Partially decoupling cell size and density as tailorable design parameters in mSLA-printed gyroid cores for sustainable sandwich panels

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Supporting Information

Statistical analysis summary

All statistical analyses reported in the main text used the per-specimen compressive data reproduced in Table S7, with n = 3 per condition unless stated otherwise. Conditions are labelled by unit cell size and nominal density (for example, 10-15 denotes a 10 mm cell at 15% nominal density). The 5 mm conditions (5-10 and 5-15) were excluded from the within-level comparisons as a degraded-fidelity processing zone (Section 3.3), so within-level tests spanned the 10, 15 and 20 mm series unless stated otherwise; the effect of including the 5 mm conditions is given in Table S2. Table S1 summarizes the inclusion set, sample size, and method for every reported test and fit. Within-level comparisons used one-way ANOVA across cell sizes with Welch's two-sample t-tests (unequal variances) for pairwise follow-ups, with significance taken at $p < 0.05$ (Table S2). Variance homogeneity was checked with Levene's test and was non-significant for every level and response basis (smallest $p = 0.30$). All p-values are reported uncorrected, and for multiplicity context, a per-level Holm correction over the full family of tests at each level (14 tests at 15%, 18 at 20%) retains none of the 15% results and retains every 20% effect with $p \leq 0.0018$, consistent with the main-text qualification that the cell-size effect on strength is robust at 20% and emerging at 15%. At n = 3 per group, no distribution-free two-sample test can return $p < 0.1$ (20 permutations), and the Kruskal–Wallis omnibus at 20% gives $p = 0.061$ against the parametric value of 3.5×10^{-6}; the pairwise values should therefore be read as parametric follow-ups to significant omnibus or regression results, not as standalone evidence.

The primary density-controlled test was a per-level analysis of covariance of log strength on cell size with log measured density as covariate (k = 9 per level). Ratio normalizations (specific strength UCS/ρ, and UCS/ρ²) were retained as sensitivity analyses, and because the ρ² basis is informed by the fitted strength exponent, the normalization was re-run for exponents between 1.5 and 2.3 (Table S4). The conclusions were unchanged across this range except the 10 vs 15 mm contrast at 20%, which is marginal for exponents above ≈2.1.

Gibson–Ashby scaling exponents were fitted by least squares on log-transformed per-specimen properties against log measured density, per cell size and pooled across the 10, 15 and 20 mm series with a common slope (Table S3). The size × log density interaction was non-significant ($p = 0.80$), justifying the common-slope model, and the cell-size main effect on the prefactor was non-significant (nested F-test, $p = 0.13$), and since at k = 36 the tests have limited power against an effect concentrated at the highest density level they are not read as affirmative evidence of absence. A quadratic term in log density was significant ($p = 1.8 \times 10^{-5}$), indicating curvature in the log–log relationship, and excluding the defect-prone 5% conditions removes the curvature ($p = 0.54$) and gives a common strength exponent of $n \approx 1.5$ (modulus $n \approx 1.3$), matching the bending-dominated collapse-strength prediction. For the density nulls at 5% and 10%, the minimum detectable pairwise effect at 80% power was 11.7% and 4.9% of the mean, respectively.

As a constitutive check on the density-only Gibson–Ashby form, the bivariate power law $\log \mathrm{UCS} = a + b \log \rho + c \log L$ was fitted to the per-specimen strengths of the 10, 15 and 20 mm

series (k = 36), with cell size L as a continuous covariate. Measured density and log cell size were effectively uncorrelated in this design (r = 0.01), so the two coefficients are separated. The density exponent (b = 1.98) recovered the common-slope value above, and a nested F-test against the per-size-intercept model showed no loss of fit (p = 0.29), so the bivariate form is a parsimonious restatement of that analysis. The cell-size coefficient was positive but not significant (c = 0.22 ± 0.12, p = 0.086, partial R^2 = 0.09, Figure S1a), corresponding to a strength factor of ≈1.17 from 10 to 20 mm at fixed measured density, and the same fit for modulus gave c = 0.16 (p = 0.24). A uniform cell-size term therefore adds little once measured density is accounted for, consistent with the within-level result that the effect is concentrated at the highest density level rather than distributed uniformly. Including the degraded 5 mm conditions returns a highly significant cell-size term (c = 0.54, p = 8×10^{-8}), but this conflates the failure-driven 5 mm prefactor deficit with the constitutive trend and is not interpreted further. The design decoupling behind this form is visualized in Figure S1b and S1c, with wall thickness rising with cell size at every nominal density whilst measured density is independent of cell size at 5% and 10%.

