

Valuing a Marketing Board's Price Guarantee and Forward Sales under Geometric Mean Reversion, Currency Risk and Non-Traded Harvest Risk: Evidence from Ghana's Cocoa Sector

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Valuing a Marketing Board's Price Guarantee and Forward Sales under Geometric Mean Reversion, Currency Risk and Non-Traded Harvest Risk: Evidence from Ghana's Cocoa Sector

Abstract

Ghana's Cocoa Board (Cocobod) fixes the farm-gate price of cocoa before each season, sells much of the projected crop forward, and, under the Ghana Cocoa Board Act, 2026 (Act 1182), guarantees farmers at least 70% of the gross free-on-board (FOB) value of the crop. These commitments are contingent claims whose values depend jointly on the world cocoa price, two exchange rates and the size of the harvest. This paper develops a closed-form framework for valuing them. The cocoa price follows the two-factor model of Schwartz and Smith (2000), whose short-term component is the geometric mean-reverting process of Atiase et al. (2021). London prices are converted into cedis through the sterling-dollar and dollar-cedi rates, and the harvest is a non-traded risk priced under the minimal martingale measure. I derive closed-form values for the share guarantee and price floor embedded in the producer price, and for the liability created when forward sales exceed the harvest. I prove that constant-volatility valuations overstate guarantee values when prices mean-revert. Using International Cocoa Organisation, Bank of Ghana and Cocobod data, I find that the 2023/24 over-commitment was worth about US\$56 million before the season, rising to US\$269 million under combined harvest and price stress. The statutory guarantee for 2026/27 was worth about 5.6% of the producer price and the price floor about 8.5%. Cedi volatility accounts for over 80% of the uncertainty in the realised FOB value. Covering the floor with 99% confidence requires a reserve of about GH¢1.8 billion per 100,000 tonnes of forecast crop.

Keywords: Geometric mean reversion · Commodity derivatives · Price guarantee · Quantity risk · Minimal martingale measure · Cocoa

JEL Classification: G13 · Q14 · Q18 · F31

1 Introduction

Cocoa is central to Ghana's economy. Cocoa bean exports are a major source of foreign exchange, and many Ghanaians depend directly or indirectly on the crop for their livelihoods. Because bean prices are highly volatile and largely determined on the London futures exchange, swings pass directly into the country's export revenues, farm incomes, and exchange rate (van Huellen et al., 2026). Ghana is also the only country that has retained its cocoa marketing board. Through the Ghana Cocoa Board (Cocobod) and its subsidiary, the Cocoa Marketing Company (CMC), the state holds a monopoly on the export of Ghanaian cocoa beans, so the price, currency and quantity risks of the national crop are concentrated in a single public institution (van Huellen et al., 2026).

These risks arise from the way prices are set. Before each main-crop season, the Producer Price Review Committee negotiates the farm-gate price based on a projected gross free-on-board value (GFOB). The GFOB is the product of three forecasts: crop size, the dollar price of beans, and the cedi-dollar exchange rate. Working rules allocate between 60% and 70% of the GFOB

to farmers, and the agreed price is usually fixed for the season (van Huellen et al., 2026). To secure the export price, the CMC sells part of the projected crop forward under standard contracts of the Federation of Cocoa Commerce, in which the London sterling price and the sterling–dollar rate are fixed at signature. For 31 years, these forward contracts also served as collateral for the syndicated loan that financed cocoa purchases, until the loan was discontinued in 2024 (van Huellen et al., 2026). Arrangements of this kind dampen price volatility within a season, but they do not shield export and producer prices from variation between seasons in world prices set on derivatives markets (Staritz et al., 2023).

The 2023/24 season showed how these commitments can be affected. Cocobod projected an output of 800,000 tonnes and committed 786,672 tonnes in contracts, but actual production was 432,145 tonnes, 46% below the projection (Ghana Cocoa Board [COCOBOD], 2026). Adverse weather, underinvestment and the expansion of gold mining reduced the crop. Because Ghana is the second-largest producer, the shortfall also contributed to a tripling of the London price between January and mid-April 2024. As a result, about half of the predicted 2024/25 crop was committed for delivery at prices well below the terminal price (van Huellen et al., 2026). Rolled-over contracts were priced at about US\$2,661 per tonne when market prices exceeded US\$8,000 (COCOBOD, 2026).

The currency created a further exposure in the following season. The 2024/25 producer price was set at US\$3,100 per tonne using an exchange rate of GH¢16 per dollar. When the cedi later traded near GH¢10.29, Cocobod maintained the cedi price by absorbing the difference (COCOBOD, 2025). In 2026, the Ghana Cocoa Board Act, 2026 (Act 1182) introduced an automatic adjustment of the producer price to movements in the world price and the exchange rate, together with a guaranteed minimum of 70% of the gross FOB price for farmers (COCOBOD, 2026; GBC, 2026). On 25 September 2026, the 2026/27 producer price was set at GH¢42,400 per tonne, stated to be 71.18% of the realised gross FOB value (3News, 2026; GBC, 2026).

A guaranteed price and a forward book are contingent claims, and option-pricing methods are the natural way to value them. Gardner (1977) likened the deficiency payments of agricultural price supports to put options written by the government (Tirupattur et al., 1997), and Marcus and Modest (1986) showed that a price support on an uncertain crop is equivalent to a random number of put options. Later studies valued averaging features, joint price–yield risk and exotic payoffs in United States farm programmes (Kang & Brorsen, 1995; Tirupattur et al., 1997), as well as revenue insurance (Stokes, 2000; Richards & Manfredo, 2003). These valuations, however, share three features that do not fit a cocoa marketing board.

First, the futures price follows geometric Brownian motion (GBM) with constant volatility. Marcus and Modest (1986) adopt this assumption for convenience, while noting Samuelson’s (1965) analysis of how volatility changes over a contract’s life. Commodity prices, by contrast, display mean reversion (Bessembinder et al., 1995), a feature built into the futures-pricing models of Schwartz (1997) and Schwartz and Smith (2000), and for which Atiase et al. (2021) derived European option prices. Second, a single currency is involved, whereas Ghana’s cocoa revenue passes through two exchange rates and the CMC is exposed to both cedi–dollar and dollar–sterling risk (van Huellen et al., 2026). Third, quantity is either fixed or tied to traded assets, whereas cocoa harvest risk is not traded. Tirupattur et al. (1997) note that valuing claims

on non-traded yield requires an additional risk adjustment that no-arbitrage arguments alone cannot supply.

This paper develops a valuation framework that removes these three limitations. I embed the geometric mean-reverting process of Atiase et al. (2021) as the short-term factor of the Schwartz–Smith model, combine it with the two exchange rates and a harvest forecast that is correlated with prices, and derive the joint distribution of these variables under a cedi pricing measure. The non-traded harvest is priced under the minimal martingale measure (Föllmer & Schweizer, 1991; Schweizer, 1995), which yields an explicit harvest risk adjustment.

On this basis, I obtain four sets of closed-form results. The first gives the values of the share guarantee and the price floor embedded in the producer price, and characterises the distribution of the floor payment, which can be used to size a reserve. The second gives the value of the liability that arises when forward sales exceed the harvest. The third shows that the formulas contain several established results as special cases: the price-support formula of Marcus and Modest (1986), the Schwartz–Smith option on futures, a discrete geometric-average option in the spirit of Kemna and Vorst (1990) and the composite option of Turvey and Yin (2002). The fourth proves two comparative-statics results: constant-volatility valuations overstate guarantee values when prices mean-revert, and a negative price–harvest correlation strictly increases the forward-sale liability.

I then estimate the model from ICCO cocoa prices for 1994–2026, Bank of Ghana exchange rates and Cocobod’s crop targets and outcomes, and apply it to the 2023/24 over-commitment and the 2026/27 guarantee. Cocoa prices are well described by the one-factor geometric mean-reverting model, with a half-life of about 1.8 years, and Cocobod’s crop outturns fell short of target in six of eight seasons, with shortfalls tending to coincide with rising prices. Using only information available before the 2023/24 season, the over-commitment was worth about US\$56 million. The eventual cost was far larger and becomes plausible only when harvest and price stress are combined. The 2026/27 guarantee is worth about 5.6% of the producer price and the price floor about 8.5%, and the cedi exchange rate, rather than the cocoa price, is the dominant source of risk.

The rest of the paper is organised as follows. Section 2 reviews the related literature. Section 3 defines the contracts, Section 4 presents the model and Section 5 derives the valuation results. Section 6 describes the data and estimation; Section 7 presents the empirical results; Section 8 discusses the findings in light of the literature and their practical implications; and Section 9 concludes. Appendix A contains the proofs of the auxiliary lemmas, and Appendix B gives further details of data construction.

2 Literature Review

2.1 Mean reversion in commodity prices

Two classical ideas underlie modern commodity-price models. The theory of storage holds that inventories yield a convenience yield that is high when stocks are scarce (Working, 1949), and Samuelson (1965) showed that futures volatility changes over a contract's life. Building on these ideas, Gibson and Schwartz (1990) modelled the convenience yield as a stochastic, mean-

reverting factor in pricing oil contingent claims, and Bessembinder et al. (1995) found evidence of mean reversion in the term structure of futures prices. Schwartz (1997) compared three models that account for mean reversion: a one-factor model in which the logarithm of the spot price follows a mean-reverting process, a second model that adds a stochastic convenience yield, and a third that also includes stochastic interest rates. He estimated them using the Kalman filter for copper, oil and gold.

Schwartz and Smith (2000) later proposed a two-factor model in which the log spot price is the sum of a short-term deviation, following a zero-mean Ornstein–Uhlenbeck process, and a long-term equilibrium level, following arithmetic Brownian motion. When futures prices are lognormal, Black’s (1976) formula values options on them. Most closely related to the present paper, Atiase et al. (2021) derived a pricing formula for a European option on an asset that follows a geometric mean-reverting model and compared it with Monte Carlo simulation. This process uses the log-price dynamics of Schwartz’s (1997) one-factor model and the short-term factor of Schwartz and Smith (2000), and this link is the starting point of my model.

2.2 Hedging under price and quantity uncertainty

A separate body of literature studies producers who face both price and output risk. McKinnon (1967) examined futures markets and buffer stocks as instruments of income stability for primary producers, Rolfo (1980) derived optimal hedges under price and quantity uncertainty for a cocoa producer, and Ho (1984) analysed intertemporal futures hedging jointly with the production decision. Karp (1987) solved a dynamic hedging problem with stochastic production, Lapan and Moschini (1994) studied hedging under price, basis and production risk, and Rao (2011) considered multiperiod hedging with mean-reverting spot prices. More recently, Nyassoke Titi et al. (2022) modelled price and output as correlated jump-diffusions and obtained closed-form hedging positions for a farmer with constant absolute risk aversion. In electricity markets, Oum and Oren (2010) derived static hedges for load-serving entities exposed to non-traded quantity risk. This literature determines optimal hedge positions; it does not value the commitments that create the exposure.

2.3 Contingent-claims valuation of agricultural guarantees

Marcus and Modest (1986) showed that the United States price support system provides beneficiaries with a random number of put options. In their model, futures prices and a demand-shift variable follow geometric Brownian motions with time-invariant volatility; a constant-elasticity demand curve determines quantity; and the result is a put option with a random exercise price in the Fischer (1978) sense. Turvey (1992) derived contingent-claim models implied by stabilisation and insurance policies, using futures contracts to form the riskless portfolio (Tirupattur et al., 1997).

Subsequent work extended these valuations to more complex programmes. Kang and Brorsen (1995) valued deficiency payments as average-price options using Black-type and GARCH models with Monte Carlo integration, assuming a fixed payment yield. Tirupattur et al. (1997) valued exotic features of United States programmes, including random contract size, payoffs driven by both price and yield, and averaging, under geometric Brownian motion and with numerical methods. They also noted that because yield is not traded, valuing yield claims requires an equilibrium model with a risk-adjusted drift. Stokes (2000) showed that Crop

Revenue Coverage behaves like an exotic put option because it relies on futures-price averaging; Richards and Manfredo (2003) measured the error from ignoring infrequent shocks when rating revenue insurance; and Aase (2004) proposed a pricing model for combined price and yield futures.

2.4 Cross-currency commodity options and Ghana's cocoa marketing system

Contracts whose underlying is quoted in one currency but paid in another are valued by quanto and change-of-numéraire techniques (Reiner, 1992; Geman et al., 1995). In agriculture, Turvey and Yin (2002) valued Canadian-dollar options on commodities referenced to United States futures, separating the quanto component, which depends on the futures price and the exchange rate, from the local basis, with all variables following geometric Brownian motion.

The literature on Ghana's cocoa system is largely institutional. Staritz et al. (2023) show that the price stabilisation systems of Côte d'Ivoire and Ghana dampen producer-price volatility within a season and, through the Living Income Differential, raise export prices, but do not shield prices from variation between seasons, so that both countries largely remain price-takers. Van Huellen et al. (2026) document the composite price received by the CMC, the projection of the GFOB, the two exchange-rate exposures and the 2023/24 over-commitment. They conclude that the pricing system leaves the CMC exposed to significant price and quantity risk, and recommend a liquidity reserve fund combined with a dedicated risk-management strategy. However, their analysis does not include a valuation model.

2.5 Research gap

Table 1 summarises the literature. Studies of commodity prices provide mean-reverting price models but do not value guarantees. Studies of hedging treat stochastic quantity but determine hedge positions rather than values. Guarantee valuations rely on constant volatility and a single currency, cross-currency valuations ignore quantity, and the Ghana literature focuses on institutions. I am not aware of a study that values a marketing board's guarantee or forward-sale commitments under mean-reverting futures dynamics, or that combines a non-traded harvest with more than one currency in a closed-form valuation. This paper fills that gap.

Table 1. Summary of the literature and the contribution of this paper

Strand	Representative studies	Price dynamics	Quantity	Currencies	Output
Commodity futures models	Schwartz (1997); Schwartz and Smith (2000); Atiase et al. (2021)	Mean-reverting	None	One	Futures and option prices

Strand	Representative studies	Price dynamics	Quantity	Currencies	Output
Hedging under	Rolfo (1980); Karp	GBM,	Stochastic	One	Hedge positions output risk (1987); Rao jump-diffusion (2011); Nyassoke or Titi et al. (2022) mean-reverting
Valuation of	Marcus and Modest	GBM or GARCH	Fixed, or tied	One	Guarantee guarantees (1986); Kang and to traded values Brorsen (1995); assets Tirupattur et al. (1997)
Cross-currency	Turvey and Yin	GBM	None	Two	Composite and options (2002) quanto options
Ghana's cocoa	Staritz et	—	Documented	Documented	Institutional system al. (2023); van analysis Huellen et al. (2026)
This paper		Two-factor mean-reverting	Non-traded, correlated with price	Three (£, US\$, GH¢)	Closed-form guarantee and liability values

3 Institutional Setting and the Contracts to be Valued

I formalise the season described in Section 1. Time t denotes the valuation date, and T is the settlement date at which the harvest is known, the forward contracts fall due, and farmers are paid. Let $F(t, T)$ be the London cocoa futures price (£ per tonne) for delivery at T , so that $F(T, T)$ is the sterling spot price at T . Let X_t be the dollar price of one pound (US\$ per £) and Y_t the cedi price of one dollar (GH¢ per US\$). The dollar value of the sterling futures quotation is $H(t) = F(t, T) X_t$. Because the sterling price and the sterling-dollar rate of a fixed-price contract are both fixed at signature (van Huellen et al., 2026), a sale signed at t_k locks in the dollar terminal price $H(t_k)$. The country premium and the Living Income Differential are represented by a proportional mark-up $\pi \geq 0$ on the dollar terminal price (see Remark 1). Forward sales are signed at dates $t < t_1 < \dots < t_n \leq T$ with weights $w_k > 0$, $\sum_{k=1}^n w_k = 1$, which are the proportions of the crop priced at each date.

