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Energy Use Efficiency in G20 Economies under Carbon Emissions Constraints: A Variable-Returns-to-Scale Allocative Luenberger Indicator Approach

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Energy Use Efficiency in G20 Economies under Carbon Emissions Constraints: A Variable-Returns-to-Scale Allocative Luenberger Indicator Approach

Abstract

Energy-use efficiency is an important indicator of primary-energy performance, but it does not show whether energy, labor, and capital are moving toward a minimum-value feasible input mix. The objective of this study is to assess energy-use efficiency and factor-allocation change jointly for 18 Group of Twenty (G20) economies over 2008–2022. We develop a biennial variable-returns-to-scale Allocative Luenberger indicator that evaluates adjacent years against a common production technology while holding carbon emissions at observed levels. Using country-year valuation coefficients on a common purchasing-power-parity basis, the indicator decomposes full-input benchmark-value change into value-share-weighted technical and benchmark-relative allocation components; a matched energy-only direction measures change in the energy gap. Under the baseline input definitions, equal-country cumulative energy-use efficiency change is 0.0389. Full-input benchmark-value change is 0.0607, comprising a technical Luenberger component of 0.0889 and an allocation component of −0.0282. These baseline results show improvements in energy-use efficiency and full-input technical performance over the sample period, whereas factor allocation moved away from the benchmark input mix. The allocation result is sensitive to input definitions, aggregation choices, and frontier composition.

Keywords: energy-use efficiency; factor allocation; Allocative Luenberger indicator; energy policy; G20 economies

1 Introduction

National energy assessments commonly ask how much primary energy an economy could save without reducing output. Carbon emissions make this comparison less direct. Economies with similar energy use and output may have different emissions profiles, while lower emissions do not necessarily imply more efficient energy use. Our comparison measures feasible energy contraction at the observed levels of desirable output and carbon emissions [25, 26].

Studies of G20 energy performance generally report eco-efficiency, total-factor energy efficiency, or environmental productivity [24–26]. These measures locate an economy relative to a production frontier. They do not show whether its joint use of energy, labor, and capital is moving toward a minimum-value feasible input mix under country-year-specific valuation vectors expressed on a common purchasing-power-parity (PPP) basis. That is an allocative question rather than an energy-efficiency question.

Energy efficiency and factor allocation need not move together. An economy may narrow its energy gap while moving farther from the benchmark input mix; it may also improve its input allocation without narrowing the energy gap. We examine both types of change for 18 G20 member economies over 2008–2022 and compare their movements across adjacent country-year observations.

The G20 provides a useful setting because it combines major developed and emerging economies with markedly different production scales, energy systems, and emissions profiles. Accordingly, we ask two empirical questions. First, how did the feasible energy-contraction gap change across the sample period when output and carbon emissions are held at their observed levels? Second, did improvements in that energy gap coincide with movements toward a lower-value feasible allocation of energy, labor, and capital?

We formulate a two-valuation, biennial variable-returns-to-scale (VRS) Allocative Luenberger indicator for national energy accounts. For each pair of adjacent years, observations from both periods form a common VRS reference technology, with carbon emissions held at the level observed for the economy being evaluated. The non-radial specification allows energy, labor, and capital to contract separately. Country-year-specific benchmark input values expressed on a common PPP basis make the full input bundle comparable. The emissions equality is a conditioning device, not a policy carbon cap.

The two questions are implemented through matched directional specifications. With $\mathbf{g}_E = (E, 0, 0)'$, only energy contracts, and the resulting local biennial indicator LPI_E measures change in the energy gap. With $\mathbf{g}_F = \mathbf{x}$, all three inputs can contract. The Allocative Luenberger indicator is the two-valuation average change in benchmark-relative allocative inefficiency on the common biennial technology. The reported full-input measures satisfy

$$CL_F = LPI_F + AL_F, \tag{1}$$

where CL_F is overall benchmark-value change, LPI_F is a value-share-weighted technical Luenberger component, and AL_F is benchmark-relative factor-allocation change.

The methodological contribution is a formulation of benchmark-relative allocation change on a common local-biennial VRS technology. Lin et al.'s cost-oriented Luenberger measure embeds allocation change within a broader cost-productivity decomposition [17]. Our formulation makes benchmark-relative allocation change the object of measurement and evaluates it symmetrically from the two endpoint valuation views on a local-biennial VRS technology. Observed-emissions conditioning and a common-PPP valuation basis make the construction usable for national energy accounts. A matched energy-only direction then permits direct comparison between change in the energy gap and change in the joint allocation of energy, labor, and capital. The empirical design tests input definitions, emissions treatment, aggregation, sample period, and frontier composition.

Under the baseline input definitions, equal-country cumulative energy-efficiency change from 2008 to 2022 is 0.0389. Overall full-input benchmark-value change is 0.0607, comprising a technical Luenberger component of 0.0889 and allocation change of −0.0282. Capital services and total labor hours retain this sign pattern, but current-PPP capital stock does not: LPI_E and LPI_F are approximately zero and AL_F is positive. CL_F remains positive across all four input definitions and all 18 one-country frontier re-estimations. The dense frontier and strong self-reference restrict these findings to descriptive accounting for the fixed sample rather than a country-performance ranking.

The remainder of the paper proceeds as follows. Section 2 positions the study in the energy-efficiency, allocative Luenberger, and biennial-technology literatures. Section 3 presents the indicators, data, valuation basis, and validation design. Section 4 reports the energy, allocation, alignment, and model-comparison results. Sections 5 and 6 discuss their interpretation and state the conclusions and limitations.

2 Related Literature

2.1 National Energy-Use Efficiency with Undesirable Outputs

Directional distance functions measure feasible input contraction in a chosen direction while desirable output is maintained [2, 3]. Environmental DEA extends this construction by including undesirable outputs in the production technology [5, 8]. The resulting framework supports both input-specific energy-efficiency measures and environmental productivity indices [19, 27, 30, 31].

Country studies do not always measure the same object. Sadorsky [24] examines eco-efficiency trends for the G18, whereas Shah et al. [25, 26] study energy-efficiency levels, technology gaps, and productivity change across G20 economies. We use the narrower energy-specific measure: feasible contraction of primary energy at the observed levels of output and carbon emissions. Movement of the complete energy–labor–capital bundle is treated as a separate allocative question.

The closest national studies differ in the object and reference technology. Sadorsky [24] estimates CRS eco-efficiency and Malmquist change for the G18 over 1997–2019 before forecasting to 2040. Shah et al. [26] use super-SBM, metafrontier, and Malmquist–Luenberger models for developed and developing G20 economies over 1995–2020, with the technology gap as a central decomposition. Their companion study [25] compares four renewable-energy, non-renewable-energy, and carbon-emissions input–output bundles over the same period. None of these three studies uses input valuation vectors to separate benchmark-value and factor-allocation change. Our 2008–2022 comparison instead uses adjacent-year VRS technologies, conditions the baseline frontier on observed emissions, and reports energy-only and value-share-weighted full-input directions for the same observations.

