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Diffraction Characteristics of the Connes-Apodized Aperture in the Presence of Tertiary Spherical Aberration

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Abstract

The diffraction behavior of a Connes-apodized circular aperture in the presence of tertiary spherical aberration (TSA) and defocus is investigated using the Fourier–Bessel diffraction formulation. Normalized and unnormalized amplitude point spread functions (PSFs) were computed for various values of the TSA coefficient and defocus parameter, and the corresponding Strehl ratios were evaluated to quantify image-quality degradation. The results show that increasing TSA and defocus progressively broaden the diffraction profile, reduce the central amplitude, and redistribute optical energy toward the outer diffraction regions. Strehl-ratio analysis reveals a monotonic decrease in optical performance with increasing aberration magnitude, although the degradation remains gradual due to the higher-order nature of TSA. A comparative study involving Connes, Bartlett, Hanning, and Blackman apodization functions demonstrates that the Connes filter provides the narrowest central diffraction peak and superior energy concentration near the optical axis, while the Blackman filter offers the strongest sidelobe suppression. The findings indicate that Connes apodization maintains favorable diffraction characteristics and exhibits good tolerance to tertiary spherical aberration, making it suitable for high-resolution optical imaging applications affected by higher-order wavefront distortions.

Keywords: Connes apodization, tertiary spherical aberration, point spread function, diffraction, defocus, Strehl ratio, optical imaging systems.

1. INTRODUCTION

The diffraction characteristics of an optical imaging system are fundamentally governed by its point spread function (PSF), which describes the response of the system to a point object. The PSF determines important imaging properties such as spatial resolution, image contrast, energy concentration, and aberration sensitivity. In an ideal diffraction-limited system, the PSF is controlled solely by the aperture geometry. However, practical optical systems invariably experience wavefront distortions arising from fabrication tolerances, alignment errors, and optical aberrations, which lead to degradation of image quality and reduction of optical performance [1–4].

Among the various approaches employed to improve diffraction performance, pupil apodization has proven to be an effective technique for suppressing sidelobes and redistributing optical energy toward the central diffraction maximum. By appropriately modifying the amplitude transmission across the pupil, apodization reduces diffraction-induced oscillations and enhances image contrast without altering the physical aperture dimensions [5–8]. Numerous apodization functions have been investigated in the literature, including Gaussian, cosine, Hanning, Hamming, Blackman, and polynomial filters. The Connes apodizer is particularly attractive because of its ability to provide significant sidelobe suppression while preserving a relatively narrow central diffraction core [7,8].

In addition to diffraction effects, optical aberrations represent a major limitation on imaging performance. Defocus is one of the most frequently encountered aberrations and introduces a quadratic phase variation across the pupil, resulting in broadening of the central diffraction maximum and reduction of peak intensity [2,3,11]. Spherical aberration constitutes another important class of monochromatic aberrations and arises when rays passing through different pupil zones fail to converge at a common image point. Higher-order spherical aberrations introduce increasingly complex wavefront distortions and can significantly alter the diffraction structure and energy distribution within the image plane [2,3,9].

Among the higher-order aberrations, tertiary spherical aberration (TSA) corresponds to an eighth-order phase variation across the pupil and becomes increasingly significant in high-performance optical systems operating with large numerical apertures and stringent image-quality requirements [2,9,10]. Because the phase error increases rapidly toward the pupil boundary, TSA can strongly influence the interference process responsible for image formation, leading to broadening of the diffraction pattern, reduction of peak intensity, and deterioration of optical resolution. The resulting degradation becomes particularly important when tertiary spherical aberration is combined with defocus, since both aberrations contribute simultaneously to wavefront mismatch across the pupil [3,11].

Previous investigations have extensively examined diffraction behavior in the presence of defocus and lower-order spherical aberrations using various pupil-apodization techniques [1–8]. Nevertheless, comparatively limited attention has been devoted to the diffraction characteristics of Connes-apodized apertures under the influence of tertiary spherical aberration. In particular, a detailed analysis of the combined effects of TSA and defocus on the amplitude point spread function and Strehl-ratio performance remains largely unexplored.

The present work investigates the diffraction characteristics of a Connes-apodized circular aperture in the presence of tertiary spherical aberration and defocus. Numerical simulations based on the Fourier–Bessel diffraction formulation are employed to evaluate both normalized and unnormalized amplitude point spread functions over a range of aberration strengths. Strehl-ratio analysis is further performed to quantify the degradation of optical performance and to assess the effectiveness of Connes apodization in mitigating the influence of higher-order wavefront distortions. The results provide insight into the diffraction behavior of highly aberrated optical systems and contribute to the understanding of image-quality degradation associated with tertiary spherical aberration. In addition, a comparative assessment involving Bartlett, Hanning, and Blackman apodization functions is presented to place the performance of the Connes filter in a broader context. The findings may be useful in the design and optimization of advanced optical imaging systems requiring improved tolerance to higher-order aberrations.

2. THEORETICAL FORMULATION

2.1 Generalized Pupil Function

The diffraction characteristics of an optical imaging system can be analyzed using scalar diffraction theory under the Fraunhofer approximation. The generalized pupil function combines the aperture transmission function with the wavefront aberration and is expressed as [1,4]

$$G(\rho) = P(\rho)\exp[-iW(\rho)] \quad (1)$$

where $P(\rho)$ denotes the pupil transmission function, $W(\rho)$ represents the wave aberration function, and ρ is the normalized radial pupil coordinate satisfying

$$0 \leq \rho \leq 1.$$

For a rotationally symmetric optical system, the amplitude point spread function (PSF) is given by the Fourier–Bessel diffraction integral [1–3]

$$B(Z) = \int_0^1 P(\rho)\exp[-iW(\rho)]J_0(Z\rho)\rho d\rho \quad (2)$$

where J_0 denotes the zeroth-order Bessel function of the first kind and Z represents the reduced image-plane coordinate.

