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Geometric Foundations of Quantum Entanglement in the Helical-Chain–Biconical Model

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Abstract

We propose a geometric framework for quantum entanglement based on a helical-chain phase field and a biconical 2 : 1 transmission ratio. Postulates A1–A7 define the underlying phase field, the 2 : 1 transmission ratio, the spinor boundary condition, apex-to-apex docking with Fermi statistics, the half-angle measurement projection, an even- N tooth-site decoherence potential, and a classical readout rule. Within this postulate system we obtain: the singlet form under a two-sector truncation (Theorem 1); the half-angle probability $P(+) = \cos^2(\theta/2)$, hence $E = -\cos \Delta$ and $|S|_{\max} = 2\sqrt{2}$ (Theorems 2–3); the classical side $|S| \leq 2$, independent of N , under the classical readout (Theorem 4a); for even N , an N -peak structure, peak spacing $2\pi/N$, $\tau_d \propto N^{-3}$, and instanton action $(8/N)\sqrt{IV_0}$ (Theorem 4b); the free $K = 1$ Luttinger-liquid limit corresponding to a Dirac fermion (Theorem 5); the GHZ Mermin quantity $|M|_{\max} = 4$ (Theorem 6); and the standard W-state reduction $(2/3)|\Phi^+\rangle\langle\Phi^+| + (1/3)|00\rangle\langle 00|$ (Theorem 7). We distinguish postulates, theorems, and standard-result correspondences. Under the combined postulates A3–A5, the 2 : 1 transmission ratio provides a geometric basis for the spinor boundary and the half-angle projection; logical necessity comes from the combination of postulates, not from A2 alone. The tooth number N (even) is a replaceable parameter of the decoherence geometry and should be determined experimentally. The present model is an equivalent ontological interpretation of standard quantum mechanics; the N -peak decoherence distribution is a candidate distinguishable signal.

Keywords: Quantum entanglement; Spin 1/2; Spinor double cover; Bell inequality; Decoherence; Tooth number

1 Introduction

Standard quantum mechanics describes entanglement formally: an entangled state cannot be decomposed into a product state, Bell inequalities are violated, and entanglement entropy quantifies correlations. Standard quantum mechanics does not explain why entangled states have the singlet form, why the correlation function is $-\cos \Delta$, why the maximum CHSH violation is $2\sqrt{2}$, or why decoherence produces classical results.

This paper develops a unified geometric framework starting from a helical-chain phase field and a biconical 2 : 1 transmission ratio. We strictly distinguish three types of statements:

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1. **Postulates:** A1–A7, including the spinor boundary condition, Fermi statistics, the half-angle projection, the even- N tooth-site potential, and the classical readout rule. These are the starting points of the model and are not called “theorems.”
2. **Theorems:** results derived from the postulates, including the singlet form, the half-angle probability, $E = -\cos \Delta$, CHSH, the classical-side bound, the N -dependent structure, and the GHZ/W structures.
3. **Standard-result correspondences:** the low-energy limit reduces to the Dirac equation, Maxwell equations, and standard quantum-mechanical predictions; these are requirements of the correspondence principle, not new derivations of the model.

The paper is organized as follows. Section 2 presents the postulates and notation. Section 3 proves the singlet form under the two-sector truncation. Section 4 derives the measurement probabilities and CHSH. Section 5 treats even- N tooth-site decoherence, split into Theorems 4a and 4b. Section 6 discusses the low-energy correspondence. Section 7 addresses multipartite entanglement. Section 8 lists distinguishable predictions and N -discrimination criteria. Section 9 summarizes and discusses.

2 Postulates and Notation

A1 (Helical-chain phase field). The underlying structure is a helical-chain phase field $\Phi(x)$, with the phase quantization condition $\oint d\Phi = 2\pi n$. A single-particle state is labeled by the lower-cone angle

$$\varphi \in [0, 2\pi).$$

A2 (Biconical 2 : 1 transmission ratio). The upper-cone angle is

$$\alpha = 2\varphi \pmod{2\pi}.$$

This is the geometric transmission setting of the model. It provides a geometric basis for the spinor boundary in A3 and the half-angle projection in A5, but does not by itself “derive” them.

A3 (Spinor boundary condition, postulate). The lower-cone wave function satisfies the antiperiodic boundary condition

$$\psi(\varphi + \pi) = -\psi(\varphi).$$

This is the manifestation of the spin-1/2 double cover on the wave function and is introduced as a postulate. A2 provides the 2 : 1 geometric intuition; A3 is its quantized consequence.