The national samples also differ despite both being described as groups of 18 G20 economies. Sadorsky [24] includes Türkiye and omits Saudi Arabia, whereas the harmonized panel used here omits Türkiye and includes Saudi Arabia. The results therefore differ in measurement object, reference technology, sample period, and country composition; they should not be interpreted as estimates of the same G18 index in different years.

2.2 Cost and Allocative Luenberger Measurement

Allocative analysis combines the production technology with a valuation vector. Cost-efficiency models compare observed input expenditure with the minimum expenditure needed to produce a given output under specified prices [1, 11]. The difference between cost and technical inefficiency gives a benchmark-relative allocation gap; dynamic measures compare these gaps across periods [12, 28]. In a non-radial setting, valuation shares can aggregate input-specific contractions rather than impose one proportional contraction on every input [21].

Lin et al. [17] use this non-radial structure to construct a cost Luenberger productivity indicator with input contractions weighted by observed cost shares. Their indicator identifies allocative-efficiency change within a broader cost-productivity decomposition. Work on non-radial profit performance and allocation indices likewise shows that the contraction rule affects the attribution of technical and allocative performance [16, 23, 32].

Building on these cost and allocative measurement approaches, we formulate a two-valuation Allocative Luenberger indicator on a common local-biennial VRS technology for national energy accounts. Emissions enter the reference technology as an observed-quantity equality, and country-year-specific valuation vectors are expressed on a common PPP basis. The resulting measure reports benchmark-relative allocation change while retaining an additive link to technical and overall benchmark-value change. It is distinct from the technology-gap ratios used to compare country groups.

2.3 Biennial Technologies, VRS, and Feasibility

The cross-period reference technology determines which observations define feasible productivity change. Conventional Malmquist programs may be infeasible under VRS, especially when undesirable outputs are held at observed levels [4, 6]. A global technology supplies one reference set by pooling all sample periods, but the resulting benchmark depends on observations that may be far from the transition being evaluated. The biennial approach instead pools two adjacent periods, placing both endpoints in the same reference set [14, 22].

Our energy-specific and full-input comparisons use the same adjacent-period technology. We use VRS because national economies operate at different scales and retain a matched CRS model for comparison.

These literatures motivate three connected design choices: distinguishing energy-specific change from benchmark-relative factor-allocation change, evaluating allocation change symmetrically from both endpoint valuations, and using a common biennial technology to retain endpoint feasibility. We combine these choices with common-PPP input valuations for national energy accounts through matched energy-only and full-input directions on the same emissions-conditioned local-biennial VRS technology.

3 Methodology and Empirical Design

We construct a standalone full-input Allocative Luenberger indicator from the directional Russell measure on a local biennial technology. Each pair of adjacent-year observations is evaluated against its pooled frontier. Annual frontiers are used for the energy-only efficiency–frontier split, while the full-input analysis uses the biennial technical-gap–allocative identity. The energy-only model is a second direction of the same measure.

3.1 Observed-Emissions-Conditioned Production Technology

Let $j = 1, \ldots, J$ index countries. Suppressing the country index, an observation in period t consists of a strictly positive input vector $\mathbf{x}^t \in \mathbb{R}^N_{++}$, desirable output $y^t \in \mathbb{R}_+$, carbon emissions $b^t \in \mathbb{R}_+$, and a strictly positive benchmark valuation-coefficient vector $\mathbf{w}^t \in \mathbb{R}^N_{++}$. In the application, $N = 3$ and $\mathbf{x}^t = (E^t, L^t, K^t)'$. For reference set R, define the production technology and input requirement set as

$$S^R = \{(\mathbf{x}, y, b) : \mathbf{x} \text{ can produce } (y, b) \text{ using observations in } R\}, \tag{2}$$

$$L^R(y, b) = \left\{\mathbf{x} : (\mathbf{x}, y, b) \in S^R\right\}. \tag{3}$$

The empirical analysis uses the VRS representation

$$S^R_{\text{VRS}} = \left\{(\mathbf{x}, y, b) : \mathbf{X}^R\boldsymbol{\lambda} \le \mathbf{x},\ \mathbf{Y}^R\boldsymbol{\lambda} \ge y,\ \mathbf{B}^R\boldsymbol{\lambda} = b,\ \mathbf{1}'\boldsymbol{\lambda} = 1,\ \boldsymbol{\lambda} \ge \mathbf{0}\right\}. \tag{4}$$

Throughout, S^R denotes S^R_{VRS} in Equation (4). The equality $\mathbf{B}^R\boldsymbol{\lambda} = b$ holds carbon emissions at the observed level; carbon is not freely disposable, and no separate abatement technology is specified. This is a conditioning restriction for comparison at the observed emissions quantity, not a carbon price, allowance, statutory ceiling, or policy counterfactual. The matched CRS model omits the convexity restriction $\mathbf{1}'\boldsymbol{\lambda} = 1$.

For an adjacent pair $(t, t+1)$, let

$$B(t, t+1) = \{t, t+1\}, \qquad S^B = S^{B(t,t+1)}, \tag{5}$$

Here S^B pools observations from t and $t+1$ under the VRS restrictions in Equation (4). Both endpoints are feasible in this common technology, and no other sample year enters the reference set.

3.2 Directional Russell, Cost, and Allocative Inefficiency

The directional Russell measure (DRM) accommodates different input-contraction directions in one program. For endpoint $s \in \{t, t+1\}$, valuation period $v \in \{t, t+1\}$, and reference technology S^R, choose a nonzero input direction $\mathbf{g}^s_x \in \mathbb{R}^N_+ \setminus \{\mathbf{0}\}$ and set the output directions to zero. The associated weights are

$$\mu_i^{s|v} = \frac{w_i^v g^s_{x_i}}{\sum_{k=1}^N w_k^v g^s_{x_k}}, \qquad \sum_{i=1}^N \mu_i^{s|v} = 1. \tag{6}$$

The superscript $s|v$ denotes endpoint s evaluated using the valuation vector from period v. Technical inefficiency in the selected direction is

$$\begin{aligned} TI_R^{s|v}(\mathbf{g}_x^s) &= \vec{R}^R(\mathbf{x}^s, y^s, b^s; \mathbf{g}_x^s, 0, 0 \mid \mathbf{w}^v) \\ &= \max_{\boldsymbol{\beta} \in \mathbb{R}_+^N} \left\{ \sum_{i=1}^N \mu_i^{s|v} \beta_i : (\mathbf{x}^s - \boldsymbol{\beta} \odot \mathbf{g}_x^s, y^s, b^s) \in S^R \right\}. \end{aligned} \tag{7}$$

The direction specifies which inputs contract, and each β_i is the corresponding contraction coefficient. The valuation vector enters through $\mu_i^{s|v}$, while feasibility remains defined by the physical quantities in S^R. A radial measure would restrict the inputs to one common contraction coefficient.

