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Aspect-ratio-dependent stall delay and spanwise separation topology of a finite NACA 2412 wing at $Re = 2.2 \times 10^6$

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Abstract

This study introduces a wall-shear-stress-based separation fraction metric, $\mathcal{S}$, and its spanwise extension, $\mathcal{S}(z/b)$, to quantify chordwise and spanwise trailing-edge separation on finite wings. The metric is demonstrated using RANS simulations of the NACA 2412 wing at $Re = 2.2 \times 10^6$ across $AR = 1, 2$, and 4 , employing the standard $k-\varepsilon$ turbulence model with standard wall functions. The two-dimensional results were validated against experimental measurements, predicting pre-stall lift within $2-3\%$ with $1-2^\circ$ stall-angle overprediction. For three-dimensional cases, the stall onset is progressively delayed with decreasing AR , from $\alpha = 20^\circ$ at $AR = 4$ to $\alpha = 30^\circ$ at $AR = 1$, compared to the two-dimensional baseline of $\alpha = 16^\circ$. This delay is attributed to tip-vortex downwash suppressing trailing-edge separation across a larger span fraction. The spanwise separation profiles reveal a smooth root-to-tip gradient at higher AR , whereas the profiles collapse into a broad inner-span plateau at $AR = 1$. These results demonstrate how aspect ratio affects chordwise and spanwise trailing-edge separation from onset to deep stall and provide a surface-resolved reference dataset for low- AR cambered-wing stall prediction and finite-wing aerodynamic design.

Keywords: NACA 2412 airfoil, finite wing, aspect ratio, stall delay, trailing-edge separation, wall-shear-stress metric, spanwise separation topology, RANS simulation

List of abbreviations

AR	Aspect ratio
CFD	Computational fluid dynamics
LLT	Lifting-line theory
MAV	Micro air vehicle
RANS	Reynolds-averaged Navier–Stokes
UAV	Unmanned aerial vehicle
c	Chord length (m)
C_D	Drag coefficient
C_L	Lift coefficient
$C_{L,\max}$	Maximum lift coefficient
C_p	Pressure coefficient
e	Oswald efficiency factor
G_k	Production of turbulent kinetic energy
I_u	Turbulence intensity (%)
k	Turbulent kinetic energy (m^2/s^2)
L_z	Wing span (mm)
M	Mach number
p	Static pressure (Pa)
Re	Reynolds number
S	Suction-surface separation fraction (%)
$S(z/b)$	Spanwise separation fraction (%)
u_i	Reynolds-averaged velocity component (m/s)
u'_i	Fluctuating velocity component (m/s)
U_∞	Freestream velocity (m/s)
x	Chordwise coordinate
x_s	Chordwise separation location
x_{TE}	Trailing-edge coordinate
y^+	Dimensionless wall distance
z/b	Normalized spanwise coordinate
α	Angle of attack ($^\circ$)
α_0	Zero-lift angle of attack ($^\circ$)
α_{stall}	Stall angle of attack ($^\circ$)
δ_{ij}	Kronecker delta
ε	Turbulent dissipation rate (m^2/s^3)
κ	von Kármán constant
μ	Dynamic viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
μ_t	Turbulent eddy viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
ρ	Fluid density (kg m^{-3})
σ_k	Turbulent Prandtl number for k
σ_ε	Turbulent Prandtl number for ε
τ_w	Wall shear stress (Pa)

1 Introduction

The accurate prediction of aerodynamic stall on finite wings is very important because of its direct implications for the design of unmanned aerial vehicles (UAVs), micro air vehicles (MAVs), small fixed-wing aircraft, and rotorcraft blades. Unlike two-dimensional (2D) airfoil configurations, real wings exhibit strong three-dimensional (3D) flow conditions governed by tip vortices, spanwise pressure differences, and aspect-ratio- (AR-) dependent induced effects. At moderate chord Reynolds numbers in the range $Re = 1 \times 10^6 - 3 \times 10^6$, trailing-edge stall is the dominant failure mode for cambered airfoils such as the NACA 2412 [1, 2]. In this regime, flow separation starts at the trailing edge and progressively migrates upstream with increasing angle of attack (α), leading to a decisive loss of lift [3]. Understanding how this separation topology develops as a function of AR is essential for reliable aerodynamic analysis of small aerial vehicles.

There are rigorous experimental and computational studies available in the present literature on the aerodynamic characteristics of the two-dimensional NACA 2412 airfoil. The foundational experimental database was established by Jacobs and Sherman [1], who reported lift and drag coefficients for an infinite-span NACA 2412 section over a range of Reynolds numbers. This initial study was later supplemented by the detailed boundary-layer measurements of Seetharam and Wentz [4]. They conducted wind-tunnel tests at $Re = 2.2 \times 10^6$ and a Mach number of $M = 0.13$, providing skin-friction distributions, pressure gradients, and hot-film measurements at angles of attack corresponding to pre-stall ($\alpha = 12.4^\circ$), stall onset ($\alpha = 14.4^\circ$), and post-stall ($\alpha = 16.4^\circ$). On the computational side, multiple RANS-based CFD studies have been performed to reproduce the experimental results using various turbulence models. Coles and Wadcock [5] provided evidence that RANS models struggle to capture the sharp post-stall lift drop, a finding later confirmed by numerous authors [6, 7]. More recently, Rumsey and Ying [6] showed that the standard $k-\varepsilon$ and $k-\omega$ models both delay the onset of trailing-edge separation relative to experiment, overpredicting the stall angle by $1-2^\circ$.

The standard $k-\varepsilon$ model proposed by Launder and Spalding [9] remains widely used in engineering CFD because of its robust convergence, low computational cost, and established performance for attached and mildly separated flows. However, its isotropic eddy-viscosity formulation limits its accuracy in flows involving strong adverse pressure gradients, curvature, and large-scale separation [10, 11]. For airfoil stall prediction, the model commonly delays trailing-edge separation and overpredicts the stall angle by about $1-2^\circ$ relative to experiment [6, 12]. Despite this limitation, when used with standard wall functions at $y^+ \approx 30-60$, it provides a computationally efficient approach that can reproduce pre-stall lift and approximate stall onset within engineering accuracy for moderate-Reynolds-number two-dimensional airfoils [13, 14].