A4 (Apex-to-apex docking, Fermi statistics, exchange operator, and two-sector truncation, postulate). When two particles share the same helical chain, apex-to-apex docking requires the relative angle

$$\varphi_A - \varphi_B = \pi n, \quad n \in \mathbb{Z}.$$

Under the two-sector truncation, only the two degenerate sectors $n = 0, 1$ are retained. The exchange operator acts on the two sectors as

$$\hat{P}_{AB}|n=0\rangle = |n=1\rangle, \quad \hat{P}_{AB}|n=1\rangle = |n=0\rangle,$$

and the physical docking state is required to satisfy

$$\hat{P}_{AB}|S\rangle = -|S\rangle.$$

This is the concrete implementation of Fermi statistics under the two-sector truncation. **Two-sector truncation (model choice):** at low energy only the two degenerate sectors $n = 0, 1$ are retained; higher- n sectors are suppressed by the soft-connection gap (order-of-magnitude estimate in Appendix E; quantitative treatment left for future work). Equal weights and the relative minus sign follow from the exchange-operator representation and $\hat{P}_{AB}|S\rangle = -|S\rangle$; they are not extra assumptions.

A5 (Half-angle measurement projection, postulate). Measurement along direction θ corresponds to the lower-cone equilibrium positions

$$\varphi_+ = \frac{\theta}{2}, \quad \varphi_- = \frac{\theta}{2} + \pi.$$

The projection states are

$$\psi_+(\varphi) = \frac{1}{\sqrt{\pi}} \cos\left(\varphi - \frac{\theta}{2}\right), \quad \psi_-(\varphi) = \frac{1}{\sqrt{\pi}} \sin\left(\varphi - \frac{\theta}{2}\right).$$

The half-angle is a setting of A5; its inner-product result is given by Theorem 2.

A6 (Even- N tooth-site decoherence potential, model choice). The decoherence potential is

$$V_N(\varphi) = V_0(1 - \cos N\varphi), \quad N = 2, 4, 6, \dots$$

with N strictly equivalent minima

$$\varphi_k = \frac{2\pi k}{N}, \quad k = 0, 1, \dots, N-1.$$

Restriction to even N : by A3, physical states satisfy the antiperiodic boundary condition, so the Hamiltonian potential must satisfy

$$V_N(\varphi + \pi) = V_N(\varphi).$$

This requires N to be even. Odd- N potentials are incompatible with the spinor boundary and are therefore not adopted.

A7 (Classical readout rule, model choice). After decoherence, each tooth site k gives the predetermined readout

$$A_k(\theta_a) = \text{sign}\left[\cos\left(\varphi_k - \frac{\theta_a}{2}\right)\right], \quad B_k(\theta_b) = -\text{sign}\left[\cos\left(\varphi_k - \frac{\theta_b}{2}\right)\right].$$

Except for boundary directions of measure zero, $A_k, B_k \in \{\pm 1\}$; at the boundary we take the regularized value $\text{sign}(0) = 0$, which does not affect the convex-combination bound. This is a classical readout model; it is not derived from the A5 projection or from the Lindblad equation.

Notation.

Symbol	Meaning
φ	lower-cone angle
$\alpha = 2\varphi$	upper-cone angle
I	lower-cone moment of inertia
V_0	tooth-site potential amplitude
N	tooth number (even)
θ	measurement direction angle
$\Delta = \theta_a - \theta_b$	angle between two measurement directions
$\omega_N = N\sqrt{V_0/I}$	local tooth-site frequency

3 Singlet Form Under the Two-Sector Truncation

Theorem 1 (Singlet form under the two-sector truncation). Under A3 (spinor boundary) and A4 (exchange operator $\hat{P}_{AB}|n=0\rangle = |n=1\rangle$, $\hat{P}_{AB}|n=1\rangle = |n=0\rangle$, with physical states satisfying $\hat{P}_{AB}|S\rangle = -|S\rangle$), the docking state uniquely takes the antisymmetric combination

$$|S\rangle = \frac{1}{\sqrt{2}} (|n=0\rangle - |n=1\rangle),$$

i.e., the standard singlet form.

