N^2 = 4N^2$. It is important to note that Baca Mendoza's sequence begins with the pair (8, 8) — corresponding to N = 2 — and continues (18, 18, 32, 32...), without explicitly considering the first binode (2, 2) corresponding to N = 1 (H, He, Li, Be). It was Gutiérrez-Samanez (2004) who completed the sequence from N = 1, establishing the full series 2, 2, 8, 8, 18, 18, 32, 32... In his posthumous 1965 work, presented originally at the VII Latin American Congress of Chemistry (Mexico City, 1959), Baca Mendoza identified that the initial pairs of new orbital transitions align along a single 45° line and predicted geometrically that Z = 173 would be the closure of the fifth binode — one of the most notable

quantitative convergences in the history of the periodic table, anticipating by 52 years the Dirac-Fock calculations of Pyykkö (2011) for finite nuclei (Z ≈ 172). Maar & Maar (2019) document this tradition in an international journal of the history of chemistry: *"Julio António Gutierrez (b. 1955) continued Mendoza's work... on the 'quantification' of Mendeleev's table."*

2.4 Gutiérrez-Samanez (2004, 2020): binodes, N, τ, and so(4,4)

An epistemological turning point in the research was the discovery, in 2004, of the coherence between two orders of description previously kept separate. On one hand, the **extended form of the four binodes** — the visual topology of the periodic table when periods are read horizontally in pairs (2,2), (8,8), (18,18), (32,32) — constituted a **formal or topological** fact. On the other hand, the parabolic function **$Y = 4N^2$** constituted an **analytical or geometric** description. The discovery was that both coincided exactly and without exception: the formal (topological) was found to be described by the analytical (geometric). This coherence — that the topology of the period pairs is exactly the parabolic function $4N^2$ — was the step that no researcher had taken until that moment, and it constitutes the original core of the Binodic Periodic System.

Building on Baca Mendoza's legacy, Gutiérrez-Samanez identified analytically in 2004 that the periods group into pairs of the form ($2N^2$, $2N^2$), and that the sum $4N^2$ quantifies the exact element capacity of each pair without exception. This analysis led to two original quantum numbers: the **binode quantum number N**, a collective eigenvalue that distinguishes period pairs, and the **period spin $\tau = \pm½$**. The cumulative sum gives the closure function $Z(N) = 2N(N + 1)(2N + 1)/3$, reproducing exactly Z = 4 (Be), Z = 20 (Ca), Z = 56 (Ba), Z = 120 (E120). These results — $Y = 4N^2$, Z(N), N and τ — were established analytically in 2004 and published in peer-reviewed form in 2020 (*Foundations of Chemistry*, vol. 22(2), pp. 235–266). The formalisation of N and τ as exact eigenvalues of the quadratic Casimir operator of the Lie algebra so(4,4) — whose spectrum $N(N + 2)$ generates multiplicities of $4N^2$ per binode — was developed in subsequent work between 2020 and 2026, posterior to the FoC article. A crucial result of this later work: what the standard interpretation calls "anomalies" of electronic configuration are not anomalies without cause or pattern, but necessary consequences of a relativistic correction that grows with Z — a sub-function anticipated geometrically by Baca Mendoza in 1959.

The public trajectory of the BPS documents its presence in the most relevant international forums since 2011. Between 31 July and 5 August 2011, the author presented paper **1191: "Binodic Periodic Table, Mathematical Function"** at the **43rd IUPAC World Congress of Chemistry** (San Juan, Puerto Rico). Immediately after, between 9 and 11 August 2011, he presented **"Periodic Table, Mathematical Function and Bridge with Quantum Mechanics"** at the **Summer Symposium of the International Society for the Philosophy of Chemistry (ISPC)** (Universidad de Los Andes, Bogotá, Colombia). This symposium brought together the most prominent figures in the philosophy of chemistry: **Rom Harré** (Oxford), **Eric Scerri** (UCLA), **Guillermo Restrepo**, **W.H. Eugen Schwarz**, **Joachim Schummer** (editor of *HYLE*) and **Pieter Thyssen** — the same researchers who appear in convergences § 2.13b and § 2.14 of this paper.

