

Supplemental Material: Reconfigurable high-power Kerr optical parametric oscillators with sub-hertz linewidths

1. Theoretical modeling

The dynamical evolution of the optical parametric oscillators (OPOs) in a microring resonator is governed by three nonlinearly interacting fields—the pump A_P , signal A_B , and idler A_C —whose intracavity dynamics are described by the coupled-mode equations:

$$t_R \frac{dA_P}{dt} = -\frac{\alpha}{2}A_P - i\Delta_P A_P + i\gamma L_R(|A_P|^2 + 2|A_B|^2 + 2|A_C|^2)A_P + i2\gamma L_R A_P^* A_B A_C + \sqrt{\theta} E_{in}, \quad (S1)$$

$$t_R \frac{dA_B}{dt} = -\frac{\alpha}{2}A_B - i\Delta_B A_B + i\gamma L_R(2|A_P|^2 + |A_B|^2 + 2|A_C|^2)A_B + i\gamma L_R A_P^2 A_C^*, \quad (S2)$$

$$t_R \frac{dA_C}{dt} = -\frac{\alpha}{2}A_C - i\Delta_C A_C + i\gamma L_R(2|A_P|^2 + 2|A_B|^2 + |A_C|^2)A_C + i\gamma L_R A_P^2 A_B^*, \quad (S3)$$

Here, α is the total cavity loss rate, and Δ_P , Δ_B , and Δ_C denote the detunings of the pump, signal, and idler, respectively. L_R is the cavity length, and $t_R = 1/\text{FSR}$ is the round-trip time, where FSR is the free spectral range. γ is the Kerr nonlinear coefficient, θ is the coupling coefficient, and E_{in} is the input field.

To capture the wavelength reconfiguration dynamics of the OPO while maintaining analytical clarity, we restrict the model to three-wave interactions among the pump, signal, and idler, neglecting Raman scattering and higher-order dispersion. The system is formulated within the Lugiato–Lefever equation (LLE) framework, augmented by a feedback term E_F to describe coherent feedback through mode-selective sideband self-injection locking (SIL):

$$t_R \frac{\partial A_\mu}{\partial t} = \left(-\frac{\alpha}{2} - i\delta_0 - it_R D_{int}(\mu) \right) A_\mu + i\gamma L \sum_{m,n} A_m A_n A_{m+n-\mu}^* + \sqrt{\theta} E_{in} \delta_{\mu,0} + \sqrt{\theta} E_F \delta_{\mu,\mu_s}, \quad (S4)$$

Here, A_μ denotes the field amplitude of mode μ , and $D_{int}(\mu)$ is the integrated dispersion. $\delta_{\mu,0}$ and δ_{μ,μ_s} are Kronecker delta functions selecting the pump and injected sideband modes, respectively. To reduce model complexity, the feedback field is approximated as a delayed replica of a selected sideband, $E_F = \sqrt{\beta} A_{n-l} e^{-i\phi_F}$, where β is the feedback injection ratio, ϕ_F is the feedback phase and l represents the feedback-loop delay.

In the simulations of wavelength reconfiguration and noise dynamics, the system

parameters are set to $P_{in} = 0.03 \text{ W}$, $Q_i = 2 \times 10^6$, $Q_{ex} = 1 \times 10^6$, $\gamma = 0.98 \text{ W}^{-1} \text{ m}^{-1}$ and $\beta_2 = -60 \text{ ps}^2/\text{km}$. A relatively large β and an extended feedback loop provide sufficient feedback gain and increase the effective photon lifetime. These choices support wavelength reconfiguration and noise suppression through sideband SIL.

