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Cosmic Rays Propagation in Plasmas with Ionization Effects in Astrophysical Settings

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ABSTRACT

Cosmic rays are a significant non-thermal component of astrophysical plasmas and can substantially affect, how magnetized media behave and exchange energy, and become ionized. Cosmic rays exchange momentum and energy with thermal plasma through self-generated Alfvén waves, while in partially ionized regions. They also experience ionization losses and are affected by ion-neutral damping. In this work, a theoretical four-fluid model is presented in which thermal plasma, cosmic rays, and forward and backward-propagating self-excited Alfvén waves are treated as distinct but massless coupled fluids. The role of ionization is incorporated through closure terms in the cosmic-ray and thermal energy equations and through effects of ionization. The numerical results show that increasing the ionization parameter significantly enhances the plasma velocity and thermal pressure, whereas the cosmic-ray pressure decreases noticeably, the forward and backward Alfvén wave pressures exhibit only minor variations. It regulates the efficiency of wave-mediated coupling between cosmic rays and the thermal plasma by modifying the ionization fraction of the background medium. The analysis indicates that the ionization controls whether cosmic rays remain coupled to the plasma through Alfvénic turbulence or decouple and propagate with fewer constraints. This framework provides a transparent basis for understanding cosmic-ray transport, and plasma heating in weakly and moderately ionized astrophysical systems such as the interstellar medium, molecular cloud envelopes, and cosmic-ray driven outflows.

Introduction

Cosmic rays (CRs) constitute one of the most energetic components of the interstellar medium (ISM). It carry energies that extend from a few MeV to beyond 10^{20} eV. Cosmic rays play a fundamental role in regulating the physical, thermal and dynamical evolution of astrophysical plasmas. In the interstellar medium, their energy density is often comparable to that of magnetic fields and thermal gas(K. M. Ferriere, 2001; D. P. Cox, 2005), which implies that they may influence the structure and evolution of diffuse and dense plasma systems alike. Their interaction with background plasma is mediated through multiple channels: direct momentum transfer, generation of Alfvénic turbulence, and crucially, ionization of neutral gas. While the former two mechanisms have received extensive theoretical attention (R. Kulsrud & W. P. Pearce, 1969; E. Parker, 1969), the direct role of CR-driven ionization as an energy transfer mechanism between the CR population and the thermal gas has received comparatively limited investigation in the context of coupled fluid dynamics. These interactions govern not only the transport of energetic particles but also the transfer of momentum and energy between cosmic rays and the background plasma(J. Skilling, 1975a,b,c; E. G. Zweibel, 2017). This makes their transport intrinsically linked to magnetohydrodynamic wave activity and affects instabilities(Parker instability, Jeans instability, and magnetorotational instability), especially Alfvén waves generated by CR streaming instabilities (E. Parker, 1969; T. Kuwabara & C.-M. Ko, 2006), and can modify the outflows(D. Breitschwerdt et al., 1991; B. Ramzan et al., 2020).

The hydrodynamic treatment of CR-plasma coupling emphasizes wave-particle interactions and quasi-linear diffusion (J. R. Jokipii, 1966).. Early studies have shown that CRs streaming faster than the local Alfvén speed can excite resonant hydromagnetic waves(P. Blasi & E. Amato, 2019), which in turn scatter the CRs, reducing their bulk streaming motion and enabling momentum and energy exchange with the background plasma(S. Lucek & A. Bell, 2000). Such feedback forms the foundation of cosmic-ray hydrodynamics and has been used in studies of shocks, outflows, cloud interfaces, and large-scale galactic transport. The theoretical foundation of cosmic-ray plasma interactions begins with the quasi-linear equation for energetic particles and presented by R. SCHLICKEISER (1989).

This naturally led to the foundational two-fluid model. A. HEAVENS (1984) develop a hydrodynamic approach for nonlinear shock acceleration in it. To test these theoretical models G. ZANK (1989) performed the first numerical simulations of cosmic-ray mediated shocks. As research progressed toward multi-fluid dynamics, (C.-M. Ko, 1992; C.-M. KO et al., 1997) emphasized the critical role of Cosmic ray-plasma system in modifying shock structures, and capture the strong interference of forward and backward Alfvén waves, (M. Malkov, 1997a,b) analyze the reactions between cosmic rays, magmatic field, and the thermal plasma, paving the way for advanced three- and four-fluid models. However, in weakly ionized astrophysical media, this coupling is modified by the physics of ionization. Low-energy CRs are a major source of ionization in dense and shielded regions where stellar ultraviolet photons cannot penetrate effectively(B. P. Flannery & W. H. Press, 1979; M. Padovani et al., 2009, 2020). This picture omits a direct energy channel; when CR protons ionize neutral atoms, they transfer kinetic energy to newly created free electrons and ions(R. Schlickeiser et al., 2016).

The understanding of cosmic ray propagation in thermal plasma as an energy transfer mechanism from few decades to the pioneering work of L. Spitzer Jr & M. G. Tomasko (1968), who first demonstrated that CR-induced ionization represents a significant heating source in diffuse interstellar gas (plasma). With increasing ionization, each event releases energy that thermalizes the plasma. The detailed physics of ionization was later elucidated by (R. Gredel et al., 1989), who provide a comprehensive calculations of ionization cross-sections and quantify the energy partition between secondary electrons and thermal excitation. It establishes that ionization cascades can multiply the effective ionization rate by 2 to 3 times as compared to primary CR interactions. In recent decades, observational constraints on ionization have improved through absorption spectroscopy H_3^+, (N. Indriolo & B. J. McCall, 2012) used these observations to measure cosmic ray ionization rates directly in diffuse clouds and provided the first reliable estimates of ionization rates from astronomical data, showing rates of order 10^{-17} per second. Comprehensive modern treatments of cosmic ray ionization in molecular clouds and circumstellar environments by M. Padovani et al. (2009, 2018) and colleagues, and shown that ionization rates depend on column density, cloud composition, and the CR spectrum, and that ionization modify chemical abundances and thermal structure in complex ways. K. Silsbee & A. V. Ivlev (2019) and V. H. M. Phan et al. (2021) have begun to address the dynamical coupling between ionization, and CR transport recognizing that ionization affects on wave-particle interactions and CR scattering. Despite this theoretical and observational foundation, the direct incorporation of ionization-driven energy transfer as a bulk forcing term in multi-fluid hydrodynamical (four fluid) models remains unexplored, (i.e.) a gap that the present work addresses through the introduction of an ionization coupling term in the fluid equations.

The ionization fraction controls the collision rates, and these collisions dampen Alfvén waves(J. E. Leake, 2006; R. Soler et al., 2013; Y. Huang, 2021). These waves dampen through non-linear Landau damping and ion-neutral friction, transferring their energy to thermal plasma (I. U. Zaheer et al., 2026). Therefore, ionization is not merely a secondary chemical consequence of CR transport; it is part of the feedback loop that determines how efficiently CRs couple to the plasma(F. Holguin et al., 2019; B. Sike et al., 2025). This energy does not pass through the wave field but directly heats the plasma and accelerates fluid. The magnitude and consequences of this direct ionization coupling have not been quantified in multi-component fluid models before. The purpose of the present work is to formulate a four-fluid model for CR interaction with plasma, in which the role of ionization is include, the four components are thermal plasma, CR fluid, and forward- and backward-propagating Alfvén waves that are counter propagating. The key innovation is the inclusion of an ionization coupling term $L_c = I_c\ \rho\ P_c$ that quantifies the rate of energy transfer from the CR to the thermal components due to ionization; here I_c is a dimensionless ionization coupling coefficient that encompasses the CR ionization cross-section, the abundance of neutral target atoms, and the energy per ionization event. We integrate the coupled equations using the Runge-Kutta 4th-order (RK4) method in MATLAB and investigate how the variation of I_c ($I_c = 0,\ 0.1,\ 0.2$) affects the evolution of all pressure components and the bulk flow velocity.

Our primary finding is that direct ionization coupling drives significant thermal pressure growth and plasma heating, for $I_c \gtrsim 0.1$. The thermal heating occurs even as the CR pressure is decreased, which implies the presence of some other mechanisms of energy transfer, not only wave damping. Forward and backward wave pressures remain small and insensitive to I_c, which suggests that wave-mediated interactions are secondary to the direct ionization effect in driving the observed fluid dynamics. The bulk plasma flow speed increases monotonically and is accelerated by the CR-ion-neutral interactions. These results have significant implications for the thermal balance of star-forming regions, the pressure support structure of molecular clouds, and the multiphase structure of the diffuse ISM. This approach preserves the analytic transparency of the four-fluid formalism while capturing the most important physical consequences of the ionization.

