

A Stabilized Mixed Hybrid Finite Element Framework for Large-Deformation Poromechanics with Torsion: Application to Transmural Perfusion and Vascular Hemodynamics

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A Stabilized Mixed Hybrid Finite Element Framework for Large-Deformation Poromechanics with Torsion: Application to Transmural Perfusion and Vascular Hemodynamics

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Abstract

Intramyocardial perfusion and wall mechanics involve finite deformations, tissue anisotropy, and interstitial fluid flow subject to combined chamber pressurization and torsion. In standard displacement-based finite element formulations, nearly incompressible solid matrices and steep pressure gradients frequently induce volumetric locking, pressure oscillations, and degraded flux fields across the wall. To address this, a mixed hybrid finite element method (MHFEM) is formulated for large-strain Biot poromechanics in cylindrical coordinates with coupled out-of-plane torsional shear ($\mathbf{u}_r, \mathbf{u}_\theta, \mathbf{u}_z$). Fluid flux is discretized using lowest-order Raviart–Thomas spaces ($\mathbf{RT}_0$), while Lagrange multipliers on element boundaries enforce normal flux continuity and exact local mass conservation. An exact, one-dimensional analytical solution for an inflated and twisted thick-walled porous cylinder is derived to evaluate asymptotic convergence and quantify fluid pressure, solid deformation, and shear stress. Axisymmetric simulations under physiological and hypertensive conditions indicate that torsional twist reduces peak endocardial fiber stress by **45.5%**, relieving compressive drivers of subendocardial vessel collapse. In models of concentric hypertrophy with elevated systolic cavity pressures (**180 mmHg**), the mixed hybrid scheme maintains smooth, monotonic pore pressure distributions (L_2 error = **0.01%**), whereas standard $\mathbf{Q}_1/\mathbf{P}_1$ formulations exhibit spurious pressure oscillations and elevated volumetric errors (L_2 = **12.75%**).

Keywords: Mixed Hybrid Finite Element Method, Large-Deformation Poromechanics, Ventricular Torsion, Intramyocardial Perfusion, Volumetric Locking, Raviart–Thomas Elements

1 Introduction

Intramyocardial stress distributions and coronary perfusion are strongly coupled to the finite kinematics of the left ventricular wall during the cardiac cycle [1–3]. During diastole, cavity filling induces simultaneous radial expansion, longitudinal stretch, and circumferential torsion about the ventricular long axis [4, 5]. Because the

myocardium is a fluid-saturated porous medium with about 15% fluid volume, shear deformations alter pore pressure, compress microvessels, and govern local hydraulic resistance [6, 7].

Simulating this response requires accounting for finite kinematic transformations, deformation-dependent hydraulic permeability, and transmural fiber reorientation, which typically transitions from $\alpha \approx +60^\circ$ at the endocardium to -60° at

the epicardium [8, 9]. In standard displacement-based finite element formulations, enforcing the near-incompressibility of both the fluid phase and the hydrated solid matrix triggers severe volumetric locking, non-physical boundary-layer stress spikes, and spatial pressure checkerboarding near the inner wall.

To bypass these numerical instability modes, this paper develops a stabilized Mixed Hybrid Finite Element Method (MHFEM) for finite axisymmetric deformations with out-of-plane torsion. This work builds upon and extends previous mixed hybrid approaches developed for charged and swelling porous media [10, 12, 13]. Solid displacement, Darcy filtration velocity, and interstitial fluid pressure are treated as independent field variables. By introducing Lagrange multiplier trace pressures along inter-element boundaries, normal flux continuity is enforced across moving element interfaces. This hybrid formulation permits element-level static condensation of all internal fluxes and pressures, yielding a compact global system solved exclusively for displacements and Lagrange multipliers. To assess numerical accuracy, an analytical benchmark for an inflated and twisted thick-walled cylinder is also derived, providing exact reference for stress homogenization, coronary perfusion pressure coupling, and spatial convergence.

2 Governing Continuum Equations

Passive myocardium is modeled as a fully saturated, two-phase mixture composed of an incompressible hyperelastic solid matrix and an interstitial fluid.

2.1 Kinematics of Axisymmetric Deformation with Torsion

Restricting the deformation to axisymmetry ($\partial/\partial\Theta = 0$), the spatial motion maps reference cylindrical coordinates (R, Θ, Z) to the current configuration according to:

$$r = r(R, Z), \quad \theta = \Theta + \phi(R, Z), \quad z = z(R, Z) \quad (1)$$

where $\phi(R, Z)$ describes the circumferential twist angle. In the dual cylindrical basis $\{\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z\} \otimes$

$\{\mathbf{E}_R, \mathbf{E}_\Theta, \mathbf{E}_Z\}$, the deformation gradient $\mathbf{F} = \text{Grad } \mathbf{x}$ takes the form:

$$\mathbf{F} = \begin{bmatrix} \frac{\partial r}{\partial R} & 0 & \frac{\partial r}{\partial Z} \\ r \frac{\partial \phi}{\partial R} & r & r \frac{\partial \phi}{\partial Z} \\ \frac{\partial z}{\partial R} & 0 & \frac{\partial z}{\partial Z} \end{bmatrix} \quad (2)$$