Proof. By the two-sector truncation of A4, the low-energy docking retains only the two degenerate sectors $n=0, 1$. The exchange operator acts as

$$\hat{P}_{AB}|n=0\rangle = |n=1\rangle, \quad \hat{P}_{AB}|n=1\rangle = |n=0\rangle.$$

Let a general superposition be $|\psi\rangle = c_0|n=0\rangle + c_1|n=1\rangle$. Requiring $\hat{P}_{AB}|\psi\rangle = -|\psi\rangle$ gives

$$\hat{P}_{AB}|\psi\rangle = c_0|n=1\rangle + c_1|n=0\rangle = -c_0|n=0\rangle - c_1|n=1\rangle.$$

Comparing coefficients yields $c_0 = -c_1$. Normalization gives

$$|S\rangle = \frac{1}{\sqrt{2}} (|n=0\rangle - |n=1\rangle).$$

This is the unique solution. Equal weights $|c_0| = |c_1| = 1/\sqrt{2}$ are a mathematical consequence of the exchange-operator representation and the antisymmetry condition, not an extra assumption. An overall phase does not affect the physical state; fixing the coefficients to be real and the relative sign to be negative makes $|S\rangle$ unique. $\square$

Remark. This theorem does not claim “uniqueness” beyond the two-sector truncation. If more sectors are retained, the antisymmetric combination is not unique; the two-sector truncation is the model choice of A4. The mapping of $|n=0\rangle, |n=1\rangle$ to spin-up/spin-down is given by the half-angle projection of A5: $|n=0\rangle$ corresponds to the ψ_+ sector, and $|n=1\rangle$ to the ψ_- sector.

4 Measurement Probabilities and CHSH

Theorem 2 (Half-angle probability). Under the half-angle projection of A5, measurement along direction θ gives

$$P(+) = \cos^2 \frac{\theta}{2}, \quad P(-) = \sin^2 \frac{\theta}{2}.$$

Proof. Take the reference state to be the spin-up state:

$$\psi_{\uparrow}(\varphi) = \frac{1}{\sqrt{\pi}} \cos \varphi.$$

Projecting onto $\psi_+ = \frac{1}{\sqrt{\pi}} \cos(\varphi - \theta/2)$ gives

$$\langle \psi_+ | \psi_{\uparrow} \rangle = \frac{1}{\pi} \int_0^{2\pi} \cos\left(\varphi - \frac{\theta}{2}\right) \cos \varphi d\varphi = \cos \frac{\theta}{2}.$$

Squaring gives $P(+)$. Normalization gives $P(-)$. The half-angle comes from the projection setting of A5. $\square$

Theorem 3 (Correlation function and CHSH). For the singlet state of Theorem 1,

$$E(\theta_a, \theta_b) = -\cos(\theta_a - \theta_b), \quad |S|_{\max} = 2\sqrt{2}.$$

Proof. By Theorem 2, let $\Delta = \theta_a - \theta_b$. Then

$$P(++) = P(--)= \frac{1}{2} \sin^2 \frac{\Delta}{2}, \quad P(+-) = P(-+) = \frac{1}{2} \cos^2 \frac{\Delta}{2}.$$

Substituting into

$$E = P(++) + P(--)- P(+-) - P(-+)$$

gives

$$E = \sin^2 \frac{\Delta}{2} - \cos^2 \frac{\Delta}{2} = -\cos \Delta.$$

Taking the optimal angles

$$a = 0^\circ, \quad a' = 90^\circ, \quad b = 45^\circ, \quad b' = 135^\circ,$$

term-by-term evaluation gives $S = -2\sqrt{2}$, hence $|S|_{\max} = 2\sqrt{2}$. $\square$

Entanglement entropy: the singlet has Schmidt coefficients $c_1 = c_2 = 1/\sqrt{2}$, so the entanglement entropy is

$$S_{\text{ent}} = -\sum_i |c_i|^2 \ln |c_i|^2 = \ln 2.$$

Remark. Theorems 2–3 rely on the half-angle projection postulate A5; A5 is geometrically motivated by the 2 : 1 ratio of A2, but the half-angle is not derived from A2 alone.

5 Even- N Tooth-Site Decoherence

5.1 Theorem 4a: Classical side $|S| \leq 2$ independent of N

Theorem 4a (Classical-side bound). Under the classical readout rule A7, if after decoherence the system is a classical convex combination

$$\rho = \sum_{k=0}^{N-1} p_k |\varphi_k\rangle\langle\varphi_k|, \quad \sum_k p_k = 1,$$

and each tooth site k gives predetermined readouts A_k, B_k , then the CHSH quantity satisfies

$$|S| \leq 2.$$

For equal probabilities $p_k = 1/N$, at the standard optimal angles $|S| = 2$ for $N = 2, 4, 6$.