The corresponding intensity point spread function is

$$I(Z) = |B(Z)|^2 \quad (3)$$

which characterizes the imaging performance of the optical system.

2.2 Connes Apodization Function

In the present investigation, the pupil is apodized using the Connes filter whose transmission function is given by [7]

$$P(\rho) = (1 - \beta^2 \rho^2)^2 \quad (4)$$

where β is the apodization parameter.

For $\beta = 0$, the aperture transmission becomes uniform and the diffraction pattern corresponds to that of a clear circular aperture. Increasing β progressively reduces the transmission near the pupil boundary, thereby suppressing secondary diffraction maxima and improving energy concentration within the central diffraction region.

Substituting Eq. (4) into Eq. (2), the amplitude PSF becomes

$$B(Z) = \int_0^1 (1 - \beta^2 \rho^2)^2 \exp[-iW(\rho)] J_0(Z\rho) \rho \, d\rho \quad (5)$$

2.3 Defocus Aberration

Defocus introduces a quadratic phase variation across the pupil and is represented by [2,3,11]

$$W_D(\rho) = \frac{\phi_d}{2} \rho^2 \quad (6)$$

where ϕ_d denotes the defocus coefficient.

The corresponding generalized pupil function becomes

$$G(\rho) = P(\rho) \exp \left[-i \left(\frac{\phi_d}{2} \rho^2 \right) \right] \quad (7)$$

The quadratic phase error reduces the peak intensity and broadens the central diffraction structure.

2.4 Tertiary Spherical Aberration (TSA)

Tertiary spherical aberration corresponds to an eighth-order wavefront deformation and becomes increasingly significant in high-numerical-aperture and precision optical systems containing residual higher-order aberrations [2,3,9–11].

The wave aberration function is expressed as

$$W_{TSA}(\rho) = \frac{\phi_d}{2} \rho^2 + \frac{TSA}{8} \rho^8 \quad (8)$$

where TSA denotes the tertiary spherical aberration coefficient.

Substitution into Eq. (5) yields

$$B(Z) = \int_0^1 (1 - \beta^2 \rho^2)^2 \exp \left[-i \left(\frac{\phi_d}{2} \rho^2 + \frac{TSA}{8} \rho^8 \right) \right] J_0(Z\rho) \rho \, d\rho \quad (9)$$

The eighth-order phase term introduces strong wavefront distortion near the pupil boundary and modifies the diffraction-energy distribution within the image plane. Since the phase error increases rapidly toward the pupil edge, tertiary spherical aberration primarily influences the

marginal pupil regions and can significantly alter diffraction behavior in high-resolution optical systems.

2.5 Strehl Ratio

The degradation of imaging performance is quantified using the Strehl ratio, defined as [2,3]

$$SR = \frac{I_{\max(\text{aberrated})}}{I_{\max(\text{reference})}} \quad (10)$$

where $I_{\max(\text{aberrated})}$ is the peak intensity of the aberrated PSF and $I_{\max(\text{reference})}$ corresponds to the peak intensity of the diffraction-limited system.

A Strehl ratio equal to unity indicates diffraction-limited performance, whereas lower values signify increasing aberration-induced degradation.

3. NUMERICAL PROCEDURE

The diffraction characteristics of the Connes-apodized circular aperture in the presence of tertiary spherical aberration were evaluated numerically using the Fourier–Bessel diffraction formulation described in Section 2. The amplitude point spread function was computed by numerically evaluating Eq. (9) for different combinations of the tertiary spherical aberration coefficient TSA and the defocus coefficient ϕ_d .

The numerical simulations were performed using a Python-based computational framework developed with NumPy, SciPy, and Matplotlib libraries. The normalized pupil coordinate was sampled uniformly over the interval

$$0 \leq \rho \leq 1$$

using 6000 integration points to ensure numerical convergence and stability of the diffraction integral. Simpson's numerical integration method was employed for evaluating the Fourier–Bessel integral, while the Bessel function J_0 was computed using standard scientific-computing routines [1].

The reduced image-plane coordinate was selected within the range

$$-15 \leq Z \leq 15$$

using 1200 uniformly spaced sampling points. This interval was found sufficient to capture both the principal diffraction maximum and the surrounding diffraction structure.

The Connes apodization parameter was fixed at

$$\beta = 0.05$$

throughout the investigation. This value provides effective sidelobe suppression while maintaining adequate energy concentration within the central diffraction region. In addition to the Connes filter, Bartlett, Hanning, and Blackman apodization functions were considered for comparative analysis under identical aberration conditions. The comparison was

performed to evaluate the relative influence of different pupil-transmission functions on the diffraction characteristics and image-quality performance of optical systems affected by tertiary spherical aberration. The comparative study was carried out for selected aberration conditions and is presented separately to highlight the effectiveness of the Connes filter relative to other commonly employed apodization functions.

The defocus coefficient was varied as

$$\phi_d = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

representing progressively increasing levels of defocus aberration.

To investigate the influence of tertiary spherical aberration, the aberration coefficient was varied according to

$$TSA = 0, 2, 4, 6, 8, 10.$$

For each parameter combination, both normalized and unnormalized amplitude point spread functions were calculated. The normalized distributions were obtained by dividing the computed amplitudes by their corresponding peak values, enabling direct comparison of diffraction-profile shapes under different aberration conditions. The unnormalized distributions were retained to quantify the actual reduction in peak amplitude caused by aberration-induced phase distortions.

