

Apparent directional failure of moment closures traced to a polarity-representation inconsistency in heterogeneous transport

Fujio Takenari

`fujio-takenari-ni@alumni.osaka-u.ac.jp`

Independent Researcher <https://orcid.org/0009-0005-8549-0022>

Research Article

Keywords: velocity-jump process, moment closure, kinetic-to-continuum transport, polar transport, nematic orientation field, polarity representation, representation consistency, anisotropic diffusion, heterogeneous transport, Monte Carlo simulation, finite-volume method, Chapman–Enskog closure, reproducible computational research, prospective specification, computational biophysics

Posted Date: October 20th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11423899/v1>

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Additional Declarations: The authors declare no competing interests.

Abstract

Continuum moment closures replace kinetic descriptions of directional transport with a small set of local fields, but a polar drift term requires information that a nematic orientation field does not supply: a sign lift of the director. We report a controlled study in which this lift, rather than closure order, dominates an apparent finite-scale closure failure. In a previously audited comparison on a strongly heterogeneous polar field, drift-diffusion, anisotropic-diffusion, Chapman-Enskog density and dynamic first-moment closures all failed a prespecified final-time adequacy criterion, with combined errors 58–64 times the replicate-noise scale and final directional errors of -74.3 and -92.6 degrees. A prospectively specified, hash-recorded polarity-seam sensitivity control, varying only the sign lift between seam-rich, intermediate and globally combed representations, produced an anomalous role-swap signature instead of a monotone seam response. Uniform-field tests then showed that the Monte Carlo sampler biases motion along the raw-angle lift of the director, antiparallel to the closure drift over 63.6% of the structured region. Rebuilding the closure drift from this verified effective lift reduces the discrepancy to 4.9–9.6 times replicate noise for the dynamic closure, with angular errors of 0.4–1.7 degrees in the locked and intermediate conditions and 10.1–13.9 degrees in the combed condition, and speed ratios of 1.05–1.15. The remaining discrepancy is not monotone in seam severity. These results indicate that representation consistency between microscopic samplers and continuum coefficients is a prerequisite for meaningful closure assessment, and that prospectively specified representation controls can expose inconsistencies that numerical audits alone do not.

Introduction

Velocity-jump processes provide a foundational kinetic description of biological migration: particles move at fixed speed and reorient stochastically at a Poisson rate toward direction distributions set by their local microenvironment [1, 26–33]. In oriented tissues, including white-matter tracts, collagen networks and aligned extracellular matrix, a common computational strategy is to close the kinetic equation at low angular moments and evolve a small set of continuum fields [2–9, 34]. The resulting drift-diffusion and anisotropic-diffusion equations underpin, among other applications, diffusion-tensor-imaging-informed models of glioma invasion [7, 8, 36–40]. The asymptotic foundations are well developed: diffusion limits are established under parabolic scaling [2, 3, 12], moment hierarchies can be constructed systematically [10, 11], and first-moment closures that retain flux dynamics have been derived and analysed [4, 13].

Asymptotic consistency, however, addresses a different question from finite-scale quantitative fidelity. A continuum closure may be asymptotically consistent and numerically converged while still failing to reproduce a kinetic trajectory over the finite spatial and temporal scales used in an application, particularly when the orientation field varies on scales comparable to the kinetic persistence length. In that regime a particle samples a finite neighbourhood during a single run, whereas a local closure represents transport through a restricted set of moments evaluated pointwise.

For polar transport there is an additional, structural subtlety that this paper places at the centre. A nematic orientation field defines a director line at every point: θ and $\theta + \pi$ describe the same structure. A polar drift term, by contrast, requires a vector, and therefore a sign lift of the director field. Any lift is extra information that the structure itself does not supply, and on a heterogeneous field any concrete lift generically produces seams: curves across which the lifted vector reverses while the underlying director line is continuous. In a previous, provenance-controlled stage of this project the lift was locked to the $n_x \geq 0$ convention; a structural audit counted 520 sign-discontinuity edges with a maximum drift jump of 0.1376, approximately twice the maximum local drift magnitude, and both initial conditions lay one grid cell from the nearest seam. Against a canonical polar velocity-jump Monte Carlo reference on that field, none of the tested local closures, including a dynamic first-moment quasi-equilibrium closure with verified fast-relaxation consistency, recovered a prespecified final-time adequacy criterion; the dynamic closure ended 58–64 replicate-noise units from the target, with final directional errors of -74.3 and - 92.6 degrees.

A lift ambiguity carries a sharper danger than seams alone: the microscopic sampler and the continuum coefficients may silently adopt different lifts, in which case the two layers of the comparison solve different transport problems. Here we show that exactly this occurred in the tested implementation, and that it dominates the apparent closure failure. We first executed the single controlled experiment recommended during an independent cross-system audit of this project: a prospectively specified, hash-recorded polarity-seam sensitivity control that varies only the sign representation of the drift field, holding the nematic structure, kinetic parameters, seeds, replicate structure, metric and analysis times fixed. The control returned a striking anomaly, a mirror-image exchange of roles between Monte Carlo and closure across conditions, rather than any monotone seam response. Tracing that anomaly identified an antiparallel inconsistency between the lift realized by the sampler and the lift used by the polar closure coefficient fields, verified by uniform-field tests with directional agreement of exactly plus one and minus one in the two half-spaces of orientation. Rebuilding the closure drift from the verified effective lift reduces the discrepancy by approximately an order of magnitude and reduces directional errors from near-orthogonal to at most 16 degrees, leaving a smaller residual discrepancy of 4.9–9.6 noise units for the dynamic closure that is not monotone in seam severity.

The study is deliberately narrow in scope. The orientation field is synthetic and is placed over a real human T1-derived support mask used only as a controlled geometry; the study is a transport stress test, not a biological or clinical validation. Two initial locations, one field and one kinetic parameter set are examined, and conclusions are stated for the tested configuration. Within that scope the results reframe the earlier finding of this project and, we suggest, carry a general lesson for kinetic-to-continuum comparisons in oriented media: representation consistency is a prerequisite that numerical convergence, asymptotic consistency and workflow audits do not by themselves establish.

