

Spectral Memory of an Evolving Localization Length – Quantum Coherence, Clock Reconstruction and Intergalactic Constraints

ANISSE ZEROUTA

a.zerouta@precogtec.com

Precog Technologies– Theoretical Physics Division <https://orcid.org/0000-0001-7966-0746>

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Spectral Memory of an Evolving Localization Length

Quantum Coherence, Clock Reconstruction and Intergalactic Constraints

Anisse Zerouta

Theoretical Physics Division, Precog Technologies, Paris, France.

a.zerouta@precogtec.com

Abstract

Can an evolving localization length leave a mechanical record from which its history can be reconstructed? We study a completely positive momentum-transfer channel in which one geometric aperture controls both event rate and transfer width. On the constant-winding area branch, the accumulated radial spectrum develops a q^{-3} reference tail whose amplitude fixes the coefficient of the associated $d^2 \ln(R/d)$ coherence law. A Gaussian finite-size response closes the channel exactly: the coherent event count generates all even transfer moments, and one derivative fixes both the small-separation coherence curvature and the nonrelativistic energy input. For an arbitrary strictly increasing aperture history, Mellin factorization reconstructs the event occupation per logarithmic scale; with a known rate functional this determines the normalized aperture clock. A logarithmic stability bound, a cumulative-chronology estimate and finite-band noise ensembles quantify what remains recoverable in practice. We then propagate a specified proton-alpha realization through an expanding plasma. Low-redshift intergalactic-medium (IGM) temperatures impose a conservative coherent-channel ceiling of about 200–260 eV per nucleus. Extending the temperature-slope fit by three aperture decades removes the former grid-edge minimum and produces a broad χ^2 plateau: the shallow minimum is $\chi^2 = 20.59$, with 18.68 contributed by the measured density slopes. The result therefore constrains the homogeneous heating realization through its linked spectral, coherence and transport predictions rather than through an independently adjustable heating amplitude.

Keywords: quantum localization, momentum transfer, finite-size response, coherence, spectral memory, inverse problem, intergalactic medium, DJAMILAR

1 Introduction

A mechanism that attenuates spatial quantum coherence also perturbs mechanical motion. In spontaneous-localization models, the perturbation is often summarized by a diffusion coefficient or a mean heating rate [1–4]. General reviews establish the theoretical and empirical context [5, 6], while non-interferometric tests connect mechanical noise, heating and radiation to the underlying channel [7]. Those moments are powerful constraints, but they do not determine the distribution carrying the energy. A dilute energetic population and a small temperature increase of an entire gas can have the same mean energy and very different subsequent evolution.

This distinction becomes a source of information when the localization length evolves. A small aperture permits broad momentum transfers; a larger aperture produces narrower transfers. If the same aperture also controls the event rate, the accumulated mechanical spectrum records how strongly each scale was occupied. The record can survive even though the instantaneous generator is Markovian and completely-positive divisible.

Translation-covariant open-system dynamics supplies the relation between momentum-transfer statistics and coherence loss [8–11]. Broad distributions generated by mixtures of narrower ones are also familiar from superstatistics [12, 13]. The physical question is therefore not whether a scale mixture can produce a broad tail, but whether one specified history fixes additional observables that cannot be tuned independently. Here the same aperture history fixes event counts, spectrum, coherence attenuation and energy moments.

The first result is a finite-size consistency structure. On the constant-winding area branch, a q^{-3} radial tail fixes the coefficient of a logarithmically enhanced quadratic coherence law. A Gaussian target response then provides an exact closure: differentiation of the coherent event count generates all even transfer moments and ties the small-separation coherence curvature to the nonrelativistic energy input. A cutoff introduced through target response therefore cannot be changed independently of the count, coherence and energy budget.

The second result is an inverse statement. Constant winding is not required for the information result. For any strictly increasing aperture history, the unfiltered point-domain spectrum, or a spectrum deconvolved by a known nonvanishing aperture-independent response, determines the event occupation per logarithmic aperture interval. When the rate functional is known, that occupation reconstructs the normalized aperture clock, and one external time calibration fixes its absolute normalization. The inversion is exponentially ill-posed, so we derive a logarithmic stability bound and a cumulative-chronology estimate, then test the practical reconstruction with finite-band noisy ensembles.

Throughout this paper, DJAMILAR denotes the research programme built around postulated microscopic filtration units called *Djamilars*, from which the aperture concept originated. The present work uses the programme only to motivate the aperture dictionary, activation law and a specified cosmological benchmark. The spectral, finite-response and inverse results are properties of the stated effective channel and do not require an independent gravitational field equation. Appendix G records the provenance of those ingredients.

After deriving the general spectral and finite-size relations, we propagate a specified proton–alpha source through an expanding homogeneous plasma. This gives a controlled forward model for the low-redshift intergalactic medium (IGM). The IGM application is deliberately used as a falsifiability test: the temperature normalization can be fitted over a broad aperture plateau, but the predicted density slopes remain the dominant residual. The separate question of how the same channel acts once a composite target is internally resolved is not needed for the theorems developed here and is treated in the submitted companion analysis of composite response and Solar-System paleo-records [14].

2 A geometric protocol with coherent target response

2.1 Aperture clock and activation

Let $r > 0$ be the effective localization aperture. The DJAMILAR state dictionary is

$$C_d = \pi n_d r^2, \quad \vartheta = \ln(C_d/C_{d,\text{ref}}), \quad \frac{\dot{r}}{r} = \frac{1}{2} \left(\dot{\vartheta} - \frac{\dot{n}_d}{n_d} \right), \quad (1)$$

where n_d denotes the proper geometric population density. We select the branch

$$r(t) = r_0 e^{\kappa(t-t_0)}, \quad \kappa = Q_w/2 > 0. \quad (2)$$

Here $Q_w \equiv \dot{\vartheta} - \dot{n}_d/n_d = 2\dot{r}/r$ is the winding rate, taken constant on this branch. A constant κ is therefore a protocol assumption, corresponding to a fixed value of the combination in Eq. (1). It need not imply a constant geometric density or a universal winding parameter in other sectors.

For a stable target of mass m and activation radius R , prescribe

$$\Gamma(r) = \frac{A r^b}{1 + (r/R)^p}, \quad A = \frac{\gamma_0}{r_0^b} \left(\frac{m}{m_u} \right)^\alpha, \quad 0 < b < p. \quad (3)$$

Here m_u is the atomic mass constant and γ_0 has units of inverse time. The branch $b = 2$ is the open-area branch of Refs. [15, 16] and is controlled by aperture area; p sets the suppression for $r \gg R$. The application uses $(\alpha, b, p) = (1, 2, 4)$. The activation radius R and the internal response size introduced below have distinct roles and need not be equal.

The benchmark exponents are constrained, but it is important to separate the constant-winding result from a general aperture history. Define

$$\omega(r) \equiv \frac{d \ln r}{dt} > 0. \quad (4)$$

For the unfiltered point-domain base channel below the activation radius, an event at aperture r transfers a mean energy proportional to $\hbar^2/(mr^2)$, while the number of

events per logarithmic aperture interval is Γ/ω . Hence

$$\frac{d\mathcal{E}}{d \ln r} \propto \frac{m^{\alpha-1} r^{b-2}}{\omega(r)}, \quad r \ll R. \quad (5)$$

For the constant-winding branch of Eq. (2), $\omega = \kappa$, and $(\alpha, b) = (1, 2)$ is the unique choice within Eq. (3) for which every e-fold below R injects the same energy into every target, independent of both aperture and target mass. This uniform-injection statement is therefore a model-selection property of the constant-winding, point-domain base rate rather than an assertion that the final finite-response target receives equal energy at every aperture: the response of Sect. 4 reshapes the event count and energy profile at $r \lesssim s$. Independently, $b = 2$ is the open-area branch of Refs. [15, 16]. Exact scale-inversion symmetry about the activation radius, $\Gamma(R^2/r) = \Gamma(r)$, then requires $p = 2b = 4$. For the constant-winding benchmark the normalized occupation of unfiltered events per logarithmic aperture interval is a hyperbolic secant centred on the crossing,

$$\frac{1}{N} \frac{dN}{d \ln y} = \frac{2}{\pi} \operatorname{sech}(2 \ln y), \quad y = \frac{r}{R}. \quad (6)$$

Within Eq. (3), these two conditions leave no free exponent; γ_0 , r_0 and the response size remain the only inputs.

For $r \gg R$, Eq. (3) falls as $\Gamma \simeq AR^p r^{b-p}$. With the benchmark parameters of Sect. 6, the present unfiltered activation rates are $\Gamma_p(r_0) = 1.03 \times 10^{-39} \text{ s}^{-1}$ per proton and $\Gamma_\alpha(r_0) = 6.50 \times 10^{-38} \text{ s}^{-1}$ per alpha particle. The nuclear channel is therefore effectively closed today and its direct benchmark content is a cosmological relic.

For the conditional cosmological realization of Sect. 6, one activation target is one nucleus; atoms, molecules and bulk matter are not treated as single activation targets, and electrons are not activated directly. This target rule is part of the specified application rather than of the spectral theorem. The connection to collective-domain prescriptions is not used in the present IGM calculation.

2.2 Local aperture clocks

Equation (1) also fixes the aperture seen by a target in a region of different geometric density. If ϑ is a universal function of time, so is C_d , and two regions with densities n_d and $n_{d,\text{loc}}$ satisfy

$$r_{\text{loc}}(t) = r(t) \left[\frac{n_d(t)}{n_{d,\text{loc}}(t)} \right]^{1/2}. \quad (7)$$

A different density changes the level of the clock, not only its rate. Which matter is activated, and when, therefore depends on the physical identity of n_d . Three readings bracket the possibilities. In the uniform reading (U), n_d is not tied to local matter and every target sees the cosmological clock. In the frozen reading (F), n_d dilutes with the expansion until a structure decouples from the Hubble flow at scale factor a_{dec} ; bound matter then has $r_{\text{loc}} = r (a_{\text{dec}}/a)^{3/2}$ and crosses later, while unbound intergalactic gas keeps the cosmological clock. In the matter reading (M), n_d follows the local matter density ρ , so that $r_{\text{loc}} = r (\bar{\rho}/\rho)^{1/2}$. Expanding gas at constant overdensity Δ then

crosses $\ln \sqrt{\Delta}/\kappa$ later than mean-density gas, and a bound region has been activated by today only if $\rho \leq \bar{\rho}(r_0/R)^2$. The benchmark of Sect. 6 assumes that intergalactic targets share the mean geometric density; readings U and M are retained below as alternative prescriptions for the diffuse-gas clock.

The local-clock distinction does not affect laboratory stationarity. At the benchmark $\kappa = Q_w/2 = 1.393 \times 10^{-16} \text{ s}^{-1}$, so the fractional aperture drift is only 4.40×10^{-9} per year. A fixed-aperture approximation is therefore excellent over laboratory timescales even though cosmological propagation retains the long history.

2.3 The effective channel

In the chosen rest frame, let

$$\begin{aligned} \dot{\rho} = & -\frac{i}{\hbar}[H(\mathbf{P}), \rho] + \Gamma(t) \int d^3q \mu_r(q) \mathcal{R}(q) (U_{\mathbf{q}} \rho U_{\mathbf{q}}^\dagger - \rho), \\ U_{\mathbf{q}} = & e^{i\mathbf{q} \cdot \mathbf{X}/\hbar}, \quad \mu_r \geq 0, \quad \int \mu_r d^3q = 1, \quad 0 \leq \mathcal{R}(q) \leq 1. \end{aligned} \quad (8)$$

The dissipator includes the response in both its gain and loss terms. Its total event rate is ΓC , where

$$k_r(q) = 4\pi q^2 \mu_r(q) = \frac{r}{\hbar} h(qr/\hbar), \quad C(r) = \int_0^\infty k_r(q) \mathcal{R}(q) dq. \quad (9)$$

The normalized radial shape satisfies $h \geq 0$ and $\int_0^\infty h(u) du = 1$. Its moments are $M_j = \int_0^\infty u^j h(u) du$. A nonnegative finite jump intensity gives completely positive, trace-preserving evolution [8, 9]. Prescribed time dependence preserves complete-positive divisibility. Equation (8) fixes ensemble dynamics; a jump representation does not select a unique state-vector ontology or establish objective reduction by itself.

An explicit rigid-target realization takes a normalized nonnegative coupling profile $f(\mathbf{x})$ and

$$\mathcal{F}(\mathbf{q}) = \int f(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{x}/\hbar} d^3x, \quad \mathcal{R}(q) = |\mathcal{F}(\mathbf{q})|^2. \quad (10)$$

Spherical symmetry is assumed. The profile is the density seen by the effective geometric coupling. Rigid-density descriptions and their dependence on target structure are standard in localization models [17]; identifying this profile with a particular nuclear charge or mass distribution requires additional microscopic input. Equation (8) retains the coherent elastic channel: transfers outside the selected response are suppressed. The present article treats this scalar completion throughout.