The transition from a two-dimensional airfoil to a three-dimensional finite wing introduces distinct flow behavior. For a finite wing, the wingtip creates vortices that contribute to induced downwash, reducing the local angle of attack near the tip and delaying flow separation relative to the root. This effect helps maintain attached trailing-edge flow and sustain lift beyond the two-dimensional stall angle [15]. As a

result, low-aspect-ratio wings ($AR < 4$) show more delayed stall and reduced separation compared to moderate- to high- AR wings due to the stronger spanwise influence of the tip vortex. Abbott and von Doenhoff [2] quantified $C_{l,\max}$ data for the NACA 2412 airfoil at $AR = 6$ and $AR = \infty$. However, a quantitative, surface-resolved metric for tracking chordwise and spanwise separation growth, rather than qualitative visualization alone, has not been established for low-aspect-ratio NACA 2412 wings at $Re = 2.2 \times 10^6$.

Ostowari and Naik [16] experimentally demonstrated that the two-dimensional sectional assumption for airfoil profiles deteriorates significantly with decreasing aspect ratio for finite rectangular wings due to strong spanwise effects. The computational study of Gopalarathnam et al. [17] showed that trailing-edge separation initiates near the root and propagates outward with increasing angle of attack, although their work mainly focused on the low-Reynolds-number regime. Ribeiro et al. [18] generated a post-stall aerodynamic database across multiple aspect ratios using SST $k-\omega$ RANS simulations for symmetric NACA profiles. Konstadinopoulos et al. [19] further clarified the aspect-ratio dependence of reversed-flow extent for NACA 0018 finite wings. However, systematic surface-resolved characterization of chordwise and spanwise separation for low-aspect-ratio NACA 2412 wings at $Re = 2.2 \times 10^6$ remains limited.

Previous studies have provided useful insight into finite-wing aerodynamic coefficients, post-stall behavior, and aspect-ratio effects. However, the chordwise distribution of separation characteristics and its spanwise growth across low-aspect-ratio NACA 2412 wings have not been systematically analyzed at $Re = 2.2 \times 10^6$. To address this gap, the present study introduces a wall-shear-stress-based separation fraction, S , and its spanwise extension, $S(z/b)$, and applies these metrics through RANS-based CFD analysis of a two-dimensional NACA 2412 baseline extended to finite wings with $AR = 1, 2$, and 4 . The experimental data of Seetharam and Wentz [4] are used to validate the two-dimensional results, while the finite-wing pre-stall lift behavior is compared with lifting-line theory to assess the reliability of the three-dimensional simulations. The study focuses on characterizing both two-dimensional and three-dimensional separation topology using a wall-shear-stress-based separation fraction, S , to quantify suction-surface separation in the two-dimensional case, which is then extended spanwise as $S(z/b)$ for the three-dimensional finite wings. Using these parameters, this study identifies how aspect ratio affects stall onset, chordwise flow-separation development, and spanwise separation distribution from initial separation to deep stall.

2 Numerical Methodology

2.1 Governing Equations

Steady, incompressible Reynolds-averaged Navier-Stokes (RANS) equations are used. In tensor notation, the continuity momentum equations are written as:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \quad (1)$$

$$\rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \rho \overline{u'_i u'_j} \right]. \quad (2)$$

Here, $\bar{u}_i$ and $\bar{p}$ are the Reynolds-averaged velocity components and pressure, u'_i denotes the fluctuating velocity component, ρ is the fluid density, and μ is the dynamic viscosity. The Reynolds stress tensor is closed using the Boussinesq eddy-viscosity hypothesis:

$$-\rho \overline{u'_i u'_j} = \mu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \frac{2}{3} \rho k \delta_{ij}, \quad (3)$$

where μ_t is the turbulent eddy viscosity, k is the turbulent kinetic energy, and δ_{ij} is the Kronecker delta. In the standard k - ε model, the eddy viscosity is evaluated as

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon}, \quad (4)$$

where ε is the turbulent dissipation rate and C_μ is an empirical model constant. Standard wall functions were used for the near-wall treatment, with the first cell centroid maintained in the log-law region, corresponding to $y^+ \approx 30$ –300. All simulations were performed using the pressure-based steady solver in ANSYS Fluent 2023 R1 [23].

2.2 Turbulence Modeling

The standard k - ε model of Launder and Spalding [9] with standard wall functions is employed. The transport equations for k and ε are:

$$\frac{\partial}{\partial x_j} (\rho \bar{u}_j k) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k - \rho \varepsilon \quad (5)$$

$$\frac{\partial}{\partial x_j} (\rho \bar{u}_j \varepsilon) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} G_k - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} \quad (6)$$

where $G_k = \mu_t (\partial \bar{u}_i / \partial x_j + \partial \bar{u}_j / \partial x_i) \partial \bar{u}_i / \partial x_j$ is the production of turbulent kinetic energy, and the turbulent eddy viscosity is computed as $\mu_t = \rho C_\mu k^2 / \varepsilon$. Standard model constants are used: $C_\mu = 0.09$, $C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.92$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$ [9].

Near-wall turbulence is handled by standard wall functions with a target $y^+ \approx 40$, placing the first cell centroid within the log-law region ($30 \leq y^+ \leq 300$). The dimensionless velocity profile in the log-law region is given by:

$$u^+ = \frac{1}{\kappa} \ln(E y^+), \quad \kappa = 0.4187, \quad E = 9.793 \quad (7)$$

where $u^+ = \bar{u} / u_\tau$, $y^+ = \rho u_\tau y / \mu$, and $u_\tau = \sqrt{\tau_w / \rho}$ is the friction velocity [23]. This wall treatment was selected over low-Reynolds-number resolution ($y^+ \sim 1$) to maintain computational economy across the full parametric sweep of three aspect ratios and seventeen angles of attack. The known tendency of standard k - ε to delay trailing-edge separation by 1–2° relative to experiment [6] is acknowledged and quantified in Section 4.

2.3 Numerical Schemes and Discretization

Pressure–velocity coupling is handled by the SIMPLE algorithm [24]. All convective terms are discretized with second-order upwind schemes. Gradients are reconstructed using the least-squares cell-based method with second-order pressure interpolation. The discretization settings are summarised in Table 1.

Table 1: Numerical discretization settings employed in all simulations.

Term	Scheme
Pressure–velocity coupling	SIMPLE
Gradient reconstruction	Least-squares cell-based
Pressure interpolation	Second-order
Momentum	Second-order upwind
Turbulent kinetic energy	Second-order upwind
Dissipation rate	Second-order upwind

2.4 Solution Algorithm and Convergence Criteria

Convergence was evaluated by monitoring the reduction of the scaled residuals for continuity, x -, y -, and z -momentum, k , and ε , along with the stability of the aerodynamic coefficients, C_L and C_D . Different iteration counts were used for different cases because solution stability varied with angle of attack and flow-separation intensity. Therefore, each case was iterated until reasonable residual levels were reached and the variations in C_L and C_D became stable over a sustained number of iterations. Under-relaxation factors were set to 0.3 for pressure, 0.7 for momentum, and 0.8 for k and ε [23]. For post-stall cases, where the flow was highly separated, residuals showed mild oscillations due to the unsteady nature of separated flow treated within a steady RANS framework. In those cases, the reported force coefficients were obtained from the mean values over the final converged iteration window [6, 18].

