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VIJAYA SHANTHI LOVARI

rkjrf2019@gmail.com

Osmania University

Karuna sagar

Osmania University

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Influence of Connes Apodization on Tailoring the Point Spread Function of Optical Imaging Systems in the Presence of Defocus and Spherical Aberrations

**Vijaya Shanthi Lovari¹, Karuna Sagar Dasari²*

Department of Physics, Osmania University^{1,2}, Telangana, India

Email: ^{*1} rkjrf2019@gmail.com, ² dasariks@osmania.ac.in

Abstract

The diffraction characteristics of a Connes-apodized circular aperture were investigated in the presence of defocus, primary spherical aberration (PSA), and secondary spherical aberration (SSA). Using a Fourier–Bessel diffraction formulation, normalized and unnormalized amplitude point spread functions were evaluated for various aberration strengths and defocus conditions. The corresponding Strehl ratios were computed to quantify aberration-induced image degradation. The results demonstrate that Connes apodization effectively suppresses diffraction sidelobes and improves energy concentration within the central diffraction region. Increasing defocus and spherical aberration coefficients progressively broaden the diffraction profile, reduce the peak amplitude, and decrease the Strehl ratio. A comparative analysis indicates that PSA produces more rapid degradation of diffraction performance than SSA, particularly at larger aberration magnitudes. The study confirms the effectiveness of Connes apodization in maintaining acceptable imaging performance under moderate wavefront distortions and provides insight into the interaction between defocus and higher-order spherical aberrations in optical imaging systems.

Keywords: Connes apodization, point spread function, diffraction, defocus aberration, primary spherical aberration, secondary spherical aberration, Strehl ratio.

1. Introduction

The imaging performance of an optical system is fundamentally governed by its point spread function (PSF), which describes the image produced by an ideal point source. The PSF determines important imaging characteristics including spatial resolution, contrast, energy concentration, and sensitivity to optical aberrations. In an ideal diffraction-limited system, the PSF is determined solely by the aperture geometry; however, practical optical systems are invariably influenced by diffraction and wavefront aberrations, resulting in degradation of image quality and reduced optical performance [1–3].

Diffraction arising from finite aperture dimensions produces secondary maxima surrounding the central diffraction peak. These sidelobes reduce image contrast and may obscure weak objects located near bright sources. To mitigate such effects, various pupil-modification techniques have been developed, among which apodization has emerged as an effective approach for controlling diffraction behavior and improving image quality [4–6]. By modifying the amplitude transmission across the pupil, apodization redistributes optical energy from the outer diffraction rings toward the central diffraction maximum, thereby suppressing sidelobes and enhancing image contrast.

A variety of apodization functions have been investigated in the literature, including Gaussian, cosine, Hanning, Hamming, Blackman, polynomial, and Connes filters [5–8]. Among these, the Connes apodizer is particularly attractive because it provides substantial sidelobe suppression while maintaining a relatively narrow central diffraction lobe. Owing to these characteristics, Connes-type apodization has found applications in astronomical imaging, optical metrology, microscopy, and high-resolution imaging systems where diffraction control is of primary importance [6,8].

In addition to diffraction effects, optical aberrations constitute a major source of image degradation. Defocus is one of the most frequently encountered aberrations and arises when the image plane deviates from the ideal focal position. The resulting quadratic phase variation across the pupil broadens the diffraction pattern and reduces the peak intensity of the PSF [2,9]. Spherical aberration represents another important wavefront error that significantly influences imaging performance, particularly in high-numerical-aperture optical systems. Depending on the order of the wavefront deformation, spherical aberration may be classified as primary spherical aberration (PSA) and secondary spherical aberration (SSA), corresponding to fourth-order and sixth-order phase variations, respectively [3,10].

The influence of apodization on diffraction characteristics has been extensively investigated within the framework of Fourier optics. Goodman [1] established the

theoretical foundation for diffraction analysis, while Mahajan [2,3] provided comprehensive treatments of aberration effects on imaging systems. Several studies have demonstrated that suitable apodization functions can effectively suppress sidelobes and improve energy concentration within the central diffraction region [4–8]. Likewise, the influence of defocus and spherical aberration on point spread functions and image quality has received considerable attention owing to its practical importance in optical design and imaging applications [9–12].

Despite these developments, relatively limited attention has been devoted to the diffraction behavior of Connes-apodized circular apertures under the combined influence of defocus and higher-order spherical aberrations. In particular, a systematic investigation of the diffraction characteristics and Strehl-ratio performance associated with primary and secondary spherical aberrations in Connes-apodized optical systems remains insufficiently explored.

The present work addresses this issue through a detailed numerical investigation of a Connes-apodized circular aperture in the presence of defocus, primary spherical aberration, and secondary spherical aberration. Fourier–Bessel diffraction integrals are employed to evaluate both normalized and unnormalized amplitude point spread functions over a wide range of aberration coefficients. The imaging performance of the system is further quantified through Strehl-ratio analysis. The results provide insight into the manner in which different orders of spherical aberration influence diffraction behavior and energy concentration in Connes-apodized optical systems, thereby contributing to the design and optimization of high-performance imaging systems operating under non-ideal wavefront conditions.

2. Theoretical Formulation

The diffraction characteristics of an optical imaging system are analyzed using scalar diffraction theory under the Fraunhofer approximation. The generalized pupil function incorporates both the pupil transmission function and the wavefront aberration and is expressed as:

$$G(\rho) = P(\rho) \exp[-iW(\rho)] \quad (1)$$

where $P(\rho)$ denotes the pupil transmission function, $W(\rho)$ represents the wave aberration function, and ρ is the normalized radial pupil coordinate satisfying $0 \leq \rho \leq 1$. For a rotationally symmetric optical system, the amplitude point spread function is given by:

$$B(Z) = \int_0^1 P(\rho) \exp[-iW(\rho)] J_0(Z\rho) \rho d\rho \quad (2)$$

where J_0 is the zeroth-order Bessel function of the first kind and Z denotes the reduced image-plane coordinate. The corresponding intensity point spread function is:

$$I(Z) = |B(Z)|^2 \quad (3)$$

2.1 Connes Apodization Function

The pupil is apodized using the Connes filter whose transmission function is defined as:

$$P(\rho) = (1 - \beta^2 \rho^2)^2 \quad (4)$$

where β is the apodization parameter. For $\beta=0$, the aperture transmission becomes uniform. Increasing β progressively reduces the transmission near the pupil boundary and suppresses the secondary diffraction maxima. Substituting Eq. (4) into Eq. (2), the amplitude point spread function becomes:

$$B(Z) = \int_0^1 (1 - \beta^2 \rho^2)^2 \exp[-iW(\rho)] J_0(Z\rho) \rho d\rho \quad (5)$$

2.2 Defocus Aberration

Defocus introduces a quadratic phase variation across the pupil and is represented by:

$$W_D(\rho) = \frac{\phi_d}{2} \rho^2 \quad (6)$$

where ϕ_d is the defocus coefficient. The generalized pupil function becomes:

$$G(\rho) = P(\rho) \exp \left[-i \frac{\phi_d}{2} \rho^2 \right] \quad (7)$$

The quadratic phase error broadens the central diffraction maximum and reduces the peak intensity.