Definition 1 (Realised GFOB per tonne). The realised gross FOB value per tonne, in cedis, is

$$G_T = (1 + \pi) A_T Y_T, \quad A_T = \prod_{k=1}^n H(t_k)^{w_k}. \quad (1)$$

A_T is the weighted geometric average of the dollar terminal prices locked in by the forward sales.

Definition 2 (Producer-price commitment). Let $K > 0$ be the cedi producer price per tonne announced at t , and let $0 < \theta \leq 1$ be the guaranteed share of the GFOB ($\theta = 0.7$ under Act 1182 (COCOBOD, 2026; GBC, 2026); $0.6 \leq \theta \leq 0.7$ under earlier working rules (van Huellen et al., 2026)). With realised harvest Q_T tonnes, the total cedi payment to farmers at T is

$$\begin{aligned} \Pi_T &= Q_T \max\{K, \theta G_T\} = Q_T K + Q_T \max\{\theta G_T - K, 0\} \\ &= Q_T \theta G_T + Q_T \max\{K - \theta G_T, 0\}. \end{aligned} \quad (2)$$

Decomposition (2) separates two embedded options. The *share guarantee*, $Q_T \max\{\theta G_T - K, 0\}$, is the cost of guaranteeing the share θ on top of a fixed price; it is paid when world prices rise. The *price floor*, $Q_T \max\{K - \theta G_T, 0\}$, is the cost of paying the fixed price K on top of pure indexation; it is paid when world prices or the exchange rate fall. The price floor has the random-number-of-puts structure of Marcus and Modest (1986).

Definition 3 (Forward-sale over-commitment). Suppose q_c tonnes have been sold forward at an average locked dollar price K_f per tonne. If the harvest falls short of q_c , the undelivered tonnes must either be bought in at the prevailing dollar price $(1 + \pi)H(T)$ or delivered later, from the next crop, at the contract price. In both cases, the economic cost per undelivered tonne is the difference between the prevailing value $(1 + \pi)H(T)$ and K_f . The cedi liability at T is

$$L_T = Y_T \max\{q_c - Q_T, 0\} [(1 + \pi)H(T) - K_f]. \quad (3)$$

This is the exposure that materialised in 2023/24, when 786,672 tonnes had been committed against actual production of 432,145 tonnes and 333,767 tonnes of undelivered contracts were rolled over into the following season (COCOBOD, 2026; van Huellen et al., 2026).

Remark 1 (Premium specification). The Living Income Differential is a fixed US\$400 premium over the terminal price (van Huellen et al. 2026). A fixed dollar premium would make G_T a sum of lognormal variables. I therefore represent the combined premium as a proportion π of the terminal price, calibrated at the start of the season. This approximation keeps all results in closed form.

4 The Model

4.1 Assumptions

Let $(\Omega, \mathcal{F}, (\mathcal{F}_s)_{s \geq 0}, \mathbb{P})$ be a filtered probability space satisfying the usual conditions. It carries standard Brownian motions $B^\chi, B^\xi, B^X, B^Y, B^Q$ with constant pairwise correlations $\rho_{ij}, i, j \in \{\chi, \xi, X, Y, Q\}$. The filtration is generated by these processes.

Assumption A1 (Cocoa price; Schwartz & Smith, 2000). The log sterling spot price of cocoa is $\ln S_s = \chi_s + \xi_s$, where

$$d\chi_s = -\kappa \chi_s ds + \sigma_\chi dB_s^\chi, \quad d\xi_s = \mu_\xi ds + \sigma_\xi dB_s^\xi, \quad (4)$$

with $\kappa > 0$, $\sigma_\chi > 0$, $\sigma_\xi \geq 0$ and $\rho_{\chi\xi} = \rho$. Here χ_s is the short-term deviation and ξ_s the long-term equilibrium level of the log price, κ is the speed of mean reversion of the short-term deviation, σ_χ and σ_ξ are the volatilities of the two factors, μ_ξ is the drift of the equilibrium level, and ρ is the correlation between the two factors. Under the sterling pricing measure $\mathbb{Q}^{uk}$ the factors follow

$$d\chi_s = (-\kappa\chi_s - \lambda_\chi) ds + \sigma_\chi d\hat{B}_s^\chi, \quad d\xi_s = (\mu_\xi - \lambda_\xi) ds + \sigma_\xi d\hat{B}_s^\xi,$$

where $\hat{B}^\chi, \hat{B}^\xi$ are $\mathbb{Q}^{uk}$ -Brownian motions and $\lambda_\chi, \lambda_\xi$ are constant market prices of risk (Schwartz & Smith, 2000).

Assumption A2 (Exchange rates). Under $\mathbb{P}$,

$$\frac{dX_s}{X_s} = \mu_X ds + \sigma_X dB_s^X, \quad \frac{dY_s}{Y_s} = \mu_Y ds + \sigma_Y dB_s^Y, \quad (5)$$

with constant drifts μ_X, μ_Y and volatilities $\sigma_X, \sigma_Y > 0$.

Assumption A3 (Harvest). Let $M_s = \mathbb{E}^\mathbb{P}[Q_T | \mathcal{F}_s]$ be the crop forecast at time s , so that $M_T = Q_T$. The forecast is revised without bias, with constant volatility $\sigma_Q \geq 0$:

$$\frac{dM_s}{M_s} = \sigma_Q dB_s^Q, \quad 0 \leq s \leq T. \quad (6)$$

Cocobod forecasts the crop from pod counts and farmer surveys (van Huellen et al., 2026). Equation (6) models the sequence of revisions, such as the 2023/24 revision from an initial projection of 850,000 tonnes to 501,434 tonnes at mid-season (COCOBOD, 2023), as a lognormal martingale.

Assumption A4 (Markets). There are money-market accounts in cedis, dollars and sterling with constant continuously compounded rates r_{gh}, r_{us}, r_{uk} . Cocoa futures for every maturity are traded in London and marked to market in sterling. Spot exchange rates are traded. The harvest is not traded, and there are no transaction costs.

Assumption A5 (Correlation structure). Let R be the 4×4 correlation matrix of the *traded* drivers $(B^\chi, B^\xi, B^X, B^Y)$, assumed positive definite. Let $\mathbf{\rho}_Q = (\rho_{Q\chi}, \rho_{Q\xi}, \rho_{QX}, \rho_{QY})^\top$ be the harvest correlations, and write $R_Q^2 = \mathbf{\rho}_Q^\top R^{-1} \mathbf{\rho}_Q \leq 1$.

Assumption A6 (Pricing rule). Values are computed in cedis under the minimal martingale measure $\mathbb{Q}$ (Föllmer & Schweizer, 1991; Schweizer, 1995), defined in Lemma 3. The arbitrage-free value at t of a cedi payoff Ψ at T is

$$J(t) = e^{-r_{gh}(T-t)} \mathbb{E}^\mathbb{Q}[\Psi | \mathcal{F}_t]. \quad (7)$$

This is the risk-neutral valuation of Theorem 1 in Atiase et al. (2021), with discounting at the domestic risk-free rate (Harrison & Pliska, 1981; Björk, 2009).

Throughout, $\tau = T - t$, $N[\cdot]$ is the standard normal distribution function and $\phi(\cdot)$ its density. For $u \in [t, T]$ I use the deterministic functions

$$h_1(u) = \frac{e^{-\kappa(T-u)} - e^{-\kappa\tau}}{\kappa}, \quad h_2(u) = \frac{e^{-2\kappa(T-u)} - e^{-2\kappa\tau}}{2\kappa}. \quad (8)$$

These satisfy $h_1(T) = (1 - e^{-\kappa\tau})/\kappa$ and $h_2(T) = (1 - e^{-2\kappa\tau})/(2\kappa)$.

Remark 2 (Link to Atiase et al., 2021). If $\sigma_\xi = 0$ and $\mu_\xi = 0$, then $\xi_s \equiv \bar{\xi}$ and (4) gives $d \ln S_s = \kappa(\bar{\xi} - \ln S_s) ds + \sigma_\chi dB_s^\chi$. This is the log-price dynamics of $dP_s = \gamma(\eta - \ln P_s)P_s ds + \varepsilon P_s dB_s$ in Atiase et al. (2021) with $\gamma = \kappa$, $\varepsilon = \sigma_\chi$ and $\eta = \bar{\xi} + \sigma_\chi^2/(2\kappa)$. The present model therefore contains the 2021 process as its short-term factor. Table 2 summarises the notation.

Table 2. Notation

Symbol	Meaning	Units
$F(t, T)$	London cocoa futures price for delivery at T	£/tonne
X_t, Y_t	Dollar–sterling and cedi–dollar exchange	US\$/£, GH¢/US\$ rates
$H(t)$ $= F(t, T)X_t$	Dollar value of the futures quotation	US\$/tonne
G_T	Realised gross FOB value per tonne	GH¢/tonne
Q_T, M_t	Realised harvest and its forecast at t	tonnes
K, θ	Producer price and guaranteed GFOB share	GH¢/tonne, —
q_c, K_f	Tonnes sold forward and average locked	tonnes, US\$/tonne price
π	Proportional country premium and Living Income Differential	—
$\kappa, \sigma_\chi, \sigma_\xi$	Reversion speed and factor volatilities	$yr^{-1}, yr^{-\frac{1}{2}}$
μ_ξ, ρ	Drift of the long-term level; correlation of the two price factors	yr^{-1} , —
$\sigma_X, \sigma_Y, \sigma_Q$	Exchange-rate and harvest-forecast volatilities	$yr^{-\frac{1}{2}}$
μ_X, μ_Y	Drifts of the exchange rates under $\mathbb{P}$	yr^{-1}
r_{gh}, r_{us}, r_{uk}	Cedi, dollar and sterling risk-free rates	yr^{-1}
ψ	Harvest risk adjustment under $\mathbb{Q}$	yr^{-1}
$\lambda_\chi, \lambda_\xi$	Market prices of factor risk under $\mathbb{Q}^{uk}$	yr^{-1}
$\boldsymbol{\vartheta}, \boldsymbol{\rho}_Q$	Drift shifts from $\mathbb{P}$ to $\mathbb{Q}$; harvest correlations with traded risks	$yr^{-\frac{1}{2}}$, —

4.2 Pricing measure and auxiliary results

This subsection establishes the dynamics of all variables under the cedi pricing measure. Lemmas 1, 2 and 4 adapt standard results to the present setting and are proved in Appendix A. Lemma 3, which gives the explicit harvest risk adjustment, is proved in the text.

Lemma 1 (Futures dynamics under the sterling measure). Under Assumption A1, the futures price is (Schwartz & Smith, 2000)

$$\ln F(s, T) = e^{-\kappa(T-s)} \chi_s + \xi_s + \mathcal{A}(T - s), \quad (9)$$

where

$$\mathcal{A}(\tau) = (\mu_\xi - \lambda_\xi)\tau - (1 - e^{-\kappa\tau}) \frac{\lambda_\chi}{\kappa} + \frac{1}{2} \left[\frac{\sigma_\chi^2}{2\kappa} (1 - e^{-2\kappa\tau}) + \sigma_\xi^2 \tau + \frac{2\rho \sigma_\chi \sigma_\xi}{\kappa} (1 - e^{-\kappa\tau}) \right].$$

Under $\mathbb{Q}^{uk}$ the futures price satisfies

$$\frac{dF(s, T)}{F(s, T)} = \sigma_\chi e^{-\kappa(T-s)} d\hat{B}_s^\chi + \sigma_\xi d\hat{B}_s^\xi. \quad (10)$$

The proof is given in Appendix A.

Lemma 2 (Dynamics under the cedi measure). Let $\mathbb{Q}$ be a cedi pricing measure with Brownian motions $\tilde{B}^i$, and let $\mathbb{Q}^{uk}$ be related to $\mathbb{Q}$ by the change of numéraire

$$\left. \frac{d\mathbb{Q}^{uk}}{d\mathbb{Q}} \right|_{\mathcal{F}_s} = D_s = \frac{X_s Y_s e^{r_{uk}s}}{X_0 Y_0 e^{r_{gh}s}} \quad (11)$$

(Geman et al., 1995). Then, under $\mathbb{Q}$,

$$\frac{dY_s}{Y_s} = (r_{gh} - r_{us}) ds + \sigma_Y d\tilde{B}_s^Y, \quad \frac{dX_s}{X_s} = (r_{us} - r_{uk} - \rho_{XY} \sigma_X \sigma_Y) ds + \sigma_X d\tilde{B}_s^X, \quad (12)$$

$$\frac{dF(s, T)}{F(s, T)} = -[c_{FX}(s) + c_{FY}(s)] ds + \sigma_\chi e^{-\kappa(T-s)} d\tilde{B}_s^\chi + \sigma_\xi d\tilde{B}_s^\xi, \quad (13)$$

where the instantaneous covariances between futures and exchange-rate returns are

$$c_{FX}(s) = \sigma_X [\rho_{\chi X} \sigma_\chi e^{-\kappa(T-s)} + \rho_{\xi X} \sigma_\xi], \quad c_{FY}(s) = \sigma_Y [\rho_{\chi Y} \sigma_\chi e^{-\kappa(T-s)} + \rho_{\xi Y} \sigma_\xi]. \quad (14)$$

The proof is given in Appendix A.

The drift in (13) is the quanto adjustment (Reiner, 1992) for the two-currency case. Because the futures volatility decays with time to maturity (the Samuelson (1965) effect), the adjustment is itself maturity-dependent. In the constant-volatility setting of Turvey and Yin (2002), by contrast, the adjustment does not depend on maturity.