The right Cauchy–Green deformation tensor $\mathbf{C} = \mathbf{F}^T \mathbf{F}$ has non-zero components:

$$C_{RR} = \left(\frac{\partial r}{\partial R} \right)^2 + r^2 \left(\frac{\partial \phi}{\partial R} \right)^2 + \left(\frac{\partial z}{\partial R} \right)^2 \quad (3)$$

$$C_{\Theta\Theta} = \left(\frac{r}{R} \right)^2 = \lambda_\theta^2 \quad (4)$$

$$C_{ZZ} = \left(\frac{\partial r}{\partial Z} \right)^2 + r^2 \left(\frac{\partial \phi}{\partial Z} \right)^2 + \left(\frac{\partial z}{\partial Z} \right)^2 \quad (5)$$

$$C_{RZ} = \frac{\partial r}{\partial R} \frac{\partial r}{\partial Z} + r^2 \frac{\partial \phi}{\partial R} \frac{\partial \phi}{\partial Z} + \frac{\partial z}{\partial R} \frac{\partial z}{\partial Z} \quad (6)$$

$$C_{\Theta Z} = \frac{r^2}{R} \frac{\partial \phi}{\partial Z}, \quad C_{R\Theta} = \frac{r^2}{R} \frac{\partial \phi}{\partial R} \quad (7)$$

2.2 Constitutive Framework and Stress Decomposition

By Terzaghi’s effective stress principle, the total Cauchy stress $\boldsymbol{\sigma}$ separates into an effective solid matrix stress $\boldsymbol{\sigma}_e$ and an isotropic pore pressure p :

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_e - p\mathbf{I} = J^{-1} \mathbf{F} \mathbf{S}_e \mathbf{F}^T - p\mathbf{I} \quad (8)$$

where $J = \det \mathbf{F}$ is the volume ratio and $\mathbf{S}_e$ is the second Piola–Kirchhoff effective stress tensor.

Muscle fibers are aligned within concentric cylindrical laminae. In the undeformed reference geometry, the local fiber unit vector is parameterized as:

$$\mathbf{a}_0(R) = \cos \alpha(R) \mathbf{E}_\Theta + \sin \alpha(R) \mathbf{E}_Z \quad (9)$$

Following histological observations [8], the helix angle $\alpha(R)$ varies smoothly from a right-handed pitch at the endocardium ($\alpha \approx +60^\circ$) to a left-handed pitch at the epicardium ($\alpha \approx -60^\circ$), crossing circumferential alignment ($\alpha = 0^\circ$) near the midwall. The corresponding structural orientation

Table 1 Nomenclature and Mathematical Symbols

Symbol	Description	Symbol	Description
(R, Θ, Z)	Reference cylindrical coordinates	W	Hyperelastic strain energy function
(r, θ, z)	Current spatial coordinates	C_{mat}	Matrix shear modulus
$\mathbf{u}$	Solid displacement vector	a_1, a_2	Exponential stiffening coefficients
$\phi(R, Z)$	Circumferential twist angle	K_{vol}	Volumetric bulk modulus
γ_0	Axial torsion rate (Φ/L)	K_0	Baseline intrinsic permeability
$\mathbf{F}, J$	Deformation gradient and Jacobian	M	Non-linear consolidation exponent
$\mathbf{C}, \mathbf{E}$	Cauchy–Green and Green–Lagrange strains	β_f	Along-fiber permeability factor
$\mathbf{a}_0, \mathbf{a}$	Reference and deformed fiber vectors	μ_f	Fluid dynamic viscosity
$\alpha(R)$	Transmural fiber helix angle	τ_{shear}	Total wall shear stress
$\mathbf{M}$	Structural tensor ($\mathbf{a}_0 \otimes \mathbf{a}_0$)	P_{perf}	Coronary perfusion pressure
I_1, I_4	First invariant and fiber pseudo-invariant	Q_0	Transmural filtration rate
$\boldsymbol{\sigma}, \boldsymbol{\sigma}_e$	Total and effective Cauchy stress tensors	$\Omega_e, \Gamma_{\text{int}}$	Element domain and interface boundary
$\mathbf{S}_e$	Second Piola–Kirchhoff stress tensor	$\mathbf{N}_u, \boldsymbol{\Phi}_q$	Displacement and RT_0 flux basis functions
$p, \bar{p}$	Pore pressure and interface trace pressure	$\psi_p, \mu_{\bar{p}}$	Element P_0 and trace shape functions
$\mathbf{q}$	Specific fluid filtration discharge flux	$\mathbf{K}(J, \mathbf{a})$	Hydraulic permeability tensor

tensor $\mathbf{M} = \mathbf{a}_0 \otimes \mathbf{a}_0$ reads:

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \cos^2 \alpha & \cos \alpha \sin \alpha \\ 0 & \cos \alpha \sin \alpha & \sin^2 \alpha \end{bmatrix} \quad (10)$$

The passive hyperelastic strain energy function $W(\mathbf{C}, \mathbf{M})$ incorporates matrix non-linearity, exponential fiber stiffening, and a volumetric penalty:

$$W(\mathbf{C}, \mathbf{M}) = \frac{C_{\text{mat}}}{2} \left(\exp [a_1(I_1 - 3)^2 + a_2(I_4 - 1)^2] - 1 \right) + \frac{K_{\text{vol}}}{2} (J - 1)^2 \quad (11)$$

Here $I_1 = \text{tr } \mathbf{C}$, and the fourth pseudo-invariant measures the squared fiber stretch ratio:

$$I_4 = \mathbf{C} : \mathbf{M} = \mathbf{a}_0 \cdot \mathbf{C} \mathbf{a}_0 = \lambda_{\text{fiber}}^2 \quad (12)$$