2.3 Primary Spherical Aberration (PSA)

Primary spherical aberration is represented by a fourth-order phase variation across the pupil and is given by:

$$W_{PSA}(\rho) = \frac{\phi_d}{2}\rho^2 + \frac{PSA}{4}\rho^4 \quad (8)$$

The corresponding amplitude point spread function is:

$$B(Z) = \int_0^1 (1 - \beta^2 \rho^2)^2 \exp \left[-i \left(\frac{\phi_d}{2} \rho^2 + \frac{PSA}{4} \rho^4 \right) \right] J_0(Z\rho) \rho d\rho \quad (9)$$

where PSA denotes the primary spherical aberration coefficient.

2.4 Secondary Spherical Aberration (SSA)

Secondary spherical aberration corresponds to a sixth-order wavefront deformation and is represented as:

$$W_{SSA}(\rho) = \frac{\phi_d}{2}\rho^2 + \frac{SSA}{6}\rho^6 \quad (10)$$

The corresponding amplitude point spread function is:

$$B(Z) = \int_0^1 (1 - \beta^2 \rho^2)^2 \exp \left[-i \left(\frac{\phi_d}{2} \rho^2 + \frac{SSA}{6} \rho^6 \right) \right] J_0(Z\rho) \rho d\rho \quad (11)$$

2.5 Strehl Ratio

The Strehl ratio is used to quantify aberration-induced degradation of image quality and is defined as:

$$SR = \frac{I_{\max(\text{aberrated})}}{I_{\max(\text{reference})}} \quad (12)$$

where $I_{\max(\text{aberrated})}$ is the maximum intensity of the aberrated point spread function and $I_{\max(\text{reference})}$ is the maximum intensity of the corresponding diffraction-limited system. A Strehl ratio equal to unity represents diffraction-limited performance, whereas smaller values indicate increasing degradation of optical quality.

3. Numerical Procedure

The diffraction characteristics of the Connes-apodized circular aperture were evaluated numerically using the Fourier–Bessel diffraction formulation described in the previous section. The amplitude point spread function was computed by evaluating Eq. (2) for different combinations of the apodization parameter β , defocus coefficient ϕ_d , and spherical aberration coefficients PSA and SSA.

The numerical simulations were carried out using a Python-based computational framework developed with NumPy, SciPy, and Matplotlib libraries. The normalized pupil coordinate was sampled uniformly over the interval $0 \leq \rho \leq 1$ using 6000 integration points to ensure numerical convergence of the diffraction integral. Simpson's integration technique was employed for numerical evaluation, while the Bessel function J_0 was computed using standard scientific-computing routines.

The reduced image-plane coordinate was selected within the range:

$$-15 \leq Z \leq 15$$

using 1200 equally spaced sampling points. This interval was sufficient to capture both the principal diffraction maximum and the surrounding sidelobe structure.

The Connes apodization function $P(\rho) = (1 - \beta^2 \rho^2)^2$ was employed throughout the investigation. The apodization parameter was fixed at $\beta = 0.05$, which provided effective sidelobe suppression while maintaining adequate energy concentration within the central diffraction maximum.

The defocus coefficient was varied as:

$$\phi_d = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

representing progressively increasing levels of defocus aberration.

To investigate the influence of spherical aberrations, the aberration coefficients were varied according to:

$$PSA, SSA = 0, 2, 4, 6, 8, 10$$

For each parameter combination, both normalized and unnormalized amplitude point spread functions were computed. The normalized distributions were obtained by dividing the calculated amplitudes by their corresponding peak values, enabling direct comparison of diffraction-profile shapes. The unnormalized distributions were retained to evaluate the actual reduction in peak amplitude resulting from aberration-induced phase errors.

The imaging performance of the optical system was further quantified using the Strehl ratio. Strehl-ratio calculations were performed for all investigated values of PSA, SSA, and ϕ_d , allowing quantitative assessment of diffraction degradation and comparison of the relative influence of primary and secondary spherical aberrations.

The numerical procedure was repeated systematically for all investigated aberration conditions, and the resulting diffraction profiles and Strehl-ratio variations form the basis of the results and discussion presented in the subsequent sections.

