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Programmable Coherent Memory Kernels for Quantum Reservoir Computing in Waveguide QED

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Abstract

Reservoir computing requires memory distributed over task-relevant timescales, not merely long memory. In most quantum reservoirs, however, the memory profile emerges from intrinsic dynamics and cannot be engineered directly. Here we introduce a waveguide-QED quantum reservoir in which a driven Kerr resonator couples to a one-dimensional waveguide at spatially separated points. The unmeasured propagating field returns past-input information after a geometry-defined delay, forming a coherent non-Markovian feedback channel without intermediate measurement or classical reinjection. Delay-resolved memory benchmarks reveal two complementary roles: propagation sets when information returns, while Kerr dynamics convert the returned field into nonlinear temporal features accessible to a linear readout. In NARMA- n tasks, the optimal delay shifts systematically with task order, approximately tracking the explicit input-product lag. These results establish coherent delayed feedback as a mechanism for engineering task-relevant memory in quantum reservoir computing.

Introduction

Reservoir computing (RC) processes temporal information by embedding input histories into the high-dimensional dynamical state of a complex system, while training only a linear readout layer [1–11]. Building on this framework, quantum reservoir computing (QRC) exploits coherence, intrinsic interactions, and large accessible

quantum state spaces to generate rich temporal features, motivating extensive theoretical proposals and a growing number of experimental demonstrations across photonic, superconducting-circuit, trapped-ion, spin-based, and related platforms [12–25]. Despite these advances, realizing the full computational potential of QRC requires not only rich quantum dynamics, but also the ability to retain and transform information from past inputs.

Memory is a central resource in RC: a reservoir must preserve task-relevant information from past inputs while progressively discarding obsolete history [26–30]. However, the usefulness of memory is not determined by duration alone. Standard temporal tasks, including short-term memory (STM) [31], delayed-product [29], delayed-XOR [32] and NARMA- n prediction [30], require information retained at specific past lags or over characteristic temporal windows. The relevant computational resource is therefore not simply memory depth, but a delay-resolved memory profile aligned with the temporal structure of the task. In QRC, quantum coherence and intrinsic interactions offer distinct physical mechanisms for retaining past information and transforming it into nonlinear temporal features accessible to the readout [27–29].

This task-matching perspective has been explored extensively in classical RC through task-specific capacity analysis [33] and delay-based architectures, where self-feedback loops or inter-node propagation delays reshape lag-resolved memory profiles and information-processing capacities [10, 32, 34, 35]. The coherent quantum counterpart, however, remains less developed. In a time-delay quantum reservoir, past information must be carried by an unmeasured propagating field and returned coherently to the reservoir, rather than being converted into a classical record and subsequently reinjected.

Measurement-feedback QRC schemes offer an explicit route to memory reinjection, but typically rely on repeated measurements and the classical processing of the outcomes [19, 24, 28, 36]. Non-Markovian quantum reservoirs, by contrast, can preserve coherent dynamics and enhance memory without intermediate measurement, yet their temporal memory profiles are typically fixed by the intrinsic reservoir dynamics rather than engineered for specific task-relevant timescales [29, 37]. The central challenge is therefore to engineer the temporal structure of quantum-reservoir memory through coherent feedback, without intermediate measurement or classical reinjection, and to match the resulting memory profile to task-relevant temporal dependencies.

In this work, we propose a non-Markovian waveguide-QED reservoir consisting of a driven giant Kerr-nonlinear resonator coupled nonlocally

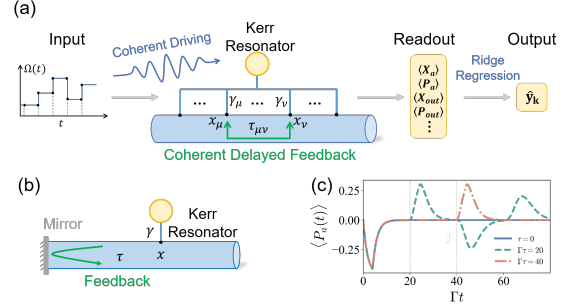


Fig. 1 Non-Markovian waveguide-QED reservoir. (a) A driven Kerr resonator couples to a 1D waveguide at multiple points, generating coherent delayed feedback with propagation times $\tau_{\mu\nu}$. The drive $\Omega(t)$ encodes the input, while the measured reservoir features are mapped to $\hat{y}_k$ through a trained linear readout. (b) Minimal single-delay geometry, equivalently represented by a resonator coupled to a semi-infinite waveguide, with coherent memory-return delay τ . (c) Resonator dynamics following a short-pulse excitation for representative delays $\Gamma\tau \in \{0, 20, 40\}$, illustrating the delay-controlled return of information to the resonator.

to a one-dimensional (1D) waveguide at spatially separated points (Fig. 1). The required ingredients—nonlocal coupling, finite propagation times, and nonlinear resonator dynamics—are compatible with existing giant-atom and waveguide-QED platforms [38–42]. The emitted field carries information away from the resonator and coherently returns it after a geometry-defined delay, so that memory is encoded in the joint resonator–waveguide dynamics rather than stored solely in the local resonator. This architecture realizes a quantum time-delay reservoir in which the unmeasured propagating field serves as a coherent memory carrier, returning information about past inputs to the nonlinear resonator. Waveguide propagation determines when information about past inputs becomes dynamically available again, whereas the Kerr interaction mixes the returning field with the current resonator state and converts it into nonlinear temporal features accessible through a linear readout. Delay-resolved STM and delayed-product capacities identify these two roles independently. We further show that, in NARMA- n prediction, the optimal feedback delay shifts systematically with task order and tracks the explicit input-product lag. These results establish waveguide-mediated coherent feedback as a hardware-level mechanism for engineering task-relevant memory in QRC.