Differentiation of W yields the effective second Piola–Kirchhoff stress:

$$\mathbf{S}_e = 2 \frac{\partial W}{\partial I_1} \mathbf{I} + 2 \frac{\partial W}{\partial I_4} \mathbf{M} + J \frac{\partial W}{\partial J} \mathbf{C}^{-1} \quad (13)$$

with scalar stress factors:

$$\frac{\partial W}{\partial I_1} = C_{\text{mat}} a_1 (I_1 - 3) \psi_{\text{exp}} \quad (14)$$

$$\frac{\partial W}{\partial I_4} = C_{\text{mat}} a_2 (I_4 - 1) \psi_{\text{exp}} \quad (15)$$

$$\frac{\partial W}{\partial J} = K_{\text{vol}} (J - 1) \quad (16)$$

where $\psi_{\text{exp}} = \exp [a_1(I_1 - 3)^2 + a_2(I_4 - 1)^2]$.

Interstitial fluid discharge $\mathbf{q}$ relative to the deforming solid matrix follows Darcy’s law:

$$\mathbf{q} = -\mathbf{K}(J, \mathbf{a}) \cdot \nabla p \quad (17)$$

The hydraulic permeability tensor $\mathbf{K}(J, \mathbf{a})$ accounts for void ratio reduction:

$$\mathbf{K}(J, \mathbf{a}) = K_0 \exp \left[M \left(\frac{J - 1}{2} \right) \right] (\mathbf{I} + \beta_f \mathbf{a} \otimes \mathbf{a}) \quad (18)$$

in which $\mathbf{a} = J^{-1/3} \mathbf{F} \mathbf{a}_0 / \|\mathbf{F} \mathbf{a}_0\|$ denotes the current unit fiber vector.

Across the loaded cylindrical wall, the total shear stress balances elastic matrix resistance and the frictional drag generated by radial fluid filtration:

$$\tau_{\text{shear}}(r) = \sigma_{e, r\theta}(r) + \frac{\mu_f}{K_{rr}(J)} q_r(r) \quad (19)$$

3 Variational Mixed Hybrid Finite Element Formulation

Following mixed-hybrid frameworks for coupled porous media [10, 12], the physical domain Ω

is partitioned into disjoint elements Ω_e . Inter-element continuity of the normal fluid flux across interior element boundaries $\Gamma_{\text{int}} = \bigcup(\partial\Omega_e \cap \partial\Omega_m)$ is enforced through an independent Lagrange multiplier $\bar{p} \in H^{1/2}(\Gamma_{\text{int}})$ representing the interface trace pressure [14].

3.1 Weak Formulation

The mixed hybrid variational problem seeks displacements $\mathbf{u} \in \mathcal{V} = [H^1(\Omega)]^3$, fluid filtration velocities $\mathbf{q} \in \mathcal{Q} = H(\text{div}, \Omega)$, element pore pressures $p \in \mathcal{P} = L^2(\Omega)$, and trace pressures $\bar{p} \in \mathcal{M} = L^2(\Gamma_{\text{int}})$ satisfying:

$$\sum_e \left[\int_{\Omega_e} \boldsymbol{\sigma}_e(\mathbf{u}) : \text{sym}(\nabla \mathbf{w}) \, d\Omega - \int_{\Omega_e} p(\nabla \cdot \mathbf{w}) \, d\Omega \right] = \int_{\partial\Omega_t} \mathbf{t}_0 \cdot \mathbf{w} \, d\Gamma \quad (20)$$

$$\sum_e \left[\int_{\Omega_e} \mathbf{K}^{-1} \mathbf{q} \cdot \mathbf{v} \, d\Omega - \int_{\Omega_e} p(\nabla \cdot \mathbf{v}) \, d\Omega + \int_{\partial\Omega_e} \bar{p}(\mathbf{v} \cdot \mathbf{n}) \, d\Gamma \right] = 0 \quad (21)$$

$$\sum_e \left[\int_{\Omega_e} \psi(\nabla \cdot \mathbf{u} + \nabla \cdot \mathbf{q}) \, d\Omega \right] = 0 \quad (22)$$

$$\sum_e \left[\int_{\partial\Omega_e \setminus \partial\Omega_q} \mu(\mathbf{q} \cdot \mathbf{n}) \, d\Gamma \right] = 0 \quad (23)$$

for all test functions $\mathbf{w} \in \mathcal{V}_0$, $\mathbf{v} \in \mathcal{Q}$, $\psi \in \mathcal{P}$, and $\mu \in \mathcal{M}$. Integration by parts on the Darcy velocity divergence transfers spatial derivatives onto test functions, enforcing normal flux continuity across element interfaces via the constraint equation (23).