Results

A locked heterogeneous polar transport problem and its apparent closure failure

The kinetic reference is a canonical polar velocity-jump process (Methods): particles move at speed $s = 0.09$ and reorient at rate $\lambda = 10$ toward a von Mises distribution centred on the local polar direction, with concentration $2.5 c(x)$ set by the local coherence. The structural field (Fig. 1a) is generated deterministically over a real human T1 coronal slice that supplies an anatomical support mask within the unit square; the orientation field itself is synthetic. Two initial-condition cells are used: a high-coherence off-singularity seed at $(0.5, 0.45)$ and an on-singularity seed at the nematic defect core $(0.5, 0.5792\dots)$. The locked $n_x \geq 0$ lift of the director defines the closure drift field $u = s A_1(k) p$ with $p = n \text{ sign}(n_x)$ (Fig. 1b,c); the frozen evaluation metric combines the final-density RMSE with a scaled final centre-of-mass (COM) velocity error, and adequacy is the ratio of this combined error to the Monte Carlo replicate-noise scale (Methods).

For the production parameters the nominal kinetic run length $s/\lambda = 0.009$ is comparable to the 0.0099 seed-to-seam distance, so the configuration probes a finite-scale regime in which a particle can traverse appreciable directional variation within a single run. In the previous stage of this project, none of the tested local closures recovered adequacy on this problem. We reproduced that comparison here with an independent implementation of the dynamic first-moment closure and of the Chapman-Enskog density closure; every reported headline value was recovered to the printed precision (Table 1): combined errors of 0.4611 and 0.3855 (63.7 and 57.8 noise units), final angular errors of -74.3 and -92.6 degrees, and closure speeds 2.8 - 2.9 times slower than the Monte Carlo reference. One feature of the locked comparison already hints that something other than closure order is at work: the drift-free anisotropic-diffusion comparator achieved combined errors of 0.358 and 0.342 , outperforming every closure that includes the polar drift term.

A prospectively specified polarity-seam control yields a role swap, not a monotone response

An independent cross-system audit of this project recommended exactly one additional controlled experiment for the present revision: a polarity-seam sensitivity control. The construction and endpoints were specified and hash-recorded before any closure-versus-Monte-Carlo endpoint was computed for the non-locked conditions (Methods). Three deterministic sign lifts of the same nematic field were compared (Fig. 2a-c): the locked $n_x \geq 0$ lift (106 drift-carrying seam edges; integrated seam strength, defined as the summed drift jump across sign-flip faces, of 8.41); an intermediate lift combed independently within 16×16 blocks (55 edges; 5.65); and a globally combed lift (0 drift-carrying edges; 0.05). For each condition and cell, three Monte Carlo replicates were generated with the same fixed seeds as the locked stage, and the dynamic closure (primary) and density closure (secondary) were evaluated under the frozen metric, with replicate noise recomputed per condition by the same estimator.

The pre-specified endpoints were the combined metric and adequacy, the COM trajectory at ten mandatory times, the final angular error and the closure-to-Monte-Carlo speed ratio, with no success threshold defined at any point.

The outcome was not a monotone seam response (Fig. 2d-f). Adequacy of the dynamic closure moved from 63.7 and 57.8 noise units (locked, off and on cells) to 33.9 and 29.0 (intermediate) and back up to 54.0 and 53.3 (combed). The final angular error reversed sign across the ladder, from -74.3 and -92.6 degrees under the locked lift to +88.9 and +103.1 degrees under the combed lift, while the speed ratio crossed from strong undershoot (0.36 and 0.34) to strong overshoot (3.34 and 3.28). Most strikingly, the conditions exchanged roles: the combed-condition Monte Carlo behaved like the locked-condition closure (final heading +5.3 degrees at speed 0.0091, against +19.2 degrees at 0.0103), while the combed-condition closure behaved like the locked-condition Monte Carlo (+94.2 degrees at 0.0304, against +93.5 degrees at 0.0285). A mirror-image exchange of this kind is not explained by any mechanism in which seam severity simply modulates a closure deficiency; it indicates that the two layers of the comparison were not solving the same transport problem.

The role swap identifies an antiparallel polarity-lift inconsistency

Tracing the anomaly through the sampling pipeline identified the mechanism. The Monte Carlo sampler evaluates the local director by nematic double-angle interpolation, which necessarily returns the $n_x \geq 0$ representative n_p of the director line; the polar branch then rotates the turning-kernel centre by π wherever the supplied sign field is negative. For the locked lift $s = \text{sign}(n_x)$, the supplied sign flips coincide with the representative's construction flips, and the two cancel: the kernel centre realized by the sampler is the raw-angle lift $e(\theta) = (\cos \theta, \sin \theta)$ with θ in $[0, \pi]$, a field with $e_y \geq 0$ everywhere. The closure coefficients, by contrast, were built from $p = n \text{sign}(n_x)$, a field with $p_x \geq 0$ everywhere. The two lifts describe the same director lines but are antiparallel wherever θ exceeds $\pi/2$, which occupies 63.6% of the positive-coherence support of this field. The nearest cells in that half-space lie approximately one grid spacing from each initial location (Fig. 3b). More generally, for any supplied sign field $s(x)$ the sampler's effective lift is $s n_p$ away from sign-interpolation transitions, so supplied and effective lifts need not coincide.

Two uniform-field tests verify this directly (Fig. 3a): at $\theta = 1.0$ the Monte Carlo drift direction agrees with the closure drift to a normalized inner product of +1.000, while at $\theta = 2.0$ the inner product is -1.000, with identical drift magnitude 0.0656 in both cases (4.7% below the infinite-relaxation value $s A_1 = 0.0688$, as expected at finite λ). The inconsistency also re-orders the seam ladder that the sampler actually experienced (Fig. 3c): the effective lift of the locked condition is nearly seam-free (one drift-carrying edge; integrated strength 0.003), whereas the effective lifts of the intermediate and combed conditions carry integrated strengths of 14.07 and 8.46. The antiparallel half-space occupies 63.6% of the positive-coherence support, and its nearest cells lie approximately one grid spacing from

both initial locations. Under this re-ordering, the role swap, sign reversal and undershoot-overshoot crossover observed in the pre-specified control are all consistent with the two layers transporting along different lifts of the same director field.