For the allocative analysis, we set the full-input direction to $\mathbf{g}_F^s = \mathbf{x}^s$. We write $TI_R^{s|v}$ for $TI_R^{s|v}(\mathbf{g}_F^s)$ from this point onward; the weights in Equation (6) are the value shares of the endpoint input bundle evaluated at the period-v valuation vector. They coincide with the observed benchmark-value shares when $s = v$; cross-period evaluations use the other endpoint's valuation vector.

For the same output and emissions, the value function selects the feasible input bundle with the lowest benchmark value:

$$c^R(\mathbf{w}^v, y^s, b^s) = \min_{\mathbf{x}} \left\{ \mathbf{w}^{v\prime}\mathbf{x} : (\mathbf{x}, y^s, b^s) \in S^R,\ \mathbf{w}^v > \mathbf{0} \right\}. \tag{8}$$

Under the full-input specification, $\mathbf{w}^{v\prime}\mathbf{x}^s$ is the common-PPP benchmark value of the observed bundle. Normalized cost inefficiency is

$$CI_R^{s|v} = \frac{\sum_{i=1}^N w_i^v x_i^s - c^R(\mathbf{w}^v, y^s, b^s)}{\sum_{i=1}^N w_i^v g_{x_i}^s}. \tag{9}$$

With $\mathbf{g}_x^s = \mathbf{x}^s$, Equation (9) reduces to $1 - c^R(\mathbf{w}^v, y^s, b^s)/(\mathbf{w}^{v\prime}\mathbf{x}^s)$. Allocative inefficiency is the gap between cost and technical inefficiency:

$$AI_R^{s|v} = CI_R^{s|v} - TI_R^{s|v}, \qquad CI_R^{s|v} = TI_R^{s|v} + AI_R^{s|v}. \tag{10}$$

3.3 Biennial Allocative Luenberger Indicator

All dynamic comparisons evaluate both endpoints on the common biennial technology S^B. For a fixed valuation period v, allocation change is

$$AL^v = AI_B^{t|v} - AI_B^{t+1|v}. \tag{11}$$

We define the biennial Allocative Luenberger indicator as the average of the two valuation views:

$$AL^{t,t+1} = \frac{1}{2}\Big[\big(CI_B^{t|t} - TI_B^{t|t}\big) - \big(CI_B^{t+1|t} - TI_B^{t+1|t}\big) + \big(CI_B^{t|t+1} - TI_B^{t|t+1}\big) - \big(CI_B^{t+1|t+1} - TI_B^{t+1|t+1}\big)\Big]. \tag{12}$$

More generally, for $H \in \{CI, TI, AI\}$, let

$$\Delta_B H^{t,t+1} = \frac{1}{2}\left[(H_B^{t|t} - H_B^{t+1|t}) + (H_B^{t|t+1} - H_B^{t+1|t+1})\right]. \tag{13}$$

We use $CL^{t,t+1} = \Delta_B CI^{t,t+1}$, $LPI^{t,t+1} = \Delta_B TI^{t,t+1}$, and $AL^{t,t+1} = \Delta_B AI^{t,t+1}$. Because $CI = TI + AI$ for every endpoint–valuation pair,

$$CL^{t,t+1} = LPI^{t,t+1} + AL^{t,t+1}. \tag{14}$$

A positive AL means that the observed input mix moves closer to the minimum-value benchmark; a negative value means that it moves farther away. Its measurement object is the change in benchmark-relative allocative inefficiency. Averaging the period-t and period-$(t+1)$ valuation views preserves valuation symmetry, while the common biennial technology holds the feasible set fixed across endpoints. Equations (11)– (14) constitute the standalone biennial VRS Allocative Luenberger indicator proposed here.

Proposition 1 (Static ordering) *Suppose* $\mathbf{x}^s, \mathbf{w}^v > \mathbf{0}$*, the target is feasible in* S^R*, and* $\mathbf{g}_x^s = \mathbf{x}^s$*. Let* $\mathbf{x}_T$ *be any optimal technical projection and* $\mathbf{x}_C$ *any minimum-value bundle, and write* $W = \mathbf{w}^{v\prime}\mathbf{x}^s$*. Then*

$$TI_R^{s|v} = \frac{W - \mathbf{w}^{v\prime}\mathbf{x}_T}{W}, \qquad CI_R^{s|v} = \frac{W - \mathbf{w}^{v\prime}\mathbf{x}_C}{W}, \qquad AI_R^{s|v} = \frac{\mathbf{w}^{v\prime}\mathbf{x}_T - \mathbf{w}^{v\prime}\mathbf{x}_C}{W} \geq 0. \tag{15}$$

Consequently, $0 \leq TI_R^{s|v} \leq CI_R^{s|v} \leq 1$.

Proof Under the full-input direction, $\sum_i \mu_i^{s|v}\beta_i = (W - \mathbf{w}^{v\prime}\mathbf{x}_T)/W$. The projected technical bundle is feasible for the minimum-value program, so $\mathbf{w}^{v\prime}\mathbf{x}_C \leq \mathbf{w}^{v\prime}\mathbf{x}_T$, which proves the ordering and non-negativity statements. □

Section S3 of the Supplementary Information reports a secondary decomposition that separates valuation-vector changes from changes evaluated at fixed valuations, together with additivity checks.

3.4 Data and Sample

The balanced panel covers 18 G20 economies annually from 2008 to 2022, giving 270 country-year observations and 252 adjacent country-year transitions. The common window is set by the 2008–2022 coverage of the maintained energy-valuation file together with the availability of the remaining required energy, PWT, and emissions

fields. Türkiye is absent from the harmonized source panel, Saudi Arabia is retained, and regional aggregates are excluded. Table 1 reports the variables, sources, and roles in the model. Frontier quantities retain their source units. Before optimization, PWT monetary quantities and employment are converted from millions; positive reciprocal rescaling improves numerical conditioning without changing $w_i x_i$.

Table 1 Variables, sources, and roles in the empirical model

Symbol	Definition	Main source	Role
E	Total primary energy use (kgoe)	Energy Institute and OWID [7, 20]	Active input under the energy-only direction; first input under the full-input direction
L	Persons engaged	PWT 11.0 [9, 13]	Non-energy input
K	Real capital stock at constant national prices	PWT 11.0	Non-energy input
y	Output-side real GDP	PWT 11.0	Desirable output
b	Annual CO_2 emissions	OWID	Undesirable output, imposed as an equality
C_E, C_L, C_K	Benchmark input values expressed on a common PPP basis	PWT 11.0; author-constructed energy-price proxy using Energy Institute fuel shares and World Bank Pink Sheet prices	Valuation coefficients, direction weights, and value-minimization objective in full BVAL

3.5 Common-PPP Valuation Basis

We express the country-year-specific valuation vectors on a common PPP basis:

$$C_E = \tilde{p}_E E, \qquad C_L = \alpha_L \tilde{Y}, \qquad C_K = r_K \tilde{K}. \tag{16}$$

Here $\tilde{p}_E = p_E^N / pl_{gdpo}$ is the benchmark valuation coefficient for the fixed 2008 fuel mix. The nominal energy-price proxy is an author-constructed weighted series: country-specific 2008 oil, coal, and natural-gas shares come from the Energy Institute's *Statistical Review of World Energy*, while annual Brent crude, Australian coal, and U.S. natural-gas quotations come from the World Bank Pink Sheet. The component prices are converted to USD per kgoe before applying the fixed 2008 shares. The resulting workbook is a research-assembled input, not a standalone dataset issued by either provider [7, 29]. For labor, $\alpha_L = labsh$ and $\tilde{Y} = cgdpo \times 10^6$; for capital, $r_K = irr + \delta > 0$ and $\tilde{K} = cn \times 10^6$.

