Supplementary Material for "Stratospheric enhancement of tropical cyclone high-level warm cores"

S1 Sensitivity analysis for SII

The classification of intruding and non-intruding TCs, presented in Section 2.2, is based on SII values computed within a circle of radius 2.5° centred on the TC. This radius was determined through a sensitivity analysis examining how the number of TCs classified as intruding changes as the radius used to compute SII increases (Fig.S1). We selected 2.5° as the threshold radius because it is the last radius for which the increase in the number of intruding TCs between adjacent radii exceeds 10 TCs per grid-point step. Beyond this radius, further enlarging the domain increases the potential for false positives in the classification.

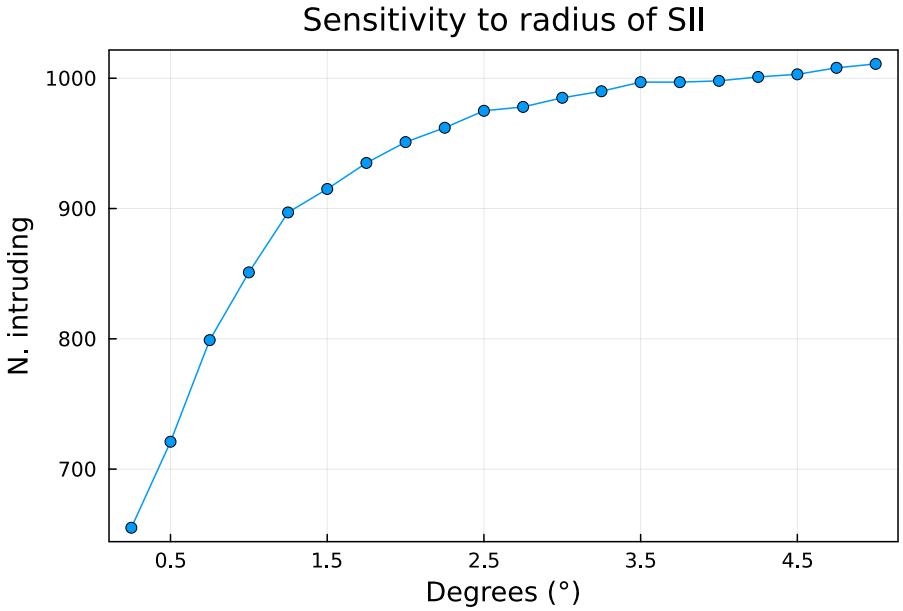


Figure S1: Sensitivity analysis for SII: number of TCs classified as intruding as a function of the radius of the area considered around the storm.

S2 Global distribution of HWC and non-HWC storms

Cyclogenesis location are found to be similar across basins, with a SE-HWC occurrence rate ranging from the 40% of EP basin, to the 50% of the WP basin.

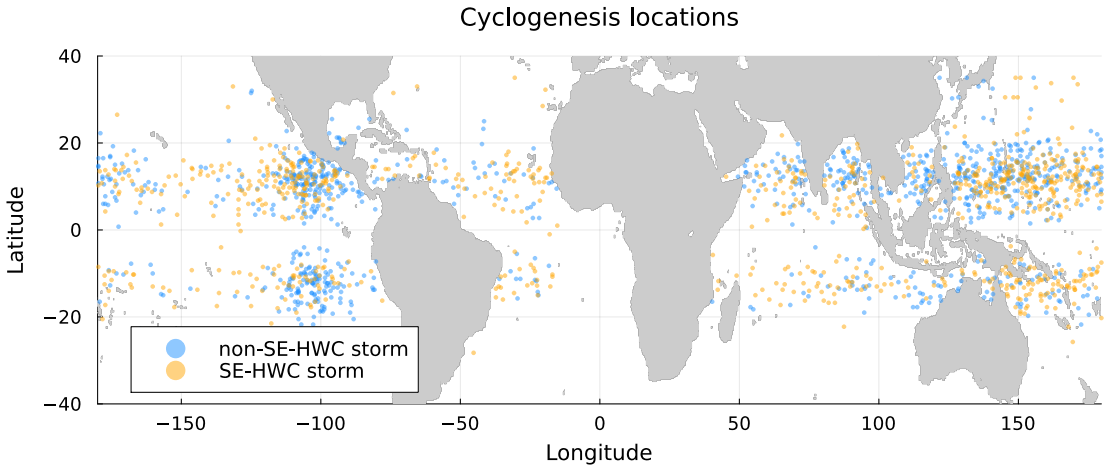


Figure S2: Distribution of the cyclogenesis locations of the 2112 cyclones analysed in the main text. Each dot represents a single TC.

