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Intermittent Adaptive Sliding Mode Control for a Chaotic Lorenz System with a Hamiltonian Energy-Based Hidden Attractor

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Abstract: This paper presents a novel intermittent adaptive sliding mode control (IASMC) strategy for regulating the Hamiltonian energy of a generalized chaotic Lorenz system exhibiting hidden attractors. Conventional sliding mode control (SMC) suffers from severe chattering and high steady-state errors, while standard adaptive fuzzy approaches often demand continuous parameter updates, leading to excessive computational costs. To overcome these limitations, the proposed method integrates a model-reference adaptive law with an intermittent updating mechanism that activates the control action only when the Hamiltonian energy deviates beyond a predefined threshold. This hybrid design not only preserves the robustness of SMC against uncertainties but also significantly reduces the computational burden and chattering amplitude. The sliding surface coefficients are optimally tuned online using a nonlinear optimization routine in MATLAB. The effectiveness of the proposed IASMC is validated through numerical simulations on the generalized Lorenz system. A quantitative comparison with conventional SMC and a recent adaptive fuzzy controller reveals that the proposed method achieves a mean squared error (MSE) of only **0.0049** in sinusoidal tracking, which is **70-fold lower** than conventional SMC (MSE = 0.3468) and **twice as accurate** as the adaptive fuzzy approach (MSE = 0.0107). These results demonstrate the superior tracking precision, reduced chattering, and computational efficiency of the proposed intermittent

This paper proposes an intermittent adaptive sliding mode controller for the Hamiltonian energy control of a generalized chaotic Lorenz system with hidden attractors. To drive the chaotic system toward the desired Hamiltonian energy level, a sliding mode controller is designed. This controller is then extended in a model reference adaptive manner, combining the advantages of both control strategies. The resulting approach exhibits robustness against uncertainties while optimizing sliding mode control parameters through online adjustment of control actions. To achieve this goal, the existing nonlinear optimization method in MATLAB software is utilized for tuning the sliding mode controller parameters. To reduce computational cost and simulation time, an intermittent control scheme is employed, where parameters are adjusted only when the intermittent threshold condition is satisfied. Considering the desired energy level, the proposed intermittent adaptive sliding mode control scheme is applied to the generalized chaotic Lorenz system, demonstrating its effective performance.

Keywords: Chaotic system, Hamiltonian energy, Hidden attractors, Adaptive sliding mode control, Intermittent control

1. Introduction

Chaotic systems are systems that are highly sensitive to initial conditions. They appear random but possess deterministic behavior internally. In various systems such as chemical, physical, and biological systems, chaos and hyperchaos with complex dynamics can be observed [1,2].

Another characteristic of chaotic systems is the emergence of a positive Lyapunov exponent, which can be examined through nonlinear analysis in their linearized

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systems [3]. In chaotic systems, the existence of unstable equilibrium points is crucial for stimulating self-excited attractors and maintaining sustained oscillatory behaviors [1]. However, in some systems, attractors are hidden, and special strategies are required to discover them. Attractors of systems that have no equilibrium points or only stable equilibria are considered hidden [4]. Research on hidden attractors can be found in the works of Leonov et al [5,6].

Sufficient energy is vital to support a dynamical system. If this energy is not supplied, the chaotic oscillator will gradually decay to a quiescent state [1]. Circuits and oscillators can be controlled by regulating their energy performance. Therefore, defining a dimensionless Hamiltonian energy for dynamical systems is essential. One application of Hamiltonian energy analysis is chaos control; chaotic behaviors can be suppressed and controlled using negative feedback based on Hamiltonian energy. For example, in [7] , a symmetric megastable oscillator with a growing attractor and its Hamiltonian energy level was approximated. Dynamics analysis and Hamiltonian energy control of a class of Filippov neural models were studied in [8] Hamiltonian energy, complex dynamical analysis, and information patterns of a new neural network were investigated in [9]. Chaotic dynamics analysis of a double inverted pendulum with large oscillation angles based on the Hamiltonian function was carried out in [10]. In [11] a class of Hamiltonian conservative chaotic systems with transient quasi-periodicity and multistability was modeled and implemented in hardware. An innovative method for Hamiltonian energy generation of a new hyperchaotic nonlinear model and its control was presented in [12]. In [13], energy conservation and singular orbits of two new chaotic systems were examined, and FPGA

implementation was performed for both systems. Modeling, synchronization, and FPGA implementation of Hamiltonian conservative hyperchaos were accomplished in [14]. In [15], Hamiltonian chaotic systems with hidden attractors were stabilized using adaptive terminal sliding mode control. A new megastable oscillator and its Hamiltonian energy: infinitely many hidden attractors and self-excited ones were studied in [16]. Dynamics analysis and Hamiltonian energy control of a generalized Lorenz system with a hidden attractor were performed in [17].

It has been proven in [17] that one chaotic dynamic can be transformed into another chaotic dynamic by online tuning of the energy feedback gain. One application of chaos-to-chaos control is in the control of biological systems, such as the treatment of epileptic seizures [17]. On the other hand, energy-based feedback control strategies can be employed for chaos-to-chaos control. In recent years, fuzzy systems have been used as model-free controllers for chaotic systems [18-20]. However, fuzzy controllers contain parameters that are difficult to determine. One way is to use evolutionary optimization algorithms to tune these parameters. In [21], a hierarchical fuzzy optimal controller for Hamiltonian energy control of a generalized chaotic Lorenz system with a hidden attractor using a genetic evolutionary algorithm was presented by [21]. Jia et al. in 2025 introduced a new class of locally Hamiltonian conservative chaotic systems [22]. These chaotic systems are generated by using piecewise functions instead of smooth Hamiltonian functions. They showed that the Hamiltonian energy in these systems is locally conserved, and by adjusting the piecewise function, multi-scroll dynamics can be controlled. Finally, they implemented these systems using FPGA technology, which, besides physically demonstrating complex dynamics, serve as new pseudo-random signal generators for chaotic applications.

Yu et al. in 2025 introduced a new five-dimensional Hamiltonian conservative chaotic system that, by using piecewise functions, produces multi-directional multi-scroll attractors in one, two, or three axes [23]. They showed that by adjusting the parameters of the piecewise function and initial conditions, the number, size, and energy level of the scrolls can be precisely controlled. Moreover, the system passed NIST tests in terms of pseudo-random signal quality, and its hardware implementation using FPGA confirmed its physical feasibility.

Vivekanandhan et al. in 2026 introduced a new megastable hyperchaotic oscillator without an external driving force, using a combination of signum and trigonometric functions [24]. By analyzing Hamiltonian energy, phase portraits, basins of attraction, and Lyapunov spectra, they investigated the dynamical characteristics and sensitivity of the system to initial conditions, demonstrating its potential for applications such as random number generation and secure communications. Langlett et al in 2025 studied quantum chaos at finite non-zero temperature in local spin Hamiltonians [25]. They showed that the statistical properties of finite-temperature eigenstates can be accurately described by pure random states subject to a local charge constraint. In this reference, by normalizing the energy density using a Hamiltonian-dependent parameter, an excellent correlation was shown between the entanglement entropy of eigenstates and constrained random states. It was concluded that the concept of "maximal chaos" can be defined even in pure quantum regimes and at finite temperature. Iqbal and Wang in 2025 introduced a new three-dimensional fractional-order chaotic system with a simple algebraic structure [26]. This chaotic system generates attractors similar to classical models such as Lorenz. In this reference, the dynamical properties of the system were investigated through theoretical analyses and dynamical simulations (including the Lyapunov exponent spectrum and fractal dimensions) as well as the extraction of the Hamiltonian energy function. Finally, using an active control method, synchronization of the system was implemented and its application in secure transmission of real audio signals for secure communications was demonstrated.