The tilded series enter the valuation layer only. The programs use benchmark valuation coefficients $w_i = C_i/x_i$, so $w_i x_i = C_i$ for each input, while the DEA constraints retain the frontier quantities in their source units.

The baseline capital quantity is PWT `rnna`. PWT reports this series at constant 2021 national prices in millions of 2021 US dollars. Because `rnna` and current-PPP capital stock `cn` coincide in 2021, the maintained quantity can equivalently be written as

$$K_{it} = cn_{i,2021} \frac{rnna_{it}}{rnna_{i,2021}} \times 10^6. \tag{17}$$

Its cross-country level is therefore anchored to the 2021 PPP comparison, while its time path follows each country's constant-price capital-stock growth. It is not a nominal capital amount in local currency.

Three input-definition comparisons retain the same benchmark input values. The first substitutes current-PPP capital stock $K^P_{it} = cn_{it} \times 10^6$ and uses C_K/K^P as its paired valuation coefficient. This comparison changes the cross-country capital levels while leaving the capital benchmark value unchanged. The second uses a capital-services quantity $K^S_{it} = ck_{i,2021}\, rkna_{it}$, which joins the 2021 current-PPP service level to the within-country constant-price service index. The third uses total labor hours $H_{it} = emp_{it} \times 10^6 \times avh_{it}$. The corresponding coefficients are C_K/K^S and C_L/H, so all paired valuation products remain unchanged. PWT reports all required observations except average hours for South Africa in 2015; that value is linearly interpolated between 2014 and 2016.

Table 2 reports the resulting value shares. The medians are 4.37% for energy, 46.27% for labor, and 48.77% for capital. These are own-period shares ($s = v$); cross-period full-input evaluations recalculate the shares using Equation (6). Under the energy-only direction, energy is the sole active component and receives unit weight.

Table 2 Distribution of common-PPP benchmark-value shares, 2008–2022 (%)

Input	Mean	Median	Minimum	Maximum
Energy	5.30	4.37	0.88	22.62
Labor	46.97	46.27	20.71	65.39
Capital	47.73	48.77	23.23	69.70

3.6 Empirical Direction Specifications

We estimate the general DRM in Equation (7) under an energy-only and a full-input direction. The energy-only specification sets

$$\mathbf{g}^s_E = (E^s, 0, 0)'. \tag{18}$$

Equation (6) assigns $\mu_E^{s|v} = 1$ and $\mu_L^{s|v} = \mu_K^{s|v} = 0$ under either valuation period, so technical inefficiency does not depend on $\mathbf{w}^v$. Define $D_E^R(z^s) \equiv TI_R^{s|v}(\mathbf{g}_E^s)$, where $z^s = (\mathbf{x}^s, y^s, b^s)$. Under VRS, D_E^R is obtained from

$$\begin{aligned} D_E^R(\mathbf{x}, y, b) = \max_{\beta, \boldsymbol{\lambda}} \quad & \beta \\ \text{s.t.} \quad & \mathbf{X}^R\boldsymbol{\lambda} \leq \big(E(1-\beta), L, K\big)', \\ & \mathbf{Y}^R\boldsymbol{\lambda} \geq y, \qquad \mathbf{B}^R\boldsymbol{\lambda} = b, \\ & \mathbf{1}'\boldsymbol{\lambda} = 1, \qquad \boldsymbol{\lambda} \geq \mathbf{0}, \quad \beta \geq 0. \end{aligned} \tag{19}$$

The annual efficiency score and biennial change are

$$EE^t = 1 - D_E^t(z^t), \tag{20}$$

$$LPI_E^{t,t+1} = D_E^B(z^t) - D_E^B(z^{t+1}). \tag{21}$$

A positive LPI_E indicates that the energy inefficiency gap is smaller at the second endpoint. We separate this change into efficiency and frontier components:

$$\Delta EE_E^{t,t+1} = D_E^t(z^t) - D_E^{t+1}(z^{t+1}), \tag{22}$$

$$\Delta F_E^{t,t+1} = \left[D_E^B(z^t) - D_E^t(z^t)\right] - \left[D_E^B(z^{t+1}) - D_E^{t+1}(z^{t+1})\right]. \tag{23}$$

These components satisfy $LPI_E = \Delta EE_E + \Delta F_E$. Labor and capital remain bounded by their observed quantities through the input inequalities. The energy-only distance is independent of the valuation vector; no allocative decomposition is reported for this specification.

For the full-input specification, $\mathbf{g}_F^s = \mathbf{x}^s$, and Equation (6) weights the input contractions by the value shares of the endpoint bundle under each period's common-PPP valuation vector. We write the associated change measures as

$$(CL_F, LPI_F, AL_F) \equiv (CL, LPI, AL)\big|_{\mathbf{g}_x = \mathbf{g}_F}.$$

3.7 Empirical Comparisons and Validation

The baseline uses non-radial B-VRS with the observed-emissions equality. Four matched comparisons change one feature at a time. They replace VRS with B-CRS, remove emissions from the technology, replace the equality with the ceiling

$$\mathbf{B}^R\boldsymbol{\lambda} \leq b, \tag{24}$$

or impose a radial full-input direction while holding the biennial value programs and endpoint valuation vectors fixed. The ceiling permits a reference combination with no more emissions than the evaluated observation. We apply the radial comparison only to the full-input specification because the energy-only direction has a single active contraction component.

The three input-definition comparisons use current-PPP capital stock, PPP-anchored capital services, or total labor hours. Each changes one quantity and its paired valuation coefficient, leaving the observed benchmark input values, directional specification rules, emissions treatment, returns to scale, and biennial reference-set membership unchanged. The direction vectors follow the modified quantities, and cross-observation valuations can change.

Two influence diagnostics distinguish aggregation effects from changes in frontier support. Fixed-score leave-one-country-out aggregation drops one country's scores from the equal-country mean without re-estimating the frontiers. Country-deletion re-estimation also removes that country from each affected biennial reference set before recomputing the remaining scores. We also report GDP- and energy-use-weighted means, country medians and trimmed means, and cumulative estimates for windows beginning in 2008, 2011, 2015, and 2019.

Frontier-discrimination diagnostics report annual and biennial effective observations, the shares of zero distances and near-zero changes, positive reference intensities, peer-set concentration, and self-reference. An annual leave-own-observation-out exercise tests whether each target remains feasible against the other 17 economies. These are fixed-sample diagnostics. We do not use DEA bootstrap inference, and the indicators are not interpreted as population estimates or country rankings.