S3 Mean *SST*

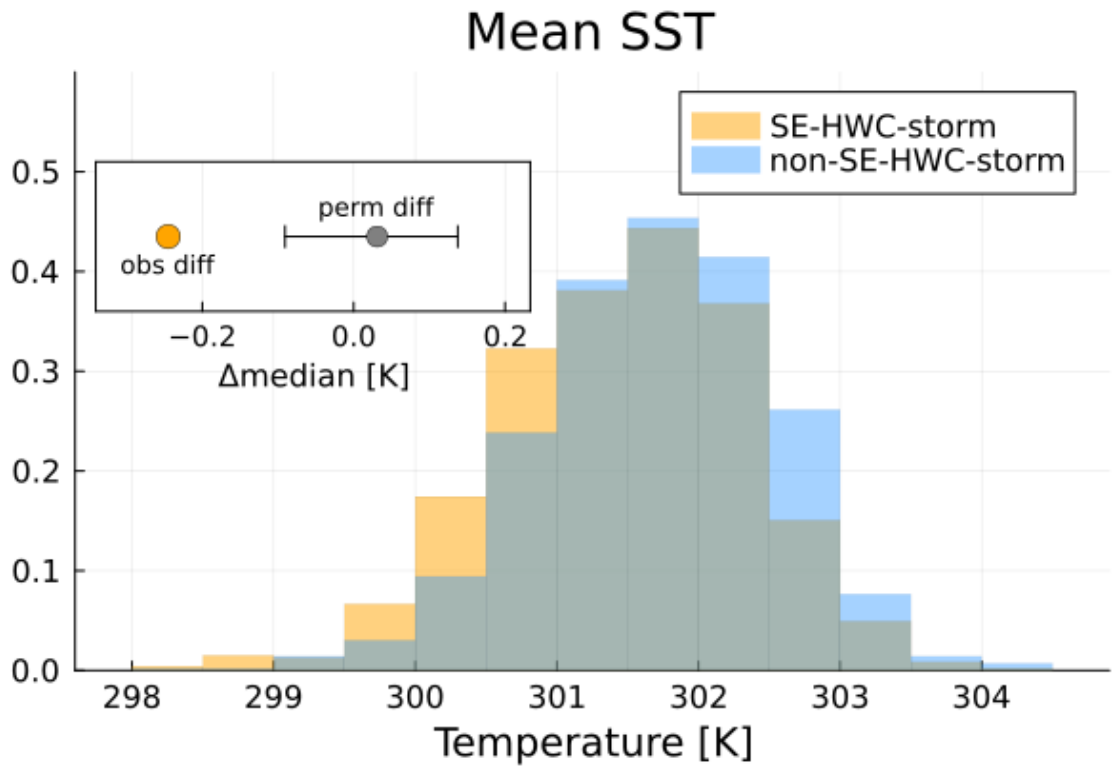


Figure S3: As in Fig. 3 of the main text, for the time-averaged *SST* between TS onset and peak intensity.

S4 Pre-storm environmental conditions at maximum PI

According to Emanuel's formulation of potential intensity, PI depends on SST, T_o, and Δk. Of these three variables, only SST exhibits a statistically significant difference at the time of maximum PI, with HWC storms developing over slightly colder oceans (approximately 0.15 K).