The optical performance of the system was further quantified through Strehl-ratio calculations using Eq. (10). Strehl ratios were evaluated for all investigated values of TSA and ϕ_d , providing a quantitative measure of the degradation produced by the combined influence of tertiary spherical aberration and defocus.

The numerical procedure was repeated systematically for all investigated parameter combinations, and the resulting normalized amplitude point spread functions, unnormalized amplitude point spread functions, and Strehl-ratio variations constitute the basis of the results and discussion presented in the subsequent section.

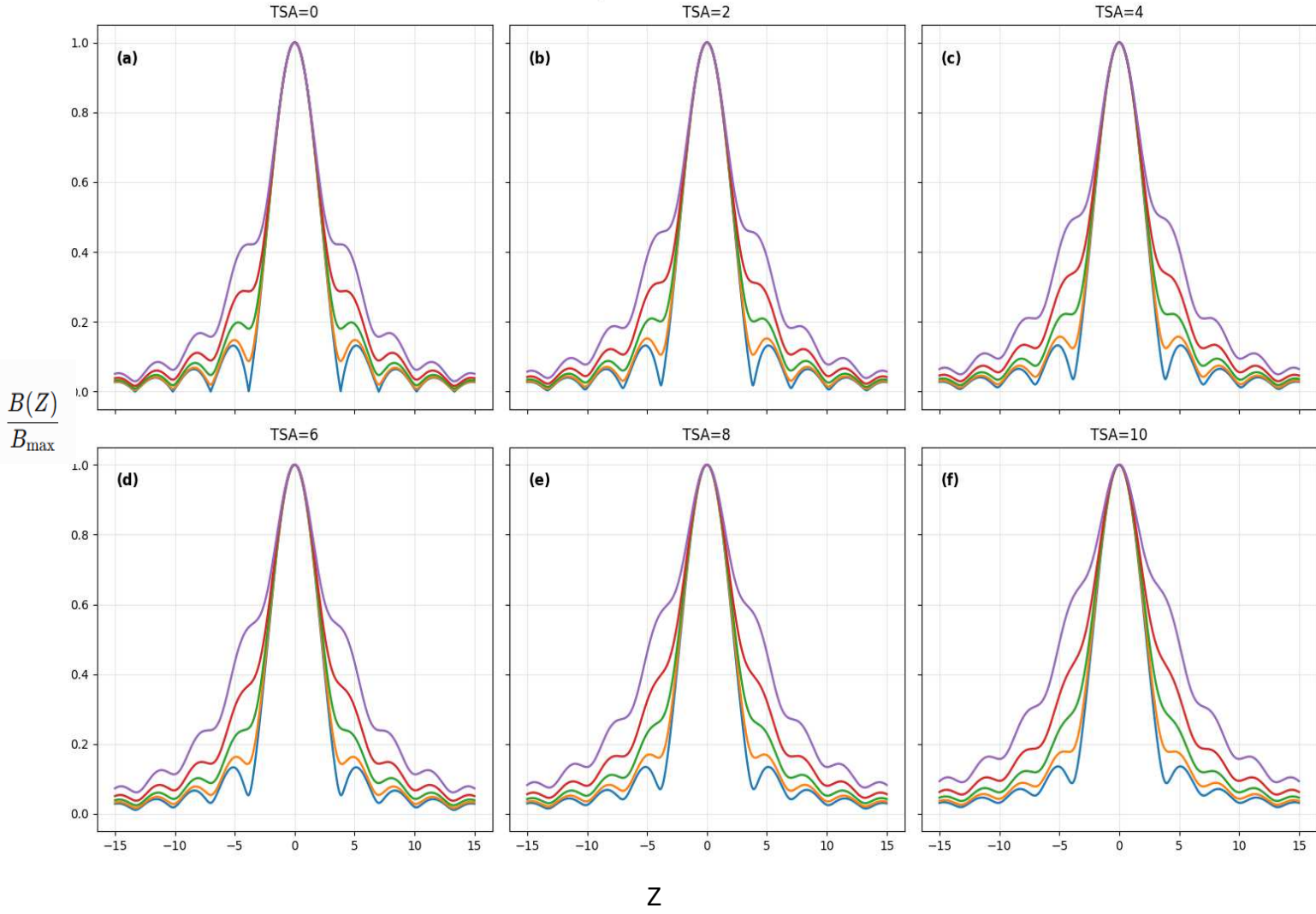


Figure 1. Normalized Amplitude Point Spread Functions of the Connes-Apodized Aperture in the Presence of Tertiary Spherical Aberration.

4. Results and Discussion

4.1 Normalized Amplitude Point Spread Functions

Figure 1(a–f) shows the normalized amplitude point spread functions of the Connes-apodized circular aperture ($\beta = 0.05$) for tertiary spherical aberration coefficients ranging from $TSA = 0$ to $TSA = 10$ and defocus coefficients $\phi_d = 0, \pi/2, \pi, 3\pi/2$, and 2π . Since the amplitudes are normalized to their respective maxima, the figure highlights changes in diffraction-profile shape, diffraction width, and sidelobe structure independently of peak-amplitude variations.

In the absence of tertiary spherical aberration [Fig. 1(a)], the diffraction behavior is governed entirely by defocus. Increasing ϕ_d produces a gradual broadening of the central diffraction lobe together with enhanced sidelobe amplitudes, indicating redistribution of optical energy away from the optical axis. The focused condition ($\phi_d = 0$) exhibits the narrowest diffraction profile and the strongest energy localization.