Results

Waveguide-QED reservoir and memory kernel

We consider the waveguide-QED reservoir illustrated in Fig. 1(a), consisting of a driven Kerr-nonlinear resonator coupled to a 1D waveguide at M spatially separated coupling points $x_1, \dots, x_M$. In a frame rotating at the drive frequency ω_d , and in the interaction picture with respect to the free waveguide Hamiltonian, the total Hamiltonian reads

$$H(t) = \Delta a^\dagger a + \frac{U}{2} a^{\dagger 2} a^2 + i[\Omega(t)a^\dagger - \Omega^*(t)a] + \sum_{\mu=1}^M \sum_{\alpha=L,R} \int d\omega [g_{\mu\alpha\omega} e^{-i\delta_\omega t} a^\dagger b_\alpha(\omega) + \text{H.c.}], \quad (1)$$

where a annihilates an excitation of the resonator and $b_\alpha(\omega)$ annihilates a waveguide mode of frequency ω propagating in direction $\alpha = L, R$. The parameters $\Delta = \omega_r - \omega_d$ and $\delta_\omega = \omega - \omega_d$ denote the resonator and waveguide detunings in the rotating frame, respectively, U is the Kerr nonlinearity, and $\Omega(t)$ is the input-dependent coherent drive. For symmetric coupling to left- and right-propagating modes, we take $g_{\mu\alpha\omega} = \sqrt{\gamma_\mu/4\pi} e^{is_\alpha \omega x_\mu/v_g}$, with $s_R = +1$ and $s_L = -1$, so that γ_μ is the total radiative decay rate of coupling point μ .

Eliminating the waveguide degrees of freedom yields a nonlocal Heisenberg–Langevin equation for the resonator operator [43]. For resonant driving ($\Delta = 0$), it takes the form

$$\dot{a}(t) = -iUa^\dagger(t)a^2(t) + \Omega(t) + \xi(t) - \int_0^t K(t-t')a(t')dt', \quad (2)$$

where $\xi(t)$ is the zero-mean vacuum input noise and $K(t)$ is the (effective) memory kernel generated by waveguide propagation. Within a local Markov approximation at each coupling point, while retaining the finite propagation delays between spatially separated points [43–45], the

memory kernel can be written as

$$K(t) \simeq \frac{\Gamma}{2} \delta(t) + \sum_{\mu \neq \nu} \frac{\sqrt{\gamma_\mu \gamma_\nu}}{2} e^{i\phi_{\mu\nu}} \delta(t - \tau_{\mu\nu}), \quad (3)$$

where $\Gamma = \sum_\mu \gamma_\mu$ is the total radiative decay rate, $\tau_{\mu\nu} = |x_\mu - x_\nu|/v_g$ is the propagation time between coupling points, and $\phi_{\mu\nu} \simeq \omega_d \tau_{\mu\nu}$ is the corresponding propagation phase. The first, instantaneous term in Eq. (3) describes the local Markovian decay, whereas each delayed term represents a geometry-induced coherent feedback channel. Through this channel, a field emitted by the resonator propagates through the waveguide and coherently re-enters the resonator after a finite propagation time $\tau_{\mu\nu}$, with feedback strength $\sqrt{\gamma_\mu \gamma_\nu}/2$, without intermediate measurement or classical reinjection.

To isolate the elementary mechanism of waveguide-mediated coherent memory, we focus on the minimal nontrivial configuration consisting of two coupling points separated by a single propagation delay $\tau = \tau_{12}$. Despite its simplicity, this geometry already captures the complete physical cycle underlying the proposed reservoir: information carried by an unmeasured propagating field leaves the resonator, is temporarily stored in the waveguide, returns after a finite propagation time, and is subsequently transformed by the local Kerr nonlinearity into nonlinear temporal features. It therefore provides the simplest setting in which memory transport can be cleanly separated from memory processing. The same effective single-delay feedback channel can be implemented experimentally by placing the resonator in front of a mirror, or equivalently by coupling it to a semi-infinite waveguide, as illustrated in Fig. 1(b). In this representation, the feedback delay is simply determined by the resonator–mirror distance. Following a short-pulse excitation, the returning field produces a delayed echo in the resonator response [Fig. 1(c)], providing a direct dynamical signature of the coherent memory channel. The multi-point geometry considered in Eq. (3) extends this minimal construction to multiple independently programmable memory-return times.

The effective kernel in Eq. (3) naturally separates two physical ingredients of the reservoir. Waveguide propagation determines when information carried by the emitted field coherently returns

to the resonator, whereas the Kerr interaction determines how this returning field is transformed into nonlinear computational features. Relative to the input update interval Δt , the ratio $\tau/\Delta t$ fixes the temporal location of this coherent memory return in discrete input steps. To distinguish these two roles, we compare a time-delay linear reference reservoir (TDLRC, $U = 0$), which isolates coherent memory transport, with a time-delay Kerr quantum reservoir (TDQRC, $U = 30\Gamma$), in which coherent memory transport and nonlinear memory processing coexist. All remaining parameters are kept identical. The TDLRC is simulated by solving the corresponding delay-differential equation (DDE) [43], while the TDQRC is simulated using the time-bin matrix-product-state (MPS) method [46–48], which explicitly represents the propagating waveguide field responsible for coherent delayed feedback. Implementation details are provided in Methods, and system–field correlations are discussed in Supplementary Section S1. Unless stated otherwise, all results are obtained for balanced couplings $\gamma_1 = \gamma_2 = \Gamma/2$ and constructive propagation phases $\phi_{\mu\nu} = 2\pi m$. The effects of propagation phase and coupling asymmetry are discussed in Supplementary Sections S2 and S3, respectively.