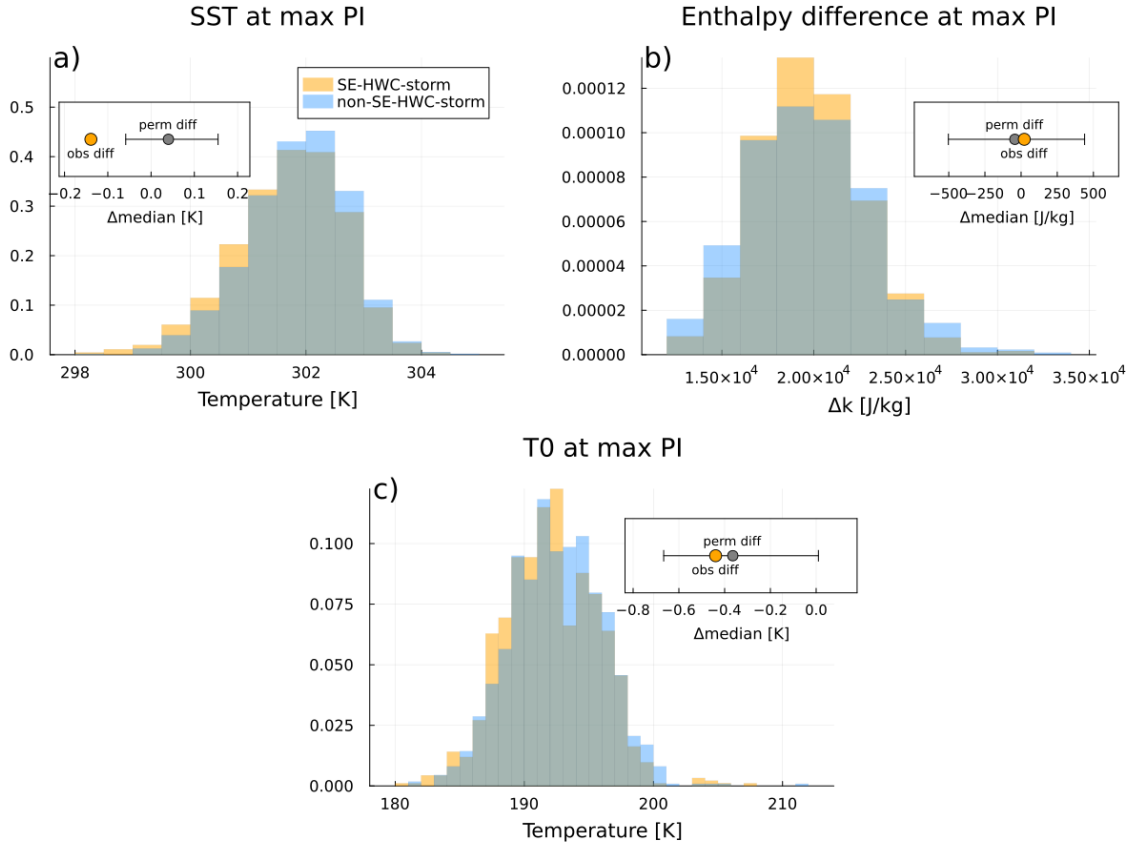


Figure S4: As in Fig. 3 of the main text, for (a) SST, (b) Δk, and (c) T_o, all evaluated at the time of maximum PI.

S5 Complement to mass flow rate

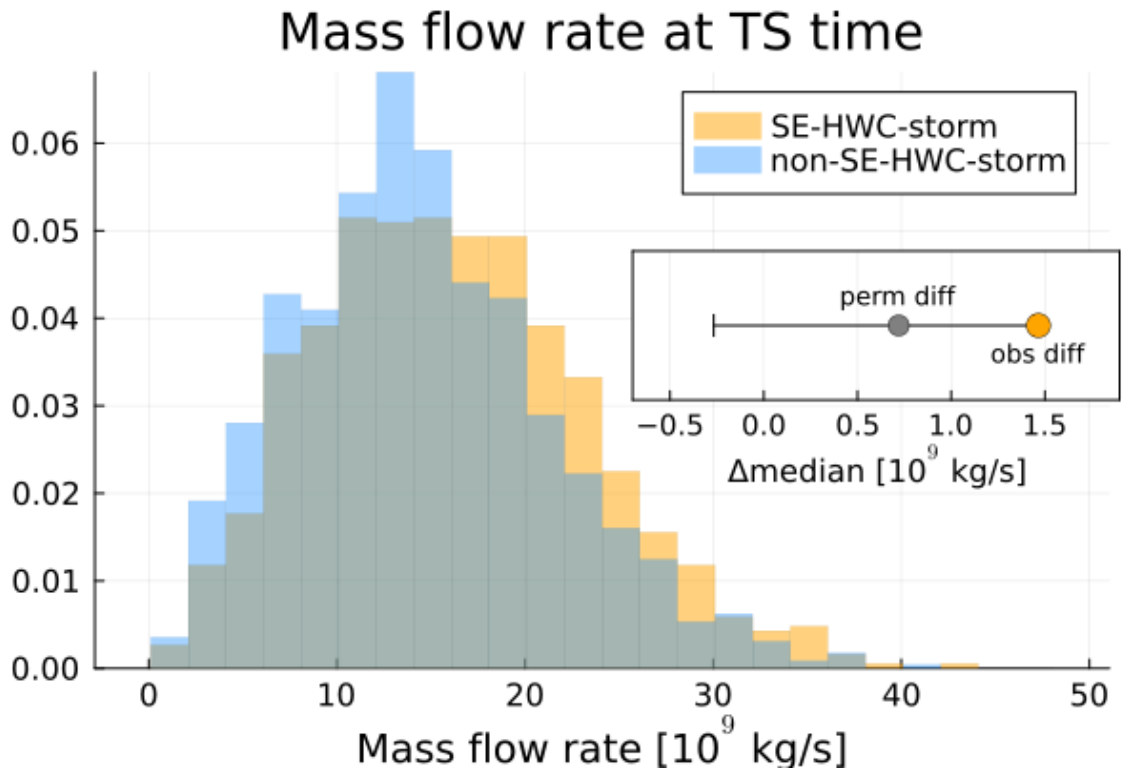


Figure S5: As in Fig. 3 of the main text, for the mass flow rate at TS time.