The introduction of tertiary spherical aberration progressively modifies the diffraction structure. For $TSA = 2$ and $TSA = 4$ [Fig. 1(b,c)], only modest changes are observed, suggesting that small eighth-order phase distortions have limited influence on the normalized diffraction profile. Nevertheless, the diffraction envelopes associated with larger defocus coefficients become slightly broader, indicating the onset of aberration-induced energy redistribution.

As the aberration coefficient increases to $TSA = 6$ and $TSA = 8$ [Fig. 1(d,e)], the broadening becomes increasingly evident. The diffraction profiles corresponding to $\phi_d = 3\pi/2$ and 2π develop extended shoulders surrounding the central maximum, while the sidelobe structure becomes more pronounced. These features indicate increasing phase mismatch among wavefront contributions originating from different pupil regions and a consequent reduction in energy localization near the optical axis.

The most significant modifications occur for $TSA = 10$ [Fig. 1(f)]. Under this condition, the diffraction profiles exhibit substantial broadening, particularly for the larger defocus coefficients. The optical energy becomes distributed over a wider range of image-plane coordinates, and the sidelobes extend farther from the diffraction center. Although the distributions remain symmetric about the optical axis, the resulting diffraction structure corresponds to a considerably less localized point response.

A comparison of Fig. 1(a–f) demonstrates that the influence of tertiary spherical aberration increases systematically with aberration magnitude. For a fixed value of ϕ_d , increasing TSA produces broader diffraction envelopes and stronger sidelobe structures. Similarly, for a fixed TSA value, increasing defocus enhances the redistribution of optical energy and further widens the diffraction profile. These observations indicate that defocus and tertiary spherical aberration act cooperatively in degrading diffraction performance.

The relatively gradual evolution of the diffraction profiles with increasing TSA can be attributed to the eighth-order nature of the aberration term. Since the phase error varies as ρ^8 , significant wavefront distortion occurs primarily near the pupil boundary. The Connes apodization function suppresses the contribution of these outer pupil regions, thereby reducing the sensitivity of the diffraction pattern to tertiary spherical aberration. Consequently, noticeable degradation appears only at larger aberration coefficients.

The normalized results therefore indicate that tertiary spherical aberration primarily affects the spatial distribution of optical energy rather than the normalized peak value itself. Its principal effect is manifested through progressive broadening of the diffraction envelope and enhancement of the surrounding diffraction structure, effects that become increasingly pronounced with increasing defocus.