This paper is organized as follows: Section 2 presents the theoretical framework of the four-fluid model, which incorporates the effects of ionization. Section 3 presents the simulation results, with particular emphasis on plasma

flow, ionization, and energy transfer among various components of the model. Section 4 interprets these results in the context of astrophysical environments and discusses their implications. Section 5 highlights future research directions, and the last section 6 summarizes the main findings and presents the general conclusions.

Four-Fluid Hydrodynamical Model with Cosmic Rays and Waves

In this section we present the comprehensive set of governing equations that form the foundation of the theoretical model used to describe the transport of energy, and interaction of cosmic rays in a magnetized plasma. The system is based on a hydrodynamic model first proposed by (C.-M. Ko, 1992, 2001) that describes the cosmic ray plasma system. The system comprises four components: cosmic rays, thermal plasma, backward, and forward self-propagating Alfven Waves, which are treated as fluids and embedded together to model the cosmic ray plasma system(C. Ko et al., 2021; B. Ramzan et al., 2020, 2021, 2023; K. Irshad et al., 2023; B. Ramzan et al., 2024; I. U. Zaheer et al., 2026). The physical model incorporates the conservation laws for mass, momentum, and energy, as well as coupling terms that represent cosmic-ray streaming, stochastic acceleration, and energy exchange processes. In this model, we also consider the effects of ionization due to cosmic ray propagation. We also assumed a steady-state condition in the hydrodynamic model. Based on this, five interrelated equations are derived and categorized to capture the cosmic ray plasma system behavior with the ionization effect. At the mathematical core of our analysis these governing equations offer a rigorous and detailed investigation, how cosmic rays interact and exchange energy through nonlinear feedback with the background plasma. We define the following set of equations that explain the physical significance of the system.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0 \tag{1}$$

$$\rho \frac{\partial U}{\partial t} + \rho (U \cdot \nabla) U = -\nabla \left(P_{th} + P_c + P_w^+ + P_w^- \right) \tag{2}$$

$$\frac{\partial E_k}{\partial t} + \nabla \cdot F_k = -U \cdot \nabla \left(P_{th} + P_c + P_w^+ + P_w^- \right) \tag{3}$$

$$\frac{\partial E_{th}}{\partial t} + \nabla \cdot F_{th} = U \cdot \nabla P_{th} + L_c \tag{4}$$

$$\frac{\partial E_c}{\partial t} + \nabla \cdot F_c = U \cdot \nabla P_c + (e_+ - e_-) V_A \cdot \nabla P_c + \frac{P_c}{\tau_c} - L_c \tag{5}$$

$$\frac{\partial E_w^{\pm}}{\partial t} + \nabla \cdot F_w^{\pm} = U \cdot \nabla P_w^{\pm} \mp e_{\pm} V_A \cdot \nabla P_c - \frac{2 P_c}{\tau_c} \tag{6}$$

1 & 2 are the conservation of mass (continuity equation) and momentum equations. Where U is the plasma velocity and ρ is the fluid mass density. The left-hand side of 2 represents the change in momentum, while the right-hand side is the net force acting on the fluid due to the gradients of thermal, cosmic ray, and Alfvén wave pressures P_{th}, P_c, P_w^+,and P_w^- respectivily. Equation 3 describes the evolution of the kinetic energy of the bulk fluid in terms of the flux and density of kinetic energy. Equation 4 describes how the thermal energy flux density evolves in space and steady state flow. Equations 5 and 6 are equations for the gradient of the Cosmic Ray energy flux density and the gradient of the wave energy flux density. The first term in 6 on the right represents the work done by the plasma on the Alfven waves. The last two terms ($e_{\pm} V_A \cdot \nabla P_c$) and $\frac{P_c}{\tau_c}$ represent the streaming instabilities and stochastic acceleration.V_A is tha Alfvén speed and $V_A = \frac{B}{\sqrt{\mu_0 \rho}}$.

$$F_k = E_k \cdot U \tag{7}$$

$$E_k = \frac{1}{2} P_k \tag{8}$$

$$F_k = \frac{1}{2} \rho U^2 \tag{9}$$

In terms of kinetic energy, F_k, E_k, and P_k represent the kinetic energy flux, kinetic energy density, and kinetic pressure assosiated with plasma, respectively.

$$F_{th} = (E_{th} + P_{th})\, U, \qquad E_{th} = \frac{P_{th}}{\gamma_g - 1} \tag{10}$$

$$F_{th} = \frac{\gamma_g}{\gamma_g - 1} P_{th}\, U \tag{11}$$

where F_{th}, E_{th}, and P_{th} correspond to the thermal flux, thermal energy density, and thermal pressure of the plasma, respectively, and $\gamma_g \approx 5/3$ is the polytropic index, which relates the thermal pressure to the thermal energy density through the ideal gas equation of state.

The cosmic ray energy flux and energy density are given by

$$F_c = (E_c + P_c) \cdot [U + (e'_+ - e'_-)V_A] - \kappa \frac{\partial E_c}{\partial x} \tag{12}$$

$$E_c = \frac{P_c}{\gamma_c - 1} \tag{13}$$

Substituting Eq. 13 into Eq. 12, we obtain

$$F_c = \frac{\gamma_c P_c}{\gamma_c - 1}[U + (e'_+ - e'_-)V_A] - \frac{\kappa}{\gamma_c - 1}\frac{dP_c}{dx} \tag{14}$$

where F_c, E_c, and P_c are the cosmic ray flux, energy density, and pressure, respectively; γ_c is the cosmic ray adiabatic index; κ is the diffusion coefficient. The wave energy flux, energy density, and pressure are related by

$$F_w^{\pm} = E_w^{\pm}(U \pm V_A) + P_w^{\pm} U \tag{15}$$

$$E_w^{\pm} = 2P_w^{\pm} \tag{16}$$

Substituting Eq. 16 into Eq. 15, we obtain

$$F_w^{\pm} = P_w^{\pm}(3U \pm 2V_A) \tag{17}$$

where $F_w^{\pm}$, $E_w^{\pm}$, and $P_w^{\pm}$ are the wave flux, energy density, and pressure, respectively. The $\pm$ signs correspond to forward and backward propagating Alfvén waves.

$$F_w^{+} = E_w^{+}(U + V_A) + P_w^{+} U \qquad E_w^{+} = 2P_w^{+} \tag{18}$$

$$F_w^{+} = P_w^{+}(3U + 2V_A) \tag{19}$$

$$F_{w^-} = E_w^{-}(U - V_A) + P_w^{-} U \qquad E_w^{-} = 2P_w^{-} \tag{20}$$

$$F_{w^-} = P_w^{-}(3U - 2V_A) \tag{21}$$

The wave-related parameters are defined as follows:

$$e_{\pm} = e'_{\pm} = \frac{P_w^{\pm}}{P_w^{+} + P_w^{-}} \tag{22}$$

$$\frac{1}{\tau} = 16\alpha \frac{V_A^2}{c^2} \frac{P_w^{+} P_w^{-}}{P_w^{+} + P_w^{-}} \tag{23}$$

$$\kappa = \frac{c^2}{3\alpha(P_w^{+} + P_w^{-})} \tag{24}$$

where $e_{\pm}$ and $e'_{\pm}$ represent the normalized wave pressures, τ is the nonlinear interaction time, α is a constant, V_A is the Alfvén speed, c is the speed of light, and κ is the diffusion coefficient.