Our solvable specialization uses

$$\mu_r(\mathbf{q}) = \frac{r^3}{\pi^{3/2} \hbar^3} e^{-r^2 q^2 / \hbar^2}, \quad h_G(u) = \frac{4}{\sqrt{\pi}} u^2 e^{-u^2}, \quad M_2 = \frac{3}{2}. \quad (11)$$

For a Gaussian f of root-mean-square radius s , Eq. (10) gives

$$\mathcal{R}_s(q) = \exp\left[-\frac{s^2 q^2}{3\hbar^2}\right]. \quad (12)$$

Here s is a coupling-response radius and need not coincide with the electromagnetic charge radius or matter radius of a real target. Equations (2), (3), (8), (11) and (12) fully specify the finite-size effective channel. The point-domain reference is $s = 0$, denoted by a subscript pt.

2.4 Gravitational and relativistic status

The current static reduction of the motivating gravitational sector has the form

$$G_{\text{eff}}(\mathbf{x}, t) = \frac{\hbar c^5}{4\pi D_{\text{stat}}(\mathbf{x}, t)}, \quad (13)$$

where D_{stat} is a collective stiffness. None of the spectral, coherence, transport or energy results below uses Eq. (13). The aperture dependence of that stiffness is not specified by the transfer channel. We retain state-dependent G_{eff} and do not infer $G_{\text{eff}} \propto r^2$ or a universal constant Newton coupling from Eq. (1).

For cosmological propagation the preferred rest frame is explicitly the homogeneous Friedmann–Lemaître–Robertson–Walker (FLRW) four-velocity. Free propagation uses the Hamiltonian $H = \sqrt{m^2 c^4 + c^2 P^2}$ in that frame; the nonrelativistic limit is used only where stated. This is a specified preferred-frame effective dynamics. It does not inherit the Lorentz covariance of relativistic collapse constructions [18, 19]. Internal response at relativistic momentum, a microscopic reservoir and a gravitational backreaction law are not derived by choosing this Hamiltonian. They are separate matching questions for a more fundamental realization.

3 Spectral memory and its coherence amplitude

3.1 Exact statistics and the rare-event limit

For any finite history, define

$$\nu(B) = \int dt \Gamma(t) \int_B \mu_{r(t)}(\mathbf{q}) \mathcal{R}(q) d^3 q, \quad N = \nu(\mathbb{R}^3). \quad (14)$$

Since a free Hamiltonian depending only on momentum does not change momentum populations, the total impulse has the compound-Poisson law

$$\Phi(\mathbf{k}) = \exp\left[\int (e^{i\mathbf{k}\cdot\mathbf{q}} - 1) \nu(d^3 q)\right], \quad P = e^{-N} \sum_{j=0}^{\infty} \frac{\nu^{*j}}{j!}. \quad (15)$$

Here $\mathbf{k}$ has inverse-momentum units and $\nu^{*0} = \delta_0$. An initial momentum distribution is convolved with P . The radial intensity $J(q) = dN/dq$ describes individual transfers; it

is also the rare transferred-particle spectrum when $N \ll 1$. More precisely, for $N \leq 1$,

$$P_1 = (1 - N)\delta_{\mathbf{0}} + \nu, \quad \|P - P_1\|_{\text{TV}} \leq N(1 - e^{-N}) \leq N^2. \quad (16)$$

For finite measures σ and τ , $\|\sigma - \tau\|_{\text{TV}} \equiv \sup_A |\sigma(A) - \tau(A)|$. Appendix D gives the bound and its finite-history counterpart. No diffusion approximation is used.

3.2 Aperture-generated spectral law

The unfiltered reference intensity on $r_i \leq r \leq r_f$ is

$$J_{\text{pt}}(q) = \frac{A}{\kappa \hbar} \int_{r_i}^{r_f} \frac{r^b h(qr/\hbar)}{1 + (r/R)^p} dr. \quad (17)$$

An aperture-independent response factorizes at fixed physical transfer momentum:

$$J(q) = \mathcal{R}(q) J_{\text{pt}}(q). \quad (18)$$

This equation concerns the injection-frame accumulation. Its cosmological replacement is derived in Sect. 5.

Theorem 1 (Spectral law) *Let $0 < b < p$, $\kappa > 0$ and h be normalized and nonnegative. For the ideal aperture interval $(0, \infty)$ and $x = qR/\hbar$,*

$$J_{\text{pt}}(q) = \frac{A\hbar^b}{\kappa} q^{-b-1} \int_0^\infty \frac{u^b h(u)}{1 + (u/x)^p} du, \quad (19)$$

$$N_{\text{pt}} = \frac{AR^b}{\kappa} \frac{\pi}{p} \csc\left(\frac{\pi b}{p}\right). \quad (20)$$

If $M_b < \infty$, then

$$J_{\text{pt}}(q) \sim K_b q^{-b-1}, \quad K_b = \frac{A\hbar^b M_b}{\kappa}, \quad x \rightarrow \infty. \quad (21)$$

If $M_{b-p} < \infty$, the low-momentum branch is

$$J_{\text{pt}}(q) \sim \frac{AR^p \hbar^{b-p} M_{b-p}}{\kappa} q^{p-b-1}. \quad (22)$$

For $b - p < j < b$ and finite M_j ,

$$\frac{1}{N_{\text{pt}}} \int_0^\infty q^j J_{\text{pt}}(q) dq = \left(\frac{\hbar}{R}\right)^j M_j \frac{\sin(\pi b/p)}{\sin[\pi(b-j)/p]}. \quad (23)$$

The proof, including the low-momentum marginal cases, is in Appendix A. The mechanism is transparent: events per logarithmic aperture interval scale as r^b for $r \ll R$, while their characteristic momentum scales as r^{-1} . The occupation of scales therefore produces a q^{-b-1} spectrum. The mathematics is a change of variable and a beta integral; the physical prescription fixes their measure.

For a finite history, the high-momentum law is an intermediate asymptotic requiring $\hbar/R \ll q \ll \hbar/r_i$. A coherent target also requires an interval over which $\mathcal{R}(q)$ is

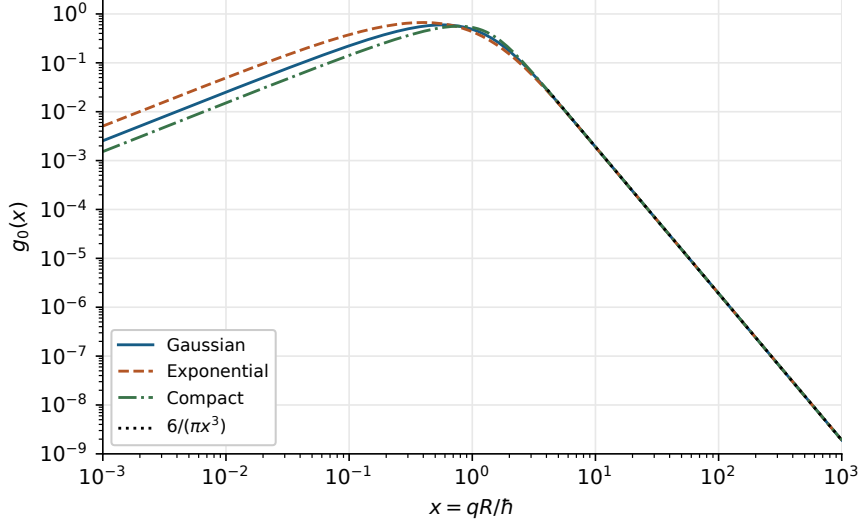


Fig. 1 The area-controlled reference spectrum $g_0(x) = \hbar J_{\text{pt}}(\hbar x/R)/(RN_{\text{pt}})$ for three isotropic kernels with $M_2 = 3/2$: Gaussian; $h(u) = (\sqrt{8})^3 u^2 \exp(-\sqrt{8}u)/2$; and a uniform ball, $h(u) = 3u^2/U^3$ on $0 < u < U = \sqrt{5}/2$. All approach $6/(\pi x^3)$ although their shoulders differ. Curves use the ideal aperture interval and no internal response.

approximately constant. For Eq. (12), the nonrelativistic algebraic window must fit inside

$$\frac{\hbar}{R} \ll q \ll \min\left(\frac{\hbar}{r_i}, \frac{\sqrt{3}\hbar}{s}, mc\right). \quad (24)$$

These are scale-separation conditions, not sharp step cutoffs. If the interval is narrow, Eq. (17) is the prediction.

For $b = 2$, $p = 4$, $\alpha = 1$ and Eq. (11),

$$N_{\text{pt},i} = \frac{\pi\gamma_0}{2Q_w} \frac{m_i R_i^2}{m_u r_0^2}, \quad (25)$$

$$J_{\text{pt},i}(q) \sim \frac{3\gamma_0 \hbar^2}{Q_w r_0^2} \frac{m_i}{m_u} q^{-3}, \quad \frac{dN_i}{dE} \sim \frac{3\gamma_0 \hbar^2}{4m_u Q_w r_0^2} E^{-2}. \quad (26)$$

The energy relation uses $E = q^2/(2m_i)$ in the window of Eq. (24). Its amplitude is species independent per original target at fixed total kinetic energy, before response and transport. Integrated counts and shoulder energies remain species dependent.

3.3 A common amplitude for the spectrum and coherence

Consider a controlled pair of paths at constant physical separation d in the nonexpanding protocol. Angular averaging of the same transfer measure gives the visibility factor

$e^{-\chi(d)}$, with

$$\chi(d) = \int_0^\infty J(q) [1 - \text{sinc}(qd/\hbar)] dq, \quad \text{sinc } z = \frac{\sin z}{z}. \quad (27)$$

This is the established transfer-coherence transform [10]; the prescribed aperture history gives it a definite spectrum and normalization.

Proposition 2 (Marginal amplitude relation) *Let $J \geq 0$ have finite total intensity and $J(q) \sim Kq^{-3}$ at large q , with $K > 0$. For any fixed reference length $R > 0$,*

$$\lim_{d \rightarrow 0} \frac{\chi(d)}{d^2 \ln(R/d)} = \frac{K}{6\hbar^2}. \quad (28)$$

If instead $\int q^2 J(q) dq < \infty$, then

$$\lim_{d \rightarrow 0} \frac{\chi(d)}{d^2} = \frac{1}{6\hbar^2} \int_0^\infty q^2 J(q) dq. \quad (29)$$

The proof uses the small-argument expansion of $1 - \text{sinc } z$ and is given in Appendix B. Equation (28) holds beyond Gaussian kernels. Once the tail amplitude is fixed, its logarithmic coherence coefficient has no additional normalization to fit. For the Gaussian area branch, with $\eta = d/R$,

$$\frac{\chi_{\text{pt}}(d)}{N_{\text{pt}}} = \frac{\eta^2}{\pi} \left[\ln\left(\frac{2}{\eta}\right) + \frac{1 - \gamma_E}{2} \right] + O(\eta^4), \quad (30)$$

where γ_E is Euler's constant. A finite response changes the true small- d limit to Eq. (29); a logarithmic interval can remain between the internal and activation scales. This crossover is calculated next.

4 A closed Gaussian finite-size realization

4.1 Exact closure and displaced production

Multiplying the two Gaussians in Eqs. (11) and (12) gives the exact identity

$$\mu_r(\mathbf{q}) \mathcal{R}_s(q) = C_s(r) \mu_{r_s}(\mathbf{q}), \quad r_s = \sqrt{r^2 + s^2/3}, \quad C_s(r) = \left(\frac{r}{r_s}\right)^3. \quad (31)$$

Thus the coherent channel has rate $\Gamma_s = \Gamma C_s$ and aperture r_s . The internal size both suppresses broad events and increases the effective aperture. No separate spectral cutoff is added after solving the channel.