Proof. Except for boundary directions of measure zero, $A_k, B_k \in \{\pm 1\}$; at the boundary $\text{sign}(0) = 0$, which does not affect the convex-combination bound. For each k , the CHSH quantity

$$S_k = A_k(a)B_k(b) - A_k(a)B_k(b') + A_k(a')B_k(b) + A_k(a')B_k(b')$$

satisfies $|S_k| \leq 2$ (the standard classical inequality). The total correlation is the convex combination

$$S = \sum_k p_k S_k.$$

By the triangle inequality,

$$|S| \leq \sum_k p_k |S_k| \leq 2.$$

For equal probabilities, term-by-term evaluation gives $S_k = -2$ for $N = 2, 4, 6$ (Table 1), hence $S = -2$ and $|S| = 2$. $\square$

5.2 Theorem 4b: Decoherence structure for even N

Theorem 4b (N -dependent results, N even). For $V_N(\varphi) = V_0(1 - \cos N\varphi)$ with N even:

1. **Peak number:** in the $\varphi \in [0, 2\pi)$ expansion, decoherence produces N equivalent peaks; in the fundamental domain $[0, \pi)$, the number of independent peaks is $N/2$.
2. **Peak spacing:** in $[0, 2\pi)$, $d_N = 2\pi/N$; in $[0, \pi)$, the spacing is the same but the peak number is halved.
3. **Local frequency:** $\omega_N = N\sqrt{V_0/I}$.
4. **Zero-point width:**

$$\sigma_\varphi = \sqrt{\frac{\hbar}{2N\sqrt{IV_0}}}.$$

5. **Resolvability condition:**

$$\sqrt{IV_0} \gg \frac{\hbar N}{8\pi^2}.$$

Table 1: Term-by-term S_k for $N = 2, 4, 6$ (standard optimal angles, A7 readout rule). This table is computed term by term using A7 with $a = 0^\circ, a' = 90^\circ, b = 45^\circ, b' = 135^\circ$. The entries $S_k = -2$ are the results under A7 and the standard optimal angles; $\text{sign}(0) = 0$ affects only the intermediate terms for $N = 4, k = 1, 3$, and does not change the final $S = -2$; for $N = 6$ there are no boundary terms.

N	k	φ_k	S_k
2	0	0°	-2
2	1	180°	-2
4	0	0°	-2
4	1	90°	-2 (with $\text{sign}(0) = 0$ intermediate term)
4	2	180°	-2
4	3	270°	-2 (with $\text{sign}(0) = 0$ intermediate term)
6	0	0°	-2
6	1	60°	-2
6	2	120°	-2
6	3	180°	-2
6	4	240°	-2
6	5	300°	-2

6. Instanton action:

$$S_{\text{inst}}^{(N)} = \frac{8}{N} \sqrt{IV_0}.$$

7. Decoherence time (phenomenological rate approximation, model result):

$$\tau_d^{(N)} \approx \frac{8c_s\sqrt{I}\hbar}{N^3V_0^{3/2}L} \propto N^{-3}.$$

Proof. The peak number, spacing, frequency, and width follow directly from the minima structure of V_N . The instanton action is

$$S_{\text{inst}}^{(N)} = \int_0^{2\pi/N} d\varphi \sqrt{2IV_N(\varphi)} = \frac{8}{N} \sqrt{IV_0}.$$

The decoherence time uses the phenomenological rate approximation

$$\gamma_N \approx \frac{(N^2V_0)^2L}{8Ic_s\omega_N\hbar} = \frac{N^3V_0^{3/2}L}{8c_s\sqrt{I}\hbar},$$

whose inverse gives $\tau_d^{(N)}$. Item 7 is a model result within the phenomenological rate approximation; the ratio of γ_N to the Lindblad γ in Appendix B is determined by the specific coupling model and is not derived rigorously from the Lindblad equation. $\square$

N -parity compatibility. By A3, physical states satisfy $\psi(\varphi + \pi) = -\psi(\varphi)$. The Hamiltonian potential must preserve this subspace, so it must satisfy

$$V_N(\varphi + \pi) = V_N(\varphi).$$

For $V_N = V_0(1 - \cos N\varphi)$,

$$V_N(\varphi + \pi) = V_0(1 - (-1)^N \cos N\varphi).$$

This equals $V_N(\varphi)$ only for even N . Therefore the model adopts only even N .

Peak-number definition. Since $|\psi|^2$ has period π , the number of independent peaks in the fundamental physical domain $[0, \pi)$ is $N/2$. An experiment “measuring the peak number to determine N ” should specify whether it measures the N peaks in the $[0, 2\pi)$ lifted distribution or the $N/2$ peaks in the $[0, \pi)$ physical domain. The two conventions differ by a factor of 2, but both can be used to discriminate N .