This also explains why the inconsistency escaped a sixteen-stage fail-closed audit harness in the earlier stage. The numerical qualification suite verified coefficient identities, homogeneous relaxation, asymptotic recovery of the density closure and conservation properties, all on fields whose directors lay in the consistent half-space; the identity of the two lifts was an unstated assumption shared by the specifications that the gates enforced. A representation control varies precisely what such gates hold fixed.

Lift-consistent closures collapse the discrepancy by an order of magnitude

The diffusion tensor and the quasi-equilibrium second moment are quadratic in the lift and therefore identical under either convention; only the drift $u = s A1(k) p$ changes sign. Rebuilding u from the verified effective lift of each condition and repeating the full comparison, with the metric, targets and noise estimator unchanged, substantially reduces the discrepancy (Fig. 4; Table 2). For the dynamic closure the combined error falls to 0.033-0.075, corresponding to 4.9-9.6 noise units across all six condition-cell pairs, against 29.0-63.7 under the inconsistent lift. Final angular errors fall from the range -92.6 to +103.1 degrees to 0.4-1.7 degrees in the locked and intermediate conditions and 10.1-13.9 degrees in the combed condition; speed ratios move from 0.34-3.34 to 1.05-1.15. The density closure behaves similarly, with angular errors of 0.1-1.2 degrees in the locked and intermediate conditions and 11.8-16.0 degrees in the combed condition (Table 2). The lift-consistent drift closures now also outperform the drift-free anisotropic-diffusion comparator by a factor of approximately five to ten in combined error, restoring the expected ordering for this tested polar-transport configuration.

An internal identity provides a strong correctness check: in the drift-carrying region the effective lift of the combed condition coincides with the supplied lift of the locked condition, so the lift-consistent closure of the combed condition must reproduce the inconsistent closure of the locked condition exactly. It does, to all printed digits (final headings +19.2 and -0.6 degrees in the off and on cells).

A residual discrepancy remains after lift alignment

The corrected comparison leaves a residual discrepancy of 4.9-9.6 noise units for the dynamic closure. This is a measured difference between the tested local closure and the kinetic reference after lift alignment, not by itself evidence for a unique residual mechanism. Its structure is non-monotone in aggregate seam severity of the effective lift: the intermediate condition carries the heaviest effective seam load (integrated strength 14.07) yet shows angular errors of only 1.7 degrees, while the combed condition (8.46) retains larger directional residuals of 13.9 and 10.1 degrees. This pattern is compatible

with a role for the spatial placement of seams relative to the transported density, but three conditions are insufficient to establish that explanation. Speed ratios after alignment range from 0.98 to 1.15 across the two closures. A kinetic relaxation lag is one possible contributor to the remaining mismatch because particles retain path history over a time scale of order $1/\lambda$ whereas a local closure responds to local coefficients, but the present data do not isolate that mechanism. The finite-scale question of how faithfully a local closure tracks kinetic transport when structural variation is comparable to the run length therefore remains open after the representation layers are made consistent.

Numerical verification and provenance

All quantitative claims rest on verified reproductions. The separate dynamic-closure implementation reproduced the previous stage's production time-step bound to five significant figures ($4.1267\text{e-}4$) and every headline value of the locked comparison to the printed precision (Table 1); mass was conserved to a relative change of at most $3.1\text{e-}16$ with no negativity clipping. The density-closure solver reproduced the previous stage's final-time comparators to 0.05% (on cell) and 0.6% (off cell), and a single internally consistent solver family was used across all conditions. The Monte Carlo variant that injects an arbitrary sign field was validated bit-for-bit against the canonical sampler on the locked condition (identical trajectories and histograms), and the canonical five source files of the previous stage were hash-verified before use. The seam-control construction and endpoints were prospectively specified and hash-recorded before closure-versus-Monte-Carlo endpoints were computed for non-locked conditions according to the retained workflow record; the lift-inconsistency diagnosis and lift-consistent re-analysis were performed post hoc. The XJ and XC solver hashes recorded in the prospective plan correspond to the pre-diagnostic solver versions. The final package versions add only an optional `p_override` interface for the post-hoc lift-consistent analysis; removing that interface reconstructs the recorded solver files exactly, and rerunning the pre-specified endpoints with those exact bytes reproduces the science payload.

Discussion

For the heterogeneous polar transport problem studied here, the apparent finite-scale failure of an entire ladder of local moment closures is dominated not by closure order but by a silent inconsistency between two lifts of the same nematic director field: the raw-angle lift realized by the kinetic sampler and the $n_x \geq 0$ lift used to build the polar closure coefficients. Once the closure drift is constructed from the verified effective lift, the same closures that appeared to fail by 58–64 noise units with near-orthogonal directional errors instead track the kinetic reference to 4.9–9.6 noise units for the dynamic closure, with directional errors at or below 16 degrees and near-unit speed ratios. The earlier interpretation of this project must therefore be superseded: the large directional mismatch was dominated by representation inconsistency rather than demonstrating an intrinsic failure of the tested moment closures.

We believe the general lesson is worth stating plainly. In kinetic-to-continuum comparisons over oriented media, the polar lift of a nematic structure is not part of the structure; it is a modelling choice that must be declared once and realized consistently in every layer of the computation. Asymptotic consistency, numerical convergence, conservation checks and even an aggressive fail-closed audit architecture do not by themselves test that identity, because each layer can be individually correct with respect to its own lift. The sixteen-stage harness of the previous stage verified what its specifications stated; the shared, unstated assumption between specifications was exactly where the inconsistency lived. What exposed it here was a low-cost, prospectively specified representation control: three deterministic re-lifts of the same field with fixed endpoints produced a role-swap pattern that was inconsistent with a simple monotone seam-severity explanation. Representation-sensitivity controls and homogeneous-field sweeps across both orientation half-spaces may therefore be useful qualification checks for polar transport comparisons.

