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Property Rights, Productive Sufficiency, and the Reconstitution of Economic Power. Contractual Substitution, Institutional Capture, and a Conditional Feasibility Frontier

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Research Article

Keywords: property rights, economic power, institutional capture, incomplete contracts, institutional persistence

Posted Date: November 19th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11485169/v1

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Additional Declarations: No competing interests reported.

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Contractual Substitution, Institutional Capture, and a Conditional Feasibility Frontier

Abstract

Rights of control can protect specific investment and finance, yet also transmit economic power across periods. Whether productive functions can be preserved while reducing this transmission depends not only on contractual substitutes but also on what those substitutes recreate and on the capacity of powerful actors to reshape institutional rules. This paper proposes a joint feasibility test that combines the investment and pledgeability functions of rights, the effective control recreated by contractual substitutes, and the endogenous response of agents influencing institutional rules under a shared enforcement budget. A bounded model of investment and three-state wealth–power–rule feedback yields an exact local stability criterion and reduces feasible institutional comparisons to a one-dimensional search. Contractual protections may themselves recreate wealth-linked control, while financing constraints can make the least proprietary arrangement dynamically inferior to another feasible arrangement. Numerical counterexamples show that stable outcomes under fixed rules can become unstable when rule capture is admitted. A separate stock–flow argument establishes why local stability does not imply elimination of inherited rights or prevention of re-entry. These are conditional model results, not empirical estimates or a universal case against ownership.

Keywords: property rights; economic power; institutional capture; incomplete contracts; institutional persistence
JEL classifications: D23; D31; D72; G32; P14.

1. Introduction

The economic functions of property and its role in reproducing authority are neither identical nor automatically separable. A residual right may induce an investor to make an otherwise noncontractible investment, support a loan by increasing pledgeable income, and secure adaptation to unforeseen circumstances. The same right may convert an inherited resource advantage into an advantageous bargaining position and enable its holder to shape the terms of future transactions. These are distinct mechanisms, with distinct conditions of necessity. A policy that reduces control may preserve productive activity if other enforceable instruments replace its incentive function. Yet what happens if the replacement instrument itself creates enduring claims over others, if financing cannot be reproduced without collateral, or if actors who retain power can influence the rule governing future control?

The relevant proposition is therefore more demanding than an assertion that ownership and control are distinguishable. **Under what conditions can productive sufficiency, financing viability and a durable attenuation of wealth-linked power be satisfied by the same institution, when functional substitutes and institutions themselves respond to incentives?** The difficulty is that maintaining investment today, curtailing

tomorrow's transmission of authority, and protecting the rule from being recaptured are different tasks.

Three literatures motivate the question. First, incomplete-contracting theory explains why residual control matters to relationship-specific investment (Grossman and Hart, 1986; Hart and Moore, 1990), and financial contracting explains when control can be allocated contingently (Aghion and Bolton, 1992). The alternative of regulating access to critical resources rather than assigning ownership is explicitly analyzed by Rajan and Zingales (1998); relational commitments and formal versus real authority add further institutional dimensions (Aghion and Tirole, 1997; Baker, Gibbons and Murphy, 2002). This paper does **not** claim priority for separating ownership, access or compensation.

Second, initially unequal resources can affect future production and opportunities through financial constraints (Banerjee and Newman, 1993; Galor and Zeira, 1993; Holmström and Tirole, 1997; Kiyotaki and Moore, 1997). But wealth persistence and power persistence are not synonymous. Wealth can reproduce without conferring control over others; conversely, authority can persist through offices, networks or formal rules when the holder owns little wealth. Third, institutional arrangements can reinforce themselves and be altered by actors' responses to them (Greif and Laitin, 2004). Changes in de jure rights can elicit investment in de facto power, potentially leaving substantive influence intact (Acemoglu and Robinson, 2008). The enterprise of designing rights that are both functionally adequate and resistant to reconstitution must acknowledge these mechanisms rather than treat the new rule as permanently exogenous.

A closely related recent contribution reinforces the need for careful priority claims. Cao, Lagunoff and Xu (2026) analyze how locally dominant groups use legal weaknesses and public assets to shift property claims and reproduce wealth advantages through political control. Their model establishes a substantial connection between property-rights institutions, political asymmetries and wealth dynamics; that connection is therefore not novel here. The present model addresses a different, deliberately narrower problem: the **same arrangement** must pass specific-investment and loan-pledgeability tests, its contractual substitutions must be evaluated for effective reconstitution of control, and its operating-rule feedback must fit a common resource constraint. The institutional-capture coefficient is not a substitute for their richer theory of local politics. Rather, the aim is to specify an implementability test that would be incomplete if the relevant political mechanisms were ignored.

The position of the paper relative to these literatures is consequently asymmetric. Incomplete-contract theory supplies the productive constraint; financial-contracting theory supplies an additional viability constraint; the law-and-organization literature motivates aggregation across legally separate claims; institutional-persistence theory motivates endogenous rule drift. The paper's candidate contribution is the **joint conditional restriction** imposed when they operate together. If a referee shows that this exact joint result is already available, the mathematical exercise would remain valid but its publication claim would have to be narrowed further.

The paper develops one deliberately bounded comparative model. An agent chooses costly investment when offered a mix of wealth-linked residual control and enforceable protection. An additional pledgeability condition protects financing. Contractual protection need not be innocuous: it can produce claims whose aggregate economic effects recreate part of the wealth-linked power ostensibly removed. A third institutional instrument limits the effect of investments in rule capture. The resulting three-state, locally linear model distinguishes persistent relational power, deployable wealth and effective rule drift. An exact loop-gain calculation then leads to a theorem characterizing joint functional and dynamic feasibility subject to scarce enforcement resources.

The contribution claimed is not a new matrix theorem, a general argument against private property, or the first demonstration that political power can reproduce wealth. It is a **single joint implementability test**: the same alternative rights bundle must (i) sustain investment and financing, (ii) avoid rebuilding effective wealth-linked control through its substitute contracts, and (iii) remain locally stable when economically advantaged actors can influence its operating rules, all subject to one real-resource constraint. These are separate restrictions on the same candidate institution, not three interchangeable measures of institutional quality. Two consequences are especially relevant. First, successful substitution on the productive margin need not attenuate effective wealth-linked power if the substitute recreates enduring control. Second, even in a model with monotone primitive effects, the minimum legally proprietary control need not minimize dynamic amplification once binding financing requirements are admitted. A precise counterexample establishes the latter rather than relying on a verbal conjecture. A supplementary stock–flow account clarifies why a locally stable feedback is not equivalent to the disappearance of pre-existing rights or the prevention of newly created ones.

The article is a **conditional theory of feasible institutional comparisons**, not a claim that all property rights are unnecessary.