5.3 Explicit results for $N = 2, 4, 6$

Table 2: Equal-probability CHSH values.

N	$[0, 2\pi)$ peaks	$[0, \pi)$ independent peaks	Peak spacing	Equal-probability $ S $
2	2	1	π	2
4	4	2	$\pi/2$	2
6	6	3	$\pi/3$	2

Table 3: Instanton and hierarchy suppression.

N	$S_{\text{inst}}^{(N)}/\sqrt{IV_0}$	Hierarchy suppression factor
2	4	$e^{-4\sqrt{IV_0}/\hbar}$
4	2	$e^{-2\sqrt{IV_0}/\hbar}$
6	$4/3 \approx 1.33$	$e^{-1.33\sqrt{IV_0}/\hbar}$

If the **input assumption** requires $V_0/E_0 \sim 10^{-31}$, i.e. $S_{\text{inst}}^{(N)}/\hbar \approx 71.4$, then

$$\frac{\sqrt{IV_0}}{\hbar} \approx 8.93 N.$$

Table 4: Required $\sqrt{IV_0}/\hbar$ for hierarchy self-consistency.

N	Required $\sqrt{IV_0}/\hbar$
2	17.9
4	35.7
6	53.6

This is a “self-consistency relation under an input assumption,” not a model prediction of 10^{-31} .

5.4 Non-equal-probability deviation

Let

$$p_k = \frac{1}{N} + \delta p_k, \quad \sum_k \delta p_k = 0.$$

The deviation of the CHSH quantity with respect to δp_k is

$$\delta S = \sum_k \delta p_k S_k,$$

where S_k is the CHSH value contributed by tooth site k alone. Since $\sum_k \delta p_k = 0$, the constant part does not contribute. δS depends on the correlation between δp_k and S_k ; it is not simply $4\delta p$. The explicit value must be computed term by term for each N and readout rule.

Table 5: Decoherence-time scaling.

N	Relative scaling of $\tau_d^{(N)}$
2	$\propto 1/8$
4	$\propto 1/64$
6	$\propto 1/216$

Table 6: Resolvability condition.

N	$\sqrt{TV_0}/\hbar \gg$
2	$2/(8\pi^2) \approx 0.025$
4	$4/(8\pi^2) \approx 0.051$
6	$6/(8\pi^2) \approx 0.076$

6 Low-Energy Correspondence

Theorem 5 (Free Luttinger-liquid limit $\rightarrow$ Dirac fermion). In the $1+1$ -dimensional continuum limit, the lower-cone angle field gives the Luttinger-liquid action

$$S_\varphi = \frac{K}{2} \int dt dx \left[\frac{1}{c_s} (\partial_t \varphi)^2 - c_s (\partial_x \varphi)^2 \right].$$

In the free limit $K = 1$ (or after an appropriate rescaling/compactification), bosonization corresponds to a massless Dirac fermion:

$$S_{\text{Dirac}}^{(1+1)} = \int d^2x \bar{\Psi} i \gamma^\mu \partial_\mu \Psi.$$

The spinor boundary condition $\varphi \rightarrow \varphi + \pi$ corresponds to $\Psi \rightarrow -\Psi$, as given by A3.

Proof. Standard Luttinger-liquid bosonization gives

$$\begin{aligned} \varphi &= \phi_R + \phi_L, & \Pi &= \partial_x(\phi_R - \phi_L), \\ \Psi_R &\sim e^{i\phi_R}, & \Psi_L &\sim e^{-i\phi_L}. \end{aligned}$$

For general $K \neq 1$, the fermion field is interacting and not a free Dirac field. Only when $K = 1$ or interactions are negligible does it correspond to a free massless Dirac fermion. $\square$

3 + 1-dimensional lift (embedding assumption). The transverse displacement fields u_1, u_2 give two circular-polarization modes:

$$A_R = \frac{u_1 + iu_2}{\sqrt{2}}, \quad A_L = \frac{u_1 - iu_2}{\sqrt{2}}.$$

At low energy these are lifted to a $U(1)$ gauge field with action

$$S_{\text{gauge}} = -\frac{1}{4} \int d^4x F_{\mu\nu} F^{\mu\nu}.$$

The transverse defect excitations give a two-component spinor field; the low-energy action is the Dirac action, and the tree-level $g = 2$ emerges automatically; the anomalous magnetic moment is given by QED loop corrections as required by the correspondence principle. This step is an embedding assumption, not a rigorous derivation from the $1+1$ -dimensional chain.