Before all of them, the author presented the binodic function $Y = 4N^2$, the closure function $Z(N)$ and the bridge with quantum mechanics.

The following year, the author organised the **Third International Conference on the Periodic Table** (Cusco, Peru, 2012), named in honour of Dr. Oswaldo Baca Mendoza. This conference gave direct rise to the volume *Mendeleev to Oganesson* (Scerri & Restrepo, eds., OUP, 2018) — which contains the chapter by Ceulemans and Thyssen on the chemical mechanics of Wheeler (§ 2.13 of this paper), and which in its introduction acknowledges: *"The organizer, Julio Gutierrez Samanez, has had a lifelong interest in the periodic table, especially its mathematical aspects."* Between 2011 and 2020, the author continued developing and publishing the BPS in Meta-Synthesis, Google Scholar and Academia.edu, until in 2020 *Foundations of Chemistry* published the peer-reviewed article. The recognition by Scerri in Editorial 65 of FoC (2020, vol. 22, pp. 135–136) is explicit: *"His article in this issue provides a mathematical analysis of the periodic table that highlights the doubling of period lengths, which the author attributes to a 'binodic function'."* The priority of the BPS in the analytical identification of $Y = 4N^2$, $Z(N)$, N and τ is documented in the records of the IUPAC (paper 1191, 2011), the ISPC (Bogotá, 2011) and *Foundations of Chemistry* (2020).

The extension of the BPS to chemical properties was developed by Japanese researcher Nawa Nagayasu, who — thanks to the international dissemination of the author's work by Dmitry Weise (Russia) through the PT-L discussion list — mapped electronegativity (Pauling scale) onto the **Julio's Binodic Spiral**, producing "Julio's environment map of electronegativity" and "Julio's diagram of electronegativity", catalogued at Meta-Synthesis (PT_id = 1241, 2021, together with contributions by René Vernon). Mark Leach describes explicitly: *"A discussion on the Google Groups Periodic Table Discussion List, involving René Vernon, Nawa Nagayasu & Julio Samanez (all contributors this database) lead to the development of the representations below."* Nagayasu's maps constitute the empirical demonstration of the chain $N \rightarrow Z \rightarrow$ chemical properties that the BPS proposes: electronegativity of the 118 elements distributes continuously and predictably over the binodic geometry — precisely what the Schwarz et al. paradigm seeks without being able to obtain from the vertical reading of the LSPT.

2.5 Stowe (1989) and Scerri & Rihani (2011, 2018): the square cross

Timothy Stowe published in 1989 "A Physicist's Periodic Table" (PT_id = 38, Meta-Synthesis), organised in a square cross arrangement — structurally analogous to the Andean Chakana — with the sequence n = 1: 2 elements, n = 2: 8, n = 3: 18, n = 4: 32, n = 5: 32, n = 6: 18, n = 7: 8, n = 8: 2, i.e. 2, 8, 18, 32, 32, 18, 8, 2. This visual form of symmetric pairs of square crosses was published by Gutiérrez-Samanez in his 2004 book as Binodic Chakanas, inspired by the Chakana function of W. Núñez del Prado presented in that work. Eric Scerri and Jeries A. Rihani produced the Stowe-Janet-Scerri table (PT_id = 414, 2011) and its extended version (PT_id = 779, 2018). Both exhibit the paired period structure visually but do not provide the generative function $Y = 4N^2$.

2.6 Katz (2001): the eight-period table

Gary Katz published in 2001 "The Periodic Table: An Eight Period Table For The 21st Century" (*The Chemical Educator*, 6(6), 324–332), correctly identifying the eight-period structure corresponding to four binodes, but without identifying the deep cause — the azimuthal emergence rule — or providing the generative function $Y = 4N^2$.

2.7 Muradjan (c. 2015): the double numerical structure

A.Z. Muradjan published "Mathematical structure for the Periodic System Table" (Global Science Journals), proposing a double numerical structure based on the formula $N = 2n^2$. His work recognises the mathematical formula $2n^2$, but does not sum it to produce the binodic closure function, nor does it introduce formal quantum numbers for the paired periods.