Read against the closest models, the added comparison is precise. Rajan and Zingales (1998) establish how access can support specific investment without conventional ownership, but do not impose the present three-state rule-drift test on a jointly financeable substitute. Acemoglu and Robinson (2008) explain persistence of elite power through compensating de facto investments, whereas the present design treats the enforcement of a candidate operating arrangement as a scarce, explicitly budgeted input. Cao, Lagunoff and Xu (2026) model politically mediated appropriation with richer local political structure, while the present setting asks whether the *replacement* bundle is jointly admissible before making any claim about its observed distributional effects. These distinctions locate an incremental model contribution, not priority over their underlying mechanisms. Its thresholds are formulated for one investing agent, a reduced-form lender constraint, a locally interior capture decision, and a fixed institutional architecture in which rule *effects* adjust endogenously. The model does not solve the full political equilibrium of constitutional amendment, all parties' incentive problems, or general-equilibrium welfare. Those are identified extensions, not conclusions smuggled into the analysis.

2. Rights, functions and effective reconstitution

2.1. Separating an economic function from the complete legal bundle

A firm's governance arrangement may specify claims to income, authority over use, exclusion, duration, transfer, security interests, and rules for revision. These components can be legally separated, but legal separation does not entail economic separation. For example, a combination of long-term financing covenants, veto rights, an exclusive licence and a renewable management contract may reproduce an incumbent's discretion even if no single agreement contains a conventional shareholding right. Conversely, an asset's productive use can sometimes be maintained by temporary access or bounded contractual protection without granting equivalent control over its future resale or inheritance.

The proper comparison is therefore between **effective bundles of rights**, not just the headline ownership label. Let $I = (q, g, z)$ denote a particular three-instrument representation within a broader institutional space. Here $q \in [0,1]$ denotes the intensity of the designated **wealth-linked** residual decision right; $g \in [0, G]$ is a credible, bounded investment-protecting substitute; and $z \in [0, Z]$, $Z \leq 1$, is a safeguard against translation of current economic advantage into favorable rule drift. The three coordinates are institutional primitives, not indexes of justice. In applications they must be defined through enforceable decisions, eligible participants, and the mechanism assigning rights. The model never equates wealth alone with authority.

To allow effective reconstitution, g may generate an additional wealth-to-power response $b_g g$ even when its legal form is not residual ownership. This is a **stress-test parameter**, not an assumption that all protection is repatrimonializing: $b_g = 0$ covers protections that do not recreate this channel. The distinction is supported by the literature on separation of cash-flow and control rights, which shows that formally fragmented legal claims can still entrench effective control (Bebchuk, Kraakman and Triantis, 2000). In the present model, the institution's net effect must be evaluated after adding all economically effective claims, not by assuming that a non-share contract is non-proprietary in substance.

This interpretation places an important limit on how the paper can be used. Suppose a legal reform substitutes an unconditional, indefinitely renewable covenant for a share-based veto. Both the investor's investment incentive and the incumbent financier's fallback bargaining position may remain unchanged. In that case the reform changes the *documentation* of power, not its effective incidence. Conversely, a bounded access agreement conditional on current investment, with no transferable veto or right to capitalize future appropriations, may protect productive incentives without creating the same enduring power. The sign and magnitude of b_g summarize a locally identified portion of that difference, not a legally self-evident classification.

Functionally equivalent rights also need not have equivalent distributional implications for third parties. A loan covenant that protects a lender may reduce workers' ability to renegotiate conditions, while an enforceable access right could preserve the investor's commitment without shifting the same risk onto others. This is why the analysis restricts attention to *admissibility under explicitly protected functions*. It would be misleading to infer

from an investment floor alone that workers, financiers or consumers are unaffected. The joint frontier can locate candidates for further appraisal, but does not complete that appraisal.

2.2. Which productive functions are protected?

The article uses a **protected functional vector**, but solves only two explicit components: a minimum relationship-specific investment, $\underline{i} > 0$, and a lender's minimum pledgeable amount, $\underline{L} > 0$. These constraints are not sufficient conditions for social efficiency. In a full comparison, counterpart investment, safety, innovation, liquidity, administrative cost, legal credibility and distributional consequences might also be protected. The present restricted set isolates a falsifiable boundary. If an alternative fails one protected constraint, it is ruled out in the relevant class; if it satisfies both, that does not establish Pareto dominance.

The distinction matters especially for institutional proposals involving worker, intermediary or collective holdings.

A protected threshold is a screening condition, not a welfare objective. The focal investor may invest at least $\underline{i}$ even when the contract imposes costly restrictions on co-investors or shifts bargaining losses to employees. Conversely, an alternative may have a lower local wealth–power multiplier while costing more to administer. For this reason the paper asks whether the joint constraint set is nonempty; it does not rank all admissible institutions by surplus, justice or legitimacy. A change in the holder of voting authority is not itself proof of improved productivity. The allocation of financial rights, contributions to loss absorption, who actually exercises control, and whether the guarantees are credible must be specified separately. The governance costs emphasized by Hansmann (1996), and the multiple rules distinguished by Ostrom (2005), are reasons not to assume a zero-cost substitution.

3. Investment, finance and the feasible instruments

3.1. Investment choice

An investor chooses relationship-specific investment $i \geq 0$ after the applicable rights are announced and before the relevant adaptation takes place. Consider the locally quadratic objective

$$U(i; q, g) = (s + \beta q + \gamma g)i - \frac{k}{2}i^2, \qquad s \geq 0, \quad \beta, \gamma, k > 0. \qquad (1)$$

The term s denotes independently appropriable marginal benefits. A unit of q improves the agent's protected share of the return to a noncontractible investment; a unit of g provides some separately enforceable protection. Both are bounded because noncontractibility, enforcement and participation limit what they can accomplish. Strict concavity delivers the unique best response

$$i(q, g) = \frac{s + \beta q + \gamma g}{k}. \qquad (2)$$

The investment condition is thus

$$\beta q + \gamma g \geq D, \qquad D = k\underline{i} - s. \qquad (3)$$

This payoff can be derived locally from a bargaining problem in which the investor's marginal appropriable return depends on relevant residual authority and contractual guarantees. It is not a fully specified Grossman–Hart bargaining equilibrium; the coefficients require independent evidence in applications. If a critical residual decision is noncontractible, γ can be effectively zero, yielding the familiar hold-up boundary. Noncontractible control cannot simply be replicated by declaring a new contract credible.

3.2. Financing and enforcement scarcity

The lender's maximum enforceable repayment capacity is represented by $L_0 + \ell_q q + \ell_g g$, with $\ell_q, \ell_g \geq 0$. A project is financeable in the reduced-form lender test only if

$$\ell_q q + \ell_g g \geq F, \qquad F = \underline{L} - L_0. \qquad (4)$$

When $\ell_q > 0$ and $\ell_g > 0$, these channels provide partial substitutes; otherwise, one may be indispensable. Equation (4) is intentionally an *upper-bound feasibility test* for pledgeability, not a model of competitive credit pricing. It reflects the difficulty that financial contracting and collateral theories assign to enforceability (Holmström and Tirole, 1997; Kiyotaki and Moore, 1997). A financially infeasible alternative is excluded even if its investment incentives appear satisfactory.

Implementing g uses real resources $C_g(g) = \frac{c_g}{2} g^2$, where $c_g > 0$. Protecting the rule from capture costs $C_z(z) = c_z z$, $c_z > 0$. Let $B \geq 0$ be the total institutional-resource ceiling. Thus

$$\frac{c_g}{2} g^2 + c_z z \leq B, \qquad 0 \leq q \leq 1,\ 0 \leq g \leq G,\ 0 \leq z \leq Z \leq 1. \qquad (5)$$

The budget is common across safeguards and substitutes: spending that makes guarantees credible is unavailable for anti-capture enforcement. These are **real costs**; the analysis does not treat compensation payments or transfers as social costs without further specification. We assume the institutional planner can choose the initial rules, while the subsequent *effects* of capture arise from agents' best responses.

