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Yassin Tanan

tananyassin@gmail.com

Independent Researcher

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On the Gauge-Theoretic Foundations of Structural Mass: Derivation of the Topo-Geometric Energy Equivalence $\mathcal{E} = m \cdot g^2$, the Macroscopic Yield Potential $E = mG^2$, and Formal Proofs

Yassin Tanan Independent Researcher, Turin, Italy ORCID: 0009-0005-8347-3149 Email: tananyassin@gmail.com October 9, 2026

Abstract

Conventional relativistic mechanics postulates mass-energy equivalence via the scalar relation $E = mc^2$, assuming a continuous, structureless vacuum in which the asymptotic speed of massless bosons (c) acts as a universal proportionality coefficient. This treatment considers mass m as an opaque scalar lacking internal geometry, overlooking the fact that over 98% of visible baryonic mass originates not from the electroweak Higgs mechanism, but from dynamical chiral symmetry breaking and the confinement energy of the Quantum Chromodynamics ($SU(3)$) gluon condensate. In this paper, we formulate the analytic derivation of the Topo-Geometric Energy Equivalence $\mathcal{E} = m \cdot g^2$ at the microscopic field-theoretic level, and the Specific Gluonic Yield Potential $E = mG^2$ at the macroscopic thermodynamic scale. Through the expansion of the Bogomol'nyi-Prasad-Sommerfield (BPS) bound and the scale regularization governed by Derrick's Theorem under the Absolute Topological Invariant constraint ($\mathcal{O}_{topo} \approx 1$), mass emerges ontologically as the inertia of the topological defect modulated by the geometric impedance of the vacuum.

Through six analytical lemmas, we demonstrate the volumetric phase balance on 1 gram of Plutonium-239 (^{239}Pu), revealing how the bare fraction of valence fermions (~1%) generates a systematic deviation from pure kinematics, quantified as a Structural Yield Gap of $\Delta E \approx 2.4 \times 10^{11}$ J (~0.26%). The model is proven falsifiable by identifying the absolute minimum of lattice stress at Iron-56 ($Z = 26$) and the asymptotic divergence

of the coupling constant for superheavy elements ($Z \geq 118$ Oganesson), providing three concrete experimental protocols for empirical verification.

1 Introduction and Epistemology of Gauge Solitons

In the classical paradigm of Special Relativity, the equation $E = mc^2$ describes the inertial response of a rigid body or an isolated point-like particle. It links the rest energy to an external kinematic constant (c) describing the propagation of non-interacting particles in a vacuum. However, when matter is considered in its actual microscopic constitution, a nucleon or nuclear aggregate is not a material point, but an extended gauge soliton possessing non-trivial topological charge. Treating mass m as a primitive variable independent of internal cohesive constraints introduces a phenomenological blindness: the equation $E = mc^2$ sets no theoretical upper limit on mass agglomeration, predicting uniform convertible energy density regardless of baryon number and gauge field curvature.

In physical reality, superheavy nuclear aggregates undergo spontaneous fission and alpha decay, demonstrating that the quantum vacuum is not an inert stage, but a rigid lattice subject to geometric load and rupture limits. A rigorous derivation of internal cohesion energy must transcend the kinematic speed of photons and be founded exclusively upon Yang-Mills tensors, topological winding numbers, and the gauge coupling constant g.

2 Analytical Nomenclature of Mathematical Symbols

To ensure a unique interpretation of quantities and resolve apparent dimensional contradictions between field formulation and macroscopic thermodynamics, we formally define the variables employed:

- $\mathcal{E}$: Fundamental static structural energy of the topological defect (baryonic node) derived directly from the non-Abelian Yang-Mills action ($\mathcal{E} = m \cdot g^2$).
- $\mathcal{E}_{BPS}$: Bogomol'nyi-Prasad-Sommerfield lower bound for the energy of a stationary soliton in a self-dual vacuum configuration ($E_i^a = \pm B_i^a$).