To assess whether the larger F_{max} values of HWC-storms stem from a larger area over which F is computed (i.e., the convective area) or from a more intense mass flux, we compared the convective area and the mean mass flux between the two classes. Both variables are significantly larger for HWC-storms (Fig. S6). To quantify their relative importance, we expressed the class differences as ratios relative to the mean value over the whole population of 2112 TCs, and compared them with the relative difference in F_{max}. The F_{max} of HWC-storms is about 25% larger than that of non-HWC-storms. This difference is associated with a convective area about 7% larger and a mean mass flux about 18% more intense, indicating that the strength of the updrafts contributes more than the extent of the convective area to the larger F_{max} values.

S6 Permutation analysis by category

The longer intensification phase of SE-HWC-storms could simply result from the higher intensities they achieve, since reaching a higher peak generally requires more time. To disentangle the two quantities, we repeated the permutation test described in Section 2.4 separately for each TC category, shuffling the class labels among TCs of the same category regardless of their basin and year

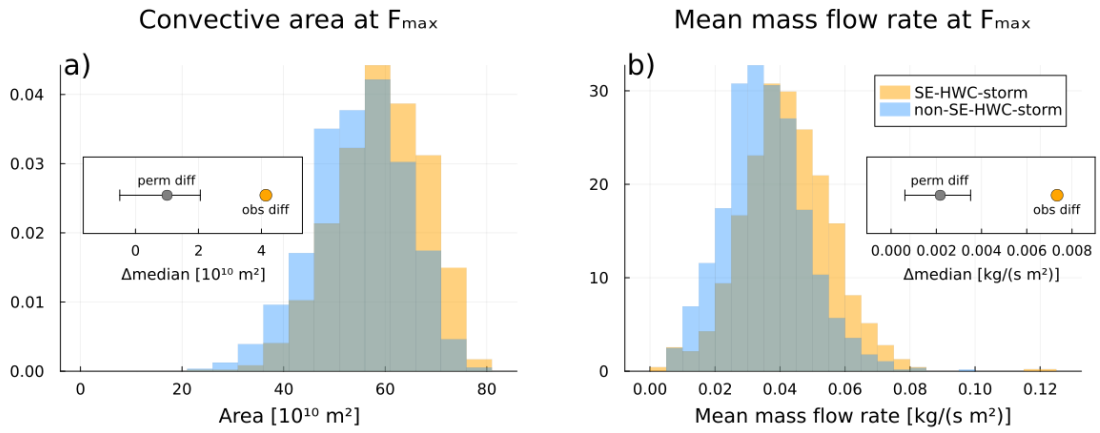


Figure S6: As in Fig. 3 of the main text, for (a) convective area and (b) mean mass flow rate, both at time of peak mass flow rate.

of occurrence.

As expected, this removed the v_{max} difference between SE-HWC-storms and non-SE-HWC-storms in all categories except tropical storms (Table 1). This exception is likely due to the large number of non-SE-HWC-storms concentrated at the weakest end of TS category.

The intensification phase, instead, remained significantly longer for SE-HWC-storms in every category, with differences ranging from 0.75 days for tropical storms and Category 1-3 TCs to more than one day for Category 4 and 5 TCs. Within the same category, SE-HWC-storms therefore reached their peak intensity over a longer time than non-SE-HWC-storms, indicating that their longer intensification phase is not a mere consequence of their higher intensity.

	TS	**Cat. 1**	**Cat. 2**	**Cat. 3**	**Cat. 4**	**Cat. 5**
n (SE-HWC)	798 (154)	364 (173)	203 (128)	217 (133)	374 (258)	155 (120)
v_{max} **(m/s)**						
Observed diff.	5.14	0.00	0.00	0.00	0.00	0.00
Null mean	−0.49	0.04	0.00	0.11	0.39	−0.61
p-value	<0.0001 ✓	1.0 ✗	1.0 ✗	1.0 ✗	0.981 ✗	1.0 ✗
Time to peak (days)						
Observed diff.	0.75	0.75	0.75	0.75	1.13	1.25
Null mean	−0.06	−0.01	0.03	−0.01	0.02	−0.05
p-value	<0.0001 ✓	<0.0001 ✓	0.0013 ✓	0.0006 ✓	<0.0001 ✓	0.0002 ✓

Table 1: Results of the permutation test stratified by TC category. For each category, n is the total number of TCs, with the number of SE-HWC-storms in brackets. *Observed diff.* is the difference between the class medians (SE-HWC-storms minus non-SE-HWC-storms), *Null mean* is the mean difference of the empirical null distribution obtained from 10^4 label permutations within each category, and the p-value is two-sided. ✓ and ✗ indicate significance and non-significance at the 99% confidence level. The null differences in v_{max} reflect identical class medians, likely owing to the 5-kt resolution of best-track winds.