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Linear Control System of a 6-DOF Parallel Motion Platform for Controlled Dynamic EEG Phantom Experiments with Actuator Dynamics

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ABSTRACT This paper presents a Type-1 linear controller for a six-degree-of-freedom Stewart motion platform intended for controlled dynamic electroencephalography phantom experiments. The platform is intended to provide repeatable mechanical motion so that electrode signals can later be compared under static and dynamic conditions. A simplified task-space model is formed about a nominal equilibrium pose. First-order actuator dynamics represent input lag, and one integral error state is added for each controlled coordinate. The resulting model contains 24 states: 12 platform states, six actuator states, and six integral states. A null-space method is used to formulate the state-feedback gain for prescribed closed-loop eigenvalues. An analytical controllability check establishes rank 24 for the augmented model. The MATLAB responses approach the prescribed translational and rotational step commands with approximately zero steady-state error. The rotational responses also show substantial transient motion, including roll overshoot. These results demonstrate tracking within the simplified linear model; the command amplitudes do not establish the validity of a small-motion approximation for a physical platform. The study provides an initial control framework for future experiments on motion artifacts, signal quality, and source-localization stability. Physical actuator allocation, workspace limits, and experimental electroencephalography validation remain to be addressed.

INDEX TERMS Actuator dynamics, electroencephalography phantom, eigenvalue assignment, integral control, motion platform, state-space model, Stewart platform.

I. INTRODUCTION

THE Stewart platform is a parallel mechanism that produces motion in six degrees of freedom (DOF) [1]. It has a fixed base, a moving platform, and six actuated legs. Its stiffness, positioning capability, and load-carrying structure make it useful for motion simulation, robotic positioning, vibration isolation, and aerospace applications. Hexapod motion-cueing systems are one example in which coordinated platform motion is central to the application [2].

Control is more involved than for a single actuator because all six legs contribute to the pose of the moving platform. A movement along one axis may require several actuators to act together. The physical mechanism is therefore a coupled, multiple-input multiple-output (MIMO) system. State-space models provide a convenient basis for its analysis. The nonlinear dynamics can be expressed in the standard rigid-body form [3]–[5]

$$\begin{aligned} M(q_{\text{abs}})\ddot{q}_{\text{abs}} + C(q_{\text{abs}}, \dot{q}_{\text{abs}})\dot{q}_{\text{abs}} \\ + G(q_{\text{abs}}) = J^T(q_{\text{abs}})u_{\ell} + d', \end{aligned} \quad (1)$$

where q_{abs} is the absolute platform pose, M is the inertia matrix, $C(q_{\text{abs}}, \dot{q}_{\text{abs}})\dot{q}_{\text{abs}}$ contains Coriolis and centrifugal terms, G is the gravity vector, J is the Jacobian, u_{ℓ} contains the leg forces, and d' is a generalized disturbance. The subscript

on u_{ℓ} distinguishes physical leg forces from the task-space input used later.

The motivation here is controlled motion for electroencephalography (EEG) phantom experiments. Motion may affect electrode recordings through cable movement, changes at the electrode interface, movement of the conductive medium, or changes in source-localization estimates. A repeatable mechanical input would allow these effects to be studied under defined static and dynamic conditions. Prescribing the motion amplitude and frequency also makes comparisons between test conditions more meaningful.

Related robotic studies illustrate why sensing and control must be considered in the context of the intended task. Kashyap *et al.* evaluated quadrotor obstacle avoidance with Vicon, real-time kinematic GPS, and camera measurements, and identified camera errors associated with forward velocity and yaw [6]. Samir and Chakravarthy used dynamic-inversion joint tracking and a steering loop for a simulated wheel-less snake robot [7]. Other task-specific systems include the PHOENIX human-assistance rover [8] and a rover subsystem that combines soil collection with onboard biomolecule analysis [9]. These studies provide context for coordinating actuation, sensing, and a defined experimental task. The present work focuses on a linear motion-control model for a future



FIGURE 1. Representative six-degree-of-freedom motion platform for controlled dynamic motion. Adapted from Motion Systems [10].

EEG test platform.

The objective is to design a Type-1 state-feedback controller that tracks step commands while accounting for actuator lag. A 12-state platform model is augmented by six actuator states and six integral states. The resulting 24-state system is used for null-space eigenvalue assignment and MATLAB simulation. Throughout this paper, *Type-1* refers to the MIMO integral-tracking architecture with one error integrator per output.

II. MODELING OF THE STEWART PLATFORM

A representative six-DOF platform is shown in Fig. 1. Changing the forces or lengths of its six legs produces three translations and three rotations. This image illustrates the mechanism; it is not a specification of the particular mass, inertia, or workspace used in the mathematical model. The present analysis uses task-space coordinates rather than individual leg coordinates.

A. TASK-SPACE COORDINATES

Let q_0 denote the nominal absolute pose. The coordinate deviation from that pose is $q = q_{\text{abs}} - q_0$, written as

$$q = [p_x \ p_y \ p_z \ \alpha \ \beta \ \gamma]^T, \quad (2)$$

where p_x, p_y , and p_z are translational deviations, and α, β , and γ are roll, pitch, and yaw deviations. The nominal orientation is zero, so the angular deviations equal the corresponding absolute orientation coordinates. This choice follows the task-space formulation used for Stewart-platform dynamics [3]. The platform state vector is

$$x = [p_x \ p_y \ p_z \ \alpha \ \beta \ \gamma \ \dot{p}_x \ \dot{p}_y \ \dot{p}_z \ \dot{\alpha} \ \dot{\beta} \ \dot{\gamma}]^T. \quad (3)$$

It contains six coordinates and six velocities, giving 12 platform states before actuator and integral states are introduced.