Input encoding and readout

The reservoir is driven by a scalar input sequence $\{u_k\}$, encoded in the coherent drive via a zero-order-hold protocol,

$$\Omega(t) = \epsilon u_k, \quad (k-1)\Delta t \leq t < k\Delta t, \quad (4)$$

where ϵ sets the input amplitude and Δt is the input update interval. Each input value is therefore held constant for one reservoir-evolution step, while the waveguide-mediated dynamics retain and process information from preceding steps.

The reservoir state is accessed through first-order quadrature moments of both the local resonator and the outgoing waveguide field,

$$\mathbf{r}(t) = (\langle X_a(t) \rangle, \langle P_a(t) \rangle, \langle X_{\text{out}}(t) \rangle, \langle P_{\text{out}}(t) \rangle)^{\mathbf{T}}, \quad (5)$$

where $X_a = a + a^\dagger$, $P_a = i(a^\dagger - a)$, $X_{\text{out}} = b_{\text{out}} + b_{\text{out}}^\dagger$, $P_{\text{out}} = i(b_{\text{out}}^\dagger - b_{\text{out}})$, and b_{out} is the waveguide output field operator. Because the readout

contains only first-order moments and is subsequently processed by a linear regression layer, any nonlinear temporal features accessible to the output must be generated by the intrinsic reservoir dynamics.

In order to enrich the accessible dynamical features without introducing an additional memory channel, we adopt temporal multiplexing [3]. During each input interval k , the response is sampled at $t_{k,m} = k\Delta t - m\theta$ ($m = 0, \dots, N_v - 1$), where θ is the virtual-node spacing and N_v is the number of virtual nodes. The resulting feature vector is

$$\mathbf{x}_k = (1, \mathbf{r}^{\mathbf{T}}(t_{k,0}), \dots, \mathbf{r}^{\mathbf{T}}(t_{k,N_v-1}))^{\mathbf{T}}, \quad (6)$$

where the leading constant provides the readout bias. We impose $(N_v - 1)\theta \leq \Delta t$ so that all virtual nodes lie within a single input interval. Temporal multiplexing therefore resolves the intra-step reservoir trajectory but does not explicitly embed responses from earlier input intervals. Memory extending beyond one input step must consequently originate from the resonator–waveguide dynamics. The predicted output is obtained from the linear readout

$$\hat{y}_k = W_{\text{out}} \mathbf{x}_k. \quad (7)$$

The weights W_{out} are trained (and thus optimized) via ridge regression [30] after discarding an initial washout period, and are evaluated on independent test data. The resonator is initially in its ground state and the waveguide is in vacuum. The decay of feature-level dependence on the initial resonator state, and hence the echo-state property in the operating regime considered here, is verified in Supplementary Section S4.

Together, the geometry-induced memory kernel, coherent input encoding, and linear readout establish a reservoir in which coherent waveguide propagation controls the temporal placement of memory, while intrinsic Kerr dynamics govern its nonlinear processing. The following sections quantify these two roles independently using delay-resolved memory benchmarks.

Linear and nonlinear memory

Having established the geometry-induced memory kernel above, we now use delay-resolved benchmark tasks to identify the distinct roles of coherent

waveguide propagation and Kerr nonlinearity. The short-term-memory task resolves when information about a past input becomes accessible to the readout, whereas the delayed-product task tests whether this returned information is nonlinearly combined with the current input.

The STM target is

$$y_{k,\text{STM}}^{(d)} = u_{k-d}, \quad (8)$$

where d is the input lag and the inputs are sampled independently as $u_k \sim \mathcal{U}(0, 1)$. To probe nonlinear delayed-memory processing, we use the delayed-product target

$$y_{k,\text{prod}}^{(d)} = (u_k - \bar{u})(u_{k-d} - \bar{u}), \quad (9)$$

where $\bar{u}$ is the input mean. This task requires both the retention of u_{k-d} and its multiplicative mixing with the current input u_k . For each task, the delay-resolved capacity is defined as

$$C_d^\eta = \frac{\text{Cov}^2(\hat{y}_{k,\eta}^{(d)}, y_{k,\eta}^{(d)})}{\text{Var}(\hat{y}_{k,\eta}^{(d)})\text{Var}(y_{k,\eta}^{(d)})}, \quad \eta \in \{\text{STM}, \text{prod}\}, \quad (10)$$

with all quantities evaluated on held-out test data after washout and training. The capacity satisfies $0 \leq C_d^\eta \leq 1$, with unity corresponding to perfect linear reconstruction of the target from the reservoir features.

The STM capacity directly resolves the temporal memory structure induced by the memory kernel in Eq. (3). In the absence of a finite feedback delay, the instantaneous kernel contribution produces a local fading-memory envelope, as seen for $\tau/\Delta t = 0$ in Fig. 2(a). In this regime, the TDQRC exhibits a broader short-lag response than the TDLRC, indicating that the Kerr dynamics reshape the local reservoir relaxation even before a temporally separated return is introduced.

For a finite propagation delay τ , the emitted field coherently re-enters the resonator after a time τ . In units of the input update interval Δt , this produces a primary memory revival near

$$d_\tau \simeq \frac{\tau}{\Delta t}. \quad (11)$$

At the shortest finite delay [Fig. 2(b)], the returning field overlaps with the local fading-memory