3 Computational Setup

3.1 Computational Domain and Geometry

The NACA 2412 airfoil was selected as the baseline geometry due to its widespread use in general aviation and small UAV applications. This cambered airfoil has 2% maximum camber at 40% chord and 12% maximum thickness at 30% chord. Airfoil coordinates were generated using the standard NACA four-digit series formulation with a reference chord length $c = 731$ mm, as shown in Fig. 1a. The three-dimensional wing geometry extruded from this profile, exemplified here for $AR = 2$, is shown in Fig. 1b.

For the two-dimensional computational domain, a C-type topology centered around the NACA 2412 airfoil profile was considered. The inlet boundary was placed $10c$

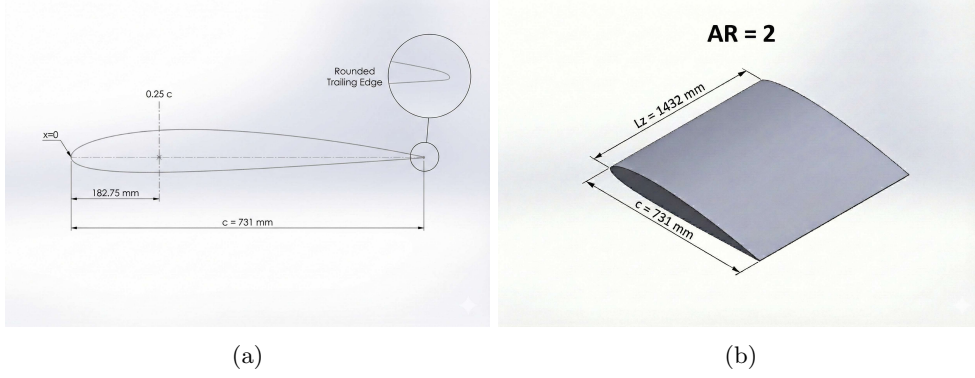


Fig. 1: (a) NACA 2412 airfoil geometry showing chord length ($c = 731$ mm), quarter-chord location ($x/c = 0.25$), and rounded trailing edge detail. (b) Three-dimensional wing geometry for $AR = 2$ ($L_z = 1,462$ mm) showing the extruded NACA 2412 profile.

upstream of the leading edge, the pressure outlet was located $20c$ downstream of the trailing edge, and the lateral far-field boundaries were positioned $10c$ from the airfoil center, as shown in Fig. 2a. For the finite-wing cases, the chordwise and normal domain extents were kept identical to those used in the two-dimensional simulations. However, the domain was extended in the spanwise direction to include the full wing span, with an $8c$ clearance from the wingtip to the lateral far-field boundary, as shown in Fig. 2b.

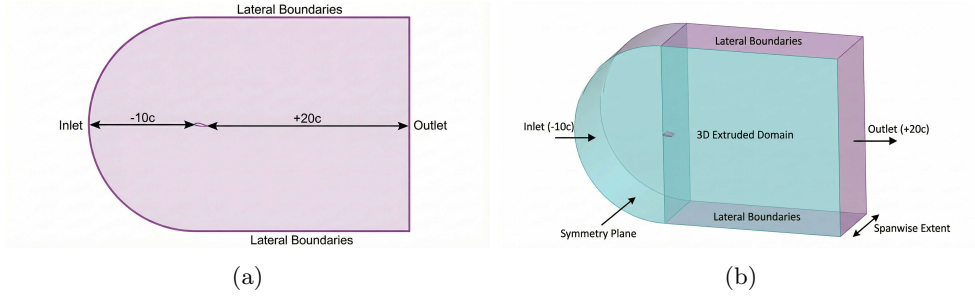


Fig. 2: Computational domain configurations. (a) Two-dimensional C-type domain showing the velocity inlet at $10c$ upstream, pressure outlet at $20c$ downstream, and lateral far-field boundaries. (b) Three-dimensional extruded domain showing the symmetry plane at the root, lateral boundaries, inlet ($-10c$), outlet ($+20c$), and spanwise extent accommodating the wing half-span with $8c$ clearance beyond the wingtip.

3.2 Boundary Conditions

For this study, $Re = 2.2 \times 10^6$ was examined, calculated based on chord length and freestream velocity. The associated freestream velocity is $U_\infty =$

44.2 m/s, with air properties evaluated at standard sea-level conditions ($\rho = 1.225 \text{ kg/m}^3$, $\mu = 1.789 \times 10^{-5} \text{ kg/(m} \cdot \text{s)}$). This Reynolds number is indicative of small UAVs, general aviation aircraft, and horizontal-axis wind turbine blades [1, 2]. To capture the complete behavior from pre-stall, to stall onset, to post-stall regimes, the angle of attack was varied after every $\alpha = 2^\circ$.

At the inlet, a constant velocity was applied with a turbulence intensity of $I_u = 5\%$ and a turbulent viscosity ratio of $\mu_t/\mu = 10$. At the pressure outlet, a zero-gauge static pressure condition was prescribed. The airfoil and wing surfaces were defined as no-slip adiabatic walls in all cases.

For the two-dimensional cases, the upper and lower far-field boundaries were treated as pressure far-field conditions. For the three-dimensional cases, the root plane was set as a symmetry boundary condition, allowing only half of the finite wing to be modeled while representing the full wing through symmetry. The wingtip surface and the remaining wing geometry were modeled as no-slip walls, as in the two-dimensional cases. The upper, lower, and side boundaries were treated as pressure far-field boundaries, similar to those of the two-dimensional configuration. The boundary conditions applied to all simulations are summarised in Table 2.

Table 2: Boundary conditions applied to all computational domains.