TABLE 1. Parameters used for the Stewart-platform model

Parameter	Symbol	Value
Platform mass	m	32 kg
Moment of inertia about x	I_x	2 kg m ²
Moment of inertia about y	I_y	2 kg m ²
Moment of inertia about z	I_z	4 kg m ²
Nominal platform height	z_0	0.55 m
Gravitational acceleration	g	9.81 m/s ²
Actuator time constant	τ_a	0.05 s

B. NONLINEAR DYNAMIC MODEL

The nonlinear model underlying the linear approximation is

$$M(q_{\text{abs}})\ddot{q}_{\text{abs}} + C(q_{\text{abs}}, \dot{q}_{\text{abs}})\dot{q}_{\text{abs}} + G(q_{\text{abs}}) = J^T(q_{\text{abs}})u_\ell + d'. \quad (4)$$

This form separates inertia, velocity-dependent effects, gravity, and the applied leg forces. The disturbance term may represent external loading or model uncertainty. A detailed physical model requires the platform geometry and the pose-dependent Jacobian [4], [5]. The present controller instead uses a simplified model about a nominal equilibrium pose.

C. PHYSICAL PARAMETERS

Table 1 lists the mass, inertias, equilibrium height, gravitational acceleration, and actuator time constant used in the model. A diagonal inertia approximation is adopted at the nominal pose, and the same first-order lag is applied to all six generalized input channels.

The nominal pose and velocity are

$$q_0 = [0 \ 0 \ 0.55 \ 0 \ 0 \ 0]^T, \quad \dot{q}_0 = 0. \quad (5)$$

The platform is centered at a height of 0.55 m with zero roll, pitch, and yaw. At this configuration, the approximate inertia matrix is

$$M_0 = \text{diag}(32, 32, 32, 2, 2, 4). \quad (6)$$

The first three entries correspond to translational mass and the last three to rotational inertia; their units are given in Table 1.

III. LINEARIZED LTI MODEL

For motion near the nominal pose, $M(q_{\text{abs}})$ is approximated by M_0 and velocity-dependent and higher-order terms are neglected. Linearization is a local approximation [11]. The relation $q_{\text{abs}} = q_0 + q$ connects the absolute pose to the deviation coordinates used in q and x . In particular, p_z is a height change from equilibrium; the absolute height is $z_0 + p_z$.

The simplified acceleration equation is

$$\ddot{q} = M_0^{-1}u, \quad (7)$$

where

$$u = [F_x \ F_y \ F_z \ T_x \ T_y \ T_z]^T. \quad (8)$$

Here F_x, F_y , and F_z are generalized forces, and T_x, T_y , and T_z are generalized torques about the nominal axes. The vector

u is the net incremental task-space input about a gravity-balanced equilibrium. The equilibrium supporting force is separate from u , so (7) describes deviations after the nominal gravity load has been balanced.

The model assumes that the generalized input can be applied directly. A physical implementation would need to map it to leg forces through J^T and account for the equilibrium load. Consequently, the coupled mechanism is represented here by a simplified, decoupled task-space plant; the model does not resolve the six leg lengths or leg forces.

Using (3), the linear time-invariant (LTI) system is

$$\dot{x} = A_n x + B_n u, \quad (9)$$

with

$$A_n = \begin{bmatrix} 0_{6 \times 6} & I_{6 \times 6} \\ 0_{6 \times 6} & 0_{6 \times 6} \end{bmatrix}, \quad B_n = \begin{bmatrix} 0_{6 \times 6} \\ M_0^{-1} \end{bmatrix}. \quad (10)$$

The first six equations relate coordinates to velocities. The remaining six describe the accelerations produced by the generalized input.

IV. FIRST-ORDER ACTUATOR DYNAMICS

The model in (9) would apply a commanded input immediately. A first-order lag is introduced to represent finite actuator response:

$$\tau_a \dot{u} + u = u_c, \quad (11)$$

where u_c is the commanded generalized input and u is the input applied to the platform model. The time constant is

$$\tau_a = 0.05 \text{ s}. \quad (12)$$

Thus,

$$\dot{u} = -\frac{1}{\tau_a} u + \frac{1}{\tau_a} u_c. \quad (13)$$

Six input channels require six actuator states. The system order therefore rises from 12 to 18 before adding the integral states. This lag is applied at the generalized-input level; it is not a separately identified model of each physical leg actuator.

V. TYPE-1 CONTROLLER DESIGN

A. OUTPUT SELECTION

The controlled outputs are the six platform coordinates:

$$y = [p_x \ p_y \ p_z \ \alpha \ \beta \ \gamma]^T. \quad (14)$$

The output equation is

$$y = Cx, \quad (15)$$

where

$$C = [I_{6 \times 6} \ 0_{6 \times 6}]. \quad (16)$$

B. INTEGRAL ERROR STATES

Integral action is included to remove the steady-state error for constant reference commands when the closed-loop system is stable [12]. Define

$$\dot{\eta} = r - y, \quad (17)$$

where r is the reference vector. The six integral states are

$$\eta = [\eta_1 \ \eta_2 \ \eta_3 \ \eta_4 \ \eta_5 \ \eta_6]^T. \quad (18)$$

At a constant-reference equilibrium, $\dot{\eta} = 0$ requires $y = r$. This step-tracking property assumes that the augmented feedback system is stable and that the required input is feasible. Saturation and anti-windup are not included in the reported model.