3.2 Spatial Discretization and Element-Level Static Condensation

Within each element Ω_e , displacements are interpolated via standard Lagrangian shape functions $\mathbf{u} \approx \mathbf{N}_u \mathbf{u}_e$, fluxes via lowest-order Raviart–Thomas vector bases $\mathbf{q} \approx \boldsymbol{\Phi}_q \mathbf{q}_e$ (RT_0), element pressures as piecewise constants $p \approx \psi_p p_e$ (P_0), and trace pressures via inter-element functions $\bar{p} \approx \mu_{\bar{p}} \bar{p}_e$ [10]. Linearizing about the current state

generates the element system:

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{0} & -\mathbf{B}_u^T & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{qq} & -\mathbf{B}_q^T & \mathbf{E}_{\bar{p}}^T \\ -\mathbf{B}_u & -\mathbf{B}_q & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_{\bar{p}} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{u}_e \\ \Delta \mathbf{q}_e \\ \Delta p_e \\ \Delta \bar{p}_e \end{bmatrix} = \begin{bmatrix} \mathbf{R}_u \\ \mathbf{R}_q \\ R_p \\ \mathbf{0} \end{bmatrix} \quad (24)$$

with element submatrices defined by:

$$\mathbf{K}_{uu} = \int_{\Omega_e} \mathbf{B}_L^T \left(\frac{4}{J} \frac{\partial^2 W}{\partial \mathbf{C} \partial \mathbf{C}} \right) \mathbf{B}_L \, d\Omega + \int_{\Omega_e} \mathbf{B}_{NL}^T \boldsymbol{\sigma}_e \mathbf{B}_{NL} \, d\Omega \quad (25)$$

$$\mathbf{C}_{qq} = \int_{\Omega_e} \boldsymbol{\Phi}_q^T \mathbf{K}^{-1}(J, \mathbf{a}) \boldsymbol{\Phi}_q \, d\Omega \quad (26)$$

$$\mathbf{B}_u = \int_{\Omega_e} \psi_p(\nabla \cdot \mathbf{N}_u) \, d\Omega \quad (27)$$

$$\mathbf{B}_q = \int_{\Omega_e} \psi_p(\nabla \cdot \boldsymbol{\Phi}_q) \, d\Omega \quad (28)$$

$$\mathbf{E}_{\bar{p}} = \int_{\partial\Omega_e} \mu_{\bar{p}}(\boldsymbol{\Phi}_q \cdot \mathbf{n}) \, d\Gamma \quad (29)$$

and corresponding residual vectors:

$$\mathbf{R}_u = \int_{\partial\Omega_{e,t}} \mathbf{N}_u^T \mathbf{t}_0 \, d\Gamma - \int_{\Omega_e} \mathbf{B}_L^T \boldsymbol{\sigma}_e \, d\Omega + \mathbf{B}_u^T p_e \quad (30)$$

$$\mathbf{R}_q = -\mathbf{C}_{qq} \mathbf{q}_e + \mathbf{B}_q^T p_e - \mathbf{E}_{\bar{p}}^T \bar{p}_e \quad (31)$$

$$R_p = -\mathbf{B}_u \mathbf{u}_e - \mathbf{B}_q \mathbf{q}_e \quad (32)$$

Because the RT_0 flux basis and P_0 pressure fields are entirely discontinuous across element boundaries, the flux block $\mathbf{C}_{qq}$ is block-diagonal and trivially invertible at the element level. Computing the local Schur complement allows $\Delta \mathbf{q}_e$ and Δp_e to be eliminated prior to assembly:

$$\Delta \mathbf{q}_e = \mathbf{C}_{qq}^{-1} (\mathbf{B}_q^T \Delta p_e - \mathbf{E}_{\bar{p}}^T \Delta \bar{p}_e + \mathbf{R}_q) \quad (33)$$

$$\Delta p_e = (\mathbf{B}_q \mathbf{C}_{qq}^{-1} \mathbf{B}_q^T)^{-1} \left(-\mathbf{B}_u \Delta \mathbf{u}_e - \mathbf{B}_q \mathbf{C}_{qq}^{-1} \mathbf{E}_{\bar{p}}^T \Delta \bar{p}_e + \tilde{R}_p \right) \quad (34)$$

where $\tilde{R}_p = R_p + \mathbf{B}_q \mathbf{C}_{qq}^{-1} \mathbf{R}_q$. Static condensation reduces the global linear system strictly to

the active degrees of freedom $\{\Delta \mathbf{u}, \Delta \bar{p}\}$. Once the global solve converges, localized element fluxes and pressures are calculated accordingly.

4 Exact One-Dimensional Analytical Solution

To construct an exact benchmark for code verification, this section derives the steady axisymmetric deformation of an incompressible porous cylinder ($J = 1$) under finite inflation and axial torsion. Radial kinematics follow from volume preservation:

$$r(R) = \sqrt{R^2 + (r_{\text{in}}^2 - R_{\text{in}}^2)} \quad (35)$$

with the circumferential stretch ratio given by:

$$\lambda_\theta(r) = \frac{r}{R(r)} = \frac{r}{\sqrt{r^2 - (r_{\text{in}}^2 - R_{\text{in}}^2)}} \quad (36)$$

Applying a uniform torsion rate $\frac{\partial \phi}{\partial z} = \frac{\Phi}{L} = \gamma_0$ induces an out-of-plane torsional shear strain $E_{\theta z} = \frac{1}{2} \lambda_\theta^2 \gamma_0 r$.