For $b = 2$, $p = 4$, set $y = r/R$, $\xi = s/R$ and $\lambda = \xi^2/3$. The point-domain reference event occupation is

$$w(y) = \frac{4}{\pi} \frac{y}{1 + y^4}, \quad \int_0^\infty w(y) dy = 1. \quad (32)$$

Table 1 Finite-response predictions for the Gaussian area branch. Counts refer to the ideal history; $B/6$ is the small-separation coefficient $\chi_s/[N_{\text{pt}}(d/R)^2]$. Numerical values follow from Eqs. (33)–(36).

s/R	N_s/N_{pt}	r_{pic}/R	$B(\lambda)$	$B(\lambda)/6$
0.1	0.982716	1.002483	4.236302	0.706050
0.3	0.906018	1.021206	2.226675	0.371113
1.0	0.601506	1.164734	0.551804	0.091967

The coherent count and its second dimensionless moment are controlled by

$$F(\lambda) = \frac{N_s}{N_{\text{pt}}} = \int_0^\infty w(y) \left(1 + \frac{\lambda}{y^2}\right)^{-3/2} dy, \quad (33)$$

$$B(\lambda) = \frac{3}{2} \int_0^\infty \frac{w(y)}{y^2 + \lambda} \left(1 + \frac{\lambda}{y^2}\right)^{-3/2} dy = -F'(\lambda). \quad (34)$$

For $\lambda > 0$ both are finite. A closed form is

$$F(\lambda) = \frac{4}{\pi} \text{Im} \left\{ \frac{\text{atanh} \sqrt{1 + i\lambda}/\sqrt{1 + i\lambda} - 1}{1 + i\lambda} \right\}, \quad (35)$$

with principal complex branches and the continuous limit $F(0) = 1$. Appendix C derives this expression and its limiting forms. No complex physical parameter is introduced; Eq. (35) is an elementary representation of a positive real integral.

The maximum of the production rate $\Gamma_s(t)$ is shifted. Writing $v = y_{\text{pic}}^2$, its unique positive root satisfies

$$2v^3 - \lambda v^2 - 2v - 5\lambda = 0. \quad (36)$$

For every $\lambda > 0$, $y_{\text{pic}} > 1$. The delay relative to the point-domain reference crossing is $\Delta t = \ln(y_{\text{pic}})/\kappa$. Finite target size therefore changes the physical event clock as well as the tail.

4.2 Counts generate coherence and energy

The derivative in Eq. (34) is part of a more general relation that does not require a Gaussian point-domain reference kernel or an exponential clock.

Proposition 3 (Response-moment identities) *Let J_{pt} be any finite nonnegative radial transfer intensity. Define, for $\beta > 0$,*

$$J_\beta(q) = e^{-\beta q^2} J_{\text{pt}}(q), \quad N_\beta = \int_0^\infty J_\beta(q) dq. \quad (37)$$

Then all even moments exist and

$$(-1)^n \partial_\beta^n N_\beta = \int_0^\infty q^{2n} J_\beta(q) dq, \quad n = 0, 1, \dots, \quad (38)$$

$$\lim_{d \rightarrow 0} \frac{\chi_\beta(d)}{d^2} = -\frac{\partial_\beta N_\beta}{6\hbar^2}, \quad \mathcal{E}_\beta^{\text{NR}} = -\frac{\partial_\beta N_\beta}{2m}. \quad (39)$$

Here $\mathcal{E}_\beta^{\text{NR}}$ is the accumulated nonrelativistic energy input per original target in the nonexpanding free channel.

The response parameter $\beta = s^2/(3\hbar^2)$ has inverse squared-momentum units; it is not an inverse temperature. The derivatives hold at fixed J_{pt} , hence at fixed clock and activation law. They describe a response family; comparing different species without matching those other parameters is not an implementation of the derivative. Appendix C proves the identities by dominated differentiation. Their mathematical structure is that of a Laplace transform. Their physical use here is to connect three responses of the same specified geometric channel.

For the area branch, Eq. (39) becomes

$$\lim_{d \rightarrow 0} \frac{\chi_s(d)}{N_{\text{pt}}(d/R)^2} = \frac{B(\lambda)}{6}, \quad \mathcal{E}_s^{\text{NR}} = N_{\text{pt}} \frac{\hbar^2}{2mR^2} B(\lambda). \quad (40)$$

Equivalently, $\lim_{d \rightarrow 0} \chi_s/d^2 = -\frac{1}{2} \partial_{s^2} N_s$. The complete coherence function is

$$\frac{\chi_s(d)}{N_{\text{pt}}} = \int_0^\infty w(y) \left(1 + \frac{\lambda}{y^2}\right)^{-3/2} \left[1 - \exp\left(-\frac{\eta^2}{4(y^2 + \lambda)}\right)\right] dy. \quad (41)$$

Equations (18), (33) and (41) determine the finite-size spectral, count and coherence responses with a single λ . Figure 2 displays their crossover. At $s \ll d \ll R$, Eq. (30) is recovered to the accuracy allowed by scale separation. At $d \ll s$, the coefficient saturates to $B/6$. If $s \sim R$, neither a long algebraic tail nor a long logarithmic interval is expected; the full curves remain well-defined predictions. For well-separated scales, coherence measurements restricted to $s \ll d \ll R$ may agree with the point-domain prediction within experimental precision, even though the finite internal response regularizes the second momentum-transfer moment and the accumulated energy input. Measurements at $d \ll s$ probe the finite quadratic coefficient fixed by Eq. (40).

The small-size limit also has a universal marginal structure. For this Gaussian realization,

$$F(\lambda) = 1 - \frac{\lambda}{\pi} \left[3 \ln\left(\frac{4}{\lambda}\right) - 5\right] + O(\lambda^2), \quad \lambda \rightarrow 0^+. \quad (42)$$

Thus the small deficit of coherent events is logarithmically enhanced, while its derivative controls the energy and coherence curvature. More generally, combining Eq. (28) with $J_s = \exp[-s^2 q^2/(3\hbar^2)] J_{\text{pt}}$ and a Kq^{-3} tail gives

$$N_{\text{pt}} - N_s \sim \frac{K}{3\hbar^2} s^2 \ln(R/s) \sim 2\chi_{\text{pt}}(s), \quad s \rightarrow 0. \quad (43)$$

The coefficient is fixed by the same tail amplitude. Equation (43) is a leading asymptotic relation, not an equality at $s \sim R$.

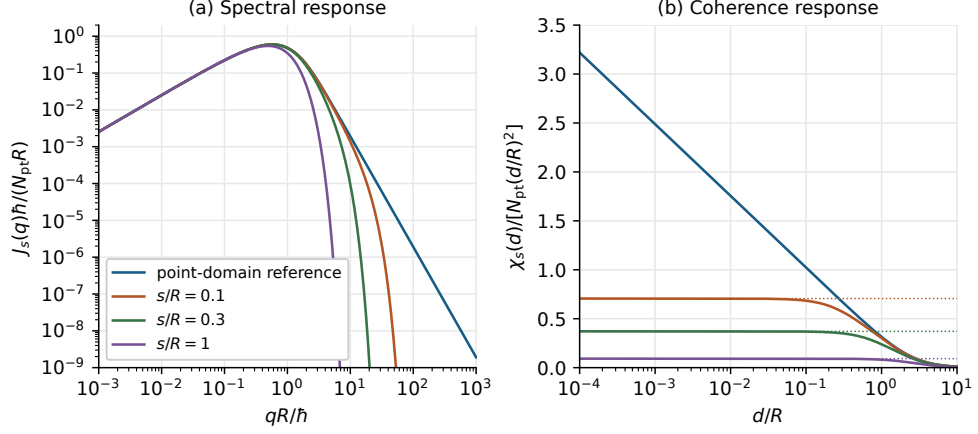


Fig. 2 Joint finite-size prediction for Gaussian transfers and the response $\mathcal{R}_s = \exp[-s^2 q^2 / (3\hbar^2)]$, with $b = 2, p = 4$. (a) The unnormalized coherent spectrum, expressed per ideal point-domain reference event. (b) The coherence curvature as a function of separation; horizontal dotted lines are $B(\lambda)/6$ and contain no independently fitted coefficient. Both panels use exactly the same four values of s/R .

4.3 An energy relation across target species

Substituting Eq. (25) into Eq. (40) gives

$$\mathcal{E}_{s,i}^{\text{NR}} = \frac{\pi\gamma_0\hbar^2}{4m_u Q_w r_0^2} B(\lambda_i). \quad (44)$$

For common s_i/R_i , the mean nonrelativistic injection per original target is therefore independent of species, even though the number of transferred targets and their conditional mean energies differ. The cancellation uses $\alpha = 1$ and $b = 2$, the uniform-injection principle of Eq. (5), together with the common kernel and the same broad history. It is not a thermal equipartition statement and does not survive arbitrary species-dependent responses or subsequent losses.

This equality concerns energy input, not temperature. In the rare-event regime most targets receive no transfer. The coherent population carries a conditional mean energy $\mathcal{E}_{s,i}^{\text{NR}}/N_{s,i}$, which scales as $(m_i R_i^2)^{-1}$ at fixed λ . The amplitude and species relations provide more discriminating content than the isolated exponent E^{-2} . The species independence is a property of the coherent scalar response specified in Eq. (8); alternative many-body completions are outside the scope of the present inverse and IGM analysis.

5 Expansion and identifiable spectral information

5.1 Response acts at the injection momentum

Let $a(t_0) = 1$. A free physical impulse injected at time t becomes $\mathbf{q}_0 = a(t)\mathbf{q}$ today. Consequently,

$$J_{s,\text{obs}}(q_0) = \int dt \frac{\Gamma(t)}{a(t)} k_{r(t)} \left(\frac{q_0}{a(t)} \right) \mathcal{R}_s \left(\frac{q_0}{a(t)} \right). \quad (45)$$

The factor $1/a$ is the radial Jacobian. An aperture-independent response does not generally factor out of this integral, since its physical momentum argument varies with injection time. A response multiplying the final spectrum is instead an event-weighted effective function, not the intrinsic $\mathcal{R}_s(q_0)$.

For the Gaussian realization, Eq. (31) gives the exact characteristic exponent

$$\ln \Phi_{\text{obs}}(\mathbf{k}) = \int dt \Gamma(t) C_s[r(t)] \left\{ \exp \left[-\frac{\hbar^2 a(t)^2 k^2}{4r_s(t)^2} \right] - 1 \right\}. \quad (46)$$

Expansion changes apparent scales, not event counts. The effective observed aperture is

$$\ell_s(t) = \frac{\sqrt{r(t)^2 + s^2/3}}{a(t)}, \quad \frac{\dot{\ell}_s}{\ell_s} = \kappa \frac{r^2}{r^2 + s^2/3} - H(t), \quad (47)$$

for constant proper internal size s . In the unresolved regime $r \gg s$, this becomes $\dot{\ell}/\ell = \kappa - H$. A local scaling argument with approximately constant $H < \kappa$ then changes the point-domain reference high-momentum index to $1 + b\kappa/(\kappa - H)$. For the benchmark, $H/\kappa \simeq 0.0188$ at $z \simeq 0.34$, so the local point-domain index is about 3.04 rather than 3. Near the internal scale the extra factor in Eq. (47) matters; Eq. (45) covers that regime without assigning a power law.

The response-moment identity also survives expansion in a weighted form. Differentiation at fixed aperture history gives

$$-\partial_\beta N_\beta = \int dt \int_0^\infty \frac{q_0^2}{a(t)^2} \frac{dJ_{\beta,\text{obs}}(q_0)}{dt} dq_0. \quad (48)$$

It generates the injection moment, not the unweighted present moment. When $a_{\min} \leq a(t) \leq a_{\max}$ over production,

$$a_{\min}^2 (-\partial_\beta N_\beta) \leq \int q_0^2 J_{\beta,\text{obs}}(q_0) dq_0 \leq a_{\max}^2 (-\partial_\beta N_\beta). \quad (49)$$

These bounds separate response physics from the known kinematic loss during expansion.

5.2 What reconstruction determines

Write $v = q_0^2$ and let $S_s(v)$ denote the isotropic vector density of the observed transfer measure, so that $J_{s,\text{obs}}(q_0) = 4\pi q_0^2 S_s(q_0^2)$. Define

$$z_s(t) = \frac{r(t)^2 + s^2/3}{a(t)^2 \hbar^2}, \quad \mathcal{M}_s(B) = \int \Gamma(t) C_s[r(t)] \mathbf{1}_B[z_s(t)] dt. \quad (50)$$

Then

$$S_s(v) = \frac{1}{\pi^{3/2}} \int_0^\infty z^{3/2} e^{-zv} \mathcal{M}_s(dz). \quad (51)$$

Proposition 4 (Identifiable combined-scale measure) *For a finite positive history with $0 < z_{\min} \leq z_s(t) \leq z_{\max} < \infty$, ideal knowledge of $S_s(v)$ for $v > 0$ uniquely determines $\mathcal{M}_s$. Necessarily,*

$$(-1)^n \frac{d^n S_s}{dv^n} \geq 0, \quad n = 0, 1, \dots \quad (52)$$

The recovered measure is the event-weighted occupation of ℓ_s , not the chronological order of scale visits or the separate functions r , a and s .

Uniqueness follows from the Laplace transform of the finite positive measure $z^{3/2} \mathcal{M}_s(dz)$. A known monotonic clock and known response allow the measure to be assigned to physical times. An unknown internal size creates an additional geometric ambiguity. For a general non-Gaussian internal response, Eq. (51) need not hold; a failure of complete monotonicity then excludes this Gaussian combined-scale representation, not every possible aperture model.