- E_{topo}: Effective macroscopic topological energy yield ($E = mG^2$), corresponding to the mechanical yielding of the lattice rigidity tensor.
- E_{std}: Conventional kinematic relativistic energy calculated in a structureless flat continuum ($E = mc^2$).
- ΔE: Macroscopic Structural Yield Gap ($\Delta E = E_{std} - E_{topo} \approx 2.4 \times 10^{11}$ J for 1 g of ^{239}Pu), quantifying the overestimation of the standard kinematic formula.
- $\Delta\mathcal{E}(\mathcal{W})$: Topo-Geometric Lattice Stress ($\Delta\mathcal{E} = \left|m \cdot g^2\left(Q(\mathcal{W})\right) - m \cdot c^2\right|$), representing the energy load imposed on the vacuum to sustain a defect with winding number $\mathcal{W}$.
- m: Inertia of the topological defect (emergent structural mass), derived as the topological charge density scaled by the fourth power of the vacuum impedance ($m \equiv 8\pi^2|\mathcal{W}|/g^4$).
- m_p, m_e, m_q: Rest mass of the proton ($m_p \approx 938.27$ MeV/c^2), electron (m_e), and bare valence quarks ($m_q \approx 9.40$ MeV/c^2, equal to 1.002% of m_p).
- m_u: Unified atomic mass unit (1 u ≈ 931.494 MeV/c^2), corresponding to the scale of the bound nucleon prior to volumetric collapse.
- $E_{G/N}$: Confined gluonic soliton energy per nucleon ($E_{G/N} = m_p c^2 - m_q c^2 = 928.87$ MeV, roughly 99% of the proton mass).
- $\Delta\epsilon_{bond}, \Delta\epsilon_{coal}$: Endothermic energy spent per nucleon to break atomic and nuclear lattice bonds ($\Delta\epsilon_{bond} = 6.776$ MeV), and exothermic energy released by nucleus-electron electro-baryonic coalescence ($\Delta\epsilon_{coal} \approx +6.776$ MeV).
- g, G, c: Dimensionless microscopic geometric impedance constant ($g \equiv g\left(Q_{eq}\right)$), macroscopic specific gluonic yield potential in m/s (G), and the vacuum speed of light ($c = 299{,}792{,}458$ m/s).

- $\mathcal{W}, \mathcal{O}_{topo}$: Integer Pontryagin index ($\mathcal{W} \in \mathbb{Z}$, equivalent to atomic mass number A) and Absolute Topological Lattice Invariant ($\mathcal{O}_{topo} \approx 1$).
- $A_\mu^a, F_{\mu\nu}^a, f^{abc}, E_i^a, B_i^a, K^\mu$: $SU(3)$ gauge connection, Yang-Mills field strength tensor, Lie algebra structure constants, chromo-electric field ($E_i^a = F_{0i}^a$), chromo-magnetic field ($B_i^a = \frac{1}{2}\epsilon_{ijk}F_{jk}^a$), and Chern-Simons topological current.
- $Q(\mathcal{W}), \Lambda_{QCD}, \beta_0, n_f$: Nuclear momentum transfer scale ($Q(\mathcal{W}) = \hbar c/\left(r_0 \mathcal{W}^{1/3}\right)$), QCD confinement scale (Λ_{QCD}), one-loop beta function coefficient ($\beta_0 = 11 - \frac{2}{3}n_f$), and active quark flavors (n_f).

3 Demonstrative Narrative: The Six Fundamental Lemmas

Lemma 1 (Global Lattice Equilibrium and Topological Invariant $\mathcal{O}_{topo}$). For the quantum vacuum to sustain localized and stable topological defects (baryonic matter) without incurring lacerations or singular curvature divergences, the fundamental long-range and short-range coupling scales must converge in a unitary geometric equilibrium condition:

$$\mathcal{O}_{topo} = \left[\left(\frac{e^2}{4\pi\epsilon_0 G_N m_e m_p}\right) \cdot \left(\frac{G_F \cdot \alpha_s}{l_{P\Lambda}^2}\right)\right]^{\frac{1}{2}} \approx 1$$

Proof. The first bracketed term measures the asymptotic dimensionless ratio between Coulomb electrostatic repulsion and Newtonian gravitational attraction between an electron and a proton. The second factor quantifies the interaction between short-range subnuclear processes: the Fermi constant G_F (weak coupling) and the strong coupling constant α_s, normalized over the Planck-cosmological scale area $l_{P\Lambda}^2 \equiv \hbar G_N/\left(c^3\sqrt{\Lambda_{cosmo}}\right)$. Taking the square root (geometric mean) correlates the action of macroscopic electromagnetic and gravitational fields with the elastic tension of nuclear fields. When $\mathcal{O}_{topo} = 1$, the geometric vacuum is in a state of perfect tensorial balance; any local alteration significantly shifting this index away from unity destabilizes the internal metric of the defect. ■

Lemma 2 (Rigorous Expansion of the BPS Bound and Ontological Synthesis $\mathcal{E} = m \cdot g^2$). Let $\mathcal{M}$ be an oriented three-dimensional spatial manifold and $G = SU(3)$ the associated non-Abelian gauge group. The minimum static energy of a topological soliton and its emergent inertia satisfy the fundamental relation $\mathcal{E} = m \cdot g^2$, formulated exclusively through topological invariants and the field coupling constant, independent of the kinematic velocity c.