Boundary	Type	Condition
Inlet	Velocity inlet	U_∞ , $I_u = 5\%$, $l_t = 0.01c$
Outlet	Pressure outlet	$p_{\text{gauge}} = 0 \text{ Pa}$
Airfoil/wing surface	No-slip wall	$u = v = w = 0$
Far-field lateral (2D and 3D)	Pressure far-field	Freestream Mach gradient
Root plane (3D)	Symmetry	Zero normal gradient
Tip surface (3D)	No-slip wall	$u = v = w = 0$

3.3 Mesh Generation and Grid Independence Study

Unstructured triangular meshes were generated using the built-in meshing tool in ANSYS Fluent for two-dimensional domains, with a surface mesh size of 0.005 m along the airfoil surface, and a trailing-edge region with local refinement using five divisions to improve resolution in the high-gradient flow region. Prismatic inflation layers were generated along the airfoil profile to resolve the near-wall turbulent boundary layer in the log-law region. The inflation layer consisted of 10 layers with a growth ratio of 1.17, with a first cell height of $\Delta y = 0.02 \text{ mm}$ to maintain $y^+ \approx 40$. Representative views of the two-dimensional meshes are shown in Fig. 3

A grid-independence study was performed on the two-dimensional domain by monitoring the lift coefficient, C_L , across three mesh counts. Two angles of attack were considered: $\alpha = 12.4^\circ$ for the pre-stall condition and $\alpha = 16^\circ$ for the post-stall condition, to ensure that the mesh densities were sufficient for both regimes. The mesh-sensitivity results are presented in Table 3. In both cases, the relative change in

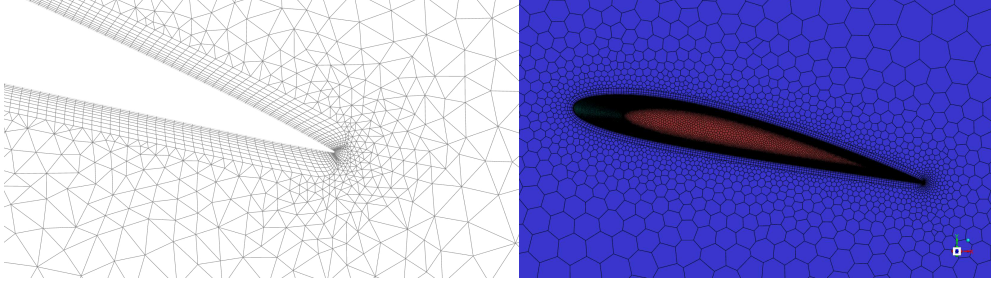


Fig. 3: computational meshes employed in the present study: (a) 2D triangular mesh with trailing edge refinement and (b) 3D polyhedral mesh with near-wall inflation layers.

C_L between the medium and fine meshes was below 1% (0.6% for the pre-stall case and 0.98% for the post-stall case), satisfying the grid-independence criterion [26]. The medium mesh, with approximately 10,800 cells, was selected as the final mesh and used for all two-dimensional simulations.

Table 3: Two-dimensional grid-independence study: C_L at $Re = 2.2 \times 10^6$.

Case	Mesh	Cells	α ($^\circ$)	C_L	Relative change (%)
Pre-stall	Coarse	8,542	12.4	1.3384	—
	Medium	10,816	12.4	1.3615	+1.73
	Fine	26,815	12.4	1.3533	-0.60
Post-stall	Coarse	8,446	16.0	1.4127	—
	Medium	10,806	16.0	1.4340	+1.51
	Fine	26,529	16.0	1.4480	+0.98

For the three-dimensional configurations, polyhedral meshes were generated using ANSYS Fluent. The airfoil surface mesh size was kept the same as in the two-dimensional cases at 0.005 m. However, the trailing-edge region was further refined to 0.001 m to better capture the sharp three-dimensional geometric feature. The inflation-layer settings were also kept consistent with the two-dimensional mesh, using 10 layers with a growth ratio of 1.17. The spanwise extent of the computational domain was determined by the aspect ratio in each case. The total cell count for each aspect-ratio configuration is given in Table 5.

To obtain mesh-independent results, a grid-independence study was also conducted for the three-dimensional case at $AR = 2$, $\alpha = 10^\circ$, and $Re = 2.2 \times 10^6$. The summary is given in Table 4. The difference in C_L between the medium and fine meshes was 0.87%, which satisfies the 1% grid-independence criterion. Therefore, the medium mesh configuration with approximately 946,000 cells was selected as the final mesh for all

finite-wing simulations, with the aspect ratio varied according to the corresponding wingspan.

Table 4: Three-dimensional grid-independence study: C_L at $AR = 2$, $\alpha = 10^\circ$, and $Re = 2.2 \times 10^6$.

Mesh	Cells	C_L	Relative change (%)
Coarse	417,989	0.6100	—
Medium	946,346	0.5863	−3.89
Fine	1,670,010	0.5812	−0.87

Table 5: Three-dimensional mesh statistics for each aspect-ratio configuration.

AR	Span L_z (mm)	Total cells
1	731	0.5 million
2	1,462	0.9 million
4	2,924	1.8 million

4 Numerical Validation

To check the predictive reliability of the numerical framework, a validation study was conducted. First, the two-dimensional NACA 2412 results were compared against the wind-tunnel experimental measurements of Seetharam and Wentz [4] at $Re = 2.2 \times 10^6$ and Mach number, $M = 0.13$. For three-dimensional cases, experimental or computational results datasets for low-aspect-ratio NACA 2412 wings at $AR = 1, 2$, and 4 at $Re = 2.2 \times 10^6$, are not readily available in the open literature. Thereafter, classical lifting-line theory (LLT) was used to compare pre-stall lift behavior to find out the consistency of the three-dimensional results.

Fig. 4a compares the two-dimensional computed lift coefficient, C_L , with experimental data conducted by Seetharam and Wentz [4] over the angle of attack varying from $\alpha = -4^\circ$ to $\alpha = 20^\circ$ at $Re = 2.2 \times 10^6$. In the pre-stall regime ($\alpha \leq 10^\circ$), the computed results match closely with the experimental trend within less than 3%. It indicates that, in terms of the attached-flow aerodynamics, the framework is well-suited.

In the near-stall and stall regime, the computational result yields a maximum lift coefficient of $C_{L,\max} = 1.434$ at α_{stall} close to 16° , compared to the experimental value