7 Multipartite Entanglement

Theorem 6 (GHZ Mermin quantity). For the GHZ state

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle),$$

the Mermin quantity satisfies

$$|M|_{\max} = 4.$$

Proof. The standard GHZ three-body correlation is

$$E(\theta_a, \theta_b, \theta_c) = \cos(\theta_a + \theta_b + \theta_c).$$

Using the σ_x, σ_y combination,

$$M = \langle \sigma_x^A \sigma_x^B \sigma_x^C \rangle - \langle \sigma_x^A \sigma_y^B \sigma_y^C \rangle - \langle \sigma_y^A \sigma_x^B \sigma_y^C \rangle - \langle \sigma_y^A \sigma_y^B \sigma_x^C \rangle.$$

Term-by-term evaluation for the GHZ state gives $M = 1 - (-1) - (-1) - (-1) = 4$. The half-angle projection within the model gives the same three-body correlation, hence $|M|_{\max} = 4$. This is the standard result as a correspondence within the model, not a new prediction of the model. $\square$

Theorem 7 (Standard W state as a correspondence within the model). For the standard W state

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle),$$

the two-body reduction is

$$\rho_{AB} = \frac{2}{3}|\Phi^+\rangle\langle\Phi^+| + \frac{1}{3}|00\rangle\langle 00|,$$

where

$$|\Phi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}.$$

Proof. This follows directly from the standard W-state expansion: when C is in $|0\rangle$, the AB part is

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}} = |\Phi^+\rangle,$$

with weight 2/3 (two sectors); when C is in $|1\rangle$, AB is in $|00\rangle$, with weight 1/3 (one sector). Hence

$$\rho_{AB} = \frac{2}{3}|\Phi^+\rangle\langle\Phi^+| + \frac{1}{3}|00\rangle\langle 00|.$$

$\square$

Remark. This theorem is the standard W state as a correspondence within the model; it is not derived from A1–A7. Whether the model naturally produces the W state requires additional three-body docking dynamics and is left for future work.

8 Distinguishable Predictions and N -Discrimination Criteria

8.1 Predictions independent of N

The following results hold for any even N and cannot be used to discriminate N :

1. $E(\theta_a, \theta_b) = -\cos(\theta_a - \theta_b)$;
2. $|S|_{\max} = 2\sqrt{2}$;
3. entanglement entropy $\ln 2$;
4. GHZ Mermin quantity $|M|_{\max} = 4$;
5. W-state weights $2/3, 1/3$;
6. after decoherence, the classical side $|S| \leq 2$, and for equal probabilities under the A7 readout, $|S| = 2$.

8.2 Predictions depending on N (even N)

Table 7: N -dependent observables.

Observable	$N = 2$	$N = 4$	$N = 6$
$[0, 2\pi)$ peaks	2	4	6
$[0, \pi)$ independent peaks	1	2	3
Peak spacing	π	$\pi/2$	$\pi/3$
Resolvability condition $\sqrt{TV_0}/\hbar \gg$	$2/(8\pi^2)$	$4/(8\pi^2)$	$6/(8\pi^2)$
Relative scaling of $\tau_d^{(N)}$	$1/8$	$1/64$	$1/216$
Instanton suppression factor	$e^{-4\sqrt{TV_0}/\hbar}$	$e^{-2\sqrt{TV_0}/\hbar}$	$e^{-1.33\sqrt{TV_0}/\hbar}$
Required $\sqrt{TV_0}/\hbar$ for hierarchy self-consistency	17.9	35.7	53.6

Experimental criteria:

1. **Measure the peak number after decoherence:** in the $[0, 2\pi)$ lifted distribution this gives N ; in the $[0, \pi)$ physical domain it gives $N/2$. If the experiment can access only the fundamental domain $[0, \pi)$, the measured independent peak number is $N/2$, which must be multiplied by 2 to obtain N ; either convention can discriminate even N .
2. **Measure the peak spacing:** verify $2\pi/N$.
3. **Measure the scaling of τ_d with parameters:** verify the N^{-3} dependence (phenomenological rate approximation).
4. **Measure the relation between the CHSH deviation δS and the distribution of δp_k :** verify the classical convex-combination structure, though this does not simply discriminate N .
5. **Measure the hierarchy suppression factor** (if accessible): discriminate $N = 2, 4, 6$.

8.3 Numerical window

Taking $E_0 = 1 \text{ eV}$, for typical parameters

$$\tau_d^{(N)} \sim \frac{10^{-14}}{(N/2)^3} \text{ s} \quad \text{to} \quad \frac{10^{-17}}{(N/2)^3} \text{ s}.$$

If E_0 is lower (e.g., meV), τ_d becomes correspondingly longer. The N -peak resolvability condition is $\sqrt{IV_0} \gg \hbar N / (8\pi^2)$.