Outside the rare-event regime, recovery of the transfer measure first requires decompounding Eq. (15) and accounting for the initial distribution. A known nonvanishing initial characteristic function permits this operation in ideal data; inverse Laplace transformation and decompounding are ill-conditioned with finite noisy data. The proposition is an identifiability result. Spectral memory denotes retained occupation information, without implying information backflow or an unrestricted reconstruction of G_{eff} .

5.3 Clock reconstruction beyond exponential winding

The occupation result can be strengthened when the aperture history itself is strictly increasing. First consider the nonexpanding, unfiltered point-domain radial channel and write $\varphi = \ln r$ and

$$W(\varphi) \equiv \frac{dN}{d\varphi} = \frac{\Gamma(e^\varphi)}{\omega(e^\varphi)}. \quad (53)$$

With $k_r(q) = (r/\hbar)h(qr/\hbar)$, the accumulated radial spectrum is

$$J(q) = \int_{-\infty}^{\infty} W(\varphi) \frac{e^\varphi}{\hbar} h\left(\frac{qe^\varphi}{\hbar}\right) d\varphi. \quad (54)$$

Let

$$\mathfrak{M}_J(\zeta) = \int_0^\infty q^{\zeta-1} J(q) dq, \quad H(\zeta) = \int_0^\infty u^{\zeta-1} h(u) du. \quad (55)$$

Direct substitution gives the exact factorization

$$\boxed{\mathfrak{M}_J(\zeta) = \hbar^{\zeta-1} H(\zeta) \int_{-\infty}^\infty W(\varphi) e^{-(\zeta-1)\varphi} d\varphi.} \quad (56)$$

Thus the unfiltered transfer spectrum is a multiplicative convolution in physical scale, or an ordinary convolution in logarithmic scale. If the measured nonexpanding spectrum is multiplied by a known, aperture-independent response $\mathcal{R}(q)$ that is nonzero over the inversion range, the same theorem applies to $J_{\text{obs}}(q)/\mathcal{R}(q)$. An unknown response, an aperture-dependent response, or response zeros introduce an additional degeneracy and prevent unconditional recovery of W from the spectrum alone.

Proposition 5 (Monotone clock reconstruction) *Suppose $r(t)$ is strictly increasing on a finite production interval, the unfiltered single-event radial kernel is known, and $H(\zeta)$ has no zero on a Mellin inversion line. Ideal knowledge of $J(q)$ then uniquely determines $W(\ln r)$. If, in addition,*

$$\Gamma(r) = \gamma_\star G(r) \quad (57)$$

with known G , then

$$t(r) - t_i = \frac{1}{\gamma_\star} \int_{\ln r_i}^{\ln r} \frac{W(\varphi)}{G(e^\varphi)} d\varphi. \quad (58)$$

Consequently the normalized chronology

$$F_t(r) = \frac{\int_{\ln r_i}^{\ln r} W(\varphi) G(e^\varphi)^{-1} d\varphi}{\int_{\ln r_i}^{\ln r_f} W(\varphi) G(e^\varphi)^{-1} d\varphi} \quad (59)$$

is fixed even when $\gamma_\star$ is unknown. If the elapsed production time Δt is independently known, then

$$\gamma_\star = \frac{1}{\Delta t} \int_{\ln r_i}^{\ln r_f} \frac{W(\varphi)}{G(e^\varphi)} d\varphi, \quad \omega(r) = \gamma_\star \frac{G(r)}{W(\ln r)}. \quad (60)$$

For the Gaussian radial kernel used here,

$$h_G(u) = \frac{4}{\sqrt{\pi}} u^2 e^{-u^2}, \quad H_G(\zeta) = \frac{2}{\sqrt{\pi}} \Gamma_E\left(\frac{\zeta+2}{2}\right), \quad (61)$$

which has no zeros. The ideal logarithmic occupation is therefore identifiable on every admissible inversion line.

A useful exactly soluble deformation of the benchmark is

$$\omega(r) = \kappa \left(\frac{r}{R}\right)^\delta, \quad \Gamma(r) = \frac{Ar^b}{1 + (r/R)^p}. \quad (62)$$

Then $W \propto r^{b-\delta}/[1 + (r/R)^p]$, so the point-domain theorem is obtained from the constant-winding result by $b \mapsto b - \delta$. In particular,

$$J_{\text{high}}(q) \propto q^{-(1+b-\delta)}. \quad (63)$$

When the low-momentum moment condition of Appendix A also holds, $p < b - \delta + 3$,

$$J_{\text{low}}(q) \propto q^{p-b+\delta-1}. \quad (64)$$

For the area branch $b = 2, p = 4$, the two algebraic branches therefore read

$$J_{\text{high}} \propto q^{-3+\delta}, \quad J_{\text{low}} \propto q^{1+\delta}, \quad -2 < \delta < 1, \quad (65)$$

and their signed logarithmic slopes satisfy

$$\sigma_{\text{low}} - \sigma_{\text{high}} = 4, \quad \delta = \sigma_{\text{high}} + 3 = \sigma_{\text{low}} - 1. \quad (66)$$

At $\delta = 1$ the low branch is $q^2 \ln(q_*/q)$ for a fixed reference scale q_* , and for $1 < \delta < 2$ it saturates at q^2 ; the slope-difference diagnostic is therefore restricted to the algebraic-moment regime in Eq. (65). Because the spectrum depends on the rate and the winding only through $W = \Gamma/\omega$, the two signed slopes fix p and the combination $b - \delta$, not b and δ separately. A q^{-3} tail therefore identifies locally constant winding only once the area branch $b = 2$ is assumed; an accelerating clock on a steeper activation branch produces the same tail.

The expanding Gaussian problem admits a chronological completion under one additional condition. If $z_s(t)$ in Eq. (50) is strictly monotone and $\mathcal{M}_s(dz) = m(z)dz$, then

$$\left| \frac{dt}{dz} \right| = \frac{m(z)}{\Gamma[r(z, t), t] C_s[r(z, t)]}, \quad r(z, t) = \sqrt{a(t)^2 \hbar^2 z - s^2/3}. \quad (67)$$

A known branch orientation and one boundary condition fix the sign and integration constant. If $z_s(t)$ is not monotone, repeated visits to the same combined scale cannot be chronologically ordered by a single spectrum; only the cumulative occupation of Proposition 4 remains identifiable. Equation (67) is therefore a conditional clock-reconstruction result, not an unrestricted inversion of arbitrary histories.

5.4 Stability and resolution of the reconstruction

Proposition 5 is an identifiability statement. Its inversion divides by $H(\zeta)$, which for the Gaussian kernel decays as $e^{-\pi|\varpi|/4}$ along an inversion line $\zeta = c + i\varpi$, so the problem is exponentially ill-posed. The following result quantifies what a noisy spectrum can recover. Measure lengths in units of a fixed reference length and momenta in the corresponding units of $\hbar$, so that J and W are dimensionless and $\hbar = 1$. Fix $c > -2$ and set $w_c(\varphi) = W(\varphi)e^{-(c-1)\varphi}$, with Fourier transform $\hat{w}_c(\varpi) = \int w_c(\varphi)e^{-i\varpi\varphi}d\varphi$, so that Eq. (56) reads $\mathfrak{M}_J(c + i\varpi) = H(c + i\varpi)\hat{w}_c(\varpi)$. Define the data norm $\|f\|_c^2 = \int_0^\infty q^{2c-1}|f(q)|^2dq$ and the Sobolev norm $\|w\|_{H^\theta}^2 = (2\pi)^{-1} \int (1 + \varpi^2)^\theta |\hat{w}(\varpi)|^2 d\varpi$.

Proposition 6 (Logarithmic stability) *Let W have compact support and $\|w_c\|_{H^\theta} \leq \Theta$ for some $\theta > 0$, and let a measured spectrum satisfy $\|J^\varepsilon - J\|_c \leq \varepsilon$. Define the spectral cutoff reconstruction w_c^ε by $\hat{w}_c^\varepsilon(\varpi) = \mathfrak{M}_{J^\varepsilon}(c + i\varpi)/H_G(c + i\varpi)$ for $|\varpi| \leq \varpi_c$ and $\hat{w}_c^\varepsilon(\varpi) = 0$ otherwise. Then*

$$\|w_c^\varepsilon - w_c\|_{L^2} \leq \frac{\varepsilon}{|H_G(c + i\varpi_c)|} + \Theta(1 + \varpi_c^2)^{-\theta/2}. \quad (68)$$

With $\varpi_c = (2/\pi) \ln(1/\varepsilon)$ the error is $O(\varepsilon^{1/2} [\ln \varepsilon^{-1}]^{-(c+1)/2}) + \Theta[\pi/(2 \ln \varepsilon^{-1})]^\theta$, that is $O([\ln \varepsilon^{-1}]^{-\theta})$ as $\varepsilon \rightarrow 0$.

Proof The Mellin–Plancherel identity gives $\int |\mathfrak{M}_f(c + i\varpi)|^2 d\varpi = 2\pi \|f\|_c^2$, and Plancherel’s identity in φ gives $\|w\|_{L^2}^2 = (2\pi)^{-1} \int |\hat{w}|^2 d\varpi$. Split the Fourier-domain error at $|\varpi| = \varpi_c$. For $|\varpi| \leq \varpi_c$ the noise contribution to the squared error is at most $\varepsilon^2 / \min_{|\varpi| \leq \varpi_c} |H_G(c + i\varpi)|^2$. Because $|\Gamma_E(x + iy)|^2 = \Gamma_E(x)^2 \prod_{n \geq 0} [1 + y^2/(x+n)^2]^{-1}$ decreases in $|y|$, the minimum is attained at $|\varpi| = \varpi_c$. The truncated tail contributes at most $\Theta^2(1 + \varpi_c^2)^{-\theta}$. Stirling’s formula, $|H_G(c + i\varpi)| \simeq (2/\sqrt{\pi})\sqrt{2\pi}(\varpi/2)^{(c+1)/2}e^{-\pi\varpi/4}$, gives the rate. $\square$

Logarithmic rates of this kind are characteristic of exponentially ill-posed problems [20]. The logarithmic bound controls a weighted occupation error. For the displayed cutoff, the associated half-wavelength is

$$\Delta\varphi_c \simeq \frac{\pi}{\varpi_c} = \frac{\pi^2}{2 \ln(1/\varepsilon)}, \quad (69)$$

or 1.1, 0.71 and 0.36 e-folds at normalized error levels 10^{-2} , 10^{-3} and 10^{-6} . This is a scale of this estimator, not a universal resolution bound or a guarantee that an arbitrary transient of that width can be recovered. The absolute norm error of Proposition 6 becomes a relative error only after fixing a data normalization.

Independent sampling does not reduce that relative data-norm error. For $J_j^\varepsilon = J_j(1 + e_{\text{rel}}\xi_j)$ with centred, unit-variance independent ξ_j and fixed positive quadrature weights w_j ,

$$\mathbb{E} \left[\frac{\sum_j w_j |J_j^\varepsilon - J_j|^2}{\sum_j w_j |J_j|^2} \right] = e_{\text{rel}}^2. \quad (70)$$

Additional samples can reduce the uncertainty of fitted modes or averaged observables, but do not imply a $1/\sqrt{N}$ reduction of the norm in the stability theorem.

Corollary 7 (Stability of normalized chronology) *Let $I = [\varphi_-, \varphi_+]$ have length L , and let the known positive rate shape $G_{\log}(\varphi) = G(e^\varphi)$ from Eq. (59) satisfy $G_{\log} \geq g_{\min} > 0$ on I . For nonnegative occupations $W, \tilde{W}$, define*

$$Z = \int_I \frac{W}{G_{\log}} d\varphi, \quad F_W(x) = Z^{-1} \int_{\varphi_-}^x \frac{W}{G_{\log}} d\varphi. \quad (71)$$

If $\delta = \sqrt{L} \|\tilde{W} - W\|_{L^2(I)}/g_{\min} < Z$, then

$$\|F_{\tilde{W}} - F_W\|_\infty \leq \frac{2\delta}{Z - \delta}. \quad (72)$$

Table 2 Synthetic inversion errors: median [5th, 95th percentile] over 256 noise realizations. Broad-band cases use 400 momentum points over $10^{-4} \leq qR/h \leq 10^4$ with ideal relative-only errors; the finite-band case uses 120 points over 0.1–10, plus an additive standard-deviation floor $10^{-3} \max J$. These are repeated-noise sensitivities of stated examples, not general resolution guarantees.