Lemma 3 (Harvest under the minimal martingale measure). Let $\mathfrak{g} \in \mathbb{R}^4$ be the constant drift shifts that take the traded drivers from $\mathbb{P}$ to $\mathbb{Q}$, so that $\tilde{B}_s^i = B_s^i + \vartheta_i s$ for $i \in \{\chi, \xi, X, Y\}$. They are

$$\begin{aligned}\vartheta_\chi &= \frac{\lambda_\chi}{\sigma_\chi} + \rho_{\chi X}\sigma_X + \rho_{\chi Y}\sigma_Y, & \vartheta_\xi &= \frac{\lambda_\xi}{\sigma_\xi} + \rho_{\xi X}\sigma_X + \rho_{\xi Y}\sigma_Y, \\ \vartheta_X &= \frac{\mu_X - r_{us} + r_{uk} + \rho_{XY}\sigma_X\sigma_Y}{\sigma_X}, & \vartheta_Y &= \frac{\mu_Y - r_{gh} + r_{us}}{\sigma_Y}.\end{aligned}\quad (15)$$

If a volatility is zero (for example $\sigma_\xi = 0$), the corresponding driver carries no traded risk and is omitted from R , $\boldsymbol{\rho}_Q$ and $\boldsymbol{\vartheta}$. Under the minimal martingale measure $\mathbb{Q}$, the harvest forecast satisfies

$$\frac{dM_s}{M_s} = -\psi ds + \sigma_Q d\tilde{B}_s^Q, \quad \psi = \sigma_Q \boldsymbol{\rho}_Q^\top R^{-1} \boldsymbol{\vartheta}, \quad (16)$$

where $\tilde{B}^Q$ is a $\mathbb{Q}$ -Brownian motion with the same correlations as B^Q . Consequently

$$\bar{Q}_t := \mathbb{E}^\mathbb{Q}[Q_T \mid \mathcal{F}_t] = M_t e^{-\psi\tau}. \quad (17)$$

Proof. The expressions (15) follow by comparing drifts. For χ , the $\mathbb{P} \rightarrow \mathbb{Q}^{uk}$ shift is λ_χ/σ_χ (Assumption A1), and the $\mathbb{Q}^{uk} \rightarrow \mathbb{Q}$ shift is $\rho_{\chi X}\sigma_X + \rho_{\chi Y}\sigma_Y$ (see the proof of Lemma 2 in Appendix A); the argument for ξ is the same. For X and Y , subtract the $\mathbb{Q}$ -drifts in (12) from the $\mathbb{P}$ -drifts in (5) and divide by the volatility.

Next, decompose the harvest driver into its projection on the traded drivers and an orthogonal remainder:

$$dB_s^Q = \boldsymbol{\rho}_Q^\top R^{-1} d\mathbf{B}_s^{tr} + \sqrt{1 - R_Q^2} dB_s^\perp,$$

where $\mathbf{B}^{tr} = (B^\chi, B^\xi, B^X, B^Y)^\top$ and $B^\perp$ is a Brownian motion independent of $\mathbf{B}^{tr}$. The minimal martingale measure changes drifts only in the directions spanned by the traded assets; martingales orthogonal to the traded assets remain martingales (Föllmer & Schweizer, 1991; Schweizer, 1995). Hence $B^\perp$ is a $\mathbb{Q}$ -Brownian motion, while $d\mathbf{B}_s^{tr} = d\tilde{\mathbf{B}}_s^{tr} - \boldsymbol{\vartheta} ds$. Substituting gives

$$dB_s^Q = d\tilde{B}_s^Q - \boldsymbol{\rho}_Q^\top R^{-1} \boldsymbol{\vartheta} ds, \quad d\tilde{B}_s^Q := \boldsymbol{\rho}_Q^\top R^{-1} d\tilde{\mathbf{B}}_s^{tr} + \sqrt{1 - R_Q^2} dB_s^\perp.$$

By construction $\tilde{B}^Q$ has unit quadratic variation and the same correlations as B^Q . Inserting this into (6) gives (16). Finally, $M_T = Q_T$, and $M_s e^{\psi s}$ is a $\mathbb{Q}$ -martingale, which gives (17). ■

Two limiting cases clarify (16). If the harvest is uncorrelated with the traded risks ($\boldsymbol{\rho}_Q = \mathbf{0}$), then $\psi = 0$ and the pricing measure uses the unbiased crop forecast. If, instead, the harvest is spanned by traded risks ($R_Q^2 = 1$), the market is complete in the relevant directions and ψ is the unique no-arbitrage adjustment, as in Marcus and Modest (1986). In all cases ψ is an explicit function of the correlations and of the market prices of traded risk, so it provides an explicit counterpart to the risk-adjusted drift that Tirupattur et al. (1997) require for non-traded yield.

Lemma 4 (Tilted lognormal expectation). Let (U, V) be bivariate normal with means μ_U, μ_V , variances $s_U^2, s_V^2 > 0$ and covariance c . Let $a > 0$ and $V_0 > 0$. Then

$$\mathbb{E}[e^U \max\{ae^V - V_0, 0\}] = e^{\mu_U + \frac{1}{2}s_U^2} (F^* N[d_1] - V_0 N[d_2]), \quad (18)$$

$$\mathbb{E}[e^U \max\{V_0 - ae^V, 0\}] = e^{\mu_U + \frac{1}{2}s_U^2} (V_0 N[-d_2] - F^* N[-d_1]), \quad (19)$$

where

$$F^* = a e^{\mu_V + \frac{1}{2}s_V^2 + c}, \quad d_1 = \frac{\ln(F^*/V_0) + \frac{1}{2}s_V^2}{s_V}, \quad d_2 = d_1 - s_V. \quad (20)$$

The proof is given in Appendix A.

Lemma 4 generalises the integral in Theorem 2 of Atiase et al. (2021) by the weight e^U . The weight shifts the mean of V by the covariance c , and this is the mechanism through which the random harvest enters every formula below. For a single variable the same device underlies the random-exercise-price formula of Fischer (1978) used by Marcus and Modest (1986).

5 Valuation Results

5.1 The joint distribution

For $u \in [t, T]$, define the variance and covariance functions

$$\Phi_F(u) = \sigma_\chi^2 h_2(u) + \sigma_\xi^2(u - t) + 2\rho \sigma_\chi \sigma_\xi h_1(u), \quad (21)$$

$$\begin{aligned} \Psi_X(u) &= \sigma_X [\rho_{\chi X} \sigma_\chi h_1(u) + \rho_{\xi X} \sigma_\xi(u - t)], & \Psi_Y(u) \\ &= \sigma_Y [\rho_{\chi Y} \sigma_\chi h_1(u) + \rho_{\xi Y} \sigma_\xi(u - t)], \end{aligned} \quad (22)$$

$$\Lambda(u) = \sigma_Q [\rho_{Q\chi} \sigma_\chi h_1(u) + \rho_{Q\xi} \sigma_\xi(u - t)], \quad (23)$$

and the combined functions

$$\begin{aligned} \Phi_H(u) &= \Phi_F(u) + \sigma_X^2(u - t) + 2\Psi_X(u), & \Gamma_Y(u) &= \Psi_Y(u) + \rho_{XY} \sigma_X \sigma_Y(u - t), & \Gamma_Q(u) \\ & & &= \Lambda(u) + \rho_{QX} \sigma_Q \sigma_X(u - t). \end{aligned} \quad (24)$$

Theorem 1 (Joint law of price, exchange rate and harvest). Under Assumptions A1–A6 and conditional on $\mathcal{F}_t$, the following hold under $\mathbb{Q}$.

(i) For $u \in [t, T]$,

$$\ln H(u) = \ln H(t) + (r_{us} - r_{uk})(u - t) - \Gamma_Y(u) - \frac{1}{2} \Phi_H(u) + \mathcal{M}_H(u), \quad (25)$$

where $\mathcal{M}_H(u) = \int_t^u [\sigma_\chi e^{-\kappa(T-s)} d\tilde{B}_s^\chi + \sigma_\xi d\tilde{B}_s^\xi + \sigma_X d\tilde{B}_s^X]$.

(ii) The exchange rate and harvest are

$$\ln Y_T = \ln Y_t + \left(r_{gh} - r_{us} - \frac{1}{2} \sigma_Y^2 \right) \tau + \sigma_Y (\tilde{B}_T^Y - \tilde{B}_t^Y), \quad (26)$$

$$\ln Q_T = \ln M_t - \left(\psi + \frac{1}{2} \sigma_Q^2 \right) \tau + \sigma_Q (\tilde{B}_T^Q - \tilde{B}_t^Q). \quad (27)$$

(iii) The family $\{\ln H(u) : u \in [t, T]\} \cup \{\ln Y_T, \ln Q_T\}$ is jointly Gaussian, with

$$\text{Cov}(\ln H(u), \ln H(v)) = \Phi_H(u \wedge v), \quad \text{Cov}(\ln H(u), \ln Y_T) = \Gamma_Y(u), \quad (28)$$

$$\text{Cov}(\ln H(u), \ln Q_T) = \Gamma_Q(u), \quad \text{Cov}(\ln Y_T, \ln Q_T) = \rho_{QY} \sigma_Q \sigma_Y \tau. \quad (29)$$

Proof. Step 1 (drifts). By Lemma 2 and Itô's formula,

$$d \ln F = - \left[c_{FX} + c_{FY} + \frac{1}{2} v_F^2 \right] ds + \sigma_X e^{-\kappa(T-s)} d\tilde{B}_s^X + \sigma_\xi d\tilde{B}_s^\xi,$$

with $v_F^2(s) = \sigma_X^2 e^{-2\kappa(T-s)} + \sigma_\xi^2 + 2\rho\sigma_X\sigma_\xi e^{-\kappa(T-s)}$, and

$$d \ln X = \left(r_{us} - r_{uk} - \rho_{XY} \sigma_X \sigma_Y - \frac{1}{2} \sigma_X^2 \right) ds + \sigma_X d\tilde{B}_s^X.$$

Since $\ln H = \ln F + \ln X$, integrating from t to u requires the elementary integrals

$$\int_t^u e^{-\kappa(T-s)} ds = h_1(u), \quad \int_t^u e^{-2\kappa(T-s)} ds = h_2(u).$$

These give $\int_t^u v_F^2 ds = \Phi_F(u)$, $\int_t^u c_{FX} ds = \Psi_X(u)$ and $\int_t^u c_{FY} ds = \Psi_Y(u)$. Collecting terms, the integrated drift of $\ln H$ is

$$(r_{us} - r_{uk})(u - t) - \Psi_X - \Psi_Y - \rho_{XY} \sigma_X \sigma_Y (u - t) - \frac{1}{2} \Phi_F - \frac{1}{2} \sigma_X^2 (u - t).$$

By (24) this equals $(r_{us} - r_{uk})(u - t) - \Gamma_Y(u) - \frac{1}{2} \Phi_H(u)$, which proves (25). Equations (26) and (27) follow by integrating (12) and (16).

Step 2 (Gaussianity). Each stochastic term is a Wiener integral of a deterministic integrand with respect to correlated Brownian motions. Any finite linear combination of such integrals is again a Wiener integral of a deterministic function, and hence normal. The family is therefore jointly Gaussian and independent of $\mathcal{F}_t$.

Step 3 (covariances). By the Itô isometry and $d\langle \tilde{B}^i, \tilde{B}^j \rangle_s = \rho_{ij} ds$, for $u \leq v$,

$$\text{Cov}(\mathcal{M}_H(u), \mathcal{M}_H(v)) = \int_t^u [v_F^2(s) + \sigma_X^2 + 2c_{FX}(s)] ds = \Phi_H(u).$$

In the same way,

$$\text{Cov}(\mathcal{M}_H(u), \sigma_Y \tilde{B}_T^Y) = \int_t^u [c_{FY}(s) + \rho_{XY} \sigma_X \sigma_Y] ds = \Gamma_Y(u),$$

$$\text{Cov}(\mathcal{M}_H(u), \sigma_Q \tilde{B}_T^Q) = \int_t^u \sigma_Q [\rho_{QX} \sigma_X e^{-\kappa(T-s)} + \rho_{Q\xi} \sigma_\xi + \rho_{QX} \sigma_X] ds = \Gamma_Q(u).$$

The last covariance, $\rho_{QY} \sigma_Q \sigma_Y \tau$, is immediate. ■

Corollary 1 (Distribution of the realised GFOB). Let $\ell_T = \ln G_T$ with G_T as in (1). Then, under $\mathbb{Q}$ and conditional on $\mathcal{F}_t$, $\ell_T \sim N(m_G, s_G^2)$, with $m_G = \ln \mathbb{G}_t - \frac{1}{2} s_G^2$, and $(\ell_T, \ln Q_T)$ is bivariate normal, where

$$s_G^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \Phi_H(t_i \wedge t_j) + \sigma_Y^2 \tau + 2 \sum_{k=1}^n w_k \Gamma_Y(t_k), \quad (30)$$

$$\mathbb{G}_t := \mathbb{E}^{\mathbb{Q}}[G_T | \mathcal{F}_t] = (1 + \pi) H(t) Y_t \exp \left\{ (r_{gh} - r_{us})\tau + (r_{us} - r_{uk})\bar{\tau} - \frac{1}{2} \Delta \right\}, \quad (31)$$

$$C_G := \text{Cov}(\ell_T, \ln Q_T) = \sum_{k=1}^n w_k \Gamma_Q(t_k) + \rho_{QY} \sigma_Q \sigma_Y \tau, \quad (32)$$

with

$$\bar{\tau} = \sum_{k=1}^n w_k (t_k - t), \quad \Delta = \sum_{k=1}^n w_k \Phi_H(t_k) - \sum_{i=1}^n \sum_{j=1}^n w_i w_j \Phi_H(t_i \wedge t_j) \geq 0.$$

Proof. By (1), $\ell_T = \ln(1 + \pi) + \sum_k w_k \ln H(t_k) + \ln Y_T$, a linear combination of the Gaussian family in Theorem 1. The variance (30) and covariance (32) follow from (28)–(29) by bilinearity.

For the mean, (25)–(26) give

$$m_G = \ln[(1 + \pi)H(t)Y_t] + (r_{us} - r_{uk})\bar{\tau} + \left(r_{gh} - r_{us} - \frac{1}{2}\sigma_Y^2\right)\tau - \sum_k w_k \left[\Gamma_Y(t_k) + \frac{1}{2}\Phi_H(t_k)\right].$$

Adding $\frac{1}{2}s_G^2$, the terms $\sum_k w_k \Gamma_Y(t_k)$ and $\frac{1}{2}\sigma_Y^2\tau$ cancel, leaving $m_G + \frac{1}{2}s_G^2 = \ln \mathbb{G}_t$. This proves (31).

Finally, Φ_H is non-decreasing, so $\Phi_H(t_i \wedge t_j) = \min\{\Phi_H(t_i), \Phi_H(t_j)\} \leq \frac{1}{2}[\Phi_H(t_i) + \Phi_H(t_j)]$. Summing with weights $w_i w_j$ gives $\Delta \geq 0$. ■

Two features of Corollary 1 are worth noting. First, the currency covariances Γ_Y cancel from the expected GFOB (31): $\mathbb{G}_t$ is the cedi forward value of the composite price, reduced only by the averaging convexity term Δ . Second, with a single sale at settlement ($n = 1$, $t_1 = T$), $\Delta = 0$ and $\mathbb{G}_t = (1 + \pi)F(t, T)X_t Y_t e^{(r_{gh} - r_{uk})\tau}$, the cedi composite forward price.