Connes Apodization Filter

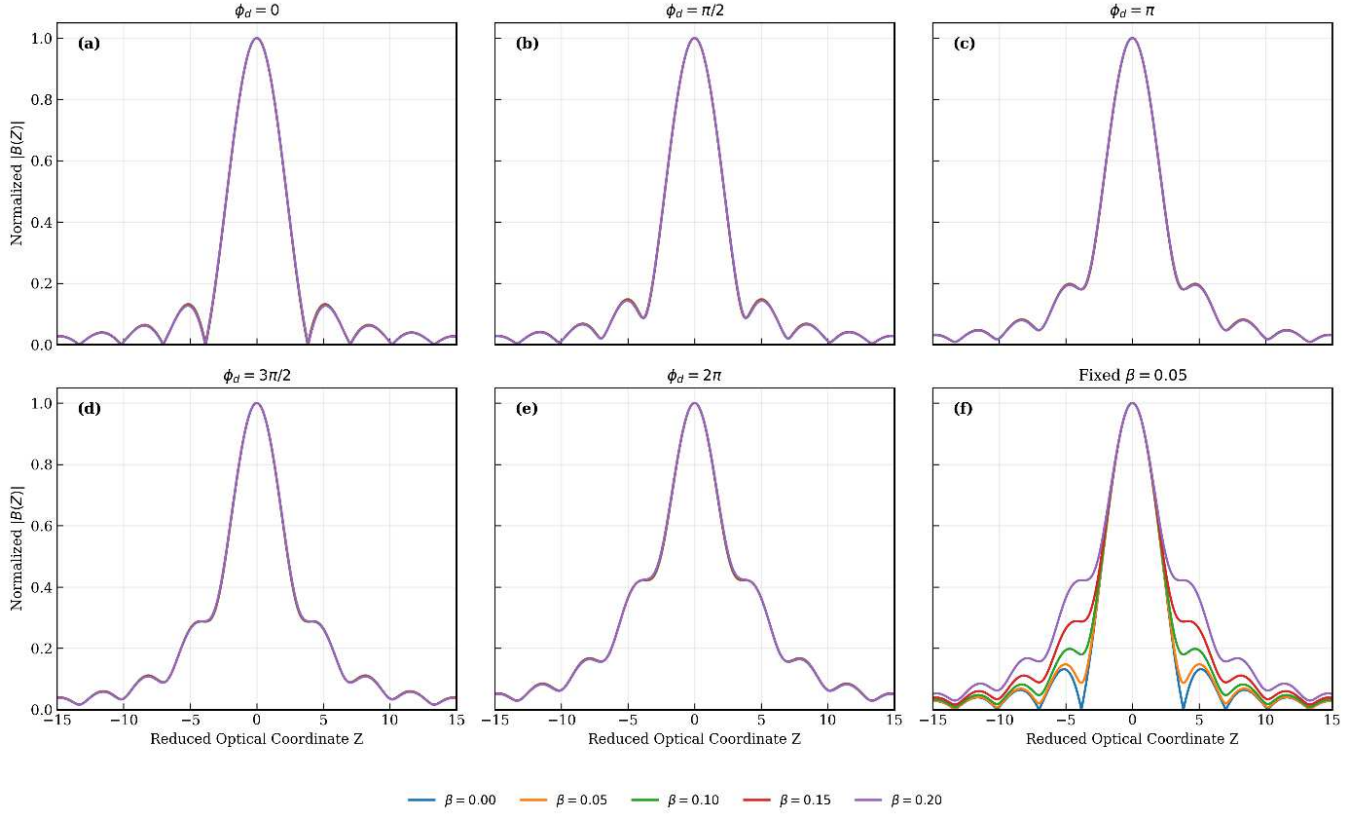


Figure: 1

4.1 Diffraction Characteristics of the Connes-Apodized Circular Aperture

Figure 1 illustrates the normalized amplitude point spread functions $|B(Z)|$ of a Connes-apodized circular aperture for different values of the apodization parameter β and defocus coefficient ϕ_d . The Connes transmission function is given by

$$A(\rho) = (1 - \beta^2 \rho^2)^2.$$

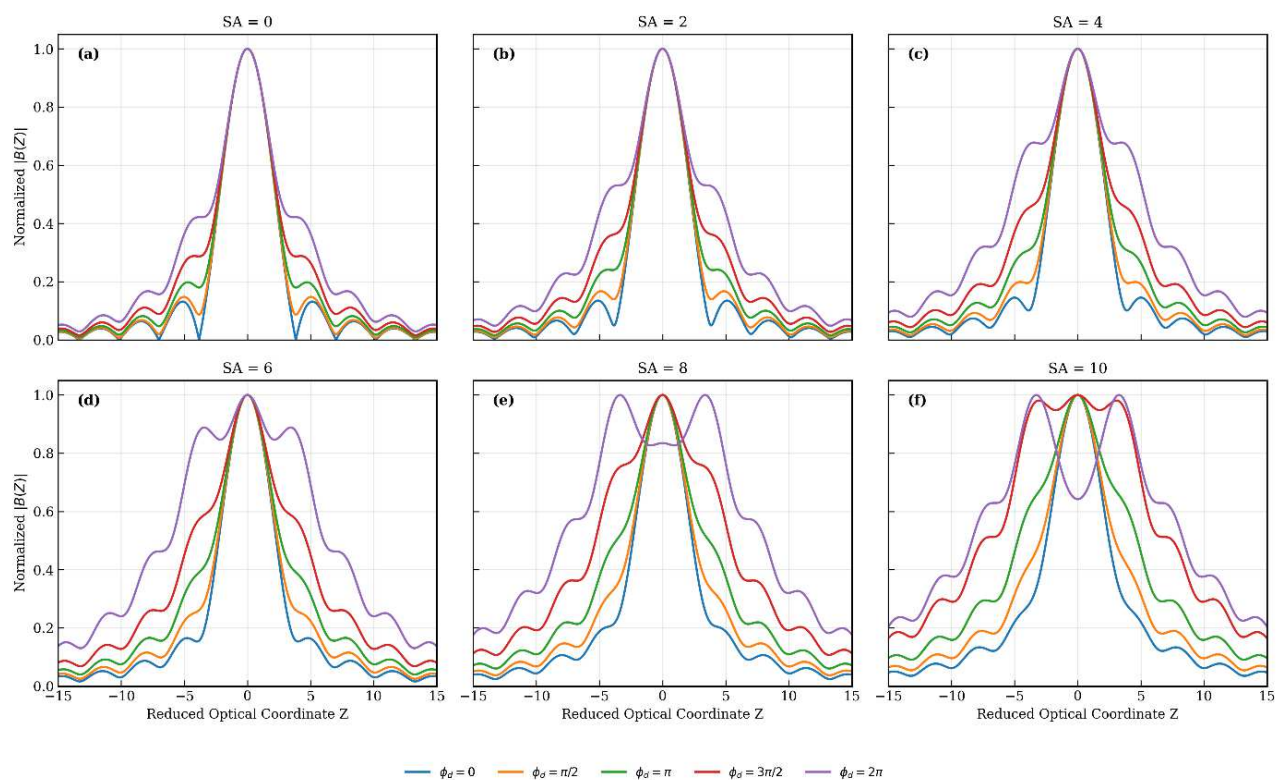
For the aberration-free case ($\phi_d = 0$), the diffraction pattern exhibits the classical Airy-type response with a well-defined central maximum and pronounced sidelobes. As the apodization parameter increases, the transmission near the pupil edge is progressively reduced, resulting in suppression of the secondary maxima and a slight broadening of the central diffraction lobe. This behavior indicates a redistribution of optical energy from the sidelobes toward the principal maximum.

The introduction of defocus ($\phi_d = \pi/2$ to 2π) produces a gradual reduction in the on-axis response and broadening of the diffraction profile due to the increasing phase mismatch across the pupil. As the defocus coefficient increases, optical energy is redistributed away from the optical axis, leading to degradation of the diffraction-limited performance. Nevertheless, the beneficial influence of Connes apodization remains

evident throughout the investigated range. Larger values of β consistently reduce sidelobe amplitudes and produce smoother diffraction envelopes, thereby mitigating diffraction-induced oscillations.

Figure 1(f) summarizes the effect of varying defocus for a fixed apodization parameter ($\beta = 0.05$). A systematic decrease in the central peak and progressive broadening of the diffraction structure are observed with increasing ϕ_d . These results indicate that defocus is the primary factor responsible for degradation of the point spread function, whereas Connes apodization acts as an effective sidelobe-suppression mechanism. Overall, the filter improves energy concentration within the central diffraction region and maintains smoother diffraction profiles even under significant defocus conditions.

Connes Filter with Primary Spherical Aberration (PSA)



4.2 Effect of Primary Spherical Aberration on the Diffraction Characteristics of the Connes-Apodized Aperture

Figure 2 presents the normalized amplitude point spread functions $|B(Z)|$ for a Connes-apodized circular aperture ($\beta = 0.05$) in the presence of primary spherical aberration. The spherical aberration coefficient is varied from $SA = 0$ to $SA = 10$, while the defocus coefficient assumes the values $\phi_d = 0, \pi/2, \pi, 3\pi/2$, and 2π . The figure illustrates the combined influence of defocus and primary spherical aberration on the diffraction structure of the optical system.