2. Sideband-selective self-injection locking suppresses cluster-comb formation

At high pump power, the OPO exhibits not only cascaded nonlinear processes but also cluster comb formation, which prevents further power growth of the primary sidebands (see Fig. S1(a)). Cluster-comb suppression and higher-order sideband enhancement arise from the coherent feedback induced increase in the effective photon lifetime of the injected mode, which lowers its effective loss relative to the other cavity modes within the parametric gain bandwidth. As a result, the injected mode dominates the intracavity energy transfer, promotes cascaded four-wave mixing (FWM) with the pump, and suppresses energy transfer through nondegenerate FWM to adjacent modes, thereby inhibiting cluster-comb formation and boosting higher-order sideband power (see Fig. S1(b)). We further simulate the cluster-comb suppression mechanism. Once the cluster-comb emerges, injection locking is applied to a mode adjacent to the primary sideband, triggering pronounced mode competition and reconstruction of the OPO state. The cluster-comb is subsequently suppressed, leaving the selected signal-idler pair as the dominant oscillation state (see Fig. S1(c)). These simulations illustrate nonlinear mode selection and energy transfer under active feedback control.

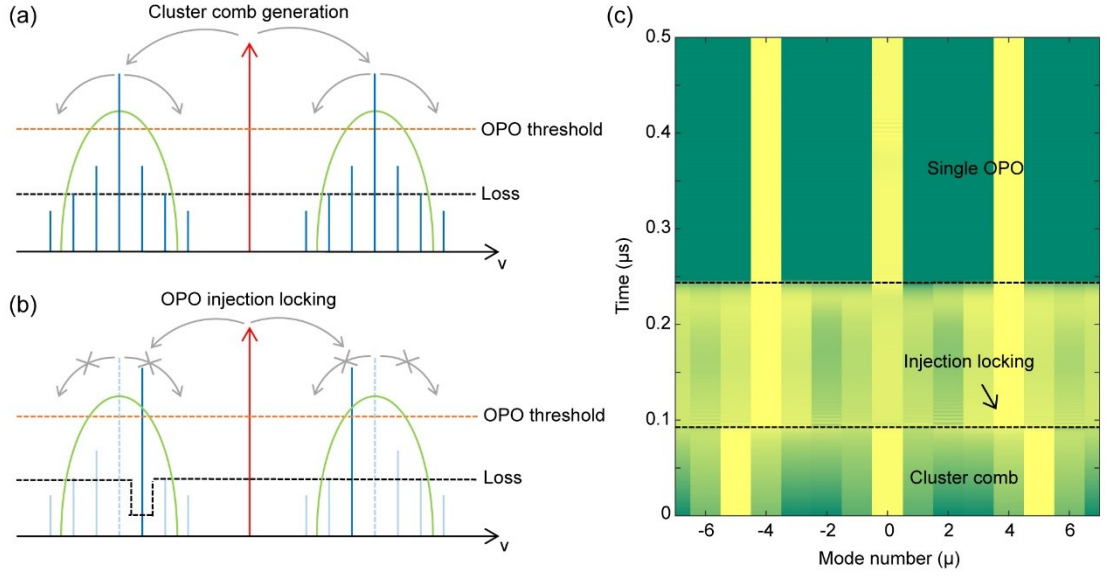


Fig. S1. Cluster-comb formation and suppression by sideband SIL. (a) Schematic of cluster-comb generation in a free-running OPO at high pump powers. (b) Schematic of cluster-comb suppression by sideband SIL. (c) Simulated cluster-comb suppression via injection locking of a mode adjacent to the primary sideband.

3. Characterization of the microresonator

We characterize a silicon nitride (Si_3N_4) microring resonator designed for injection-locked OPO operation. The microring has a radius of $R = 115 \text{ } \mu\text{m}$, with a waveguide cross-section of $W \times H = 1.8 \text{ } \mu\text{m} \times 0.72 \text{ } \mu\text{m}$. The device adopts a pulley waveguide coupling scheme, with a 700 nm coupling gap between the microring and the pulley waveguide. By fitting the transmission spectrum of the fundamental TE_{00} mode, we extract an intrinsic quality factor of the pump mode as $Q_i \approx 1.2 \times 10^6$ (see Fig. S2).