Table S1. Analysis basis for every statistical test and fit reported in the main text: response variable, inclusion set, sample size, and method.

Analysis	Response	Inclusion set	n / k	Method
Density across cell sizes, 5% level	ρ	10-5, 15-5, 20-5	3 × 3	One-way ANOVA
Density across cell sizes, 10% level	ρ	10-10, 15-10, 20-10 (5-10 excluded)	3 × 3	One-way ANOVA
Density across cell sizes, 15% level	ρ	10-15, 15-15, 20-15 (5-15 excluded)	3 × 3	One-way ANOVA
Density across cell sizes, 20% level	ρ	10-20, 15-20, 20-20	3 × 3	One-way ANOVA
Strength across cell sizes, per level	UCS	As rows 1 to 4	3 × 3	One-way ANOVA, Welch pairwise
Specific-strength basis, 15% and 20%	UCS/ρ	10, 15, 20 mm at each level	3 × 3	One-way ANOVA, Welch pairwise
ρ^2-normalized basis, 15% and 20%	UCS/ρ^2	10, 15, 20 mm at each level	3 × 3	One-way ANOVA, Welch pairwise
Per-level density control (primary), 15% and 20%	log UCS	10, 15, 20 mm at each level	k = 9	ANCOVA, log ρ covariate
Normalization-exponent sensitivity	UCS/ρ^n, n = 1.5–2.3	15% and 20% levels	3 × 3	One-way ANOVA, Welch pairwise
Modulus across cell sizes, 20% level	E_c	10-20, 15-20, 20-20	3 × 3	One-way ANOVA, Welch pairwise
Per-size scaling fits	log UCS and log E_c vs log ρ	Four density levels per cell size	k = 12	Log–log least squares
Pooled scaling fits	log UCS and log E_c vs log ρ	10, 15, 20 mm × 5–20%	k = 36	Common-slope ANCOVA
Scaling curvature test	log UCS	As pooled fits	k = 36	Nested F-test, quadratic term
Minimum detectable effect, 5% and 10%	ρ	10, 15, 20 mm at each level	3 × 3	Two-sample t power analysis
Variance homogeneity	All responses	All tested levels and bases	3 × 3	Levene's test

Table S2. Within-level statistical results on all analysis bases (n = 3 per condition; 10, 15 and 20 mm series). Omnibus values are one-way ANOVA p-values except the ANCOVA rows, which give the partial F-test for cell size with log measured density as covariate; pairwise values are Welch's t-tests. Parenthesized omnibus values include the 5 mm condition at that level. The final column records whether the omnibus result survives a per-level Holm correction over the full family of tests at that level (14 tests at 15%, 18 at 20%), where no 15% result survives, and every 20% effect with $p \leq 0.0018$ does.