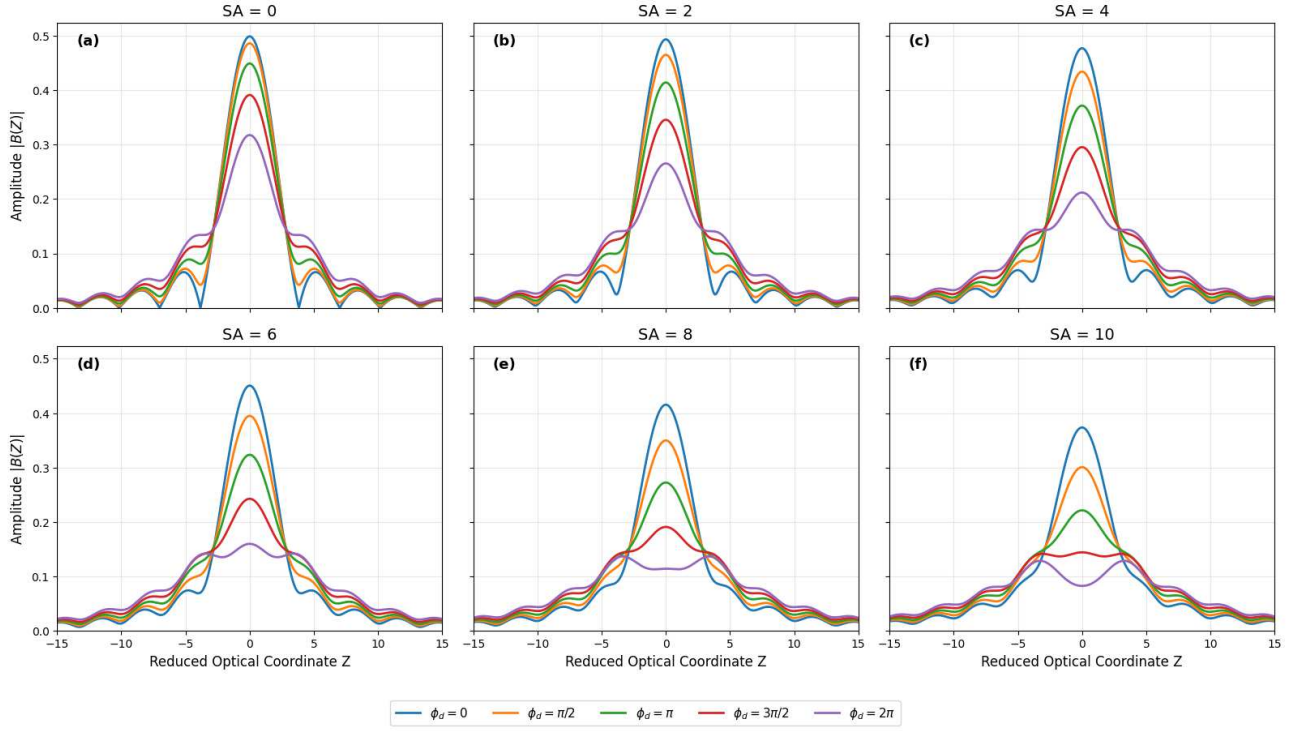
For $SA = 0$, the diffraction pattern exhibits a dominant central maximum with relatively weak sidelobes. Increasing defocus produces moderate broadening of the diffraction profile and redistribution of optical energy away from the optical axis. The influence of the Connes apodization remains evident through the smooth diffraction envelope and suppression of high sidelobe levels.

As the spherical aberration coefficient increases from $SA = 2$ to $SA = 6$, noticeable modifications occur in the diffraction structure. The sidelobes increase in amplitude, the diffraction envelope broadens, and a progressively larger fraction of optical energy is transferred from the central diffraction core to off-axis regions. Although the central maximum remains dominant at moderate aberration levels, the diffraction pattern departs gradually from the near-diffraction-limited response observed for small aberration coefficients.

For larger aberration strengths ($SA = 8$ and 10), the effect of primary spherical aberration becomes particularly pronounced. Strong off-axis maxima emerge, and substantial energy redistribution occurs throughout the diffraction profile. At $SA = 10$, peak splitting is observed for larger defocus values, indicating severe wavefront distortion and the onset of strongly aberrated imaging conditions. The appearance of multiple maxima demonstrates that spherical aberration dominates the diffraction behavior and significantly reduces the concentration of optical energy near the optical axis.

A comparison of Figs. 2(a)–2(f) shows that primary spherical aberration and defocus act cooperatively in degrading the point spread function. Increasing values of both SA and ϕ_d broaden the diffraction profile and enhance sidelobe formation, leading to reduced image sharpness and energy concentration. Although the Connes filter maintains relatively smooth diffraction envelopes throughout the investigated range, its ability to suppress diffraction effects becomes progressively limited as the spherical aberration strength increases. Consequently, primary spherical aberration emerges as the dominant factor governing the deterioration of diffraction performance at large aberration levels.

Connes Filter with Primary Spherical Aberration (PSA): Unnormalized Amplitude PSFs



4.3 Unnormalized Diffraction Characteristics of the Connes-Apodized Circular Aperture in the Presence of Primary Spherical Aberration

Figure 3 presents the unnormalized amplitude point spread functions $|B(Z)|$ for a Connes-apodized circular aperture ($\beta = 0.05$) in the presence of primary spherical aberration. Unlike the normalized distributions shown in Figure 2, the present results preserve the actual peak amplitudes and therefore provide direct information regarding the loss of energy concentration caused by aberration. The spherical aberration coefficient is varied from $SA = 0$ to $SA = 10$, while the defocus coefficient assumes the values $\phi_d = 0, \pi/2, \pi, 3\pi/2$, and 2π .

For $SA = 0$, the diffraction-limited condition is observed, with the largest on-axis amplitude occurring at $\phi_d = 0$. Increasing defocus progressively reduces the peak amplitude and broadens the diffraction profile, indicating redistribution of optical energy away from the optical axis. As primary spherical aberration is introduced and subsequently increased, the reduction in peak amplitude becomes increasingly pronounced. Simultaneously, the diffraction envelope broadens and the sidelobes gain relative prominence, demonstrating a transfer of optical energy from the principal diffraction maximum into off-axis regions.