The ionization loss term is defined as

$$L_c = I_c \rho P_c \tag{25}$$

where L_c represents the ionization loss rate, I_c is the ionization loss coefficient, ρ is the plasma mass density, and P_c is the cosmic ray pressure. As L_c appears as a sink in the cosmic ray energy equation (Eq. 5) and as a source in the thermal energy equation (Eq. 4). Physically, this term accounts for the energy transfer from cosmic rays to the

thermal plasma through ionization processes. Consequently, the cosmic ray pressure decreases while the thermal pressure increases.

$$\phi_B = B \cdot A \tag{26}$$

$$V_A = \frac{B}{\sqrt{\mu_0 \rho}} \tag{27}$$

$$\phi_M = \rho U \tag{28}$$

$$A_g = P_{th} \cdot \rho^{-\gamma_g} \tag{29}$$

$$F_t = F_k + F_{th} + F_c + F_{w+} + F_{w-} \tag{30}$$

$$P_{tot} = P_k + P_{th} + P_c + P_w^+ + P_w^- \tag{31}$$

$$W_A = \left[F_c + \frac{(U+V_A)^2}{V_A} E_w^+ - \frac{(U-V_A)^2}{V_A} E_w^- \right] \tag{32}$$

Equations (26) through (32) represent the integration constants that are utilized in solving the equations. These constants define the assumptions and boundary conditions for integration. We studied the constant flux tube geometry and considered $A = 1$. Therefore, the magnetic flux taken as constant is written as $\phi_B = B$. Equation (27) is a mathematical expression for the Alfvén speed. Equation (28) is a mathematical expression for the mass flux, equation (29) is a constant that defines the thermal pressure behavior, equation (30) expresses the total flux, which is an algebraic sum of all the fluxes, equation (31) is a mathematical expression for the total pressure, which is the algebraic sum of all the pressures. Finally, we used a constant W_A that relates the cosmic flux and energies of the forward and backward self-propagating Alfvén waves.

Using Equations (26) to (32), we derived a set of five autonomous interlinked equations that were solved numerically in MATLAB using the RK-45 approach. These equations involve the gradients of the wind flow, thermal pressure, cosmic ray pressure, and self-propagating forward and backward Alfvén waves.

Flow Analysis and Results

We present the numerical solutions of the autonomous system of equations derived in the previous section.To solve the four-fluid steady-state model, wherein all time-dependent terms are neglected, the parameters defined in Equations (7)-(32) are substituted into the general governing equations (1)-(6).The governing equations are reduced to a system of first-order ordinary differential equations that describe the behavior of the pressure components and the plasma flow velocity in a four-fluid system, incorporating the ionization effects. This system comprises seven variables: the spatial coordinate x, the velocity squared U^2, the thermal pressure P_{th}, the cosmic ray pressure P_c, the forward and backward Alfvén wave pressures P_w^+ and P_w^-, and the diffusive flux term ($k \frac{dE_c}{dx}$). These equations are solved under steady-state conditions i.e ($\frac{\partial}{\partial t} = 0$), with x serving as the independent spatial variable. The resulting set of six coupled autonomous ODEs governing cosmic ray plasma dynamics in the four-fluid framework is presented below(K. Irshad et al., 2023; B. Ramzan et al., 2024).

$$\left(1-M^{-2}\right)\frac{1}{2}\frac{dU^2}{dx}=-\frac{1}{\rho}\frac{dP_c}{dx}\left[\frac{e_+\left(1+\frac{1}{2}M_A^{-1}\right)}{(1+M_A^{-1})}+\frac{e_-\left(1-\frac{1}{2}M_A^{-1}\right)}{(1-M_A^{-1})}\right]+\frac{1}{\rho U}\left[\frac{P_c}{2(1-M_A^{-1})^2}-(\gamma_g-1)\ L_c\right], \tag{33}$$

where $M=\dfrac{U}{a_{eff}}$, M is mach number and

$$a_{eff}^2=\frac{\gamma_g P_t h}{\rho}+\frac{\gamma_c P_c}{\rho}\frac{(1+\frac{1}{2}M_A^{-1})^2}{(1+M_A^{-1})^2}+\frac{3P_w^+}{2\rho}\frac{(1+\frac{1}{3}M_A^{-1})}{(1+M_A^{-1})}+\frac{3P_w^-}{2\rho}\frac{(1-\frac{1}{3}M_A^{-1})}{(1-M_A^{-1})} \tag{34}$$

$$\frac{dP_{th}}{dx}=-\frac{\gamma_g P_{th}}{2U^2}\left(\frac{dU^2}{dx}\right)+\frac{(\gamma_g-1)}{U}\ L_c, \tag{35}$$

$$\frac{dP_c}{dx}=\frac{(\gamma_c-1)}{\kappa}\left(\kappa\frac{dE_c}{dx}\right) \tag{36}$$

$$\left(\kappa\frac{dE_c}{dx}\right)=-(\frac{P_c}{\tau}-L_c)+\left\{\frac{U\left(1+(e^+-e^-)M_A^{-1}\right)\rho^{\gamma_c}}{(\gamma_c-1)\left(1+(e_+-e_-)M_A^{-1}\right)^{\gamma_c}}\right\} *\left\{\frac{d}{dx}\left[P_c\left|\frac{1+(e^+-e^-)M_A^{-1}}{\rho}\right|^{\gamma_c}\right]\right\}, \tag{37}$$

$$\frac{dP_w^+}{dx}=-\frac{3P_w^+\left(1+\frac{1}{3}M_A^{-1}\right)}{2U^2(1+M_A^{-1})}\frac{dU^2}{dx}-\frac{e_+M_A^{-1}}{2(1+M_A^{-1})}\frac{dP_c}{dx}-\frac{P_c}{4U(1+M_A^{-1})^2}, \tag{38}$$

$$\frac{dP_w^-}{dx}=-\frac{3P_w^-\left(1-\frac{1}{3}M_A^{-1}\right)}{2U^2(1-M_A^{-1})}\frac{dU^2}{dx}+\frac{e_-M_A^{-1}}{2(1-M_A^{-1})}\frac{dP_c}{dx}-\frac{P_c}{4U(1-M_A^{-1})^2} \tag{39}$$

Equation (33) describes the spatial evolution of U^2 due to cosmic ray pressure gradients and stochastic acceleration, where $M_A = U/V_A$ is the Alfvénic Mach number and M is mach number. Equation (35) describe the thermal pressure P_{th} with the ionization loss term I_c. Equations (36) and (37) account for the cosmic ray pressure P_c evolution, including wave interactions, diffusion, and ionization losses, (38) and (39) describe the forward and backward Alfvén wave pressures P_w^+ and P_w^- respectively including the adiabatic and coupling effects.We investigate the dynamics of the four-fluid cosmic-ray plasma system numerically by integrating the governing equations from (33) to (39), adjusted to include ionization. We solve these equations numerically using the fourth-order Runge-Kutta (RK45) methods implemented in MATLAB, and the numerical integration yields physically admissible steady shock-type solutions that satisfy the prescribed boundary conditions. The results exhibit the characteristic of sharp transition associated with a shock, demonstrating the influence of ionization on the flow dynamics. Away from the shock region(downstream) , the flow variables gradually approach their corresponding equilibrium states, ensuring that the numerical solutions remain consistent with the underlying physical model. In the present analysis, the adiabatic indices are $\gamma_g = 5/3$ for the thermal plasma and $\gamma_c = 4/3$ for the cosmic-ray fluid. The mass and magnetic fluxes are fixed at $\phi_M = 1.6$ and $\phi_B = 1.0$ respectively, whereas the total conserved flux is $F_t = 29.99$. The numerical calculations are initiated from $U_0^2 = 16$, $P_{c0} = 0.6$, $P_{\rm th0} = 0.6$, $P_{w0}^+ = 0.1$, and $P_{w0}^- = 0.2$ as the boundary values. The resulting flow structure is analyzed through the spatial distributions of the velocity and pressure components, which characterize the dynamical response of the plasma system. There are three different cases for ionization, are considered to examine the influence on the velocity and pressure distributions: (i) the without ionization ($I_c = 0$), (ii) weak ionization case ($I_c = 0.1$), and (iii) stronger ionization case ($I_c = 0.2$). The resulting profiles of $U(x)$, P_c, $P_{\rm th}$, P_w^-, and P_w^+ exhibit the shock-type structure under the adopted normalized closure parameters, which allow the propagation of both forward- and backward-propagating Alfvén waves. As the ionization strength increases, noticeable variations appear in the flow speed and pressures, demonstrating the role of ionization in the plasma dynamics. At the downstream region all physical variables(velocity and pressures) approaches to their respective asymptotic states, indicating that the computed solutions remain consistent with the given boundary conditions and are representative of astrophysical shock environments.

Impact on Plasma Flow With and Without Ionization

Figure 1 illustrates the plasma velocity profile $U(x)$, for three different ionization strengths. The solid curve corresponds without ionization case ($I_c = 0$), whereas the dashed and dash-dotted curves represent ionization strengths of $I_c = 0.1$ and $I_c = 0.2$ respectively. In each case, the plasma velocity increases from the upstream region

Table 1. Parameters for the boundary conditions.