Liang et al. in 2026 conducted an in-depth analysis of the (3+1)-dimensional BKPE equation in fluid dynamics [27]. In this reference, by using the semi-inverse method (SIM) and traveling wave transformation, the variational principle and Hamiltonian of the system were derived. Then, through symmetry analysis and phase diagrams, the conditions for the existence of various wave solutions and chaotic behaviors were examined, and by employing approaches based on the variational principle (VA) and Hamiltonian (HBA), various types of wave solutions such as solitary and periodic waves were obtained. Pan et al. in 2025 proposed a new method for constructing Hamiltonian conservative chaotic systems using the rotation product of the displacement group [28]. They implemented a 5D Hamiltonian conservative system. In this reference, the dynamical characteristics of the system, including multi-mode coexistence and the ability to generate hyperchaotic states with high Lyapunov exponents, were analyzed, and

through NIST tests and hardware implementation on a DSP platform, the system's capability to generate high-complexity pseudo-random sequences and its physical feasibility for secure cryptographic applications were confirmed.

Sliding mode control is a robust nonlinear control method that is resistant to uncertainties and external disturbances and can provide a fast dynamic response. It is also simple to implement. Adaptive control of the controller parameters can manage the behavior of the system. The optimization operation in adaptive control leads to a response with minimum error and high tracking accuracy. Given these advantages, a combined adaptive sliding mode controller is designed in this paper. Since this combination alone imposes heavy computational and time costs, an intermittent controller is designed in this paper. To reduce simulation time and computational cost, the optimization operation in the adaptive sliding mode controller is executed only when the error exceeds a threshold value; otherwise, the sliding mode parameters are kept at their previous step values. Here, the proposed intermittent adaptive sliding mode controller is employed to determine an appropriate control signal for steering the hidden attractor dynamics toward the desired dynamics based on processing the Hamiltonian energy as a quantity from the time-varying dynamics of the hidden attractor.

This paper is organized as follows: Section 2 presents the generalized chaotic Lorenz system, computes the Hamiltonian energy of the system, and discusses Hamiltonian energy control. Section 3 introduces and describes the design procedure of the intermittent adaptive sliding mode controller. Section 4 provides the simulations and results. Section 5 presents the conclusion.

- **2. Generalized Chaotic Lorenz System (GCLS)**

The dynamic equations of the generalized chaotic Lorenz system (GCLS) [29], which is derived from the Robinovich system [30] , can be written as follows:

$$\begin{cases} \dot{x} = -a(x-y) - byz \\ \dot{y} = cx - y - xz \\ \dot{z} = -dz + xy \end{cases} \quad (1)$$

where x,y,z are the state variables, and a,b,c,d are the constant parameters of the system. The aforementioned chaotic dynamical system possesses a hidden attractor in its initial state [30].

Chaotic attractors may be self-excited or hidden [30]. The basin of attraction of a self-excited attractor is associated with an unstable equilibrium, whereas the basin of attraction of a hidden attractor does not intersect with small neighborhoods of equilibrium points. There exist known hidden attractors associated with unstable equilibria [29]. The Lorenz attractor [30] examined in this study is a classical attractor that arises from an unstable equilibrium.

Thus, the behavior of the GCLS varies depending on its parameters and the initial values of the state variables. For the system to exhibit chaotic features, system (1) must be dissipative [22], which requires that the divergence be negative, as shown:

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a - 1 - d \tag{2}$$

2-1. Hamiltonian Energy Function

To find the Hamiltonian energy function (1), the autonomous dynamical system (1) can be considered as follows:

$$\dot{X} = f(X) \tag{3}$$

Where $X \in \mathbb{R}^n$ is the state vector, n is the dimension of the state space, and f(x) is a smooth function. Therefore, based on the fundamental theorem of vector calculus, the Helmholtz representation (Yu et al. 2025), f(x) is decomposed into a conservative field $f_c(x)$ and a dissipative field $f_d(x)$:

$$f(X) = f_c(X) + f_d(X) \tag{4}$$

In the context of energy problems, based on Liouville's theorem, the Hamiltonian energy function can be computed for system (1) in the decomposed form (4) by considering the following equations [1]:

$$\begin{cases} \nabla H^T f_c(X) = 0 \\ \dot{H} = \nabla H^T f_d(X) \end{cases} \tag{5}$$

Where for system (1), $f_c(x)$ and $f_d(x)$ are obtained as follows:

$$f_c(X) = \begin{pmatrix} ay-byz \\ cx-xz \\ xy \end{pmatrix}, f_d(X) = \begin{pmatrix} -ax \\ -y \\ -dz \end{pmatrix} \quad (6)$$

Based on equations (5) and (6):

$$(ay - byz)\frac{\partial H}{\partial x} + (cx - xz)\frac{\partial H}{\partial y} + xy\frac{\partial H}{\partial z} = 0 \quad (7)$$

$$(-ax)\frac{\partial H}{\partial x} + (-y)\frac{\partial H}{\partial y} + (-dz)\frac{\partial H}{\partial z} = \dot{H} \quad (8)$$

Since $\dot{H} = \frac{\partial H}{\partial x}\dot{x} + \frac{\partial H}{\partial y}\dot{y} + \frac{\partial H}{\partial z}\dot{z}$, solving equations (7) and (8) yields the Hamiltonian energy function and its first-order derivative as follows:

$$H = \frac{1}{2}\left[x^2 - \frac{a}{c}y^2 + (b - \frac{a}{c})z^2\right] \quad (9)$$

$$\dot{H} = -ax^2 + \frac{a}{c}y^2 - d\left(b - \frac{a}{c}\right)z^2 = \nabla H^T f_d(X) \quad (10)$$

2-2- Hamiltonian energy control

Energy is the most important issue for sustained oscillations in any dynamical system. A new way to control chaotic periodic oscillations in dynamical systems through energy feedback has been established. These energy feedback control methods can be applicable for controlling chaotic and hyperchaotic systems. An effective method for controlling chaotic and oscillatory behaviors can be energy control. The Hamiltonian energy function can establish an effective relationship between the state space and the system's energy. The main goal of this paper is to develop a controller for GCLS such that its Hamiltonian energy tracks a reference value, called the desired Hamiltonian energy level $H_d(t)$.

Assumption: Considering practical considerations, the desired Hamiltonian energy often does not change. As a result of this assumption, the reference trajectory is considered as a zero trajectory in one case, and in another case, it is generated as a sinusoidal signal representing the desired energy level. In order to force tracking of this trajectory, a control signal u(t) is applied to the system as follows:

$$\begin{cases} \dot{x} = -a(x-y) - byz + u \\ \dot{y} = cx - y - xz \\ \dot{z} = -dz + xy \end{cases} \tag{11}$$

In (11), u must be designed such that the Hamiltonian energy computed from the system reaches the desired energy. Further explanations regarding controller design are provided.

3- Design of the Intermittent Adaptive Sliding Mode Controller

Here, a generalized sliding mode controller with intermittent adaptive control is employed to determine the control signal *u*. Initially, a basic sliding mode controller is designed, which is then extended in a predictive manner. Since the execution time and computational cost are high, an intermittent controller is designed based on a threshold condition. This controller operates according to the threshold condition: if the threshold condition, which here is the mean squared error, exceeds a certain value, the parameter tuning and adaptive control computations are performed; otherwise, the previous parameters are retained and no new computations are carried out.