Figure 2 shows the feedback structure. The controller uses the platform states, actuator states, and integral states to generate u_c through their corresponding feedback gains.

C. FINAL 24-STATE AUGMENTED MODEL

The complete state vector is

$$\bar{X} = \begin{bmatrix} x \\ u \\ \eta \end{bmatrix}. \quad (19)$$

Its entries, written as a single vector over several lines, are

$$\begin{aligned} \bar{X} = [& p_x, p_y, p_z, \alpha, \beta, \gamma, \\ & \dot{p}_x, \dot{p}_y, \dot{p}_z, \dot{\alpha}, \dot{\beta}, \dot{\gamma}, \\ & u_1, u_2, u_3, u_4, u_5, u_6, \\ & \eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6]^T. \end{aligned} \quad (20)$$

The 24 states consist of 12 platform states, six actuator states, and six integral error states.

For feedback design, the reference is set to zero in the homogeneous augmented system:

$$\dot{\bar{X}} = \bar{A}\bar{X} + \bar{B}u_c, \quad (21)$$

where

$$\bar{A} = \begin{bmatrix} A_n & B_n & 0_{12 \times 6} \\ 0_{6 \times 12} & -\tau_a^{-1} I_{6 \times 6} & 0_{6 \times 6} \\ -C & 0_{6 \times 6} & 0_{6 \times 6} \end{bmatrix} \quad (22)$$

and

$$\bar{B} = \begin{bmatrix} 0_{12 \times 6} \\ \tau_a^{-1} I_{6 \times 6} \\ 0_{6 \times 6} \end{bmatrix}. \quad (23)$$

The state-feedback law is

$$u_c = -K\bar{X}, \quad (24)$$

with

$$K \in \mathbb{R}^{6 \times 24}. \quad (25)$$

Partitioning the gain gives

$$K = [K_x \ K_u \ K_i], \quad (26)$$

where $K_x \in \mathbb{R}^{6 \times 12}$ acts on the platform states, and $K_u, K_i \in \mathbb{R}^{6 \times 6}$ act on the actuator and integral states, respectively. The reference is restored during simulation through (17); its explicit state-space input matrix is given in Appendix A.

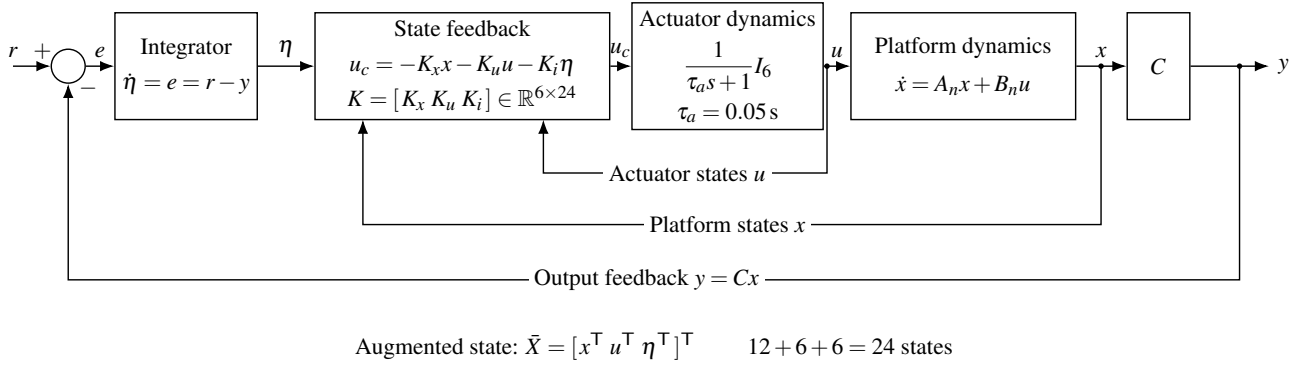


FIGURE 2. Type-1 state-feedback control structure. The integrator supplies η , the actuator block supplies u , and the platform model supplies x . The three signals enter the corresponding feedback gains. The first-order lag accounts for actuator response.

VI. CONTROLLABILITY AND NULL-SPACE GAIN CALCULATION

The controllability matrix for the augmented model is

$$\mathcal{P} = [\bar{B} \quad \bar{A}\bar{B} \quad \bar{A}^2\bar{B} \quad \cdots \quad \bar{A}^{23}\bar{B}]. \quad (27)$$

Full controllability requires

$$\text{rank}(\mathcal{P}) = 24. \quad (28)$$

The stated augmented model satisfies this condition. Appendix A proves controllability by decomposing the system into six controllable four-state subsystems. The shorter matrix $[\bar{B}, \bar{A}\bar{B}, \bar{A}^2\bar{B}, \bar{A}^3\bar{B}]$ already has rank 24. High powers of $\bar{A}$ cause severe numerical scaling in the full matrix in (27), so an unscaled numerical rank calculation can underestimate the exact rank. The analytical result avoids this tolerance-dependent ambiguity.