2.8 Vyatkin (2019): the spin-orbital exclusion principle

Viktor Vyatkin (Kuban State Agrarian University, Russia) demonstrated in "Spin-orbital exclusion principle and the periodic system" (*Pure and Applied Chemistry*, DOI: 10.1515/pac-2019-0702), presented at the 4th International Conference — Mendeleev 150, ITMO, St. Petersburg, July 2019, that the real sequence of new sublevels develops in cycles of two mirror-image periods — exactly the BPS binodic structure. However, Vyatkin is not aware of the azimuthal emergence rule.

2.9 The Dyads proposal of Muradjan (c. 2019): a new quantum number for paired periods

Aco Muradjan (Macedonia) published approximately in 2019, in the context of the International Year of the Periodic Table, a research paper in which he proposes a new quantum number *nd* for what he calls "Dyads", recovering the term coined by Charles Janet in 1929. He proposes a modified quantum number set {nd, n, l, ml, ms} where (l) ≤ nd−1, and notes that the two periods within each dyad carry a positive/negative or left/right distinction — equivalent to $\tau = \pm½$. The structural convergence with the BPS is notable: *nd* corresponds exactly to N; the left/right distinction corresponds to $\tau = \pm½$; and the redefinition of (l) relative to nd is identical to the BPS constraint $l \leq N-1$. The fundamental difference is that Muradjan's proposal arrives phenomenologically, without the generative function $Y = 4N^2$ or a formal algebraic derivation.

2.10 Vernon (2024): bilateral symmetry of the BPS

René Vernon (University of Canberra, Australia) developed in 2024 "Bilateral Symmetry in the Periodic Binodic Table" (PT_id = 1303, Meta-Synthesis), explicitly building on Gutiérrez-Samanez (2020). Vernon adapts $4N^2$ to reveal the bilateral symmetry in a "pancake" format making $\tau = \pm½$ geometrically visible. However, Vernon does not adopt the azimuthal emergence rule and completes the first period of the first binode with subatomic particles (electron, neutron; H, He), instead of the chemical elements H→He and Li→Be. Vernon also documents that Rydberg (1913) already used $4N^2$ for double period lengths.

2.11 Varlamov (2025): group algebra SO(4,4)

V.V. Varlamov (Siberian State Industrial University, Russia), in a series culminating in arXiv:2501.18272 (2025), independently arrives at the Lie algebra SO(4,4) as the appropriate symmetry group for the periodic table. Varlamov constructs from the group algebra of so(4,2) and extends it to so(4,4), producing two three-dimensional projections of the weight diagram corresponding exactly to the two semi-binodes α and β of the BPS. The equivalence between the BPS notation (N, τ, l, pos) and Varlamov's Cartan labels constitutes the most rigorous independent algebraic validation of the binodic structure. Regarding priority: N and τ appear in Gutiérrez-Samanez (2020) with analytical foundations dating to 2004; the formalisation of N and τ as eigenvalues of so(4,4) was developed by Gutiérrez-Samanez between 2020 and 2026, while Varlamov's independent extension to so(4,4) dates to 2025.

2.12 Kudan (2022): the phenomenological path from mass spectrometry

Pavel V. Kudan (Russia), specialist in mass spectrometry, published in 2022 "Kudan's Periodic System" (PT_id = 1285, Meta-Synthesis), a helical variant with structural features convergent with the BPS: (1) four periods of elements analogous to the four binodes N = 1,2,3,4; (2) two semi-periods of opposite polarity (−/+) analogous to τ = −½/+½; (3) small energy barriers between blocks S-F, F-D, D-P; and (4) large energy barriers between P and S corresponding to binode closures. According to personal communication with the author, Kudan was inspired by the "Genome of Matter" (PT_id = 1221, Meta-Synthesis, 2016) of Gutiérrez-Samanez. The convergence is here partially derived: Kudan starts from a representation of the author and arrives at the same structural conclusions phenomenologically. Kudan has multiple entries in Leach's database: PT_id = 1230, 1234, 1285, and 1298.