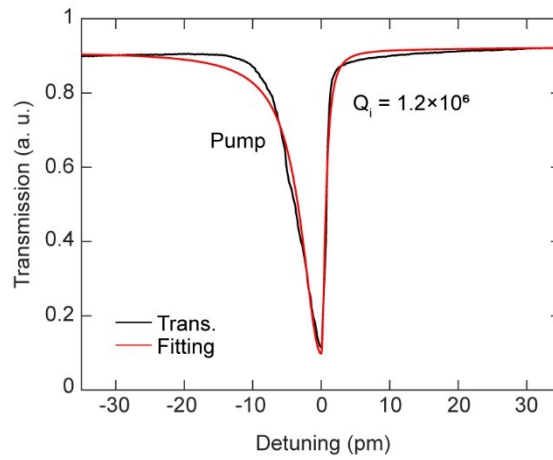


Fig. S2. Measured transmission spectrum and fit of the pump resonance, yielding an intrinsic quality factor $Q_i \approx 1.2 \times 10^6$.

4. Experimental setup

The experimental configuration for OPO injection locking and the associated measurement scheme are presented (see Fig. S3). A commercial distributed-feedback (DFB) laser serves as the pump laser. After amplification by an erbium-doped fiber amplifier (EDFA), the pump passes through a polarization controller (PC1) and is coupled into one port of a 50:50 coupler. Light is coupled in and out of the Si_3N_4 chip via lensed fibers, with a total coupling loss of ~ 5.05 dB. The pump is aligned to the fundamental TE_{00} mode to excite the OPO. The OPO output is split by a 90:10 coupler. Approximately 90% of the power is sent to an optical spectrum analyzer (OSA, AQ6370D-01-L-0/FC/RFC) for real-time spectral monitoring, while the remaining 10% is routed into the feedback loop. Within the loop, an EDFA (small-signal gain ~ 45 dB) compensates for propagation losses, and a tunable bandpass filter (TBPF) is used to select the injection mode. A variable optical attenuator (VOA) provides precise control of β . For wavelength-tuning measurements under injection locking, a selected sideband is filtered by the TBPF, amplified, and polarization-conditioned via a second polarization controller (PC2), and re-injected into the microcavity through a 90:10 coupler. Approximately 90% of the optical power is coupled back into the cavity. By tuning the VOA, β can be systematically varied, enabling quantitative analysis of its impact on injection-locked wavelength tuning.

To analyze the noise-suppression mechanism without and with injection locking, the selected OPO signal is amplified, filtered, and characterized using a delayed self-heterodyne interferometer. The optical field is split into two paths by a 50:50 coupler. One arm is frequency-shifted by an acousto-optic modulator (AOM, 100 MHz), while the other is delayed by approximately 100 m of single-mode fiber. The resulting delayed self-heterodyne beat note, centered at 100 MHz, is recorded to extract the power spectral density (PSD) of the signal.

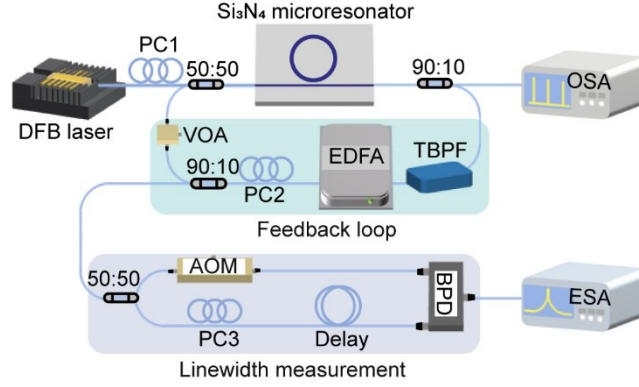


Fig. S3. Experimental setup and optical layout for mode-selective sideband SIL. PC, polarization controller; TBPF, tunable bandpass filter; AOM, acousto-optic modulator; VOA, variable optical attenuator; BPD, balanced photodetector; OSA, optical spectrum analyzer; ESA, electrical spectrum analyzer.

5. Fine frequency tuning via integrated electrodes

The frequency tunability of the injection-locked OPO is characterized by using integrated electrodes on the microcavity. During electrode-induced thermal tuning, the injection-locked sidebands maintain stable output power, with a tuning range of ~ 23.5 GHz (see Fig. S4). This range is primarily limited by the resistance of the integrated electrodes, which constrains the achievable thermo-optic tuning efficiency. To further evaluate the dynamic tuning capability, a triangular modulation signal is applied to the electrodes. Under injection-locked operation, a frequency sweep of 6 GHz is achieved (see Fig. S5). The observed tuning speed is currently limited by the sampling rate of the ESA. In principle, further optimization of the system configuration is expected to enable higher tuning rates, opening potential applications in LiDAR and precision sensing.