For the MIMO feedback design, a null-space method is used to assign the desired eigenvalues of

$$A_{cl} = \bar{A} - \bar{B}K. \quad (29)$$

For each target eigenvalue λ_i , the method solves

$$[\lambda_i I - \bar{A} \quad \bar{B}] \begin{bmatrix} v_i \\ k_{v_i} \end{bmatrix} = 0. \quad (30)$$

Here $v_i \in \mathbb{R}^{24}$ is a candidate closed-loop eigenvector and $k_{v_i} \in \mathbb{R}^6$ is its associated gain vector. After selecting 24 suitable null-space vectors, form

$$V = [v_1 \quad v_2 \quad \cdots \quad v_{24}] \quad (31)$$

and

$$K_v = [k_{v_1} \quad k_{v_2} \quad \cdots \quad k_{v_{24}}]. \quad (32)$$

If V is nonsingular, the gain is

$$K = K_v V^{-1}. \quad (33)$$

Equation (30) then implies $(\bar{A} - \bar{B}K)v_i = \lambda_i v_i$. Because multiple input directions are available, each null space offers several candidate vectors. Their selection must produce a full-rank, adequately conditioned V ; merely computing a null space does not complete the assignment.

TABLE 2. Desired closed-loop eigenvalues in s^{-1}

Pole	Value	Pole	Value	Pole	Value
λ_1	-1.0	λ_9	-2.6	λ_{17}	-7.0
λ_2	-1.2	λ_{10}	-2.8	λ_{18}	-7.5
λ_3	-1.4	λ_{11}	-3.0	λ_{19}	-0.6
λ_4	-1.6	λ_{12}	-3.2	λ_{20}	-0.7
λ_5	-1.8	λ_{13}	-5.0	λ_{21}	-0.8
λ_6	-2.0	λ_{14}	-5.5	λ_{22}	-0.9
λ_7	-2.2	λ_{15}	-6.0	λ_{23}	-1.1
λ_8	-2.4	λ_{16}	-6.5	λ_{24}	-1.3

A. DESIRED EIGENVALUES

Table 2 lists all 24 target eigenvalues. The first 12 were selected with platform motion in mind, the next six with actuator response, and the last six with integral tracking. These groups describe the design intention; full-state feedback can mix the corresponding closed-loop modes.

All targets lie in the open left-half plane. Successful assignment therefore gives an asymptotically stable LTI closed loop. In an implementation, this must be checked using the eigenvalues of the computed A_{cl} , together with the rank and conditioning of V .

VII. SIMULATION SETUP

The closed-loop model was simulated in MATLAB with the designed feedback gain. The step command was

$$r = [0.4 \quad 0.3 \quad 0.5 \quad 0.9 \quad 0.6 \quad 0.3]^T. \quad (34)$$

The first three entries are in meters and the last three in radians. The translational commands are therefore 0.4 m in p_x , 0.3 m in p_y , and 0.5 m in p_z , all measured from equilibrium. The corresponding steady-state absolute height represented by the model is

$$p_{z,abs} = z_0 + p_z = 0.55 + 0.5 = 1.05 \text{ m}. \quad (35)$$

These translation values also appear in the reference lines and initial errors in Figs. 3 and 5.

The roll, pitch, and yaw commands are 0.9, 0.6, and 0.3 rad. The simulation duration is 15 s, with a time step of 0.01 s. The plotted coordinates begin at zero deviation from the nominal pose.

The steps serve as an initial tracking test rather than a complete human head-motion profile. They test whether the mathematical system approaches fixed translational and rotational references. Sinusoidal trajectories and frequency sweeps are possible later tests for controlled dynamic EEG experiments, but their responses are not presented in this study. The large step amplitudes also require the model-validity qualification discussed below.

VIII. RESULTS AND DISCUSSION

The MATLAB results include translational response, rotational response, tracking errors, applied generalized inputs, and commanded inputs, as shown in Figs. 3–8.

A. TRANSLATIONAL STEP RESPONSE

Figure 3 shows p_x , p_y , and p_z approaching their respective references. The final values are close to the commands over the displayed 15 s interval. The integral states provide the mechanism for eliminating constant-reference error in a stable closed loop. The particular transient shape depends on the assigned eigenvalues and eigenvectors; negative real poles alone do not require every output to be monotonic.

B. ROTATIONAL STEP RESPONSE

The roll, pitch, and yaw responses are shown in Fig. 4. All three approach their references. However, the roll response has substantial overshoot, and the pitch and yaw responses are nonmonotonic. The roll curve reaches approximately 1.8 rad for a 0.9 rad command, as estimated from the figure. The result supports orientation tracking in the LTI simulation, while showing that transient motion needs further attention before an EEG experiment is specified.

C. TRACKING ERROR

The tracking error is

$$e = r - y. \quad (36)$$

Figures 5 and 6 show the translational and rotational errors. The errors approach approximately zero over the plotted interval. The negative roll-error excursion is consistent with the overshoot in Fig. 4. These observations describe the plotted responses; no numerical settling-time criterion or final error tolerance is imposed.