Table 1 illustrates the operational interpretation of the three institutional controls. Its entries describe *potential* implementations, not established estimates of their effectiveness.

Table 1. Institutional interpretation of the model's choice variables. The terms are operational analogies; the parameters are not measured in the present paper.

Choice	What the parameter represents	Observable institutional counterpart	Main limitation
q	Wealth-linked residual decision authority	Capital-based voting or residual veto rights	Formal votes may not equal effective authority
g	Credible protection of a specific investment	Enforceable access, performance covenant, contingent creditor protection	Long-lived covenants may reconstruct exclusion or veto power
z	Effective restriction on the conversion of influence into favorable operating rules	Independent review, enforceable conflict-of-interest controls, protected adjudication	Statutory independence need not be effective independence

3.3. The least expensive functional substitute at a given control intensity

Fix any $q \in [0,1]$. Because both protected constraints are nondecreasing in g, the least guarantee meeting them is

$$g_{\min}(q) = \max\left\{0, \frac{D - \beta q}{\gamma}, \frac{F - \ell_q q}{\ell_g}\right\}. \qquad (6)$$

For clarity, the main theorems assume $\ell_g > 0$; when $\ell_g = 0$, equation (4) is imposed separately and its quotient removed. The **functionally admissible control set** is

$$\mathcal{Q} = \left\{q \in [0,1]: g_{\min}(q) \le G, \quad c_g/2\, g_{\min}(q)^2 \le B\right\}. \qquad (7)$$

Since $g_{\min}$ is continuous, $\mathcal{Q}$ is compact and may be empty. The maximum feasible safeguard remaining at a given $q \in \mathcal{Q}$ is

$$z_{\max}(q) = \min\left\{Z, \frac{B - c_g g_{\min}(q)^2/2}{c_z}\right\}. \qquad (8)$$

These simple formulas already reveal a missed issue in a model containing only an investment requirement. When financing binds, reducing q can force a large increase in g. That increase may induce investment well above the specified floor and may itself restore some of the very economic power that the reform sought to attenuate. Minimum q and minimum reproduction need not be synonymous.

4. Endogenous economic and institutional reproduction

4.1. Wealth, power and the rule state

Let p_t, w_t and m_t denote small deviations, respectively, in an agent's effective relational power, deployable wealth and the extent of institutional rule drift favoring that agent, around a regime-specific interior reference point. Power is not identified with ownership, and rule drift is not automatically a formally enacted statute. The institutional state can encompass enforceable exceptions, discretionary permissions, favored access or revised operational rules that influence future bargaining positions. The reference regime $I = (q, g, z)$ is fixed by the comparison; **capture of its effective operation is endogenous**.

Figure 1 depicts the marginal transmission channels. The arrows express derivatives around a specified reference regime; they do not represent an assertion that every wealth holder actually controls the rules. The coefficient $b(q, g)$ includes both the effect of the targeted legal right and any control function reconstructed by g.

Consider the local first-order system

$$\begin{pmatrix} p_{t+1} \\ w_{t+1} \\ m_{t+1} \end{pmatrix} = J(q, g, z) \begin{pmatrix} p_t \\ w_t \\ m_t \end{pmatrix} + \varepsilon_{t+1}.$$

$$J = \begin{pmatrix} a & b(q,g) & h \\ d\big(i(q,g)\big) & c & 0 \\ (1-z)u & (1-z)v & r \end{pmatrix} \qquad (9)$$

with $0 \leq a, c, r < 1, h, u, v \geq 0$ (with $h > 0$ and at least one of u, v positive for an active capture path), and

$$b(q, g) = b_0 + b_q q + b_g g, \qquad d(i) = d_0 + d_i i, \qquad (10)$$

where $b_0, b_g, d_i \geq 0$ and $b_q, d_0 > 0$. The b_0 component captures residual wealth-based bargaining advantages independent of the designated control right. The $b_q q$ term captures the targeted wealth-linked rights. The $b_g g$ term is a potential functional reconstitution of similar advantages via contracts, guarantees or vetoes. The coefficient $d(i)$ measures the conversion of current power into next-period deployable wealth through appropriable results; investment can increase the surplus on which the bargaining outcome operates. The hm_t term represents effective benefits of favorable institutional drift. The variables are locally measured in deviations and may have a nonzero reference level; a stable Jacobian does not imply equal wealth or equal power.

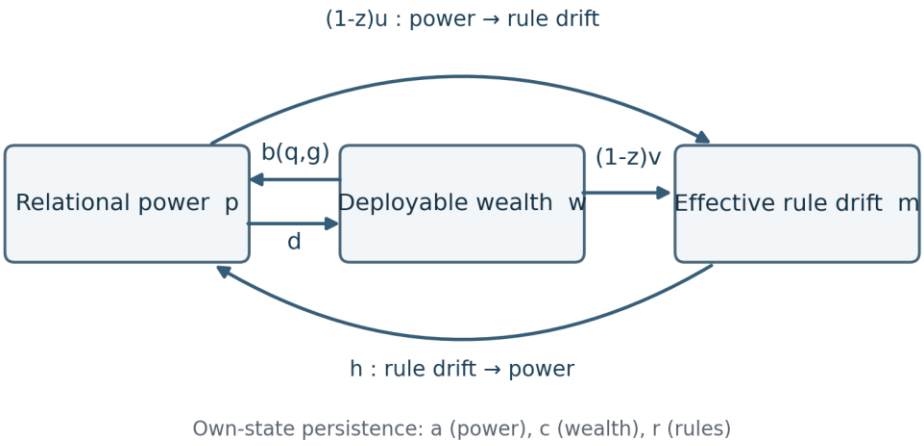


Figure 1. Economic and institutional channels in the three-state Jacobian. The arrows are directional marginal effects; a, c and r are own-state persistence parameters. Source: model equations (9)–(10). Figure generated programmatically in Python using code developed with generative-AI assistance (OpenAI ChatGPT, GPT-6; October 2026).

The institutional mechanism is given a parsimonious behavioral basis. Suppose an actor can expend influence effort $e_t \geq 0$ with payoff, around an interior reference point,

$$V_e(e_t; p_t, w_t) = e_t\big(A_0 + A_p p_t + A_w w_t\big) - \frac{\kappa}{2} e_t^2, \quad \kappa > 0,\ A_0 > 0. \qquad (11)$$

Its best response is $e_t^\star = \big[\big(A_0 + A_p p_t + A_w w_t\big)/\kappa\big]_+$. Interior linearization gives marginal effort $up_t + vw_t$, where $u = A_p/\kappa$, $v = A_w/\kappa$. Let the rule-effect state follow $m_{t+1} = rm_t + (1 - z)(e_t^\star - e^\star)$ around its reference; this yields the last row of (9). The safeguard z reduces the fraction of privately exerted influence translated into rule advantages; it need not prohibit petitioning or participation. We assume m is a supplementary privileged-access margin that does not itself alter the investor's marginal payoff in (1) at first order. If it also changes project control rights, an additional investment-state derivative must be incorporated, and the theorem is not directly transportable.