CBC solves all linear programs through PuLP [10, 18]. Primal and dual feasibility tolerances are 10^{-7}, the internal zero threshold is 10^{-10}, and scores within 10^{-7} of zero are reported as zero. We rebuild the 270-row panel from registered raw files, check their SHA-256 hashes, and verify the transformed data after reciprocal scaling.

Every biennial endpoint is feasible. The largest absolute residuals in $CL_F - LPI_F - AL_F$ and $LPI_E - \Delta EE_E - \Delta F_E$ are below 2×10^{-16}. For selected high-influence transitions, HiGHS dual-simplex and interior-point solutions differ from the CBC estimates by at most 1.30×10^{-8}.

4 Results

Unless otherwise stated, the results in this section use the baseline input definitions.

4.1 National Energy-Use Efficiency

The mean annual energy-use efficiency score is 0.9466. The annual mean rises from 0.9527 in 2008 to 0.9562 in 2015, then falls to 0.9321 in 2022 (Figure 1, blue line and left axis). The median remains one throughout the sample because many countries lie on the annual emissions-conditioned VRS frontier. Annual frontier observations decline from 14 in 2008 to 12 in 2022. Across all years, 208 of the 270 annual energy distances (77.0%) are zero at the 10^{-7} reporting tolerance. The corresponding biennial change is near zero in 129 of the 252 transitions (51.2%).

Under the baseline input definitions, equal-country-weighted energy-efficiency change over 2008–2022 is 0.0389 (Figure 1, red dashed line and right axis), with the decomposition

$$LPI_E \approx 0.0389 \approx \underbrace{-0.0206}_{\Delta EE_E} + \underbrace{0.0595}_{\Delta F_E}. \tag{25}$$

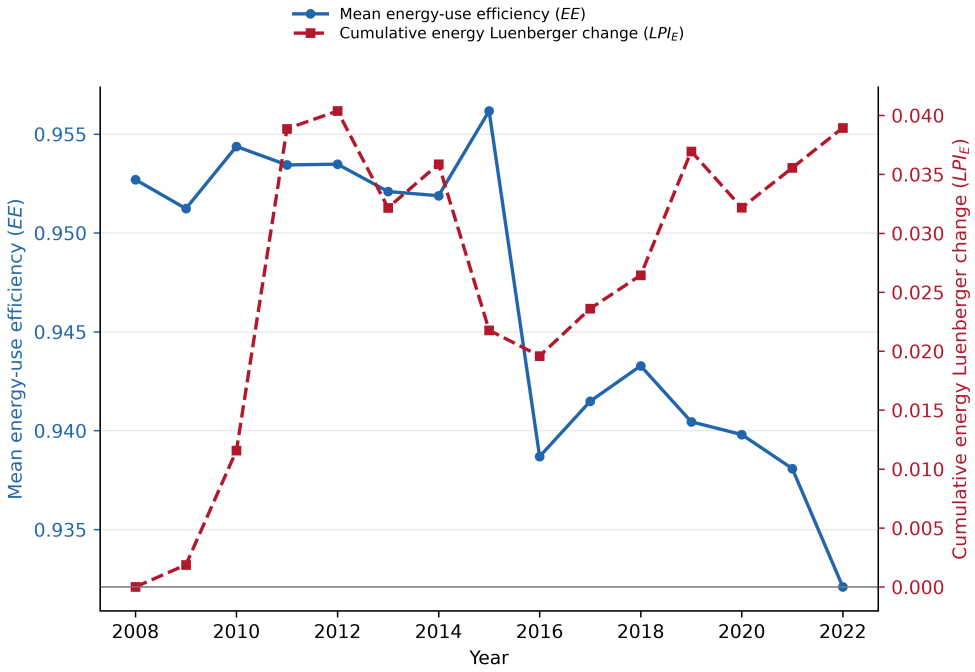


Fig. 1 National energy-use efficiency under the baseline input definitions and emissions-conditioned B-VRS model. The blue line uses the left axis and reports the equal-country mean of annual EE; the red dashed line uses the right axis and cumulates the equal-country mean of adjacent-period LPI_E from a 2008 baseline of zero

The positive frontier component outweighs the decline in own-period energy efficiency. Annual EE is measured against single-year frontiers, whereas cumulative LPI_E uses biennial frontiers. The displayed components are rounded separately to four decimal places.

Seven country-level cumulative LPI_E values are positive, eight are negative, and three are numerically close to zero. France has the largest positive change (0.5795), followed by the United Kingdom (0.1310), Italy (0.1260), Indonesia (0.1001), and India (0.0834). The largest declines occur in Brazil (−0.1905), Japan (−0.0575), the Russian Federation (−0.0482), and Canada (−0.0388).

France accounts for 82.7% of the net country sum. Even without France, the fixed-score equal-country mean remains positive at 0.0071. GDP- and energy-use-weighted cumulative values are also positive, at 0.0267 and 0.0119, respectively. The sign is more stable than the magnitude.

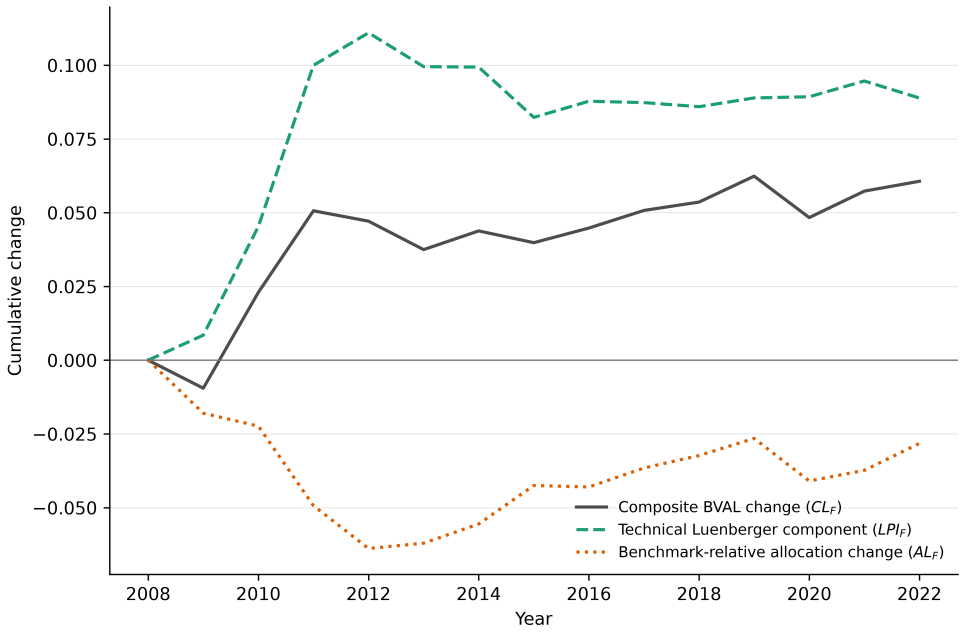


Fig. 2 Cumulative full-input BVAL decomposition under the baseline emissions-conditioned B-VRS specification. Values are equal-country-weighted means cumulated from 2008

4.2 Full-Input Technical and Factor-Allocation Change

Under the baseline input definitions, the equal-country mean of full-input benchmark-value performance improves over 2008–2022, while the allocation component is negative:

$$CL_F \approx 0.0607 \approx \underbrace{0.0889}_{LPI_F} + \underbrace{(-0.0282)}_{AL_F}. \tag{26}$$

The allocation loss offsets 31.8% of the value-share-weighted technical Luenberger component. The secondary decomposition in Section S3 comprises an own-valuation allocative-efficiency component of 0.0070, a fixed-valuation frontier-gap component of −0.0428, and a valuation-vector effect of 0.0075. Components are rounded separately.