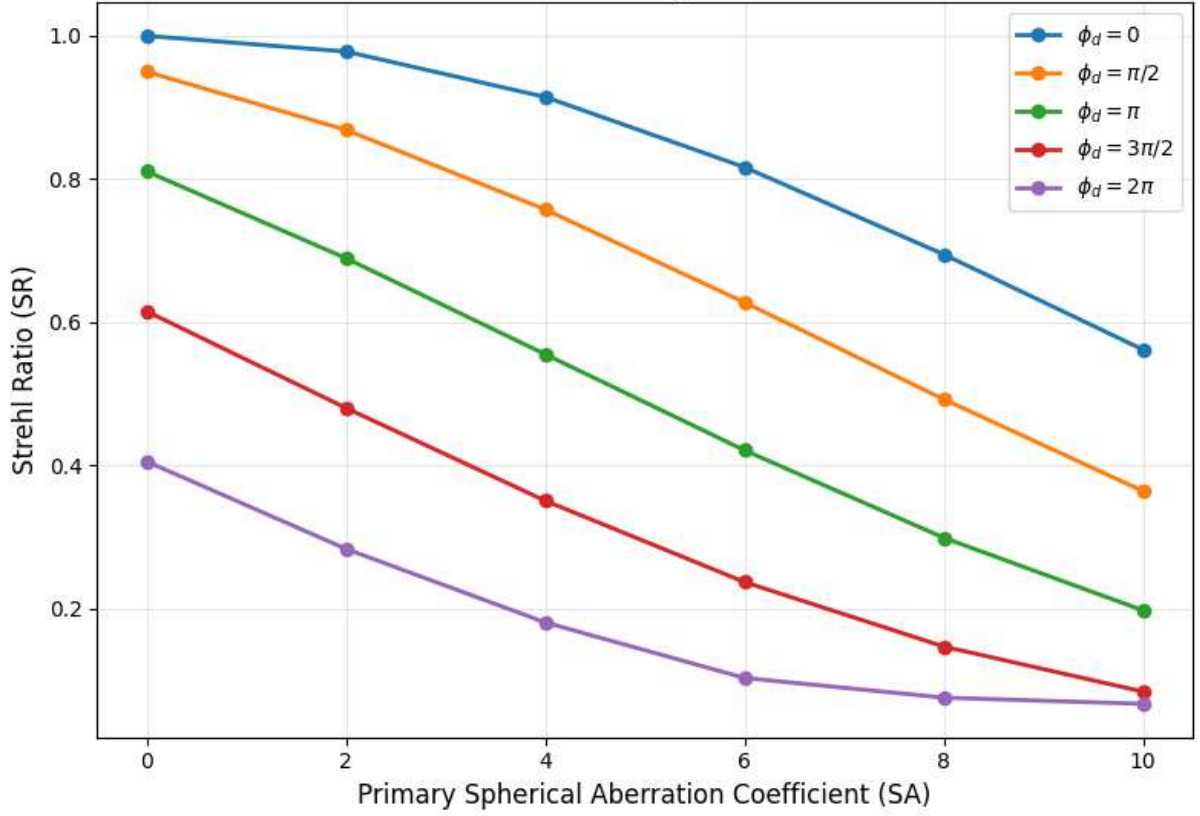
At moderate aberration levels ($SA = 4$ and 6), substantial degradation of the central response is observed for all investigated values of defocus. The combined action of

spherical aberration and defocus produces broader diffraction profiles and reduced energy concentration near the optical axis. For larger aberration strengths ($SA = 8$ and 10), the diffraction structure undergoes significant modification, with the appearance of multiple competing maxima and, in some cases, clear peak splitting. This behavior indicates severe wavefront distortion and substantial redistribution of optical energy throughout the image plane.

A comparison of Figs. 3(a–f) demonstrates that both primary spherical aberration and defocus contribute to a systematic reduction of the central diffraction amplitude. The degradation becomes increasingly severe as either parameter increases, confirming that the two aberrations act cooperatively in reducing image quality. While the Connes filter maintains relatively smooth diffraction envelopes, the loss of on-axis energy concentration ultimately becomes the dominant feature at large aberration strengths.

Comparison of Figures 2 and 3 further highlights the distinction between normalized and unnormalized diffraction analysis. Figure 2 emphasizes changes in diffraction shape and sidelobe structure, whereas Figure 3 reveals the actual reduction in peak amplitude associated with increasing aberration. Together, the two figures show that primary spherical aberration degrades imaging performance through both energy redistribution into off-axis regions and a significant reduction in the central diffraction amplitude. These effects lead directly to reduced image sharpness, contrast, and resolving capability in Connes-apodized optical systems.

Figure 4: Strehl Ratio vs Primary Spherical Aberration
Connes Filter ($\beta = 0.05$)



4.4 Strehl Ratio Analysis of the Connes-Apodized Circular Aperture in the Presence of Primary Spherical Aberration

The Strehl ratio is one of the most important quantitative measures used to evaluate optical system performance in the presence of aberrations. It represents the ratio of the peak intensity of an aberrated point spread function to that of the corresponding diffraction-limited system and therefore provides a direct indication of image degradation caused by wavefront distortions. For the present investigation, the Strehl ratio was computed as

$$SR = \frac{I_{\max}(SA, \phi_d)}{I_{\max}(SA = 0, \phi_d = 0)}$$

where $I_{\max}$ denotes the maximum intensity of the point spread function.

Figure 4 illustrates the variation of the Strehl ratio as a function of the primary spherical aberration coefficient SA for different values of the defocus parameter ϕ_d . The results provide a quantitative assessment of the combined influence of spherical aberration and defocus on the diffraction performance of the Connes-apodized optical system.

4.4.1 Aberration-Free Condition

Figure 4 illustrates the variation of the Strehl ratio (SR) with the primary spherical aberration coefficient SA for a Connes-apodized circular aperture ($\beta = 0.05$) under different defocus conditions ($\phi_d = 0, \pi/2, \pi, 3\pi/2, 2\pi$). The Strehl ratio provides a quantitative measure of image degradation by comparing the peak intensity of the aberrated point spread function with that of the corresponding diffraction-limited system.

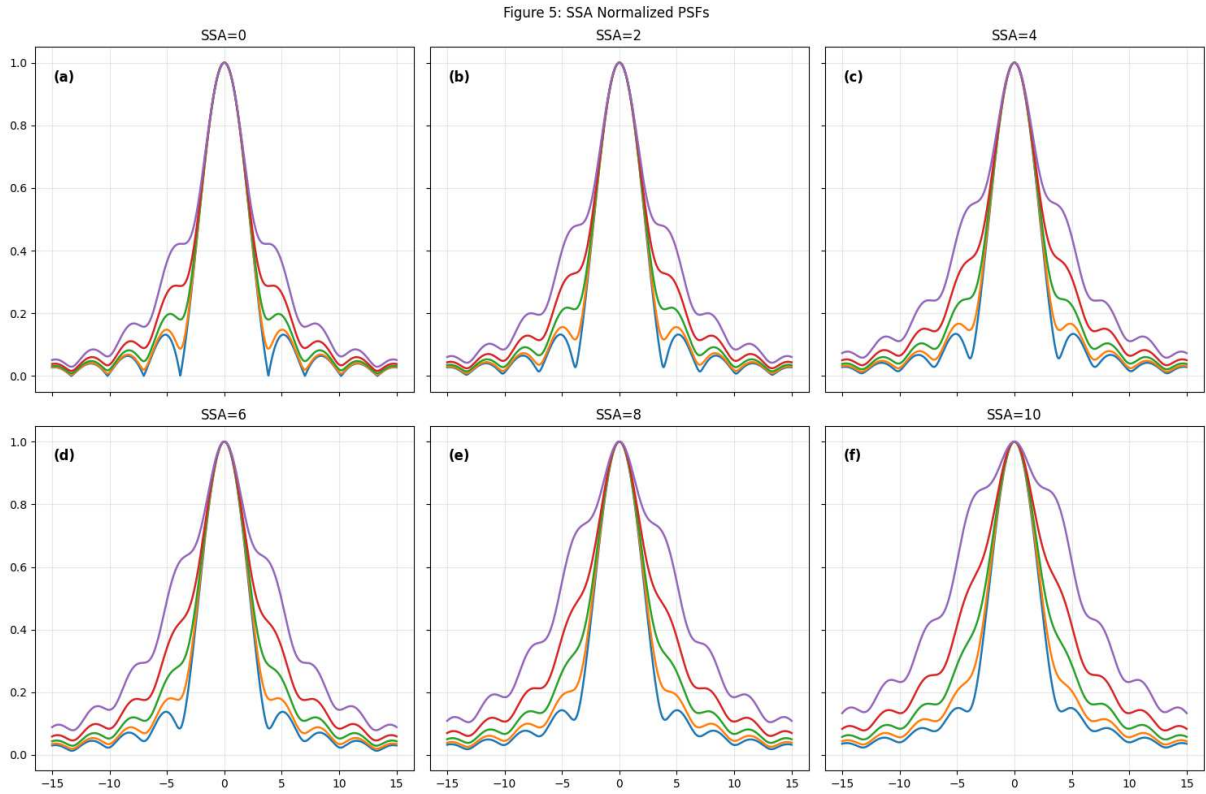
For the aberration-free condition ($SA = 0, \phi_d = 0$), the Strehl ratio is unity, indicating diffraction-limited performance. Even in the absence of spherical aberration, increasing defocus produces a reduction in the Strehl ratio, demonstrating that defocus alone causes a significant loss of optical energy concentration at the image center.