8.4 Hierarchy relation

The instanton mechanism gives

$$\frac{S_{\text{inst}}^{(N)}}{\hbar} = \frac{8}{N} \frac{\sqrt{IV_0}}{\hbar}.$$

If the **input assumption** requires $V_0/E_0 \sim 10^{-31}$, i.e. $S_{\text{inst}}^{(N)}/\hbar \approx 71.4$, then

$$\frac{\sqrt{IV_0}}{\hbar} \approx 8.93 N.$$

This is a self-consistency relation under an input assumption, not an automatic resolution of the hierarchy problem by the model. Its significance is that, if such hierarchy suppression is required, the required $\sqrt{IV_0}/\hbar$ scales linearly with N , which can serve as an indirect N -discrimination criterion.

9 Conclusions and Discussion

Within the explicit postulate system, the helical-chain–biconical model yields the following results:

- **Postulates:** A1 phase field; A2 2 : 1 transmission ratio; A3 spinor boundary condition; A4 apex-to-apex docking, Fermi statistics, exchange operator, and two-sector truncation; A5 half-angle measurement projection; A6 even- N tooth-site decoherence potential; A7 classical readout rule.
- **Theorems:** singlet form under the two-sector truncation (Theorem 1); half-angle probability (Theorem 2); $E = -\cos \Delta$, $|S|_{\text{max}} = 2\sqrt{2}$ (Theorem 3); classical side $|S| \leq 2$ (Theorem 4a); even- N decoherence structure (Theorem 4b); free $K = 1$ Luttinger-liquid limit corresponding to a Dirac fermion (Theorem 5); GHZ Mermin quantity $|M|_{\text{max}} = 4$ (Theorem 6); standard W state as a correspondence within the model, $|\Phi^+\rangle$ (Theorem 7).
- **Standard-result correspondences:** the low-energy limit reduces to the Dirac equation, Maxwell equations, and standard quantum-mechanical predictions; these are requirements of the correspondence principle, not new derivations of the model.

On the 2 : 1 transmission ratio. Under the combined postulates A3–A5, the 2 : 1 transmission ratio provides a geometric basis for the spinor boundary and the half-angle projection; logical necessity comes from the combination of postulates, not from A2 alone. Standard quantum mechanics is the low-energy effective description; standard quantum mechanics “does not need” the 2 : 1 ratio because it does not commit to an underlying ontology. This is an ontological statement, not a new physical prediction.

On the tooth number N . N must be even, as determined by the compatibility of the A3 spinor boundary with V_N . N is not a logical necessity of the core entanglement structure; the

core results (Theorems 1–3, Theorem 4a, Theorems 6–7) hold for any even N . N is a replaceable parameter of the decoherence geometry and should be determined experimentally through the peak number, peak spacing, τ_d scaling, and instanton suppression factor. We give explicit calculations for $N = 2, 4, 6$ as discrimination criteria.

Current status. The present model is an equivalent ontological interpretation of standard quantum mechanics; it unifies the singlet, $-\cos \Delta$, $2\sqrt{2}$, $\ln 2$, the classical side $|S| \leq 2$, and the GHZ/W structures within the helical-chain–biconical geometry. The N -peak decoherence distribution is a candidate distinguishable signal awaiting experimental test.

Statements and Declarations

Competing Interests. The author declares that he has no competing interests.

Author Contributions. Qi-Dong Ran conceived the model, performed all derivations, and wrote the manuscript.

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A Bosonization Details

Luttinger-liquid parameters:

$$K = \sqrt{\frac{\mu}{T}}, \quad c_s = \sqrt{\frac{T}{\mu}}.$$

Bosonization mapping:

$$\begin{aligned} \varphi &= \phi_R + \phi_L, & \Pi &= \partial_x(\phi_R - \phi_L), \\ \Psi_R &= \frac{1}{\sqrt{2\pi a}} e^{i\phi_R}, & \Psi_L &= \frac{1}{\sqrt{2\pi a}} e^{-i\phi_L}. \end{aligned}$$

The free limit $K = 1$ corresponds to a massless Dirac fermion. The spinor boundary condition $\varphi \rightarrow \varphi + \pi$ corresponds to $\Psi \rightarrow -\Psi$.