D. ACTUATOR AND COMMANDED INPUTS

The first-order actuator model distinguishes u_c , generated by the controller, from u , applied to the platform model. Figures 7 and 8 show these two sets of signals. The largest input excursions occur during the early transient, when the platform is moving toward its references. The inputs then decrease toward zero as the motion settles. The lag in (11)

accounts for the difference between commanded and actual input.

The plotted signals are generalized inputs: channels 1–3 are forces in newtons, and channels 4–6 are torques in newton-meters. The equilibrium gravity load is separate from these incremental inputs, so the platform still requires a supporting load when the plotted values approach zero.

E. RELEVANCE TO CONTROLLED EEG PHANTOM EXPERIMENTS

In a static phantom experiment, electrode voltages depend on the source arrangement, conductive medium, and electrode geometry. Moving the phantom introduces additional possible effects, including cable motion, interface changes, and changes in localization estimates. A controlled motion source is intended to make these test conditions repeatable and to help separate algorithm behavior from uncontrolled mechanical variation.

The proposed controller is an initial step toward that goal. Fixed displacements and, after further validation, prescribed oscillatory or roll, pitch, and heave motions could be compared with no-motion recordings. The resulting EEG data could then be used to examine signal stability, artifact levels, and source-localization accuracy. These EEG outcomes are proposed applications; they were not measured in the present control study.

Hardware tests should measure pose accuracy and repeatability separately, as in the evaluation of a three-DOF tendon-driven wrist by Samir *et al.* [13]. Sensor calibration and controlled comparisons are also central to experimental design. Hasan *et al.*, for example, compared free and swirling jets using nozzles with the same exit diameter and calibrated yaw-meter measurements [14]. For the proposed phantom setup, these experimental principles translate to calibrated motion measurements and repeated recordings under a documented protocol.

F. SCOPE OF THE LINEAR RESULTS

The responses demonstrate tracking for the stated LTI equations. A 0.5 m height change, a 0.9 rad roll command, and the larger transient roll excursion have not been validated as small perturbations about the nominal pose. Nonlinear validation is needed to establish whether a physical Stewart platform can follow the same commands with acceptable accuracy.

The model also omits leg geometry, Jacobian variation, actuator stroke and force limits, joint limits, and state estimation. No saturation treatment, disturbance-rejection study, or experimental EEG result is presented. A physical implementation needs leg-force allocation, gravity compensation, workspace checks, and a feasible sensing arrangement. Full-state availability is assumed in the feedback law.

The numerical gain, selected null-space vectors, achieved closed-loop poles, full initial augmented state, solver settings, and raw response arrays are not available with the simulation figures. These details are needed to reproduce the trajectories

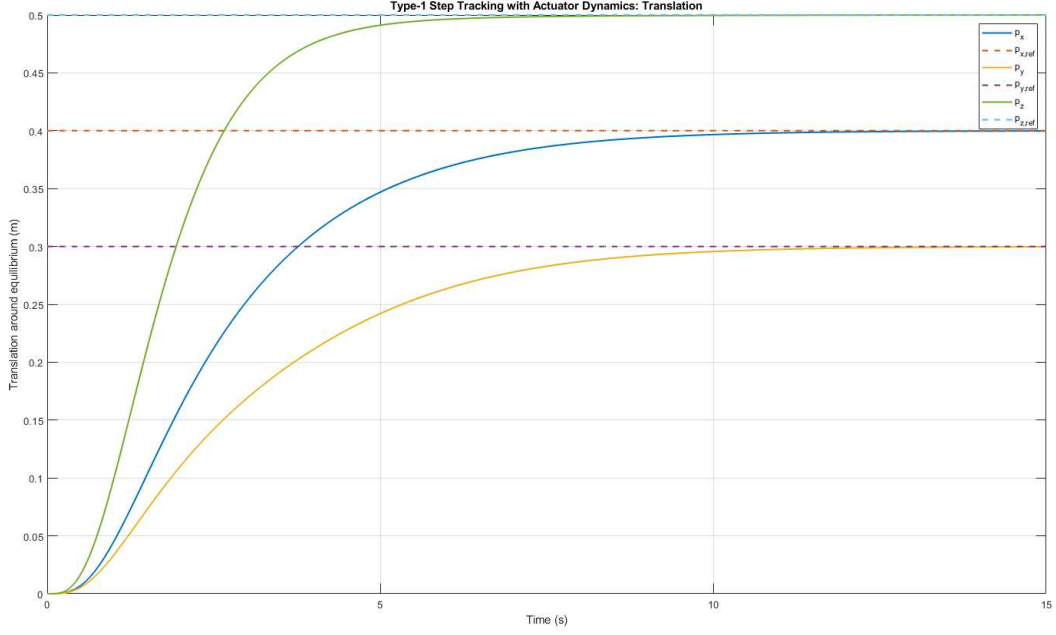


FIGURE 3. Translational step responses. The reference changes are $p_x = 0.4$ m, $p_y = 0.3$ m, and $p_z = 0.5$ m, measured from the nominal pose.