The influence effort thus responds endogenously to the resources and power generated by the economic system. The exercise is narrower than a full theory of political equilibrium: rule drift is an intermediary with local persistence and bounded responsiveness, not a model of constitutional choice. It nevertheless closes a loop left open in a fixed-rule analysis, consistent with the general concerns identified by Greif and Laitin (2004) and Acemoglu and Robinson (2008), without claiming their mechanism as a discovery.

It is useful to separate three different interpretations of the institutional instrument z. First, a legal rule can assign authority to review contested decisions. Second, the reviewing body can have an independent budget and information sufficient to exercise that authority. Third, the actor whose decision is reviewed may find another channel of influence, rendering the formal safeguard ineffective. Only the **second, actually effective margin** belongs in the numerical value of z. Assigning a high z on the basis of a statutory title alone

would assume the outcome being investigated. The parameter r likewise concerns persistence of realized privileged access, not the duration of a legal document.

The model maintains a sharp difference between influence efforts *exerted* and influence efforts *converted into control*. Even when the optimal private lobbying effort $e_t^\star$ rises with p_t and w_t, an institution with credible review may attenuate the marginal translation into m_{t+1}. This is an accounting of a behavioral mechanism, not a normative argument against legitimate participation by economic actors. Alternative arrangements might redistribute lawful access to rule-making rather than simply reduce total participation; in such applications the scalar reduced form would require replacement by an explicit multi-actor governance game.

4.2. An exact local reproduction test with institutional capture

For the nonnegative Jacobian (9), define $K = (1-a)(1-c) > 0$. The **economic feedback ratio** is

$$R_E(q,g) = \frac{b(q,g)d\big(i(q,g)\big)}{K}. \qquad (12)$$

The additional **institutional feedback ratio** is

$$R_I(q,g,z) = \frac{h(1-z)\big[(1-c)u + v\,d\big(i(q,g)\big)\big]}{(1-r)K}. \qquad (13)$$

Their sum

$$\boxed{R(q,g,z) = R_E(q,g) + R_I(q,g,z)} \qquad (14)$$

measures the combined local feedback. It includes two distinct capture routes: $p \to m \to p$, and $p \to w \to m \to p$. The denominators adjust for persistence in wealth, power and the rule state. These expressions remain invariant under independent positive changes of units of p, w, m, since each complete loop is scale-free.

Proposition 1 (three-state local stability). For the deterministic, autonomous local system with innovations set to zero, under (9)–(10) the reference state is locally asymptotically stable when $R(q,g,z) < 1$ and has an unstable linear mode when $R(q,g,z) > 1$. For the specified nonnegative Jacobian, $\rho(J) < 1$ if and only if $R < 1$. At $R = 1$, the linearized system has an eigenvalue equal to one; stability of any nonlinear system at the boundary requires separate analysis. When exogenous shocks continue, the same eigenvalue condition describes decay of transitory deviations, not disappearance of responses to new shocks.

A proof, based on a nonnegative-matrix Schur complement, is provided in Appendix A. The spectral test itself is standard; separating the economic and rule-drift routes gives the relevant institutional interpretation. The condition has an important negative implication: $R_E < 1$ **is not sufficient for stability once influence on the operating rules is admitted**. A reform can pass a fixed-rule test and fail when power-holder responses are included.

Economically, R_E traces the conversion of power into investable resources and back into power. The additional R_I captures two routes through institutions: a direct response of influence effort to power, and an indirect response operating through the wealth that past power helps generate. The latter can dominate a favorable direct effect even when the legal allocation of q is unchanged. This is why the statement "$R_E < 1$" cannot be read as an institutional stability certification.

Nor is strict stability sufficient for substantive closure. Constant resource advantages, recurrent inflows, inherited rights and institutional replacement can all support positive stationary levels under a stable Jacobian. We return to this distinction in Section 6.

5. A joint functional–institutional feasibility theorem

5.1. Reduction of the constrained institutional problem

For fixed q, greater g weakly increases the invested amount and the effective wealth-to-power channel b; it also consumes budget. Under the nonnegative sign assumptions, choosing $g > g_{\min}(q)$ therefore cannot reduce R_E, and leaves no more funds for the safeguard. Likewise, at fixed (q, g), larger z weakly reduces the institutional capture term. These monotonicities yield a reduction from a three-dimensional institutional problem to a one-dimensional search. Define

$$\mathcal{R}(q) = R\big(q, g_{\min}(q), z_{\max}(q)\big), \quad q \in \mathcal{Q}. \qquad (15)$$

Theorem 1 (joint functional viability and local anti-capture feasibility). Assume (1)–(11), $\ell_g > 0$, $c_g, c_z > 0$ and the stated nonnegative coefficients. Then:

(a) The investment, financing, instrument-bound and common-budget requirements admit an architecture if and only if $\mathcal{Q} \neq \varnothing$.

(b) For every admissible q, the least attainable combined reproduction ratio is $\mathcal{R}(q)$.

(c) An architecture simultaneously meeting the investment and financing requirements, respecting the budget and satisfying $\rho(J) < 1$ exists **if and only if**

$$\boxed{\mathcal{Q} \neq \varnothing \quad \text{and} \quad \min_{q \in \mathcal{Q}} \mathcal{R}(q) < 1.} \qquad (16)$$

(d) If a benchmark regime is functionally and financially feasible, the same criterion can be applied to the subset of alternatives satisfying $q < q_B$; a successful regime is an attenuation of the targeted legal right, but the **total effective wealth-based channel** must be separately compared through $b(q, g)$.

The mathematical reduction is elementary but substantively restricts feasible claims: an apparently favorable reform must have a credible joint financing instrument, a funded capture safeguard, and no overlooked repatrimonializing effect from the substitute. The theorem proves neither that $\mathcal{Q}$ is always nonempty nor that the resulting institution dominates the benchmark in welfare or distributive terms.

The proof is in Appendix B. The strict inequality is consequential: $R = 1$ is not local asymptotic stability. Since $\mathcal{Q}$ is compact and $\mathcal{R}$ continuous on it, the minimum exists. The result can be used without imposing a globally monotone relationship between legal ownership and effective control.

The theorem identifies a scarce-resource trade-off rather than an optimal constitution. At each candidate q, two independent constraints determine the smallest g that will keep investment and pledgeability above their thresholds. Purchasing any additional g cannot improve the modeled reproduction ratio, because it increases either effective wealth-linked protection or the investment-derived appropriation margin and simultaneously competes with z for budget. The remaining budget is assigned to effective safeguards. The one-dimensional comparison is therefore a *frontier for admissible operation*, not a recommendation always to select the smallest q or to maximize institutional supervision regardless of cost.