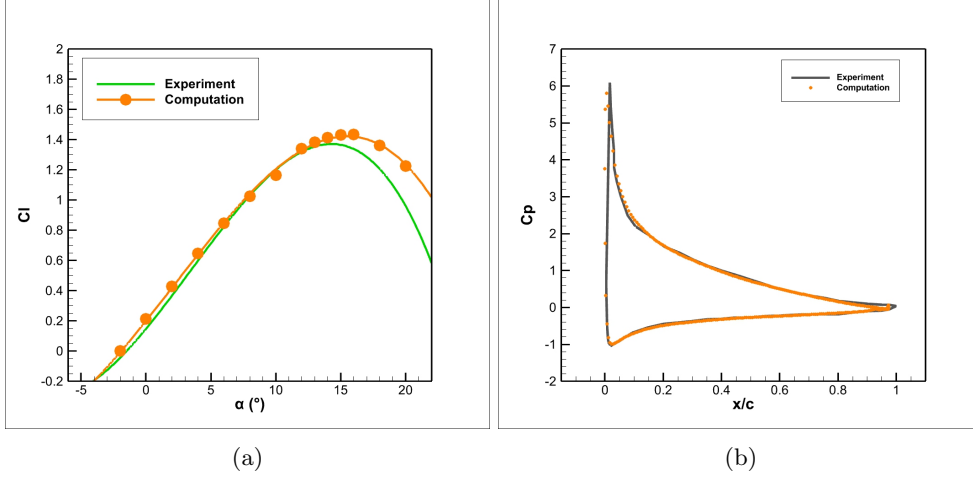


Fig. 4: Two-dimensional validation of the NACA 2412 airfoil at $Re = 2.2 \times 10^6$ against the experimental data: (a) lift coefficient C_L as a function of the angle of attack α ; (b) chordwise pressure coefficient distribution at $\alpha = 12.4^\circ$.

of $C_{L,max} = 1.446$ at $\alpha_{stall} \approx 14.4^\circ$. This indicates that the numerical framework captures the magnitude of the peak lift accurately, although the onset of stall is slightly delayed, consistent with the known behavior of steady RANS simulations for trailing-edge stall prediction [6].

Post-stall, the computation captures the qualitative trend of lift reduction, although the rate of decrease in C_l is underpredicted. This behavior is consistent with the known limitations of steady RANS simulations for massively separated flows. Overall, the pre-stall lift prediction is sufficiently accurate for the present parametric study, while the observed 1–2° stall-angle offset is acknowledged as a limitation of the numerical framework.

Surface pressure coefficient is also compared with experimental results Fig. 4b at $\alpha = 12.4^\circ$. It shows a good agreement with the experimental data on both the pressure and suction surfaces across the chord. This close agreement confirms that the framework can also reliably reproduce surface pressure distribution variables, providing confidence for the subsequent wall-shear-stress-based separation analysis.

For three-dimensional simulations, a consistency check was performed by comparing lift coefficients at $AR = 2$ against Prandtl lifting-line theory (LLT) in the pre-stall regime [2]. From the experimental measurements [4], the two-dimensional lift curve slope $a_0 = 6.073 \text{ rad}^{-1}$ and zero-lift angle $\alpha_0 = -1.18^\circ$ were extracted using linear regression over the attached flow regime. The LLT finite-wing lift curve is then calculated as:

$$a = \frac{a_0}{1 + \frac{a_0}{\pi e AR}} = \frac{6.073}{1 + \frac{6.073}{\pi \times 0.85 \times 2}} = 2.888 \text{ rad}^{-1} \quad (8)$$

where a representative span efficiency factor of $e = 0.85$ is adopted for the unswept rectangular wing planform [27]. The resulting LLT lift expression becomes:

$$C_L^{\text{LLT}} = 2.888(\alpha - \alpha_0) \quad [\alpha \text{ in radians}] \quad (9)$$

The full C_L comparison across the angle of attacks is presented in Table 6. LLT values are physically applicable for $\alpha \leq 16^\circ$, beyond which the inviscid attached-flow is violated. Therefore, the LLT comparison is strictly limited to the attached flow regime.

In the pre-stall regime ($\alpha = 4^\circ\text{--}16^\circ$), the CFD results give a consistent 2.6–3.7% agreement with LLT. This falls well within the accepted engineering tolerance for RANS versus LLT comparison [16, 18]. The deviation is larger at $\alpha = 0^\circ$ and 2° , respectively 16.3% and 5.9%, attributable to the relatively greater sensitivity of low-incidence C_L to the precise value of α_0 and the small absolute lift magnitudes involved, small absolute errors appear as large percentages.

The $AR = 2$ wing continues generating lift beyond the two-dimensional stall angle of 16° , up to $C_{L,\text{max}} = 1.212$ at $\alpha = 24^\circ$, stalling sharply after that. This approximated delay of 8° relative to the two-dimensional baseline shows the influence of tip vortex downwash suppressing the trailing-edge separation [15, 18]. The close agreement to the pre-stall LLT and physically consistent post-stall behaviour together suggest that the three-dimensional solver is suited for the purpose of the study in Section 5.

Table 6: Comparison of computed C_L ($AR = 2$, $Re = 2.2 \times 10^6$) with Prandtl lifting-line theory. Dashes indicate post-stall conditions where LLT is inapplicable.

α ($^\circ$)	C_L (CFD)	C_L (LLT)	ΔC_L (%)
0	0.1017	0.0875	+16.3
2	0.1950	0.1842	+5.9
4	0.2914	0.2809	+3.7
6	0.3872	0.3776	+2.6
8	0.4868	0.4743	+2.6
10	0.5858	0.5710	+2.6
12	0.6890	0.6677	+3.2
14	0.7896	0.7644	+3.3
16	0.8890	0.8611	+3.2
18	0.9856	—	—
20	1.0760	—	—
22	1.1632	—	—
24	1.2124	—	—
26	0.9718	—	—
30	0.7658	—	—

5 Results and Discussion

5.1 Two-Dimensional Baseline Stall Mechanism

The two-dimensional airfoil’s performance is assessed from how its lift and drag coefficients vary across the full range of angles of attack. From this we obtain the stall angle α_{stall} and the maximum lift coefficient $C_{l,\text{max}}$, which serve as the main reference values for the aspect-ratio study that follows.

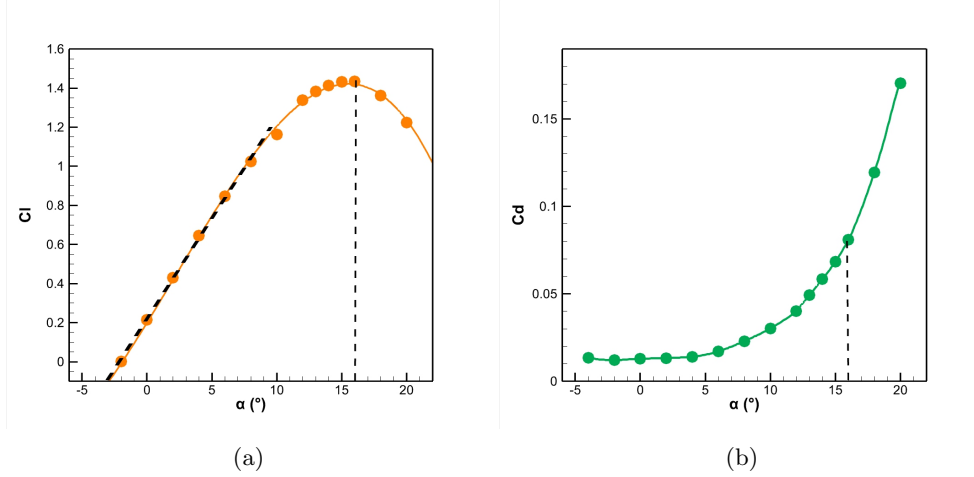


Fig. 5: Two-dimensional aerodynamic coefficients for the NACA 2412 airfoil at $Re = 2.2 \times 10^6$: (a) lift coefficient C_l versus angle of attack α , with the dashed line showing the linear lift curve slope; (b) drag coefficient C_d versus angle of attack α . The vertical dashed line in both panels denotes the stall angle $\alpha_{\text{stall}} = 16^\circ$.