3-1- Design of the Sliding Mode Controller Based on Desired Hamiltonian Energy

The Hamiltonian energy of system (11) was given in equation (9). However, to calculate its derivative in the presence of *u*, it is computed as equation (12).

$$\dot{H} = -ax^2 + xu + \frac{a}{c}y^2 - bz^2 + \frac{a}{c}z^2 \tag{12}$$

The Hamiltonian energy tracking error is defined as:

$$E_h = H - H_d \tag{13}$$

Where H_d is the desired Hamiltonian energy. The derivative of the Hamiltonian energy tracking error is obtained as:

$$E_h = \dot{H} - \dot{H}_d \tag{14}$$

Here, an integral sliding mode is used. The sliding surface *s* is designed as:

$$s = \lambda E_h + \int E_h \tag{15}$$

Where $\lambda>0$ is a constant parameter. The derivative of the sliding surface (15) is defined as:

$$\dot{s} = \lambda \dot{E}_h + E_h \tag{16}$$

To investigate the asymptotic stability of the proposed sliding mode controller using Lyapunov stability theory, a positive definite Lyapunov function is defined as:

$$V = \frac{1}{2} s^2 \tag{17}$$

The derivative of the Lyapunov function (17) is calculated as:

$$\dot{V} = s\dot{s} = (\lambda E_h + \int E_h)(-\lambda a x^2 + \lambda \frac{a}{c} y^2 - \lambda b z^2 + \lambda \frac{a}{c} z^2 + E_h + xu - \lambda \dot{H}_d) \tag{18}$$

To make $\dot{V}$ negative definite, the control effort u is defined as:

$$\begin{aligned} u_1 &= \frac{1}{(x+\varepsilon)} (\lambda a x^2 - \frac{\lambda a}{c} y^2 + \lambda b z^2 - \frac{\lambda a}{c} z^2 + \dot{H}_d) \\ u_2 &= \frac{1}{\lambda (x+\varepsilon)} (-\lambda E_h - \int E_h) \\ u_s &= -(K \tanh(s)) \end{aligned} \tag{19}$$

where u_s is the switching control term. The parameter K is the sliding mode controller gain. Substituting (19) into (18) yields equation (20):

$$\dot{V} \approx -\lambda E_h^2 - K s^2 \le 0 \tag{20}$$

According to equation (20), $\dot{V} \le 0$; therefore, the system is asymptotically stable.

3-2- Design of the Combined Adaptive Sliding Mode Controller

In this stage, the designed sliding mode controller is considered. We extend this controller to a adaptive control framework. In the proposed scheme, the sliding mode control parameters, including λ and K, are updated with size Np samples. Then, with the newly obtained parameters, future control efforts are computed. Based on the obtained control effort, the system states are also calculated with these new controller parameters. This process continues until all time

samples are exhausted. These parameter updates, help the controller achieve better tracking error performance; moreover, this scheme enables the controller to perform more successfully in the face of uncertainties and disturbances.

Since the parameter tuning computations are performed, we expect faster convergence to the target and improved accuracy from the controller. However, the computational cost, and especially the execution time of this controller, is high. In the next section, to address this issue, we employ an intermittent controller.

As an example, suppose we have a total of 10,000 samples. Assume number of controller parameters samples is set to 50. In this case, every 50 samples, the parameter tuning computations are carried out, and new states, including control effort and system state equations, are calculated. This operation continues until all samples are exhausted. In this example, there is a loop where, for every 50 samples, new computations are performed and the system states are stored. Given that the total number of samples is 10,000 and the number of estimated samples of the controller parameters in each simulation period is 50, this loop executes new computations 200 times during the simulation. Since the simulation error is acceptable most of the time, new computations are not always necessary. Therefore, in this paper, we will employ an intermittent control scheme. We use the fmincon toolbox for the optimization and parameter tuning part. The cost function of the proposed adaptive sliding mode controller is as follows:

$$J = \frac{1}{N}\sum_{i=1}^{N}(E_h^2) + \frac{0.01}{N}\sum_{i=1}^{N}(U^2) \tag{21}$$

Where N is the total number of samples in the simulation, E_h is the Hamiltonian energy error, and u is the control effort. The small coefficient 0.01 is due to the lower importance of minimizing control effort compared to the Hamiltonian energy error.

3-3- Design of the Combined Intermittent Adaptive Sliding Mode Controller

In this section, we design an intermittent controller combined with an adaptive sliding mode controller. In an intermittent controller, unlike traditional time-based control methods, control commands are updated only at essential and specific moments, known as triggering instants. The intermittent control approach leads to a reduction in computational burden without a significant degradation in the overall performance of the baseline controller. In this paper, we employ this approach to reduce simulation execution time and computational cost. The intermittent controller operates based on a threshold condition. In this paper, the threshold condition is considered based on the mean squared tracking error. If the mean squared tracking error exceeds 0.05, new computations for tuning the sliding mode parameters are performed; otherwise, the previous parameters are retained. This scheme results in reduced simulation execution time and computational costs. Figure 1 shows the block diagram of the proposed intermittent adaptive sliding mode control scheme for driving the Hamiltonian energy to the desired value.

Fig. 1- Block diagram of the proposed intermittent adaptive sliding mode control scheme

4- Simulations and Results

Here, the proposed combined predictive event-triggered sliding mode controller is applied to the generalized chaotic Lorenz system (GCLS) with parameters $a = 2.98, b = -0.438, c = 6.8, d = 1$. With these parameters, the GCLS exhibits chaotic behavior. The initial conditions of the system are $x(0) = 3.8, y(0) = 2.1, z(0) = -3.1$. Consequently, the initial Hamiltonian energy is $H(0) = \frac{1}{2}\left[x(0)^2 - \frac{a}{c} y(0)^2 + (b - \frac{a}{c}) z(0)^2 \right] = 2.0434$ and its first-order derivative is $\dot{H}(0) = \nabla H(0)^T f_d(X(0)) = -32.678$. In the simulations, the desired Hamiltonian energy is considered in two cases. In the first case, it is assumed that H_d=0, and in the second case, a sinusoidal function $5(1+\sin(t/10))$ is considered. The initial values of the sliding mode controller parameters are set to $\lambda = 20K = 1.45$. The search range for these parameters is considered as follows:

$$0.5 < \lambda < 150, \quad 0.01 < K < 1 \tag{22}$$

a) Simulation results in the zero-target tracking mode

In this case, the desired Hamiltonian energy is assumed to be zero. To evaluate the proposed method, its results are compared with similar controllers. Figure 2 shows the Hamiltonian energy curves obtained from the sliding mode controller, the proposed intermittent adaptive sliding mode controller, and the adaptive fuzzy controller [31].

Fig. 2- Tracking error curves obtained from the sliding mode controller, the intermittent adaptive sliding mode controller, and the adaptive fuzzy controller [31] in the zero-tracking mode.

According to Figure 2, it is observed that the tracking error obtained from the intermittent adaptive sliding mode controller is better in terms of error magnitude compared to the other two controllers. From this figure, it can be seen that from the start up to ten seconds, the performance of the adaptive fuzzy controller has been good, but after 10 seconds, at certain time intervals, the error of the adaptive fuzzy controller has increased. However, the proposed controller achieves a lower error over time. Figure 3 shows the Hamiltonian energy curves of the three mentioned controllers.

Fig. 3- Hamiltonian energy curves obtained from the sliding mode controller, the intermittent adaptive sliding mode controller, and the adaptive fuzzy controller [31] in the zero-tracking mode.

From Figure 3, it is observed that overall, the Hamiltonian energy obtained from the proposed controller exhibits less oscillation. Moreover, the proposed method quickly tracks the target trajectory. Figure 4 shows the control effort curves obtained from the mentioned controllers.

Fig. 4- Control effort curves obtained from the sliding mode controller, the intermittent adaptive sliding mode controller, and the adaptive fuzzy controller [31] in the zero-tracking mode.

From Figure 4, it is observed that the oscillation obtained from the proposed method is lower compared to the fuzzy controller method. It is also seen that the conventional SMC controller consumes a larger control effort at the initial simulation times. Figure 5 shows the parameter K curve and Figure 6 shows the parameter λ curve obtained from the intermittent adaptive sliding mode controller.

Fig. 5- Parameter K curve obtained from the intermittent adaptive sliding mode controller in the zero-tracking mode.

Fig. 6- Parameter λ curve obtained from the intermittent adaptive sliding mode controller in the zero-tracking mode.