5.2. A conditional no-reconstitution requirement

A legal reduction $q' < q_B$ does not establish a reduction in patrimonial power when the substitute recreates relevant claims. For instance, preserving i through a long-term creditor veto may leave the creditor with similar outside-option advantages to those previously conferred through voting stock. Under the model, a strict reduction of the **direct** wealth-to-power conversion relative to a benchmark requires

$$b_q(q_B - q') > b_g(g' - g_B). \qquad (17)$$

This is an **effective non-reconstitution test** for one measured transmission channel, not a full legal definition of non-proprietary rights. Failure of (17) means that q fell while b did not. Conversely, satisfying (17) does not guarantee that other dimensions of control, income, transferability and exclusion have not been reconstructed. Applied institutional research must inventory and aggregate those rights explicitly; the result is strongest when the empirically relevant channel can be identified.

Corollary 1 (invariance through substitute leakage). If the increase $b_g(g' - g_B)$ equals $b_q(q_B - q')$, then the direct wealth-to-power coefficient b is unchanged despite a strict reduction in designated ownership control. If investment and the capture parameters also remain unchanged, the full linearized dynamics are unchanged. If the substitute causes higher investment or more rule capture, total feedback can instead increase.

The corollary makes operational a central caution from rights-bundle analysis: formal dissociation is not the same thing as functional dissociation. The equality is a deliberately stringent counterexample, not a general empirical prediction.

Corollary 2 (a pre-specified attenuation target). Let a feasible benchmark (q_B, g_B, z_B) be given, and pre-specify positive margins ϵ, δ, σ. Define the closed subset $\mathcal{Q}_\Delta = \{q \in \mathcal{Q}: q \leq q_B - \epsilon,\ b\big(q, g_{\min}(q)\big) \leq b(q_B, g_B) - \delta\}$. There exists a feasible regime that decreases designated wealth-linked control by at least ϵ, reduces effective direct wealth-to-power conversion by at least δ, lowers combined feedback by at least σ, and is strictly locally stable if and only if $\mathcal{Q}_\Delta$ is nonempty and

$$\min_{q\in\mathcal{Q}_\Delta}\mathcal{R}(q) < 1, \qquad \min_{q\in\mathcal{Q}_\Delta}\mathcal{R}(q) \le R(q_B, g_B, z_B) - \sigma. \qquad (17a)$$

The claim follows from Theorem 1 because, at any fixed q, replacing g with $g_{\min}(q)$ can only lower b and the feedback ratio, and expanding z to $z_{\max}(q)$ can only lower that ratio. Strict stability and a weak pre-specified numerical improvement are deliberately kept distinct. This additional test matters because *crossing a stability threshold* and *reducing wealth-derived control relative to a stated benchmark* are logically different achievements. It also avoids treating a policy that merely relabels a control right as a successful reduction of power.

5.3. Financing may reverse the ordering of institutions

Without financing constraints, the lower envelope $g_{\min}(q)$ is determined by the investment floor alone. With pledgeability, however, an inexpensive reduction of q can force a large increase in g to reach $\underline{L}$. If that guarantee both raises investment and recreates bargaining rights, the total wealth–power feedback need not be smallest at the smallest feasible q.

Proposition 2 (a reversal under binding pledgeability). Within the admissible class there exist parameter values and two functionally and financially feasible institutions $q_1 < q_2$ for which $R_E\big(q_1, g_{\min}(q_1)\big) > R_E\big(q_2, g_{\min}(q_2)\big)$. The proposition remains possible even when all primitive derivatives $\beta, \gamma, b_q, b_g, d_i, \ell_q, \ell_g$ are positive.

The counterexample in Section 7 is explicit and independently reproducible. This result is stronger than repeating that institutional alternatives can be costly: it establishes that **an ordering by the amount of legal property control can conflict with the ordering by actual reproduction of economic power**. The conflict arises because financial substitutability, productive response and reconstitution are jointly endogenous to the chosen instruments.

5.4. Capture creates a second impossibility region

Even if a functionally viable arrangement satisfies $R_E < 1$, capture can rule out local stabilization when safeguards are too weak. Given (q, g) with $R_E < 1$, (13) implies that stability requires

$$(1 - z)\, h[(1 - c)u + vd(i)] < (1 - r)K[1 - R_E(q, g)]. \qquad (18)$$

If the right-hand side is nonpositive, no choice of z affects the offending economic loop enough. If positive, (18) defines an explicit **minimum effective safeguard**; the strict inequality is retained when the threshold binds. Under the common budget, increasing z also reduces the funds available for credible productive and financial guarantees. Joint feasibility can thus fail because (i) the substitute has inadequate capacity, (ii) the necessary substitute reconstitutes control, (iii) the economic loop remains too strong, or (iv) the anti-capture protection is unaffordable after functional safeguards are financed. These are analytically distinct failure modes.

A safeguard is also an economic use of resources. Choosing z at its maximum is warranted **conditional on minimizing this particular local feedback measure**, not because higher

enforcement expenditure is always socially desirable. Where monitoring produces false positives, delays needed investment, or simply reallocates influence to a different privileged group, additional protected functions would have to enter the comparison.

6. Local stability is not closure of the patrimonial channel

The local criterion $R < 1$ concerns the response of *deviations* around a reference position. It says nothing, by itself, about whether that position is just, concentrated or inherited. In a linear system with persistent positive inflows $\nu \geq 0$, $x_{t+1} = Jx_t + \nu$, $\rho(J) < 1$ implies $x_t \to (I - J)^{-1}\nu$, which is generally positive. A stable system can therefore reproduce a persistent level of authority even when disturbances fade.

The distinction is sharper if rights can be created afresh. Let $S_t \geq 0$ count the stock of relevant wealth-linked control positions; $N_t \geq 0$ denotes new positions formed during the period; $E_t \geq 0$ denotes reconstitution on positions previously converted; $X_t \geq 0$ denotes voluntary conversions or expirations arranged institutionally; and $\delta \in [0,1]$ denotes ordinary expiry. Accounting yields

$$S_{t+1} = (1 - \delta)S_t + N_t + E_t - X_t. \qquad (19)$$

The accounting identity assumes physically and legally admissible conversion flows, in particular $0 \leq X_t \leq (1 - \delta)S_t + N_t + E_t$ in this timing convention, so that the stock cannot be negative. Relabeling pre-existing claims does not count as X_t if it preserves an economically equivalent claim.

Proposition 3 (stock–flow closure requirement). For any horizon T, equation (19) implies

$$S_T = S_0 + \sum_{t=0}^{T-1} (N_t + E_t - X_t - \delta S_t)\,. \qquad (20)$$

Therefore the condition $S_T \leq \bar{S}$ is equivalent to cumulative conversions and expirations at least matching inherited excess stock plus cumulative new and reconstituted positions. In the stationary special case $X_t = 0$, $N_t + E_t = \omega > 0$ and $0 < \delta \leq 1$, S_t tends to $\omega/\delta > 0$, irrespective of whether the local power–wealth Jacobian is stable. Consequently, a stability test cannot by itself certify *closure* of the targeted patrimonial channel.

This identity applies a stock–flow accounting framework to the present institutional problem and is mathematically straightforward. Its contribution here is methodological: it prevents the overinterpretation of the dynamic theorem as an account of eliminating inherited control positions or preventing new legal or contractual equivalents. Robust institutional comparison must distinguish the **functional sufficiency** of a replacement, its **dynamic resistance to capture**, and the **entry/conversion/anti-reconstitution** measures required for closure over a stated horizon.