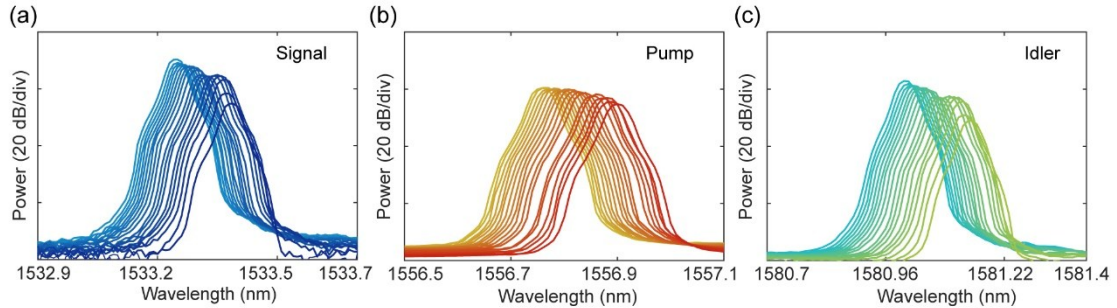


Fig. S4. Electrode-induced wavelength tuning of the injection-locked OPO. (a)-(c) The signal, pump, and idler exhibit stable and continuous tuning over a range of ~ 23.5 GHz under injection-locked operation.

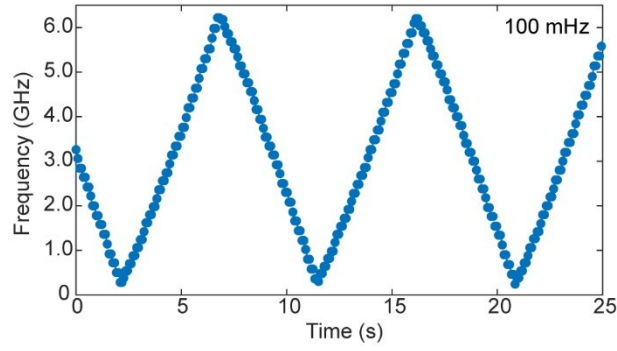


Fig. S5. Frequency sweeping of the injection-locked OPO. A 6 GHz frequency sweep is achieved within 5 s under injection-locked operation.

6. Feedback injection ratio dependence of wavelength tuning

We characterize wavelength reconfiguration by tuning the relative effective gain of competing parametric channels through the β . As β increases from 0 to 0.734 (Fig. S6), the signal wavelength is tuned from 1530.9 nm to 1532.45 nm. At $\beta = 0.471$, the effective gain of the injection mode remains below that of the initial mode and FWM interaction between the injection mode and the initial sideband occurs and gives rise to pronounced mode competition. At $\beta = 0.734$, the injection mode exceeds the initial mode in gain, suppressing it and forming a new injection-locked sideband.

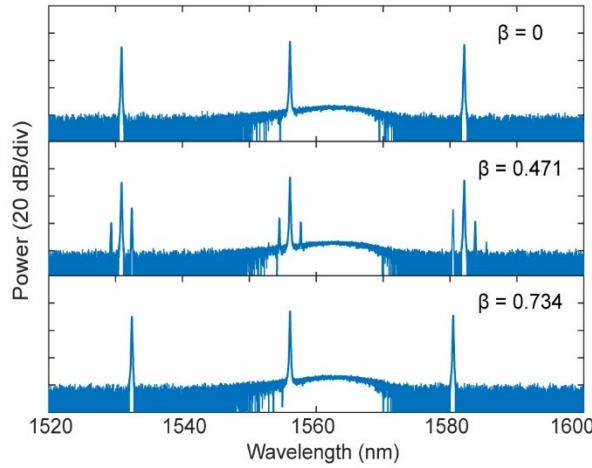


Fig. S6. Injection-locked wavelength tuning under varying β (0–0.734). The signal wavelength shifts from 1530.9 nm to 1532.45 nm.