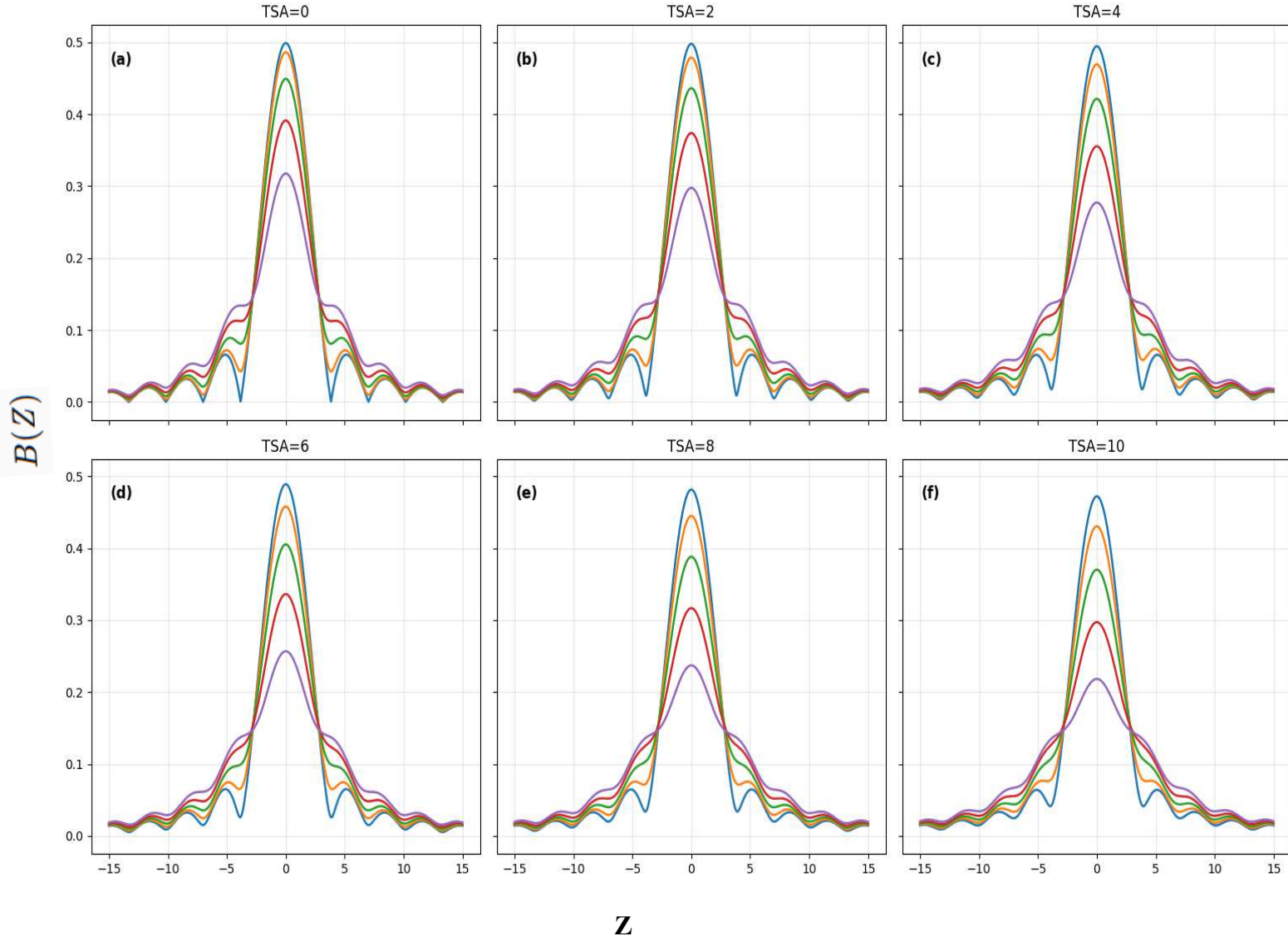


Figure 2. Unnormalized Amplitude Point Spread Functions in the Presence of Tertiary Spherical Aberration

4.2 Unnormalized Amplitude Point Spread Functions

Figure 2(a–f) presents the unnormalized amplitude point spread functions of the Connes-apodized circular aperture ($\beta = 0.05$) in the presence of tertiary spherical aberration (TSA) and defocus. In contrast to the normalized distributions shown in Fig. 1, the present results preserve the actual amplitude magnitude and therefore provide direct information regarding the loss of energy concentration at the optical axis. The unnormalized profiles reveal the extent to which tertiary spherical aberration and defocus reduce the peak diffraction response and degrade imaging performance.

For the aberration-free case [Fig. 2(a)], the maximum amplitude is obtained for the focused condition ($\phi_d = 0$), where the peak amplitude is approximately 0.50. Increasing the defocus coefficient produces a systematic reduction in the central amplitude accompanied by a gradual broadening of the diffraction profile. At $\phi_d = 2\pi$, the peak amplitude is reduced substantially relative to the focused system, indicating that defocus alone can significantly diminish energy concentration within the central diffraction maximum.

The introduction of tertiary spherical aberration progressively modifies the diffraction behavior. For $TSA = 2$ and $TSA = 4$ [Fig. 2(b,c)], the reduction in peak amplitude remains relatively small, and the diffraction structure retains much of the character observed in the aberration-free case. Nevertheless, the diffraction envelopes become slightly broader, particularly for larger values of the defocus coefficient, indicating the onset of aberration-induced energy redistribution.

As the aberration coefficient increases to $TSA = 6$ and $TSA = 8$ [Fig. 2(d,e)], a more pronounced reduction in peak amplitude becomes evident for all defocus conditions. The diffraction profiles broaden significantly and the surrounding diffraction structure becomes increasingly diffuse. These changes indicate that the combined action of defocus and eighth-order wavefront distortion reduces constructive interference at the optical axis and redistributes optical energy over a wider region of the image plane.

The strongest degradation is observed for $TSA = 10$ [Fig. 2(f)]. Under this condition, the central amplitude decreases considerably for every investigated value of ϕ_d . The effect is particularly evident for $\phi_d = 3\pi/2$ and 2π , where the diffraction envelopes become substantially broader and the energy concentration near the optical axis is markedly reduced. The resulting point response exhibits significantly poorer localization, reflecting a considerable departure from diffraction-limited performance.

A consistent ordering of the diffraction amplitudes is observed throughout Fig. 2, namely

$$\phi_d = 0 > \phi_d = \frac{\pi}{2} > \phi_d = \pi > \phi_d = \frac{3\pi}{2} > \phi_d = 2\pi.$$

This trend demonstrates that increasing defocus systematically reduces the peak diffraction response irrespective of the magnitude of tertiary spherical aberration. The quadratic phase variation associated with defocus weakens constructive interference at the image center, resulting in a monotonic decrease in the central amplitude.

The influence of tertiary spherical aberration is likewise evident when the diffraction profiles corresponding to a fixed defocus coefficient are compared across Fig. 2(a–f). Increasing TSA produces a gradual reduction in peak amplitude together with progressive broadening of the diffraction envelope. Because the aberration term varies as ρ^8 , the associated phase error becomes significant primarily near the pupil boundary. Consequently, the degradation introduced by TSA develops more gradually than that typically observed for lower-order spherical aberrations.

The relatively moderate sensitivity of the diffraction pattern to TSA may be attributed to the Connes apodization function, which suppresses the transmission of marginal pupil regions where the eighth-order phase distortion is largest. As a result, the influence of tertiary spherical aberration on the central diffraction response is partially mitigated, even at relatively large aberration coefficients.

The unnormalized amplitude distributions therefore demonstrate that tertiary spherical aberration degrades optical performance through simultaneous reduction of the central diffraction amplitude and redistribution of optical energy away from the optical axis.

Although the degradation becomes increasingly pronounced with increasing aberration strength and defocus, the gradual nature of the observed changes indicates that the Connes-apodized aperture maintains a comparatively high degree of tolerance to eighth-order spherical aberration. These results provide the physical basis for the Strehl-ratio analysis presented in the following section.