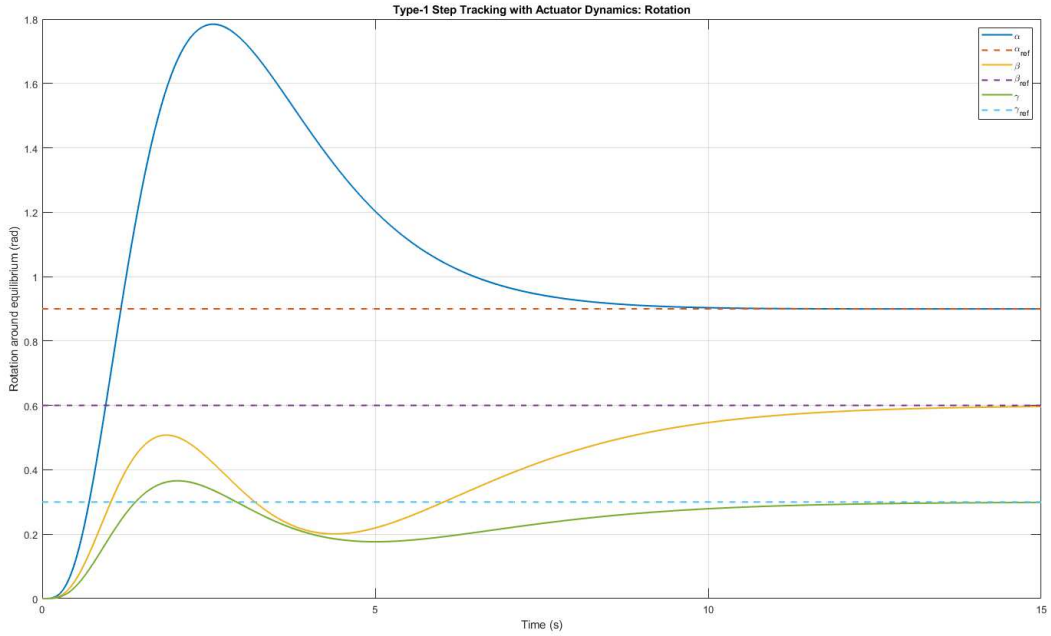


FIGURE 4. Rotational step responses for roll α , pitch β , and yaw γ . The commands are 0.9, 0.6, and 0.3 rad, respectively.

and compute exact performance metrics. The analytical controllability and reference-input results can be checked directly from the stated matrices.

IX. MATLAB IMPLEMENTATION

The implementation has four main stages: construct the 12-state platform model, add actuator lag, add six integral error states, and compute a feedback gain through the null-space method. The augmented matrices are assembled as follows:

```
Abar = [A_n, B_n, zeros(12,6);
        zeros(6,12), ...
        -(1/tau_a)*eye(6), zeros(6,6);
        -C, zeros(6,6), zeros(6,6)];

Bbar = [zeros(12,6);
        (1/tau_a)*eye(6);
        zeros(6,6)];
```

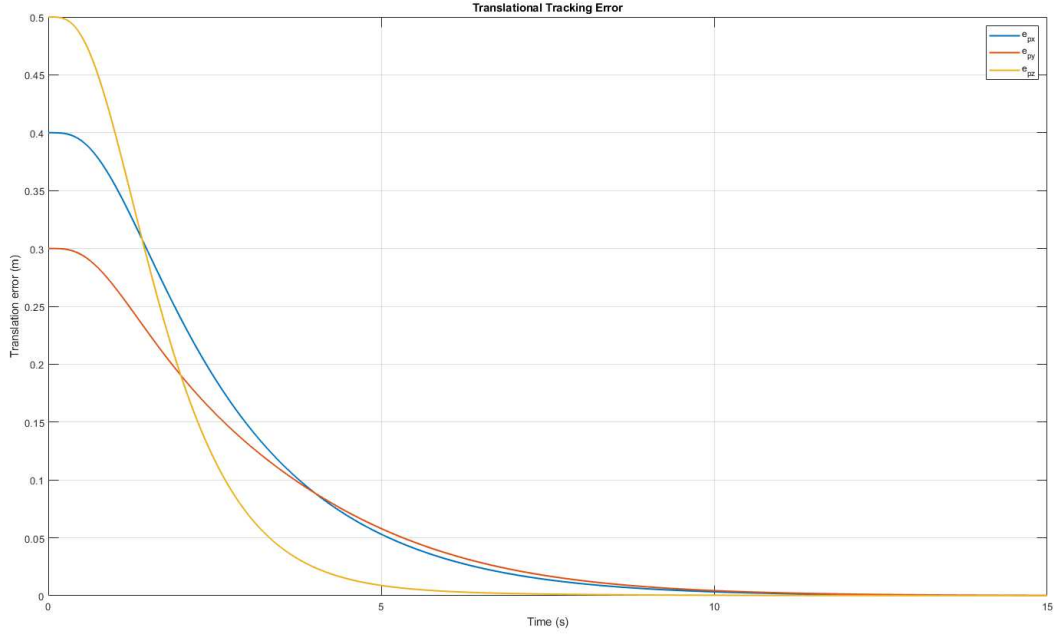


FIGURE 5. Translational tracking errors. Their initial values agree with the three translational entries in (34).