5.2 The producer-price guarantee

Theorem 2 (Value of the producer-price commitment). Under Assumptions A1–A6, define

$$\mathbb{F}_t = \theta \mathbb{G}_t e^{C_G}, \quad d_1 = \frac{\ln(\mathbb{F}_t/K) + \frac{1}{2}s_G^2}{s_G}, \quad d_2 = d_1 - s_G, \quad (33)$$

with $\mathbb{G}_t, C_G, s_G$ from Corollary 1 and $\bar{Q}_t$ from (17). Then the time- t values of the share guarantee and of the price floor in (2) are

$$J^C(t) = e^{-r_{gh}\tau} \mathbb{E}^Q[Q_T \max\{\theta G_T - K, 0\} | \mathcal{F}_t] = e^{-r_{gh}\tau} \bar{Q}_t (\mathbb{F}_t N[d_1] - K N[d_2]), \quad (34)$$

$$\begin{aligned} J^P(t) &= e^{-r_{gh}\tau} \mathbb{E}^Q[Q_T \max\{K - \theta G_T, 0\} | \mathcal{F}_t] \\ &= e^{-r_{gh}\tau} \bar{Q}_t (K N[-d_2] - \mathbb{F}_t N[-d_1]). \end{aligned} \quad (35)$$

The total farm-gate commitment Π_T in (2) has value

$$J^\Pi(t) = e^{-r_{gh}\tau} \bar{Q}_t K + J^C(t) = e^{-r_{gh}\tau} \bar{Q}_t \mathbb{F}_t + J^P(t), \quad (36)$$

and the two embedded options satisfy the parity relation

$$J^C(t) - J^P(t) = e^{-r_{gh}\tau} \bar{Q}_t (\mathbb{F}_t - K). \quad (37)$$

Proof. By Assumption A6, $J^C(t) = e^{-r_{gh}\tau} \mathbb{E}^Q[e^U \max\{\theta e^V - K, 0\}]$ with $U = \ln Q_T$ and $V = \ell_T$. By Corollary 1, (U, V) is bivariate normal with covariance $c = C_G$. Moreover, $e^{\mu_U + \frac{1}{2}s_U^2} = \mathbb{E}^Q[Q_T] = \bar{Q}_t$ by (17), and $e^{\mu_V + \frac{1}{2}s_V^2} = \mathbb{G}_t$ by (31).

Apply Lemma 4 with $a = \theta$, $V_0 = K$ and $s_V = s_G$. Then $F^* = \theta \mathbb{G}_t e^{C_G} = \mathbb{F}_t$, and (18) gives (34); (19) gives (35) in the same way.

Taking expectations of the two decompositions in (2), and using $\mathbb{E}^Q[Q_T \theta G_T] = \bar{Q}_t \theta \mathbb{G}_t e^{C_G} = \bar{Q}_t \mathbb{F}_t$ (Step 2 of the proof of Lemma 4 in Appendix A, with $g(v) = \theta e^v$), gives (36). Subtracting the two expressions in (36) gives (37). ■

Theorem 2 has a direct economic reading. The quantity $e^{-r_{gh}\tau} \bar{Q}_t \mathbb{F}_t / \theta$ is the present value of the cedi revenue from the crop, and $e^{-r_{gh}\tau} \bar{Q}_t K$ is the present value of paying the fixed producer price on the whole crop. This mirrors the quantities V_0 and V_1 of Marcus and Modest (1986).

The harvest enters the formulas in two ways: through the risk-adjusted crop $\bar{Q}_t$, which involves ψ , and through the tilt e^{C_G} . When price and harvest are negatively correlated, as the 2023/24 season illustrates, $C_G < 0$, and the tilt lowers the effective forward $\mathbb{F}_t$. Intuitively, large harvests coincide with low prices, which raises the value of the floor J^P and lowers that of the share guarantee J^C .

Remark 3 (Arithmetic averaging). Forward-sale prices are averaged arithmetically in practice. By the inequality of arithmetic and geometric means, the arithmetic average is at least the geometric average A_T . Hence (34) is a lower bound for the share guarantee and (35) an upper bound for the price floor under arithmetic averaging. As in Kemna and Vorst (1990), the geometric formulas also serve as control variates for Monte Carlo valuation of the arithmetic contracts, the approach taken by Kang and Brorsen (1995), Tirupattur et al. (1997) and Stokes (2000).

5.3 The forward-sale over-commitment liability

Theorem 3 (Value of the over-commitment liability). Under Assumptions A1–A6, define for any constant C

$$\begin{aligned} \mathcal{S}(C) &= q_c N[-e_2(C)] - \bar{Q}_t e^C N[-e_1(C)], & e_2(C) &= \frac{\ln(\bar{Q}_t/q_c) + C - \frac{1}{2}\sigma_Q^2\tau}{\sigma_Q\sqrt{\tau}}, & e_1(C) \\ & & &= e_2(C) + \sigma_Q\sqrt{\tau}, \end{aligned} \quad (38)$$

and the two covariances

$$C_1 = \Gamma_Q(T) + \rho_{QY}\sigma_Q\sigma_Y\tau, \quad C_2 = \rho_{QY}\sigma_Q\sigma_Y\tau. \quad (39)$$

Then the value of the liability (3) is

$$\mathcal{J}^L(t) = e^{-r_{gh}\tau} \mathbb{E}^\mathbb{Q}[L_T | \mathcal{F}_t] = (1 + \pi) H(t) Y_t e^{-r_{uk}\tau} \mathcal{S}(C_1) - K_f Y_t e^{-r_{us}\tau} \mathcal{S}(C_2). \quad (40)$$

Proof. Write

$$L_T = (1 + \pi) e^{U_1} \max\{q_c - e^V, 0\} - K_f e^{U_2} \max\{q_c - e^V, 0\},$$

with $U_1 = \ln(Y_T H(T))$, $U_2 = \ln Y_T$ and $V = \ln Q_T$. By Theorem 1, each pair (U_j, V) is bivariate normal. The covariances are

$$\text{Cov}(U_1, V) = \Gamma_Q(T) + \rho_{QY}\sigma_Q\sigma_Y\tau = C_1, \quad \text{Cov}(U_2, V) = C_2,$$

and $\text{Var}(V) = \sigma_Q^2\tau$.

The weights have expectations

$$\mathbb{E}^\mathbb{Q}[e^{U_1}] = H(t) Y_t e^{(r_{gh} - r_{uk})\tau}, \quad \mathbb{E}^\mathbb{Q}[e^{U_2}] = Y_t e^{(r_{gh} - r_{us})\tau}.$$

The first follows from (25)–(26) at $u = T$, where the terms in $\Gamma_Y(T)$ and $\Phi_H(T)$ cancel on adding one half of the variance $\Phi_H(T) + \sigma_Y^2\tau + 2\Gamma_Y(T)$. The second is immediate from (26).

Apply (19) of Lemma 4 with $a = 1$, $V_0 = q_c$ and $e^{\mu_V + \frac{1}{2}\sigma_V^2} = \bar{Q}_t$, so that $F^* = \bar{Q}_t e^{C_j}$. This gives

$$\mathbb{E}^\mathbb{Q}[e^{U_j} \max\{q_c - Q_T, 0\}] = \mathbb{E}^\mathbb{Q}[e^{U_j}] \mathcal{S}(C_j),$$

because $d_1 = e_1(C_j)$ and $d_2 = e_2(C_j)$ in the notation of (38). Discounting at r_{gh} and combining the two terms gives (40). ■

Corollary 2 (Independent harvest). If $\Gamma_Q(T) = 0$ and $\rho_{QY} = 0$, then $C_1 = C_2 = 0$ and

$$\mathcal{J}^L(t) = [(1 + \pi)H(t)Y_t e^{-r_{uk}\tau} - K_f Y_t e^{-r_{us}\tau}] \mathcal{S}(0). \quad (41)$$

The liability factorises into the cedi present value of the price gap per tonne and the expected shortfall $\mathcal{S}(0) = \mathbb{E}^\mathbb{Q}[\max\{q_c - Q_T, 0\}]$, a put on the harvest.

Proof. Set $C_1 = C_2 = 0$ in (40) and note that $\mathcal{S}(0)$ is the Black-type put value of Lemma 4 with $U \equiv 0$. ■

Corollary 3 (Negative price–harvest correlation increases the liability). Holding all other inputs fixed, $\mathcal{J}^L(t)$ is strictly decreasing in C_1 :

$$\frac{\partial \mathcal{J}^L(t)}{\partial C_1} = -(1 + \pi)H(t)Y_t e^{-r_{uk}\tau} \bar{Q}_t e^{C_1} N[-e_1(C_1)] < 0. \quad (42)$$

In particular, since C_1 contains the term $\sigma_Q[\rho_{Q\chi}\sigma_\chi h_1(T) + \rho_{Q\xi}\sigma_\xi\tau]$, making the correlation between the harvest and either price factor more negative raises the value of the over-commitment liability for a given risk-adjusted forecast $\bar{Q}_t$.

Proof. Write $s = \sigma_Q\sqrt{\tau}$ and differentiate (38). Since $\partial e_1 / \partial C = \partial e_2 / \partial C = 1/s$,

$$\frac{dS}{dC} = -\frac{q_c \phi(e_2)}{s} - \bar{Q}_t e^C N[-e_1] + \frac{\bar{Q}_t e^C \phi(e_1)}{s}.$$

Now $e_1^2 - e_2^2 = s(e_1 + e_2) = 2[\ln(\bar{Q}_t/q_c) + C]$, so

$$\frac{\phi(e_1)}{\phi(e_2)} = \exp\left\{-\frac{e_1^2 - e_2^2}{2}\right\} = \frac{q_c}{\bar{Q}_t e^C}, \quad \text{that is,} \quad \bar{Q}_t e^C \phi(e_1) = q_c \phi(e_2).$$

The first and third terms therefore cancel, leaving $dS/dC = -\bar{Q}_t e^C N[-e_1] < 0$. Multiplying by the positive factor $(1 + \pi)H(t)Y_t e^{-r_{uk}\tau}$ gives (42). The final statement follows because $h_1(T) > 0$ and $\tau > 0$. ■

Corollary 3 formalises the mechanism observed in 2023/24. When a poor harvest raises world prices, as Ghana's shortfall contributed to the rise in London prices in 2024 (van Huellen et al., 2026), the tonnes that must be covered are valued precisely when prices are high. The negative price–quantity correlation, which dampens the revenue risk of an unhedged producer and lowers the optimal hedge (McKinnon, 1967; Rolfo, 1980), therefore increases the cost of a forward book that exceeds the harvest.

5.4 Special cases and the effect of mean reversion

Proposition 1 (Nested models). Theorem 2 contains the following results as special cases.

- (a) **Options on futures (Black, 1976; Schwartz & Smith, 2000).** Take a deterministic harvest ($\sigma_Q = 0$, $M_t = q$), a single currency ($\sigma_X = \sigma_Y = 0$, $X \equiv Y \equiv 1$, $r_{gh} = r_{us} = r_{uk} = r$), $\pi = 0$, $\theta = 1$ and a single sale at settlement ($n = 1$, $t_1 = T$). Then

$$\mathcal{J}^C(t) = q e^{-r\tau}(F(t, T)N[d_1] - KN[d_2]), \quad s_G^2 = \Phi_F(T),$$

which is Black's formula with the Schwartz–Smith variance. If, in addition, $\sigma_\xi = 0$, the variance reduces to $\frac{\sigma_\chi^2}{2\kappa}(1 - e^{-2\kappa\tau})$ of the one-factor model. With $r = \eta$ (in Atiase et al., 2021, η is both the interest rate and the reversion level in the drift) and $\lambda_\chi = 0$, this recovers Theorems 2 and 3 of Atiase et al. (2021) (see Remark 2).

- (b) **Random number of puts (Marcus & Modest, 1986).** Take a single currency, $\pi = 0$, $\theta = 1$, $n = 1$, $t_1 = T$, constant futures volatility ($\sigma_\chi = 0$, $\sigma_\xi = \sigma_F$), and a harvest spanned by traded risks ($R_Q^2 = 1$). Then (35) becomes

$$\begin{aligned} J^P(t) &= V_1 N[-d_2] - V_0 N[-d_1], & V_0 &= e^{-r\tau} \mathbb{E}^{\mathbb{Q}}[Q_T F(T, T)], & V_1 \\ & & & = e^{-r\tau} \mathbb{E}^{\mathbb{Q}}[Q_T] K, \end{aligned} \quad (43)$$

with $d_1 = \left[\ln(V_0/V_1) + \frac{1}{2} \sigma_F^2 \tau \right] / (\sigma_F \sqrt{\tau})$. This is equation (14) of Marcus and Modest (1986) with support price K .

- (c) **Geometric-average option (Kemna & Vorst, 1990).** Take a deterministic harvest, a single currency and constant volatility ($\sigma_\chi = 0$). Then (34) is the value of a call on the discretely sampled geometric average of futures prices, the discrete analogue of the geometric-average option of Kemna and Vorst (1990).
- (d) **Composite cross-currency option (Turvey & Yin, 2002).** Take a deterministic harvest, a single foreign currency ($\sigma_X = 0, X \equiv 1, r_{us} = r_{uk}$), constant volatility, $n = 1$ and $t_1 = T$. Then (34) is the value of an option on the cedi value $F(T, T)Y_T$ of a dollar-denominated futures contract, the composite option studied by Turvey and Yin (2002) in the absence of basis risk.

Proof. (a) With the stated restrictions, $\bar{Q}_t = q$, $C_G = 0$, $\Delta = 0$ and $\mathbb{G}_t = F(t, T)$. By (21), $s_G^2 = \Phi_F(T) = \frac{\sigma_\chi^2}{2\kappa} (1 - e^{-2\kappa\tau}) + \sigma_\xi^2 \tau + \frac{2\rho\sigma_\chi\sigma_\xi}{\kappa} (1 - e^{-\kappa\tau})$, the variance of the Schwartz–Smith futures price at maturity. Setting $\sigma_\xi = 0$ leaves the first term. The link to Atiase et al. (2021) follows from Remark 2 and the observation that a claim on $F(T, T) = S_T$ is a claim on the spot.

- (b) Here $s_G^2 = \sigma_F^2 \tau$, and by Lemma 4 (Step 2), $e^{-r\tau} \bar{Q}_t \mathbb{F}_t = e^{-r\tau} \mathbb{E}^{\mathbb{Q}}[Q_T F(T, T)] = V_0$ and $e^{-r\tau} \bar{Q}_t K = V_1$. Substituting into (35) and (33) gives (43). Marcus and Modest (1986) write the support value as $W = V_0 N(d_1) - V_1 N(d_2) - (V_0 - V_1)$. Since $1 - N[x] = N[-x]$, this equals $V_1 N[-d_2] - V_0 N[-d_1]$, which is (43). When $R_Q^2 = 1$, the harvest is a function of traded risks, and the minimal martingale measure coincides with the unique martingale measure used in their complete-market derivation.
- (c) With $\sigma_\chi = 0$ and one currency, $\Phi_H(u) = \sigma_\xi^2 (u - t)$. Then $s_G^2 = \sigma_\xi^2 \sum_{i,j} w_i w_j (t_i \wedge t_j - t)$, which is the variance of the log geometric average of a geometric Brownian motion sampled at $t_1, \dots, t_n$. Formula (34) is Black’s formula applied to this lognormal average.
- (d) With these restrictions, $G_T = F(T, T)Y_T$ and $s_G^2 = \sigma_\xi^2 \tau + \sigma_Y^2 \tau + 2\rho_{\xi Y} \sigma_\xi \sigma_Y \tau$, the variance of the log composite price, and (34) is the composite option value. ■

Proposition 2 (Mean reversion lowers the time value of the guarantee). Consider a single currency, $n = 1$ and $t_1 = T$, and suppose $\rho = \rho_{\chi\xi} \geq 0$. Then:

- (i) The variance $s_G^2 = \Phi_F(T)$ is strictly decreasing in κ on $(0, \infty)$, with

$$\lim_{\kappa \rightarrow 0} \Phi_F(T) = \sigma_0^2 \tau, \quad \sigma_0^2 = \sigma_\chi^2 + \sigma_\xi^2 + 2\rho\sigma_\chi\sigma_\xi, \quad \lim_{\kappa \rightarrow \infty} \Phi_F(T) = \sigma_\xi^2 \tau. \quad (44)$$

- (ii) Holding $\bar{Q}_t$ and $\mathbb{F}_t$ fixed, both guarantee values are strictly increasing in s_G :

$$\frac{\partial J^C}{\partial s_G} = \frac{\partial J^P}{\partial s_G} = e^{-r_{gh}\tau} \bar{Q}_t \mathbb{F}_t \phi(d_1) > 0. \quad (45)$$

- (iii) Consequently, for every $\kappa > 0$, a valuation that treats the futures price as geometric Brownian motion with the instantaneous volatility σ_0 and the same $\bar{Q}_t$ and $\mathbb{F}_t$ strictly overstates both $\mathcal{J}^C$ and $\mathcal{J}^P$.