Table 1- Comparison of the three methods: sliding mode controller, intermittent adaptive sliding mode controller, and adaptive fuzzy controller [31] in the zero-tracking mode in terms of mean squared error.

Method	Mean Squared Error
Adaptive fuzzy [31]	0.0208
Sliding mode controller	0.1235
Intermittent adaptive sliding mode controller	0.0024

According to Table 1, it is observed that the mean squared error obtained from the proposed intermittent adaptive sliding mode controller is lower compared to the other two methods.

b) Sinusoidal curve tracking mode

In this case, a sinusoidal curve is considered as the tracking target. The remaining designs are similar to the previous case. Figure 7 shows the curves obtained from the sliding mode controller, the intermittent adaptive sliding mode controller, and the adaptive fuzzy controller [31] in the sinusoidal Hamiltonian energy tracking mode.

Fig. 7- Hamiltonian energy curves obtained from the sliding mode controller, the intermittent adaptive sliding mode controller, and the adaptive fuzzy controller [31] in the sinusoidal curve tracking mode.

From Figure 7, it is observed that the error obtained from the proposed controller is lower than the other two methods. It is seen that the error of the proposed method converges to zero sooner than the other two methods. From the middle plot, it is observed that the proposed method tracks

the sinusoidal curve with less oscillation and faster speed compared to the other two methods. From the bottom plot, it is observed that the oscillation obtained from the fuzzy controller is greater than the other two methods. It is observed that the proposed method has less oscillation compared to the other two methods. Figure 8 shows the K curve and Figure 9 shows the λ curve versus time.

Fig. 8- Parameter K curve obtained from the intermittent adaptive sliding mode controller in the sinusoidal tracking mode.

Fig. 9- Parameter λ curve obtained from the intermittent adaptive sliding mode controller in the sinusoidal tracking mode.

Table 2- Comparison of the three methods: sliding mode controller, intermittent adaptive sliding mode controller, and adaptive fuzzy controller [31] in the sinusoidal tracking mode in terms of mean squared error.

Method	Mean Squared Error
Adaptive fuzzy [31]	0.0107
Sliding mode controller	0.3468
intermittent adaptive sliding mode controller	0.0049

From Table 2, it is observed that the proposed intermittent adaptive sliding mode controller achieves a lower mean squared error compared to the other two methods. It should also be noted that using the event-triggered control approach reduces the simulation execution time compared to the case without it, because when the error is below the threshold value, the previous computations are retained and no new computations are performed.

Conclusion

In this paper, the objective is to drive the dynamics of a system with existing (undesirable) chaos toward the dynamics of a system with desirable chaos. Here, the concept of dynamics refers to the attractor pattern. Given that in chaotic systems a small change in initial conditions causes a large change in system behavior, many different desired trajectories can emerge. Therefore, a single desired trajectory cannot be defined. In this research, the desired trajectory was proposed in the form of a desired dynamics. One of the well-known approaches in this field is the use of the Hamiltonian energy function. In this paper, two desired Hamiltonian energy functions were considered: one zero Hamiltonian energy and the other sinusoidal. An intermittent adaptive sliding mode controller was proposed to drive the Hamiltonian energy toward the desired values. Among the advantages of this controller are its robustness against uncertainties, the ability to predict future behavior, and the optimization of the baseline sliding mode controller parameters. Furthermore, the intermittent control method reduces computational cost and simulation

execution time. Simulation results showed that in both desired Hamiltonian energy trajectories, the proposed method performed better in terms of mean squared error.

Declaration of conflicting interests

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Author Contributions

G.A. conceived and designed the study, performed the simulations, conducted the formal analysis, investigated the data, and wrote the original draft of the manuscript. M.Y. and H.R.K. supervised the project, provided critical guidance, and contributed to reviewing and editing the manuscript to improve its scientific quality and clarity. All authors have read and approved the final manuscript.

Data Availability Statement

All data generated and analyzed during this study are included in this published article and its supplementary information files.