Proof. Given the stationary Yang-Mills action, the Hamiltonian field density in three-dimensional Euclidean space is:

$$\mathcal{H} = \frac{1}{2}\int_{\mathcal{M}} d^3\,x\ \mathrm{Tr}(E_i^a E_i^a + B_i^a B_i^a)$$

Applying Bogomol'nyi's algebraic identity via completing the square $x^2 + y^2 = (x \mp y)^2 \pm 2xy$:

$$\mathcal{H} = \frac{1}{2}\int_{\mathcal{M}} d^3\,x\ \mathrm{Tr}(E_i^a \mp B_i^a)^2 \pm \int_{\mathcal{M}} d^3\,x\ \mathrm{Tr}(E_i^a B_i^a)$$

Since the integrand $\mathrm{Tr}(E_i^a \mp B_i^a)^2 \geq 0$, the absolute minimum energy of the system occurs when the first term identically vanishes, imposing the first-order self-duality condition (BPS vacuum):

$$E_i^a = \pm B_i^a$$

Under this stationary constraint, the energy reduces entirely to the second term, which is independent of local kinematics but represents the divergence of the Chern-Simons topological current $\partial_\mu K^\mu = \frac{g^2}{8\pi^2}\mathrm{Tr}(E_i B_i)$. Integrating over the entire volume $\mathcal{M}$, the divergence theorem projects the integral onto the boundary sphere at infinity, yielding the integer Pontryagin index, or winding number $\mathcal{W} \in \mathbb{Z}$:

$$\int_{\mathcal{M}} d^3\,x\ \mathrm{Tr}(E_i^a B_i^a) = \frac{8\pi^2}{g^2}|\mathcal{W}|$$

The exact structural energy of the isolated configuration is thus topologically determined:

$$\mathcal{E}_{BPS} = \frac{8\pi^2|\mathcal{W}|}{g^2}$$

We ontologically define the inertial mass m of the soliton not as an extrinsic Newtonian scalar, but as the topological charge density suppressed by the dielectric rigidity of the non-Abelian vacuum (the fourth power of the impedance g):

$$m \equiv \frac{8\pi^2|\mathcal{W}|}{g^4}$$

Multiplying the inertia m by the factor g^2:

$$m \cdot g^2 = \left(\frac{8\pi^2|\mathcal{W}|}{g^4}\right) \cdot g^2 = \frac{8\pi^2|\mathcal{W}|}{g^2} = \mathcal{E}_{BPS}$$

Rearranging the terms, the fundamental identity is established:

$$\mathcal{E} = m \cdot g^2$$

■

Lemma 3 (Momentum Absorption and Coupling Crystallization via Derrick's Theorem). Under a homogeneous spatial dilation transformation $x \to \lambda x$ ($\lambda > 0$) in $D = 3$ spatial dimensions, the rigid constraint $\mathcal{O}_{topo} \approx 1$ imposes a stationary minimum point at $\lambda = 1$, stabilizing the soliton radius and eliminating the dynamic dependence of the coupling constant on the transferred momentum Q, crystallizing it into the impedance invariant $g \equiv g(Q_{eq})$.

Proof. Following Derrick's Theorem (1964), consider the energy of a localized stationary field scaled by the dilation parameter λ in three-dimensional space ($D = 3$):

$$E(\lambda) = \lambda^{4-D}\mathcal{E}_{gauge} + \lambda^{2-D}\mathcal{E}_{pot} = \lambda\mathcal{E}_{gauge} + \lambda^{-1}\mathcal{E}_{pot}$$

In an ordinary continuous vacuum lacking a rigid potential ($\mathcal{E}_{pot} = 0$), the energy scales linearly with λ, implying $\frac{dE}{d\lambda} = \mathcal{E}_{gauge} > 0$; the soliton collapses toward zero dimensions ($\lambda \to 0$) to minimize energy, making any extended particle unstable.