2.13 Scerri (1991–2021) and Restrepo (2007–2019): those who came closest without crossing the threshold

Eric Scerri's 1991 foundational article (*British Journal for the Philosophy of Science*, 42, 309–325) establishes that quantum mechanics cannot derive the electronic configuration of many-electron atoms without approximations that are not rigorous derivations — opening exactly the epistemological space that the BPS occupies. In his 2007 book (*The Periodic Table: Its Story and Its Significance*, OUP), Scerri defends the LSPT as the most fundamental form of the table, noting that *"although many attempts have been made to explain what is sometimes referred to as the doubling of period lengths (except for the first period), none of them have been successful"* — the explicit recognition of the Löwdin Challenge in its most direct form.

However, in the 2019 second edition, Scerri abandons the LSPT because placing He above Be destroys the atomic number triad He(2)-Ne(10)-Ar(18). This shift reveals the geometric obstacle: Scerri cannot sustain the LSPT because his reading is **vertical** — He above Be — and that reading is chemically indefensible. The BPS shows that the correct reading is **horizontal**: H→He as first semi-binode α and Li→Be as first semi-binode β, without He ever being "above" Be. In 2021, Scerri confirms the problem remains open: *"Although many attempts have been made to explain what is sometimes referred to as the doubling of period lengths (except for the first period), none of them have been successful."* The Editorial

65 of *Foundations of Chemistry* (2020) describes the author's article precisely as providing *"a mathematical analysis of the periodic table that highlights the doubling of period lengths, which the author attributes to a 'binodic function'"* — the answer to the problem Scerri has recognised as pending since 2007.

Guillermo Restrepo has developed the most rigorous programme of mathematisation of chemistry in contemporary philosophy of chemistry. In his foundational article (Restrepo & Pachón, 2007, *Foundations of Chemistry*, 9(2), 189–214), he concludes that a formal mathematical structure for the periodic law is possible without reducing the concept of chemical element to the quantum atomic formalism. In his 2019 synthesis (*Chemistry — A European Journal*, 25, 15430–15440), he proposes that periodic systems possess the mathematical structure of **ordered hypergraphs**. The fundamental difference from the BPS is one of algebraic depth: Restrepo proposes a superstructure that describes the observed periodic system, but does not provide the **generative mathematical function** that explains why the periods have exactly the lengths they have, why they double, and why closures occur at Z = 4, 20, 56, 120. Restrepo's superstructure is the phenomenological image of what the BPS derives algebraically from so(4,4).

2.14 Ceulemans and Thyssen (2018): chemical mechanics and the Bertrand paradox

Arnout Ceulemans (KU Leuven, Chemistry) and Pieter Thyssen (KU Leuven, Philosophy) published in 2018 "The Chemical Mechanics of the Periodic Table" in *Mendeleev to Oganesson* (Scerri & Restrepo, eds., OUP, pp. 104–121). The work begins from Wheeler's (1971) vision of a "chemical mechanics" and articulates the **"Löwdin Challenge"**: the derivation of the Madelung rule from first principles, "the oldest and largest standing problem in quantum chemistry". They defend the third interpretation of the Madelung rule as the only universally valid one — as predictor of the onset of blocks — and document the sequence 2 – 2–8 – 8–18 – 18–32 – 32. They describe Wheeler's **"chemical double orbit"**: a force law where the angular frequency is double the radial frequency, so the classical particle visits the point of closest approach twice per revolution. Their conclusion is that the only escape from the Bertrand paradox is to adopt the perspective of **Lie algebras**.

The convergence with the BPS is profound but reveals a decisive conceptual difference. Ceulemans and Thyssen read the doubling **vertically** — one period above the other in the LSPT — whereas the BPS reads it **horizontally** — semi-binode α followed by β in continuous linear sequence. This difference prevents them from reaching $Y = 4N^2$ and from formalising Wheeler's chemical double orbit via a generative function. The BPS provides exactly what they seek and do not find: the mathematical function that makes deductive the chemical mechanics that Wheeler envisioned in 1971.

There is an additional element of intellectual importance that must be recorded. The conclusion of Ceulemans and Thyssen (2018) — that the only escape from the Bertrand paradox and the Löwdin Challenge was to rethink the premises from the perspective of **Lie algebras** — was precisely the stimulus that led the author to search for the specific Lie algebra that chemical periodicity required. Starting from the quantum numbers N and τ already identified analytically in 2004 and consolidated in the FoC article

(2020), the author initiated between 2020 and 2026 the search for the Lie algebra whose quadratic Casimir generated exactly the observed multiplicities $4N^2$, finding that algebra to be so(4,4) with Casimir spectrum $N(N+2)$. The convergence between the Ceulemans & Thyssen chapter and the BPS is therefore not only an independent coincidence: it is also a **causal chain**. They pointed the way toward the Lie algebra; the author followed it to the end.