7. Phase-insensitive self-injection locked OPO

We simulate the frequency-noise PSD at a 10^6 Hz offset for a fixed feedback phase ϕ_F across a range of β , and compare the results with experimental measurements (Fig.

S7). The simulations show that noise suppression is modest at low injection ratios, becomes more pronounced once β exceeds 0.2, and reaches an optimum near $\beta \approx 0.6$. Beyond this point, the larger injection ratio induces coherence collapse, causing the noise to increase. The simulations broadly reproduce the experimental trends.

We further simulate the frequency-noise PSD of the injection-locked mode versus ϕ_F and β (Fig. S8). At fixed β , ϕ_F variations negligibly affect noise suppression. The phase insensitivity persists across all investigated injection ratios, with injection-locking performance remaining essentially unchanged.

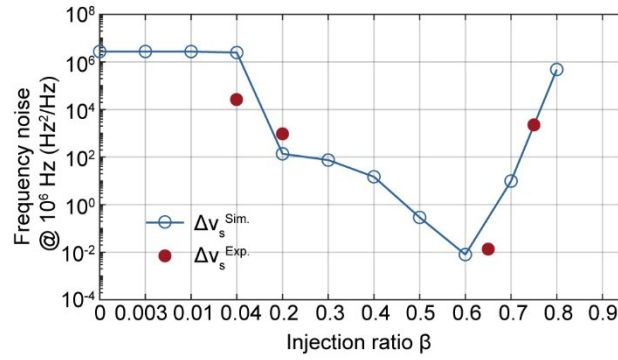


Fig. S7. Measured and simulated results of the injection-locked OPO frequency-noise PSD at a 10^6 Hz offset versus β .

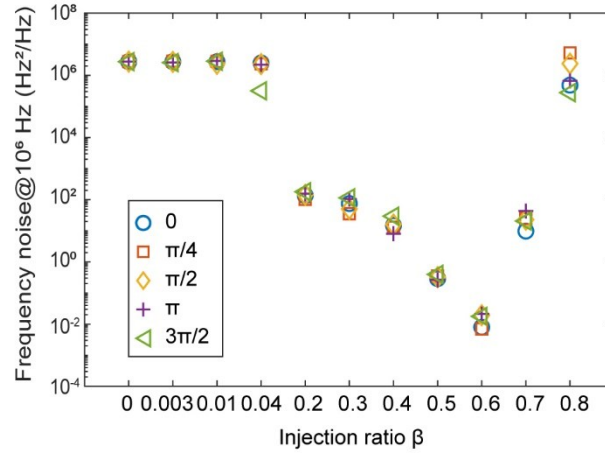


Fig. S8. Simulated frequency-noise PSD at a 10^6 Hz offset versus β for different ϕ_F .

8. Signal-idler linewidth anti-correlation

We measure the intrinsic linewidth of the injection-locked signal and the free-running idler across a range of injection-locked wavelengths (Fig. S9). The signal and idler linewidths are anti-correlated: a narrower signal linewidth accompanies a broader idler linewidth. Notably, the signal linewidth narrows more sharply as the injection-locked

wavelength approaches the pump. Although injection locking suppresses noise effectively, it cannot override the intrinsic detuning dependence of OPO noise.

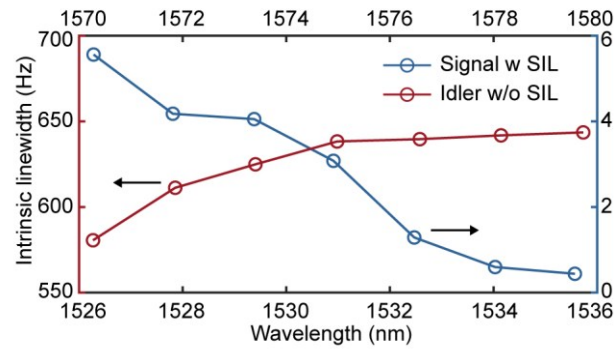


Fig. S9. Intrinsic linewidths comparison between the injection-locked signal and the free-running idler at different wavelengths (see Fig. 3(a)). Linewidth narrowing becomes more pronounced as the signal approaches the pump.