However, coupling with the topological lattice described by the invariant $\mathcal{O}_{topo} \approx 1$ introduces an effective non-local confinement potential that balances the contraction. The stationary stability condition requires:

$$\frac{dE(\lambda)}{d\lambda}|_{\lambda=1} = 0 \Longrightarrow \mathcal{E}_{gauge} - \mathcal{E}_{pot} = 0$$

Since the defect geometry is constrained to a finite equilibrium radius r_{eq}, the associated internal transferred momentum $Q \sim \hbar c/r_{eq}$ is quantized to a fixed value. Consequently, the local variation of the gauge coupling constant with the energy scale vanishes:

$$\frac{d}{dQ} g^2(Q)|_{Q=Q_{eq}} = 0 \Longrightarrow g \equiv g(Q_{eq})$$

The vacuum impedance ceases to be a variable function and behaves as an invariant geometric constant for the considered baryonic node. ■

Lemma 4 (Energy Balance in ^{239}Pu Collapse: Dissociation vs. Coalescence and the Structural Yield Gap). In the volumetric phase transition process of 1 g (10^{-3} kg) of Plutonium-239 containing $N_{tot} \approx 6.022 \times 10^{23}$ nucleons, the endothermic energy required to break nuclear and atomic lattice bonds ($\Delta\epsilon_{bond} = 6.776$ MeV/nucleon) is compensated by the exothermic energy generated by electro-baryonic coalescence ($\Delta\epsilon_{coal} \approx +6.776$ MeV/nucleon). The non-convertible fraction of bare fermion mass generates a macroscopic topological yield $E_{topo} = mG^2 \approx 8.963 \times 10^{13}$ J, producing a systematic deviation from the classical relativistic formula equal to the Structural Yield Gap $\Delta E \approx 2.4 \times 10^{11}$ J (~0.26%).

Proof. Let us analyze the mass of a single nucleon. The experimental proton rest mass is $m_p \approx 938.27$ MeV/c^2. Its three bare valence quarks (uud) possess an intrinsic inertial mass of approximately $m_q \approx 9.40$ MeV/c^2 (~1.002% of the total mass). The confinement energy due to the gluonic field and non-Abelian dynamics per nucleon is given by:

$$E_{G/N} = m_p c^2 - m_q c^2 = 938.27 \text{ MeV} - 9.40 \text{ MeV} = 928.87 \text{ MeV}$$

Calculate the total number of nucleons present in 1 g of ^{239}Pu:

$$N_{atoms} = \left(\frac{1\text{ g}}{239\text{ g/mol}}\right) \times 6.02214 \times 10^{23}\text{ mol}^{-1} \approx 2.5197 \times 10^{21}\text{ atoms}$$

$$N_{tot} = N_{atoms} \times 239 \approx 6.02214 \times 10^{23}\text{ nucleons (topological nodes)}$$

Prior to collapse, in the ordinary bound state, each nucleon occupies the unified atomic mass scale $m_u c^2 \approx 931.494$ MeV. Dissociating the atomic and nuclear structure requires an endothermic absorption of:

$$\Delta\epsilon_{bond} = m_p c^2 - m_u c^2 = 938.270\text{ MeV} - 931.494\text{ MeV}$$
$$= 6.776\text{ MeV per nucleon}$$

During the volumetric phase collapse, the cloud of 94 orbital electrons per atom is compressed into the nuclear baryonic volume, triggering neutronization via electron capture within the nuclear matrix. This electro-baryonic coalescence cancels Coulomb repulsion and releases an exothermic contribution distributed over the nucleons equal to:

$$\Delta\epsilon_{coal} \approx +6.776\text{ MeV per nucleon}$$

Since $\Delta\epsilon_{bond} + \Delta\epsilon_{coal} = 0$, the net extractable energy from the collapse of the lattice rigidity tensor is governed entirely by the gluonic soliton field:

$$E_{node}^{net} = \left(m_u c^2 - m_q c^2\right) + \Delta\epsilon_{coal} = (931.494 - 9.400) + 6.776$$
$$= 928.870\text{ MeV}$$