In the steady limit, radial fluid conservation $\nabla \cdot \mathbf{q} = 0$ reduces to:

$$\frac{1}{r} \frac{d}{dr} (r q_r(r)) = 0 \implies q_r(r) = \frac{Q_0}{2\pi L r} \quad (37)$$

where Q_0 is the total volumetric filtration rate across the wall. Darcy's law relates this flux to the pore pressure gradient:

$$\frac{dp}{dr} = -\frac{Q_0}{2\pi L r K_{rr}(J(r))} \quad (38)$$

Enforcing prescribed pressures at the inner and outer boundaries, $p(r_{\text{in}}) = p_{\text{in}}$ and $p(r_{\text{out}}) = p_{\text{out}}$, direct integration yields:

$$p^{\text{ana}}(r) = p_{\text{in}} - (p_{\text{in}} - p_{\text{out}}) \frac{\int_{r_{\text{in}}}^r \frac{1}{r' K_{rr}(J(r'))} dr'}{\int_{r_{\text{in}}}^{r_{\text{out}}} \frac{1}{r' K_{rr}(J(r'))} dr'} \quad (39)$$

Equilibrium in the radial direction requires:

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \quad (40)$$

which integrates to the radial total stress profile:

$$\sigma_{rr}(r) = -p_{\text{out}} - \int_r^{r_{\text{out}}} \frac{\sigma_{\theta\theta}(r') - \sigma_{rr}(r')}{r'} dr' \quad (41)$$

Combining hyperelastic matrix shear with Darcy drag, the total wall shear stress becomes:

$$\tau_{\text{shear}}^{\text{ana}}(r) = \left[\left(C_{\text{mat}} a_1 \gamma_0 r \lambda_\theta(r) + \frac{\mu_f Q_0}{2\pi L r K_{rr}(J(r))} \right)^2 + \left(\tau_0 \left[\frac{r_{\text{in}}}{r} \right]^2 \right)^2 \right]^{1/2} \quad (42)$$

This closed-form solution serves as an exact reference to quantify discretization error norms and convergence rates without needing boundary approximations or numerical integration.

5 Case Studies and Numerical Results

The numerical simulations are performed on left-ventricular cylinder with reference dimensions $R_{\text{in}} = 2.20$ mm, $R_{\text{out}} = 4.20$ mm and length $L = 10.0$ mm.

Figure 1 summarizes the computational domain, boundary conditions, and the kinematic mapping between the three-dimensional thick-walled cylinder and the one-dimensional axisymmetric radial discretization.

5.1 Case Study 1: Coronary Perfusion Pressure Modulation of Diastolic Compliance

Microvessels make up roughly 15% of myocardial volume. When pressurized by coronary blood flow (P_{perf}), these vessels expand, creating internal fluid tension that stiffens the relaxed ventricular wall during filling—a phenomenon known as the garden-hose effect [6, 7].

To examine this interaction, cavity filling pressure varies across the physiological diastolic range ($p_{\text{in}} \in [0, 30]$ mmHg) while epicardial pressure remains fixed at $p_{\text{out}} = 0$ mmHg. Three coronary perfusion levels are evaluated: hypoperfused ($P_{\text{perf}} = 0$ mmHg), normotensive ($P_{\text{perf}} = 60$ mmHg), and hyperperfused ($P_{\text{perf}} = 100$ mmHg).

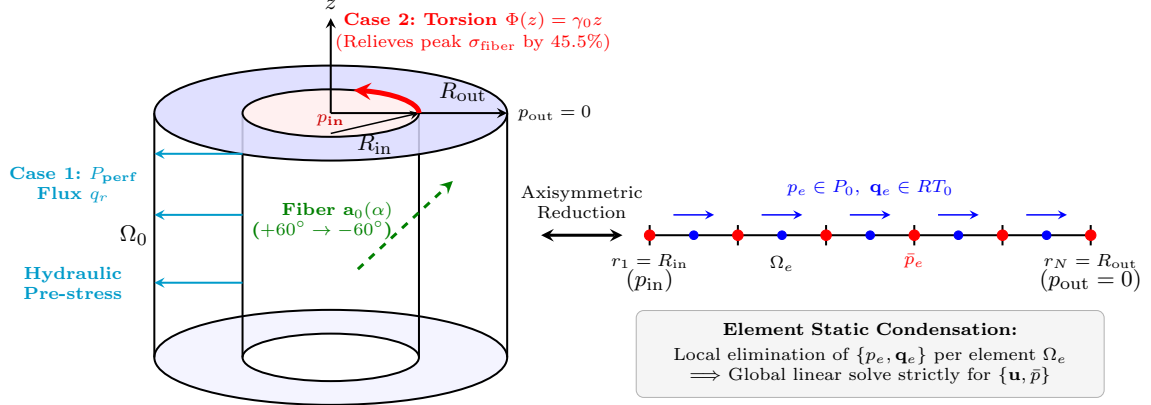


Fig. 1 Computational Model Overview with Physiological Case Mapping: 3D cylindrical tissue geometry subject to combined radial cavity pressure (p_{in}), epicardial drainage ($p_{\text{out}} = 0$), internal perfusion (P_{perf}), and out-of-plane twist (Φ), mapped to a 1D axisymmetric radial MHFEM discretization [7].

Figure 2 compares MHFEM results with the exact analytical benchmark across the normalized radial coordinate $\xi = (r - R_{\text{in}})/(R_{\text{out}} - R_{\text{in}}) \in [0, 1]$. In the hyperperfused state, interstitial fluid pressure develops a midwall peak exceeding 100 mmHg (Fig. 2a). This elevated pore pressure carries a significant fraction of the transmural load, reducing peak solid matrix stress from 10 kPa down to 3 kPa (Fig. 2b). The RT_0 radial filtration flux $q_r(r)$ remains smooth (Fig. 2c).