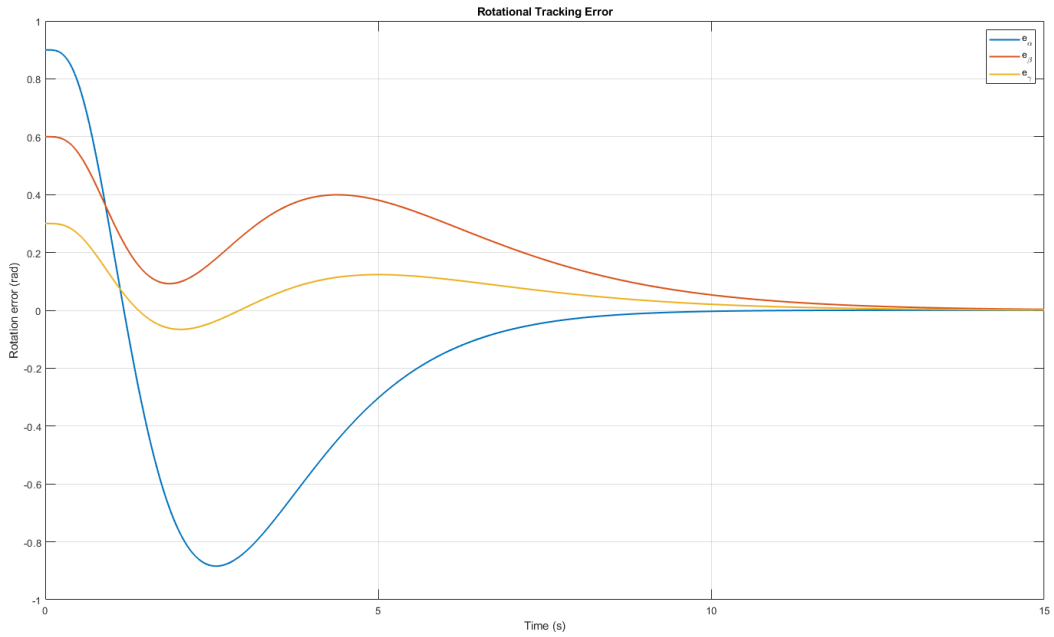


FIGURE 6. Rotational tracking errors. Integral action drives the constant-reference errors toward zero in the reported response.

For each desired eigenvalue, the null-space calculation is

```
mat = [lamda(i)*eye(n)-Abar Bbar];
nullspace = null(mat,'r');
```

Here $n = 24$, and lamda contains the desired poles. Once suitable vectors have been selected to form V and K_v , the gain relation in (33) is evaluated using

```
Knew = Kv/v;
```

where v stores the matrix V in (31). Right matrix division

solves $K_{\text{new}}V = K_v$ without explicitly forming V^{-1} . The matrix V must have full rank and acceptable conditioning. The final controller is

$$u_c = -K_{\text{new}}\bar{X}. \quad (37)$$

These excerpts specify the matrix construction and gain relation. A complete numerical implementation must also select suitable vectors from each null space as described in Section VI, verify the assigned poles, and include the reference input in Appendix A. The numerical information

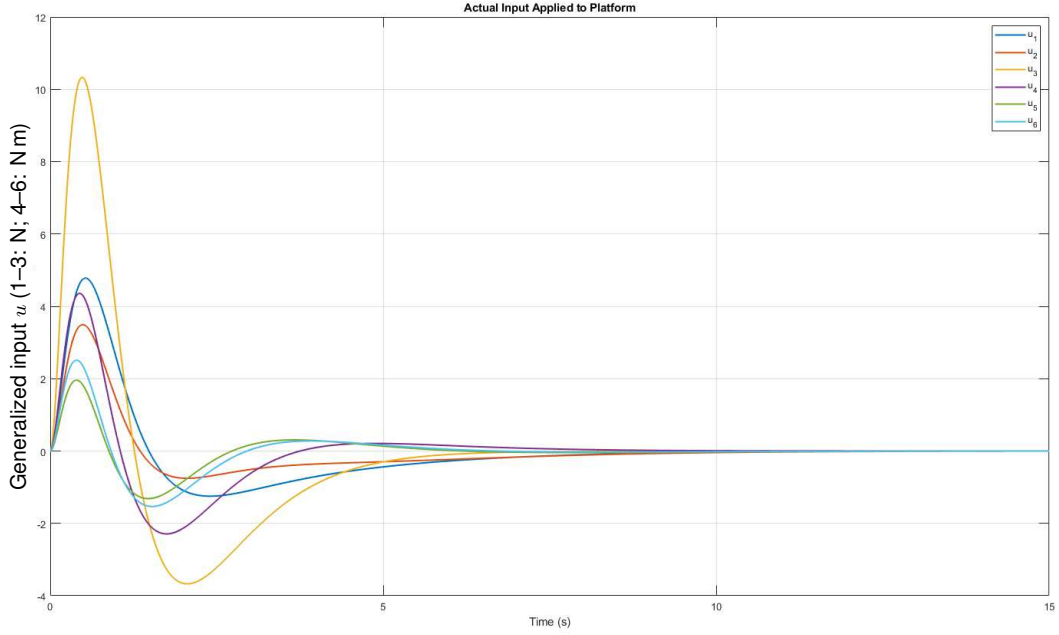


FIGURE 7. Generalized inputs u applied to the platform model after the first-order lag. Channels 1–3 are forces in N, and channels 4–6 are torques in N m.