Setting	σ	noise	relative $L^2(W)$	$\ F_{\text{rec}} - F_{\text{true}}\ _\infty$
Broad band	0.8	1%	.048 [.047,.077]	.015 [.010,.026]
Broad band	0.25	1%	.127 [.098,.129]	.047 [.023,.061]
Broad band	0.25	0.1%	.042 [.041,.054]	.016 [.011,.020]
Finite band	0.25	1% + floor	.069 [.062,.577]	.029 [.014,.148]

Proof Cauchy–Schwarz bounds both the numerator error and $|\tilde{Z} - Z|$ by δ . Since $\tilde{Z} \geq Z - \delta$ and the true cumulative numerator lies in $[0, Z]$, subtraction of the two normalized ratios gives Eq. (72). $\square$

On a fixed compact interval the weighted and unweighted occupation norms are equivalent. Thus Proposition 6 transfers to a normalized chronology when the rate is bounded away from zero. It does not similarly control the instantaneous winding $\omega \propto G_{\log}/W$ near zeros of W ; cumulative elapsed time and local clock speed are different reconstruction targets.

We test this distinction with 256 independent noise realizations for each of four synthetic settings. The true occupation is $(2/\pi) \text{sech}(2y)[1 - 0.6 \exp(-(y-0.5)^2/(2\sigma^2))]$, with $y = \varphi - \ln R$. The inversion uses truncated singular values of a noise-whitened forward matrix, the smallest rank with residual at most $1.05N$, and projection onto nonnegative occupations. It does not fit a prescribed transient template. The noise covariance is supplied by the synthetic experiment. The cumulative clock is normalized on $-2 \leq y \leq 2$ with known $G(y) = \text{sech}(2y)$.

The broad-band cases recover useful occupation and cumulative-time information. Their ideal relative precision extends into extremely faint tails and is not an instrument specification. Restricting the band and adding a noise floor produces a substantially broader error distribution (Table 2). A small global occupation error also does not certify the recovered width or height of an individual burst. These tests make the practical content of spectral memory quantitative while exposing its dependence on bandwidth, noise calibration and the known-rate assumption.

6 A conditional cosmological realization

The nuclear realization in this section is not required for Theorem 1, Propositions 2–6 or Corollary 7, which hold under their separately stated assumptions. It is a fully specified conditional benchmark of that construction.

6.1 A specified nuclear source

We choose a flat matter-plus- Λ expansion with $H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.315$ and $\Omega_\Lambda = 0.685$, using the Planck benchmark [21]. This is a prescribed low-redshift

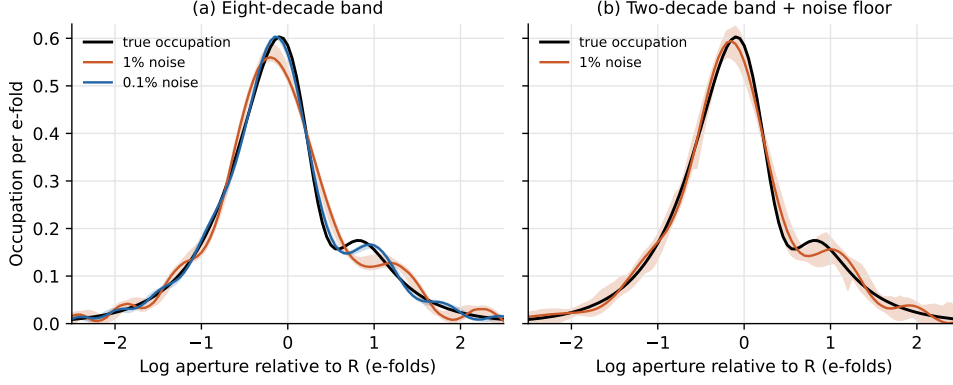


Fig. 3 Nonnegative occupation reconstructions from independent noise ensembles. Curves show medians and shading the pointwise 5th–95th percentile range. The broad-band panel compares two noise levels for the narrow transient; the finite-band panel adds an instrumental noise floor. Pointwise shading is not a simultaneous confidence band. Table 2 reports global errors.

background, not a derived DJAMILAR cosmology. The aperture inputs are $r_0 = 25$ nm, $\gamma_0 = 8 \times 10^{-10} \text{ s}^{-1}$ and

$$r_b = \sqrt{16\pi\hbar G_{\text{loc}}/c^3}, \quad Q_w = \frac{2}{t_0} \ln(r_0/r_b) = 2.786 \times 10^{-16} \text{ s}^{-1}. \quad (73)$$

The background age is $t_0 = 13.796$ Gyr. G_{loc} is the measured present reference-domain coupling. Equation (73) is the aperture boundary hypothesis of Ref. [16] with $\kappa_b = 1$ (Ref. [16] propagates $0.3 \leq \kappa_b \leq 3$ as a systematic), not a statement that G has that value at the early boundary or in every region. The channel is evaluated only during late production, when the targets exist.

Fully ionized protons and alpha particles are treated as separate stable effective targets with $R_p = 0.84075$ fm and $R_\alpha = 1.6785$ fm. These are the radius inputs of Ref. [16]. Their assignment to geometric activation, the matching of local and cosmological aperture clocks (Sect. 2.2), and the illustrative electron rule of no direct activation are additional assumptions. For the specified Gaussian coherent response benchmark we set $s_i = R_i$. This specifies a Gaussian rigid coupling profile; it is not inferred from electromagnetic form-factor data. The normalization γ_0 is likewise a scenario input rather than a measurement. Counts, energy densities and pressure scale linearly with it. For the benchmark $s_i = R_i$, the formal point-domain reference E^{-2} branch is therefore not interpreted as an extended physical nuclear spectrum; the Gaussian coherent response predicts a shoulder followed by a cutoff.

The point-domain reference crossing $r(t_i^*) = R_i$ defines reference scales

$$t_i^* = t_0 + \kappa^{-1} \ln(R_i/r_0), \quad q_i^* = \frac{a(t_i^*)\hbar}{R_i}, \quad E_i^* = \sqrt{m_i^2 c^4 + c^2 q_i^{*2}} - m_i c^2. \quad (74)$$

Table 3 Specified proton–alpha realization with $s_i = R_i$. Event counts are per original target and use the ideal-history count for reference. The coherent peak is calculated from Eq. (36).

Quantity	Proton	Alpha particle
$R_i = s_i$ (fm)	0.84075	1.6785
Crossing redshift	0.3406	0.3233
Crossing lookback time (Gyr)	3.9145	3.7572
Coherent peak lookback time (Gyr)	3.8798	3.7225
$N_{\text{pt},i}$	5.14×10^{-9}	8.14×10^{-8}
$N_{s,i}$	3.09×10^{-9}	4.89×10^{-8}
$q_i^* c$ (MeV)	175.08	88.84
E_i^* (MeV)	16.19	1.059

Since $\kappa = \ln(r_0/r_b)/t_0$, the point-domain crossing lookback time has the closed form

$$t_{\text{lb},i}^* = t_0 \frac{\ln(r_0/R_i)}{\ln(r_0/r_b)}.$$

It reproduces the 3.9145 and 3.7572 Gyr values in Table 3. The proton crossing redshift, 0.3406, agrees with Ref. [16]. Around the benchmark, one e-fold change in r_0 shifts the crossing by about 0.16 Gyr and one e-fold change in r_b by about 0.06 Gyr, so the epoch is only logarithmically sensitive to the boundary hypothesis in Eq. (73). The scales in Eq. (74) are useful coordinates for comparison; E_i^* is not asserted to be the exact mode of either response-filtered spectrum. Table 3 includes the actual shift of the coherent production maximum. For $s = R$, the central 80% of coherent events occur at $0.66297 < r/R < 3.20964$, lasting 0.3588 Gyr with this clock.

The coherent population is deeply in the rare-event regime; the bound in Eq. (16) is below 2.4×10^{-15} even for alpha particles. A very small fraction of original nuclei carries the generated energy. In particular, the benchmark does not predict strong coherence attenuation of every original nucleus: for this Gaussian channel $0 \leq \chi_s(d) \leq N_s$.

Figure 4 uses $0.03 < r/R_i < 100$. The retained coherent count is $0.601442N_{\text{pt}}$, compared with $0.601506N_{\text{pt}}$ for the ideal history. The difference is predominantly the late-aperture endpoint. Internal response regularizes the early, broad-transfer part of the model. Evaluating that response at the observed momentum instead would overestimate the proton intensity by factors 1.159, 1.847 and 4.141 at $q_0/q_p^* = 1, 2, 3$, respectively. This illustrates the importance of the order of physical operations in Eq. (45).

For a common s_i/R_i , the coherent count ratio retains $N_{s,\alpha}/N_{s,p} = m_\alpha R_\alpha^2/(m_p R_p^2) = 15.83$. Choosing target abundance ratio $n_\alpha/n_p = 1/12$, corresponding approximately to primordial mass fractions $Y = 0.25$, $X = 0.75$, gives a produced alpha-to-proton event ratio 1.3195. This ratio is not a freely specified composition. Its survival depends on the next stage.

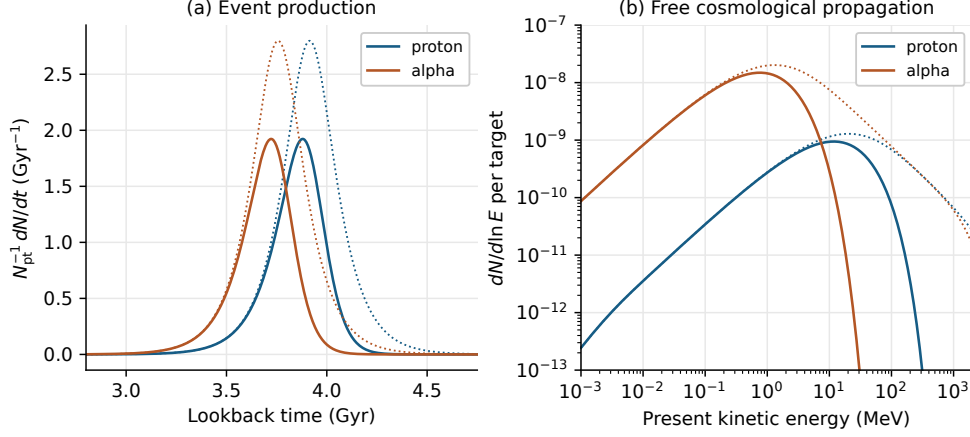


Fig. 4 Conditional nuclear source. Solid curves include $s_i = R_i$ and dotted curves show the point-domain reference. (a) Production rates per ideal point-domain reference event. (b) Present spectra after free redshift, calculated with the response at injection using Eq. (45). Energy uses the specified preferred-frame Hamiltonian. Source integration covers $0.03 < r/R_i < 100$; curves are not renormalized to that interval. The finite-size realization produces a shoulder and cutoff, rather than an extended algebraic interval for $s_i \sim R_i$.

6.2 Transport calculated by energy characteristics

Propagate the source through a homogeneous fully ionized plasma with $T_g = 10^4$ K and $n_e(t) = n_{e0}a^{-3}$. We use $n_{e0} = 0.05, 0.2, 0.8 \text{ m}^{-3}$ to expose environmental dependence. The test-particle evolution is

$$\dot{E} = -H(t) \frac{E(E + 2m_i c^2)}{E + m_i c^2} - b_C(E, t), \quad (75)$$

where b_C is an electron Coulomb stopping prescription [22], specified in Appendix F. Spatial escape, magnetic confinement, collective instabilities, nuclear reactions and thermal feedback are outside this homogeneous calculation. The model answers a definite kinetic question for the stipulated source and environment.

For a present threshold E_{th} , integrate Eq. (75) backward to find the minimum required injection energy $E_{\text{req}}(t; E_{\text{th}})$. Let

$$u_{\text{req}}(t) = \frac{r_s(t)}{\hbar c} \sqrt{E_{\text{req}}(E_{\text{req}} + 2m_i c^2)}, \quad \mathcal{T}(u) = \text{erfc } u + \frac{2u}{\sqrt{\pi}} e^{-u^2}. \quad (76)$$

The fraction surviving above threshold is then

$$\mathcal{S}_i(E_{\text{th}}) = \frac{1}{N_{s,i}^{\text{win}}} \int_{\text{win}} \Gamma_i(t) C_{s_i}[r(t)] \mathcal{T}[u_{\text{req}}(t)] dt. \quad (77)$$

This expression integrates the Gaussian transfer distribution analytically. It avoids discretizing the injected-energy variable at an absorbing threshold and gives a smooth

Table 4 Fraction of injected events remaining above 1 keV today. Both source models are propagated through the same plasma; the Gaussian columns use $s_i = R_i$ and the reference columns use $s_i = 0$. Values are quoted to four decimals; the central convergence checks in Appendix F are much tighter, while the environment itself is illustrative.

n_{e0} (m^{-3})	Proton		Alpha particle	
	Reference	Gaussian	Reference	Gaussian
0.05	0.9851	0.9753	0.5555	0.3378
0.20	0.9635	0.9396	0.3265	0.1007
0.80	0.9146	0.8601	0.1516	0.0094

Table 5 Sensitivity of the central $n_{e0} = 0.2 \text{ m}^{-3}$ benchmark to a common response size s_i/R_i . The nonrelativistic energy input is per original target. Survival fractions refer to events remaining above 1 keV today. The final column is the retained nonthermal pressure divided by the prescribed thermal pressure.

s_i/R_i	$\mathcal{E}_{s,i}^{\text{NR}}$ (eV)	Proton survival	Alpha survival	Surviving α/p	$P_{\text{nt}}/P_{\text{th}}$
0.5	0.2111	0.9552	0.2194	0.303	3.80%
1.0	0.08323	0.9396	0.1007	0.141	1.54%
2.0	0.02021	0.8945	0.00793	0.0117	0.373%

survival calculation. Equation (77) uses targets at rest before injection and at most one transfer per target. The first approximation is adequate here because thermal momenta at 10^4 K are negligible relative to the MeV-scale benchmark transfers, while the second follows from $N_{s,i} < 5 \times 10^{-8}$. The operational removal boundary is 1 keV; it labels departure from the retained nonthermal population, not thermal equilibrium at that energy.