Proof. (i) Let $g(x) = (1 - e^{-x})/x$ for $x > 0$. Then

$$\Phi_F(T) = \sigma_\chi^2 \tau g(2\kappa\tau) + \sigma_\xi^2 \tau + 2\rho\sigma_\chi\sigma_\xi\tau g(\kappa\tau).$$

Differentiating, $g'(x) = [e^{-x}(1+x) - 1]/x^2 < 0$ because $e^x > 1+x$ for $x > 0$, so g is strictly decreasing, with $g(x) \rightarrow 1$ as $x \downarrow 0$ and $g(x) \rightarrow 0$ as $x \rightarrow \infty$. With $\rho \geq 0$ and $\sigma_\chi > 0$, both κ -dependent terms are strictly decreasing in κ , and the limits (44) follow.

- (ii) From (33), $\partial d_1 / \partial s_G = 1 - d_1/s_G$ and $\partial d_2 / \partial s_G = -d_1/s_G$. Differentiating (34),

$$\frac{\partial \mathcal{J}^C}{\partial s_G} = e^{-r_{gh}\tau} \bar{Q}_t \left[\mathbb{F}_t \phi(d_1) \left(1 - \frac{d_1}{s_G} \right) + K \phi(d_2) \frac{d_1}{s_G} \right].$$

The identity $\mathbb{F}_t \phi(d_1) = K \phi(d_2)$ follows as in the proof of Corollary 3, since $d_1^2 - d_2^2 = 2 \ln(\mathbb{F}_t/K)$. With it the expression reduces to $e^{-r_{gh}\tau} \bar{Q}_t \mathbb{F}_t \phi(d_1)$. Because the right-hand side of the parity (37) does not depend on s_G , the put has the same derivative.

- (iii) The GBM valuation uses the variance $\sigma_0^2 \tau$, which by (i) strictly exceeds $\Phi_F(T)$ for every $\kappa > 0$. The claim then follows from (ii). ■

Proposition 2 isolates the variance channel through which mean reversion acts. In the full model, κ also enters $\mathbb{F}_t$ through the averaging term Δ and the tilt C_G ; these effects are quantified numerically in the empirical sections. Marcus and Modest (1986) adopted time-invariant futures volatility for convenience, noting Samuelson's (1965) analysis of how volatility evolves over a contract's life. Proposition 2 shows that, when prices mean-revert, relaxing this assumption lowers the value of the guarantee.

5.5 The distribution of the commitments

Theorems 2 and 3 give expected values. For reserve planning, the whole distribution of a commitment is needed. Because Corollary 1 delivers a bivariate normal law, the distribution of the floor payment reduces to a one-dimensional integral.

Corollary 4 (Distribution of the floor payment). Let $m_G = \ln \mathbb{G}_t - \frac{1}{2} s_G^2$ and $m_Q = \ln M_t - (\psi + \frac{1}{2} \sigma_Q^2) \tau$, and write $s_Q^2 = \sigma_Q^2 \tau$. Then the floor payment $Z_T = Q_T \max\{K - \theta G_T, 0\}$ satisfies $\mathbb{Q}(Z_T > 0) = N[(\ln(K/\theta) - m_G)/s_G]$ and, for every $\mathcal{R} > 0$,

$$\mathbb{Q}(Z_T > \mathcal{R}) = \int_{-\infty}^{\ln(K/\theta)} \frac{1}{s_G} \phi\left(\frac{\ell - m_G}{s_G}\right) N\left[\frac{m_{Q|\ell} - \ln \mathcal{R} + \ln(K - \theta e^\ell)}{s_{Q|\ell}}\right] d\ell, \quad (46)$$

where $m_{Q|\ell} = m_Q + (C_G/s_G^2)(\ell - m_G)$ and $s_{Q|\ell}^2 = s_Q^2 - C_G^2/s_G^2$. For $\alpha > 1 - \mathbb{Q}(Z_T > 0)$, the reserve that covers the floor payment with probability α is the unique $\mathcal{R}_\alpha > 0$ that solves $\mathbb{Q}(Z_T > \mathcal{R}_\alpha) = 1 - \alpha$.

Proof. By Corollary 1 and (27), $(\ell_T, \ln Q_T)$ is bivariate normal with means m_G and m_Q , variances s_G^2 and s_Q^2 , and covariance C_G ; the mean m_G follows from $\mathbb{E}^\mathbb{Q}[e^{\ell_T}] = \mathbb{G}_t$. The floor payment is positive exactly when $\theta e^{\ell_T} < K$, that is, when $\ell_T < \ln(K/\theta)$, which gives $\mathbb{Q}(Z_T > 0)$.

For $\mathcal{R} > 0$, $Z_T > \mathcal{R}$ holds if and only if $\ell_T < \ln(K/\theta)$ and $\ln Q_T > \ln \mathcal{R} - \ln(K - \theta e^{\ell_T})$. Conditional on $\ell_T = \ell$, $\ln Q_T$ is normal with mean $m_{Q|\ell}$ and variance $s_{Q|\ell}^2$, so the conditional probability of the second event is $N\left[\left(m_{Q|\ell} - \ln \mathcal{R} + \ln(K - \theta e^\ell)\right)/s_{Q|\ell}\right]$. Integrating against the density of ℓ_T gives (46).

The right-hand side of (46) is continuous and strictly decreasing in $\mathcal{R}$, tends to $\mathbb{Q}(Z_T > 0)$ as $\mathcal{R} \downarrow 0$ and to zero as $\mathcal{R} \rightarrow \infty$. A unique solution $\mathcal{R}_\alpha$ therefore exists whenever $1 - \alpha < \mathbb{Q}(Z_T > 0)$. ■

Corollary 4 is stated under the pricing measure. When the market prices of traded risk under $\mathbb{Q}$, that is, the drift shifts $\boldsymbol{\theta}$ in (15), are zero, as assumed in the base case of Section 7, the pricing and physical distributions coincide and $\mathcal{R}_\alpha$ is a physical-probability reserve.

6 Data and Estimation

6.1 Data

Table 3 lists the data. Cocoa prices are the daily series of the International Cocoa Organisation (ICCO): the London futures price (£ per tonne), the New York futures price (US\$ per tonne) and the ICCO daily price (US\$ per tonne), from 3 October 1994 to 24 September 2026 (8,211 trading days). Daily interbank exchange rates of the cedi against the dollar and the pound, and weekly auction rates for 91-day and 182-day Treasury bills, are published by the Bank of Ghana. The foreign risk-free rates are the 3-month United States Treasury bill rate and the 3-month United Kingdom interbank rate.

Cocobod's crop targets and outcomes are taken from its annual reports, its Turnaround Strategy Paper and its 2026 press release. Production by season is taken from a report of the USDA Foreign Agricultural Service [USDA FAS], whose chart reproduces Cocobod data. Appendix B describes the construction of each series. The complete dataset, the code and the extraction script are provided as Online Resource 1.

Table 3. Data sources

Series	Frequency	Period	Source
London and New York cocoa futures; ICCO daily price	Daily	Oct 1994 – Sep 2026	ICCO (n.d.)
GH¢ per US\$ and GH¢	Daily	Jan 1996 –	Bank of Ghana (2026a) per £ (interbank mid Sep 2026 rates)
Ghana 91- and	Weekly	Jun 2017 –	Bank of Ghana (2026b) 182-day Treasury Aug 2026 bills

Series	Frequency	Period	Source
US 3-month Treasury	Daily	Jan 2000 –	Board of Governors of the bill Sep 2026 Federal Reserve System (2026)
UK 3-month interbank	Quarterly	2014 – 2025	OECD (2026) rate
Cocobod crop targets	Seasonal	2015/16 –	COCOBOD (2016, 2018, 2019, and outcomes 2024/25 2020, 2021, 2023, 2026); USDA FAS (2025a, 2025b)
Ghana cocoa	Seasonal	2009/10 –	USDA FAS (2025b) production 2024/25
Season parameters (Table 9)	—	2023/24 and 2026/27	COCOBOD (2023, 2026); van Huellen et al. (2026); 3News (2026); GBC (2026)

6.2 The sterling–dollar rate

The ICCO daily price is the average of the quotations of the nearest three active futures months in London and New York, with the London component converted into dollars (ICCO, n.d.). Writing Lon_t and NY_t for the London (£) and New York (US\$) averages and I_t for the ICCO price (US\$), it follows that $I_t = \frac{1}{2}(Lon_t X_t + NY_t)$, and hence

$$X_t = \frac{2I_t - NY_t}{Lon_t}. \quad (47)$$

Equation (47) gives a daily sterling–dollar rate that is exactly aligned in time with the cocoa prices. It agrees closely with the Bank of Ghana cross rate (GH¢ per £ divided by GH¢ per US\$). Over 7,473 common trading days, the two series have a correlation of 0.9986 in levels and 0.81 in weekly changes, with a mean absolute difference of 0.008 US\$ per £ (Fig. 1). The dollar value of the London quotation is then $H_t = Lon_t X_t$.

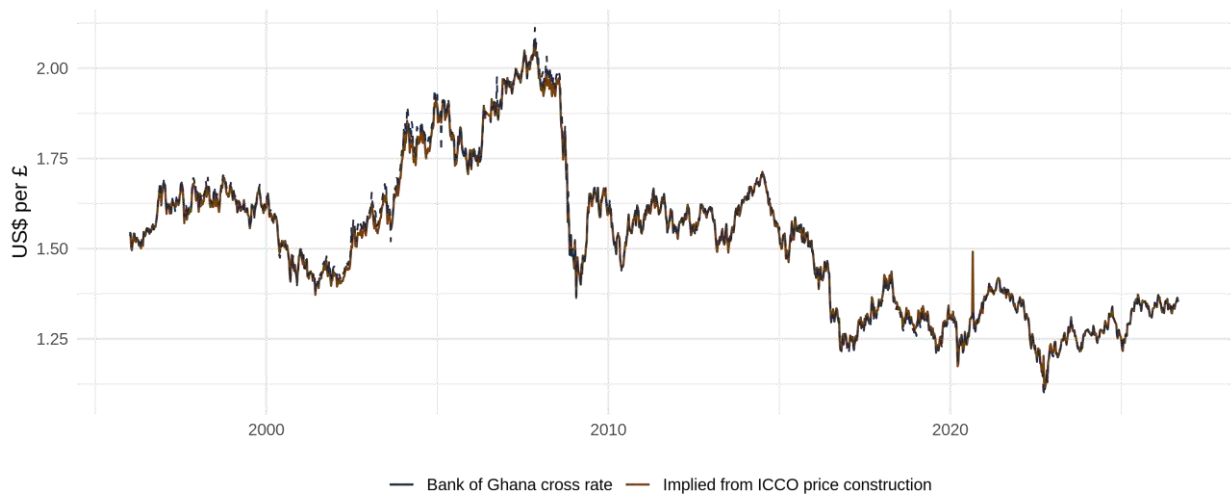


Fig. 1 Sterling–dollar rate implied by equation (47) and the Bank of Ghana cross rate, weekly, 1996–2026 (solid line: implied rate; dashed line: Bank of Ghana cross rate)

6.3 Estimation of the price dynamics

The London quotation averages the three nearest contracts, whose average maturity I denote by τ_m . By (9), the weekly log price satisfies

$$y_t = c + e^{-\kappa\tau_m}\chi_t + \xi_t + e_t, \quad (48)$$

where e_t is a small measurement error. Equation (48), together with the exact transition equations of (χ_t, ξ_t) , forms a linear Gaussian state-space model. I estimate it by maximum likelihood using the Kalman filter (Harvey, 1989; Schwartz, 1997), with weekly observations and $\tau_m = 0.33$ years. The short-term volatility identified in (48) is $\sigma_\chi e^{-\kappa\tau_m}$, and I report σ_χ at zero maturity. With a single price series, the correlation $\rho_{\chi\xi}$ is not separately identified and is set to zero.

Two samples are used: the full sample, and a pre-season sample ending in September 2023 that contains only information available at the start of the 2023/24 season. Table 4 compares the two-factor model with its nested special cases: the one-factor model of Atiase et al. (2021) ($\sigma_\xi = 0$, with μ_ξ unrestricted) and geometric Brownian motion.

Table 4. Kalman-filter estimates for the weekly log London cocoa price

Sample	Model	κ	σ_χ	σ_ξ	μ_ξ	Half-life (years)	Log-likelihood	AIC
1994–2026	Two-factor	0.376 (0.174)	0.325 (0.020)	0.0005	0.046	1.84	3019.21	−6028.4
1994–2026	One-factor	0.376	0.325	0	0.046	1.84	3019.21	−6030.4
1994–2026	GBM	—	—	0.286	0.088	—	3017.09	−6030.2
1994–2023	Two-factor	0.445	0.273	0.0002	0.036	1.56	3032.28	−6054.6 (0.197) (0.019)
1994–2023	One-factor	0.445	0.273	0	0.036	1.56	3032.28	−6056.6
1994–2023	GBM	—	—	0.236	0.066	—	3029.99	−6056.0

Notes: Standard errors in parentheses. For GBM, the column σ_ξ reports the volatility and μ_ξ the drift.

The estimated volatility of the long-term factor is essentially zero in both samples. Fixing it at 0.05, 0.10, 0.15 and 0.20 lowers the log-likelihood by 0.24, 0.64, 1.05 and 1.45. The data therefore select the one-factor model of Atiase et al. (2021), with a deterministic trend of 4.6% a year in the equilibrium level. Short-term deviations revert with a half-life of 1.84 years (1.56 years before 2023/24).

The statistical preference over geometric Brownian motion is modest. The likelihood-ratio statistic is 4.24 on the full sample and 4.58 on the pre-season sample, and monthly unit-root and variance-ratio tests (Dickey & Fuller, 1979; Lo & MacKinlay, 1988) do not reject a random walk (ADF statistic -0.96 ; $VR(36) = 0.88$, $p = 0.79$). This is consistent with the low power of such tests when half-lives are long. For valuation, the relevant channel is the variance channel of Proposition 2: at a one-year horizon, the estimated reversion lowers the variance of the log price by the factor $(1 - e^{-2\kappa})/(2\kappa) = 0.70$ relative to GBM with the same instantaneous volatility.