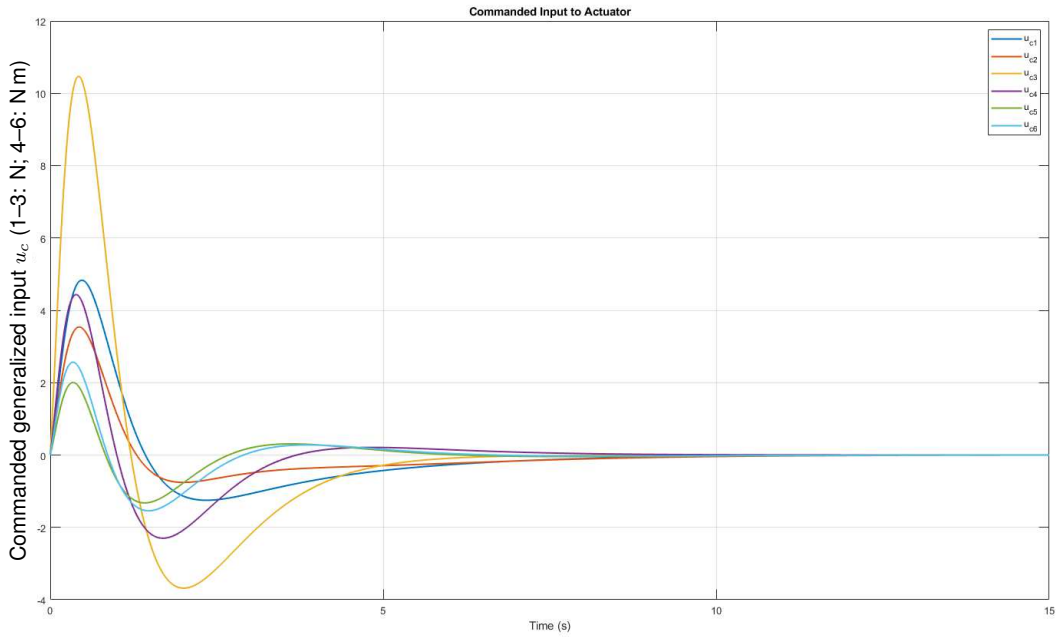


FIGURE 8. Commanded generalized inputs u_c produced by the controller. Channels 1–3 are in N and channels 4–6 are in N m; equilibrium support loads are separate.

needed to reproduce the displayed trajectories is identified in Section VIII-F.

X. CONCLUSION

A Type-1 linear state-feedback controller has been formulated for a six-DOF motion platform with first-order actuator dynamics. The intended application is controlled motion for future EEG phantom experiments. The 12-state platform model is augmented by six actuator states and six integral

states, producing a 24-state system. The stated model is analytically controllable, and a null-space eigenvalue-assignment formulation provides the feedback gain when the selected eigenvector matrix is nonsingular.

The MATLAB responses approach the translational and rotational step commands with approximately zero steady-state error. They also show substantial rotational transients. The results support the controller as an initial LTI tracking study, subject to the limits of the simplified task-space model. The

physical operating range remains to be established through nonlinear analysis and hardware tests.

Further work should connect the generalized inputs to physical legs, account for workspace and actuator constraints, and test appropriate motion amplitudes on hardware. Selected heave, roll, pitch, sinusoidal, and frequency-sweep trajectories can then be evaluated. Combining validated platform motion with EEG phantom recordings would allow no-motion and motion conditions to be compared and would support a later assessment of source-localization robustness.

APPENDIX A ANALYTICAL CONTROLLABILITY AND REFERENCE INPUT

Define $a = 1/\tau_a$ and $b_j = 1/(M_0)_{jj}$ for coordinate j . A permutation of the augmented states groups each coordinate into $\xi_j = [q_j, \dot{q}_j, u_j, \eta_j]^T$. For zero reference, its four-state subsystem is

$$A_j = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & b_j & 0 \\ 0 & 0 & -a & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \quad B_j = \begin{bmatrix} 0 \\ 0 \\ a \\ 0 \end{bmatrix}. \quad (38)$$

The corresponding controllability matrix is

$$\mathcal{P}_j = \begin{bmatrix} 0 & 0 & ab_j & -a^2b_j \\ 0 & ab_j & -a^2b_j & a^3b_j \\ a & -a^2 & a^3 & -a^4 \\ 0 & 0 & 0 & -ab_j \end{bmatrix}. \quad (39)$$

Its determinant is

$$\det(\mathcal{P}_j) = a^4b_j^3 \neq 0. \quad (40)$$

Since $\tau_a > 0$ and each diagonal entry of M_0 is positive, all six four-state subsystems are controllable. The controllability matrix of the complete augmented system therefore has rank 24. This proves controllability without relying on a tolerance applied to powers through $\bar{A}^{23}$. With $a = 20 \text{ s}^{-1}$, those high powers also explain the severe scaling of the full numerical controllability matrix.

The reference input omitted from the homogeneous design equation (21) enters through the integral states. The forced closed-loop equations are

$$\dot{\bar{X}} = (\bar{A} - \bar{B}K)\bar{X} + Er, \quad E = \begin{bmatrix} 0_{12 \times 6} \\ 0_{6 \times 6} \\ I_{6 \times 6} \end{bmatrix}, \quad (41)$$

with output

$$y = \begin{bmatrix} C & 0_{6 \times 6} & 0_{6 \times 6} \end{bmatrix} \bar{X}. \quad (42)$$

These expressions follow directly from $\dot{\eta} = r - Cx$ and use the state ordering in (20).

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