B Master Equation and Classicalization

Lindblad equation:

$$\dot{\rho} = -\frac{i}{\hbar}[H_0 + V_N, \rho] - \gamma \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right),$$

where $L_k = |\varphi_k\rangle\langle\varphi_k|$. The steady state is

$$\rho_{\text{st}} = \sum_k p_k |\varphi_k\rangle\langle\varphi_k|.$$

The classical readout is given by A7:

$$A_k(\theta_a) = \text{sign} \left[\cos \left(\varphi_k - \frac{\theta_a}{2} \right) \right], \quad B_k(\theta_b) = -\text{sign} \left[\cos \left(\varphi_k - \frac{\theta_b}{2} \right) \right].$$

Equal probabilities $p_k = 1/N$ give $|S| = 2$ of Theorem 4a. The γ_N in Theorem 4b is a phenomenological rate approximation; its ratio to the Lindblad γ is determined by the specific coupling model.

C Numerical Table Remarks

A complete numerical table requires specifying the physical values of E_0 and η . Here we give only relative scalings:

Table 8: Relative scalings.

N	v_0	I_0	t	Relative τ_d	Resolvability condition
2	0.1	1	0.1	1/8	$\sqrt{IV_0} \gg 0.025\hbar$
4	0.1	1	0.1	1/64	$\sqrt{IV_0} \gg 0.051\hbar$
6	0.1	1	0.1	1/216	$\sqrt{IV_0} \gg 0.076\hbar$

D Instanton Calculation

For even N , the instanton action is

$$S_{\text{inst}}^{(N)} = \int_0^{2\pi/N} d\varphi \sqrt{2IV_N(\varphi)}.$$

Substituting $V_N = 2V_0 \sin^2(N\varphi/2)$,

$$S_{\text{inst}}^{(N)} = \sqrt{4IV_0} \cdot \frac{2}{N} \int_0^\pi \sin u \, du = \frac{8}{N} \sqrt{IV_0}.$$

The self-consistency condition is

$$\frac{S_{\text{inst}}^{(N)}}{\hbar} = \frac{8}{N} \frac{\sqrt{IV_0}}{\hbar} \approx 71.4 \implies \frac{\sqrt{IV_0}}{\hbar} \approx 8.93 N.$$

The instanton integral is taken over the period domain $[0, 2\pi/N)$ on the antiperiodic bundle; the boundary condition is guaranteed by A3.

E Gap Estimate for the Two-Sector Truncation (Order-of-Magnitude)

The two-sector truncation of A4 (retaining only $n = 0, 1$) is a model choice. Its physical motivation is that higher- n sectors correspond to larger relative angles; the apex-to-apex docking mismatch energy is estimated as

$$E_{\text{misfit}} \sim k_\phi(\pi n)^2,$$

growing as n^2 . If the soft-connection stiffness k_ϕ is sufficiently large, higher- n sectors are suppressed by the gap, and only $n = 0, 1$ remain at low energy. Equal weights and the relative minus sign follow from the exchange-operator representation and the antisymmetry condition (Theorem 1). This is an order-of-magnitude estimate; quantitative treatment is left for future work. It is currently written into the postulates as the model choice of A4.

F List of Theorems and Postulates

Table 9: List of theorems and postulates.

Item	Content	Type	N -dependence	Status
A1	helical-chain phase field	postulate	no	—
A2	biconical 2 : 1 transmission ratio	postulate	no	—
A3	spinor boundary condition	postulate	no	—
A4	apex-to-apex docking, Fermi statistics, exchange operator, two-sector truncation	postulate/model choice	no	—
A5	half-angle measurement projection	postulate	no	—
A6	even- N tooth-site potential	postulate/model choice	yes	N even
A7	classical readout rule	postulate/model choice	no	—
Theorem 1	singlet form under two-sector truncation	theorem	no	proved
Theorem 2	half-angle probability	theorem (A5 inner product)	no	proved
Theorem 3	$E = -\cos \Delta$, $ S _{\text{max}} = 2\sqrt{2}$	theorem	no	proved
Theorem 4a	classical side $ S \leq 2$	theorem (A7 read-out)	no	proved
Theorem 4b	even- N decoherence structure	theorem/phenomenological approximation	yes	proved
Theorem 5	free $K = 1$ limit $\rightarrow$ Dirac	theorem	no	proved
Theorem 6	GHZ Mermin quantity 4	standard-result correspondence	no	proved
Theorem 7	standard W state as a correspondence within the model	standard-result correspondence	no	proved
embedding assumption	3 + 1-dimensional gauge-field lift	assumption	no	—
hierarchy relation	$V_0/E_0 \sim 10^{-31}$	self-consistency under an input assumption	yes ($\propto N$)	quantified

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