2.15 Cao, Vernon, Schwarz and Li (2021): the limit-convergence — the geometric-conceptual obstacle

Cao, Vernon, Schwarz and Li published in 2021 "Understanding Periodic and Non-periodic Chemistry in Periodic Tables" (*Frontiers in Chemistry*, DOI: 10.3389/fchem.2020.00813). It constitutes the closest convergence to the algebraic foundation of the BPS that any contemporary empirical-theoretical work has achieved, and simultaneously illustrates with clinical precision why that foundation cannot be reached from the standard chemical perspective. It is, in that sense, a **limit-convergence**: the work that comes closest without crossing the threshold.

The point of maximum proximity: Schwarz explains that "due to the quantum constraint $\ell \leq n - 1$, new electronic orbitals in screened atomic potentials appear in double steps. Therefore, there are two periods each of length 8 for 2sp and 3sp, of length 18 for 3d4sp and 4d5sp, and of length 32 for 4f5d6sp and 5f6d7sp." This is exactly the **azimuthal emergence rule** of the BPS: each new binode arises because a new azimuthal quantum number l appears, under the condition $l \leq N-1$ that so(4,4) formalises. Schwarz describes it with the same words, the same constraint and the same doubling sequence 2, 2, 8, 8, 18, 18, 32, 32 — but does not take the next step: no collective quantum number N, no function $Y = 4N^2$, no closure function Z(N). The Löwdin Challenge remains unanswered.

The deep reason for this limit is **geometric-conceptual**, not algebraic or physical. Schwarz and co-authors read the period doubling **vertically** — one period above the other in the LSPT — and that reading presents the first binode as "He above Be", which seems chemically and physically unacceptable. The authors state explicitly: "apparently there is no physico-chemical reason for there to be two ns periods before the first $(sp)^8$ period, except the desire for more symmetry and beauty in the generated periodic table." What the BPS does — and what Schwarz and co-authors cannot see — is to read the same first binode **horizontally**: H→He (first semi-binode α, $\tau = +½$) followed by Li→Be (first semi-binode β, $\tau = -½$).

Without that perspective, the sum $Y = 4N^2$ does not emerge, and without $Y = 4N^2$ there is no closure function, no binode quantum number N, no period spin τ, no algebra so(4,4) — **and no possibility of relating N, the new binode quantum number, to the atomic number Z, and therefore relating Z to the chemical properties of the elements**. That chain — properties → Z → N → so(4,4) — is the law of mathematical functions of functions that the vertical paradigm of Schwarz cannot construct.

There is in this work an element that deserves special attention: **René Vernon is a co-author**. The same Vernon who in 2024 published the bilateral symmetry table explicitly based on the BPS. Vernon's trajectory between 2021 and 2024 — from co-author of the Schwarz empirical article to independent

validator of the BPS — is the most eloquent demonstration that there is something in the BPS that the Schwarz paradigm cannot provide: the generative mathematical function that converts the observation of doubling into a deductive law.

The visual demonstration of what the Schwarz paradigm cannot reach was provided by Gutiérrez-Samanez himself in "Periodicidad Armónica de las Propiedades Químicas de los Elementos" (Academia.edu, 2025), where chemical properties — encoded according to Henry Bent's (2006) colour scheme — are represented as genetic spirals on a paraboloid of revolution. The electronegativity maps of Nagayasu on the Binodic Spiral (PT_id = 1241, 2021) demonstrate the same for electronegativity of the 118 elements. Schwarz is one of the most rigorous theorists of the periodic table. His epistemological position is explicit: the periodic law is "a contingent rule valid in chemistry under human conditions" and "its epistemological status is not comparable" to the laws of physics. The BPS proposes exactly the opposite.