Fig. 5 shows how the lift and drag coefficients vary across the full angle-of-attack sweep. Before stall, C_l rises linearly with α , much as classical thin-aerofoil theory predicts, which points to mostly attached flow on the suction surface. Beyond $\alpha \approx 10^\circ$, the lift stops climbing as quickly: the boundary layer thickens and trailing-edge separation begins, both of which cut the effective camber. This yields a maximum lift coefficient of $C_{l,\text{max}} = 1.434$ at $\alpha_{\text{stall}} = 16^\circ$, the angle taken as the definition of stall throughout this study. Past stall, C_l falls sharply as large-scale separation spreads over the suction side and pressure recovery on the upper surface collapses.

At low to moderate angles, C_d rises gradually, consistent with the boundary-layer growth expected in attached flow. Near α_{stall} , the drag climbs more steeply as the wake thickens and separated flow adds pressure drag. Post-stall, C_d keeps rising steadily as the separated wake grows wider. That $C_{l,\text{max}}$ and the onset of this drag knee occur at the same angle confirms $\alpha_{\text{stall}} = 16^\circ$ as a physically consistent stall criterion.

The wall shear stress distribution on the suction surface is examined across the same angle-of-attack sweep. The separation point x_s/c is taken as the chordwise location where the wall shear stress first turns negative, signalling local flow reversal. The separation fraction S is defined as:

$$S(\alpha) = \frac{x_{TE} - x_s}{x_{TE}} \times 100\% \quad (10)$$

where $S(\alpha)$ is the suction-surface separation fraction (%), α is the angle of attack, x_{TE} is the chordwise coordinate of the trailing edge measured from the leading edge, and x_s is the chordwise location of the separation point on the suction surface measured from the leading edge.

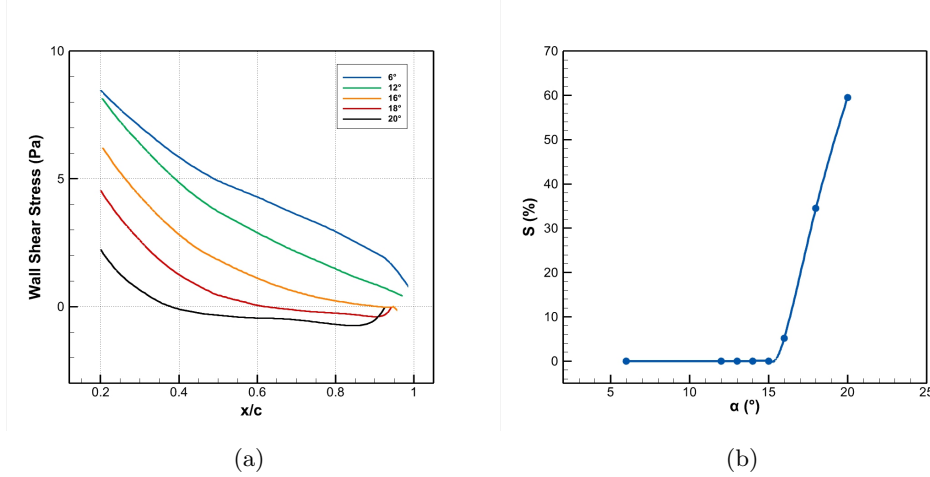


Fig. 6: Two-dimensional NACA 2412 suction-surface separation at $Re = 2.2 \times 10^6$: (a) Chordwise wall shear stress, where $\tau_w < 0$ indicates flow reversal and the zero crossing defines x_s/c ; (b) separation fraction $S(\alpha)$ versus angle of attack. Trailing-edge oscillations are excluded from the analysis.

Fig. 6a presents the chordwise wall shear stress distribution. At $\alpha = 6^\circ$ and 12° , τ_w stays positive over the entire chord, consistent with the linear C_l growth seen in Fig. 5a. At $\alpha = 16^\circ$, τ_w first turns negative at $x_s/c = 0.908$, marking the onset of trailing-edge separation that coincides with $C_{l,max}$. Beyond stall, the separation point moves rapidly upstream, from $x_s/c = 0.621$ at $\alpha = 18^\circ$ to $x_s/c = 0.379$ at $\alpha = 20^\circ$. A region of partial reattachment near $x/c = 0.93$ at $\alpha = 20^\circ$ points to a closed recirculation topology typical of fully stalled flow.

The progressive nature of this separation process is quantified by the $S(\alpha)$ curve in Fig. 6b. Its most significant feature is the rapid post-stall growth where S jumps from 5.2% at $\alpha = 16^\circ$ to 34.5% at $\alpha = 18^\circ$, reaching 59.5% at $\alpha = 20^\circ$. This sharp knee in the $S(\alpha)$ curve at $\alpha_{stall} = 16^\circ$, followed by a rapid upstream march of the separation point, is the defining signature of a progressive trailing-edge stall mechanism.

5.2 Effect of Aspect Ratio on Lift and Stall Angle

The effect of aspect ratio on lift generation and stall behavior is shown in Fig. 7. In the pre-stall regime ($\alpha \leq 10^\circ$), every finite-wing configuration has a lower lift-curve slope than the two-dimensional baseline which is consistent with the downwash-induced drop in effective angle of attack predicted by lifting-line theory.

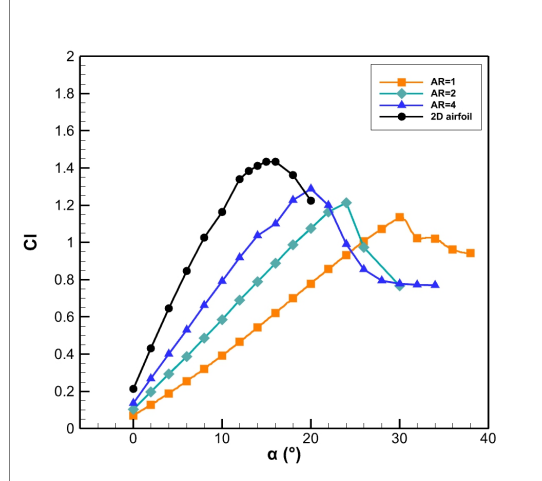


Fig. 7: Lift coefficient C_L as a function of angle of attack α for the NACA 2412 airfoil at $Re = 2.2 \times 10^6$ across all configurations: two-dimensional baseline and finite wings at $AR = 1, 2$, and 4 .