The corrected comparison also sharpens, rather than disposes of, the finite-scale question that motivated this study. On a field whose structural variation is comparable to the kinetic run length, lift-consistent local closures perform far better than the inconsistent comparison suggested, but a residual of 4.9–9.6 noise units remains for the dynamic closure. Its directional component is largest in the combed condition, whereas the intermediate condition has the largest aggregate effective seam strength but much smaller angular errors. Thus the three tested conditions do not support a monotone relation between domain-integrated seam severity and residual discrepancy. Spatial seam placement, kinetic relaxation memory, higher angular moments, nonlocal effects or other finite-scale features are possible contributors that require separate controlled tests; the present configuration, now with matched representations, provides a defined benchmark for such work.

These findings are relevant to applied settings in which polar drift terms are constructed from nematic structure data. Diffusion-tensor imaging supplies director lines rather than oriented vectors, and any model that adds an advective component or samples a polar direction must introduce a lift convention. The present results show that, in the tested implementation, two individually qualified components can disagree by a drift reversal over a large fraction of the structured field while conventional numerical checks still pass. Explicit lift declaration, cross-layer homogeneous tests and representation controls are therefore inexpensive safeguards against this class of implementation mismatch.

The remaining discrepancy should not be over-interpreted. The study examines one synthetic orientation field over a T1-derived anatomical support mask, two initial conditions and one kinetic parameter set. The aligned comparison still differs from the Monte Carlo reference by several replicate-noise units, but the present experiment was designed to detect representation sensitivity, not to identify the cause of that residual. Aggregate seam severity is non-monotone with the residual, and the suggested roles of seam placement or relaxation memory remain hypotheses for future controlled tests. Finally, although the diagnosis explains the earlier result, the earlier numbers themselves remain exactly reproducible and are retained here as the inconsistent-lift arm of the comparison; this study changes their interpretation, not their numerical provenance.

Within these limits, the study converts an apparent negative result about moment closures into two sharper statements: a representation-consistency requirement that belongs in the qualification layer of polar kinetic-continuum comparisons, and a corrected, much smaller residual discrepancy that now poses a well-defined benchmark question for closure theory in strongly heterogeneous oriented media.

Methods

Kinetic model and structural field

Particles at position x in the unit square with heading e on the unit circle satisfy a velocity-jump process with speed $s = 0.09$ and turning rate $\lambda = 10$; at each turn the new heading is drawn from a von Mises distribution $q(x, e)$ proportional to $\exp[k(x) p_{\text{eff}}(x, e)]$ with concentration $k(x) = 2.5 c(x)$. The structural field $(\theta(x), c(x))$ is generated deterministically as in the previous stage: a real human T1 coronal slice from the dipy sample data [24] (SHA-256 verified at runtime) is thresholded and resampled to an anatomical support mask on an $N = 101$ cell-centred grid, over which a synthetic coronal orientation field is constructed; the mask is used only for support and diagnostics, never as a solver boundary. The two seeds are $(0.5, 0.45)$ and $(0.5, 0.5792079207920792)$, the latter re-derived at every run from the locked minimum-coherence rule. The five canonical source files of the previous stage were used unmodified and hash-verified (SHA-256) before every run.

Monte Carlo targets and the sign-field variant

For each condition and cell the target is three replicates of 30,000 particles with fixed seeds (993000 + 137k, $k = 0, 1, 2$), Monte Carlo time step 0.005, horizon $T = 6$ and no proliferation. Final histograms are Gaussian-smoothed ($\sigma = 2$ cells) and max-normalized exactly as in the upstream workflow; the replicate-noise scale is the mean pairwise distance among replicates in the combined metric, recomputed per condition. The COM trajectory is recorded at the ten mandatory times 0, 0.025, 0.05, 0.1, 0.2, 0.5, 1, 2, 4 and 6. To vary the polarity representation, a variant sampler was derived from the canonical function whose single change is that the polarity sign field is supplied by the caller; because the sign construction consumes no random numbers, the variant reproduces the canonical sampler bit-for-bit when supplied with the locked sign field, which was verified directly (identical center-of-mass traces and final histograms).

The sampler evaluates the local director by double-angle interpolation, $\cos(2\theta)$ and $\sin(2\theta)$, which returns the $n_x \geq 0$ representative n_p of the director line; the polar branch then rotates the kernel centre by π where the supplied sign s is negative. The kernel centre realized in region interiors is therefore $s n_p$: for the locked lift $s = \text{sign}(n_x)$ this equals the raw-angle lift $e(\theta) = (\cos \theta, \sin \theta)$, θ in $[0, \pi]$. This identity was confirmed by the uniform-field tests below and defines the effective lift used in the consistent re-analysis.

Closure ladder and coefficients

The Chapman-Enskog density closure evolves the density with drift $u = s A_1(k) p$ and diffusion tensor $D = (s^2/\lambda)[(1 - A_2)/2 I + (A_2 - A_1^2) p p^T]$, where $A_j(k) = I_j(k)/I_0(k)$ are modified-Bessel ratios [14]. The dynamic first-moment quasi-equilibrium closure evolves density ρ and flux J with $d\rho/dt + \text{div } J = 0$ and $dJ/dt + \text{div } \Pi = \lambda(\rho u - J)$, the second moment closed at quasi-equilibrium, $\Pi = \rho M$ with $M = \lambda D + u u^T = s^2[(1 - A_2)/2 I + A_2 p p^T]$; $J(x, 0) = 0$ matches the uniform initial heading distribution. No parameter is fitted in either closure. Both D and M are quadratic in the lift and hence identical under either convention; only u changes. In the lift-consistent re-analysis u is built from the verified effective lift $s_{n,p}$ of each condition; in the inconsistent (previous-stage) arm it is built from s_n .