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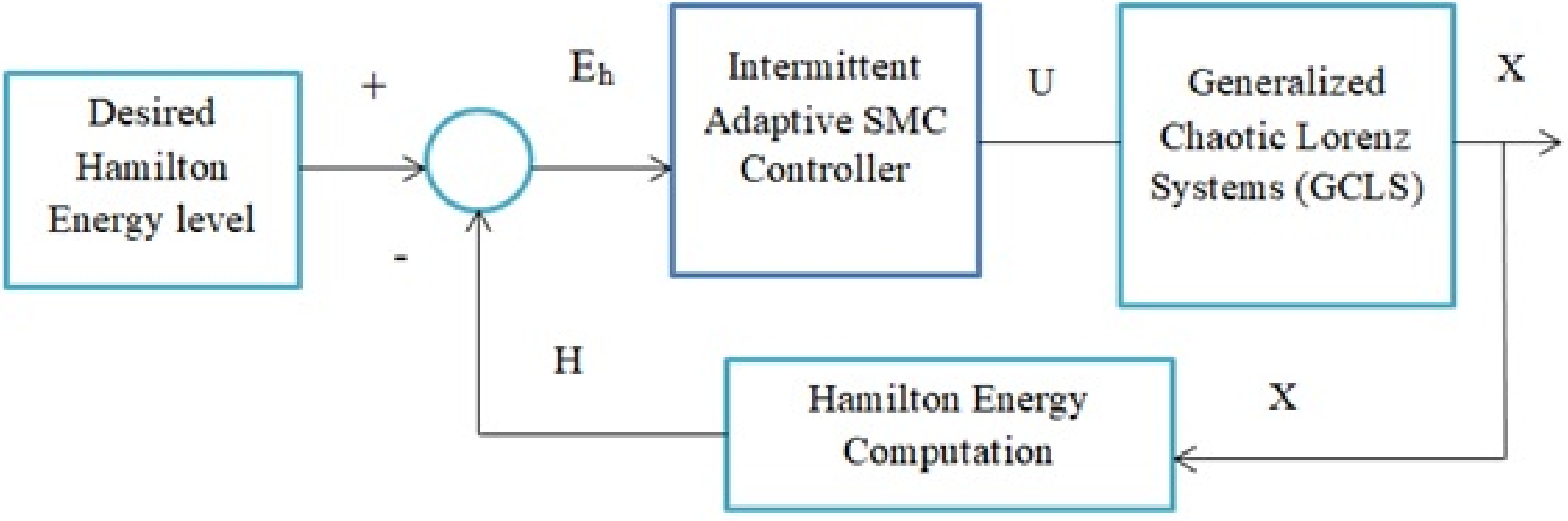
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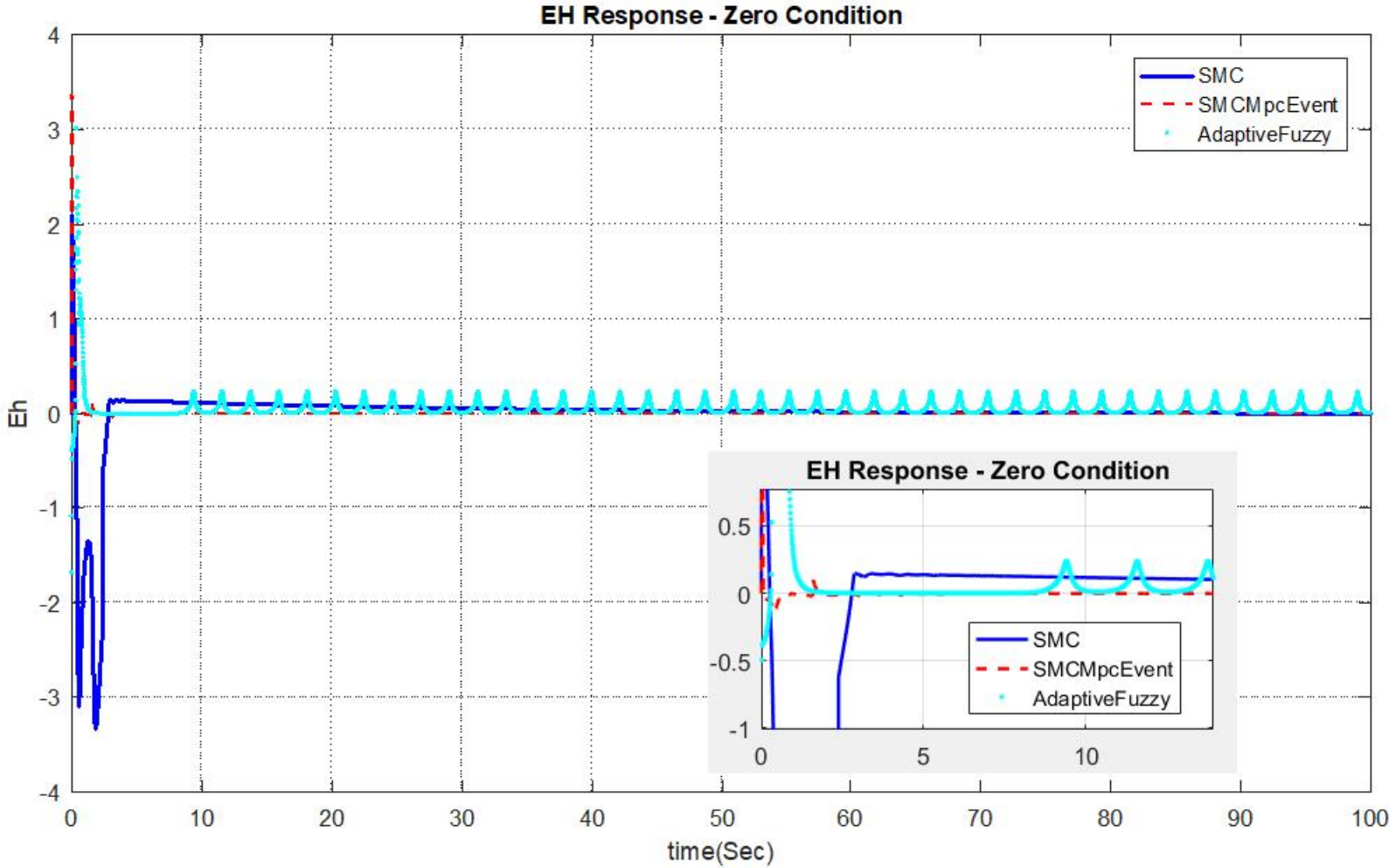
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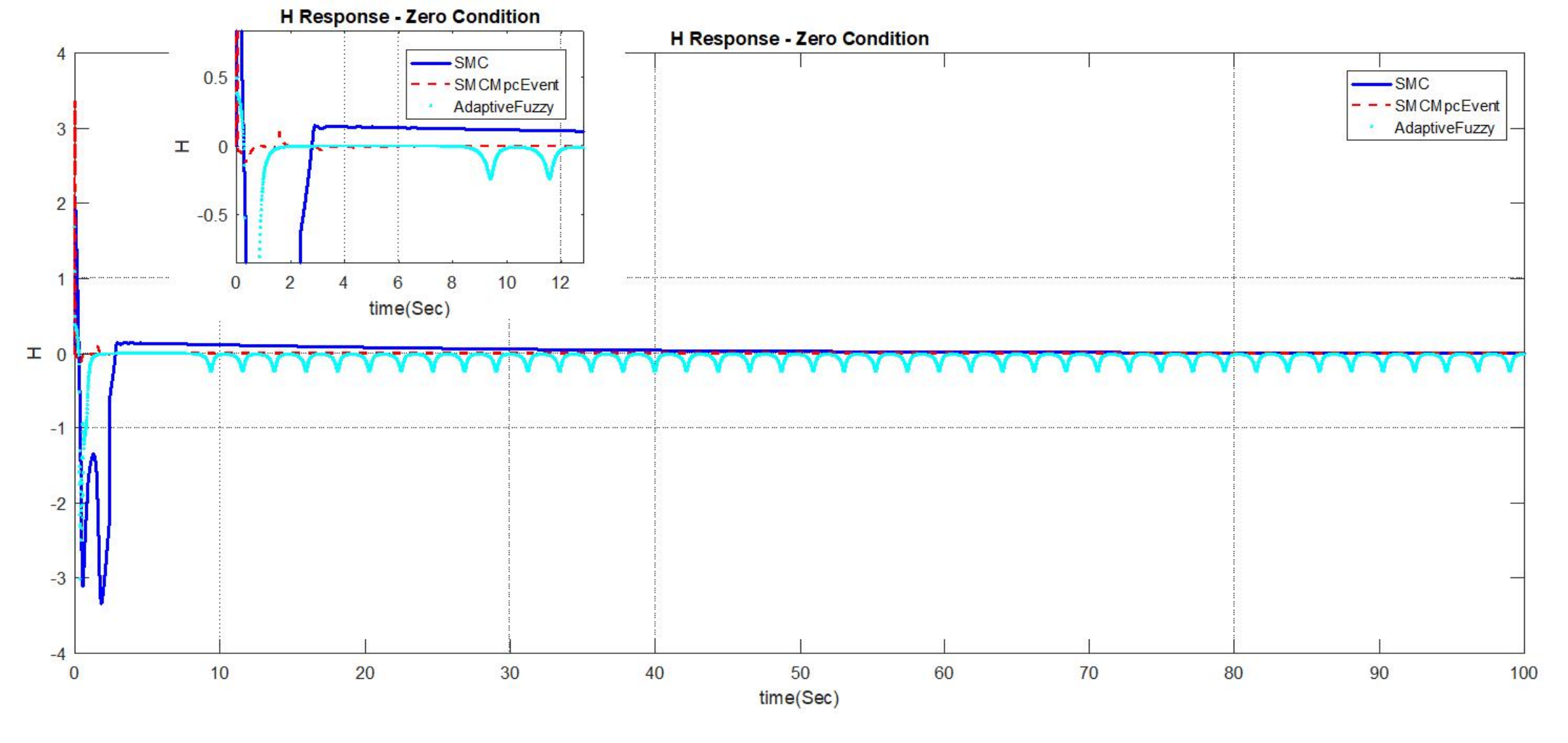