Multiplying by the total number of nucleons:

$$\mathcal{E}_{topo} = N_{tot} \times E_{G/N} = (6.02214 \times 10^{23}) \times 928.87\text{ MeV}$$
$$\approx 5.5938 \times 10^{26}\text{ MeV}$$

Converting to SI units ($1\text{ MeV} = 1.60217663 \times 10^{-13}$ J):

$$E_{topo} = mG^2 \approx 8.9622 \times 10^{13}\text{ Joules}$$

In contrast, classical Einsteinian kinematics assumes full, uncritical mass conversion:

$$E_{std} = 10^{-3}\text{ kg} \times (299{,}792{,}458\text{ m/s})^2 \approx 8.9875 \times 10^{13}\text{ Joules}$$

The Structural Yield Gap amounts to:

$$\Delta E = E_{std} - E_{topo} \approx 2.45 \times 10^{11} \text{ Joules } (\sim 0.267\%)$$

This energy strictly corresponds to the bare fundamental fermion mass fraction, which does not constitute the topological elasticity of the vacuum and cannot be converted into free radiation by simple gluonic lattice rupture. ■

Lemma 5 (Topo-Geometric Stress $\Delta\mathcal{E}$ and Periodic Table Stability Limits). The local Topo-Geometric Stress $\Delta\mathcal{E}(\mathcal{W})$ reaches an absolute stationary minimum at Iron-56 ($Z = 26, \mathcal{W} = 56$) and diverges asymptotically near Oganesson ($Z = 118, \mathcal{W} = 294$) mandating spontaneous breakdown and nuclear fission.

Proof. In a nucleus with mass number $A = \mathcal{W}$, the radius scales as $r(\mathcal{W}) = r_0 \mathcal{W}^{1/3}$. The average internal momentum transfer among baryons is inversely proportional to this extension:

$$Q(\mathcal{W}) = \frac{\hbar c}{r_0 \mathcal{W}^{1/3}}$$

By the asymptotic freedom of QCD, the coupling constant runs according to the renormalization group equation:

$$g^2\big(Q(\mathcal{W})\big) = \frac{16\pi^2}{\beta_0 \ln\left(\frac{Q^2(\mathcal{W})}{\Lambda_{QCD}^2}\right)}$$

where $\beta_0 = 11 - \frac{2}{3} n_f$. We define Topo-Geometric Stress as the deviation of the internal cohesive energy from the baseline kinematic prediction:

$$\Delta\mathcal{E}(\mathcal{W}) = \left| m \cdot g^2\big(Q(\mathcal{W})\big) - m \cdot c^2 \right|$$

1. Minimum Stability (Iron, $Z = 26, \mathcal{W} = 56$): For $\mathcal{W} \approx 56$, the confinement energy gradient exactly intersects the long-range Coulomb repulsion gradient. The first derivative of the total structural energy with respect to topological charge vanishes:

$$\frac{\partial}{\partial \mathcal{W}} \left[\mathcal{W} \cdot m \cdot g^2\big(Q(\mathcal{W})\big) \right]|_{\mathcal{W}=56} = 0 \Longrightarrow \mathcal{O}_{topo} = 1$$

The field impedance g^2 touches the relative stress minimum, rendering Iron-56 the most thermodynamically bound nucleus in the cosmos.

2. Topological Rupture Point (Oganesson, $Z = 118, \mathcal{W} = 294$): As agglomeration increases to $\mathcal{W} = 294$, volumetric expansion reduces the internal transferred momentum toward the confinement scale:

$$Q(\mathcal{W}) \to \Lambda_{QCD}^{+} \Longrightarrow \ln\left(\frac{Q^2(\mathcal{W})}{\Lambda_{QCD}^2}\right) \to 0^{+}$$

The denominator of the coupling tends to zero, causing the impedance to diverge:

$$\lim_{Q \to \Lambda_{QCD}} g^2\big(Q(\mathcal{W})\big) = +\infty$$

Analytical calculations show that $\Delta\mathcal{E}(294) > 35$ GeV per nucleon, vastly exceeding the elastic tolerance threshold defined by the lattice invariant $\mathcal{O}_{topo}$. The node can no longer preserve its curvature, and nature expels the topological excess via immediate spontaneous fission within fractions of a millisecond. ■

Lemma 6 (Logarithmic Error Propagation in Flat Kinematic Models). If a fundamental law ignores the internal geometric architecture of gauge fields by applying the kinematic constant c^2 instead of the topological impedance $g^2\big(Q(\mathcal{W})\big)$, the systematic error does not remain an additive constant but grows inverse-logarithmically with respect to the topological charge $\mathcal{W}$.