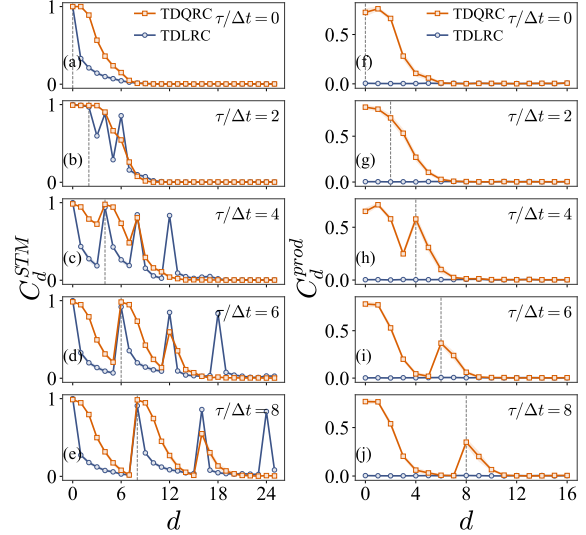


Fig. 2 Delay-resolved linear and nonlinear memory. The left and right columns show the STM capacity C_d^{STM} for reconstructing u_{k-d} and the delayed-product capacity C_d^{prod} for reconstructing $(u_k - \bar{u})(u_{k-d} - \bar{u})$, respectively. Rows correspond to feedback delays $\tau/\Delta t = 0, 2, 4, 6, 8$; vertical dashed lines mark $d = \tau/\Delta t$. TDQRC denotes the Kerr-nonlinear time-delay quantum reservoir, while TDLRC denotes the linear reference reservoir with the same feedback geometry. Parameters are $\Delta t = 2$, $\epsilon = 1$, $\gamma_1 = \gamma_2 = 0.1$ ($\Gamma = 0.2$), $N_v = 20$, and $\theta = 0.1$. Washout, training, and test sequences contain 100, 600, and 200 input steps, respectively. Shaded regions indicate one standard deviation over independent input realizations.

response, resulting in a broadened low-lag profile. As the delay increases [Fig. 2(c)–(e)], the returned component separates from the local response and forms a distinct STM revival. Its position follows Eq. (11) in both the TDLRC and TDQRC, demonstrating that the timing of the primary memory return is governed mainly by waveguide propagation rather than by the local nonlinearity.

In addition, secondary revivals appear at later lags as a consequence of repeated coherent reinteractions between the resonator and the delayed field. In the parameter regime considered here, these echoes are more pronounced in the TDLRC, where the returning field evolves linearly and can accumulate coherently over successive round trips. In the TDQRC, Kerr nonlinearity mixes such returning components with the instantaneous resonator state, thereby reshaping the echo profiles and reducing the amount of information that remains accessible as purely linear delayed recall. This behavior reflects the familiar redistribution

of reservoir capacity between linear memory and nonlinear processing [49–52]. Waveguide propagation therefore sets the temporal positions of the coherent returns, whereas Kerr dynamics redistribute their information content among linear and nonlinear features.

Next, we use the delayed-product task to determine whether the coherently returned information is converted into nonlinear temporal features. Despite its clear STM revivals, the TDLRC exhibits negligible C_d^{prod} under the present first-order-moment and linear-readout protocol [Fig. 2(f)–(j)]. A linear reservoir can retain u_{k-d} , but it cannot efficiently generate the multiplicative feature $u_k u_{k-d}$ from the same first-order observables. The TDQRC behaves qualitatively differently. At the short finite delay [Fig. 2(g)], the delayed-product response extends over neighboring lags because the coherent return overlaps with the local nonlinear response. For larger delays, the two contributions become temporally separated, and a distinct product-capacity feature emerges near $d \simeq \tau/\Delta t$ [Eq. (11)]. The coincidence between the STM and delayed-product revival positions shows that the waveguide first transports information about u_{k-d} back to the resonator, after which the Kerr interaction combines it with the current reservoir response and renders the resulting nonlinear feature accessible to a linear readout. The integer values of $\tau/\Delta t$ used in Fig. 2 provide a transparent illustration of delay-programmed memory. More generally, when $\tau/\Delta t$ is non-integer, the coherent memory return is distributed across the nearest discrete input lags rather than being localized at a single delay, as discussed in Supplementary Section S5. Likewise, extending the giant resonator to multiple coupling points introduces several coherent feedback paths and thus multiple programmable memory-return times, whose resulting memory structure is analyzed in Supplementary Section S6.

These benchmarks establish a two-stage mechanism for delay-based quantum reservoir processing. Coherent waveguide propagation controls when past-input information becomes available again, while the Kerr interaction determines whether that information remains accessible as linear recall or is transformed into nonlinear delayed features. This separation suggests

a direct task-matching principle: temporal prediction should improve when a coherent memory return is positioned near a task-relevant lag and is processed by sufficiently strong intrinsic nonlinearity. We test this principle next using NARMA- n prediction.

Task-matched time-series prediction

Now, we test whether the delay-resolved memory mechanism identified in the preceding memory analysis can be translated into improved nonlinear time-series prediction. In contrast to the STM and delayed-product benchmarks, which probe specific temporal features at a prescribed lag, nonlinear forecasting tasks combine multiple delayed dependencies and recurrent nonlinear transformations. They therefore provide a more stringent test of whether coherent memory return can be matched to task-relevant temporal structure.

Prediction accuracy is quantified by the normalized root-mean-square error (NRMSE):

$$\text{NRMSE} = \left[\frac{\sum_k (y_k - \hat{y}_k)^2}{\sum_k (y_k - \bar{y})^2} \right]^{1/2}, \quad (12)$$

where $\bar{y}$ is the mean of the target outputs over the test set.

We consider the NARMA- n benchmark, which is widely used to assess the combined nonlinear-processing and finite-memory capabilities of reservoir computers [3]. The target sequence obeys

$$y_{k+1} = 0.3 y_k + 0.05 y_k \sum_{j=0}^{n-1} y_{k-j} + 1.5 \tilde{u}_k \tilde{u}_{k-n+1} + 0.1, \quad (13)$$

with $\tilde{u}_k = u_k/5 \in [0, 0.2]$. The explicit product $\tilde{u}_k \tilde{u}_{k-n+1}$ couples the current input to a past input at lag $d_{\text{task}} = n - 1$. The recurrent terms involving the previous outputs y_k introduce additional, distributed temporal dependencies, but the product term provides a clearly identifiable nonlinear input–input correlation that can be directly compared with the delay-induced nonlinear memory characterized in the preceding memory analysis.