Numerical schemes and verification

The dynamic closure is solved with a Rusanov (local Lax-Friedrichs) finite-volume flux with local wave speeds $\sqrt{M_{xx}}$ and $\sqrt{M_{yy}}$ [15, 41–44], an integrator that treats the stiff relaxation term exactly, and reflecting ghost cells (density Neumann; antisymmetric normal flux; symmetric tangential flux), at the production resolution $CFL = 0.003125$. The independent implementation written for this study reproduces the previous stage's step bound ($4.1267e-4$) to five significant figures and every headline value of the locked comparison to the printed precision (Table 1); relative mass change was at most $3.1e-16$ with no clipping. The density closure is solved with the canonical conservative finite-volume solver of the previous stage (full tensor with cross terms; upwind advection); on the locked condition it reproduces the previous stage's final-time comparators to 0.05% (on cell) and 0.6% (off cell). Uniform-field verification runs ($c = 1$, $\theta = 1.0$ and 2.0 ; 20,000 particles, $T = 2$) gave Monte Carlo drift directions with normalized inner products of $+1.000$ and -1.000 against the closure drift, at magnitude 0.0656 , 4.7% below the infinite-relaxation value $s A_1(2.5) = 0.0688$.

Frozen metric and endpoints

The frozen primary metric is $E_{\text{combined}} = E_{\text{density}} + 0.5 E_{\text{COM}}$, where E_{density} is the RMSE between max-normalized final densities (closure field against the mean of the three replicate densities) and E_{COM} is the final COM-velocity error normalized by $V_{\text{SCALE}} = 0.04$, taken against the replicate-mean Monte Carlo COM velocity. Adequacy is E_{combined} divided by the per-condition replicate noise. Endpoints specified prospectively in the hash-recorded control plan were E_{combined} and adequacy; the COM trajectory at the ten mandatory times; the final angular error between closure and Monte Carlo COM-velocity headings; and the speed ratio. No success threshold was defined.

Seam constructions and audits

Three deterministic sign lifts were constructed before any outcome was computed. Locked: $s = \text{sign}(\cos \theta)$ with $\text{sign}(0) = +1$. Combed: greedy priority-first combing that processes cells in decreasing coherence and chooses each sign to align the lifted vector with the mean over already-assigned neighbours. Intermediate: the same combing applied independently within 16×16 blocks, chosen on structural statistics alone to give approximately half the locked drift-carrying seam count. Seam audits count 4-adjacent sign-flip faces (the all-face count reproduces the previous stage's 520-edge audit and 0.1376 maximum drift jump exactly), the drift-carrying subset with coherence positive on both sides, the

maximum drift jump and the integrated seam strength (summed drift jump). The same audit applied to the effective lift s_{n_p} gives the seam structure the sampler experienced: locked 1 carrying edge (integrated strength 0.003), intermediate 160 (14.07), combed 105 (8.46). In the drift-carrying region the effective lift of the combed condition equals the supplied lift of the locked condition, giving the internal identity check reported in Results.

Prospective specification and post hoc labelling

The control's constructions, qualification evidence, endpoints and analysis rule were recorded in a prospective analysis-plan document (SHA-256 5ed5e53c913099edce5a6df3268472d172fbe1c871f1451758fdf59d350fb68a) before closure-versus-Monte-Carlo endpoints were computed for non-locked conditions, according to the retained workflow record. Monte Carlo generation logs containing COM velocities had already been emitted and are disclosed as such; no closure-versus-Monte-Carlo discrepancy endpoint for the non-locked conditions had been computed at that point. The plan was internally hash-recorded rather than deposited in an external timestamped registry, so we describe the experiment as prospectively specified rather than formally preregistered. The plan records XJ and XC solver hashes f36b9b91152947b853fb7a7dc332342208878072c7cab22ba31dbd9843978813 and c4f9d16ed23a2a663816b3d96ca0db27ca879884cf5c15b451719b3b47b3cb94. The final solver files add only the optional `p_override` interface required for the post-hoc lift-consistent re-analysis; deleting that interface reconstructs the plan-recorded files byte-for-byte, and the pre-specified endpoint payload is reproduced with those reconstructed versions. All lift-inconsistency diagnosis, effective-lift audits and lift-consistent re-analysis were post hoc. A null or opposite outcome of the control would have been retained; no success threshold was introduced.

Software environment, roles and statistics

Computations used Python with NumPy 2.4.4, SciPy 1.17.1 and dipy 1.12.1 [22–24], matching the environment identity of the previous provenance-controlled stage (matplotlib 3.10.9 against 3.10.8 for figure rendering only [53]); the T1 input hash matched exactly. Anthropic Claude supported implementation and execution of the prospectively specified control and the subsequent diagnosis and re-analysis under human direction; OpenAI ChatGPT provided a separate cross-system audit, recommended the seam-sensitivity control and revision structure, and subsequently independently reproduced the frozen endpoint payload and the lift-consistent science payload. All AI-assisted outputs were subject to human review; the human authors retain responsibility for the scientific content. Uncertainty is summarized by per-condition replicate-noise scales; adequacy ratios and noise-unit effect sizes are reported rather than null-hypothesis significance tests, and no fitting or threshold adjustment was performed at any stage.

Declarations

Data availability

The complete package underlying this study, canonical source files, the seam-control and diagnosis code, the prospective analysis-plan document, result ledgers with all endpoint values, per-file SHA-256 manifests and figure-generation scripts, accompanies this submission and will be deposited in a public repository. The T1 input is the standard dipy sample dataset [24].

Code availability

All execution-critical code used for the Monte Carlo targets, both closure solvers, the seam constructions and audits, the effective-lift diagnosis, the metrics and the figures is included in the same package and is sufficient to reproduce every reported value in the stated software environment.

Author contributions

Takenari Fujio conceived the study, specified and enforced the audit and prospective specification protocol, brokered transfers between the two AI systems, supervised the control experiment and its adjudication, and drafted and revised the manuscript. Hiromi Fujio provided critical administrative and logistical support for protocol coordination, data archiving and independent cross-validation oversight. Both authors approved the submitted version and accept responsibility for their respective contributions. AI-system roles are described in Methods and in the AI usage statement; no AI system is listed as an author.

Competing interests

The authors declare no competing interests.