At the central density, the coherent source retains 93.96% of its protons and 10.07% of its alpha particles above threshold. Their surviving number ratio becomes 0.1414. A response that suppresses the energetic tail increases the importance of losses, especially for the lower-energy alpha source. Thus internal structure and transport do not act as independent normalization corrections. They jointly determine which part of the spectral record remains observable.

The surviving composition is particularly sensitive to the internal response. Table 5 varies a common s_i/R_i while keeping the clock, abundance and plasma prescription fixed. The produced alpha-to-proton event ratio remains 1.3195, but the surviving ratio changes by more than an order of magnitude across this modest response range. Composition therefore constrains the internal response more strongly than the asymptotic spectral slope alone.

The same calculation provides an observable moment. At $n_{e0} = 0.2 \text{ m}^{-3}$ with $n_\alpha/n_p = 1/12$, the present retained population has kinetic-energy density $7.83 \times 10^{-3} \text{ eV m}^{-3}$ and pressure $5.12 \times 10^{-3} \text{ eV m}^{-3}$. The latter is 1.54% of the prescribed electron-plus-ion thermal pressure at 10^4 K . These are conditional forward-model outputs, not measurements or empirical bounds. They identify a direct route to existing

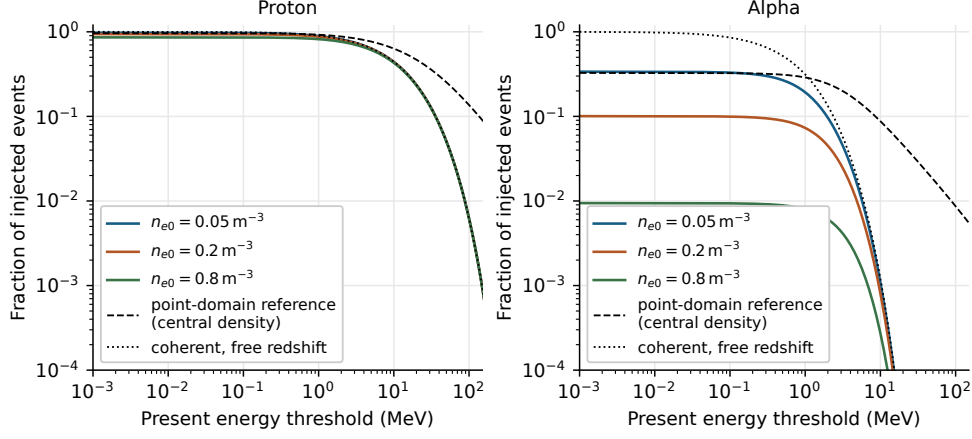


Fig. 5 Survival functions of the Gaussian finite-size source. Coloured curves give Eq. (77) at three electron densities. The black dashed curve is the point-domain reference source at $n_{e0} = 0.2 \text{ m}^{-3}$; the dotted curve is the finite-size source with free redshift only. Fractions are normalized to each source's injected count in the stated aperture window. The source response is included before propagation.

pressure, ionization and emission analyses once their environments and competing sources are supplied.

7 Energy transfer and a finite positive effective reservoir

7.1 Injection, deposition and a uniform upper bound

Let $f_i(\mathbf{p}, t)$ be a normalized target momentum density and $T_i(\mathbf{p}) = \sqrt{m_i^2 c^4 + c^2 p^2} - m_i c^2$. The channel injects

$$\mathcal{Q}_i = n_i \Gamma_i \int d^3 p f_i(\mathbf{p}, t) \int d^3 q \mu_r(\mathbf{q}) \mathcal{R}_{s_i}(q) [T_i(\mathbf{p} + \mathbf{q}) - T_i(\mathbf{p})]. \quad (78)$$

Convexity and zero mean transfer imply $\mathcal{Q}_i \geq 0$. The Hessian of T_i is bounded above by I/m_i , so

$$0 \leq \mathcal{Q}_i \leq \mathcal{Q}_i^{\text{NR}} = n_i \Gamma_i C_{s_i}(r) \frac{3\hbar^2}{4m_i r_{s_i}^2}. \quad (79)$$

The upper bound is independent of f_i . Its proof is given in Appendix E. It is exact as a nonrelativistic injection formula and a sufficient relativistic energy budget for the retained channel.

For a nonthermal kinetic-energy density ε_{nt} and pressure P_{nt} , the homogeneous energy moment is

$$\dot{\varepsilon}_{\text{nt}} + 3H(\varepsilon_{\text{nt}} + P_{\text{nt}}) = \mathcal{Q}_{\text{inj}} - \mathcal{Q}_{\text{dep}}. \quad (80)$$

Collisional losses and residual energy at the removal boundary contribute to $\mathcal{Q}_{\text{dep}}$. The thermal sector receives this deposition. Substituting the injection directly into a temperature equation would omit both the retained population and its expansion losses.

Energy conservation has long motivated microscopic interpretations and dissipative extensions of localization dynamics [23, 24]. Recent nonequilibrium analysis of gravitational collapse models further distinguishes unbounded heating from genuine dissipative thermalization [25]. Finite target size solves a different problem here: it makes the integrated energy requirement finite for the specified aperture history. It does not by itself turn the jump generator into a dissipative thermalizing channel.

7.2 An explicit effective homogeneous reservoir realization

Assume conserved target numbers, $n_i(t) = n_{i0}a^{-3}$. The integrated nonrelativistic injection per present comoving volume is

$$\mathcal{W}_{\text{NR}} = \int a^3 \mathcal{Q}_{\text{inj}}^{\text{NR}} dt = \sum_i n_{i0} \mathcal{E}_{s,i}^{\text{NR}}. \quad (81)$$

It is finite for every $s_i > 0$ in the Gaussian model. In the benchmark, Eq. (44) gives $\mathcal{E}_{s,i}^{\text{NR}} = 0.08323 \text{ eV}$ per original target for both species. Direct integration with the chosen relativistic Hamiltonian and initially cold targets gives 0.08036 eV for protons and 0.08318 eV for alpha particles, below the uniform bound.

To exhibit a closed ensemble-averaged homogeneous exchange budget, introduce an effective reservoir of mean energy density ε_D and prescribe zero pressure for its exchanging component over the production interval:

$$\dot{\varepsilon}_D + 3H\varepsilon_D = -\mathcal{Q}_{\text{inj}}. \quad (82)$$

This is an explicit constitutive assumption for the effective reservoir, not an identification with a particular DJAMILAR charge. Its solution is

$$\varepsilon_D(t) = a(t)^{-3} \left[\mathcal{U}_{D,i} - \int_{t_i}^t a(t')^3 \mathcal{Q}_{\text{inj}}(t') dt' \right], \quad \mathcal{U}_{D,i} = a(t_i)^3 \varepsilon_D(t_i). \quad (83)$$

Remark 1 (Sufficient positive effective reservoir) For the retained Gaussian coherent channel with $s_i > 0$, any initial comoving reserve $\mathcal{U}_{D,i} > \mathcal{W}_{\text{NR}}$ makes Eq. (83) positive throughout the production history. Its mean energy loss exactly balances the matter injection. The statement holds for arbitrary initial target momentum distributions for which the effective kinetic description applies.

The proof follows from Eq. (79) and monotonicity of the accumulated nonnegative input. In units $c = 1$, with $u_\mu u^\mu = -1$, the corresponding homogeneous transfer is $\nabla_\mu T_{\text{matter}}^{\mu\nu} = \mathcal{Q}_{\text{inj}} u^\nu$ and $\nabla_\mu T_D^{\mu\nu} = -\mathcal{Q}_{\text{inj}} u^\nu$. Internal redistribution between thermal and nonthermal matter cancels from the total exchange.

For the central benchmark, $n_{p0} = 6n_{e0}/7$ and $n_{\alpha 0} = n_{e0}/14$, Eq. (81) yields

$$\mathcal{W}_{\text{NR}} = 1.546 \times 10^{-2} \text{ eV m}^{-3}, \quad \frac{\mathcal{W}_{\text{NR}}}{\sum_i n_{i0} m_i c^2} = 7.22 \times 10^{-11}. \quad (84)$$

An initial comoving reserve of $2 \times 10^{-2} \text{ eV m}^{-3}$, for example, is positive throughout and sufficient for this retained channel. The exchange demand is small relative to the benchmark baryonic rest energy, supporting its treatment as a perturbation of the prescribed homogeneous background. This comparison does not calculate the gravitational response of a microscopic geometric reservoir; that requires the state dependence in Eq. (13).

The result supplies a finite positive solution of the mean homogeneous exchange problem. It does not bound energy fluctuations in individual transfer histories. A microscopic Hamiltonian producing the clock, response and reservoir must also specify those fluctuations. The conditional transfer relations and their mean energy budget are fully determined without that further identification.

7.3 Thermal deposition and intergalactic constraints

The Coulomb term used in the transport calculation, together with the residual kinetic energy deposited when a trajectory reaches the 1 keV removal boundary, also determines the heat transferred to the thermal gas. For each species let $D_{w,i}(a)$ denote the deposited energy per original target, weighted by a^2 so that subsequent free thermal evolution follows $T \propto a^{-2}$. For conserved present-day target densities, the associated temperature increment is

$$\Delta T(a) = \frac{2}{3k_B n_{\text{tot},0} a^2} \sum_i n_{i0} D_{w,i}(a), \quad n_{\text{tot},0} = n_{e0} + n_{p0} + n_{\alpha 0}. \quad (85)$$

Forward characteristics using the same Coulomb prescription and source window as Sect. 6 give, for $n_{e0} = 0.2 \text{ m}^{-3}$, close to the cosmic mean electron density ($\simeq 0.22 \text{ m}^{-3}$ for a fully ionized primordial plasma with Planck-like baryon density [21]), and $s_i = R_i$,

$$\Delta T_0 \simeq 12 \text{ K} \left(\frac{\gamma_0}{8 \times 10^{-10} \text{ s}^{-1}} \right).$$

The alpha contribution dominates this deposited heat (about 85–90% in the numerical quadrature). Equation (85) makes the bookkeeping explicit; the linear scaling follows because the source intensity is proportional to γ_0 while the single-particle transport problem is not.

Density dependence.

Gas at overdensity Δ has $n_e = \Delta n_{e0} a^{-3}$. Keeping Δ fixed along each fluid element and repeating the forward calculation, the increment at $z = 0.1$ grows as $\Delta^{0.34}$: $\Delta T = 8.5, 12.1, 18.0, 26.3$ and 41.4 K for $\Delta = 0.5, 1, 3, 10$ and 30 . Denser gas stops a larger share of the injected nuclei. The delay of reading M in Sect. 2.2 is complete by $z = 0.1$ and changes these values by less than 7%. The logarithmic slope stays between 0.33 and 0.35 for every aperture scale we tested, from 25 nm to 2.5 mm, and in both readings. Gas heated mainly by the channel therefore has a temperature–density slope

$$\gamma_{\text{ch}} \simeq 1.34, \quad (86)$$

within this fixed-density plasma and coherent-channel transport model.

A ceiling for coherent suppression.