A monotonic decrease in the Strehl ratio is observed for all defocus conditions as the primary spherical aberration coefficient increases from $SA = 0$ to $SA = 10$. The reduction is relatively gradual for small aberration strengths ($SA \leq 4$), indicating that the optical system retains a substantial fraction of its diffraction-limited performance. However, beyond $SA \approx 4$, the decline becomes more pronounced, particularly for larger values of the defocus coefficient.

The influence of spherical aberration is strongly amplified by defocus. For every value of SA , the Strehl ratio decreases systematically as ϕ_d increases, confirming the cooperative action of the quadratic phase error associated with defocus and the fourth-order phase distortion introduced by primary spherical aberration. This combined wavefront distortion weakens constructive interference at the optical axis and redistributes optical energy into surrounding diffraction regions.

The most severe degradation occurs for $\phi_d = 2\pi$, where the Strehl ratio falls to approximately 0.06 at $SA = 10$, indicating that only a small fraction of the incident optical energy remains concentrated within the principal diffraction maximum. In contrast, the focused system ($\phi_d = 0$) retains a Strehl ratio of approximately 0.56 at the same aberration level, demonstrating the comparatively stronger influence of defocus on image degradation.

The Strehl-ratio trends are consistent with the normalized and unnormalized point spread functions presented in Figs. 2 and 3. The gradual reduction in SR at low aberration strengths corresponds to moderate diffraction broadening, whereas the rapid decline observed at larger values of SA coincides with significant energy redistribution, sidelobe enhancement, and peak splitting. Therefore, the Strehl ratio provides quantitative confirmation of the diffraction behavior and establishes primary spherical aberration, particularly when combined with defocus, as a major factor limiting imaging performance in Connes-apodized optical systems.



4.3 Influence of Secondary Spherical Aberration on the Normalized Amplitude PSF

Figure 5(a–f) presents the normalized amplitude point spread functions $|B(Z)|$ for a Connes-apodized circular aperture ($\beta = 0.05$) in the presence of secondary spherical aberration. The secondary spherical aberration coefficient was varied from $SSA = 0$ to $SSA = 10$, while the defocus coefficient was considered at $\phi_d = 0, \pi/2, \pi, 3\pi/2$, and 2π . Since the diffraction profiles are normalized with respect to their peak values, the figure emphasizes modifications in diffraction shape and energy redistribution rather than changes in absolute peak intensity.

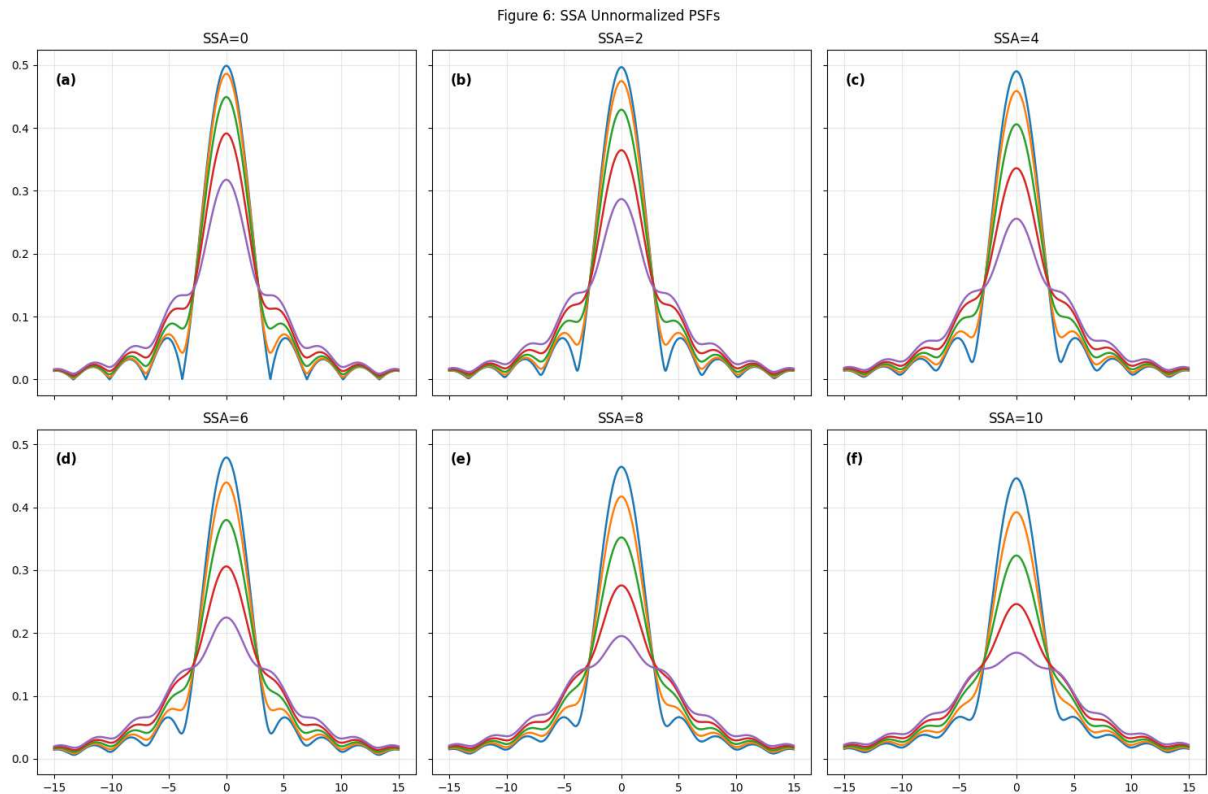
For $SSA = 0$, the diffraction behavior is governed entirely by defocus. Increasing values of ϕ_d produce a gradual broadening of the central diffraction region together with enhanced sidelobe development. The diffraction profiles remain symmetric about the optical axis, consistent with the rotational symmetry of the pupil function and aberration term.