The allocation loss occurs mainly before 2011 (Figure 2). By 2011, cumulative LPI_F is 0.1000 and cumulative AL_F is −0.0493. Over 2011–2022, AL_F is positive at 0.0211, so the later gain offsets only part of the earlier decline.

Country-level allocation changes vary widely. At the 10^{-7} reporting tolerance, ten cumulative AL_F values are positive, six are negative, and Canada and Korea are numerically zero. The largest negative changes occur in Indonesia (−0.6166), India (−0.2954), and Italy (−0.1076); Brazil (0.3007), Germany (0.1197), and China (0.0601) have the largest positive values.

The country median is 0.0034, while trimming one observation from each tail gives a mean of −0.0120. Fixed-score leave-one-country-out means range from −0.0476 to 0.0064 and become positive only when Indonesia is omitted. GDP weighting gives $AL_F = -0.0132$, compared with 0.0031 under energy-use weighting. The baseline mean is driven by a few country paths and changes sign under some aggregation rules.

The positive baseline LPI_F result is less sensitive to aggregation and country deletion, although Indonesia, India, and Italy account for 96.4% of its net country sum. The fixed-score mean remains positive when Indonesia is omitted (0.0486), as do the GDP-weighted (0.0681) and energy-use-weighted (0.0403) estimates.

Country deletion can change both the reported mean and the estimated frontier. Re-estimating each B-VRS biennial frontier after a one-country deletion gives $AL_F =$ 0.0067 without Indonesia, 0.0231 without the United States, and 0.0229 without Argentina. These are the only three sign reversals among the 18 cases. CL_F and LPI_F remain positive in every refit; both reach their minima, 0.0229 and 0.00001, when Argentina is omitted. The AL_F sign varies with country composition and frontier support.

4.3 Do Energy Improvements Coincide with Better Allocation?

Energy-efficiency and allocation changes show little descriptive alignment. Across the 252 adjacent country-year comparisons from 2008 to 2022, their Spearman rank correlation is −0.229 after changes within 10^{-7} of zero are treated as zero. At least one component is numerically zero in 188 comparisons. Of the remaining 64, 24 have matching signs, giving a concordance rate of 37.5%. The rate is 47.9% under a 10^{-9} threshold and 37.5% under the maintained 10^{-7} threshold. At the country-cumulative level, the Spearman rank correlation is −0.208, indicating weak descriptive alignment. Table 3 reports the transition classification at the maintained tolerance.

Table 3 Transition-level alignment between energy-efficiency and factor-allocation change

Category	Number	Share (%)
Near-zero component	188	74.60
Energy improvement / allocation deterioration	23	9.13
Joint improvement	17	6.75
Energy deterioration / allocation improvement	17	6.75
Joint deterioration	7	2.78

Figure 3 shows the corresponding country-level sign combinations.

France and the United Kingdom record positive changes in both dimensions. India, Indonesia, and Italy combine energy improvement with allocation deterioration, whereas Brazil and Japan lie in the opposite quadrant.

4.4 Targeted Model Comparisons

Table 4 compares the baseline results with four alternative model specifications.

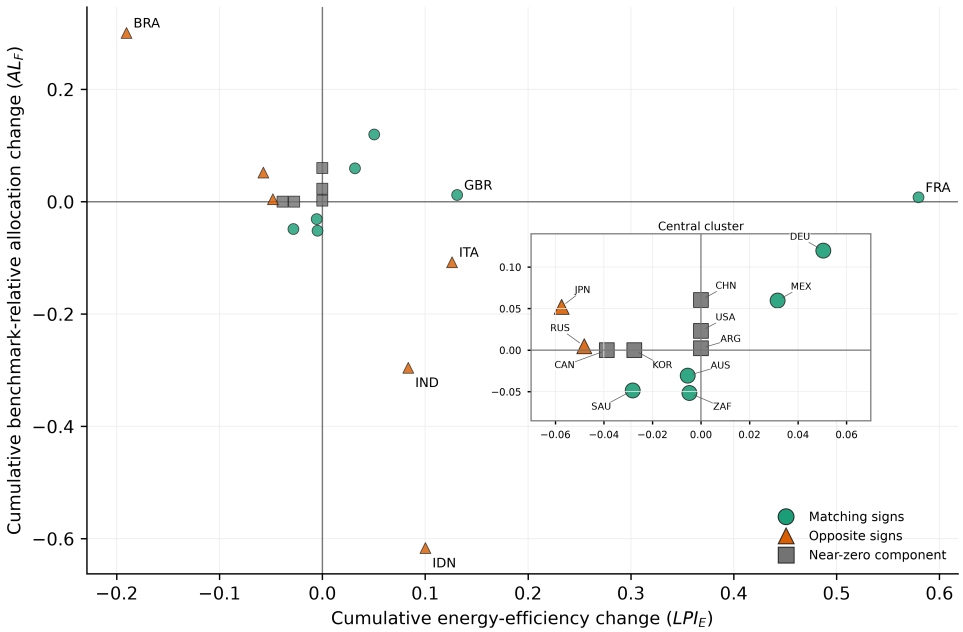


Fig. 3 Country-level cumulative energy-efficiency change and factor-allocation change under the baseline input definitions, 2008–2022. Circles indicate matching signs, triangles indicate opposite signs, and squares indicate at least one component within 10^{-7} of zero. The same marker shapes are used in the inset. The inset enlarges the central cluster; country labels use ISO-3 codes. OpenAI Codex assisted with the plotting-code change that adds marker shapes to distinguish these categories, using the authors' existing country-level outputs; the plotted coordinates are unchanged.