Recent low-redshift Ly α -forest inference from 82 HST/COS spectra reports $\log_{10} T_0$ and γ in four bins between $z = 0.06$ and 0.48 , with $\log_{10}(T_0/\text{K}) = 4.45^{+0.08}_{-0.12}$ at $z \simeq 0.1$ [26]; here γ is the standard temperature–density slope of the intergalactic medium, unrelated to the normalization γ_0 . As a deliberately conservative ceiling, require the DJAMILAR contribution by itself not to exceed an upper envelope obtained by adding twice the quoted upper 1σ offset in $\log_{10} T_0$. The $z \simeq 0.2$ bin is the most restrictive and gives $\gamma_0 \lesssim 2.5 \times 10^{-6} \text{ s}^{-1}$ for the benchmark clock, response and plasma. The heating amplitude scales with the injection of Eq. (44), proportional to $\gamma_0/(Q_w r_0^2)$, while a larger aperture moves the onset earlier. Scanning r_0 from 25 nm to 2.5 mm, the ceiling rises roughly as r_0^2 , and in energy units it is nearly universal: the injection must stay below 200–260 eV per nucleus (Table 6). No standard photoheating, shocks or feedback is subtracted, and unresolved turbulence, which the observational analysis discusses as an alternative, could only make the measured T_0 an upper bound on the gas temperature [26]. The use of IGM heating to constrain localization models has precedent in CSL analyses [4].

The coherent-channel fit to the hot IGM.

The same measurements find the low-redshift IGM about seven times hotter than canonical photoheating predicts, and an independent analysis reports a related excess in line widths [27]. We add the channel to canonical photoheating, $T(\Delta, z) = T_{\text{ph}} \Delta^{\gamma_{\text{ph}}-1} + \Delta T(\Delta, z)$ with $T_{\text{ph}} = 4000 \text{ K}$ and $\gamma_{\text{ph}} = 1.6$, read $T_0 = T(1, z)$ and $\gamma = 1 + \log_{10}[T(10, z)/T(1, z)]$, and fit the eight measured values with their asymmetric 1σ errors. The original six-point aperture scan already showed that matching the four temperatures drives the fit toward large apertures while retaining a slope mismatch. To test whether the previous upper-grid minimum was artificial, we extend the fit-only transport scan by three decades, through $r_0 = 25 \text{ mm}$, 250 mm and 2.5 m , without changing the forward model or likelihood. The enlarged scan removes the former boundary artifact but does not select a preferred physical aperture. For reading M the profile is broad: the minimum scores are 21.53, 20.75, 20.59 and 21.02 at $r_0 = 2.5 \text{ mm}$, 25 mm , 0.25 m and 2.5 m , respectively. Thus $\Delta\chi^2 < 1$ across three decades. The shallow minimum occurs at $r_0 = 0.25 \text{ m}$ with $\gamma_0 = 1.58 \times 10^8 \text{ s}^{-1}$ and

$$\chi_{\text{min}}^2 = 20.59 = 1.91 \text{ (temperatures)} + 18.68 \text{ (slopes)}. \quad (87)$$

The role of the extension is therefore to demonstrate a plateau and eliminate the previous grid-edge interpretation, not to motivate 0.25 m as a distinguished scale. Treating r_0 and γ_0 as two fitted parameters gives a nominal six-degree-of-freedom diagnostic $P = 2.17 \times 10^{-3}$. The physical conclusion is unchanged and sharpened: the temperature normalization can be fitted over the plateau, but the measured density slopes dominate the residual. The conservative ceiling remains nearly universal in injected energy, about 200–260 eV per nucleus over the original five-decade scan. The full covariance of the jointly inferred thermal parameters is not tabulated in Ref. [26],

Table 6 Coherent-channel intergalactic constraints from the four bins of Ref. [26] on the original six-point aperture grid. The ceiling requires the channel heating alone to stay below the upper 2σ envelope of T_0 ; it is identical in readings U and M. The fit adds canonical photoheating; best-fit values are for reading M and differ from reading U by less than 20%. The extended fit-only scan of Eq. (87) removes the former upper-grid minimum without changing this core table. t_{lb}^* is the proton crossing lookback time.

r_0	t_{lb}^* (Gyr)	γ_0^{max} (s^{-1})	$\mathcal{E}^{\text{max}}$ (eV)	Best γ_0 (s^{-1})	$\mathcal{E}$ (eV)	χ_{min}^2 (M / U)
25 nm	3.91	2.5×10^{-6}	262	1.9×10^{-6}	199	91.9 / 113.1
250 nm	4.28	2.3×10^{-4}	235	1.6×10^{-4}	162	90.0 / 93.0
2.5 μm	4.61	2.2×10^{-2}	216	1.9×10^{-2}	185	65.0 / 81.1
25 μm	4.92	2.2	208	1.8	166	26.2 / 35.9
250 μm	5.22	2.2×10^2	202	1.6×10^2	148	23.8 / 26.0
2.5 mm	5.49	2.3×10^4	197	1.6×10^4	138	21.5 / 22.7

so the published-marginal split-Gaussian score remains the directly reproducible test. Canonical photoheating alone has $\chi^2 = 211$, and $T_{\text{ph}} = 5000 \text{ K}$ changes the best fit by less than 2. Reference [16] estimated the thermal signal by counting the mean injected energy as immediate heat. In the present calculation that energy is instead carried by rare relativistic transfers and thermalizes only through the transport equation, so the transport treatment above replaces the instantaneous-deposition estimate of Ref. [16] for quantitative thermal predictions.

Sensitivity to unreported covariance.

As a diagnostic, keep redshift bins independent and assign the same within-bin correlation $\rho_{T\gamma}$ to the standardized temperature and slope residuals u_z, v_z . Replace their independent contribution by

$$\mathcal{S}_\rho = \sum_z \frac{u_z^2 - 2\rho_{T\gamma}u_zv_z + v_z^2}{1 - \rho_{T\gamma}^2}. \quad (88)$$

Refitting the normalization on the extended aperture scan gives minima 23.62, 21.33, 20.59, 23.41 and 34.10 for $\rho_{T\gamma} = -0.8, -0.5, 0, 0.5, 0.8$. The $\rho = -0.8$ minimum occurs at $r_0 = 25 \text{ mm}$ and the other four at $r_0 = 250 \text{ mm}$; none lies on the boundary of the enlarged 25 nm–2.5 m scan. The slope mismatch therefore survives both the extension in aperture and this specified covariance family. These assumed correlations are not a recovered covariance matrix, and the split-error score is not a substitute for the original joint likelihood. No calibrated significance is assigned to these alternative scores, nor is the coherent-channel slope test transferred to the composite-target completions analyzed in the companion paper [14] without their nonelastic transport.

8 Discussion

The construction isolates a precise meaning of spectral memory. The instantaneous dynamics can remain Markovian and CP-divisible while the final transfer distribution retains the event-weighted occupation of an evolving scale. In the point-domain area

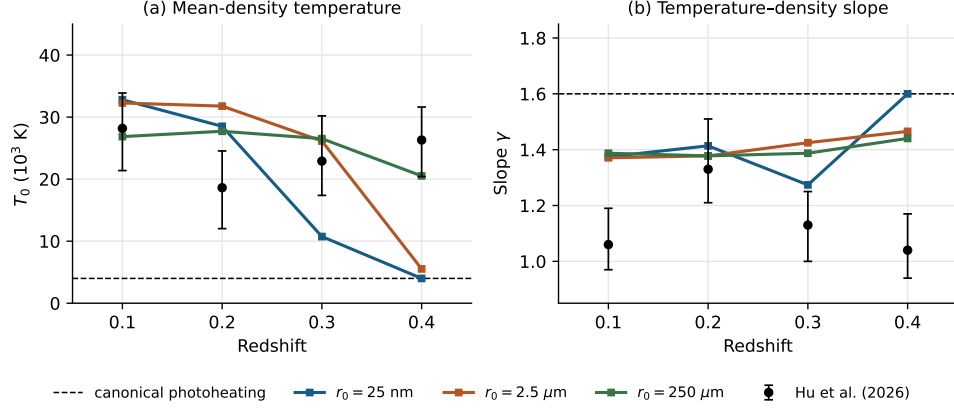


Fig. 6 Canonical photoheating plus the coherent channel (reading M), fitted to the low-redshift measurements of Ref. [26]. (a) A late onset ($r_0 = 25$ nm) cannot heat the $z = 0.4$ gas; apertures of order 250 μ m reproduce all four temperatures. (b) Every fit predicts $\gamma \simeq 1.4$, steeper than measured.

branch, the q^{-3} spectral amplitude and the logarithmically enhanced coherence coefficient are two transforms of the same scale accumulation. A finite Gaussian target response replaces the ideal marginal behaviour by an exact crossover and fixes its energetic cost through the count derivatives.

The inverse result adds a second direction to this forward consistency structure. Ideal spectra identify the logarithmic aperture occupation under the stated kernel assumptions, and a known rate functional converts that occupation into normalized chronology. The logarithmic stability theorem makes the limitation equally explicit: high-frequency structure in log-aperture is exponentially expensive to recover. The synthetic ensembles therefore serve as sensitivity demonstrations, not as universal instrument forecasts. Their main role is to distinguish mathematical identifiability from recoverability with finite band and noise.

The IGM test is useful precisely because it does not merely constrain one mean heating number. The same source history that fixes the injected energy also fixes the onset and density dependence of deposition after transport. Extending the aperture scan removes the former numerical boundary minimum and leaves a broad plateau rather than a preferred aperture scale. Across that plateau the measured temperature normalization can be approached, but the density slopes dominate the residual. This negative result constrains the homogeneous realization while leaving the general spectral and inverse theorems untouched.

The remaining microscopic interface is the response of a genuinely composite target once internal degrees of freedom are resolved. That problem requires a many-body generator rather than a scalar coherent form factor and leads to distinct thermal and paleo-observables. It is intentionally separated from the present paper so that the spectral theorem, inverse problem and IGM transport can be assessed without importing a particular composite-target completion; the submitted companion paper develops that response problem and its Solar-System tests [14].

9 Conclusion

An evolving localization aperture can leave a reconstructible mechanical record. For the specified completely positive scalar channel, one aperture history fixes event counts, momentum spectrum, coherence attenuation and finite-size energy moments. On the constant-winding area branch, the q^{-3} reference amplitude fixes the coefficient of the $d^2 \ln(R/d)$ coherence law, while the Gaussian response closes the count–moment hierarchy exactly.

For a general strictly increasing aperture history, Mellin factorization reconstructs the event occupation per logarithmic scale. With a known rate functional this determines normalized chronology, and one external time calibration fixes the absolute clock. The inverse problem is exponentially ill-posed; the logarithmic stability theorem, cumulative-chronology bound and finite-band ensembles quantify the information that can survive realistic noise and bandwidth restrictions.

The conditional cosmological realization provides a forward test of the same structure. Explicit proton–alpha transport gives a coherent-channel IGM ceiling of about 200–260 eV per nucleus. After extending the aperture fit by three decades, χ^2 varies by less than one from 2.5 mm to 2.5 m and has only a shallow minimum $\chi^2 = 20.59$ at $r_0 = 0.25$ m; 18.68 of that score comes from the density slopes. The relevant conclusion is therefore the broad slope mismatch, not a preferred aperture value. Spectral memory is consequently a linked forward-and-inverse framework whose internal consistency can be tested without treating the heating amplitude as an independent parameter.

Statements and declarations

Data and code availability

No new observational data are introduced. The accompanying source archive contains the LaTeX manuscript, figures, tabulated spectra and transport outputs, together with the scripts required to reproduce the spectral, finite-response, inversion and IGM calculations. The command `python reproduce.py --verify` checks the core spectral and transport results. The scripts `extensions/clock_inverse.py`, `extensions/inversion_stability.py`, `extensions/inversion_ensemble.py`, `extensions/igm_extended.py`, `extensions/igm_covariance.py` and `extensions/make_figures.py` reproduce the inverse-problem and extended IGM diagnostics used here. The versioned reproducibility archive is available on Zenodo at <https://doi.org/10.5281/zenodo.23451185>.

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Competing interests

The author declares no competing interests.

Appendix A Spectral theorem and endpoint control

Substituting $dt = dr/(\kappa r)$ into the radial channel gives Eq. (17). On the ideal interval, $u = qr/\hbar$ gives Eq. (19). Normalization of h and Tonelli's theorem yield

$$N_{\text{pt}} = \frac{A}{\kappa} \int_0^\infty \frac{r^{b-1}}{1 + (r/R)^p} dr = \frac{AR^b}{\kappa} \frac{\pi}{p} \csc(\pi b/p), \quad (\text{A1})$$

using $0 < b < p$. Monotone convergence of the kernel integral proves Eq. (21). For the low-momentum limit, rewrite it as

$$\int_0^\infty \frac{u^b h(u)}{1 + (u/x)^p} du = x^p \int_0^\infty \frac{u^{b-p} h(u)}{1 + (x/u)^p} du. \quad (\text{A2})$$

The second integral tends to M_{b-p} when finite. For a regular three-dimensional kernel $h(u) \sim c_0 u^2$, the low- q cases are $J_{\text{pt}} \propto q^{p-b-1}$ for $p < b+3$, $q^2 \ln(\hbar/qR)$ for $p = b+3$, and q^2 for $p > b+3$. These follow by splitting the integral at $u \sim x$.