The introduction of secondary spherical aberration modifies the diffraction structure primarily through changes in the sidelobe region. At $SSA = 2$ and $SSA = 4$, the central maximum remains relatively unchanged, whereas the surrounding diffraction rings broaden and merge into elevated shoulders. This behavior indicates the onset of energy transfer from the central diffraction core toward larger image-plane coordinates. The effect becomes increasingly pronounced as the defocus coefficient increases, demonstrating the interaction between sixth-order wavefront distortion and quadratic phase error.

For $SSA = 6$, noticeable broadening of the diffraction envelope is observed, particularly for $\phi_d = 3\pi/2$ and 2π . The diffraction profiles develop extended pedestal-like structures surrounding the principal maximum, indicating reduced localization of optical energy. At higher aberration strengths ($SSA = 8$ and 10), the broadening becomes substantial and the diffraction pattern departs significantly from the Airy-like form observed at lower aberration levels. The profiles associated with larger defocus coefficients exhibit wide central regions accompanied by elevated off-axis diffraction structures, reflecting strong redistribution of energy throughout the image plane.

The progressive modification of the diffraction profiles with increasing SSA indicates that secondary spherical aberration influences the diffraction response more gradually than primary spherical aberration. The initial degradation appears predominantly in the sidelobe structure, while significant alteration of the central diffraction region occurs only at larger aberration coefficients. The effect of secondary spherical aberration is further enhanced by increasing defocus, resulting in broader diffraction envelopes and increased energy distribution away from the optical axis.

The diffraction patterns presented in Fig. 5 therefore demonstrate that secondary spherical aberration produces a continuous redistribution of optical energy from the central diffraction region into surrounding image-plane locations. As the aberration coefficient increases, the diffraction response becomes progressively broader, indicating a reduction in the spatial localization capability of the optical system.



4.4 Unnormalized Amplitude Point Spread Functions in the Presence of Secondary Spherical Aberration

Figure 6(a–f) presents the unnormalized amplitude point spread functions $|B(Z)|$ obtained for the Connes-apodized circular aperture ($\beta = 0.05$) in the presence of secondary spherical aberration (SSA). The secondary spherical aberration coefficient was varied from $SSA = 0$ to $SSA = 10$, while the defocus coefficient was considered at $\phi_d = 0, \pi/2, \pi, 3\pi/2$, and 2π . Unlike the normalized distributions shown in Figure 5, the present representation preserves the actual amplitude values and therefore provides a direct measure of the loss of energy concentration within the central diffraction region.

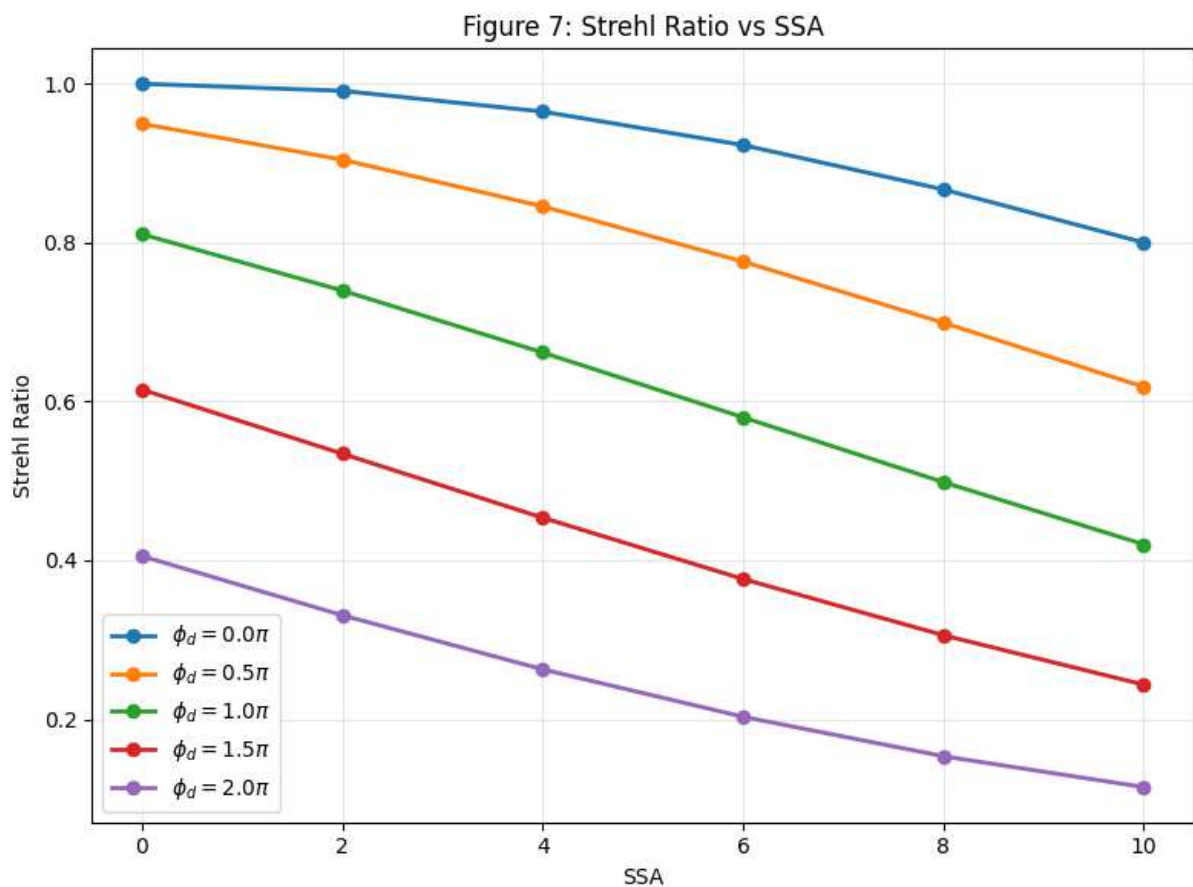
For $SSA = 0$, the largest on-axis amplitude occurs for the focused condition ($\phi_d = 0$). Increasing defocus produces a systematic reduction in peak amplitude accompanied by broadening of the diffraction profile, indicating progressive redistribution of optical energy away from the optical axis. The reduction in central amplitude becomes increasingly pronounced as the defocus coefficient approaches 2π .

The introduction of secondary spherical aberration produces a continuous decrease in the central diffraction amplitude for all investigated defocus conditions. At lower aberration levels ($SSA = 2$ and 4), the diffraction profiles retain their general structure, although a noticeable reduction in peak amplitude and enhancement of the surrounding diffraction region are observed. As the aberration coefficient increases further ($SSA = 6, 8, 10$), the diffraction envelope broadens substantially and the central response decreases significantly, particularly for larger values of the defocus coefficient.

The combined influence of secondary spherical aberration and defocus becomes evident through the progressive formation of broad pedestal-like structures surrounding the principal maximum. These features indicate increased phase mismatch among wavefront contributions originating from different pupil zones and correspond to a transfer of optical energy from the central diffraction core into off-axis regions. At $SSA = 10$, the diffraction profiles become considerably wider and flatter near the optical axis, reflecting severe degradation of spatial energy localization.