Table 4 Cumulative results under the baseline and four targeted comparisons

Specification	LPI_E	CL_F	LPI_F	AL_F
B-VRS, CO_2, non-radial	0.0389	0.0607	0.0889	−0.0282
B-CRS, CO_2, non-radial	0.0638	0.0815	0.1875	−0.1060
B-VRS, CO_2 ceiling, non-radial	0.1187	0.1811	0.2365	−0.0554
B-VRS, no CO_2, non-radial	0.1243	0.1139	0.2736	−0.1597
B-VRS, CO_2, radial BVAL	—	0.0607	0.0258	0.0349

Under CRS, cumulative LPI_E rises from 0.0389 to 0.0638 and AL_F becomes more negative. The scale assumption affects both the energy frontier and the technical-gap–allocative split. Replacing the observed-emissions equality by $B\lambda \leq b$ gives $(LPI_E, CL_F, LPI_F, AL_F) = (0.1187, 0.1811, 0.2365, -0.0554)$. Dropping emissions altogether gives $(0.1243, 0.1139, 0.2736, -0.1597)$. The baseline signs are retained under the ceiling, but the magnitudes are not. The equality result is therefore specific to comparison at observed emissions.

The matched radial BVAL leaves CL_F at 0.0607 because it uses the same value-minimization program. Proportional input contraction lowers LPI_F to 0.0258 and raises AL_F to 0.0349, changing the technical-gap–allocative attribution of the same overall improvement.

The input-definition comparisons reported in the Supplementary Information show a sharper qualification. With current-PPP capital stock, the cumulative vector is $(-0.0003, 0.0086, 0.0011, 0.0074)$, so the energy and allocation signs differ from the baseline and the technical Luenberger component is close to zero. PPP-anchored capital services give $LPI_E = 0.0576$, $CL_F = 0.0386$, $LPI_F = 0.0765$, and $AL_F = -0.0379$. With total labor hours, the corresponding values are 0.0137, 0.0546, 0.0862, and -0.0316. CL_F remains positive under all four input definitions, ranging from 0.0086 to 0.0607; neither the magnitude of the technical component nor the sign of AL_F is invariant to the capital quantity used in the cross-country frontier.

Frontier-discrimination diagnostics indicate a dense reference technology. At the maintained tolerance, 208 of 270 annual technical-gap scores and 129 of 252 dynamic LPI_F values are zero or near zero. The median positive peer count across recorded evaluations is one, and 57.3% of recorded reference solutions include the target observation. Once the target is excluded from its annual reference set, 62 of 270 energy and technical programs and 197 of 270 benchmark-value programs remain feasible. These diagnostics characterize the fixed 18-economy reference system.

5 Discussion

5.1 What the Two Direction Specifications Add

The energy-only and full-input directions measure different gaps. The first contracts primary energy while holding labor, capital, output, and emissions at the evaluated levels; the second permits all three inputs to contract and aggregates the input-specific gaps with benchmark-value shares. Since energy's median share is 4.37%, LPI_F and AL_F need not follow LPI_E. The baseline estimates bear out this distinction: energy-use efficiency improves modestly over 2008–2022, whereas the full-input decomposition gives a positive technical Luenberger component and a negative equal-country allocation component. The transition-level concordance rate is only 37.5% after solver-scale changes are reported as zero. This is a descriptive difference between measurement directions, not evidence that one movement causes the other.

The aggregate paths are concentrated. France accounts for 82.7% of the net country sum of LPI_E, and Indonesia, India, and Italy account for 96.4% of the net sum of LPI_F. The country median, energy-use-weighted estimate, and 2011–2022 estimate of AL_F are positive although its baseline equal-country mean is negative. The three-part diagnostic locates most of that negative mean in the fixed-valuation frontier gap; it does not identify an adjustment-cost or institutional mechanism.

Comparison with existing G18 and G20 evidence illustrates why the measurement objects should be kept separate. Sadorsky [24] identifies France and Great Britain as eco-efficiency leaders and Canada, China, India, and Indonesia as laggards over 1997–2019; Shah et al. [25, 26] instead emphasize energy-productivity change and technology gaps. In our 2008–2022 accounting, France and the United Kingdom have positive

cumulative energy and allocation changes, and Canada has negative cumulative energy change. India and Indonesia, however, combine positive energy change with negative allocation change, while Brazil combines negative energy change with positive allocation change. Agreement for some countries and reversal for others reflect differences in the outcome, period, sample composition, and reference technology. Eco-efficiency, change in the energy gap, and benchmark-relative allocation change are not interchangeable.

5.2 Specification and Measurement Dependence

The matched comparisons show where the interpretation is stable and where it is not. The observed-emissions ceiling preserves the baseline signs, but all four magnitudes increase. CRS raises the two technical Luenberger components, and radial contraction reverses the allocation sign while leaving CL_F unchanged. The technical-gap–allocative split is therefore conditional on the scale assumption, emissions treatment, and contraction direction. In particular, the baseline emissions equality compares observations at the evaluated emissions quantity; it does not represent a carbon budget, carbon price, or policy response. The ceiling and no-emissions specifications are sensitivity checks on that conditioning rule rather than policy counterfactuals.

Input definitions matter at least as much. Capital services and total labor hours retain the baseline sign pattern. Current-PPP capital stock does not: LPI_E is approximately zero, LPI_F falls from 0.0889 to 0.0011, and AL_F changes from -0.0282 to 0.0074. Both CL_F and LPI_F remain positive under all four input definitions, although LPI_F falls to 0.0011 with current-PPP capital stock. Estimates of CL_F range from 0.0086 to 0.0607; the sign of AL_F is not invariant. The current-PPP comparison changes cross-country capital levels and their paired valuation coefficients while preserving each observation's benchmark capital value. The resulting differences therefore reflect changes in both the reference technology and cross-observation valuation.

5.3 Policy Use and Interpretation

National statistical offices and energy, finance, and industry authorities could use the indicators as screening measures. A country–period with a large energy or allocation movement may warrant a sectoral audit of the underlying energy mix, capital vintage, labor use, and data definitions. The indicators should be read with absolute emissions, carbon intensity, the renewable-energy share, energy affordability, and employment measures. An input-specific contraction denotes a benchmark gap rather than a recommendation to reduce labor, productive capacity, or low-carbon investment, and a favorable AL_F does not establish that the observed input mix is socially optimal.

For practical reporting, the reference technology, emissions treatment, valuation basis, input definitions, aggregation rule, and sample window should accompany LPI_E, LPI_F, and AL_F. Comparisons that change any of these features do not measure the same benchmark, so a single sign change should not trigger intervention.

6 Conclusions

This study formulates a two-valuation, biennial VRS Allocative Luenberger indicator and applies it to 18 G20 member economies over 2008–2022. The indicator measures the two-valuation average change in benchmark-relative allocative inefficiency on a common local-biennial technology. Its additive identity, $CL_F = LPI_F + AL_F$, separates overall full-input benchmark-value change into technical and allocation components. A matched energy-only direction measures change in the energy gap. Both directions condition the reference technology on observed carbon emissions, and the full-input measure uses country-year-specific valuation vectors expressed on a common PPP basis.

Under the baseline input definitions, equal-country cumulative LPI_E is 0.0389. Full-input benchmark-value change is 0.0607, comprising a technical Luenberger component of 0.0889 and an allocation component of -0.0282. These totals do not describe a common national path. The energy and technical components are concentrated in a few countries, while the allocation sign changes with aggregation, the sample window, and three country-deletion frontier re-estimations.