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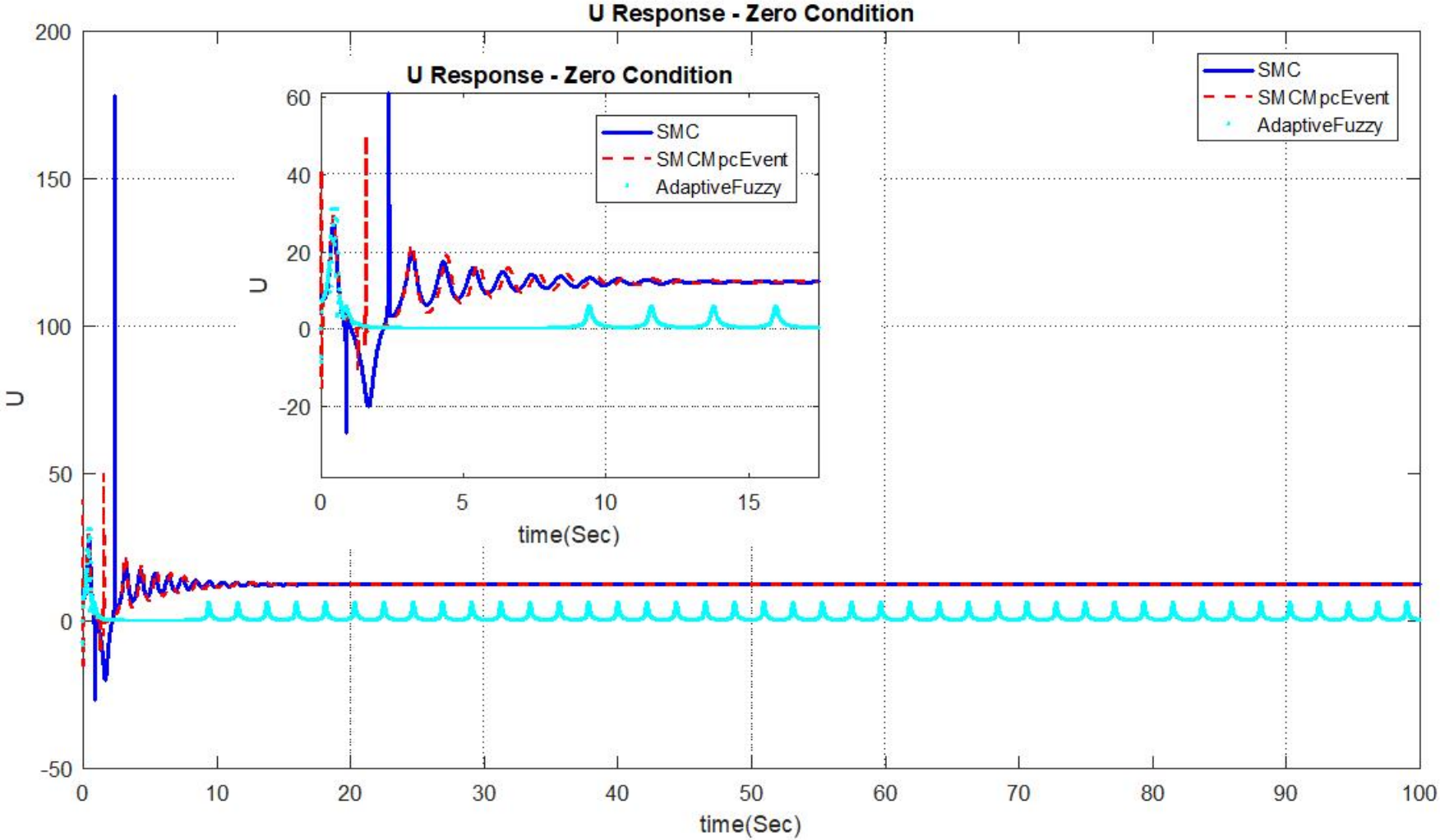
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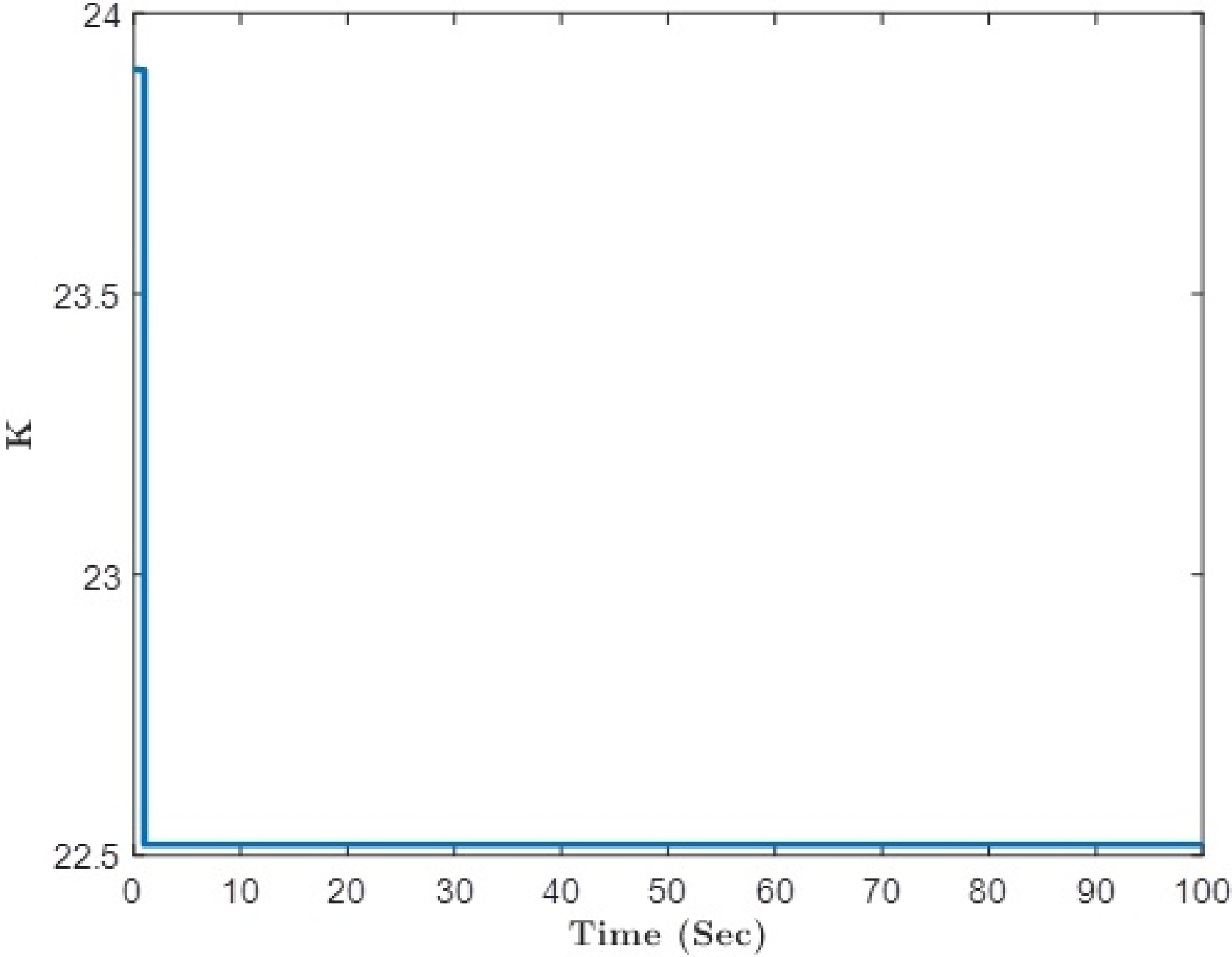
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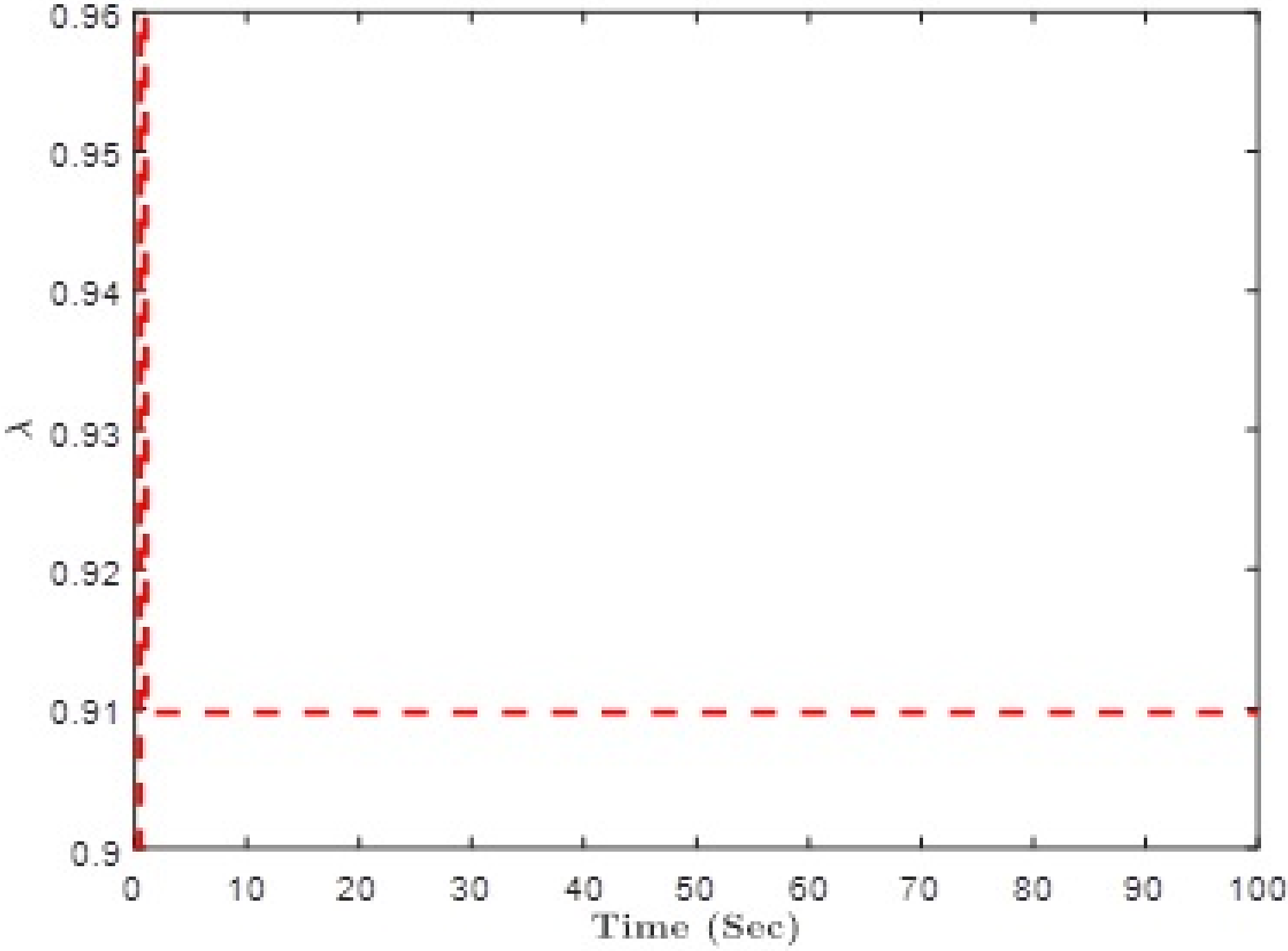