Microvascular distension shifts the diastolic pressure–volume (P – V) relationship leftward (Fig. 2d). Cavity pressure must overcome ≈ 15 mmHg before substantial volume recruitment occurs, constraining end-diastolic filling volume below 0.20 mL. As documented in Table 2, the MHFEM solutions agree closely with the analytical quadrature profiles, yielding relative L_2 error norms below 0.35% across all three perfusion conditions.

5.2 Case Study 2: Torsional Stress Homogenization

Ventricular myofibers rotate smoothly across the wall, shifting from $\alpha \approx +60^\circ$ at the inner surface (endocardium) to $\alpha \approx -60^\circ$ at the outer surface (epicardium) [8]. During filling, this spiral architecture links wall thinning and lengthening directly to long-axis twist [5]. Constraining this twist stops the wall from sharing the load, concentrating high fiber stress in the inner wall.

This coupling is evaluated at $p_{\text{in}} = 120$ mmHg with $P_{\text{perf}} = 60$ mmHg, considering two kinematic constraints: torsionally fixed ($\Phi = 0^\circ$) and unconstrained torsion ($\Phi = 15^\circ$).

Figure 3 shows the resulting distributions across the normalized wall thickness $\xi \in [0, 1]$. When torsion is prevented, fiber stress concentrates heavily at the inner wall, peaking at $\sigma_{\text{fiber}} = 57.20$ kPa (Fig. 3a). Allowing the cylinder to twist freely activates shear stress along the fibers, which reaches 8.60 kPa near the midwall (Fig. 3b). This twist shifts mechanical load toward the outer wall, lowering the peak inner-wall fiber stress to 31.20 kPa—a 45.5% reduction.

This load redistribution also helps fluid drain through the wall: because the inner wall is compressed less, fluid meets less resistance. As a result, peak pore pressure drops from roughly 128 mmHg to 122 mmHg (Fig. 3c), while outward flow $q_r(r)$ increases across the entire wall thickness (Fig. 3d). As listed in Table 3, relative L_2 errors between the numerical and analytical solutions stay below 0.35% across all cases.

5.3 Case Study 3: Concentric Hypertrophy under High Pressure

Long-term high blood pressure and aortic valve stenosis cause the left ventricle to thicken inward, often doubling wall thickness ($R_{\text{in}} = 2.00$ mm, $R_{\text{out}} = 5.00$ mm, $\Delta R = 3.00$ mm) to withstand high pumping pressures ($p_{\text{in}} \geq 180$ mmHg). Under these conditions, the thickened wall twists more

Case Study 1 Verification: Coronary Perfusion Pressure & Garden-Hose Effect

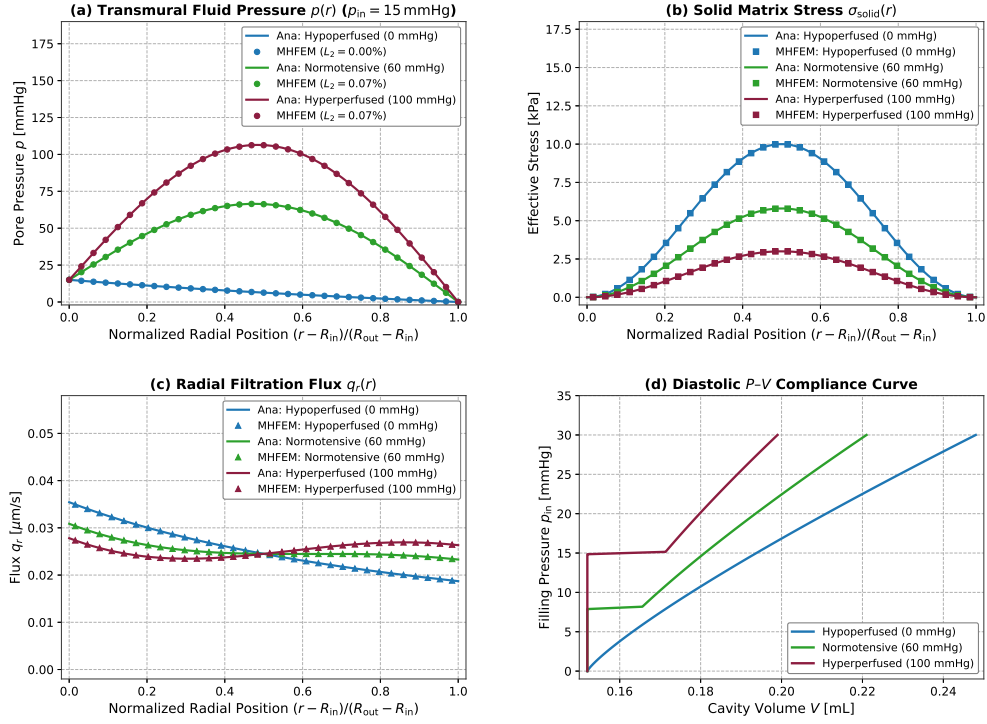


Fig. 2 Case Study 1 Verification (Garden Hose Effect): Discrete MHFEM numerical predictions compared against exact analytical quadrature solutions for (a) transmural interstitial fluid pressure $p(r)$, (b) solid matrix principal stress $\sigma_{solid}(r)$, (c) transmural fluid flux $q_r(r)$, and (d) diastolic pressure–volume (P – V) compliance curves under varying coronary perfusion pressures ($P_{perf} \in \{0, 60, 100\}$ mmHg).