Level	Response / basis	Omnibus p	10 vs 15 mm	15 vs 20 mm	10 vs 20 mm	Holm
5%	Density ρ	0.144	—	—	—	—
10%	Density ρ	0.697 (0.978 incl 5-10)	—	—	—	—
15%	Density ρ	0.027 (0.0037 incl 5-15)	—	—	—	No
20%	Density ρ	0.032	—	—	—	No
5%	UCS	0.551	—	—	—	—
10%	UCS	0.343 (0.0025 incl 5-10)	—	—	—	—
15%	UCS	0.034 (5.2×10^{-6} incl 5-15)	0.502	0.074	0.022	No
20%	UCS	3.5×10^{-6}	7.0×10^{-5}	0.440	9.8×10^{-5}	Yes
15%	UCS/ρ	0.030	0.208	0.032	0.083	No
20%	UCS/ρ	9.0×10^{-5}	0.012	0.175	0.0018	Yes
15%	UCS/ρ²	0.030	0.122	0.022	0.174	No
20%	UCS/ρ²	0.0018	0.049	0.063	0.011	Yes
15%	log UCS, ANCOVA (log ρ covariate)	$F = 3.3$, $p = 0.121$	—	—	—	No
20%	log UCS, ANCOVA (log ρ covariate)	$F = 133$, $p = 4.6 \times 10^{-5}$	—	—	—	Yes
20%	E_c	7.1×10^{-5}	0.496	0.0065	4.5×10^{-5}	Yes

Table S3. Gibson–Ashby scaling fits of per-specimen properties against measured relative density. Per-size fits are ordinary least squares on log-transformed values with t-based 95% confidence intervals (10 residual DoF), pooled fits are common-slope models with per-size intercepts. The size × log density interaction was non-significant ($p = 0.80$), justifying the common-slope model, and the cell-size main effect on the prefactor was non-significant (nested F-test, $p = 0.13$). A quadratic term in log density was significant ($p = 1.8 \times 10^{-5}$) over the full window and non-significant ($p = 0.54$) once the 5% level was excluded. Robustness checks are as follows. Per-size strength exponents excluding the 5% level are 1.47, 1.55 and 1.47 (modulus 1.10, 1.10 and 1.68), and excluding the 20% level are 2.12, 2.06 and 2.21, for the 10, 15 and 20 mm series. Raw-scale nonlinear least squares ($\sigma = C\rho^n$) gives strength exponents 1.46, 1.78 and 1.59.

Series	k	Strength exponent n (95% CI)	R^2	Modulus exponent n (95% CI)	R^2
10 mm	12	1.92 (1.71–2.12)	0.98	1.50 (1.32–1.68)	0.97
15 mm	12	2.00 (1.69–2.31)	0.95	1.55 (1.22–1.89)	0.91
20 mm	12	2.02 (1.77–2.28)	0.97	1.89 (1.68–2.11)	0.97
Pooled, common slope	36	1.98 (1.85–2.12)	0.96	1.64 (1.49–1.79)	0.94
Pooled, 5% level excluded	27	1.50 (1.31–1.69)	0.92	1.28 (1.08–1.48)	0.89

Table S4. Sensitivity of the density-normalized within-level comparisons to the normalization exponent (response UCS/ρ^n, one-way ANOVA omnibus and Welch pairwise tests). All conclusions are unchanged across this range except the 10 vs 15 mm contrast at 20%, which crosses $p = 0.05$ for exponents above ≈2.1.

Normalization	15% omnibus p	20% omnibus p	10 vs 15 mm, 20%	10 vs 20 mm, 20%	15 vs 20 mm, 20%
n = 1.5	0.030	4.7×10^{-4}	0.025	0.005	0.096
n = 1.7	0.030	8.4×10^{-4}	0.033	0.007	0.080
n = 2.0 (main text)	0.030	0.0018	0.049	0.011	0.063
n = 2.3	0.031	0.0037	0.074	0.015	0.051

Table S5. Measured apparent density and wall thickness of the printed lattices across the factorial. Dashes indicate print failures.

		Nominal density			
Cell (mm)	Property	5% (55)	10% (110)	15% (165)	20% (220)
5	ρ (kg/m³)	—	126.1 ± 12.0	201.3 ± 3.2	—
	t (μm)	—	131 ± 3	236 ± 7	—
10	ρ (kg/m³)	63.6 ± 0.8	126.7 ± 1.7	186.8 ± 4.7	252.2 ± 7.5
	t (μm)	132 ± 3	313 ± 4	499 ± 10	660 ± 15
15	ρ (kg/m³)	60.4 ± 2.6	127.8 ± 2.3	198.0 ± 3.4	266.3 ± 3.5
	t (μm)	214 ± 10	488 ± 8	765 ± 17	1086 ± 8
20	ρ (kg/m³)	64.9 ± 3.2	128.1 ± 2.1	185.9 ± 5.0	255.2 ± 3.1
	t (μm)	271 ± 11	670 ± 6	1034 ± 9	1404 ± 34

Table S6. Compressive properties across the factorial. Dashes indicate print failures.