Figure	ϕ_B	ϕ_m	$F_{\rm t}$	U_0^2	P_{c0}	P_{th0}	P_{w0}^+	P_{w0}^-	I_c
Solid line	1.0	1.6	29.99	16	0.6	0.6	0.1	0.2	0.0
Dashed line	1.0	1.6	30.20	16	0.6	0.6	0.1	0.2	0.1
Dash-dotted line	1.0	1.6	30.80	16	0.6	0.6	0.1	0.2	0.2

and undergoes a rise across the shock transition near $x = 0$. The velocity continues to increase downstream, and attain its peak value at $x \approx 3$. The comparison further indicates that increasing the ionization strength only modifies the velocity distribution while preserving the characteristic shock-type profile, i.e the velocity profiles gradually flatten out and approach their respective asymptotic values in the downstream region. In the upstream region ($x < -5$), the velocity remains nearly constant, indicating an undisturbed flow where $dU/dx \approx 0$. As the plasma approaches the shock transition, the velocity increases rapidly over the interval $-5 < x < 3$, implying a positive velocity gradient ($dU/dx > 0$). This behavior is governed by the combined influence of the wind flow equation (33) and the momentum equation, which together describe the acceleration of the plasma through the shock transition. Since the gradients of the thermal pressure and the forward and backward wave pressures are negative in this region, and through the minus sign, they provide a positive contribution that accelerates the flow. Near $x \approx 3$, the velocity exhibits a slight decrease due to ($dU/dx < 0$). This behavior is attributed to the increasing influence of the streaming instability, which becomes more significant than stochastic acceleration in the downstream region, the plasma flow experiences a mild deceleration before approaching its asymptotic downstream state.In this region, the amplitudes of the forward and backward propagating Alfvén waves are reduced, and the pressure gradients become negligible. The flow settles into its downstream state, exhibiting almost constant velocity profile.

The inclusion of ionization shows a significant rise in the plasma velocity U (dashed and dash-dotted curves) as compared to the case without ionization (solid curves). From Figure 1, U increases sharply in the transition region and attains a peak at approximately $x = 3$, which is higher than the corresponding peak without ionization. Beyond the peak, the plasma velocity exhibits a decrease before gradually approaching a nearly constant value in the far downstream region ($x \approx 5$). A similar behavior is observed for the ionization cases. However, with increase the ionization strength to $I_c = 0.2$ produces a higher maximum velocity and broadens the shock transition region compared with the lower ionization cases. This behavior demonstrates that stronger ionization has a greater influence on the plasma flow, show the result in enhanced flow acceleration and a modified shock-type profile.

Impact on Cosmic Ray Pressure With and Without Ionization

Figure 2 illustrates the spatial behavior of the cosmic ray pressure, which is obtained from the numerical solution of Eq. (36). The solid line corresponds to the case without ionization effects. In the region $-5 < x < 0$, the cosmic ray pressure exhibits a monotonic increase, characterized by a positive gradient ($dP_c/dx > 0$), which indicates a progressive rise in pressure. This positive gradient persists until $x \approx 0$, where the pressure attains its maximum value. Beyond this point, the pressure remains constant up to $x = 3$, after which it exhibits a slight increase before settling to a steady(constant value). In this asymptotic region, the gradient vanishes ($dP_c/dx = 0$) no further variation in pressure. The initial rise in pressure from $-5 < x < 0$ is attributed to stochastic acceleration(P_c/τ) and streaming instability($e_\pm V_A \partial Pc/\partial x$). At $x = 3$, cosmic rays undergo acceleration near the downstream region, and the subsequent increase in pressure is driven by the diffusive flux, represented by the term $\kappa dP_c/dx$. This term provides a positive feedback mechanism, enhancing the cosmic ray pressure at $x = 3$. This indicates that cosmic ray pressure is approaching equilibrium since the cosmic ray pressure gradient eventually reaches zero. This stabilization happens when the effects of streaming instability, diffusive flux, stochastic acceleration, and plasma work balance each other. In the presence of ionization, the cosmic-ray pressure profile is represented by the Dashed ($I_c = 0.1$) and dash-dotted ($I_c = 0.2$) curves in Figure 2. With ionization, cosmic-ray pressure behavior is similar to that of the case without ionization, but there are some differences, the maximum value of the cosmic-ray pressure decreases as the ionization strength increases. This behavior is primarily attributed to the ionization loss term, $L_c = I_c \rho\ P_c$, appearing in Eq.(36), which acts to reduce the cosmic-ray pressure. Beyond the peak near $x \approx 0$, the pressure exhibits a gradual decline ($dP_c/dx < 0$) as the influence of the ionization loss term becomes increasingly significant. At the same time, the weakening of the forward- and backward-propagating Alfvén waves reduces the efficiency, both stochastic acceleration and streaming instability, further contributing to the decrease in the cosmic-ray pressure. A further increase in the ionization parameter to $I_c = 0.2$ produces an even more pronounced reduction in the pressure profile throughout the shock transition region. For $x > 3$, the cosmic-ray pressure exhibits

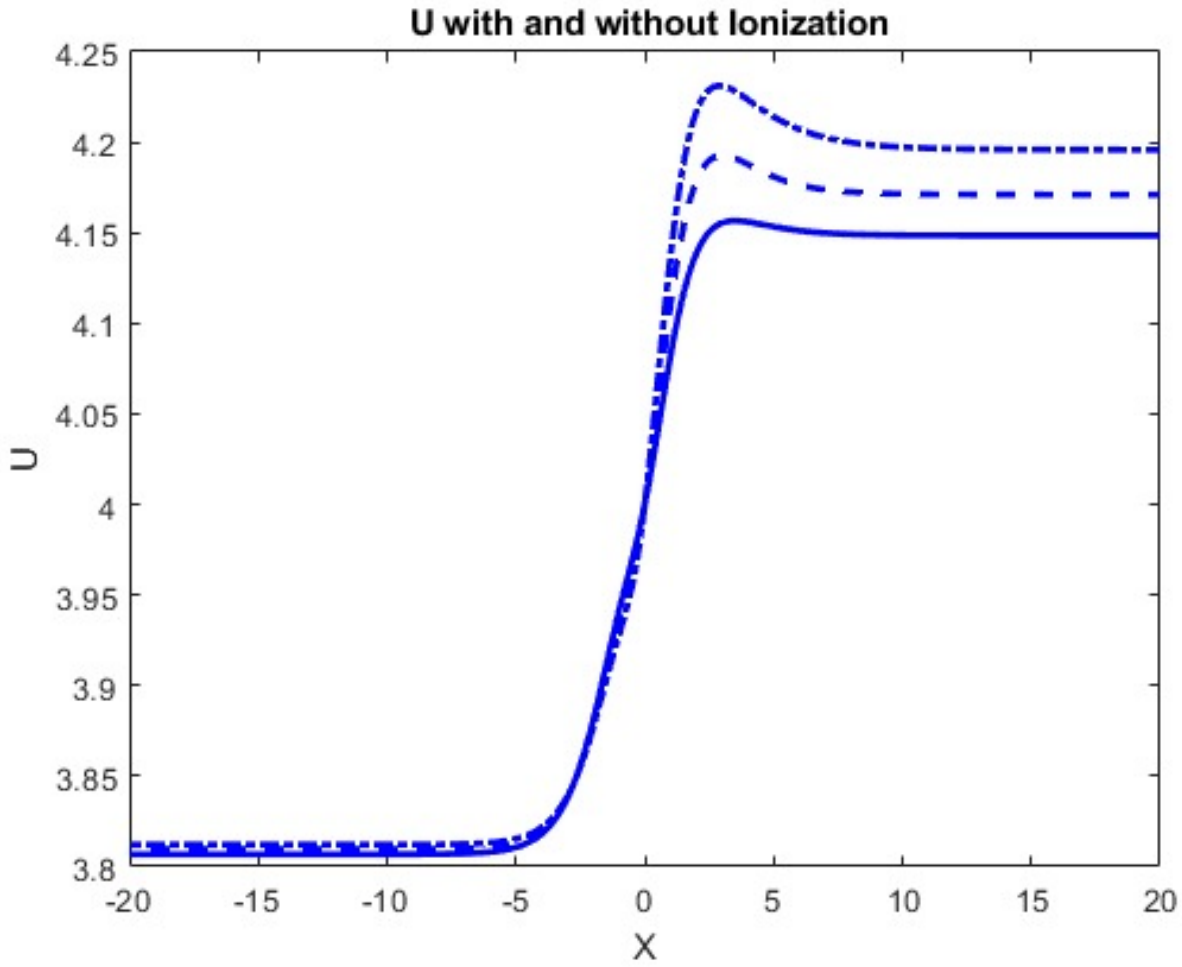


Figure 1. Plasma velocity profile, $U(x)$, illustrating the effect of ionization. The solid line denotes $I_c = 0$, while the Dashed and dash-dotted lines represent $I_c = 0.1$ and $I_c = 0.2$, respectively.

a gradual recovery, corresponding to a positive pressure gradient ($dP_c/dx > 0$). This behavior is associated with the diffusive transport of cosmic rays in the downstream region, where the amplitudes of the Alfvén waves are reduced. As a result, the cosmic-ray pressure gradually increases before approaching an approximately constant value, for which $dP_c/dx \approx 0$. Unlike the non-ionized case, the ionized profiles display a non-monotonic evolution, characterized by an initial increase, followed by a decline near the shock transition, and a subsequent recovery prior to reaching the downstream asymptotic state. Once that balance happens, the cosmic ray pressure no longer changes, indicating that the system has reached equilibrium.