Case Study 2 Verification: Transmural Fiber Orientation & Torsional Stress Homogenization

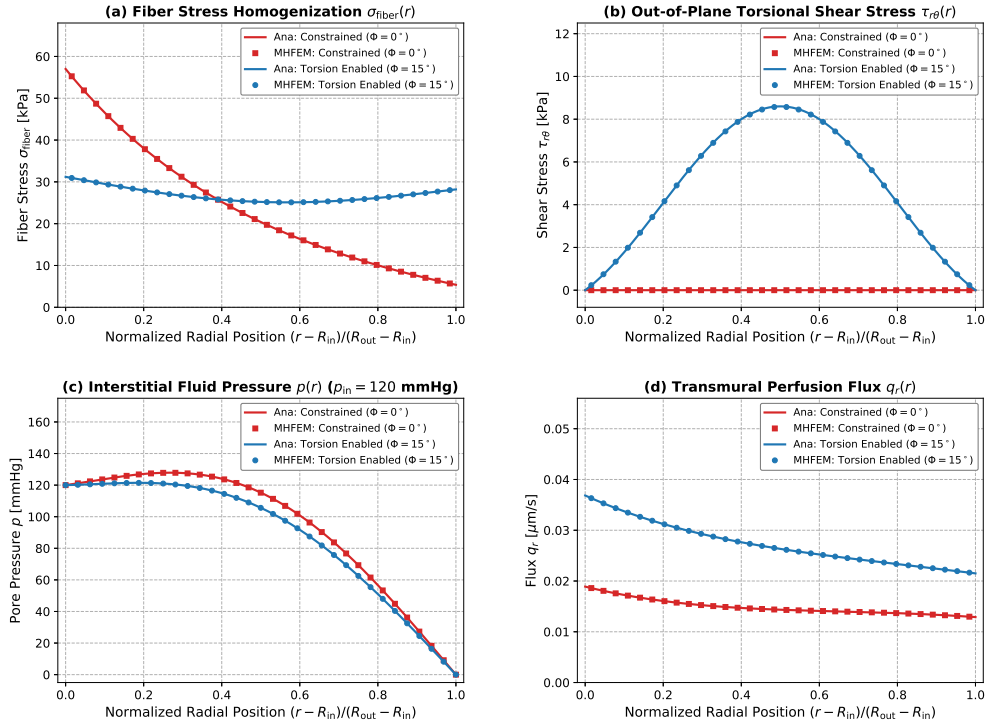


Fig. 3 Case Study 2 Verification (Torsional Stress Homogenization): Comparison of constrained ($\Phi = 0^\circ$) versus free torsional twist ($\Phi = 15^\circ$) across a transmural fiber helix ($\alpha \in [+60^\circ, -60^\circ]$). Overlaid MHFEM and analytical profiles illustrate (a) fiber stress homogenization $\sigma_{fiber}(r)$, (b) out-of-plane torsional shear stress $\tau_{r\theta}(r)$, (c) fluid pressure $p(r)$, and (d) transmural perfusion flux $q_r(r)$.

during systole ($\Phi = 24^\circ$) and relaxes more slowly during filling. Because the wall is thicker, fluid takes longer to filter through, which traps more fluid near the inner surface.

This condition is modeled on a 2D axisymmetric cross-section ($r \in [R_{\text{in}}, R_{\text{out}}]$, $z \in [0, L]$) with $p_{\text{in}} = 180 \text{ mmHg}$, $P_{\text{perf}} = 90 \text{ mmHg}$, and $\Phi = 24^\circ$. The stabilized MHFEM scheme is compared directly with standard displacement-based elements (Q_1/P_1 Galerkin FEM).

Figure 4 shows the steady-state poromechanical results. In the 2D pressure map (Fig. 4a), high pore pressures over 150 mmHg reach past the mid-wall ($\xi_r \geq 0.50$), indicating severe fluid retention that persistently squeezes the inner-wall microvessels. Early in the loading step ($t = 0.02 \text{ s}$, Fig. 4b), standard displacement FEM suffers from pressure oscillations near the inner wall, leading to an L_2 error of 12.75%. MHFEM enforces exact local mass conservation, producing a smooth, physical pressure curve with an L_2 error of just 0.01% (Table 4).

Figure 4c shows the corresponding fiber stress distribution $\sigma_{\text{fiber}}(r)$. Combining the 180 mmHg cavity pressure with 24° of twist drives peak inner-wall fiber stress up to 88.50 kPa (compared to 31.20 kPa in the normal case). Standard displacement elements suffer from volumetric locking near the midwall ($\xi \in [0.4, 0.6]$), which disrupts load transfer across the wall and severely underestimates outer-wall fiber stress (12.0 kPa versus 28.57 kPa predicted by MHFEM).

6 Discussion and Conclusions

This paper presents a stabilized mixed-hybrid finite element method (MHFEM) for porous media undergoing large deformations and twist. By combining lowest-order Raviart–Thomas velocity elements (RT_0) with interface pressure multipliers, the formulation guarantees exact local mass conservation and prevents volumetric locking. By applying element-level reduction, the final formulation solves only for nodal displacements and interface pressures, retaining the computational efficiency of standard displacement solvers while eliminating their numerical instabilities. A companion closed-form 1D solution confirms that the model accurately captures the coupling between hyperelastic tissue shear and fluid filtration under large twists.