		Nominal density			
Cell (mm)	Property	5%	10%	15%	20%
5	UCS (MPa)	—	0.43 ± 0.13	1.40 ± 0.03	—
	E_c (MPa)	—	5.8 ± 3.8	46.2 ± 2.2	—
	ε at UCS (%)	—	5.08 ± 0.71	4.75 ± 0.26	—
10	UCS (MPa)	0.24 ± 0.04	1.13 ± 0.06	2.25 ± 0.08	3.10 ± 0.07
	E_c (MPa)	13.0 ± 1.3	47.6 ± 3.5	72.8 ± 10.1	101.0 ± 3.6
	ε at UCS (%)	4.82 ± 2.03	3.35 ± 0.04	4.47 ± 0.29	4.94 ± 0.30
15	UCS (MPa)	0.21 ± 0.06	1.32 ± 0.43	2.16 ± 0.17	4.10 ± 0.06
	E_c (MPa)	11.6 ± 5.0	47.8 ± 12.0	74.6 ± 9.2	106.2 ± 10.7
	ε at UCS (%)	4.38 ± 2.68	3.97 ± 0.41	4.82 ± 0.20	5.23 ± 0.11
20	UCS (MPa)	0.27 ± 0.08	1.46 ± 0.04	2.48 ± 0.05	4.06 ± 0.08
	E_c (MPa)	12.4 ± 4.6	50.1 ± 1.6	84.3 ± 2.9	161.3 ± 4.0
	ε at UCS (%)	3.21 ± 0.36	2.64 ± 0.38	4.83 ± 0.15	4.89 ± 0.14

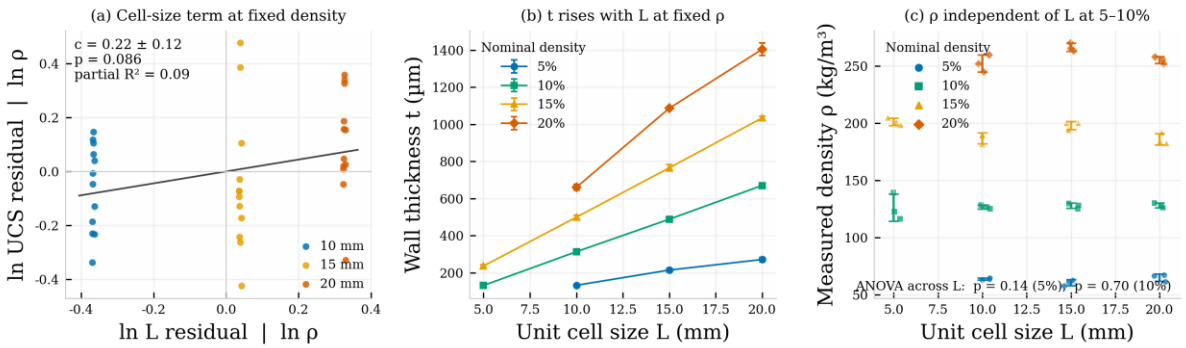


Figure S1. Constitutive and decoupling checks for the bivariate strength model. (a) Added-variable (partial-regression) plot for log cell size in the fit log UCS = a + b log ρ + c log L over the 10, 15 and 20 mm series (k = 36), where the slope is the cell-size dependence of strength at fixed measured density (c = 0.22 ± 0.12, p = 0.086). (b) Measured wall thickness against unit cell size at each nominal density, (c) Per-specimen measured density against unit cell size at each nominal density, where measured density is independent of cell size at 5% (p = 0.14) and 10% (p = 0.70).