Funding

This research received no external funding.

Ethics declaration

Not applicable. This is a purely computational study involving no human or animal subjects, no collection of primary data from participants and no clinical material. The anatomical geometry derives from a standard, publicly available sample dataset (dipy T1 sample data), and all orientation fields are synthetic; no ethics approval or participant consent was therefore required.

AI usage statement

Two AI systems were used as documented research tools under continuous human direction. Anthropic Claude supported implementation, numerical qualification, execution of the prospectively specified seam-sensitivity control, the lift-inconsistency diagnosis and factual reporting; OpenAI ChatGPT supported a separate cross-system audit, recommended the seam-sensitivity control as the single additional experiment for this revision, and independently reproduced the frozen and lift-consistent endpoint payloads from the supplied source and replicate artifacts. All AI-assisted outputs entering the

scientific record were subject to human review. Neither AI system is an author; the corresponding human author retains responsibility for the accuracy and integrity of the manuscript.

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Tables

Table 1. The locked-condition comparison: previous stage against the independent re-implementation of this study (dynamic first-moment closure). MC, Monte Carlo.

Quantity	OFF		ON	
	Previous	This work	Previous	This work
E_{density}	0.11632	0.11632	0.09908	0.09907
E_{COM}	0.68953	0.68949	0.57279	0.57274
E_{combined}	0.46109	0.46107	0.38548	0.38548
Adequacy ($\times$ noise)	63.72	63.72	57.8	57.8
Closure heading (deg)	+19.2	+19.18	-0.6	-0.63
Final angular error (deg)	-74.32	-74.31	-92.62	-92.61
Closure / MC speed	0.01031 / 0.02852	0.01031 / 0.02852	0.00731 / 0.02138	0.00731 / 0.02138

Table 2. Pre-specified control endpoints and the lift-consistent re-analysis (XJ: dynamic first-moment closure, primary; XC: Chapman-Enskog density closure, secondary). Per-condition replicate noise (off / on): locked 0.00724 / 0.00667; intermediate 0.00804 / 0.00819; combed 0.00974 / 0.00794. Rows labelled 'n_x' use the previous stage's closure drift $p = n \text{ sign}(n_x)$; rows labelled 'sampler' use the verified effective lift. The sampler-lift combed rows coincide with the n_x-lift locked rows through the internal identity described in Results.

Condition	Cell	Lift	XJ				XC			
			E_{comb}	Adeq.	Ang. (°)	Speed	E_{comb}	Adeq.	Ang. (°)	Speed
locked	off	n_x	0.4611	63.7	-74.3	0.36	0.4606	63.6	-72.2	0.34
		sampler	0.0694	9.6	+0.7	1.07	0.0540	7.5	+0.7	1.04
	on	n_x	0.3854	57.8	-92.6	0.34	0.3782	56.7	-90.9	0.31
		sampler	0.0330	4.9	+0.4	1.05	0.0431	6.5	+0.1	1.04
intermediate	off	n_x	0.2729	33.9	-38.8	0.61	0.2633	32.7	-35.6	0.59
		sampler	0.0745	9.3	+1.7	1.15	0.0619	7.7	+1.2	1.10
	on	n_x	0.2373	29.0	-47.6	0.53	0.2226	27.2	-41.1	0.52
		sampler	0.0504	6.1	+1.7	1.11	0.0643	7.8	+1.0	1.08
combed	off	n_x	0.5265	54.0	+88.9	3.34	0.4888	50.2	+88.9	3.24
		sampler	0.0598	6.1	+13.9	1.13	0.0810	8.3	+16.0	1.07
	on	n_x	0.4231	53.3	+103.1	3.28	0.4118	51.9	+102.8	3.24
		sampler	0.0424	5.3	+10.1	1.07	0.0665	8.4	+11.8	0.98

Figures

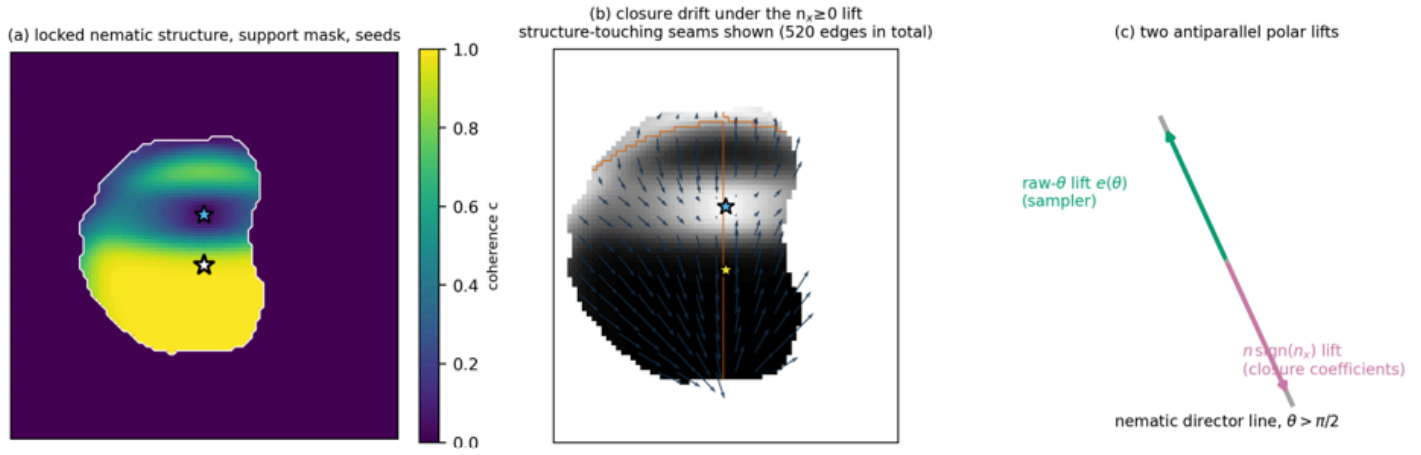


Figure 1

The locked heterogeneous polar transport problem and the polarity-lift ambiguity. (a) Orientation coherence c with the T1-derived support contour and the two seeds (white star: off-singularity; blue star: on-singularity). (b) Closure drift magnitude under the locked $n_x \geq 0$ lift with structure-touching sign-discontinuity seams (orange; 520 edges in total over the domain) and drift directions (arrows). (c) The two polar lifts of one director line for $\theta > \pi/2$: the raw-angle lift realized by the sampler and the $n_x \geq 0$ lift used by the closure coefficients are antiparallel.