3. THE CONVERGENCE TABLE AND WHAT IT REVEALS

The fourteen convergences documented above can be summarised in a comparative table:

Table 1
Fourteen independent convergences on period doubling in the periodic table

Author / Work	Year	Name for pair	New QN	Math. function	Foundation
Rydberg	1913	Double periods	None	$4N^2$ (empirical)	Spectroscopic observation
Janet	1928/1929	Left-step + Dyades (coined)	None	None	Electron configuration + first use of Dyades
Baca Mendoza	1953/1965	Binodes	N (descriptive)	$2N^2$, $2N^2$	Chemical observation + 45° line
Gutiérrez-Samanez	2004/2020	Binodes	N, $\tau=\pm½$	$Y=4N^2$, $Z(N)$ (2004/2020); so(4,4) (2020-26)	Analytical + Lie algebra so(4,4)
Stowe / Scerri & Rihani	1989/2018	Square cross (visual)	None	None	Phenomenological / visual
Katz	2001	Eight-period table	None	None	Pedagogical / empirical
Muradjan	~ 2015	Double numerical structure	None	$2n^2$ (partial)	Mathematical observation
Vyatkin	2019	Mirror-image cycles	None	None	Spin-orbital exclusion principle
Muradjan	~ 2019	Dyads (term: Janet 1929)	nd	None	Reflection symmetry (empirical)
Kudan	2022	Binodic helical system	None	None	Mass spectrometry; from Genome of Matter (2016)
Vernon	2024	Bilateral symmetry	None	$4N^2$ (cited)	Visual / historical
Ceulemans & Thyssen	2018	Chemical double orbit	$N = n + l$ (distinct)	No generative fn.	Classical mechanics + Lie; Löwdin Challenge
Cao, Vernon, Schwarz & Li	2021	Limit-convergence	$N = n + l$ (Madelung)	None	Empirical chemistry; $\ell \leq n-1$ unformalized; vertical reading
Varlamov	2025	SO(4,4) projections	4th Cartan gen.	Weight diagram	Group algebra so(4,4)

First: the structural observation has been reached from at least seven different epistemological starting points: spectroscopic empiricism (Rydberg), electron configuration (Janet, Katz, Scerri), visual symmetry (Stowe, Scerri & Rihani, Vernon), chemical observation (Baca Mendoza), spin-orbital physics (Vyatkin), group algebra (Varlamov), classical mechanics and philosophy (Ceulemans & Thyssen), phenomenological symmetry (Muradjan), mass spectrometry (Kudan), and mathematical chemistry (Restrepo).

Second: in all cases except the BPS and Varlamov (and partially Rydberg), the observation remains descriptive: the paired structure is noted but not derived from a generative principle.

Third: only the BPS provides the complete chain: empirical observation (2004) → analytical function $Y = 4N^2$ → closure function Z(N) → algebraic derivation via so(4,4) (2020–2026) → original quantum numbers N and τ.

4. THE DYADS PROPOSAL IN DETAIL: CONVERGENCES AND THE MISSING FOUNDATION

Muradjan's Dyads proposal deserves special attention as the most structurally complete of the phenomenological convergences.

4.1 What the Dyads proposal achieves

Muradjan's proposal correctly identifies that the standard quantum number set {n, l, ml, ms} is insufficient, introduces *nd* governing the pairing of periods with $l \leq nd-1$, notes the positive/negative distinction between the two periods within each dyad equivalent to $\tau = \pm½$, cites the same reference community as the BPS (Katz, Bent, Scerri, Stewart, Meta-Synthesis), and extends the framework to $Z > 120$.

4.2 What the Dyads proposal does not achieve

Muradjan's proposal does not provide the generative function $Y = 4N^2$. Without it, the number of dyads is an empirical observation rather than a mathematical necessity. It cannot predict closure atomic numbers algebraically, derive future period capacities, or account for configuration anomalies through a unified mechanism. Most fundamentally, *nd* lacks the algebraic grounding that N possesses: in the BPS, N is an eigenvalue of the quadratic Casimir $C_2[so(4,4)]$ with spectrum $N(N + 2)$.