According to Eq. (11), coherent delayed feedback returns past-input information to the resonator near $d_\tau \simeq \tau/\Delta t$. The Kerr nonlinearity can then mix this returned information with the

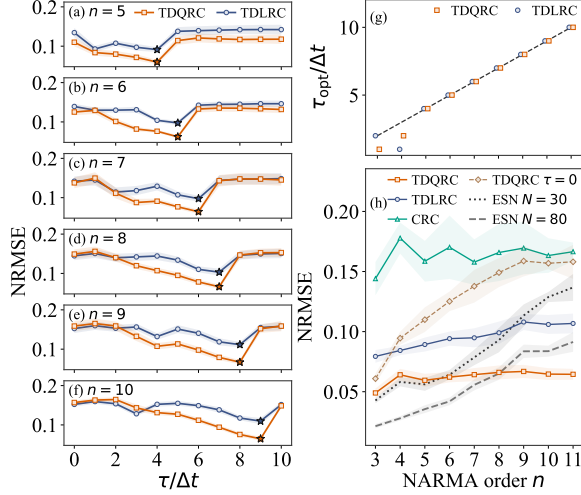


Fig. 3 Task-matched prediction results on NARMA- n sequences. (a)–(f) Test NRMSE as a function of programmed delay $\tau/\Delta t$ for $n \in \{5, 6, 7, 8, 9, 10\}$, comparing TDQRC and TDLRC. Stars indicate the minimum NRMSE for each curve. (g) Extracted optimal delay $\tau_{\text{opt}}/\Delta t$ versus the explicit input-product lag $n - 1$. The dashed identity line represents the matching condition in Eq. (14). (h) NRMSE as a function of NARMA order for the reservoir designs and baselines considered. All parameters are the same as in Fig. 2.

response to the current input. For the nonzero-mean inputs used here, $u_k = \bar{u} + \delta u_k$, with $\bar{u} = 1/2$, the expansion around the finite-drive operating point $\Omega_0 = \epsilon \bar{u}$ admits quadratic Volterra-type features proportional to $\delta u_k \delta u_{k-d\tau}$. Matching these features to the explicit NARMA input-product term suggests the task-matching condition

$$\frac{\tau_{\text{opt}}}{\Delta t} \simeq d_{\text{task}} = n - 1. \quad (14)$$

Because NARMA- n also contains recurrent and distributed memory terms, Eq. (14) should be understood as a physically motivated design rule rather than an exact identity.

As the NARMA order increases, the delay yielding the minimum prediction error shifts systematically toward larger values [Fig. 3(a)–(f)]. The extracted optimal delays closely track the explicit product lag $n - 1$ across the tested orders [Fig. 3(g)], supporting the condition in Eq. (14). This trend shows that the prediction improves when the coherent memory return is positioned near a temporal correlation explicitly required by the target dynamics. Delays that are too short return information before the relevant past input is required, whereas excessively long delays shift

the strongest memory revival beyond this task-relevant lag. This interpretation is independently supported by the delayed-product capacity evaluated at $d = n - 1$. Its delay dependence selects the same feedback region as the NARMA error minimum, as shown in Supplementary Section S7. The agreement connects the task-level prediction optimum directly to the nonlinear memory feature identified by the delay-resolved memory benchmarks.

Control comparisons separate the roles of coherent memory placement and nonlinear processing. The TDLRC retains a clear delay-dependent error landscape, demonstrating that the temporal placement of memory alone already affects prediction. However, because its first-order observables remain linear functions of the input history, it cannot efficiently generate the nonlinear delayed features required by the NARMA product term. At matched delay, the TDQRC therefore achieves lower errors, with the advantage becoming more pronounced as the NARMA order increases. The zero-delay QRC represents the complementary limit: it retains Kerr nonlinearity but lacks a temporally separated coherent memory return. Its error increases rapidly with NARMA order [Fig. 3(h)], showing that local nonlinear processing alone is insufficient when the task requires longer-range temporal correlations. High-order NARMA prediction thus benefits from the coexistence of geometry-controlled memory return and Kerr-induced nonlinear mixing.

Additional baselines test whether the observed improvement can be attributed solely to generic classical nonlinearity or reservoir size. Under the same delayed-feedback kernel and readout protocol, a scalar classical nonlinear resonator (CRC) does not reproduce the full nonlinear delayed-memory response of the TDQRC; detailed comparisons are provided in Supplementary Section S8. Standard echo-state networks (ESNs) [1] remain competitive at low NARMA orders, but their prediction errors increase more rapidly with task order under the resource and parameter settings considered here. Among the tested configurations, the matched-delay TDQRC gives the best high-order NARMA performance [Fig. 3(h)].

These results establish that NARMA prediction performance is governed not by memory depth alone, but by the alignment between the temporal structure of the reservoir memory and

that of the target task. For NARMA- n , the explicit input-product lag provides a transparent design target, and the optimal feedback delay follows this lag closely. The broader Mackey–Glass forecasting task, analyzed in Supplementary Section S9, contains distributed rather than single-lag temporal correlations. Its delay dependence is correspondingly less sharply localized, but it likewise benefits from the combination of coherent delayed feedback and Kerr nonlinear processing.

Discussion

We have demonstrated that coherent delayed feedback in waveguide QED provides a direct physical mechanism for engineering the temporal memory structure of a quantum reservoir. Information about past inputs is carried away from the nonlinear resonator by an unmeasured propagating field and coherently returned after a geometry-controlled propagation time. The waveguide therefore determines when past information becomes available again, while the local Kerr interaction determines how this returned information is transformed into nonlinear temporal features accessible through a linear readout. Unlike measurement-based feedback schemes, this memory channel requires neither intermediate detection nor classical reinjection.