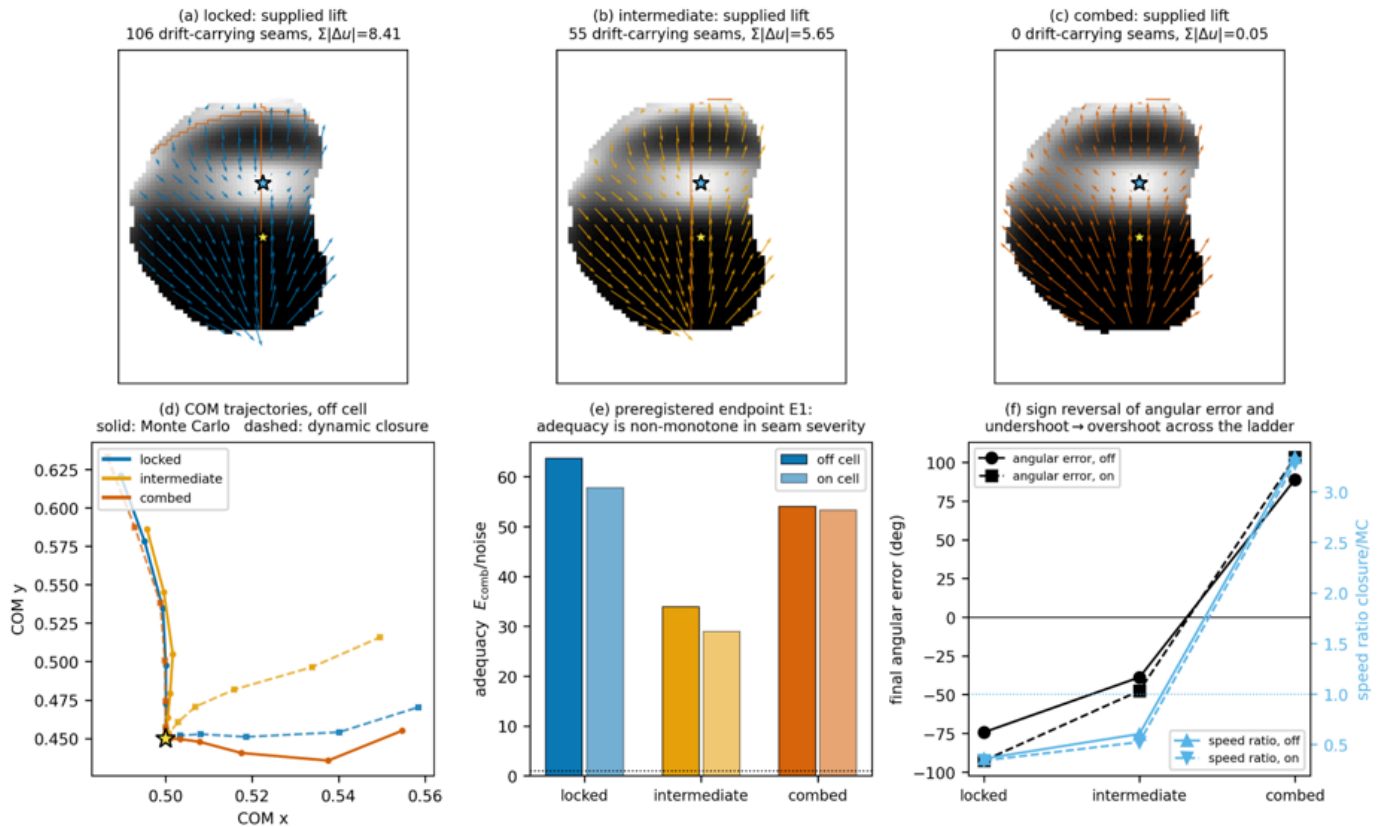


Figure 2

The pre-specified polarity-seam sensitivity control returns a role swap, not a monotone seam response. (a-c) Supplied sign lifts: locked (106 drift-carrying seams), block-combed intermediate (55) and globally combed (0), with drift directions and structure-touching seams. (d) COM trajectories in the off cell for the three conditions (solid: Monte Carlo; dashed: dynamic closure); the combed Monte Carlo mimics the locked closure and the combed closure mimics the locked Monte Carlo. (e) Pre-specified adequacy endpoint: non-monotone in seam severity. (f) Final angular error reverses sign and the speed ratio crosses from undershoot to overshoot across the ladder.