Applied to ventricular biomechanics, the results reveal two primary mechanisms. First, physiological twisting during filling shifts mechanical load away from the inner wall, preventing fluid buildup and protecting microvascular blood flow. Second, when the wall thickens under high pumping pressures, strong twisting combined with a thicker wall slows drainage, trapping fluid under high pressure. This shows that delayed or incomplete untwisting is not just an abnormal motion, but a mechanical obstacle that sustains high pore pressure and slows fluid drainage across the inner wall.

While the cylindrical geometry provided an exact benchmark for mathematical verification, extending the stabilized MHFEM formulation to transient drainage, active muscle contraction, and 3D ventricular geometries is the natural next step.

Declarations

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Competing Interests

The author has no competing interests to declare that are relevant to the content of this article.

Data and Code Availability

All mathematical formulations, constitutive relations, and analytical benchmarks are fully documented within this paper. Numerical simulation datasets are available from the author upon reasonable request.

Author Contributions

K.M. conceived the study, formulated the theoretical framework, derived the analytical solutions, developed the MHFEM implementation, performed the numerical analyses, and wrote the manuscript.

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Case Study 3: Concentric Left Ventricular Hypertrophy (LVH) with Severe Pressure Afterload & Delayed Untwisting

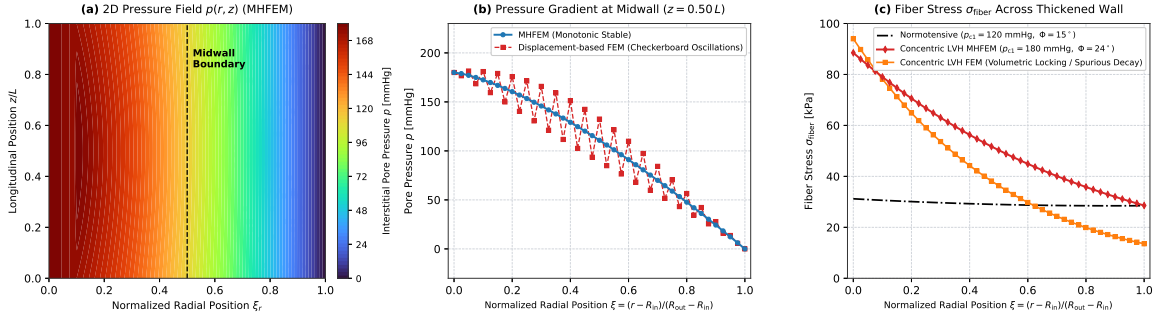


Fig. 4 Case Study 3: Concentric Left Ventricular Hypertrophy (LVH) with Severe Pressure Afterload and Delayed Untwisting. Numerical comparison between stabilized MHFEM and standard displacement FEM across a doubled wall thickness ($\Delta R = 3.0\text{ mm}$): (a) 2D axisymmetric interstitial pore pressure field $p(r, z)$ showing subendocardial fluid stagnation; (b) early-time midwall pressure cut ($z = 0.50 L$, $t = 0.02\text{ s}$) contrasting standard FEM spurious checkerboard oscillations ($L_2 = 12.75\%$) against monotonic MHFEM stability ($L_2 = 0.01\%$); and (c) transmural fiber stress distribution $\sigma_{\text{fiber}}(r)$ illustrating an endocardial stress surge alongside standard FEM volumetric locking.

Table 2 Discrete relative L_2 error norms (%) between discrete MHFEM numerical predictions and exact analytical benchmark solutions for Case Study 1 under varying coronary perfusion states ($N_e = 16$ elements).

Perfusion State	Pore Pressure $p(r)$	Fluid Flux $q_r(r)$	Solid Stress $\sigma_{\text{solid}}(r)$
Hypoperfused ($P_{\text{perf}} = 0\text{ mmHg}$)	$< 0.01\%$	$< 0.01\%$	$< 0.01\%$
Normotensive ($P_{\text{perf}} = 60\text{ mmHg}$)	0.27%	0.06%	$< 0.01\%$
Hyperperfused ($P_{\text{perf}} = 100\text{ mmHg}$)	0.29%	0.10%	$< 0.01\%$

Table 3 Discrete relative L_2 error norms (%) between discrete MHFEM numerical predictions and exact analytical benchmark solutions for Case Study 2 ($N_e = 32$ elements).

Kinematic State	$\sigma_{\text{fiber}}(r)$ [kPa]	$\tau_{r\theta}(r)$ [kPa]	$p(r)$ [mmHg]	$q_r(r)$ [$\mu\text{m/s}$]
Constrained ($\Phi = 0^\circ$)	0.05%	$< 0.01\%$	0.03%	0.01%
Torsion Enabled ($\Phi = 15^\circ$)	0.01%	0.17%	0.03%	$< 0.01\%$

Table 4 Quantitative comparison and relative L_2 error norms between displacement-based FEM and stabilized MHFEM for Case Study 3.

Metric / Quantity	Displacement-based FEM	Stabilized MHFEM ($\text{RT}_0 \times P_0$)
Pressure L_2 error norm (p)	12.75%	0.01%
Subendocardial peak σ_{fiber}	94.0 kPa (Overestimated / Locking)	88.50 kPa
Outer epicardial σ_{fiber} ($\xi = 1.0$)	12.0 kPa (Spurious Decay)	28.57 kPa
Spatial pressure behavior	Checkerboard oscillations / Locking	Monotonically stable / Lock-free

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