An institutional anti-reconstitution rule may attach limits on transferability, duration or voting authority to the relevant support, but those restrictions can themselves have costs.

The analysis therefore does not infer that perpetual constraints are always needed; it identifies which feedbacks or new-flow margins require further institutional evidence. The model intentionally does not attempt a full generational calibration or infer a socially optimal stock threshold.

7. Analytical illustrations and robustness tests

7.1. An apparently stabilizing reform defeated by rule capture

All numerical values in this section are **hypothetical**, not estimates. Let $a = 0.4$, $c = 0.3$, $r = 0.5$, $h = 0.2$, $u = 0.35$, $v = 0.10$, giving $K = 0.42$. Set $b_0 = 0.12$, $b_q = 0.75$, $b_g = 0.10$, $d_0 = 0.25$, $d_i = 0.30$. On the productive side let $s = 0.30$, $\beta = 0.50$, $\gamma = 0.80$, $k = 1$, $\underline{i} = 1$, and assume pledgeability is slack in the two regimes ($\ell_q = \ell_g = 0$ in this special illustration, with $F \leq 0$). Take $G = 0.50$, $Z = 0.80$, $c_g = 2$, $c_z = 0.20$ and $B = 0.40$.

The initial institution $(q, g, z) = (1, 0.25, 0)$ produces $i = 1$ and $b = 0.895$, with direct ratio $R_E = 0.895 \times 0.55/0.42 \approx 1.1720$, and capture $R_I = 0.2 \times (0.7 \times 0.35 + 0.1 \times 0.55)/(0.5 \times 0.42) \approx 0.2857$. Its total $R \approx 1.4577$ exceeds one.

The alternative $(q, g, z) = (0.6, 0.5, 0.7)$ also delivers $i = 1$, has $b = 0.62$, and costs $C_g + C_z = 0.25 + 0.14 = 0.39 \leq 0.40$. If the rules were artificially held exogenous ($h = 0$), the direct ratio would be $R_E \approx 0.8119 < 1$. Yet with capture left unprotected ($z = 0$), $R \approx 1.0976 > 1$. Installing $z = 0.7$ reduces capture feedback to 0.0857, so $R \approx 0.8976 < 1$. The model therefore supports a narrowly defined institutional possibility, but only **when the safeguard is financed and effective**. A reduction in ownership control alone does not explain the improvement.

7.2. A genuine counterexample to the minimum-control rule

Now suppress institutional capture for clarity by setting $h = 0$ (a nested two-state model), and let $s = 0.30$, $k = 1$, $\beta = 0.20$, $\gamma = 1$, $\underline{i} = 0.60$. Take $F = 0.60$, $\ell_q = 1$, $\ell_g = 0.10$, $G = 1$, $c_g = 1$, $B = 0.50$; all relevant substitute channels are strictly positive. Set $b_0 = 0.10$, $b_q = 0.20$, $b_g = 0.05$, $d_0 = 0.05$ and $d_i = 2$. With $a = c = 0.10$, $K = 0.81$.

At $q_1 = 0.50$, the financing constraint requires $g_{\min}(q_1) = 1$ and the resulting investment is $i_1 = 1.40$. Hence $b_1^{eff} = 0.25$, $d_1^{eff} = 2.85$, and $R_E(q_1) = 0.7125/0.81 \approx 0.8796$. The guarantee absorbs the entire real-resource budget. At $q_2 = 0.60$, financing alone would require $g \geq 0$, but the investment floor requires $g \geq 0.18$. Thus $g_{\min}(q_2) = 0.18$, $i_2 = 0.60$, $b_2^{eff} = 0.229$, $d_2^{eff} = 1.25$, and $R_E(q_2) = 0.28625/0.81 \approx 0.3534$. Both architectures are functionally and financially admissible; the one with **more** legal wealth-linked control yields substantially **less** measured wealth-mediated power feedback in this model. This is not an empirical forecast, but it refutes any general monotonic theorem under the extended admissible assumptions. Figure 2 illustrates this counterexample by plotting the feedback ratio against the legal-control share for configurations satisfying the common investment and financing constraints.

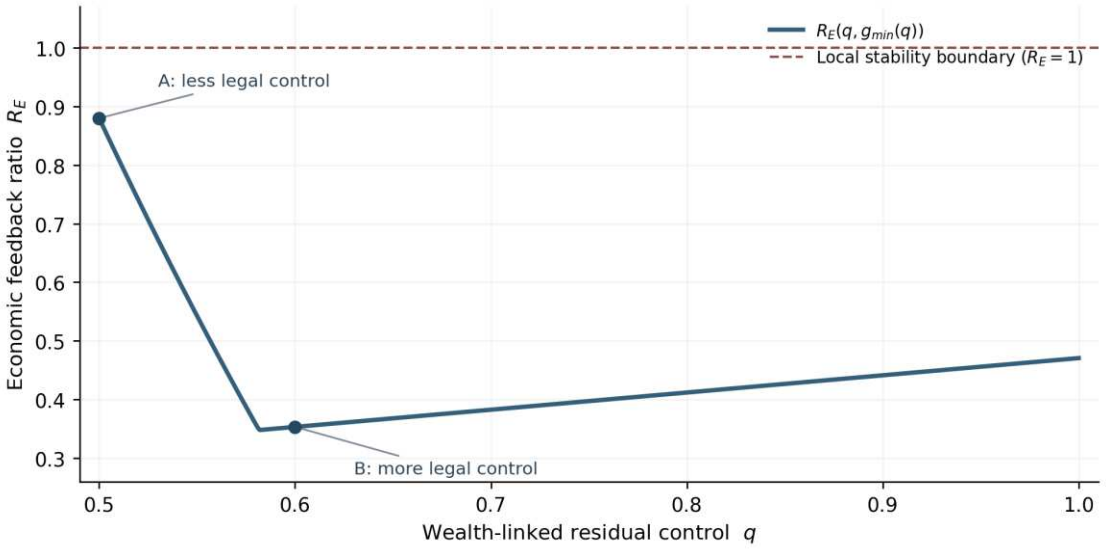


Figure 2. Feasible feedback along the financing example in Section 7.2. Each plotted point sets $g = g_{\min}(q)$, retains the common investment and loan tests, and reports R_E with $h = 0$. A higher legal-control share can produce a smaller feedback ratio within the feasible range. The unit line denotes the local stability threshold. Source: equations (2), (6) and (12), using the section's hypothetical parameters. Figure generated programmatically in Python using code developed with generative-AI assistance (OpenAI ChatGPT, GPT-6; October 2026).

If $b_g = 0$, the reversal can still occur through the effect of financing-induced overinvestment on $d(i)$, given the stated values. If $d_i = 0$ as well, the mechanism disappears in this example; the effect is therefore tied to specific behavioral and financial channels, not to a semantic preference for more control.

7.3. Sensitivity and identifiable channels

The threshold decomposes the sources of fragility. A high b_0 leaves wealth-based bargaining advantages outside the designated legal instrument. A high b_g makes a nominal substitute prone to functional reconstitution. A high d_i means that the investment response also enlarges the share of resources that can be recycled into future bargaining advantages. A high r prolongs institutional drift, and high u, v imply strong incentives for power holders to seek favorable exceptions or rules. A high z is useful only insofar as it is enforceable and financially sustainable. None of these is a universal sign or magnitude: alternative production, competition or countervailing bargaining arrangements could make the reduced-form derivatives different. The theorem is deliberately nontransportable outside its sign and timing assumptions.