The finite wings behave quite differently as stall is progressively delayed with AR , with α_{stall} shifting to roughly 20° for $AR = 4$, 24° for $AR = 2$, and about 30° for $AR = 1$. This delay follows directly from tip-vortex downwash, which lowers the local effective angle of attack near the wingtip and suppresses trailing-edge separation over a proportionally larger fraction of the span as AR falls, consistent with DeVoria and Mohseni [15] and Ribeiro et al. [18].

5.3 Separation Topology and Progressive Growth on Finite Wings

The suction-surface separation topology is examined for all three finite-wing configurations using a binary variable based on the sign of the streamwise wall shear stress: regions where $\tau_w > 0$ (attached flow) appear in blue, and regions where $\tau_w < 0$ (separated, reversed flow) appear in red. Pre-stall and post-stall angles are chosen individually for each configuration, two degrees below and two degrees above its own $C_{\{1,\text{max}\}}$ angle. This gives $\alpha = 18^\circ$ and 22° for $AR = 4$, $\alpha = 22^\circ$ and 26° for $AR = 2$, and $\alpha = 28^\circ$ and 32° for $AR = 1$, as visualised in Fig. 8.

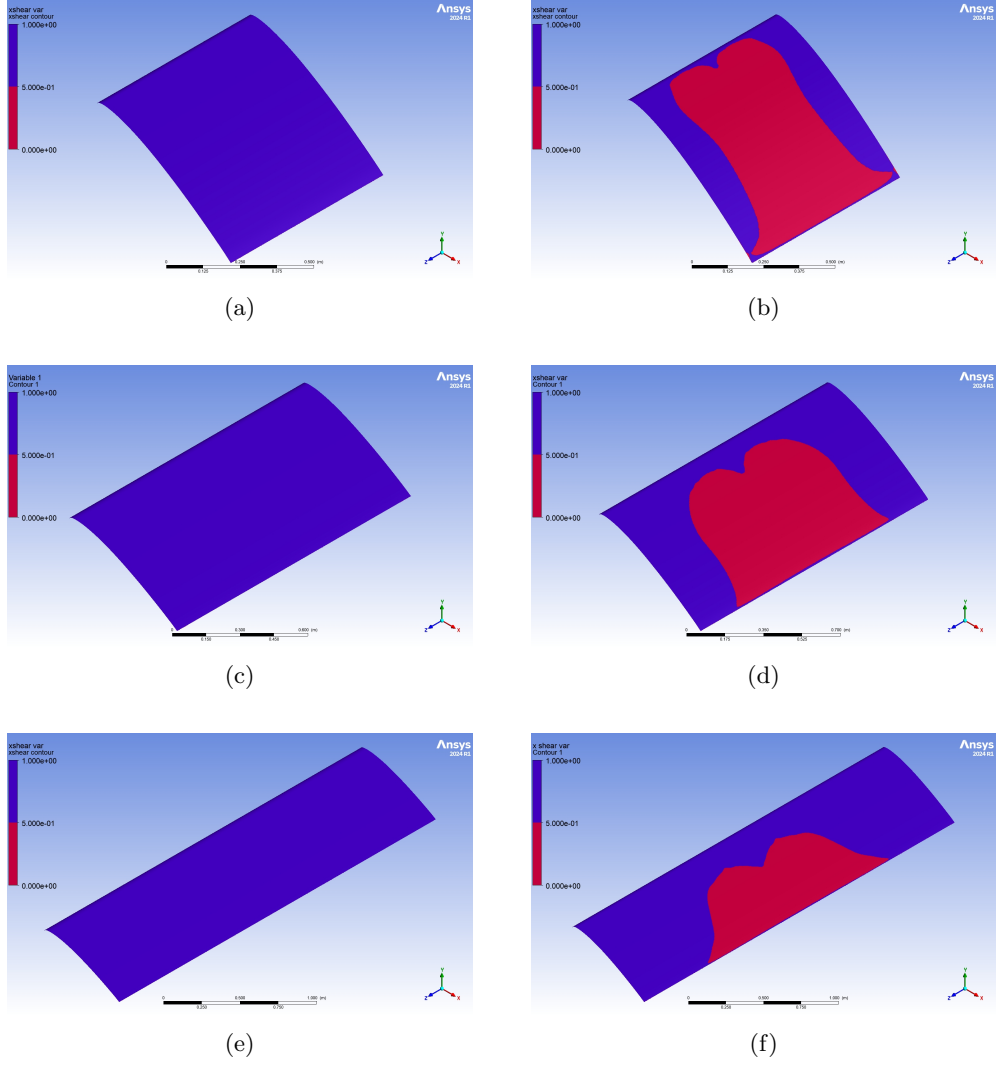


Fig. 8: Suction-surface separation topology visualised through the sign of the stream-wise wall shear stress τ_w for all three finite-wing configurations at $Re = 2.2 \times 10^6$. Blue regions indicate attached flow ($\tau_w > 0$), and red regions indicate separated flow ($\tau_w < 0$).

In the pre-stall panels, all three configurations show a uniformly blue suction surface across the full span, confirming fully attached flow below each wing's own stall angle. The post-stall panels reveal a separation pattern that is consistent in character across all three AR values but differs systematically in spatial extent. For $AR = 4$ at $\alpha = 22^\circ$, the separated zone is well-defined and root-concentrated, beginning at

the trailing edge of the root section and advancing forward, consistent with the root-initiated trailing-edge stall progression reported by Gopalarathnam et al. [17] and Konstadinopoulos et al. [19] for unswept rectangular wings. For $AR = 2$ at $\alpha = 26^\circ$, the separated zone is similarly concentrated near the center but has a more irregular boundary. For $AR = 1$ at $\alpha = 32^\circ$, the reversed-flow zone is present but proportionally more confined relative to the total surface area, as the stronger tip downwash suppresses separation over a greater portion of the span on each side.

The leading edge stays attached everywhere along the span, and the separation front advances inward from the trailing edge, confirming trailing-edge stall as the operative mechanism and matching the two-dimensional behaviour established in Section 5.1. The proportionally larger tip-attachment regions at lower AR show that the relative spanwise influence of the tip vortex grows as AR decreases, progressively limiting the extent of trailing-edge separation.