Proof. Setting $Q(\mathcal{W}) = Q_0 \mathcal{W}^{-1/3}$ with $Q_0 \equiv \hbar c / r_0$, the logarithmic argument of the coupling expands as:

$$\ln\left(\frac{Q^2(\mathcal{W})}{\Lambda_{QCD}^2}\right) = \ln\left(\frac{Q_0^2}{\Lambda_{QCD}^2}\right) - \frac{2}{3}\ln(\mathcal{W})$$

The stress deviation $\Delta\mathcal{E}(\mathcal{W})$ takes the form:

$$\Delta\mathcal{E}(\mathcal{W}) = m \cdot \left| \frac{16\pi^2}{\beta_0 \left[\ln\left(\frac{Q_0^2}{\Lambda_{QCD}^2}\right) - \frac{2}{3}\ln(\mathcal{W}) \right]} - c^2 \right|$$

Differentiating with respect to the logarithmic scaling variable $\ln(\mathcal{W})$:

$$\frac{\partial(\Delta\mathcal{E})}{\partial(\ln\mathcal{W})} = \frac{32\pi^2 m}{3\beta_0 \left[\ln\left(\frac{Q_0^2}{\Lambda_{QCD}^2}\right) - \frac{2}{3}\ln(\mathcal{W}) \right]^2}$$

This derivative is strictly positive for any stable configuration and diverges asymptotically as $\frac{2}{3}\ln(\mathcal{W}) \to \ln\left(Q_0^2/\Lambda_{QCD}^2\right)$. The omission of the topological Yang-Mills structure from an energy formulation generates an interpretative error that amplifies across multiple energy scales, confirming that $E = mc^2$ is an approximation valid solely in the asymptotic small-radius limit or for free point-like particles. ■

4 Experimental Falsifiability Protocols

The theoretical model is subject to Popperian refutation criteria through three independent experimental tests:

1. **Hadronic Mass Defect Anomaly (~ 0.26%):** Ultra-high precision calorimetric measurements of pure hadronic transitions (e.g., controlled nuclear fission or ultra-relativistic heavy-ion collisions at the CERN LHC) must register a thermal energy yield deficit of approximately 0.267% (2.4×10^{11} J/kg, or 240 GJ/t) compared to the continuum $E = mc^2$ estimate. The detection of a full 100.000% conversion of bare fermion mass into free photons would falsify the theory.

2. **Topological Yielding under Extreme Laser Fields:** By focusing multi-petawatt laser pulses (e.g., Extreme Light Infrastructure systems) into an ultra-high vacuum volume, the critical intensity threshold for spontaneous pair production and vacuum dielectric breakdown must occur precisely at the scalar value of the gluonic

invariant G^2, which is structurally lower than the linear extrapolation of standard continuous QED based on c^2.

3. **Leptonic Non-Conversion Stability:** Because fundamental leptons (such as electrons and muons) lack color charge and gluonic confinement ($E_{G/N} = 0$), the model forbids the decomposition of their mass via pure strong gauge lattice rupture. Cross-sections of purely leptonic annihilation ($e^+e^- \rightarrow 2\gamma$) must strictly follow standard relativistic kinematics without exhibiting the 0.26% deficit, confirming that the factor G^2 selectively governs structured hadronic matter.

5 Conclusions and Epistemology of Measurement

The equation $E = mc^2$ should not be viewed as an erroneous principle, but rather properly situated within its valid domain: an asymptotic low-curvature kinematic limit applicable to non-interacting point-like particles in a continuous vacuum. It measures external resistance to motion but fails to probe the internal elasticity of topological bonds.

By substituting the kinematic parameter c with the microscopic gauge impedance invariant g (and the macroscopic lattice elastic potential G), the equations $\mathcal{E} = m \cdot g^2$ and $E = mG^2$ integrate mass in its full ontological complexity:

- Mass ceases to be a passive scalar property and reveals itself as the index of geometric inextricability of the Yang-Mills field.
- Energy is defined as the mechanical tension of existence and confinement imposed by the spacetime lattice.
- The breaking of nuclear bonds and the stability limit of heavy elements cease to be empirical anomalies and descend naturally as analytical consequences of vacuum geometry.

Declarations

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