The input comparisons further narrow the conclusion. CL_F remains positive under the baseline, current-PPP capital stock, PPP-anchored capital services, and total labor hours, ranging from 0.0086 to 0.0607. With current-PPP capital stock, however, LPI_E is -0.0003, LPI_F is 0.0011, and AL_F is 0.0074. Thus neither the baseline technical magnitude nor the negative allocation sign is invariant to the capital quantity used for cross-country comparison.

Replacing the observed-emissions equality by an emissions ceiling retains the baseline signs but produces larger estimates. CRS and radial contraction also alter the decomposition. The indicators are therefore most useful for screening when reported alongside absolute emissions, carbon intensity, renewable-energy use, affordability, and employment, with the technology, valuation basis, input definitions, aggregation, and sample window stated explicitly.

The principal limitations are the fixed 18-economy sample, a dense frontier with many zero or near-zero distances, and national data that cannot identify sectoral mechanisms. The common-PPP valuation vectors are accounting benchmarks rather than domestic marginal prices or welfare weights. Consequently, the estimates do not support population inference, policy attribution, or country performance rankings.

Supplementary Information

The Supplementary Information reports the energy and BVAL linear programs, data construction, common-PPP valuation basis, feasibility and identity checks, input and emissions sensitivity, frontier-discrimination diagnostics, disaggregated results, and country-deletion re-estimations. The processed panel and analysis code will be deposited in a public repository before publication.

Statements and Declarations

Competing Interests

The authors declare that they have no competing financial or non-financial interests relevant to this work.

Funding

This research received no external funding.

Data Availability

The analysis uses public data from PWT 11.0, OWID, the Energy Institute, the World Bank Pink Sheet, and ILOSTAT. The energy-price proxy is assembled by the authors from Energy Institute fuel-use shares and World Bank commodity-price series; the corresponding workbook is not a separate dataset released by either provider. The ILOSTAT annual table `EAR_EMTA_SEX_CUR_NB_A` is retained in the analysis workflow solely to reconstruct a legacy labor-valuation diagnostic; the baseline labor valuation instead uses the PWT labor compensation share, so ILOSTAT does not determine the baseline estimates [15]. The processed 270-observation panel and analysis code are not included with this initial submission; they will be deposited in a public repository before publication, and this statement will be updated with the persistent link or DOI. Provider-controlled source files that cannot be redistributed will be excluded; the repository will identify their official sources and acquisition instructions, subject to the providers' licensing terms.

AI Use Declaration

During manuscript preparation, the authors used OpenAI Codex for English-language editing and limited wording revisions to the abstract, Figure 3 caption, and Data Availability statement. Codex also assisted with modifying the plotting code to distinguish the Figure 3 categories by marker shape, using the authors' existing country-level outputs. The authors reviewed and approved the final manuscript and are responsible for its content and reported results. AI tools were not used to generate data or re-estimate the reported results.

Abbreviations

DEA	Data envelopment analysis
DDF	Directional distance function
VRS	Variable returns to scale
CRS	Constant returns to scale
PPP	Purchasing power parity
EE	Energy-use efficiency
LPI_E	Luenberger change under the energy-only direction
BVAL	Biennial VRS–Allocative Luenberger framework
CI, TI, AI	Benchmark-value, technical, and allocative inefficiency
$s \mid v$	Endpoint s evaluated using the valuation vector from period v
CL_F	Full-input benchmark-value Luenberger change
LPI_F	Value-share-weighted full-input technical Luenberger component
AL_F	Benchmark-relative full-input allocation change
ΔEE_E, ΔF_E	Energy-efficiency and energy-frontier effects

Appendix A Linear Programs and Feasibility

For a target $z^o = (\mathbf{x}^o, y^o, b^o)$ and reference set R, the energy program used in Equation (19) is

$$\begin{aligned} \max_{\beta,\boldsymbol{\lambda}} \quad & \beta \\ \text{s.t.} \quad & \sum_{j\in R} \lambda_j E_j \le E^o(1-\beta), \\ & \sum_{j\in R} \lambda_j L_j \le L^o, \qquad \sum_{j\in R} \lambda_j K_j \le K^o, \\ & \sum_{j\in R} \lambda_j y_j \ge y^o, \qquad \sum_{j\in R} \lambda_j b_j = b^o, \\ & \sum_{j\in R} \lambda_j = 1, \qquad \boldsymbol{\lambda} \ge \mathbf{0}, \quad \beta \ge 0. \end{aligned} \tag{A1}$$

The full-input non-radial program is

$$\begin{aligned} \max_{\boldsymbol{\beta},\boldsymbol{\lambda}} \quad & \sum_{i=1}^{3} \mu_i \beta_i \\ \text{s.t.} \quad & \sum_{j\in R} \lambda_j x_{ij} \le x_i^o(1-\beta_i), \quad i = 1,2,3, \\ & \sum_{j\in R} \lambda_j y_j \ge y^o, \qquad \sum_{j\in R} \lambda_j b_j = b^o, \\ & \sum_{j\in R} \lambda_j = 1, \qquad \boldsymbol{\lambda} \ge \mathbf{0}, \quad \boldsymbol{\beta} \ge \mathbf{0}. \end{aligned} \tag{A2}$$

The value-minimization program is

$$\begin{aligned}
\min_{\mathbf{x},\boldsymbol{\lambda}} \quad & \sum_{i=1}^{3} w_i x_i \\
\text{s.t.} \quad & \sum_{j\in R} \lambda_j x_{ij} \le x_i, \quad i = 1,2,3, \\
& \sum_{j\in R} \lambda_j y_j \ge y^o, \qquad \sum_{j\in R} \lambda_j b_j = b^o, \\
& \sum_{j\in R} \lambda_j = 1, \qquad \boldsymbol{\lambda} \ge \mathbf{0}, \quad \mathbf{x} \ge \mathbf{0}.
\end{aligned} \tag{A3}$$

When $R = B(t, t+1)$, choosing the observed target itself with intensity one proves feasibility of every endpoint program. The matched CRS programs remove only the convexity equality.

Appendix B Valuation and Aggregation Diagnostics

Table B1 reports alternative aggregation weights applied to the estimated country-transition scores. The reference frontiers and country scores are held fixed across the three rows.

Table B1 Cumulative indicators under alternative country weights

Aggregation	LPI_E	AL_F
Equal country	0.0389	−0.0282
GDP weighted	0.0267	−0.0132
Energy-use weighted	0.0119	0.0031

The numerical audit covers 252 country transitions. Maximum absolute residuals are 2.78×10^{-17} for $CL_F - LPI_F - AL_F$, 1.73×10^{-18} for $LPI_E - \Delta EE_E - \Delta F_E$. The complete secondary-decomposition audit is reported in Section S3 of the Supplementary Information. Complete transition, country, annual, comparison, and valuation-share tables will be deposited with the processed panel and analysis code in the public repository before publication.

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