5. THE AZIMUTHAL EMERGENCE RULE: THE DEEP CAUSE OF PERIOD DOUBLING

All convergences documented in § 2 arrive at the observation of period doubling. This is precisely the question that the Löwdin Challenge (1969) posed and that Ceulemans and Thyssen (2018) recognise without resolving — and whose conclusion pointing to Lie algebras led the author to find so(4,4). The

BPS answers it through the **azimuthal emergence rule**: each new binode arises precisely because a new azimuthal quantum number l appears for the first time.

Binode N = 1 contains only l = 0 (s-block): capacity $4 \cdot 1^2 = 4$. Binode N = 2 introduces l = 1 (p-block): capacity $4 \cdot 2^2 = 16$. Binode N = 3 introduces l = 2 (d-block): capacity $4 \cdot 3^2 = 36$. Binode N = 4 introduces l = 3 (f-block): capacity $4 \cdot 4^2 = 64$. Binode N = 5 would introduce l = 4 (g-block): capacity $4 \cdot 5^2 = 100$. This rule, formalised by so(4,4) through $l \leq N-1$, explains simultaneously: (1) why there are exactly four binodes for Z = 1–120; (2) why each binode has exactly twice the capacity of its highest new orbital block; (3) why Muradjan correctly identifies four dyads; (4) why Katz correctly identifies the eight-period structure; and (5) why Varlamov's weight diagram produces exactly the same block structure as the BPS.

The azimuthal emergence rule also resolves the debate about the first period. The BPS places H and He in the first semi-binode α and Li, Be in the first semi-binode β of binode N = 1 — recovering Janet's binodic arrangement. The rule is unambiguous: binode N = 1 has capacity $4 \cdot 1^2 = 4$, distributed as: first semi-binode α: H (Z = 1) → He (Z = 2) horizontally; second semi-binode β: Li (Z = 3) → Be (Z = 4) horizontally.

6. THE CONVERGENCE WITH VARLAMOV: TWO ROADS TO SO(4,4)

The convergence between the BPS and Varlamov (2025) is algebraic, not merely structural. Both arrive at the same rank-4 Lie algebra so(4,4). The BPS constructs from physico-chemical observation — the pattern $(2N^2, 2N^2)$ identified analytically in 2004 — and derives the algebraic structure. Varlamov constructs from group theory and derives the periodic organisation. The equivalence is complete: binode N ↔ eigenvalue v; period spin τ ↔ fourth Cartan generator; semi-binode α ↔ first weight-diagram projection (H to Mc); semi-binode β ↔ second projection (He to Og).

The independent convergence between the Peruvian physico-geometric path and the Siberian algebraic path — separated by more than twenty years and derived from completely different starting points — is the most powerful evidence that so(4,4) is the natural symmetry algebra that the table requires.

7. EPISTEMOLOGICAL IMPLICATIONS: TOWARD A MATHEMATICAL LAW OF CHEMICAL PERIODICITY

The fourteen independent convergences have a clear epistemological implication: period doubling is not an empirical regularity without deeper foundation, but the observable consequence of the representation theory of the Lie algebra so(4,4). Mendeleev's periodic law does not explain why the periods have the lengths they have, why they double, or why closures occur at specific atomic numbers. The BPS reformulates it as a mathematical theorem: the element capacities and closure atomic numbers are exact eigenvalue consequences of so(4,4) with quantum numbers N and τ. Under this reformulation, the periodic law is a **"mathematised periodic law"** — a deductive consequence of algebraic first principles from which all 120 atomic numbers are derived without exception.

The fourteen independent convergences provide the empirical evidence for this stronger claim: when fourteen independent researchers from different traditions, countries and centuries all arrive at the same structural observation, the probability that this observation reflects an objective algebraic property is overwhelming.

8. CONCLUSIONS

First. Period doubling has been independently identified by at least fourteen groups of researchers from 1913 to 2025, working from spectroscopic, electron-configurational, visual-geometric, spin-orbital, group-theoretic, phenomenological, classical-mechanical, mass-spectrometric and empirical-chemical starting points.

Second. This multi-author convergence constitutes decisive evidence that period doubling is an objective algebraic property of chemical periodicity, not a representational convention.