Impact on Thermal Pressure With and Without Ionization

The solid curve in Figure 3 illustrates the thermal pressure profile, $P_{\rm th}(x)$, in the absence of ionization. According to Eq.35, the evolution of the thermal pressure is directly coupled to the plasma velocity gradient through the relation

$$\frac{dP_{\rm th}}{dx} = -\frac{\gamma_g P_{\rm th}}{2U^2}\left(\frac{dU^2}{dx}\right).$$

The negative sign indicates that an increase in the plasma velocity is accompanied by a corresponding decrease in the thermal pressure. Consequently, the nearly constant velocity in the upstream region ($x < -5$) produces only negligible variations in $P_{\rm th}$. As the plasma enters the shock transition region ($-5 < x < 3$), the rapid increase in velocity, shown in Figure 1, is accompanied by a pronounced reduction in the thermal pressure.

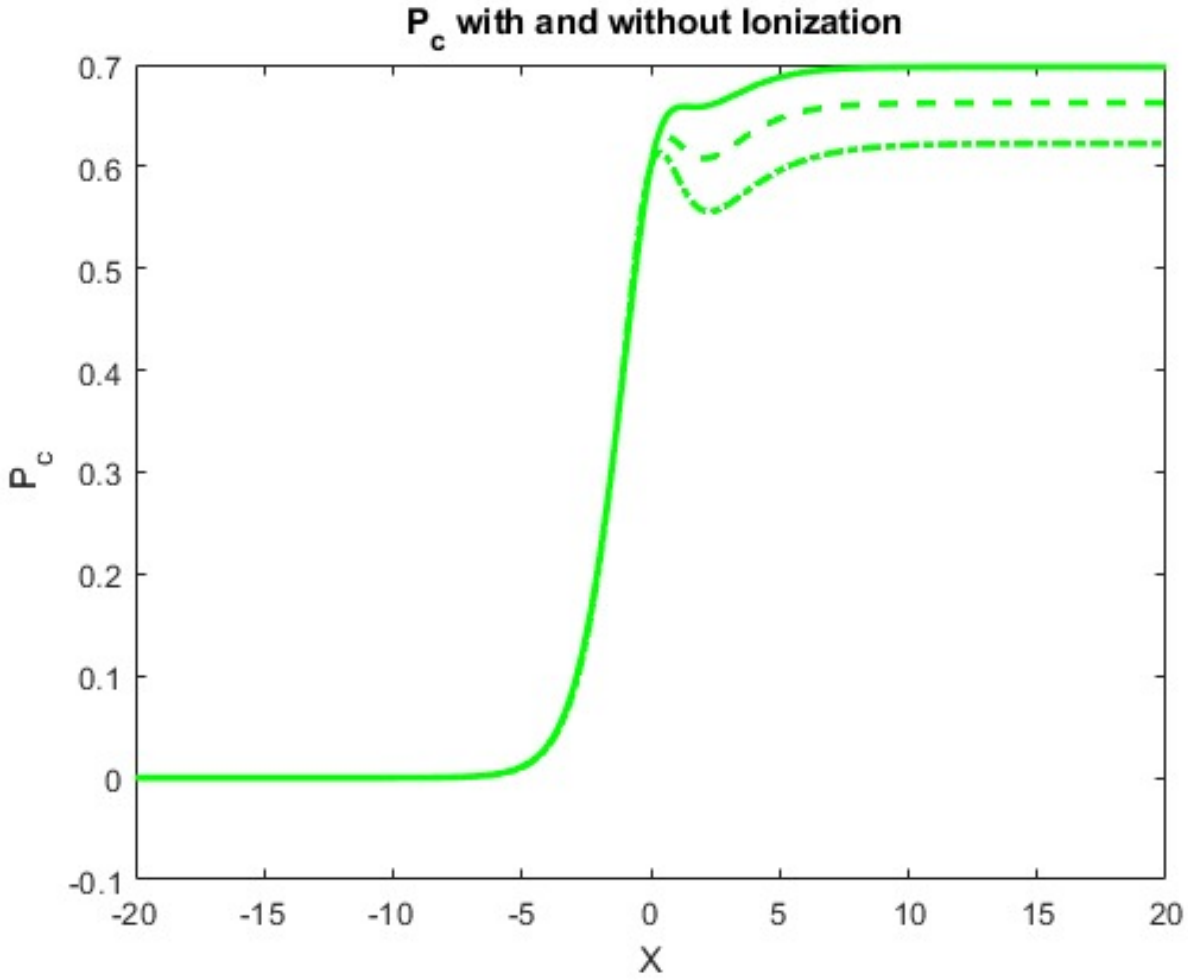


Figure 2. Spatial variation of the cosmic-ray pressure, $P_c(x)$, for different ionization strengths. The solid, Dashed, and dash-dotted curves correspond to $I_c = 0$, $I_c = 0.1$, and $I_c = 0.2$, respectively.

This behavior indicates that the total energy of the system is redistributed rather than dissipated, with energy being transferred between the kinetic and thermal components of the plasma during the shock transition. Similar to the cosmic-ray pressure, the thermal pressure gives away part of its energy to the flow. For this reason, in Figure 1 the velocity increases, and in Figure 2 the cosmic-ray pressure increases, whereas in Figure 3 the thermal pressure decreases. The thermal part acts as a reservoir that supplies energy to the other components. Hence, as the plasma velocity increases, the thermal pressure decreases. Far downstream, the pressure gradient becomes negligible, and the thermal pressure approaches a nearly constant downstream value.

The dashed and dash-dotted lines in Figure 3 correspond to the thermal pressure profiles for $I_c = 0.1$ and $I_c = 0.2$, respectively. The inclusion of ionization introduces the source term $L_c = I_c \rho P_c$ into the thermal pressure equation:

$$\frac{dP_{th}}{dx} = -\frac{\gamma_g P_{th}}{2U^2}\frac{dU^2}{dx} + L_c.$$

Physically, L_c represents the energy deposited into the thermal plasma by cosmic rays through ionization. As a result, the thermal pressure increases with increasing I_c. For $I_c = 0.1$, P_{th} is higher than the non-ionized case ($I_c = 0$), and for $I_c = 0.2$, the thermal pressure is even higher. This behavior is expected, as a larger ionization constant correspond to more efficient energy transfer from cosmic rays to the thermal pressure, that enhancing the thermal pressure. As a result, the thermal pressure increases downstream, and this increase is more pronounced for larger values of the ionization coefficient I_c. This behavior is physically consistent, as stronger ionization leads to greater energy deposition from cosmic rays into the thermal plasma, thereby elevating the thermal pressure.