The 2023/24 season also marked a change in volatility. The realised volatility of the London price was 17.8% a year in the three years before October 2023 and 58.5% from October 2023 to September 2026. This is used in the stress scenarios below.

6.4 Exchange-rate parameters

Table 5 reports the currency parameters, estimated from weekly log returns. Because the long-term factor has no variance, cocoa price shocks load on χ_t alone, and the correlation between weekly London returns and a currency is the model correlation ρ_{χ} . The sterling-dollar rate has a volatility of 9.4% a year, whereas the cedi's volatility is about twice as high: 19.5% over 2014–2026, a period that includes the debt crisis of 2021–2022 described by van Huellen et al. (2026). The correlations between the currencies and the cocoa price are small.

Table 5. Exchange-rate volatilities and correlations (weekly data)

Parameter	Estimate	Data
σ_X (US\$ per £), 1994–2026	0.094	Equation (47)
$\rho_{\chi X}$,	-0.074	Equation (47) and London price 1994–2026
σ_X ;	0.097 ; -0.066	Equation (47) and London price $\rho_{\chi X}$, 1994 – Sep 2023
σ_Y (GH¢ per	0.195	Bank of Ghana US\$), 2014–2026
σ_Y , 2007–2026;	0.160 ; 0.178	Bank of Ghana Oct 2023 – Sep 2026
$\rho_{\chi Y}$; ρ_{XY} , 2014–2026	-0.017 ; 0.053	Bank of Ghana, equation (47) and London price

6.5 Harvest parameters

Cocobod announces a crop target at the start of each season. Under Assumption A3 the forecast is a lognormal martingale, so the log forecast error over a season of length $\tau \approx 1$ year satisfies

$$\ln \frac{Q_T}{M_t} \sim N \left(-\frac{1}{2} \sigma_Q^2 \tau, \sigma_Q^2 \tau \right). \quad (49)$$

The standard deviation of the log errors therefore estimates σ_Q , and a mean well below zero indicates that outturns tend to fall short of targets. Table 6 lists the eight seasons for which both a target and an outturn are documented, and Table 7 reports the estimates. Targets for 2016/17 and 2021/22 are not reported in the documents available to the author. For 2023/24, the

Turnaround Strategy Paper (COCOBOD, 2023) reports an initial projection of 850,000 tonnes and USDA FAS (2025a) reports production of 530,873 tonnes; I use Cocobod’s 2026 figures.

Table 6. Cocobod crop targets and outturns

Season	Target (t)	Outturn (t)	Log error	Source
2015/16	850,000	778,043	−0.088	COCOBOD (2016)
2017/18	900,000	904,446	0.005	COCOBOD (2018)
2018/19	850,000	811,250	−0.047	COCOBOD (2019)
2019/20	850,000	775,488	−0.092	COCOBOD (2020)
2020/21	830,000	1,047,385	0.233	COCOBOD (2021)
2022/23	800,000	656,156	−0.198	COCOBOD (2023)
2023/24	800,000	432,145	−0.616	COCOBOD (2026)
2024/25	800,000	617,400	−0.259	USDA FAS (2025a, 2025b)

Table 7. Log forecast errors of Cocobod’s crop targets

Sample	Seasons	Mean log error	σ_Q	Mean absolute % error
Before 2023/24 (ex-ante information)	6	−0.031	0.146	11.1
All seasons, 2015/16 –	8	−0.133	0.244	16.9 2024/25
Primary Cocobod sources	7	−0.115	0.258	16.1 only
Excluding 2023/24	7	−0.064	0.158	12.8

Outturns fell short of targets in six of the eight seasons, by 13% on average in log terms. Before 2023/24, the forecast errors were modest ($\sigma_Q = 0.146$), in line with Cocobod’s statement that crop forecasts typically vary by 5% to 15% (COCOBOD, 2026). Including 2023/24, when output fell 46% below the projection, raises the estimate to 0.244.

Table 8 reports the correlation of the harvest with prices and exchange rates. The correlation ρ_{QX} is measured as the correlation between each season’s forecast error and the log change in the London price over the same season. It is corroborated by the correlation between annual changes in production and in the season-average ICCO price. Both are negative, as expected for the second-largest producer, whose shortfalls affect the world price (van Huellen et al., 2026). The correlations with the two exchange rates are also negative but small relative to their sampling error. All correlations are imprecisely estimated, so every result below is accompanied by a sensitivity analysis.

Table 8. Correlation of the harvest with prices and exchange rates

Measure	n	Correlation	95% confidence interval
Forecast error vs in-season change in London price, all seasons	8	−0.62	(−0.92, 0.16)
Same, before 2023/24	6	−0.43	(−0.92, 0.59)
Annual change in production vs change	15	−0.22	(−0.66, 0.33) in season-

Measure	n	Correlation	95% confidence interval
			average price
Forecast error vs in-season change in	8	−0.36	(−0.85, 0.46) US\$ per £
Forecast error vs in-season change in GH¢ per US\$	8	−0.40	(−0.86, 0.43)

Note: Confidence intervals from Fisher’s z-transformation.

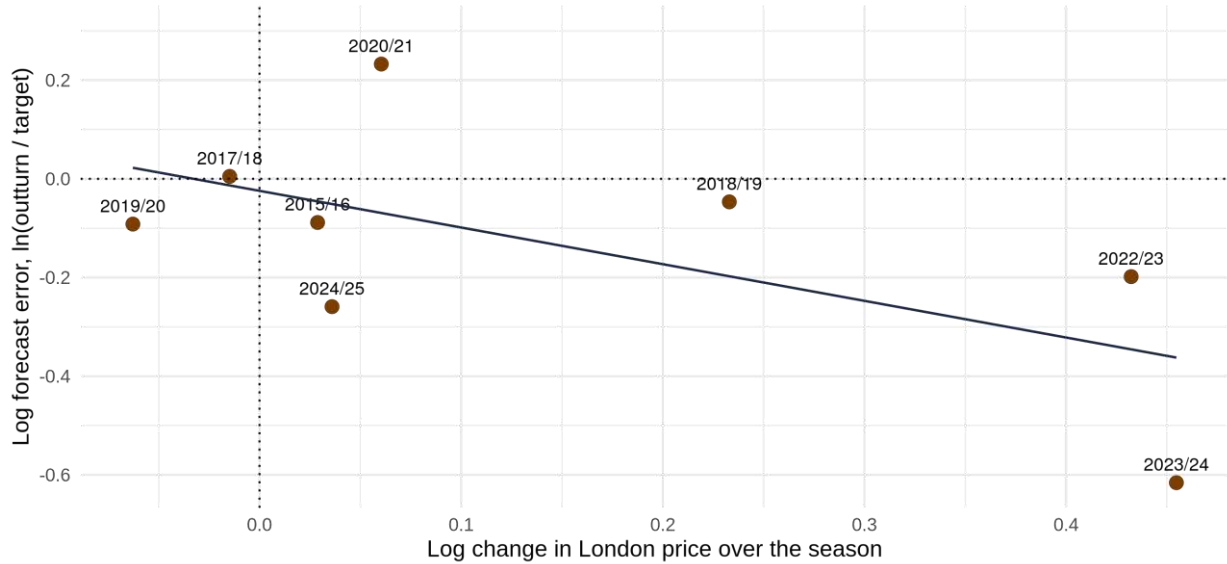


Fig. 2 Cocobod’s crop forecast errors and the change in the London cocoa price over the same season, 2015/16–2024/25 (points: seasons; line: least-squares fit)

6.6 Parameters of the two applications

Table 9 collects the parameters of the two applications. Interest rates are the latest observations on or before each valuation date, expressed with continuous compounding. The cedi rate is the 182-day Treasury bill rate, whose maturity is closest to the horizons considered.

For 2023/24 I use Cocobod’s figures: a projected output of 800,000 tonnes, 786,672 tonnes committed in contracts, actual production of 432,145 tonnes and a contract price of about US\$2,661 per tonne (COCOBOD, 2026). For 2026/27, the realised gross FOB implied by the announced producer price, $\text{GH¢}42,400/0.7118 = \text{GH¢}59,567$ per tonne, is used as the cedi forward value of the GFOB before the averaging adjustment, so that $\mathbb{G}_t = 59,567 e^{-\Delta/2}$ in (31). The valuation date, 24 September 2026, is the last day of the price sample and the day before the producer price was announced; the cedi rate is the latest Bank of Ghana observation available, that of 1 September 2026. I assume that half of the crop is already priced and that the remainder is sold in equal monthly amounts to the end of the main crop in May (van Huellen et al., 2026).

The Living Income Differential is represented by the proportional mark-up $\pi = 400/H(t)$ (Remark 1). The harvest risk adjustment is set to $\psi = 0$ in the base case; Section 7.6 reports its effect. The harvest–currency correlations are set to zero in the base case because of their imprecision, and Section 7.3 reports the effect of using the estimates in Table 8.

Table 9. Parameters of the two applications

Parameter	2023/24 over-commitment	2026/27 guarantee	Source
Valuation date t ; settlement date T	2 Oct 2023; 31 May 2024	24 Sep 2026; 31 May 2027	Main crop (van Huellen et al. 2026)
Crop forecast	800,000 t	Per tonne of	COCOBOD (2026) M_t forecast
Tonnes committed	786,672 t;	—	COCOBOD (2026) q_c ; contract US\$2,661 price K_f
Producer price K ;	—	GH¢42,400; 0.70	3News (2026); GBC guaranteed share (2026) θ
Forward value of	—	GH¢59,567 per t	$K/0.7118$ the GFOB
London price; US\$	£2,975; 1.2139	£4,310; 1.3229	ICCO; equation (47) per £ at t
GH¢ per US\$ at t	11.1492	11.2725 (1 Sep	Bank of Ghana 2026) (2026a)
r_{gh} ; r_{us} ;	28.8%; 5.46%; 5.19%	6.76%; 4.16%;	Bank of Ghana r_{uk} 3.77% (2026b); FRED; OECD
Price parameters	Pre-season sample	Full sample	Section 6.3 (κ , (Table 4) (Table 4) σ_χ)
Harvest parameters (σ_Q , $\rho_{Q\chi}$)	0.146, -0.43	0.244, -0.62	Tables 7 and 8

6.7 Computational implementation and use of AI tools

All estimations, closed-form valuations and simulations were carried out in R. Online Resource 1 contains the dataset, the R Markdown file that reproduces every table and figure in Sections 6 and 7 together with its compiled output, and the script used to extract the seasonal production series from USDA FAS (2025b). In preparing this study, the author used Claude (Anthropic), a large language model, to assist with drafting and editing of the text and codes. All derivations, computations, data and references were computed by the author, and takes full responsibility for the content of the article.

7 Results

7.1 Numerical validation

Before applying Theorems 2 and 3, I verify them numerically. The model is simulated under the sterling measure, in which futures prices are martingales, with 100,000 paths and 200 time steps. Cedi values are recovered through the change of numéraire (11). Because this check does not use Lemma 2 or Theorem 1, it tests the currency adjustments and the joint distribution independently. The test sets every correlation, all three interest rates, positive long-term

volatility, and a non-zero harvest risk adjustment to non-trivial values, so every term in the formulas is active. All closed-form values lie within Monte Carlo sampling error (Table 10).

Table 10. Closed-form values and Monte Carlo estimates under the sterling measure

Quantity	Closed form	Monte Carlo	Standard error	abs(z)
Share guarantee, equation (34)	2,662.8	2,658.0	11.7	0.40
Price floor, equation (35)	3,015.3	3,013.7	15.5	0.10
Over-commitment liability, equation (40)	857.3	860.8	2.5	1.40

7.2 The 2023/24 forward-sale over-commitment

I value, as of 2 October 2023, the exposure created by the 2023/24 forward book. The liability is measured in US dollars, which corresponds to Theorem 3 with the dollar as domestic currency. Scenario 1 uses only information available on the valuation date: price parameters estimated to September 2023 and harvest parameters from the six seasons before 2023/24. The remaining scenarios change one input at a time. Scenario 2 corrects the forecast for the average shortfall of outturns before 2023/24; Scenario 3 uses harvest parameters that include 2023/24; Scenario 4 uses the price volatility realised after October 2023; and Scenario 5 combines Scenarios 3 and 4 with the full-sample correction of the forecast. Table 11 reports the value of the exposure and the probability of an outcome at least as costly as the one realised under each scenario.

Table 11. Value of the 2023/24 over-commitment and tail probabilities

Scenario	σ_Q	$\rho_{Q\chi}$	σ_χ	Mt (tonnes)	Value (US\$ million)	P(shortfall)	P(cost $\geq$ realised)
1. Ex-ante	0.146	-0.43	0.273	800,000	56.1	46.9%	< 0.001%
2. Forecast	0.146	-0.43	0.273	775,384	74.6	57.2%	< 0.001% corrected for past shortfalls
3. Harvest	0.244	-0.62	0.273	800,000	110.3	50.7%	0.002% parameters including 2023/24
4.	0.146	-0.43	0.677	800,000	77.9	46.8%	0.059% Post- October 2023 price volatility
5. Combined stress	0.244	-0.62	0.677	700,507	269.2	75.3%	1.13%

Notes: Values are forward values at 31 May 2024. The realised economic cost is $354,527 \text{ t} \times (\text{US}\$9,106 - \text{US}\$2,661) \approx \text{US}\$2,285$ million, the market value forgone on the undelivered tonnes at the end of the main crop. COCOBOD (2026) and van Huellen et al. (2026) report that

333,767 tonnes were rolled over; on that volume the same calculation gives about US\$2,151 million. Probabilities are computed from the joint normal law of Theorem 1.

The results show, first, that the book committed almost the whole forecast crop: contracts covered 98.3% of the projected output, so that even with pre-season parameters a shortfall was almost as likely as not (46.9%). The exposure was therefore large and measurable before the season began. On information available at the time, the forward value of the over-commitment was US\$56.1 million. Allowing for the average shortfall of past outturns raises it to US\$74.6 million, and across the harvest parameters in Table 12 it ranges from US\$27 million to US\$121 million. A substantial part of this value is a correlation premium: about US\$13.7 million of the ex-ante value arises because shortfalls coincide with high prices, and the value rises monotonically as the correlation becomes more negative (Fig. 3), as Corollary 3 predicts.

Second, the cost that materialised was a joint price–quantity tail event. The London price tripled during the season (van Huellen et al., 2026), and the market value forgone on the 354,527 undelivered tonnes at the end of the main crop was about US\$2.3 billion (about US\$2.2 billion on the 333,767 tonnes reported as rolled over). Under ex-ante parameters, an outcome of this size had a probability below 0.001%. Using both the harvest uncertainty and the price volatility revealed by the crisis raises this probability to about 1.1%, and the value of the exposure to US\$269 million, almost five times the ex-ante value. Neither stress alone produces a comparable effect. The episode required a harvest failure and a price spike together, which is the combination described by Corollary 3.