Third. The term *Dyades* for the paired periods was coined by Charles Janet in 1929 — not by Muradjan's 2019 proposal. Muradjan's proposal independently introduces a quantum number *nd* formally analogous to N, but without $Y = 4N^2$ or formal algebraic derivation.

Fourth. The azimuthal emergence rule — formalised by $l \leq N-1$ in so(4,4) — is the deep causal explanation of period doubling. It explains simultaneously the element capacities, the closure atomic numbers, the eight-period structure, and the placement of H, He, Li and Be in the first binode.

Fifth. The convergence between the BPS (2004/2020) and Varlamov (2025) is algebraic. Two independent researchers arrive at the same rank-4 Lie algebra so(4,4).

Sixth. The fourteen independent convergences support the reformulation of Mendeleev's periodic law as a mathematised periodic law: element capacities and closure atomic numbers are exact eigenvalue consequences of so(4,4) with N and τ.

Seventh. The mathematisation of the periodic law is a consequence of the discovery of the eigenvalues N and τ — the "key pieces of knowledge" Einstein indicated were needed to demonstrate that Copenhagen indeterminism was epistemological and not ontological: periodicity is not indeterministic; the available conceptual framework was incomplete.

9. THE EPISTEMOLOGICAL CHALLENGE: THE FOURTEEN CONVERGENCES AND THE COPENHAGEN INTERPRETATION

The Copenhagen Interpretation implies that the distribution of elements in periods of observed lengths is irreducibly contingent: there is no mathematical function that makes them necessary from first principles. The fourteen convergences constitute accumulated empirical evidence against this position. If the distribution of elements were genuinely contingent, it would be statistically incomprehensible that

fourteen independent researchers from different centuries and traditions should all arrive at the same structural pattern: periods double in exact pairs following the law $4N^2$. Rydberg (1913) from spectroscopy; Janet (1928/1929) from electron configuration; Baca Mendoza (1953) from chemical observation; Katz (2001) from pedagogy; Stowe-Janet-Scerri (1989/2018) from visual representation; Vyatkin (2019) from the spin-orbital exclusion principle; Muradjan (c. 2019) from reflection symmetry; Vernon (2024) from the history of the formula; Varlamov (2025) from group algebra; Kudan (2022) from mass spectrometry; Ceulemans & Thyssen (2018) from classical mechanics — with Wheeler (1971) as a visionary precursor; and Schwarz et al. (2021) as limit-convergence, those who came closest without crossing the threshold due to their vertical reading of the LSPT. The international validation network — Weise (Russia, dissemination), Nagayasu (Japan, electronegativity on the Binodic Spiral), Vernon (Australia, bilateral symmetry) — documents that this algebraic truth is being recognised from three continents independently.

The BPS offers the explanation: the law $4N^2$ is the algebraically necessary consequence of so(4,4), whose quadratic Casimir eigenvalues generate exactly the observed multiplicities. The two new quantum numbers — N and $\tau = \pm½$ — are precisely the additional variables whose absence made periodicity appear contingent. In Einstein's terms, these are exactly the "key pieces of knowledge" whose absence motivated his objection to the Copenhagen Interpretation. Their discovery demonstrates that Copenhagen indeterminism was not ontological but epistemological: **the nature of periodicity was not indeterministic, but the available conceptual framework was incomplete**.

This result converges with the realist tradition in quantum mechanics (Olavo 1995, Souček, Helland, Unnikrishnan). In the domain of chemical periodicity, the BPS provides that more deterministic and complete structure: so(4,4) with N and τ as foundations. The "mathematised periodic law" — $Z(N, \tau, l, pos)$ reproducing exactly the 120 atomic numbers — is not an approximation nor an empirical rule: it is an algebraic theorem, and a falsifiable one. If any closure atomic number for $Z \leq 120$ existed that $Z(N) = 2N(N + 1)(2N + 1)/3$ did not reproduce exactly, the claim would be refuted. To date, no such exception exists. The fourteen convergences are the history of how fourteen independent researchers, without knowing it, were progressively circumscribing that same algebraic truth from different angles — until the BPS formalised it completely.

Declarations

Author Contribution

This research has no co-authors; It was conducted solely by the autor

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Data Availability

Todos los datos que respaldan los hallazgos de este estudio están disponibles en el artículo y su información complementaria

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