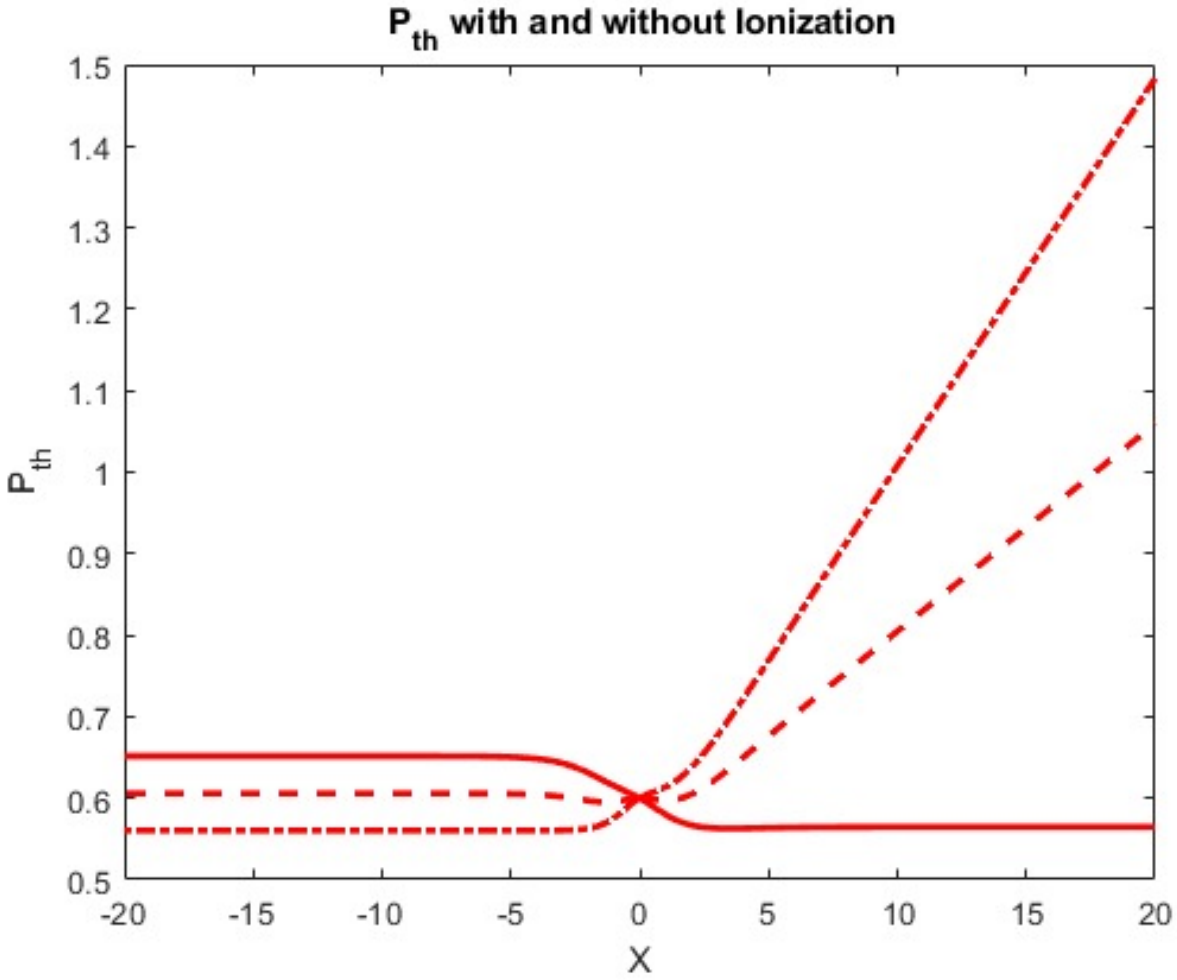


Figure 3. Spatial variation of the Thermal pressure, $P_{th}(x)$, for different ionization strengths. The solid, Dashed, and dash-dotted curves correspond to $I_c = 0$, $I_c = 0.1$, and $I_c = 0.2$, respectively.

Backward and Forward Wave Pressures

Figure 4 shows the behavior of the backward wave pressure P_w^- in the shock-type profile. On the upstream side ($x < 0$), the value almost flat at a higher level, and the pressure drops rapidly as the flow approaches $x = 0$. This decrease continues to $x = 3$, after which the wave dies at downstream. Both without and with ionization cases it follow the same trend, and the difference between them is minimal, so the ionization does not change much P_w^-. The main feature of this profile is decreasing monotonic behavior where the backward wave pressure gradientdP_w^-/dx falls sharply at the near upstream towards the shock transition and diminishes at the later downstream region.

Figure 5 shows the forward wave pressure P_w^+ for cases with and without ionization. In the upstream part ($x < 0$), the value of $P_w^+(x)$ remains nearly constant at a higher level; it maintains its level until it reaches close to the shock transition. The pressure drops quickly around the transition at $x = 0$, showing a steep decrease across the shock. The curve flattens out in the downstream region ($x > 5$) and becomes almost constant again. The solid line shows the case where ionization is absent, whereas the dashed and dash-dotted curves represent $I_c = 0.1$ and $I_c = 0.2$, respectively. All three curves follow the same general pattern, but the ionized cases settle at slightly higher values on the downstream side. Therefore, the overall behavior is that the forward wave pressure decreases through the transition, and ionization only pushes the final level slightly higher, without changing the global shape too much.

Discussion

The numerical solutions obtained from the four-fluid model using the fourth-order Runge–Kutta (RK4) method in MATLAB provide a clear picture of how ionization influences the interactions between cosmic rays and background

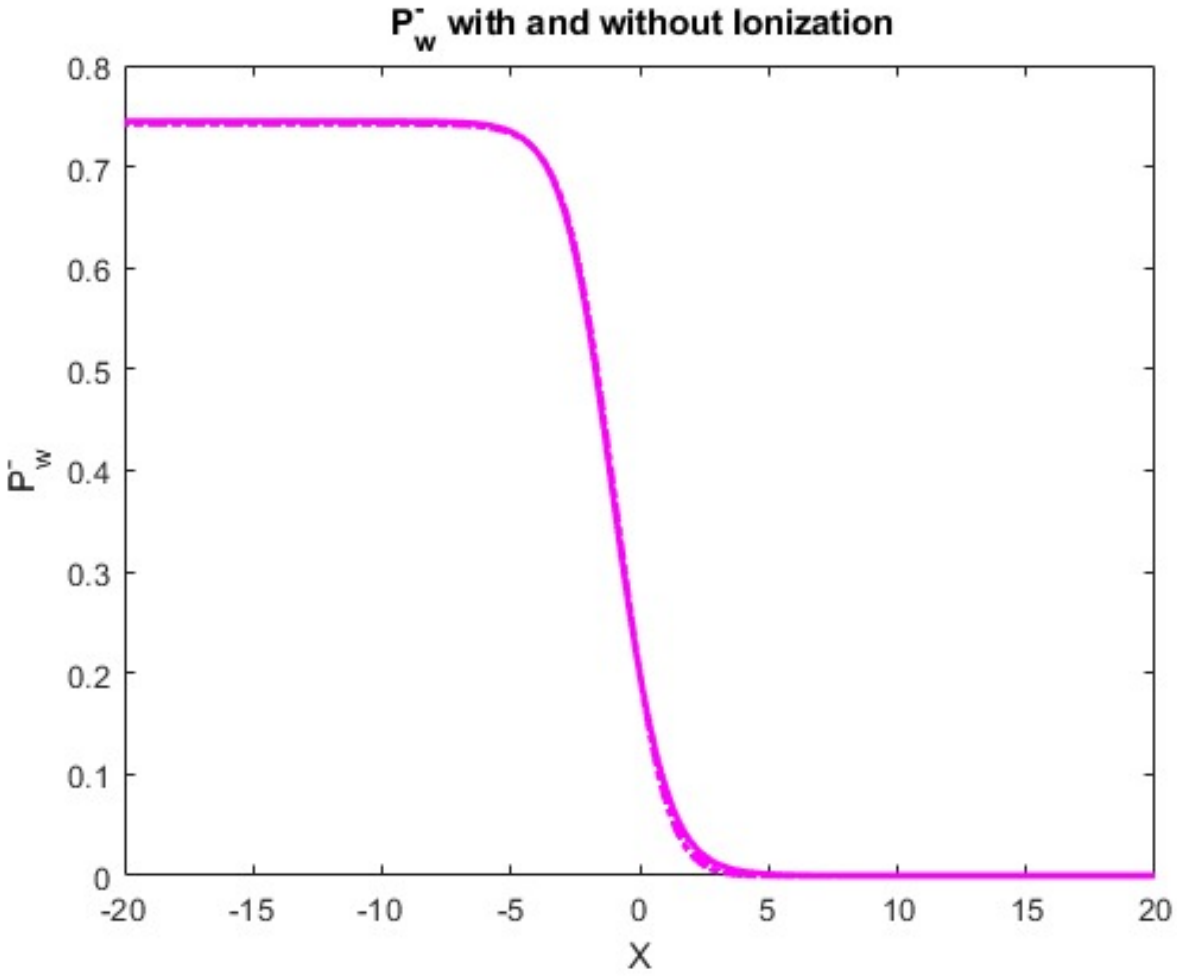


Figure 4. Profile of Backward Wave Pressure $P_w^-(x)$ for different ionization strengths. The solid, Dashed, and dash-dotted curves correspond to $I_c = 0$, $I_c = 0.1$, and $I_c = 0.2$, respectively.

plasma. The results indicated that ionization significantly altered the energy distribution among the four interacting fluids, leading to noticeable changes in the plasma dynamics and pressure evolution.The numerical profiles obtained from the present four-fluid model exhibit a clear shock-wave-type structure consisting of three distinct regions: an upstream region, a transition region, and a downstream region. For all values of the ionization parameter, the solutions remain nearly identical in the upstream region ($x < -5$), indicating that ionization does not produce a noticeable modification of the initial flow state. As the system enters the transition region, approximately $-5 \leq \mathbf{x} \leq 5$ the profiles undergo a sudden change, giving rise to a shock-like structure (K. Irshad et al., 2023; B. Ramzan et al., 2024). Beyond this region, the solutions evolve monotonically toward their downstream states. The separation between the different ionization cases becomes most apparent after the transition, showing that the primary influence of ionization is associated with the downstream evolution of the system.