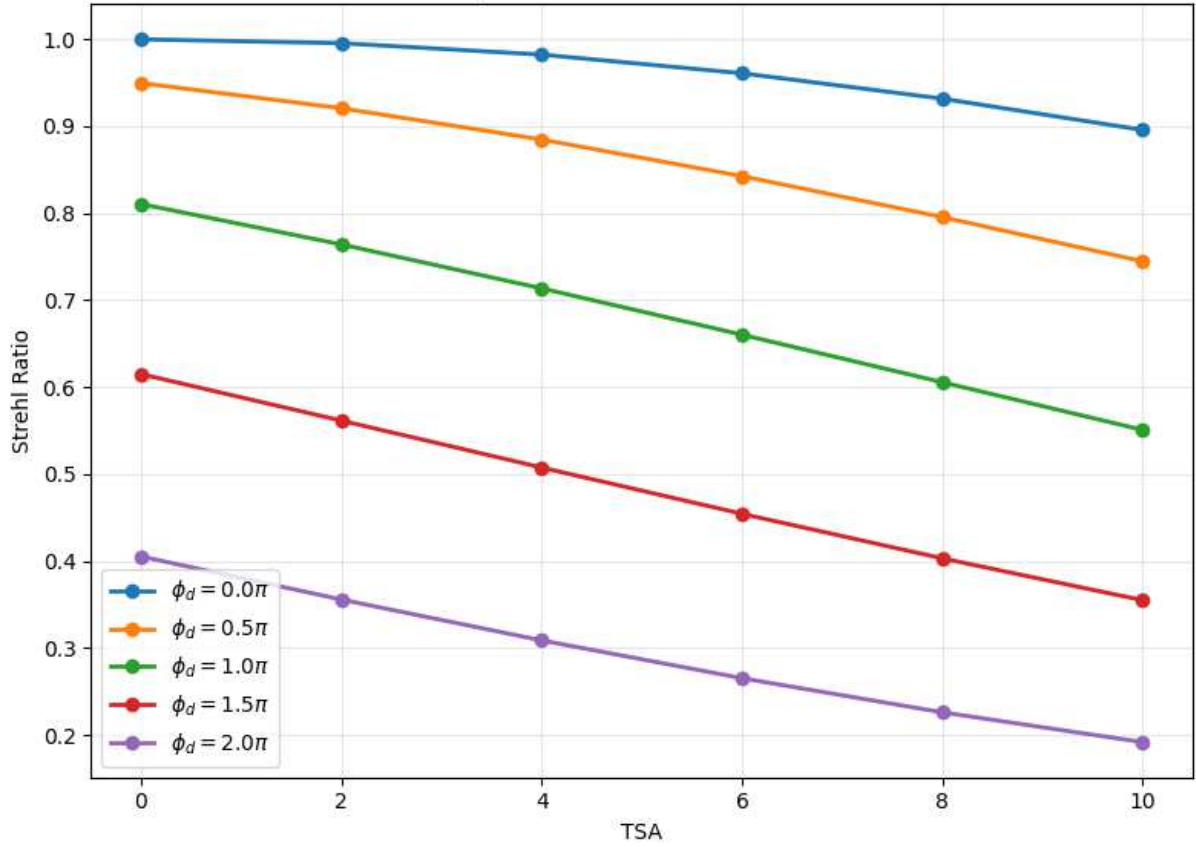


Figure 3. Strehl Ratio Variation with Tertiary Spherical Aberration

4.3 Strehl Ratio Characteristics in the Presence of Tertiary Spherical Aberration

The effect of tertiary spherical aberration (TSA) on the imaging performance of the Connes-apodized optical system is quantified through the Strehl-ratio variations shown in Fig. 3. The Strehl ratio (SR) represents the ratio of the peak intensity of the aberrated point spread function to that of the corresponding diffraction-limited system and provides a direct measure of aberration-induced degradation in image quality.

Figure 3 demonstrates a systematic reduction in Strehl ratio with increasing tertiary spherical aberration coefficient for all investigated values of the defocus parameter ϕ_d . The decrease in SR indicates progressive deterioration of the optical system's ability to concentrate energy within the central diffraction maximum. As the magnitude of the eighth-order wavefront distortion increases, constructive interference at the optical axis is reduced, leading to lower peak intensity and diminished image quality.

For the focused system ($\phi_d = 0$), the Strehl ratio remains close to unity throughout the investigated TSA range. The value decreases only from approximately 1.00 at TSA = 0 to

about 0.90 at TSA = 10. The relatively small reduction indicates that the Connes-apodized aperture exhibits substantial tolerance to tertiary spherical aberration in the absence of defocus. Even at the largest aberration coefficient considered, most of the optical energy remains concentrated within the central diffraction region.

When the defocus coefficient is increased to $\phi_d = \pi/2$, the Strehl ratio decreases more noticeably with increasing TSA, falling from approximately 0.95 at TSA = 0 to about 0.75 at TSA = 10. The combined action of quadratic and eighth-order phase errors increases the overall wavefront distortion and weakens constructive interference at the focal point. Nevertheless, the Strehl ratio remains relatively high over the entire aberration range, indicating that the system retains acceptable imaging performance under moderate aberration conditions.

For $\phi_d = \pi$, the Strehl ratio decreases from approximately 0.81 to 0.55 as TSA increases from 0 to 10. The larger reduction reflects the increasing interaction between defocus and tertiary spherical aberration. The associated wavefront distortion broadens the diffraction profile and redistributes optical energy away from the optical axis, resulting in a significant decrease in peak intensity.

The degradation becomes more pronounced for $\phi_d = 3\pi/2$, where the Strehl ratio decreases from approximately 0.61 at TSA = 0 to about 0.36 at TSA = 10. Under these conditions, the combined phase distortions substantially reduce the concentration of optical energy within the principal diffraction maximum, indicating considerable departure from diffraction-limited performance.