The delay-resolved STM and delayed-product benchmarks directly reveal these complementary roles. The primary linear-memory revival appears near $d_\tau \simeq \tau/\Delta t$, showing that the temporal position of the returned information is controlled by the propagation delay. The corresponding delayed-product response shows that Kerr dynamics convert this coherently returned information into nonlinear features involving both past and current inputs. These observations lead to a task-matching principle: reservoir performance depends not only on the total amount or depth of memory, but also on whether the memory is available at lags relevant to the target dynamics. The NARMA- n results provide a direct demonstration of this principle. The optimal feedback delay closely follows the matching condition in Eq. (14). The agreement between the task lag, the delayed-product-capacity maximum, and the minimum prediction error connects the computational optimum to a

specific physical feature of the reservoir memory kernel. The resulting improvement is therefore not attributable to longer memory alone: it arises from aligning a coherent memory return with a task-relevant nonlinear temporal dependence.

The present study focuses on a minimal single-delay geometry, first-order quadrature readout, and a local Kerr nonlinearity. These choices make it possible to isolate the elementary mechanisms of coherent memory transport and nonlinear memory processing, but they do not exhaust the design space of waveguide-QED reservoirs. Multi-point coupling can generate several independently structured return times, while propagation phases and coupling asymmetries provide additional control over the amplitudes and interference of the corresponding memory channels. Loss, dephasing, and imperfect feedback will determine how much of this coherent memory remains accessible in realistic devices. More generally, networks of coupled non-Markovian nodes may support distributed and hierarchical memory kernels that cannot be realized with a single resonator.

Finally, waveguide-QED and giant-atom platforms are particularly suited to this approach because spatially separated coupling points, finite propagation times, interference phases, and strong local nonlinearities can be engineered within the same architecture. In particular, superconducting circuit-QED platforms [40, 53–55] provide a natural implementation route, where Josephson Kerr resonators, giant artificial atoms, and microwave waveguides can be integrated with high tunability and high-fidelity quadrature readout. Our results therefore establish geometry-programmed coherent memory as a hardware-level design principle for non-Markovian quantum information processing, with QRC providing one concrete application.

Methods

Delay-differential equation for the linear reservoir

To separate coherent memory transport from nonlinear memory processing, we employ two complementary numerical descriptions for the linear and nonlinear reservoirs introduced in the main text. For the time-delay linear reference reservoir (TDLRC), we replace the resonator operator in Eq. (2) by a complex field amplitude $a(t) \rightarrow \alpha(t)$

and set the Kerr nonlinearity to zero. For the two-point geometry with constructive feedback, the resulting delay-differential equation is

$$\dot{\alpha}(t) = -\frac{\Gamma}{2}\alpha(t) - \frac{\sqrt{\gamma_1\gamma_2}}{2}\alpha(t-\tau) + \Omega(t), \quad (15)$$

with the same memory-kernel convention as in Eq. (3). This description retains the effect of waveguide propagation through the delayed amplitude $\alpha(t-\tau)$, while eliminating the waveguide degrees of freedom explicitly.

Time-bin matrix-product-state simulation

For the time-delay Kerr quantum reservoir (TDQRC), we simulate the joint resonator-waveguide quantum dynamics using the time-bin matrix-product-state (MPS) method of Ref. [46], implemented with the MPS libraries of Refs. [47, 48]. Unlike the delay-differential description, the time-bin MPS explicitly retains the propagating waveguide degrees of freedom that carry the coherent non-Markovian memory. The waveguide field is discretized into temporal modes of duration Δt_{bin} . A propagation delay is represented by an integer number of time bins, $\tau_{\mu\nu} = \ell_{\mu\nu}\Delta t_{\text{bin}}$, so that a field bin emitted at one coupling point reinteracts with the resonator after $\ell_{\mu\nu}$ time steps.

In the time-bin representation, the waveguide is mapped onto an ordered chain of field modes. At each time step, the resonator interacts with the bin, or bins, arriving at its coupling points. The relevant field modes are then advanced along the MPS chain, and the state is recompressed by singular value decomposition. This construction explicitly retains the temporally separated field modes that mediate coherent delayed feedback. Reservoir features are obtained from resonator observables and from outgoing time-bin observables after their final interaction with the resonator.

For the symmetric constructive-feedback geometries considered in the main text, we use an equivalent folded chiral representation to reduce the numerical cost. In this construction, the left- and right-propagating waveguide modes are mapped onto a single effective chiral channel while preserving the coherent delayed interaction. For the two-point geometry, the resulting model

is equivalent to a unidirectional delay line, or to a Kerr resonator coupled to a semi-infinite waveguide terminated by a mirror [46].

Unless stated otherwise, the TDQRC simulations use a time-bin duration $\Delta t_{\text{bin}} = 0.1$, a local time-bin cutoff $d_{\text{bin}} = 4$, a maximum MPS bond dimension $\chi = 24$, and a resonator Hilbert-space cutoff $d_{\text{sys}} = 12$. Numerical convergence was verified with respect to Δt_{bin} , d_{bin} , χ , and d_{sys} . The reported truncation parameters were chosen such that further increases produce no appreciable changes in the reservoir observables or benchmark performance.

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Data availability. Data are available from the authors upon reasonable request.

Code availability. Code is available from the authors upon reasonable request.