At fixed aperture, the j th radial moment is $(\hbar/r)^j M_j$. Thus

$$\int q^j J_{\text{pt}}(q) dq = \frac{A \hbar^j M_j R^{b-j}}{\kappa} \frac{\pi}{p} \csc\left[\frac{\pi(b-j)}{p}\right], \quad (\text{A3})$$

provided $b-p < j < b$ and $M_j < \infty$. Dividing by Eq. (A1) proves Eq. (23).

For the area branch, the early point-domain reference event fraction removed at $r < \epsilon R$ is

$$\Delta N_{\text{pt}}/N_{\text{pt}} = \frac{2}{\pi} \arctan(\epsilon^2) = O(\epsilon^2). \quad (\text{A4})$$

The Gaussian finite-size factor obeys $C_s \sim y^3/\lambda^{3/2}$ at $y \ll \sqrt{\lambda}$, so for fixed $\lambda > 0$,

$$\Delta N_s/N_{\text{pt}} = \frac{4\epsilon^5}{5\pi\lambda^{3/2}} + o(\epsilon^5). \quad (\text{A5})$$

Finite size improves early-history count control and bounds the transfer width. At large y , the omitted event fraction is $O(y^{-2})$ for both models. An unbounded second moment cannot be bounded by a total-variation estimate in the point-domain reference limit; the finite-size model has a finite second moment supplied by Eq. (34).

Appendix B Proof of the coherence amplitude relation

For $|z| \leq 1$, $1 - \text{sinc } z = z^2/6 + O(z^4)$. Split Eq. (27) at a fixed large q_* and at $\hbar/d$. The low- q part is $O(d^2)$ because the intensity is finite. On the middle interval a Kq^{-3} tail gives

$$\frac{Kd^2}{6\hbar^2} \int_{q_*}^{\hbar/d} \frac{dq}{q} = \frac{Kd^2}{6\hbar^2} \ln\left(\frac{\hbar}{q_*d}\right). \quad (\text{B6})$$

The fourth-order remainder contributes $O(d^2)$; the upper interval also contributes $O(d^2)$ because $1 - \text{sinc } z$ is bounded and $\int_{\hbar/d}^{\infty} q^{-3} dq = O(d^2)$. Replacing the tail by its asymptotic form changes the leading coefficient by an arbitrarily small relative amount after choosing sufficiently large q_* . This proves Eq. (28); changing R changes only a nonleading constant.

For a finite second moment, $0 \leq 1 - \text{sinc } z \leq z^2/6$. Dominated convergence proves Eq. (29). For $0 < b < 2$, the same scaling argument gives $\chi(d) \sim K_b(d/\hbar)^b c_b$, with

$$c_b = \int_0^{\infty} \frac{1 - \text{sinc } z}{z^{b+1}} dz = \frac{\pi}{2(b+1)\Gamma_E(1+b)\sin(\pi b/2)}. \quad (\text{B7})$$

Here Γ_E is the Euler gamma function. The identity follows from $1 - \text{sinc } z = \int_0^1 (1 - \cos uz) du$. The area law is the marginal member of this family.

For the Gaussian area branch, set $\delta = d^2/(4R^2)$ and integrate the aperture expression by parts. It reduces to

$$\frac{\chi_{\text{pt}}}{N_{\text{pt}}} = \frac{2\delta}{\pi} \int_0^{\infty} e^{-\delta v} \arctan(1/v) dv. \quad (\text{B8})$$

The integral equals $-\ln \delta + 1 - \gamma_E + O(\delta)$ as $\delta \rightarrow 0$. Substitution gives Eq. (30). This refinement is specific to the Gaussian shape; Proposition 2 is not.

Appendix C Gaussian response, exact count and moment identities

Equation (31) follows by completing the quadratic exponent and comparing normalized Gaussian prefactors. Under $u = y/\sqrt{y^2 + \lambda}$, the count becomes

$$F(\lambda) = \frac{4\lambda}{\pi} \int_0^1 \frac{u^4 du}{(1 - u^2)^2 + \lambda^2 u^4}. \quad (\text{C9})$$

Factor the denominator as $[1 - (1 + i\lambda)u^2][1 - (1 - i\lambda)u^2]$. Integrating the partial fractions with $\int_0^1 (1 - zu^2)^{-1} du = \text{atanh } \sqrt{z}/\sqrt{z}$ gives Eq. (35). Expanding that expression gives Eq. (42); at large λ , $F(\lambda) \sim 4/(\pi\lambda)$ and $B(\lambda) \sim 4/(\pi\lambda^2)$.

For a finite J_{pt} and β in any compact subinterval of $(0, \infty)$, $q^{2n} \exp(-\beta q^2)$ is uniformly bounded. Differentiation under the integral proves Eq. (38). The first identity in Eq. (39) follows from Eq. (29). Isotropy makes every impulse mean zero; independent impulses therefore add their nonrelativistic mean energies even outside the rare-event limit. This proves the second identity. Applying the first derivative to the dimensionless count yields Eq. (34).

The logarithmic deficit in Eq. (43) follows by splitting $\int J_{\text{pt}}(q)[1 - \exp(-s^2 q^2/(3\hbar^2))] dq$ at $q \sim \hbar/s$. On the intermediate interval its integrand is $Ks^2/(3\hbar^2 q)$; the remaining intervals are nonleading relative to $s^2 \ln(R/s)$. The factor two relative to coherence comes from the coefficients $1/3$ in the Gaussian response and $1/6$ in the angular coherence expansion.

Finally, logarithmic differentiation of $\Gamma_s \propto y^2(1+y^4)^{-1}(1+\lambda/y^2)^{-3/2}$ gives

$$\frac{d \ln \Gamma_s}{d \ln y} = 2 - \frac{4y^4}{1+y^4} + \frac{3\lambda}{y^2+\lambda}. \quad (\text{C10})$$

Its derivative with respect to y is negative. The expression falls from 5 to -2 for $\lambda > 0$, has one zero, and is positive at $y = 1$. Clearing denominators proves Eq. (36) and the delayed maximum.

Appendix D Rare events and finite-history probability control

Relative to P_1 , the one-event term in Eq. (15) decreases by the positive measure $(1 - e^{-N})\nu$. The zero-event and higher-event terms increase by total mass

$$(e^{-N} - 1 + N) + [1 - e^{-N}(1 + N)] = N(1 - e^{-N}). \quad (\text{D11})$$

Their contributions need not have disjoint support, so their total positive mass gives the upper bound in Eq. (16) without assuming that every higher-event outcome is distinguishable.

If a source interval of intensity ΔN is omitted, couple the two processes through identical events on the shared history and an independent Poisson process on the removed interval. Then

$$\|P_{\text{full}} - P_{\text{cut}}\|_{\text{TV}} \leq 1 - e^{-\Delta N} \leq \Delta N. \quad (\text{D12})$$

The same bound holds after a common deterministic redshift map. A common probability-preserving transport kernel also contracts total variation, provided the removed population is retained as an absorbing state when necessary. These probability bounds do not imply convergence of unbounded moments in an unregularized model.

Appendix E Energy bound and effective reservoir construction

For $T(\mathbf{p}) = \sqrt{m^2 c^4 + c^2 p^2} - mc^2$, the transverse Hessian eigenvalues are $c^2/\sqrt{m^2 c^4 + c^2 p^2}$ and the longitudinal eigenvalue is $m^2 c^6/(m^2 c^4 + c^2 p^2)^{3/2}$. All lie in $[0, 1/m]$. Taylor's theorem therefore gives

$$T(\mathbf{p} + \mathbf{q}) - T(\mathbf{p}) \leq \nabla T(\mathbf{p}) \cdot \mathbf{q} + \frac{q^2}{2m}. \quad (\text{E13})$$

The linear term averages to zero for the isotropic filtered transfer distribution. Jensen's inequality supplies the lower bound. Gaussian closure then gives Eq. (79). The bound does not require initially cold targets; the cold-target calculation is used only for the quoted relativistic numerical comparison.

Summing over conserved target numbers and integrating $a^3 \mathcal{Q}^{\text{NR}}$ gives Eq. (81). Inserting the upper bound into Eq. (83) proves Remark 1. A nonzero prescribed effective-reservoir equation of state can be treated with its corresponding integrating factor, but the zero-pressure component already provides an explicit positive realization. It closes homogeneous exchange bookkeeping for the chosen channel without deriving a Lorentz-covariant stochastic reservoir or its microscopic gravitational stiffness.

Appendix F Numerical specification and reproducibility

The flat matter-plus- Λ background is

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda} \right)^{1/3} \sinh^{2/3} \left(\frac{3}{2} H_0 \sqrt{\Omega_\Lambda} t \right), \quad t_0 = \frac{2 \operatorname{arsinh} \sqrt{\Omega_\Lambda / \Omega_m}}{3 H_0 \sqrt{\Omega_\Lambda}}. \quad (\text{F14})$$

The masses are $m_p = 1.67262192595 \times 10^{-27}$ kg and $m_\alpha = 6.644657345 \times 10^{-27}$ kg. Source spectra use Simpson integration on 6001 uniformly spaced $\ln(r/R_i)$ nodes in $[\ln 0.03, \ln 100]$. The radial-to-energy Jacobian is

$$\frac{dq}{dE} = \frac{E + mc^2}{c \sqrt{E(E + 2mc^2)}}. \quad (\text{F15})$$

The electron Coulomb loss prescription is

$$\begin{aligned} b_C &= \frac{4\pi n_e}{m_e v} \left(\frac{Ze^2}{4\pi\epsilon_0} \right)^2 \ln \Lambda \left[\operatorname{erf} X - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right], \\ X &= \frac{v}{\sqrt{2k_B T_g / m_e}}, \quad \ln \Lambda = \max \left\{ 2, \ln \left(\frac{2m_e c^2 \beta_v \mathcal{P}}{\hbar \omega_p} \right) - \frac{\beta_v^2}{2} \right\}, \\ \omega_p^2 &= \frac{n_e e^2}{\epsilon_0 m_e}, \quad \beta_v = v/c, \quad \gamma_L = 1 + E/(mc^2), \quad \mathcal{P} = \gamma_L \beta_v. \end{aligned} \quad (\text{F16})$$

This follows the plasma-loss form used in Ref. [22]. It is environmental input, not a localization prediction.

Backward characteristics use adaptive Runge–Kutta integration with relative tolerance 2×10^{-9} . The aperture integral uses 500 Gauss–Legendre nodes in $\ln y$. Increasing to 900 nodes and tightening the ODE tolerance to 2×10^{-11} changes the central survival fractions and selected survival-curve values by less than 1.3×10^{-9} in absolute fraction. Independent aperture and spectral quadratures agree on F to better than 2×10^{-9} over the verification set; the closed expression in Eq. (35) agrees with quadrature to better than 6×10^{-13} . Finite-difference count derivatives and the direct moment integral agree relatively within 9×10^{-9} , and the direct small-separation coherence limit agrees with $B/6$ within 3×10^{-11} for the tested response sizes.

The density-resolved IGM deposition integrates Eq. (75) forward with LSODA at relative tolerance 10^{-7} , using 48 Gauss–Legendre injection nodes in $\ln y$ over

$0.03 < y < 100$ and 32 generalized Gauss–Laguerre nodes for the Gaussian transfer distribution. At the benchmark this grid reproduces the 96×64 result to 0.4%; refinement to 144×96 changes ΔT_0 by 0.1%. Injected energy equals retained, adiabatic and deposited energy to relative accuracy 5×10^{-6} . The extended IGM fit uses direct evaluations through $r_0 = 2.5$ m, and the covariance diagnostic refits the normalization at each sampled aperture.

The inversion ensemble uses 241 occupation nodes over $-6 \leq y \leq 6$, 256 noise draws per setting and seed 20261018. Whitening uses the supplied synthetic noise standard deviations; singular values below 10^{-10} of the largest are discarded. Nonnegative projection follows truncation. The finite-band case adds the stated additive noise floor. Full arrays, percentile bands and cumulative-clock errors are included in the archive.

Appendix G Provenance of the DJAMILAR ingredients

The effective construction uses a small subset of the wider DJAMILAR programme. The aperture-state dictionary and evolving open-area branch are taken from Ref. [15]; the homogeneous cosmological clock and aperture boundary hypothesis from Ref. [16]; the state-dependent gravitational context from Ref. [28]; the Gaussian localization kernel and fixed-state size activation from Ref. [29]; and the motivation for joint mechanical/coherence reconstruction from Ref. [30]. The present spectral theorem, response-moment identities, inverse reconstruction, stability bound and IGM transport are derived from the equations stated in this manuscript. Where a programme-level interpretation is not required by those equations, it is not used as an additional assumption.

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