The lowest Strehl-ratio values are observed for $\phi_d = 2\pi$. In this case, the Strehl ratio decreases from approximately 0.40 at TSA = 0 to about 0.19 at TSA = 10. Such values indicate severe image degradation and extensive redistribution of optical energy into the surrounding diffraction structure. The results demonstrate that large defocus remains a dominant source of performance deterioration, while tertiary spherical aberration further amplifies the reduction in optical quality.

An important characteristic of Fig. 3 is the nearly monotonic behavior of all Strehl-ratio curves. No abrupt transitions or oscillatory variations are observed throughout the investigated parameter range. The smooth decline indicates that tertiary spherical aberration affects the optical system in a gradual and predictable manner, producing a continuous reduction in image quality as the aberration coefficient increases. Furthermore, the separation between successive defocus curves becomes more pronounced at larger TSA values, suggesting that the influence of defocus becomes increasingly significant as the aberration magnitude increases.

The Strehl-ratio trends are consistent with the normalized and unnormalized point spread functions presented in Figs. 1 and 2. The gradual broadening of the diffraction profiles and the reduction in central amplitude observed in those figures are reflected directly in the decreasing SR values. The reduction in Strehl ratio therefore provides quantitative confirmation of the aberration-induced redistribution of optical energy away from the central diffraction maximum.

The results demonstrate that tertiary spherical aberration produces a continuous degradation of imaging performance, although the reduction in Strehl ratio remains relatively moderate over the investigated aberration range. The gradual decline of the SR curves indicates that the Connes apodization function effectively limits the influence of the eighth-order phase distortion by reducing the contribution of highly aberrated marginal pupil regions. Consequently, the Connes-apodized aperture maintains comparatively good diffraction performance even under relatively large values of tertiary spherical aberration and defocus.

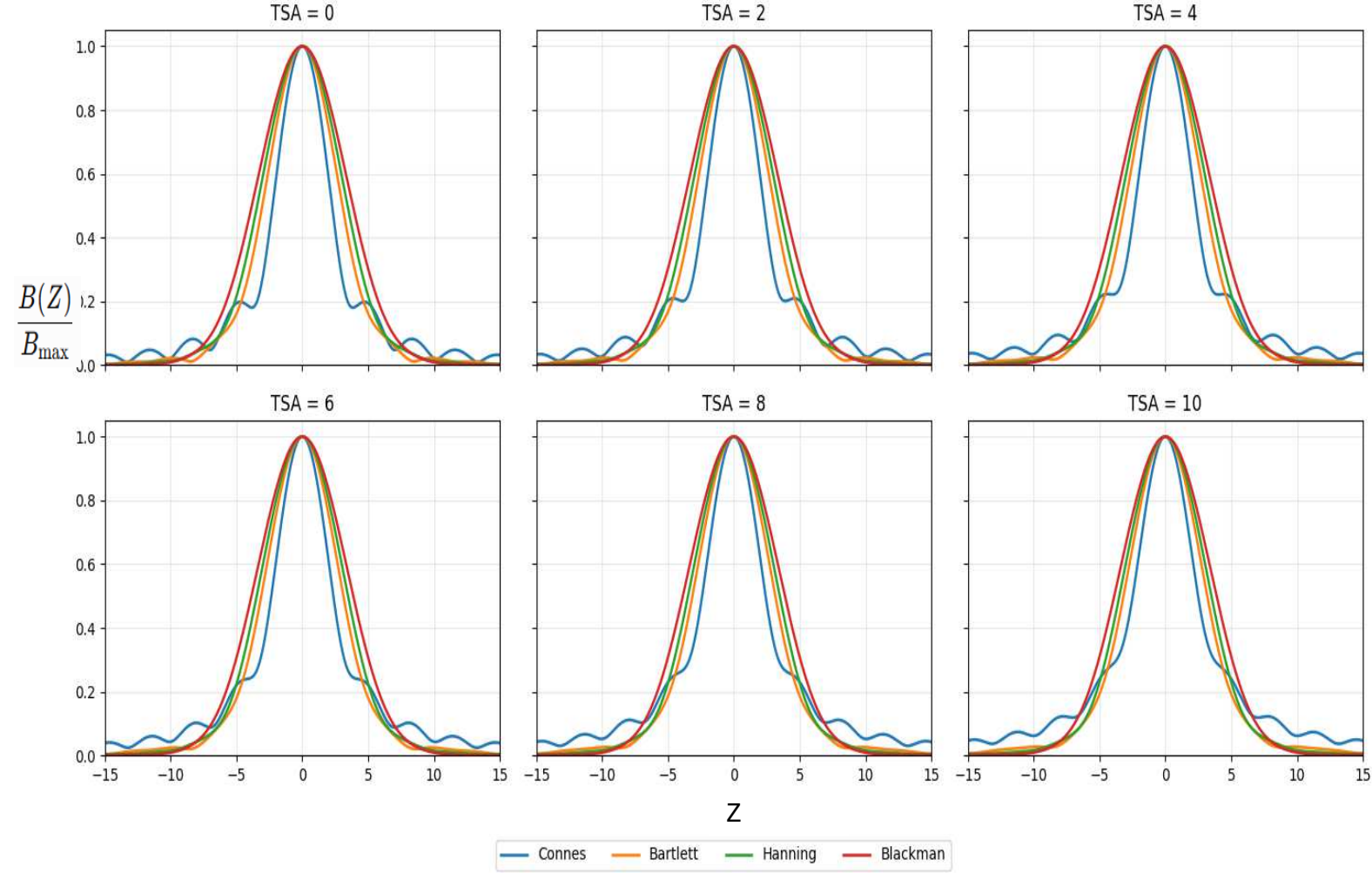


Figure 4. Comparison of Connes, Bartlett, Hanning and Blackman filters ($\phi_d = \pi$)

4.5 Comparison of Connes, Bartlett, Hanning, and Blackman Apodization Filters under Tertiary Spherical Aberration

Figure 4 presents a comparative analysis of the normalized amplitude point spread functions produced by Connes, Bartlett, Hanning, and Blackman apodization filters for a fixed defocus coefficient of $\phi_d = \pi$ and tertiary spherical aberration coefficients ranging from TSA = 0 to 10. The comparison provides insight into the relative effectiveness of different pupil-transmission functions in controlling diffraction behavior under higher-order aberration conditions.