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References

- [1] Jaeger, H. The “echo state” approach to analysing and training recurrent neural networks – with an Erratum note. *Bonn Ger. Ger. Natl. Res. Cent. Inf. Technol. Gmd Tech. Rep.* **148**, 13 (2001).
- [2] Maass, W., Natschläger, T. & Markram, H. Real-Time Computing Without Stable States: A New Framework for Neural Computation Based on Perturbations. *Neural Comput.* **14**, 2531–2560 (2002).
- [3] Appeltant, L. *et al.* Information processing using a single dynamical node as complex system. *Nat. Commun.* **2**, 468 (2011).
- [4] Larger, L. *et al.* High-Speed Photonic Reservoir Computing Using a Time-Delay-Based Architecture: Million Words per Second Classification. *Phys. Rev. X* **7**, 011015 (2017).
- [5] Tanaka, G. *et al.* Recent advances in physical reservoir computing: A review. *Neural Networks* **115**, 100–123 (2019).
- [6] Hülser, T., Köster, F., Jaurigue, L. & Lüdge, K. Role of delay-times in delay-based photonic reservoir computing [Invited]. *Opt. Mater. Express* **12**, 1214–1231 (2022).
- [7] Zhang, M., Liang, Z. & Huang, Z. R. Hardware optimization for photonic time-delay reservoir computer dynamics. *Neuromorph. Comput. Eng.* **3**, 014008 (2023).
- [8] Liang, X. *et al.* Physical reservoir computing with emerging electronics. *Nat. Electron.* **7**, 193–206 (2024).
- [9] Ren, H. *et al.* Photonic time-delayed reservoir computing based on series-coupled microring resonators with high memory capacity. *Opt. Express* **32**, 11202 (2024).
- [10] Iacob, S. & Dambre, J. Exploiting Signal Propagation Delays to Match Task Memory Requirements in Reservoir Computing. *Biomimetics* **9**, 355 (2024).
- [11] Lugnan, A., Biasi, S., Foradori, A., Bienstman, P. & Pavesi, L. Reservoir Computing with All-Optical Non-Fading Memory in a Self-Pulsing Microresonator Network. *Adv. Opt. Mater.* **13**, 2403133 (2025).
- [12] Govia, L. C. G., Ribeill, G. J., Rowlands, G. E., Krovi, H. K. & Ohki, T. A. Quantum reservoir computing with a single nonlinear oscillator. *Phys. Rev. Res.* **3**, 013077 (2021).
- [13] Mujal, P. *et al.* Analytical evidence of nonlinearity in qubits and continuous-variable quantum reservoir computing. *J. Phys. Complex.* **2**, 045008 (2021).
- [14] Kalfus, W. D. *et al.* Hilbert space as a computational resource in reservoir computing. *Phys. Rev. Res.* **4**, 033007 (2022).
- [15] Spagnolo, M. *et al.* Experimental photonic quantum memristor. *Nat. Photon.* **16**, 318–323 (2022).
- [16] Bravo, R. A., Najafi, K., Gao, X. & Yelin, S. F. Quantum Reservoir Computing Using Arrays of Rydberg Atoms. *PRX Quantum* **3**, 030325 (2022).
- [17] Senanian, A. *et al.* Microwave signal processing using an analog quantum reservoir computer. *Nat. Commun.* **15**, 7490 (2024).
- [18] Hou, Y. *et al.* High-Accuracy Temporal Prediction via Experimental Quantum Reservoir Computing in Correlated Spins. *Phys. Rev. Lett.* **136**, 120602 (2026).
- [19] Zhu, C., Ehlers, P. J., Nurdin, H. I. & Soh, D. Minimalistic and scalable quantum reservoir computing enhanced with feedback. *npj Quantum Inf.* **11**, 195 (2025).
- [20] Zhu, C., Ehlers, P. J., Nurdin, H. I. & Soh, D. Practical few-atom quantum reservoir computing. *Phys. Rev. Res.* **7**, 023290 (2025).
- [21] Das, S., Giorgi, G. L. & Zambrini, R. Quantum reservoir computing in Jaynes-Cummings models: Nonlinear memory and