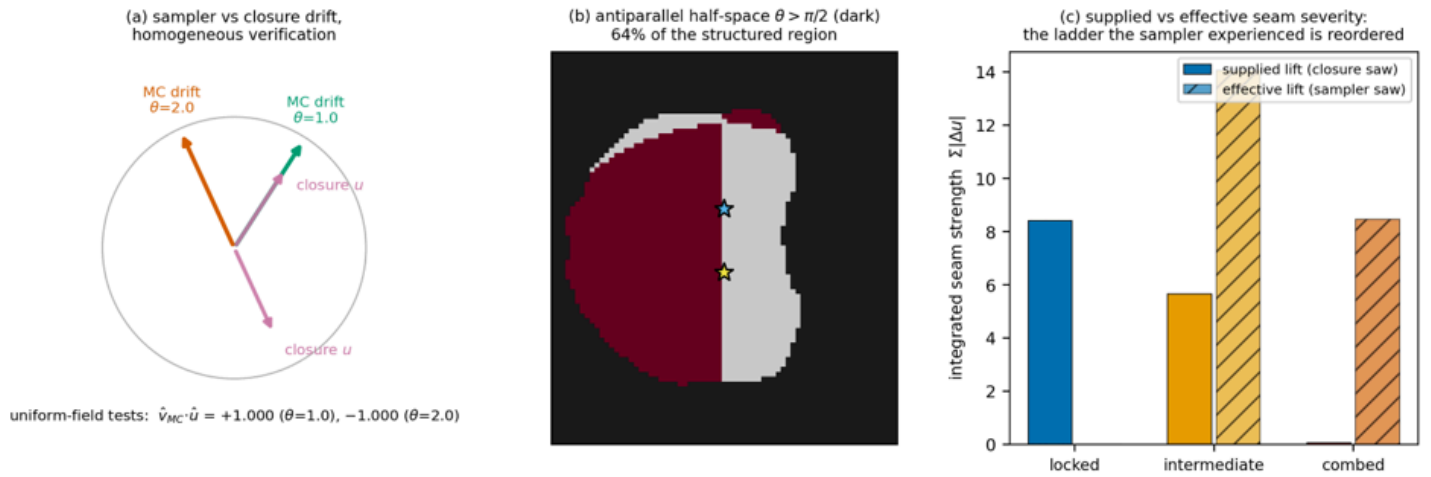


Figure 3

Diagnosis of the role swap: an antiparallel polarity-lift inconsistency. (a) Uniform-field verification: at $\theta = 1.0$ the Monte Carlo drift is parallel to the closure drift (normalized inner product $+1.000$); at $\theta = 2.0$ it is antiparallel (-1.000). (b) The half-space $\theta > \pi/2$ (dark), where the sampler's raw-angle lift and the closure's $n_x \geq 0$ lift are antiparallel, covers 64% of the structured region, including both seed neighbourhoods. (c) Integrated seam strength of the supplied lift (what the closures transported along) against the effective lift (what the sampler transported along): the ladder the sampler experienced is reordered, and the locked condition is effectively seam-free.

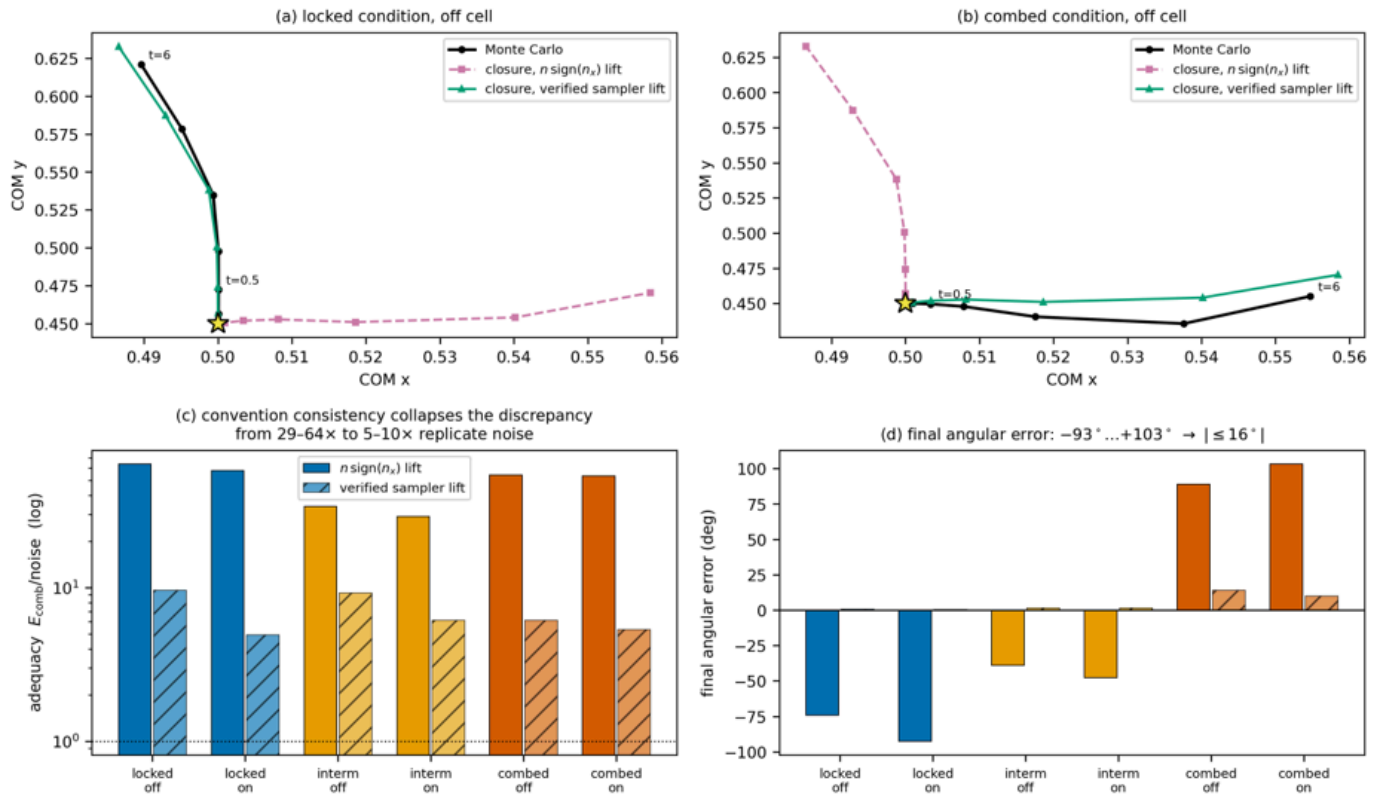


Figure 4

Lift-consistent closures collapse the discrepancy. (a) Off-cell COM trajectories in the locked condition: the closure built on the $n_x \geq 0$ lift (dashed) departs orthogonally from the Monte Carlo reference (black), whereas the closure built on the verified sampler lift (green) tracks it. (b) Same for the combed condition, where the roles of the two lifts are exchanged. (c) Adequacy for all six condition-cell pairs (dynamic closure, log scale): consistency reduces 29-64x noise to 5-10x. (d) Final angular errors fall from the range -93 to +103 degrees to at most 16 degrees.