One of the most prominent features observed in the numerical profiles is the increase in plasma velocity with increasing ionization. Although the upstream velocity remains essentially unchanged and increasing ionization leads to a higher plasma flow velocity downstream. The sudden variation across the transition therefore separates two distinct flow states, while the subsequent monotonic behavior indicates the gradual evolution of the plasma toward a new downstream configuration. This behavior suggests that ionization enhances the coupling among the charged components of the plasma, allowing momentum to be transferred more efficiently throughout the system. As the ionization parameter I_c increases from 0 to 0.2, the plasma velocity increases, while the cosmic-ray pressure decreases and the thermal pressure increases significantly. As the ionization parameter increases, the downstream cosmic-ray pressure decreases, producing a monotonic decline relative to the non-ionized case.The P_c profiles remain

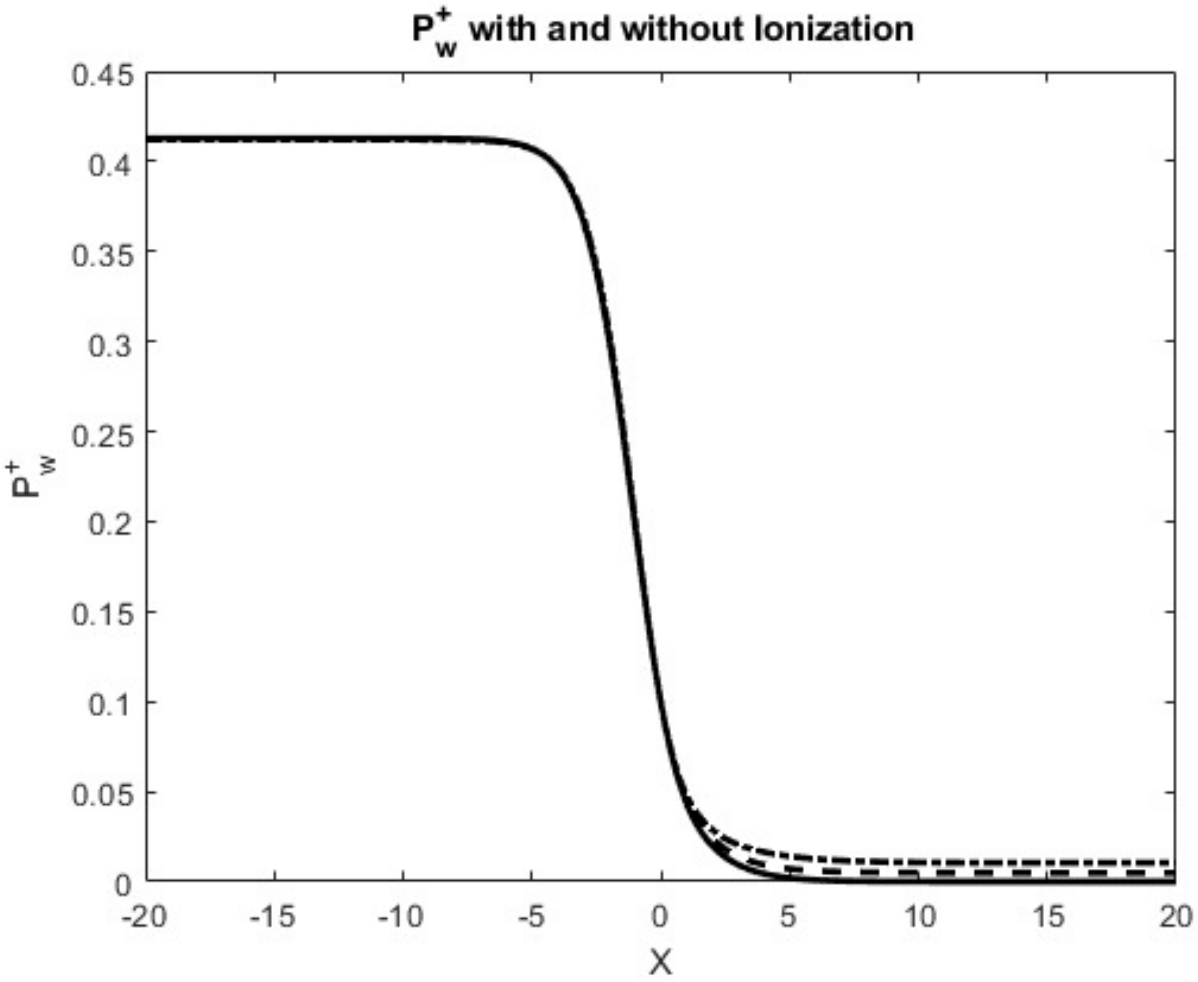


Figure 5. Profile of Forward Wave Pressure $P_w^+(x)$ for different ionization strengths. The solid, Dashed, and dash-dotted curves correspond to $I_c = 0$, $I_c = 0.1$, and $I_c = 0.2$, respectively.

comparatively unified upstream but bifurcate across the transition. Thus, while the shock-like transition remains a common feature of all solutions. This reduction indicates that a portion of the energy initially stored in the cosmic-ray component is redistributed to the thermal plasma through the coupling between the interacting fluids. Consequently, the dynamical contribution of cosmic rays becomes weaker, while the thermal plasma gains a larger share of the total energy. Similar energy redistribution between cosmic rays and the thermal plasma has been discussed in studies of cosmic ray modified flows and wave–particle interactions (J. Skilling, 1975a; E. G. Zweibel, 2017). A similar trend was observed in our previous study on nonlinear Landau wave damping (NLLWD), where increasing the damping strength resulted in higher plasma velocity, cosmic-ray pressure, and thermal pressure. In the present ionization model, the plasma velocity and thermal pressure also increased; however, the cosmic-ray pressure decreased with increasing ionization. This difference suggests that, both mechanisms enhance plasma heating, they redistribute energy within the cosmic-ray–plasma system through different physical processes (I. U. Zaheer et al., 2026).

A particularly significant result of the proposed model is the substantial increase in the thermal pressure as the ionization increases. Similar to the other profiles, only small differences are observed upstream, a substantial separation develops across the transition region. The downstream thermal pressure increases significantly with increasing ionization and subsequently approaches its asymptotic behavior monotonically. Among all the physical quantities considered, the thermal pressure exhibited the strongest response to ionization, as clearly demonstrated by the numerical profiles. This behavior indicates that ionization efficiently converts a fraction of the available nonthermal energy into thermal energy, thereby increasing the plasma temperature and internal energy. The

enhanced thermal pressure implies that ionization plays an important role in plasma heating and regulates the thermodynamic evolution of the astrophysical plasmas. An increase in the thermal pressure has several important astrophysical implications. Hotter plasma modifies the thermal balance of the interstellar medium and influences the evolution of ionized regions surrounding energetic astrophysical objects. Enhanced thermal energy can affect the propagation of shock structures, contribute to the expansion of stellar winds, and regulate interstellar gas compression. Furthermore, changes in the thermal state of the plasma influence the physical conditions under which molecular clouds evolve, thereby affecting the environment in which star formation eventually occurs. Although thermal pressure generally provides support against gravitational collapse, it is an essential parameter that governs the stability and subsequent evolution of astrophysical clouds (C. F. McKee & E. C. Ostriker, 2007). The numerical results also revealed that the forward and backward wave pressure components experienced only minor variations with increasing degree of ionization. Unlike the plasma velocity and thermal pressure, these quantities remained nearly unchanged throughout the calculations. This suggests that ionization primarily affects the exchange of energy between the cosmic-ray and thermal plasma components without significantly modifying the balance between the forward and backward propagating wave populations. Therefore, the dominant influence of ionization in the present four-fluid model is observed in the plasma flow and pressure evolution, rather than in the wave-pressure components. Their overall profiles retain the same shock-like transition and downstream behavior, but the separation between the different ionization cases remains much weaker than that observed for P_c and P_{th}. The shock-like profiles provide the point at which the ionization-dependent differences between the solutions become established. The monotonic downstream evolution then reflects the adjustment of the coupled plasma, cosmic-ray, and wave components toward their respective post-transition states. The pronounced increase in thermal pressure and plasma velocity suggests that ionization can substantially influence plasma heating and flow evolution in cosmic-ray-modified systems, while the simultaneous decline in cosmic-ray pressure highlights a change in the relative importance of the non-thermal component in the downstream region.