Desired
Hamilton
Energy level
+
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Intermittent
Adaptive SMC
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U
Generalized
Chaotic Lorenz
Systems (GCLS)
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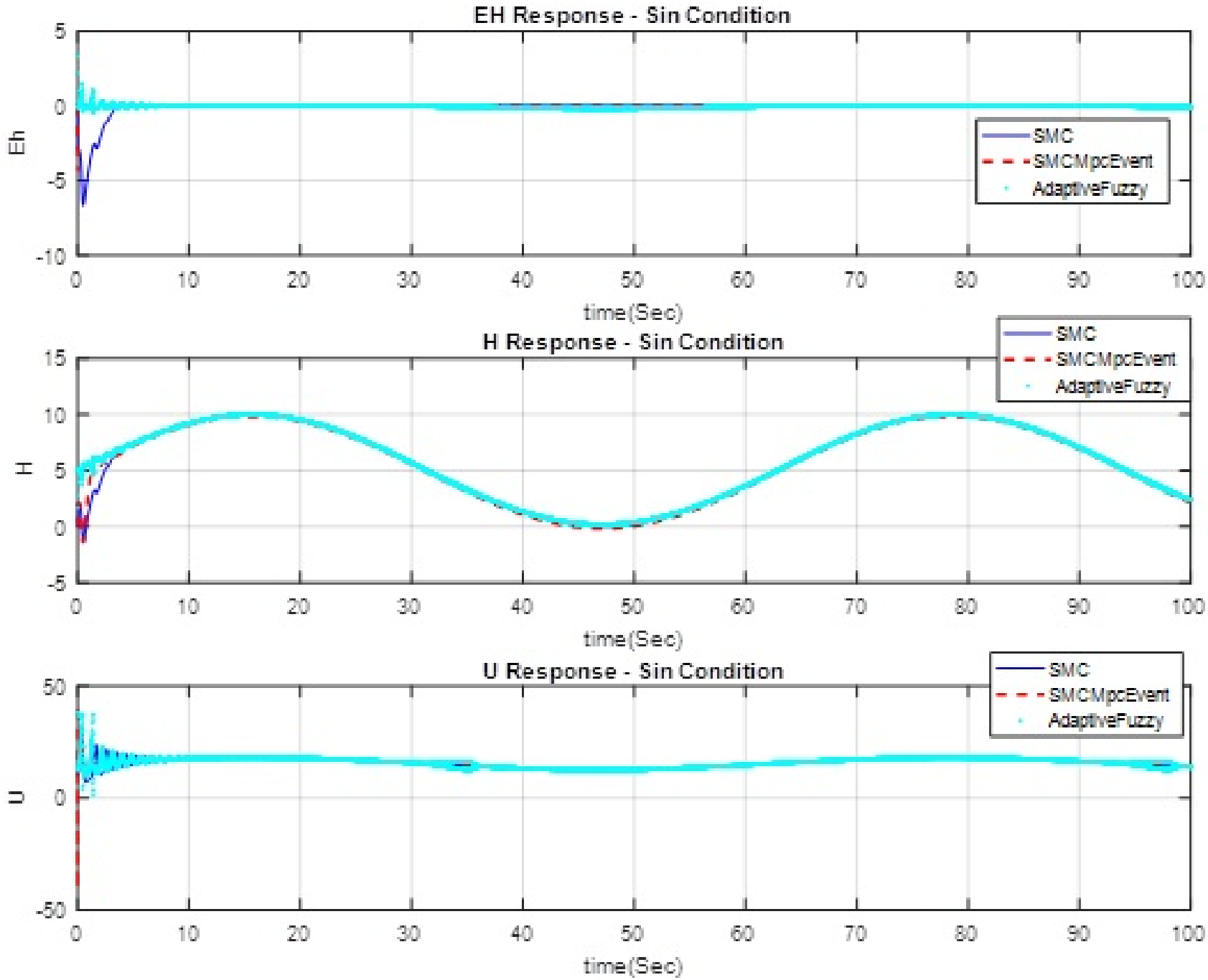