Valued in cedis, with the cedi as domestic currency and the cedi parameters estimated from January 2014 to September 2023, the same exposure has a forward value of GH¢730 million. The corresponding realised cost is GH¢32.3 billion at the Bank of Ghana rate of 31 May 2024.

Table 12. Value of the 2023/24 over-commitment by harvest parameters (US\$ million)

σ_Q	$\rho_{Q\chi} = 0$	$\rho_{Q\chi} = -0.25$	$\rho_{Q\chi} = -0.50$	$\rho_{Q\chi} = -0.75$
0.10	26.7	31.9	37.3	42.9
0.15	43.9	52.0	60.5	69.2
0.20	61.2	72.2	83.5	95.3
0.25	78.6	92.3	106.4	121.0

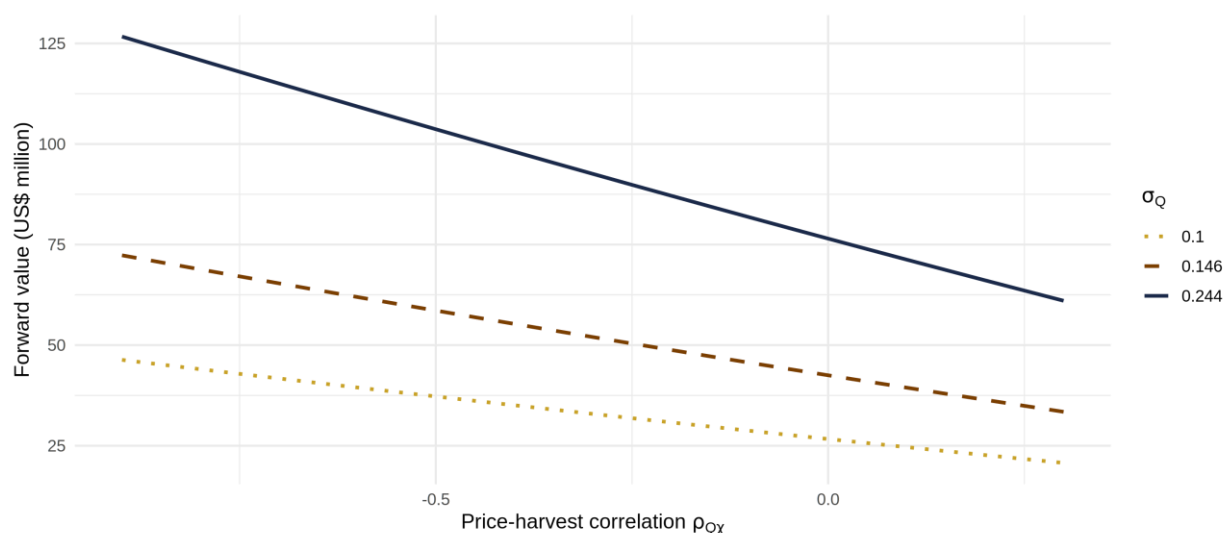


Fig. 3 Value of the 2023/24 over-commitment as a function of the price–harvest correlation, for three levels of harvest volatility (Corollary 3) (dotted line: $\sigma_Q = 0.10$; dashed line: $\sigma_Q = 0.146$, the pre-2023/24 estimate; solid line: $\sigma_Q = 0.244$, the full-sample estimate)

7.3 The 2026/27 producer-price guarantee

The announced GFOB of GH¢59,567 per tonne is 13% below the spot cedi value of the London price plus the Living Income Differential on the valuation date (GH¢68,784 per tonne). This is consistent with part of the crop having been sold forward earlier at lower prices. The producer price lies 1.7% above 70% of the announced GFOB, so the share rule binds only if the realised GFOB rises by more than 1.7%. Table 13 reports the values of the two options embedded in (2).

Table 13. Values of the 2026/27 commitments (base case)

Quantity	Forward value (GH¢ per t)	Present value (GH¢ per t)	% of producer price	Present value per 100,000 t (GH¢ million)
Share guarantee, equation (34)	2,365	2,259	5.6	226
Price floor,	3,620	3,457	8.5	346 equation (35)
Total farm-gate commitment, equation (36)	44,765	42,748	105.6	4,275

Note: Present values are discounted at the 182-day Treasury bill rate over $\tau = 0.68$ years.

Both options are economically significant. Guaranteeing farmers 70% of a GFOB that may rise is worth GH¢2,365 per tonne of forecast crop at settlement, 5.6% of the producer price. Holding the producer price fixed if the GFOB falls is worth GH¢3,620 per tonne, 8.5%. In present-value terms they amount to GH¢226 million and GH¢346 million per 100,000 tonnes.

Table 14 shows how these values depend on the assumptions. The price at which the unsold part of the crop is sold matters most: if the unsold half is sold at current market prices, the share guarantee rises to GH¢4,379 per tonne, and the 70% rule would then require an upward revision

of the producer price during the season. Cedi volatility is the next most important input, since a 50% higher volatility raises the share guarantee to GH¢3,601. The harvest parameters matter less. Using the estimated harvest–currency correlations of Table 8 changes the share guarantee to GH¢2,098 and the floor to GH¢3,945, and, as Theorem 2 predicts, a more negative correlation with the harvest lowers the share guarantee and raises the floor.

Table 14. Sensitivity of the 2026/27 values (forward, GH¢ per tonne)

Scenario	Share guarantee	Price floor
Base case	2,365	3,620
Harvest independent of price	2,525	3,445 ($\rho_{QX} = 0$)
Ex-ante harvest parameters	2,458	3,517 ($\sigma_Q = 0.146$, $\rho_{QX} = -0.43$)
Estimated harvest–currency correlations	2,098	3,945 (Table 8)
Cedi volatility since October 2023	2,163	3,418 ($\sigma_Y = 0.178$)
Cedi volatility since 2007	1,950	3,205 ($\sigma_Y = 0.160$)
Cedi volatility 50% higher	3,601	4,856 ($\sigma_Y = 0.292$)
25% of crop priced in advance	2,579	4,023
75% of crop priced in advance	2,283	3,291
Unsold half sold at current market prices	4,379	1,858

Fig. 4 illustrates Proposition 2 in the 2026/27 guarantee setting, isolating cocoa price risk. For both options, the GBM valuation with the same instantaneous volatility exceeds the mean-reverting valuation at every maturity. The overstatement is 9.7% for the share guarantee and 8.0% for the floor at six months, and 19.0% and 16.3% at one year.

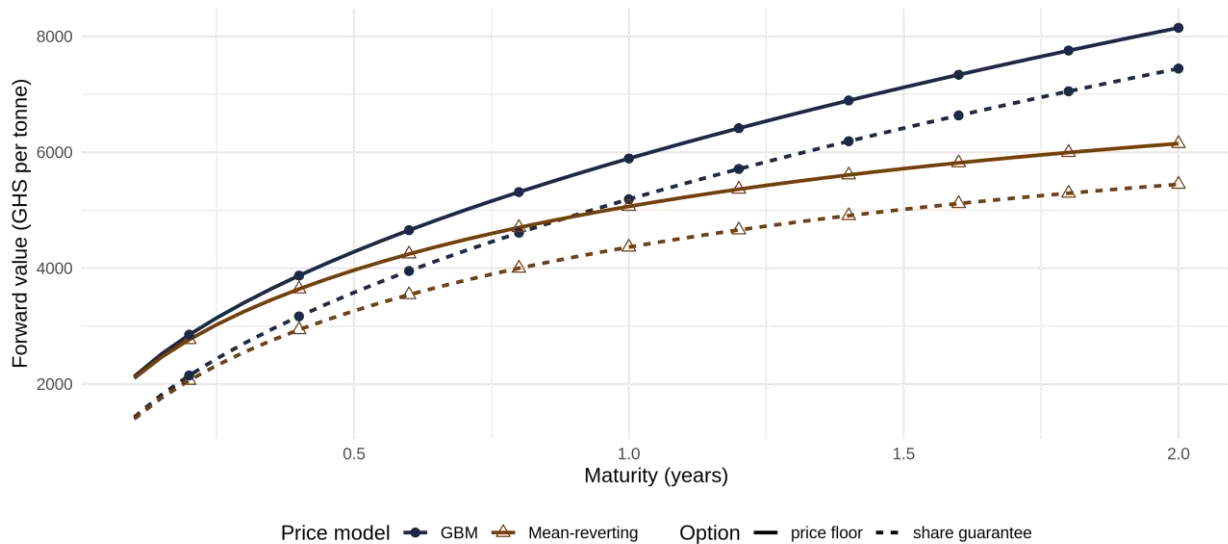


Fig. 4 Values of the share guarantee and the price floor under geometric mean reversion and under GBM with the same instantaneous volatility, as functions of maturity (Proposition 2) (solid lines: price floor; dashed lines: share guarantee; circles: GBM; triangles: mean reversion)

7.4 Sources of risk

The framework allows the value of the guarantee to be attributed to its sources. Table 15 decomposes the variance of the log realised GFOB, s_G^2 in (30), and reports the values obtained when each risk is removed in turn.

Table 15. Sources of risk in the 2026/27 guarantee

Panel A. Decomposition of the variance of the log realised GFOB, s_G^2

Component	Share of variance
Cocoa price	16.4%
Sterling-dollar rate	2.0%
Cedi-dollar rate	82.4%
Covariance terms	-0.8%
Total	100.0%

Panel B. Forward values when individual risks are removed (GH¢ per tonne)

Case	Share guarantee	Price floor
All risks (base case)	2,365	3,620
Without cocoa price risk	2,400	3,126
Without sterling risk	2,337	3,579
Without cedi risk	710	1,965
Without harvest risk	2,525	3,445
Cocoa price risk only	802	1,708

The cedi exchange rate accounts for more than four-fifths of the variance of the realised GFOB. Removing cedi risk reduces the share guarantee by 70%, from GH¢2,365 to GH¢710 per tonne.

If only cocoa price risk were considered, as in single-currency valuations, the share guarantee would be valued at GH¢802 per tonne, about one third of its value in the full model. This is the practical consequence of combining the three risks. For a producing country whose guarantee is denominated in a volatile domestic currency, a valuation based on the world price alone would substantially understate the commitment.

7.5 Reserve for the price floor

The price floor is the part of the commitment that Cocobod must finance from its own share of the FOB value, or from reserves, when the world price or the exchange rate falls. The 2024/25 currency episode illustrates this (COCOBOD, 2025). Corollary 4 gives the distribution of the floor payment, and Table 16 reports the reserve needed to cover it with 95% and 99% probability, per 100,000 tonnes of forecast crop.

Table 16. Reserve required to fund the price floor (GH¢ million per 100,000 tonnes of forecast crop, at settlement)

Scenario	Probability of a payment	Expected payment	95% reserve	99% reserve
Base case	58.4%	362	1,310	1,842
Cedi volatility 50%	58.4%	486	1,716	2,358 higher
Post-October 2023 cocoa price volatility	61.0%	478	1,671	2,336

Note: Quantiles are computed from equation (46) by numerical integration and agree with a simulation from the joint law of Theorem 1 to within 1%.

A floor payment is more likely than not (58%), but its expected size, GH¢362 million per 100,000 tonnes, is much smaller than the reserve needed for high protection. Covering the floor in 99 years out of 100 requires about GH¢1.8 billion per 100,000 tonnes, five times the expected payment, and about GH¢2.3–2.4 billion under the stress scenarios.

7.6 Robustness

Table 17 reports two further checks. Consider first the harvest risk adjustment. By Lemma 3, $\psi = \sigma_Q \rho_Q^\top R^{-1} \boldsymbol{\theta}$ depends on market prices of risk, which cannot be estimated reliably from the available data. With the full-sample estimates of σ_Q and ρ_{QX} , market prices of risk up to 0.5 in absolute value imply $|\psi| \leq 0.075$ (about 0.03 with the pre-season estimates used for 2023/24). Over the range $-0.08 \leq \psi \leq 0.08$ considered in Table 17, the value of the 2023/24 over-commitment varies between US\$31 million and US\$89 million, while the 2026/27 values change by less than 6%. Second, fixing the long-term volatility at 0.05, 0.10 or 0.15 and re-estimating the other parameters changes the 2026/27 values by less than 2%.

Table 17. Robustness of the valuations

Specification	2023/24 over-commitment (US\$ million)	2026/27 share guarantee (GH¢ per t)	2026/27 price floor (GH¢ per t)
Base case ($\psi = 0$, $\sigma_\xi = 0$)	56.1	2,365	3,620
$\psi = -0.08$	31.5	2,498	3,823
$\psi = -0.04$	42.7	2,431	3,720
$\psi = 0.04$	71.6	2,302	3,523
$\psi = 0.08$	89.1	2,240	3,428
Two-factor,	—	2,370	3,621 $\sigma_\xi = 0.05$
Two-factor,	—	2,381	3,618 $\sigma_\xi = 0.10$
Two-factor, $\sigma_\xi = 0.15$	—	2,400	3,610

8 Discussion

A marketing board such as Cocobod makes two kinds of promises before it knows how large the harvest will be or what the crop will eventually sell for. It promises farmers a price, now backed by a legal guarantee of at least 70% of the export value, and it promises buyers a quantity of cocoa at prices agreed in advance. Each promise behaves like an insurance policy that the board has written. The price guarantee protects farmers against falling prices and lets them share in rising prices, and the forward contracts protect the board against falling prices but oblige it to deliver even when the harvest disappoints. In finance, the value of such a promise today is the average of its possible future costs, weighted by how likely each outcome is and discounted to the present. This is the sense in which the paper speaks of the “value” of the guarantee and of the forward book: it is the fair premium that an insurer would charge to take over the commitment, and therefore the amount that should be set aside for it.

To compute these values, the study describes the three things that move together over a season. The world cocoa price is modelled as mean-reverting: it can move far in the short run, but, like a stretched rubber band, it tends to pull back toward a longer-run level. The sterling–dollar and dollar–cedi exchange rates convert the London price into cedis. Cocobod’s crop forecast describes the size of the harvest, is revised as the season unfolds, and tends to fall when prices rise. From this description, the paper derives exact formulas for the value of the guarantee, the value of the exposure created by selling more cocoa forward than is eventually harvested, and the probability distribution of the payments the guarantee may require. It then estimates the ingredients from three decades of public data and applies the formulas to the 2023/24 and 2026/27 seasons.

The estimated price dynamics agree with the long-standing evidence that commodity prices mean-revert (Bessembinder et al., 1995; Schwartz, 1997). For cocoa, deviations from the long-run level halve in about 1.8 years. The statistical evidence against a pure random walk is modest, however, which is typical of mean reversion that is slow relative to the sample length.

With a single price series, the long-term factor of Schwartz and Smith (2000) cannot be identified separately, and the data select the one-factor process by Atiase et al. (2021), with a trending equilibrium level. This result reflects the information contained in one price series rather than a rejection of the two-factor structure. The robustness checks confirm that allowing a moving long-run level changes the guarantee values by less than 2%.

The first theoretical result with direct policy content concerns the literature on valuing agricultural guarantees. Marcus and Modest (1986), Kang and Brorsen (1995) and Tirupattur et al. (1997) value price supports under geometric Brownian motion, in which every price shock is permanent. Proposition 2 shows that when prices mean-revert, this assumption overstates the value of a guarantee, because extreme prices at longer horizons are less likely when prices tend to drift back. In the 2026/27 guarantee setting, the overstatement is about 19% at a one-year horizon. In everyday terms, a model that ignores the rubber-band behaviour of cocoa prices would make the guarantee look more expensive than it is and could lead to larger-than-necessary reserves.