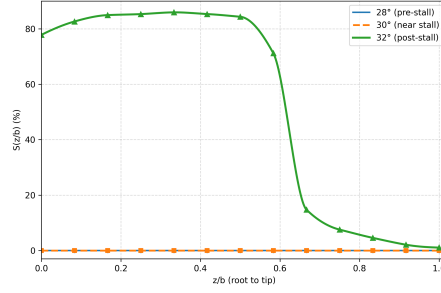
5.4 Spanwise Distribution of Separation: $S(z/b)$

The quantity $S(z/b)$ is computed at each spanwise strip as the fraction of the local suction-surface chord over which $\tau_w < 0$, by analogy with Equation 10. The resulting spanwise profiles are shown in Fig. 9a, Fig. 9b, and Fig. 9c for $AR = 1, 2$, and 4 , respectively.

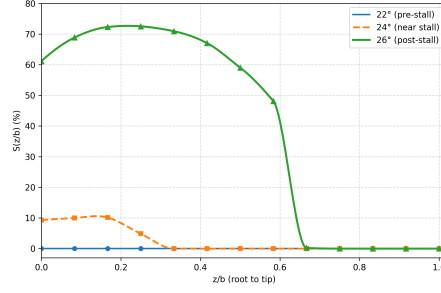
The quantity $S(z/b)$ is computed at each spanwise strip as the fraction of the local suction-surface chord over which $\tau_w < 0$, by analogy with .

The primary result is the progressive delay in stall onset as AR decreases. For $AR = 4$, measurable separation first appears at $\alpha = 20^\circ$, and for $AR = 2$ onset is delayed to $\alpha = 24^\circ$. For $AR = 1$, both the $\alpha = 28^\circ$ and $\alpha = 30^\circ$ curves lie at $S = 0$ across the full span, with separation not appearing until $\alpha = 32^\circ$. This systematic delay quantifies the tip-vortex downwash suppression mechanism and confirms that 2D stall predictions become increasingly non-conservative as AR decreases. The near-stall profiles show how incipient separation organizes spanwise before it spreads. For $AR = 4$ at $\alpha = 20^\circ$, separation is sharply localized at $z/b = 0.167$, peaking at 19.5%. For $AR = 2$ at $\alpha = 24^\circ$, it is shallower (peak 10.2%) and more smoothly spread across the inner third of the span.

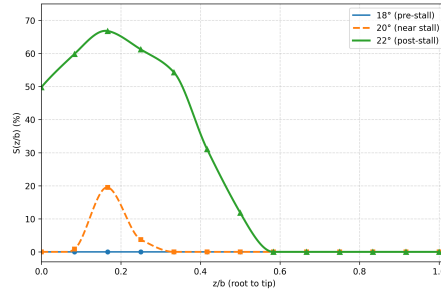
At $AR = 1$, the tip vortex cannot establish a meaningful separation gradient across the inner span at this angle; its suppressive influence is concentrated entirely in the outermost 25–30% of the semi-span. By contrast, for $AR = 4$ at $\alpha = 22^\circ$ and $AR = 2$ at $\alpha = 26^\circ$, $S(z/b)$ takes a peaked profile with a well-defined inner-span maximum that falls monotonically toward the tip. This confirms that the three-dimensional finite wing preserves the trailing-edge stall mechanism locally at each spanwise station, with the tip vortex acting to delay chordwise separation onset rather than to alter the stall mechanism itself.



(a) $AR = 1$



(b) $AR = 2$



(c) $AR = 4$

Fig. 9: Spanwise distribution of the separation fraction, $S(z/b)$, for finite NACA 2412 wings at $Re = 2.2 \times 10^6$: (a) $AR = 1$, (b) $AR = 2$, and (c) $AR = 4$. The profiles compare pre-stall, stall-onset, and post-stall conditions, illustrating the evolution of spanwise trailing-edge separation and its dependence on aspect ratio.

6 Conclusion

This study introduced a wall-shear-stress-based separation metric, S , and its spanwise extension, $S(z/b)$, to characterize the effects of aspect ratio on aerodynamic stall and spanwise separation of finite NACA 2412 wings at $Re = 2.2 \times 10^6$ using steady RANS

with the standard $k-\varepsilon$ turbulence model. The numerical setup was validated against Seetharam and Wentz’s 2D airfoil measurements, then assessed for finite wings by comparison with Prandtl lifting-line theory in attached flow.

The model reproduced pre-stall 2D airfoil characteristics accurately, predicting lift within engineering accuracy while showing the 1-2° stall-angle overshoot typical of $k-\varepsilon$. A clear stall delay appeared with a decreasing aspect ratio: stall onset rose from around 20° at AR=4 to 24° at AR=2 and 32° at AR=1, reflecting the growing role of tip-vortex downwash in suppressing trailing-edge separation.

The proposed wall-shear-stress separation fraction S captured the progressive chordwise growth of trailing-edge separation in the 2D case, while its spanwise form $S(z/b)$ provided a straightforward surface-based metric for tracking 3D separation. The spanwise distributions showed distinct behavior by aspect ratio: higher-AR wings had root-dominated separation with a sharp spanwise gradient, while lower-AR wings showed delayed, spatially confined separation due to stronger tip-vortex influence. Despite these differences, all finite-wing cases retained the same trailing-edge stall mechanism; aspect ratio changes the timing and spatial extent of separation, not the underlying physics.

The methodology offers a computationally practical framework for analyzing finite-wing stall from standard RANS and wall shear stress data. The separation metrics provide a useful tool for comparing stall behavior across geometries and could support low-AR wing design work.

Future work should move to higher-fidelity turbulence models (SST $k-\omega$, SAS, DES, LES) with experimental validation of finite-wing cases. Extending to tapered and swept platforms, dynamic stall, and unsteady flow would test how broadly the separation metrics apply.

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Authors’ Contributions

Tamal Kirttania conceived the study, developed the numerical methodology, performed the CFD simulations, analyzed the results, prepared the figures, and drafted the manuscript. Swargo Raj Saha contributed to the methodology, data analysis, interpretation of the results, and manuscript revision. Dr. Mohammad Mamun supervised the research, contributed to the conceptualization of the study, critically reviewed the manuscript, and provided overall guidance throughout the research. All authors read and approved the final manuscript.

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Data Availability

The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Declarations

Ethics Approval and Consent to Participate

Not applicable.

Consent for Publication

Not applicable.

Competing Interests

The authors declare that they have no competing interests.

Code Availability

The ANSYS Fluent simulation setup, post-processing scripts, and other computational files used in this study are available from the corresponding author upon reasonable request.

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