- time-series prediction. *Phys. Rev. Res.* **8**, 023148 (2026).
- [22] Prieto-Garcia, J. J., del Pozo-Martín, A. G. & Pino, M. Quantum Reservoir Computing for Statistical Classification in a Superconducting Quantum Circuit (2026). [arXiv:2602.15474](#).
- [23] Kobayashi, K. & Motome, Y. Edge of Many-Body Quantum Chaos in Quantum Reservoir Computing. *Phys. Rev. Lett.* **136**, 040602 (2026).
- [24] Paparelle, I. *et al.* Experimental memory control in continuous-variable optical quantum reservoir computing. *Nat. Photon.* **20**, 413–420 (2026).
- [25] Zhang, D. *et al.* Robust and efficient quantum reservoir computing with a discrete time crystal. *Phys. Rev. Appl.* **26**, 014056 (2026).
- [26] Cucchi, M., Abreu, S., Ciccone, G., Brunner, D. & Kleemann, H. Hands-on reservoir computing: a tutorial for practical implementation. *Neuromorph. Comput. Eng.* **2**, 032002 (2022).
- [27] Mujal, P., Martínez-Peña, R., Giorgi, G. L., Soriano, M. C. & Zambrini, R. Time-series quantum reservoir computing with weak and projective measurements. *npj Quantum Inf.* **9**, 16 (2023).
- [28] Kobayashi, K., Fujii, K. & Yamamoto, N. Feedback-Driven Quantum Reservoir Computing for Time-Series Analysis. *PRX Quantum* **5**, 040325 (2024).
- [29] Sannia, A., Rodríguez, R. R., Giorgi, G. L. & Zambrini, R. Non-markovianity and memory enhancement in quantum reservoir computing. *npj Quantum Inf.* **12**, 111 (2026).
- [30] te Vrugt, M. in *An introduction to reservoir computing* (ed. te Vrugt, M.) *Artificial Intelligence and Intelligent Matter* Machine Intelligence for Materials Science, 53–66 (Springer, Cham, 2026).
- [31] Fette, G. & Eggert, J. Duch, W., Kacprzyk, J., Oja, E. & Zadrożny, S. (eds) *Short term memory and pattern matching with simple echo state networks*. (eds Duch, W., Kacprzyk, J., Oja, E. & Zadrożny, S.) *Artificial Neural Networks: Biological Inspirations – ICANN 2005*, Vol. 3696 of *Lecture Notes in Computer Science*, 13–18 (Springer, Berlin, Heidelberg, 2005).
- [32] Iacob, S. & Dambre, J. Memory–nonlinearity trade-off in distance-based delay networks. *Biomimetics* **9**, 755 (2024).
- [33] Hülser, T., Köster, F., Lüdge, K. & Jaurigue, L. Deriving task specific performance from the information processing capacity of a reservoir computer. *Nanophotonics* **12**, 937–947 (2023).
- [34] Donati, G., Argyris, A., Mancinelli, M., Mirasso, C. R. & Pavesi, L. Time delay reservoir computing with a silicon microring resonator and a fiber-based optical feedback loop. *Opt. Express* **32**, 13419 (2024).
- [35] Mullarkey, P. & Marzen, S. New analytic formulae for memory and prediction functions in reservoir computers with time delays. *Chaos Interdiscip. J. Nonlinear Sci.* **35**, 103127 (2025).
- [36] Di Bartolo, R. *et al.* Time-series forecasting with multiphoton quantum states and integrated photonics. *npj Quantum Inf.* **12**, 91 (2026).
- [37] Sasaki, D. *et al.* Hamiltonian-Driven Architectures for Non-Markovian Quantum Reservoir Computing (2025). [arXiv:2505.14450](#).
- [38] Kockum, A. F., Johansson, G. & Nori, F. Decoherence-Free Interaction between Giant Atoms in Waveguide Quantum Electrodynamics. *Phys. Rev. Lett.* **120**, 140404 (2018).
- [39] Andersson, G., Suri, B., Guo, L., Aref, T. & Delsing, P. Non-exponential decay of a giant artificial atom. *Nat. Phys.* **15**, 1123–1127 (2019).

- [40] Kannan, B. *et al.* Waveguide quantum electrodynamics with superconducting artificial giant atoms. *Nature* **583**, 775–779 (2020).
- [41] Vadiraj, A. M. *et al.* Engineering the level structure of a giant artificial atom in waveguide quantum electrodynamics. *Phys. Rev. A* **103**, 023710 (2021).
- [42] Xiao, L. *et al.* Giant-atom quantum acoustodynamics in hybrid superconducting-phononic integrated circuits (2025). [arXiv:2512.16582](https://arxiv.org/abs/2512.16582).
- [43] Barahona-Pascual, C., Jiang, H., Santos, A. C. & García-Ripoll, J. J. Time-delayed collective dynamics in waveguide QED and bosonic quantum networks. *Quantum Sci. Technol.* **11**, 015016 (2026).
- [44] Grimsmo, A. L. Time-Delayed Quantum Feedback Control. *Phys. Rev. Lett.* **115**, 060402 (2015).
- [45] Chen, G. & Kockum, A. F. Efficient three-qubit gates with giant atoms (2026). [arXiv:2510.04545](https://arxiv.org/abs/2510.04545).
- [46] Pichler, H. & Zoller, P. Photonic Circuits with Time Delays and Quantum Feedback. *Phys. Rev. Lett.* **116**, 093601 (2016).
- [47] Regidor, S. A., Kozma, M. & Hughes, S. QwaveMPS: An efficient open-source Python package for simulating non-Markovian waveguide-QED using matrix product states (2026). [arXiv:2602.15826](https://arxiv.org/abs/2602.15826).
- [48] García-Molina, P., Rodríguez-Aldavero, J. J., Gidi, J. & García-Ripoll, J. J. SeeMPS: A Python-based Matrix Product State and Tensor Train Library (2026). [arXiv:2601.16734](https://arxiv.org/abs/2601.16734).
- [49] Dambre, J., Verstraeten, D., Schrauwen, B. & Massar, S. Information Processing Capacity of Dynamical Systems. *Sci. Rep.* **2**, 514 (2012).
- [50] Verstraeten, D., Dambre, J., Dutoit, X. & Schrauwen, B. Memory versus non-linearity in reservoirs. In Proceedings of the 2010 International Joint Conference on Neural Networks (IJCNN), pp. 1–8, IEEE (2010). URL <https://doi.org/10.1109/IJCNN.2010.5596492>.
- [51] Xia, J., Chu, J., Leng, S. & Ma, H. Reservoir computing decoupling memory–nonlinearity trade-off. *Chaos* **33**, 113120 (2023).
- [52] Čindrak, S., Giebeler, L., Götting, N., Gies, C. & Lüdge, K. Memory-Nonlinearity Trade-off across Quantum Reservoir Computing Frameworks (2026). [arXiv:2603.21371](https://arxiv.org/abs/2603.21371).
- [53] Bertet, P. *et al.* *Circuit quantum electrodynamics with a nonlinear resonator* (Oxford University Press, 2012).
- [54] Frisk Kockum, A., Delsing, P. & Johansson, G. Designing frequency-dependent relaxation rates and lamb shifts for a giant artificial atom. *Phys. Rev. A* **90**, 013837 (2014). URL <https://link.aps.org/doi/10.1103/PhysRevA.90.013837>.
- [55] Wu, Z.-W. & An, J.-H. Strongly coupled giant-atom waveguide quantum electrodynamics. *Phys. Rev. A* **113**, 053724 (2026). URL <https://link.aps.org/doi/10.1103/11fq-gbbl>.

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