The comparison across configurations must use a common horizon, a common definition of productive sufficiency and an identical accounting of real costs. It must also distinguish a decline in the targeted coefficient $b_q q$ from a reduction in total effective wealth-to-power conversion $b(q, g)$, and from a decline in the stationary *level* of inherited rights S_t. These are three different empirical claims.

8. Institutional scope, identification and discussion

8.1. Where the mechanisms might be observed

Financing covenants and contingent creditor control offer a concrete setting in which contractual protections can replace, but sometimes reconstruct, decision rights. Equity classes with differential voting rights illustrate why capital shares, voting rights and effective corporate influence should not be read from one another without the organizational rules (Bebchuk, Kraakman and Triantis, 2000). Cooperative and delegated governance forms offer candidate ways of changing the identity and procedures of those who exercise control, while also raising questions about investment, liquidity and accountability (Hansmann, 1996; Ostrom, 2005). These are **comparative applications**, not tested evidence for the theorem.

Related exploratory institutional work considers prototypes that separate the exercise of control from unrestricted patrimonial transfer, without treating all ownership as inefficient. For example, a multi-college enterprise design can attribute ordinary control proportionally to eligible capital contributions while assigning the exercise of some pooled blocks to worker or federated mandates. That is *not* the same as distributing all operational votes independently of subscribed capital, and it would be inaccurate to claim otherwise. Land-access arrangements, time-limited rights to recover fixed costs, and the non-exclusionary treatment of a productive monopoly suggest other domains where functional support and patrimonial persistence may be separated differently. These examples are proposed research designs. Neither their legal enforceability nor their comparative superiority follows from the present stylized model.

8.2. Empirical identification and potential refutation

A testable application requires much more than observing a reduction in voting rights. To evaluate (1)–(8), researchers must identify the investor's marginal return to q and g, the enforceability and cost of the substitute, pledgeable income under alternative claims, the required investment and the counterpart's response. To estimate (9)–(14), they must separately document the conversion of wealth into bargaining control, the conversion of current power into future deployable resources, and the effect of resources on rule-making or privileged access. Variation in voting regimes, creditor protection, ownership duration, renewal procedures, enforcement resources or limits on preferential access may offer research designs, but the exclusion and selection assumptions would need detailed justification.

A decrease in the autocorrelation of wealth cannot identify a decrease in power. An observed new formal statute cannot identify a decrease in effective rule capture; nor does convergence of measured deviations around an equilibrium establish that old titleholders have lost control. A strong empirical design would therefore report (i) functional outcomes and all-in costs, (ii) effective—not merely nominal—control and outside options, (iii) persistence and new creation of patrimonial claims, and (iv) the mechanisms of rule revision, influence and enforcement.

The theory can be rejected in multiple ways. Some safeguards may not damp the conversion of influence into rule advantages (z would lack the assumed interpretation). Protecting an investor could also reduce rather than increase others' vulnerability; in such a domain d_i or b_g might be negative. Counterpart effort might collapse despite meeting the focal investor's floor. Credit contracts might not reproduce relevant pledgeability. Finally, rule changes may generate strategic equilibrium shifts not representable by the present local linearization. Each of these outcomes restricts or invalidates transport of the theorem to that setting.

8.3. Relation to recent property-appropriation models

The recent models of Cao and Lagunoff (2023) and Cao, Lagunoff and Xu (2026) provide a particularly demanding comparison. They examine political actors' capacity to alter the allocation of property claims through gaps in effective legal protections. Such settings make it implausible to assume a reform's announced rules are necessarily its effective rules. But the question here is not the long-run distribution of appropriated properties under a specified local political process. It is the *feasibility of a replacement bundle* when the replacement must maintain productive incentives and financing and must budget for restrictions on endogenous rule drift. Consequently, the present article should not be read as offering a better prediction of local expropriation or unequal land ownership; it offers an additional test to apply **before** claiming that a candidate economic institution is viable and robust to capture.

The same comparison disciplines the interpretation of the counterexample. More formal control can coincide with a lower local feedback ratio in the stylized case because a financing substitute creates unusually high investment and hence a larger pool subject to wealth-dependent appropriation. That is not a policy argument for allocating more legal control to wealthy agents. It instead shows why an analytically adequate ranking cannot rest on a single dimension of the legal bundle. Welfare, distribution, legality and rights protection remain independently relevant, and a smaller R does not imply a better regime on those dimensions.

8.4. What the existing literature establishes—and what the test does not

The comparison should not be mistaken for three claims of novelty. **First**, Grossman and Hart (1986), Hart and Moore (1990) and Aghion and Bolton (1992) establish why residual or contingent control rights may be necessary when adaptation and financing contracts are incomplete; the quadratic investment response here is a local representation, not a replacement for their incentive analysis. Rajan and Zingales (1998) already demonstrate that access to critical resources may outperform traditional ownership as an incentive mechanism in relevant environments. The contribution here is therefore not the discovery that legal ownership can be substituted. **Second**, Greif and Laitin (2004) and Acemoglu and Robinson (2008) establish the self-reinforcement or compensation of institutional power. Their models are richer accounts of endogenous change; the present scalar influence choice is simply the minimal endogenous feedback needed for a feasibility screen. **Third**, Cao and Lagunoff (2023) and Cao, Lagunoff and Xu (2026) investigate appropriation of property claims through politically structured institutions. Their substantive mechanism is distinct

from, and in several dimensions richer than, the present constrained comparison of replacement bundles.

The article contributes an exact answer to a conditional *joint* question: for a given specification of productive functions, pledgeability, effective replacement rights, influence effort and scarce enforcement resources, is there an admissible arrangement whose local feedback is below one, and can a nominal decline in wealth-based control misrank that arrangement? The answer remains restricted to the maintained class and could be overturned in empirical domains where relevant marginal responses have different signs, missing parties impose binding incentives, or institutional changes alter the equilibrium regime discontinuously. This qualification is integral to—not a repudiation of—the derived result.

8.5. Functional sufficiency, selection and closure are distinct

A feasible configuration is not necessarily chosen. Incumbents may have veto rights, transition costs may exceed benefits for pivotal participants, and compensations may themselves recreate control. Nor does adoption entail durable institutional closure. In particular, stock conversion, new entry and reconstructed positions are separate flows, and inherited stocks can remain even in a stable regime. Our model addresses a jointly feasible **operating arrangement**; it does not establish an unforced political transition or a general optimum. The broader research separates static efficiency, investment and innovation, and distributive exposure to inherited wealth as different outcomes rather than aggregating them into one invented welfare scalar.

A full theory would endogenize selection of the new rules, counterpart incentives, conditional commitment under repeated shocks, alternative means of lobbying, cross-organization spillovers, and perhaps vector-valued control over multiple firms. The present contribution is the narrower demonstration that **functionally adequate reforms can fail through financially induced overinvestment, reconstructed contractual claims or institutional capture—and that the corresponding conditional feasibility tests can be stated and checked together**. These are additions to, not replacements for, existing theories of property and endogenous institutional persistence.