A comparison of Fig. 6(a–f) demonstrates that the reduction in peak amplitude is monotonic with increasing aberration strength. The rate of degradation becomes more pronounced at larger values of SSA , suggesting that secondary spherical aberration increasingly dominates the diffraction behavior beyond moderate aberration levels. The effect is further amplified by defocus, which accelerates the reduction of on-axis intensity and enhances the redistribution of optical energy throughout the image plane.

Comparison of Figures 5 and 6 highlights the distinction between normalized and unnormalized diffraction analysis. While Figure 5 emphasizes changes in diffraction shape and sidelobe structure, Figure 6 reveals the actual reduction in central amplitude produced by secondary spherical aberration. The results indicate that the principal consequence of increasing SSA is not only broadening of the diffraction profile but also a substantial decrease in the optical energy concentrated within the image core. This reduction in central intensity directly influences image sharpness, contrast, and resolving capability, and provides the basis for the Strehl-ratio analysis presented in the subsequent section.



4.3.3 Strehl Ratio Variation with Secondary Spherical Aberration

Figure 7 presents the variation of the Strehl ratio (SR) with the secondary spherical aberration coefficient (SSA) for different values of the defocus coefficient ϕ_d . Since the Strehl ratio represents the ratio of the peak intensity of the aberrated point spread function to that of the corresponding diffraction-limited system, it provides a quantitative measure of aberration-induced image degradation.

For all investigated defocus conditions, the Strehl ratio decreases monotonically with increasing secondary spherical aberration. The reduction in SR indicates a progressive

loss of optical energy from the central diffraction maximum and increased redistribution of energy into the surrounding diffraction structure. Consequently, the imaging system departs gradually from diffraction-limited performance as the aberration magnitude increases.

The highest Strehl-ratio values are obtained for the focused condition ($\phi_d = 0$), where the decrease remains relatively gradual throughout the investigated aberration range. The SR reduces from unity at $SSA = 0$ to approximately 0.78 at $SSA = 10$, indicating that the Connes-apodized aperture retains a significant fraction of its diffraction-limited performance even in the presence of substantial sixth-order wavefront distortion.

Increasing the defocus coefficient results in a systematic downward shift of the Strehl-ratio curves. For $\phi_d = \pi/2$ and π , the degradation becomes increasingly pronounced, while the lowest SR values are observed for $\phi_d = 3\pi/2$ and 2π . At $\phi_d = 2\pi$, the Strehl ratio decreases from approximately 0.40 at $SSA = 0$ to nearly 0.12 at $SSA = 10$, indicating severe deterioration of image quality caused by the combined action of defocus and secondary spherical aberration.

The increasing separation between the Strehl-ratio curves with increasing SSA demonstrates that the influence of secondary spherical aberration becomes progressively stronger at larger aberration magnitudes. The sixth-order phase error introduces substantial wavefront distortion near the outer regions of the pupil, while the additional quadratic phase variation associated with defocus further reduces constructive interference at the image center. The combined effect leads to significant broadening of the point spread function and a corresponding reduction in peak intensity.

A comparison with the PSA results indicates that the reduction in Strehl ratio produced by SSA is relatively gradual at lower aberration coefficients, whereas the degradation becomes increasingly pronounced at larger values of SSA. This behavior is consistent with the higher-order nature of the aberration term, whose influence becomes more significant toward the peripheral regions of the pupil.

The monotonic behavior of the Strehl-ratio curves confirms that secondary spherical aberration produces a continuous degradation of optical performance without abrupt transitions. The results further indicate that the Connes apodization function moderates the influence of higher-order wavefront distortions by reducing the contribution of highly aberrated marginal rays, thereby maintaining comparatively higher Strehl-ratio values over the investigated aberration range.

5. Conclusions

The diffraction characteristics of a Connes-apodized circular aperture have been investigated under the combined influence of defocus, primary spherical aberration (PSA), and secondary spherical aberration (SSA). The analysis was carried out using

normalized and unnormalized amplitude point spread functions together with Strehl-ratio evaluation to assess the influence of aberrations on image formation.

The diffraction analysis under defocus revealed that the Connes apodizer effectively suppresses sidelobes and promotes concentration of optical energy within the central diffraction region. Increasing defocus broadens the diffraction pattern and reduces the on-axis response; however, the apodized aperture retains smoother diffraction profiles compared with the unapodized case.

The presence of primary spherical aberration produces a progressive redistribution of optical energy from the central diffraction maximum toward the surrounding sidelobes. The normalized PSFs demonstrated increasing profile broadening and eventual peak splitting at large aberration magnitudes, while the unnormalized PSFs revealed a substantial reduction in central amplitude. The Strehl-ratio analysis confirmed a continuous degradation of imaging performance with increasing PSA, with the deterioration becoming particularly severe when combined with large defocus coefficients.

Secondary spherical aberration was found to modify the diffraction structure in a different manner. The normalized PSFs exhibited gradual broadening of the diffraction envelope and enhanced sidelobe formation, while the unnormalized distributions showed a systematic reduction in central amplitude. The corresponding Strehl-ratio curves indicated a monotonic decline in optical performance with increasing SSA. Although the degradation remained less abrupt than that observed for PSA at lower aberration magnitudes, significant reductions in image quality were observed at larger values of SSA, especially under strong defocus conditions.

Comparison of the PSA and SSA results indicates that both aberrations reduce energy concentration within the principal diffraction maximum and broaden the diffraction response. PSA produces more rapid degradation of the central diffraction structure and Strehl ratio, whereas SSA introduces a more gradual redistribution of energy that becomes increasingly significant at larger aberration coefficients.

The Strehl-ratio results showed strong agreement with the point spread function analysis, providing quantitative confirmation of the aberration-induced degradation observed in the diffraction profiles. In all cases, increasing defocus amplified the influence of spherical aberration, demonstrating the coupled nature of these wavefront errors.

The results establish that Connes apodization provides effective control of diffraction sidelobes and improves energy localization under moderate aberration conditions. Nevertheless, the benefits of apodization gradually diminish as the magnitude of spherical aberration increases, with aberration effects ultimately dominating the diffraction behavior. The present investigation demonstrates the usefulness of Connes

apodization for enhancing imaging performance in the presence of low-to-moderate wavefront distortions while also identifying the limitations imposed by higher-order aberrations.

The study provides a foundation for further investigation of higher-order aberration effects, particularly tertiary spherical aberration, where the interaction between apodization and eighth-order wavefront distortions may lead to significantly different diffraction behavior and imaging characteristics.

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Declarations

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Competing Interests

The authors declare that they have no competing interests.

Data Availability

No experimental datasets were generated or analyzed during the current study. The results are based on theoretical formulations and numerical computations described in the manuscript.

Author Contributions

Vijaya Shanthi Lovari: Conceptualization, methodology, mathematical formulation, numerical analysis, visualization, and manuscript preparation. Karuna Sagar Dasari: Supervision, validation, interpretation of results, and critical revision of the manuscript. Both authors read and approved the final manuscript.