A general observation from Fig. 4 is that all four filters maintain symmetric diffraction profiles about the optical axis for the entire range of TSA values considered. As the tertiary spherical aberration coefficient increases, only modest changes are observed in the normalized diffraction patterns, indicating that the eighth-order phase distortion affects the overall profile more gradually than lower-order aberrations. Nevertheless, clear differences exist among the four apodization functions in terms of central-lobe width and sidelobe suppression.

The Connes filter consistently produces the narrowest central diffraction maximum. In all subfigures, its diffraction profile exhibits a sharper peak and reduced main-lobe width compared with the other filters. This behavior indicates superior energy concentration near the optical axis and consequently higher resolving capability. However, the narrower main lobe is accompanied by noticeable oscillatory sidelobes, which remain visible even at larger TSA values.

The Bartlett filter provides stronger attenuation of the sidelobe structure than the Connes filter while maintaining a relatively narrow central peak. The diffraction profile is smoother and the secondary oscillations are less pronounced, indicating improved suppression of energy in the outer diffraction regions. This represents a compromise between resolution and sidelobe reduction.

The Hanning filter exhibits further broadening of the central diffraction maximum together with significant suppression of sidelobes. The smoother diffraction envelope suggests more effective redistribution of optical energy toward the central region at the expense of a moderate reduction in resolving power. The profile remains relatively stable as TSA increases, demonstrating good tolerance to higher-order wavefront distortion.

Among the four filters, the Blackman filter produces the broadest central diffraction maximum and the smoothest overall diffraction envelope. Secondary oscillations are almost completely suppressed, resulting in excellent sidelobe control. The broader main lobe indicates reduced spatial resolution, but the strong attenuation of diffraction rings may be advantageous in applications requiring high image contrast and suppression of stray diffraction energy.

An important observation is that the relative ordering of the filters remains essentially unchanged throughout the investigated TSA range. Increasing tertiary spherical aberration introduces only minor modifications to the normalized diffraction profiles, and the differences among the filters are dominated primarily by their transmission characteristics rather than by the aberration itself. This behavior confirms that tertiary spherical aberration has a comparatively weak influence on the shape of the normalized PSF, particularly when effective pupil apodization is employed.

Figure 4 clearly demonstrates that the choice of apodization function significantly influences the trade-off between diffraction-lobe width and sidelobe suppression in the presence of tertiary spherical aberration. The Connes filter consistently produces the narrowest central diffraction peak and the highest concentration of optical energy near the optical axis, thereby offering superior resolution characteristics. In contrast, the Blackman filter provides the strongest suppression of sidelobes at the expense of a broader central diffraction lobe. The

Bartlett and Hanning filters exhibit intermediate performance, balancing energy concentration and sidelobe reduction. These observations indicate that the Connes filter retains favorable diffraction characteristics under tertiary spherical aberration and is particularly suitable for applications where preservation of a narrow diffraction core and high spatial resolution are of primary importance.

5. Conclusions

The diffraction characteristics of a Connes-apodized circular aperture have been investigated in the presence of tertiary spherical aberration (TSA) and defocus using the Fourier–Bessel diffraction formulation. Numerical simulations were performed to analyze the normalized and unnormalized amplitude point spread functions as well as the corresponding Strehl-ratio variations over a wide range of aberration coefficients.

The normalized diffraction distributions revealed that increasing TSA and defocus lead to progressive broadening of the central diffraction region and enhanced redistribution of optical energy into the surrounding diffraction structure. Although the normalized peak amplitude remains unchanged, the diffraction profiles become wider with increasing aberration magnitude, indicating a reduction in image sharpness.

The unnormalized amplitude distributions demonstrated that the combined influence of TSA and defocus produces a systematic reduction in the central diffraction amplitude. The degradation becomes increasingly pronounced at larger aberration coefficients, reflecting the loss of constructive interference at the image center caused by higher-order wavefront distortions.

Strehl-ratio analysis confirmed a monotonic decrease in optical performance with increasing TSA for all investigated defocus conditions. The reduction was found to be gradual, indicating that the Connes-apodized aperture exhibits substantial tolerance to tertiary spherical aberration. Even at relatively large TSA values, the system retained a significant fraction of its diffraction-limited performance, particularly under low-defocus conditions.

A comparative assessment involving Connes, Bartlett, Hanning, and Blackman apodization functions showed that the choice of pupil transmission function strongly influences diffraction behavior. The Connes filter consistently produced the narrowest central diffraction peak and the highest concentration of optical energy near the optical axis, whereas the Blackman filter provided the strongest sidelobe suppression. Bartlett and Hanning filters exhibited intermediate characteristics.

The results demonstrate that Connes apodization provides an effective compromise between resolution preservation and diffraction control in the presence of tertiary spherical aberration. The findings may be useful in the design and optimization of high-resolution optical imaging systems where higher-order aberrations and diffraction effects play an important role in determining image quality

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