Implications of the study

This study represents the theoretical model for understanding, how ionization modifies the propagation of cosmic rays in thermal plasma with four-fluid model in cosmic ray plasma system, and numerical results demonstrate that increasing the ionization constant I_c enhances the plasma velocity and significantly increase in the thermal pressure, while the cosmic-ray pressure gradually decreases. These findings indicate that ionization plays an important role in energy exchange from the non-thermal cosmic-ray component to the thermal plasma. Such energy redistribution is expected to influence a wide range of astrophysical plasma environments in which cosmic rays, magnetic fields, and ionized matter i.e plasma coexist.

One of the most important future extensions of the present work is the simultaneous inclusion of ionization and nonlinear Landau wave damping(I. U. Zaheer et al., 2026) within the same theoretical framework, both modify the exchange of energy between cosmic rays and the thermal plasma. Studying them together should give a more realistic picture of how plasma heats up, how waves evolve, and how cosmic rays are transported in real astrophysical systems. Such a unified model would also improve our understanding of the competition between wave growth, wave dissipation, and ionization of cosmic-ray modified plasmas (E. G. Zweibel, 2017).

The enhanced thermal pressure predicted by the present model may have important implications for astrophysical outflows. In many high-energy astrophysical environments, thermal pressure is one of the driving mechanisms responsible for accelerating plasma away from the central source, as the ionization parameter increases, the substantial increase in thermal pressure provides a stronger pressure gradient that can support plasma expansion and contribute to the formation and propagation of large-scale outflows. Plasma acceleration becomes more efficient too. The increased plasma velocities obtained in the present calculations indicate that a higher degree of ionization tends to produce more effective plasma acceleration, a process that is considered fundamental in many astrophysical flow systems where the interaction between ionization and pressure gradients shapes the overall dynamics of the outflowing material.

Our results are particularly relevant to the study of astrophysical jets. Jets emerging from active galactic nuclei, young stellar objects, microquasars, and other compact astrophysical sources are generally believed to be driven by a combination of magnetic, thermal, and radiative processes. Although the present model does not explicitly describe jet formation, the observed increase in plasma velocity, together with the significant enhancement of thermal pressure, suggests that ionization can modify the physical conditions under which jets are accelerated and collimated. The reduction in cosmic-ray pressure, accompanied by stronger thermal support, indicates a mechanism of energy exchange that may influence the dynamics of jet-launching regions and the subsequent evolution of the plasma outflows. So the present results provide useful theoretical insights into the plasma conditions associated with jet

propagation rather than the jet-launching mechanism itself (R. D. Blandford & D. Payne, 1982; A. Ferrari, 1998).

The predicted increase in plasma velocity may also contribute to a better understanding of cosmic-ray-driven winds and large-scale galactic outflows. As the thermal pressure becomes increasingly dominant, the plasma acquires additional kinetic energy, facilitating its outward motion against the surroundings. Such pressure-driven acceleration has been widely recognized as an important ingredient in models of galactic winds and energetic plasma outflows, where thermal pressure, magnetic fields, and cosmic rays collectively determine the large-scale dynamics of the flow (J. E. Everett et al., 2008). The present four-fluid framework can be further extended by incorporating additional physical processes including magnetic field amplification, anisotropic cosmic-ray diffusion, radiative cooling, plasma turbulence, and multidimensional effects. These extensions would allow the model to describe more realistic astrophysical environments in which several physical mechanisms operate simultaneously. Integrating the model with magnetohydrodynamic simulations offers a significant opportunity to explore the nonlinear progression of plasmas altered by cosmic rays within realistic boundary conditions.

Future research efforts should compare to the theoretical predictions of the present model with observational data from astrophysical jets, galactic winds, and ionized plasma environments. These comparisons would help to evaluate the applicability of the proposed four-fluid model and establish strong connections between theoretical predictions and observed plasma behavior. We say that the present model may serve as a theoretical basis for developing a multi-fluid model of an ionized cosmic-ray plasma system that contributes to future studies of plasma evolution and energy transport in various astrophysical environments.

Conclusion

The current research was done to study how the effects of ionization play a role in the propagation of cosmic rays inside the intergalactic medium (IGM). A four–fluid hydrodynamic model was used in which the plasma, cosmic rays, thermal gas, and bidirectional Alfvén waves are coupled. The primary purpose was to see how energy and pressure transfer among these components when ionization is included. Chapter 3 explained the development of the mathematical model and the derivation of all governing equations. In contrast, Chapter 4 was focused on solving those equations numerically and describing what type of physical changes appear in velocity and pressure when ionization is considered. Chapter 3's first-order differential equations explained the plasma's steady state flow under advection, diffusion, and wave-particle interaction. The additional term of ionization, $L_c = I_c \rho P_c$, was included in the energy equations to represent the energy loss of cosmic rays that goes into heating the plasma.

This term is significant because it converts the non-thermal energy of cosmic rays into the thermal energy of the background plasma. The presence of forward P_w^+ and backward P_w^- Alfvén waves helped to describe the wave–particle coupling, which keeps the energy moving between different fluids(cosmic pressure, thermal plasma, forward and backward alfvén waves). These equations also include stochastic acceleration(P_c/τ) and pressure gradient work, making the model more realistic than simple two or three-fluid systems. As a result, the fundamental frame that was solved numerically was constructed using the magnetohydrodynamical model as four fluid model and solved numerically using the RK4 method. We assigned physical meanings to each term: ionization loss served as a sink for cosmic ray pressure, diffusion provided energy distribution, and advection represented mass flow.When ionization was included, the equations became more intricate and more representative of real space environments, with intense interactions between thermal plasma and magnetic fields. When these equations are solved for change ionization intensities in 3, it was discovered that the plasma's behavior also varied. The forward and backward wave pressures decreased in the absence of ionization, indicating that the waves lost energy as soon as the cosmic rays began to decline the pattern changed when ionization was included. For higher I_c values (0.1 or 0.2), the forward wave pressure P_w^+ and backward wave pressure P_w^- did not disappear but instead stabilized at new constant value. In other words, ionization prevents the wave energy from degrading too quickly and helps preserve it. Ionization caused some of the cosmic ray's energy to be lost, which resulted in a fall in cosmic ray pressure P_c, and the plasma velocity $U(x)$ rose, indicating that some pressure energy was converted into flow kinetic energy.

The numerical graphs also highlight the importance of stochastic acceleration and streaming instability and ensure energy exchanges between waves and cosmic rays. The presence of ionization keeps this coupling alive across a wider spatial region. That's why the forward and backward wave pressure curves level off instead of dropping to zero. This condition is similar to cosmic plasmas in space, where non-thermal energy persists for a long time, as in radio halos or in intracluster regions.

Acknowledgments

Author contributions statement

Inaam Ullah Zaheer conceived the project, developed the theoretical model and numerical integration scheme, flow analyses and results, and drafted the manuscript. Aqeel Afzal, SNA Qazi, and M. Jamil performed the data analysis, and contributed to the discussion on results and astrophysical implications. Bilal Ramzan supervised the overall research project, provided conceptual guidance, and thoroughly revised the manuscript. All authors reviewed and approved the final version.

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Competing Interests

The authors declare no competing interests.

Additional information

Data Availability:The datasets generated and analyzed during the current study are available from the corresponding author on reasonable request.

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