The model is positive and conditional. It does not produce a claim of Pareto improvement, general welfare dominance or durable constitutional self-enforcement. Each candidate institution must also be checked for unmodeled effects on counterpart participation, public-law constraints and the distribution of financial and control rights.

9. Conclusion

A productive justification for one right does not by itself justify the perpetuity, inheritability, accumulation or political insulation of the entire legal bundle. Equally, identifying some legal substitution does not establish that productive incentives, project finance and long-run distributions of authority remain intact. The present model gives a conditional way to keep these claims distinct. Investment incentives, lender pledgeability,

substitutes' effective reconstitution of power and actors' incentives to influence rules are jointly evaluated under an explicit enforcement budget. The exact three-state threshold shows why a system that appears stable under fixed institutions can become unstable when rule drift responds to economic advantage. The feasibility theorem provides a parsimonious test rather than an assurance of reform's success. An explicit counterexample shows that minimum legal control need not minimize power reproduction, and stock–flow accounting explains why local stability is not closure.

The broader question is not whether property is inherently desirable or undesirable, but **which bundles of rights satisfy demonstrable functions, what powers they reproduce, and whether a proposed replacement avoids recreating the same power through other channels**. Answering it in any real economic domain requires legal and organizational evidence, causal identification and the comparison of several protected outcomes. That evidence is not supplied by the mathematical examples here. The appropriate conclusion is a bounded programme of institutional comparison, not a universal superiority claim.

Statements and Declarations

Data and code availability. No empirical dataset was used. All numerical examples rely on synthetic model parameters. Reproduction scripts for the numerical consistency checks and the two figures are supplied as Online Resource 1 (anonymized Python code).

Generative-AI assistance. Generative AI (OpenAI ChatGPT, GPT-6; accessed via https://chatgpt.com; October 2026) was used in drafting and editing the manuscript, identifying relevant literature, developing and checking symbolic derivations, performing synthetic numerical consistency tests, and preparing the figures and document files. The numerical exercises are model-consistency checks, not empirical evidence. The submitting author retains responsibility for the accuracy of citations, mathematical claims and interpretations. AI is not an author.

Appendix A. Exact stability proof

Set $b = b(q,g)$, $d = d\big(i(q,g)\big)$, $u_z = (1-z)u$, $v_z = (1-z)v$. Write $J = \begin{pmatrix} a & b & h \\ d & c & 0 \\ u_z & v_z & r \end{pmatrix}$ with all entries nonnegative and $a, c, r < 1$. For a spectral parameter $\lambda > \max\{c, r\}$, the Schur-complement characteristic equation for eigenvalues outside the diagonal subsystem $\{c, r\}$ is obtained by eliminating the second and third components: $w = dp/(\lambda - c)$ and $m = [u_z + v_z d/(\lambda - c)]p/(\lambda - r)$. The resulting scalar condition is

$$\lambda = a + \frac{bd}{\lambda - c} + \frac{h}{\lambda - r}\left(u_z + \frac{v_z d}{\lambda - c}\right). \qquad (A1)$$

For completeness, the proof does not require irreducibility or a strictly positive eigenvector. Partition J into its first coordinate and the nonnegative subsystem $D =$

$\begin{pmatrix} c & 0 \\ v_z & r \end{pmatrix}$, which has $\rho(D) = \max\{c, r\} < 1$. The block inverse of $I - D$ is nonnegative. Hence, by the nonsingular M-matrix criterion for a nonnegative block matrix, $I - J$ is a nonsingular M-matrix if and only if the scalar Schur complement of $I - D$ is positive. That complement is $1 - a - bd/(1 - c) - h[u_z + v_z d/(1 - c)]/(1 - r)$. After division by $1 - a > 0$, strict positivity is exactly $R_E + R_I < 1$. If the complement is zero, $\det(I - J) = 0$ and, by continuity of the Perron root, $\rho(J) = 1$. If it is negative, $I - J$ cannot be an M-matrix and nonnegative-matrix monotonicity gives $\rho(J) > 1$. The reduction is valid with zero off-diagonal entries, including reducible J. No nonlinear conclusion is asserted at the boundary.

Appendix B. Feasibility theorem and no-reconstitution result

At fixed q, (3) and (4) demand $g \geq g_{\min}(q)$. Because $g \geq 0$, $c_g > 0$ and the cost is increasing, some $g \leq G$ respects the guarantee budget if and only if $g_{\min}(q) \leq G$ and $c_g g_{\min}(q)^2/2 \leq B$. Therefore the full functionally and financially feasible set is nonempty precisely when $\mathcal{Q} \neq \varnothing$, since choosing $z = 0$ always obeys the remaining budget. This proves (a).

The entries $b(q, g)$ and $d\big(i(q, g)\big)$ are nondecreasing in g, while the direct feedback term is their product. For fixed q, lowering g to $g_{\min}(q)$ therefore weakly reduces R_E. It weakly reduces the $vd(i)$ term in R_I, and frees budget for a weak increase of z. Since R_I decreases weakly in z, a minimizing arrangement at the selected q is $\big(g_{\min}(q), z_{\max}(q)\big)$. This proves (b). The functions $g_{\min}$ and $z_{\max}$ are continuous, so $\mathcal{Q}$ is compact and $\mathcal{R}$ is continuous on it. Consequently the minimum exists; combining the first-order criterion of Proposition 1 with (b) proves (c). Part (d) restricts the same argument to the set $\{q \in \mathcal{Q}: q < q_B\}$; its strict open benchmark constraint can be evaluated through the existence of a point in that set satisfying (16), but a minimum over that open set is not assumed to be attained. The non-reconstitution inequality (17) follows directly by subtracting $b(q', g')$ from $b(q_B, g_B)$. These arguments do not assert that minimizing R yields minimum expenditure or maximum welfare.

Appendix C. Financing reversal and stock dynamics

For the counterexample of Section 7.2, at $q_1 = 0.5$ the two lower bounds in (6) are $(0.6 - 0.3 - 0.2 \times 0.5) = 0.2$ and $(0.6 - 0.5)/0.1 = 1$. Hence $g_{\min} = 1$, $i = 0.3 + 0.2(0.5) + 1 = 1.4$, and $R_E = (0.1 + 0.2(0.5) + 0.05)\big(0.05 + 2(1.4)\big)/0.81 = 0.7125/0.81$. At $q_2 = 0.6$, the bounds are 0.18 and 0, yielding $i = 0.6$, $b = 0.1 + 0.12 + 0.05(0.18) = 0.229$, $d = 0.05 + 1.2 = 1.25$, and $R_E = 0.28625/0.81$. Both meet the financing threshold. The guarantee costs are 0.5 and 0.0162 respectively, each no more than $B = 0.5$. Thus $q_1 < q_2$ but $R_E(q_1) > R_E(q_2)$, proving Proposition 2 by construction. Proposition 3 is obtained by subtracting S_t from each side of (19) and summing the resulting identity across $t = 0, \ldots, T - 1$. With constant $\omega = N + E > 0$ and $X = 0$, its stationary scalar solution is $S^* = \omega/\delta$ when $\delta > 0$, irrespective of J.

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