The second theoretical result revisits a classic finding of the hedging literature. McKinnon (1967) and Rolfo (1980) show that when a producer's harvest and the world price move in opposite directions, the producer's revenue is naturally stabilised, and the producer should hedge less than its expected output. Corollary 3 shows the other side of the same mechanism. Once a board has sold forward more than it eventually harvests, the negative correlation works against it, because it must cover the tonnes it cannot deliver exactly when prices are high. The 2023/24 season illustrates this. Of the US\$56 million ex-ante value of the over-commitment, about US\$14 million is attributable to this correlation, and the valuation is consistent with van Huellen et al. (2026), who observed that the forward-selling system protected the board in falling markets but could not benefit from a rising one.

Cocobod's own crop forecasts form part of the same story. Outturns fell short of target in six of the eight seasons I could document, and before 2023/24 the typical forecast error of about 11% agrees with Cocobod's statement that forecasts usually vary by 5% to 15% (COCOBOD, 2026). When the forward book is sized against a forecast that tends to be too high, the exposure rises: correcting the 2023/24 forecast for the average past shortfall raises the ex-ante value of the over-commitment from US\$56 million to US\$75 million. The cost that eventually materialised, about US\$2.3 billion in market value forgone, was far larger than either figure. Under the parameters estimated from calm years, it had a probability below 0.001%, and it became plausible, at about 1%, only when a harvest failure and a price surge were considered together. This mirrors Richards and Manfredo (2003), who found that insurance priced on the assumption of smooth, continuous risk can seriously misjudge exposure to rare but severe events.

The dominance of the exchange rate is perhaps the most important new finding. Van Huellen et al. (2026) document that the CMC is exposed to both cedi-dollar and dollar-sterling risk, and Turvey and Yin (2002) show how the currency component of a cross-currency option can be separated from the commodity price. My decomposition quantifies this exposure for Ghana: the cedi-dollar rate accounts for about 82% of the uncertainty in the cedi value of the crop, the cocoa price for about 16% and sterling for about 2%. A valuation based on the world price alone, as in single-currency models, would put the 70% guarantee at about one third of its full

value. The 2024/25 episode, in which Cocobod absorbed the difference between its assumed and actual exchange rates (COCOBOD, 2025), shows that this exposure is not hypothetical.

Two further findings relate to the treatment of the harvest and to reserves. Tirupattur et al. (1997) observe that claims on non-traded yield cannot be valued by no-arbitrage arguments alone. The minimal martingale measure resolves this by producing an explicit harvest risk adjustment ψ , and my robustness analysis shows that plausible values of ψ change the 2026/27 guarantee values by less than 6%. The 2023/24 over-commitment is more sensitive to ψ , because its value is driven by the harvest shortfall itself. Finally, van Huellen et al. (2026) recommend a liquidity reserve fund, and Staritz et al. (2023) note that stabilisation schemes do not protect against variation between seasons. Corollary 4 turns this recommendation into numbers. Per 100,000 tonnes of forecast crop, the price floor has an expected cost of about GH¢362 million, and a reserve of about GH¢1.3 billion would cover it with 95% probability, or GH¢1.8 billion with 99% probability.

8.1 Practical implications

For policy-makers and managers, the results carry four practical messages. First, the guarantees given to farmers have a real and measurable cost. For 2026/27, protecting farmers against a fall in the export value is worth about 8.5% of the producer price, and the promise to share in a rise is worth about 5.6%. The protection against a fall is the one that strains Cocobod's finances, because it must be paid from Cocobod's own resources precisely when export revenue is low; its present value is about GH¢346 million for every 100,000 tonnes of cocoa. Because these costs appear in the accounts only when incurred, measuring them at the start of the season lets them be planned for rather than discovered. Second, the exchange rate deserves as much attention as the world price. Public discussion of cocoa revenue concentrates on the London price, yet most of the uncertainty in what the crop is worth in cedis comes from the cedi. Aligning the timing and currency of sales with the timing and currency of payments to farmers is therefore at least as important as hedging the cocoa price.

The third concerns forward selling, which is safest when the crop is known. Committing almost the entire forecast crop, based on forecasts that have tended to be optimistic, exposes the board to exactly the combination of events that occurred in 2023/24: a small harvest that itself pushes world prices up. Sizing forward commitments against a cautious, bias-corrected forecast, and testing them against combined harvest and price shocks, would reduce this exposure. The fourth message is that a reserve fund can be sized transparently. The figures in Table 16 scale with the size of the crop. For a crop of 600,000 tonnes, close to recent outturns, the reserve needed to cover the price floor with 99% probability would be about six times the figure per 100,000 tonnes. Such a fund would be drawn upon often, since a floor payment is more likely than not, but it would rarely be exhausted.

8.2 Limitations

These results rest on assumptions that should be kept in mind. The formulas assume that the logarithms of prices, exchange rates and the harvest forecast are normally distributed, with constant volatilities and correlations. This assumption makes exact formulas possible, but, as the 2023/24 season shows, it understates the probability of sudden regime changes, which the analysis therefore handles through stress scenarios.

Several empirical choices also shape the results. The price model is estimated from a single price series, which cannot separate short-term and long-term factors as a full futures curve could. The harvest parameters rest on only six to eight seasons of documented targets and outturns, so their estimates are imprecise, and every result is therefore reported with a sensitivity analysis. The base case sets market prices of risk to zero, so that the probabilities used for the reserve calculations coincide with real-world probabilities. The Living Income Differential is represented as a proportional rather than a fixed premium, the announced gross FOB value is used as the market forward for 2026/27, and the share of the crop already sold for that season is assumed rather than observed. None of these choices affects the closed-form results themselves, which hold for any parameter values. Each can be refined as better data, such as a full cocoa futures curve or Cocobod's internal forward-sale records, become available.

9 Conclusion

This paper values the commitments of a cocoa marketing board, namely the producer-price guarantee and the forward-sale book, in a framework that combines a geometric mean-reverting commodity price, two exchange rates and a harvest that is not traded but is correlated with prices. I derive closed-form values for the share guarantee and the price floor created by Ghana's 2026 cocoa legislation, for the liability that arises when forward sales exceed the harvest, and for the distribution of the floor payment. The formulas contain several established results as special cases, and an independent Monte Carlo check confirms them. Two theoretical results carry direct policy content: valuations based on constant volatility overstate guarantee values when prices mean-revert, and a negative price–harvest correlation strictly increases the value of a forward book that exceeds the harvest. The empirical application shows that cocoa prices over 1994–2026 are well described by the one-factor geometric mean-reverting model. Before the 2023/24 season, Cocobod's over-commitment was worth about US\$56 million, and the much higher cost that materialised required a joint harvest and price shock. The 2026/27 guarantee is worth about 5.6% and the price floor about 8.5% of the producer price, the cedi exchange rate is the dominant source of risk, and covering the price floor with 99% confidence requires a reserve of about GH¢1.8 billion per 100,000 tonnes of forecast crop. These findings provide a quantitative basis for the risk-management strategy and reserve fund recommended for Ghana's cocoa sector. Natural extensions include jumps or regime switching in prices and harvests, estimation from a full futures curve, and the joint optimisation of forward sales and reserves.

Statements and Declarations

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Competing interests The author has no relevant financial or non-financial interests to disclose.

Data availability All data analysed in this study are publicly available from the sources listed in Table 3. The final dataset is provided as Online Resource 1 and has been deposited in a public repository; the repository link is withheld for double-anonymous review and is provided on the title page.

Code availability The R Markdown file that reproduces all results and the data-extraction script are provided as Online Resource 1.

Ethics approval and consent Not applicable. The study does not involve human participants or animals.

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Appendix A Proofs of the auxiliary lemmas

A.1 Proof of Lemma 1

Equation (9) is the futures price of Schwartz and Smith (2000). It follows from $F(s, T) = \mathbb{E}^{\mathbb{Q}^{uk}}[S_T | \mathcal{F}_s]$ (Cox et al., 1981), because $\ln S_T$ is conditionally normal and the moment-generating function of a normal variable gives the result.

For (10), write $\ln F(s, T) = e^{-\kappa(T-s)}\chi_s + \xi_s + g(s)$ with $g(s) = \mathcal{A}(T - s)$. Itô's product rule gives

$$d\ln F = \kappa e^{-\kappa(T-s)}\chi_s ds + e^{-\kappa(T-s)}d\chi_s + d\xi_s + g'(s) ds.$$

After substituting the $\mathbb{Q}^{uk}$ -dynamics, the diffusion part is $\sigma_\chi e^{-\kappa(T-s)}d\hat{B}_s^\chi + \sigma_\xi d\hat{B}_s^\xi$. Because $F(\cdot, T)$ is a $\mathbb{Q}^{uk}$ -martingale, being a conditional expectation of S_T , the drift of dF/F must vanish, which gives (10). ■

A.2 Proof of Lemma 2

The cedi values of the dollar and sterling money-market accounts are $Y_s e^{r_{us}s}$ and $X_s Y_s e^{r_{uk}s}$. Under the cedi measure, both must earn r_{gh} in expectation (Harrison & Pliska, 1981). This fixes the drift of Y at $r_{gh} - r_{us}$ and the drift of XY at $r_{gh} - r_{uk}$.

By Itô's product rule, the drift of $d(XY)/(XY)$ is the drift of X plus the drift of Y plus $\rho_{XY}\sigma_X\sigma_Y$. Hence the drift of X is

$$(r_{gh} - r_{uk}) - (r_{gh} - r_{us}) - \rho_{XY}\sigma_X\sigma_Y = r_{us} - r_{uk} - \rho_{XY}\sigma_X\sigma_Y,$$

which is (12).

The density process (11) is a positive $\mathbb{Q}$ -martingale with $dD_s/D_s = \sigma_X d\tilde{B}_s^X + \sigma_Y d\tilde{B}_s^Y$. By Girsanov's theorem, for each i the process

$$\hat{B}_s^i = \tilde{B}_s^i - (\rho_{iX}\sigma_X + \rho_{iY}\sigma_Y)s$$

is a $\mathbb{Q}^{uk}$ -Brownian motion. Substituting $d\hat{B}_s^\chi$ and $d\hat{B}_s^\xi$ into (10) gives drift terms $-\sigma_\chi e^{-\kappa(T-s)}(\rho_{\chi X}\sigma_X + \rho_{\chi Y}\sigma_Y) - \sigma_\xi(\rho_{\xi X}\sigma_X + \rho_{\xi Y}\sigma_Y)$. Grouping these terms gives $-[c_{FX}(s) + c_{FY}(s)]$, which is (13). ■

A.3 Proof of Lemma 4

Step 1 (conditioning). For a bivariate normal vector, $U | V \sim N\left(\mu_U + \frac{c}{s_V^2}(V - \mu_V), s_U^2 - \frac{c^2}{s_V^2}\right)$.

The moment-generating function of a normal variable gives

$$\mathbb{E}[e^U | V] = \exp \left\{ \mu_U + \frac{1}{2} s_U^2 + \frac{c}{s_V^2} (V - \mu_V) - \frac{c^2}{2s_V^2} \right\}.$$

Step 2 (completing the square). For any integrable g , by the tower property,

$$\mathbb{E}[e^U g(V)] = e^{\mu_U + \frac{1}{2} s_U^2} \int_{-\infty}^{\infty} g(v) \frac{1}{s_V \sqrt{2\pi}} \exp \left\{ -\frac{(v - \mu_V)^2}{2s_V^2} + \frac{c(v - \mu_V)}{s_V^2} - \frac{c^2}{2s_V^2} \right\} dv.$$

The exponent equals $-\frac{(v - \mu_V - c)^2}{2s_V^2}$, so

$$\mathbb{E}[e^U g(V)] = e^{\mu_U + \frac{1}{2} s_U^2} \mathbb{E}[g(V')], \quad V' \sim N(\mu_V + c, s_V^2).$$

Step 3 (the lognormal integral). Take $g(v) = \max\{ae^v - V_0, 0\}$. The payoff is positive for $v > \ln(V_0/a)$, so, splitting the integral as in the proof of Theorem 2 in Atiase et al. (2021),

$$\mathbb{E}[g(V')] = \int_{\ln(V_0/a)}^{\infty} a e^u \varphi(u) du - V_0 \int_{\ln(V_0/a)}^{\infty} \varphi(u) du = g_1 - g_2,$$

where φ is the $N(\mu_V + c, s_V^2)$ density and u is the variable of integration. In g_1 , completing the square gives

$$u - \frac{(u - \mu_V - c)^2}{2s_V^2} = -\frac{[u - (\mu_V + c + s_V^2)]^2}{2s_V^2} + \mu_V + c + \frac{1}{2} s_V^2.$$

Substituting $x = [u - (\mu_V + c + s_V^2)]/s_V$ and using the symmetry of the normal density gives $g_1 = F^* N[d_1]$. In g_2 , the substitution $y = (u - \mu_V - c)/s_V$ gives $g_2 = V_0 N[d_2]$. This proves (18).

For (19), note that $\max\{V_0 - ae^V, 0\} - \max\{ae^V - V_0, 0\} = V_0 - ae^V$ and that, by Step 2, $\mathbb{E}[e^U ae^V] = e^{\mu_U + \frac{1}{2} s_U^2} F^*$. Then $1 - N[x] = N[-x]$ gives the result. ■

Appendix B Data construction

The ICCO series was assembled from a public copy of the historical daily file (October 1994 to August 2025) and the ICCO's recent file (October 2024 to September 2026). The two agree exactly on the 219 trading days on which they overlap and are spliced at 1 October 2024. Weekly observations are the last trading day of each week, and crop seasons run from October to September.

The Bank of Ghana exchange-rate series report old cedis before the redenomination of July 2007 (GH¢1 = 10,000 old cedis) and contain some zero entries and isolated keying errors. I therefore divided values before 3 July 2007 by 10,000, dropped the zero entries, and removed observations deviating by more than 10% from the centred seven-day median. This last step removes 8 of the 7,630 dollar observations and 2 of the sterling observations.

All interest rates are expressed with continuous compounding. An annual interest rate i on the 182-day Ghana Treasury bill is converted as $r = (365/182)\ln(1 + i \cdot 182/364)$. A discount

rate d on the US Treasury bill is converted as $r = -\ln(1 - d \cdot 91/360)/(91/365)$. The quarterly average UK interbank rate i is converted as $r = \ln(1 + i)$.

Seasonal production for 2009/10 to 2024/25 was taken from Chart 1 of USDA FAS (2025b), which reports Cocobod data. The chart is stored as vector graphics, and the values were recovered by extracting the plotted line and calibrating it against the chart's gridlines. For 2014/15 to 2022/23, the recovered values agree with the purchases declared in Cocobod's annual reports and Turnaround Strategy Paper to within 0.04%. For 2009/10 and 2010/11 they differ by -3.0% and $+2.0\%$ from the figures quoted in USDA FAS (2012), presumably because of later revisions. The extraction script is included in the supplementary material.