EH Response - Zero Condition
SMC
SMCMpcEvent
AdaptiveFuzzy
Eh
time(Sec)
4
3
2
1
0
-1
-2
-3
-4
0
10
20
30
40
50
60
70
80
90
100
EH Response - Zero Condition
SMC
SMCMpcEvent
AdaptiveFuzzy
0.5
0
-0.5
-1
0
5
10

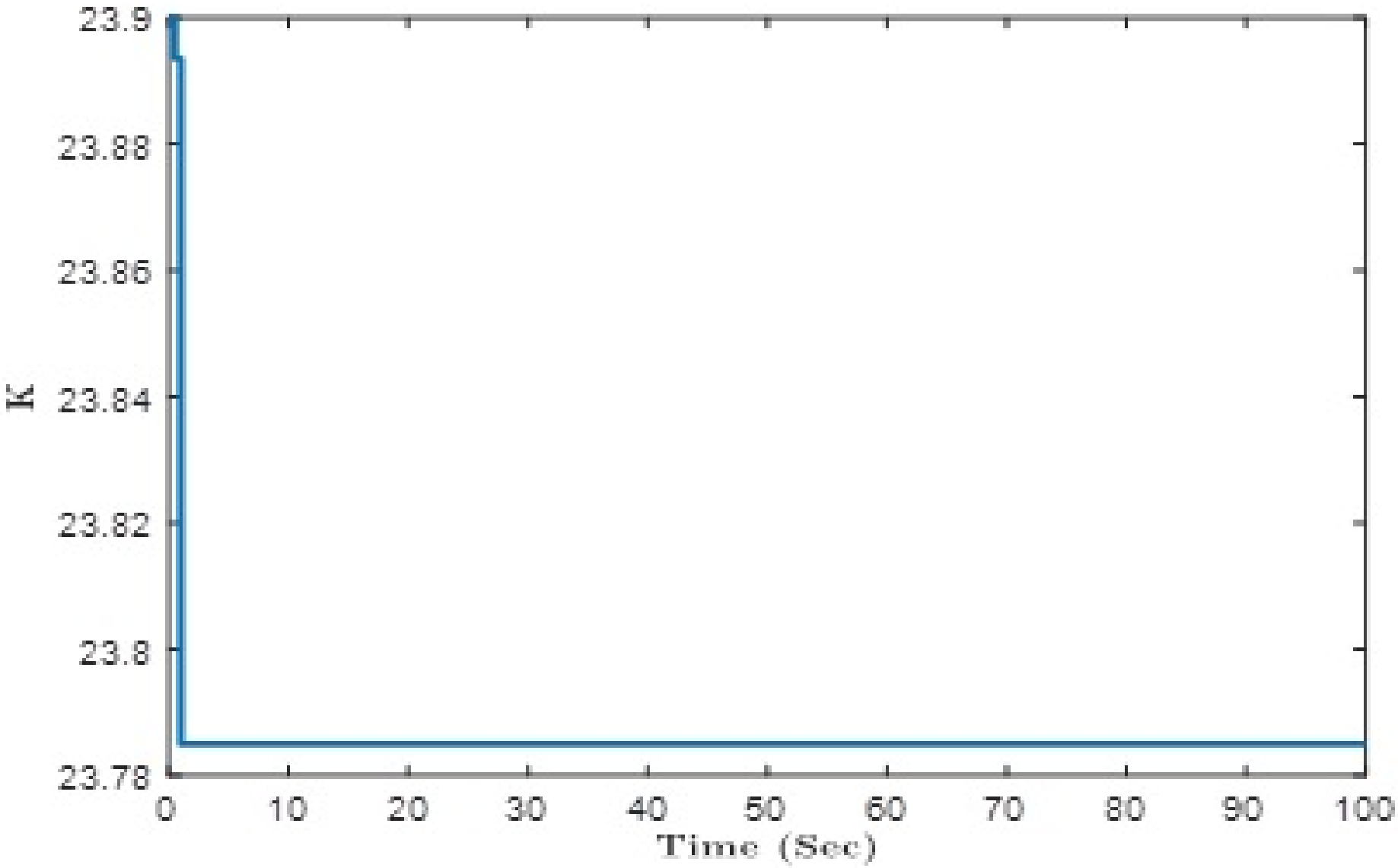

H Response - Zero Condition
SMC
SMCMpcEvent
AdaptiveFuzzy
H
time(Sec)
H Response - Zero Condition
SMC
SMCMpcEvent
AdaptiveFuzzy
H
time(Sec)

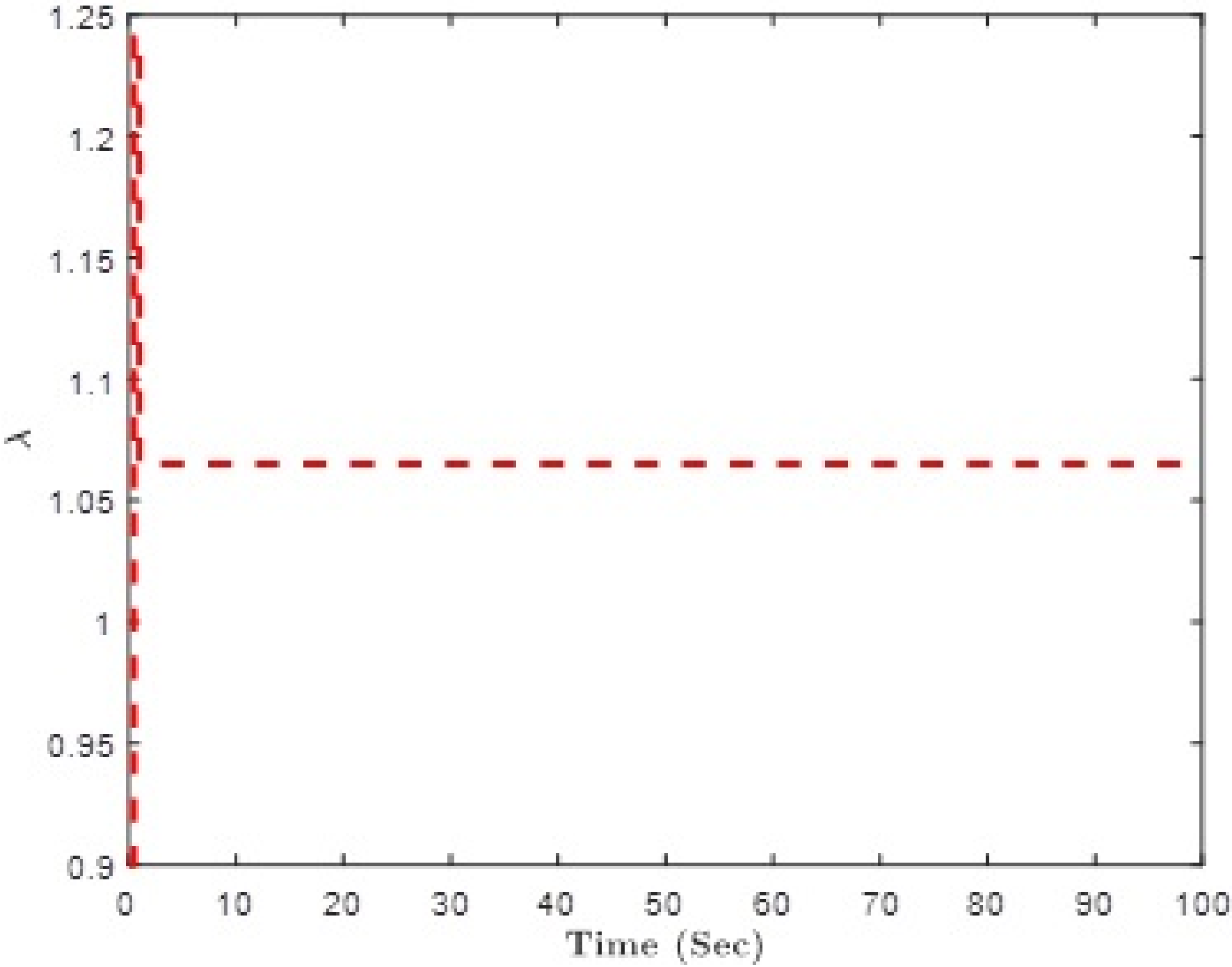

U Response - Zero Condition
SMC
SMCMpcEvent
AdaptiveFuzzy
U
time(Sec)
U Response - Zero Condition
SMC
SMCMpcEvent
AdaptiveFuzzy
U
time(Sec)


24
23.5
23
22.5
K
0
10
20
30
40
50
60
70
80
90
100
Time (Sec)


0.96
0.95
0.94
0.93
0.92
0.91
0.9
λ
0
10
20
30
40
50
60
70
80
90
100
Time (Sec)


EH Response - Sin Condition
Eh
time(Sec)
SMC
SMCMpcEvent
AdaptiveFuzzy
H Response - Sin Condition
H
time(Sec)
SMC
SMCMpcEvent
AdaptiveFuzzy
U Response - Sin Condition
U
time(Sec)
SMC
SMCMpcEvent
AdaptiveFuzzy


23.9
23.88
23.86
23.84
23.82
23.8
23.78
K
0
10
20
30
40
50
60
70
80
90
100
Time (Sec)


1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
λ
0
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20
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Time (Sec)